Bi-Level Robust Optimization of Electric Vehicle Charging Stations with Distributed Energy Resources

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***Abstract*—We develop a bi-level model, which captures strategic decision making by PEV owners, to optimize the design of a plug-in electric vehicle (PEV) charging station with distributed energy resources. The upper level of the model determines the optimal configuration of the station and pricing schemes whereas the lower level captures charging decisions by PEV owners. A robust formulation is employed to capture uncertain wholesale energy prices, renewable-resource availability, and PEV flows. The resulting bi-level robust-optimization model is transformed into an equivalent single-level optimization problem by replacing the lower-level problem with Karush-Kuhn-Tucker optimality conditions. A column-and-constraint-generation algorithm is used to solve the resultant single-level problem. Results from a realistic case study and a parameter analysis demonstrate the effectiveness of the proposed model in capturing the impacts of uncertainty and self-interested behavior by PEV owners.**

***Index Terms*—Plug-in electric vehicle, charging station, distributed energy resource, bi-level optimization, robust optimization, energy pricing**

NOMENCLATURE

Indices and Sets

d	index of representative days in set, $\mathcal{D}$
n	index of charging-demand blocks of plug-in electric vehicle (PEV) owners in set, $\mathcal{N}$
t	index of time periods in set, $\mathcal{T}$
v	index of PEV-owner types in set, $\mathcal{V}$

Parameters

c^C	annualized fixed cost of PEV-charging piles (\$/kW-year)
c^R	annualized fixed cost of renewable resource (\$/kW-year)
$c^{S,E}$	annualized fixed cost of energy storage energy capacity (\$/kWh-year)
$c^{S,P}$	annualized fixed cost of energy storage power capacity (\$/kW-year)
$u_{v,n}$	marginal utility of charging-demand block n of PEV owner type v (\$/kW)
$\bar{X}^C$	maximum capacity of PEV-charging piles that can be built (kW)
$\bar{X}^R$	maximum capacity of renewable resource that can be built (kW)
$\bar{X}^{S,E}$	maximum energy capacity of energy storage that can be built (kWh)
$\bar{X}^{S,P}$	maximum power capacity of energy storage that can be built (kW)
$\beta_d^{\gamma,\max}$	maximum uncertainty budget for $\gamma_{d,t}$ on day d (p.u.)
$\beta_d^{\gamma,\min}$	minimum uncertainty budget for $\gamma_{d,t}$ on day d (p.u.)
$\beta_d^{\pi,\max}$	maximum uncertainty budget for $\pi_{d,t}$ on day d (p.u.)
$\beta_d^{\pi,\min}$	minimum uncertainty budget for $\pi_{d,t}$ on day d (p.u.)
$\beta_{d,v}^{\Upsilon,\max}$	maximum uncertainty budget for $\Upsilon_{d,t,v}$ for type-v PEV owners on day d (p.u.)
$\beta_{d,v}^{\Upsilon,\min}$	minimum uncertainty budget for $\Upsilon_{d,t,v}$ for type-v PEV owners on day d (p.u.)
$\tilde{\gamma}_{d,t}$	reference value of $\gamma_{d,t}$ (p.u.)
$\gamma_{d,t}^{\max}$	maximum possible value of $\gamma_{d,t}$ (p.u.)
$\gamma_{d,t}^{\min}$	minimum possible value of $\gamma_{d,t}$ (p.u.)
Δ	duration of a time period (h)
$\epsilon_{d,v}^{\max}$	maximum day-d energy that can be charged into battery of type-v PEV owner (kWh)
$\epsilon_{d,v}^{\min}$	minimum day-d energy that can be charged into battery of type-v PEV owner (kWh)
$\bar{\epsilon}_{v,n}$	maximum charging demand in block n of PEV owner type v (kW)
η^C	efficiency of charging piles (p.u.)
η^S	roundtrip efficiency of energy storage (p.u.)
θ_d	weight on representative day d (days)
$\bar{\pi}$	maximum retail price of charging energy (\$/kWh)
$\tilde{\pi}_{d,t}^W$	reference value of $\pi_{d,t}^W$ (\$/kWh)
$\pi_{d,t}^{W,\max}$	maximum possible value of $\pi_{d,t}^W$ (\$/kWh)
$\pi_{d,t}^{W,\min}$	minimum possible value of $\pi_{d,t}^W$ (\$/kWh)
$\sigma_{\max}^S$	maximum state of energy (SOE) of energy storage (p.u.)
$\sigma_{\min}^S$	minimum SOE of energy storage (p.u.)
$\bar{\tau}$	capacity of distribution transformer (kW)
$\tilde{\Upsilon}_{d,t,v}$	reference value of $\Upsilon_{d,t,v}$
$\Upsilon_{d,t,v}^{\max}$	maximum possible value of $\Upsilon_{d,t,v}$
$\Upsilon_{d,t,v}^{\min}$	minimum possible value of $\Upsilon_{d,t,v}$

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Station Owner's Planning Variables

x^C capacity of PEV-charging piles built (kW)
x^R capacity of renewable resource built (kW)
$x^{S,E}$ energy capacity of energy storage built (kWh)
$x^{S,P}$ power capacity of energy storage built (kW)
$\pi_{d,t}$ price of charging energy sold to PEV owners in time period t of day d (\$/kWh)

Station Owner's Robust Variables

$\gamma_{d,t}$ capacity factor of renewable generator in time period t of day d (p.u.)
$\pi^W_{d,t}$ wholesale energy price in time period t of day d (\$/kWh)
$\Upsilon_{d,t,v}$ number of type-v PEV owners arriving to the station in time period t of day d

Station Owner's Recourse Variables

$y_{d,t}$ renewable energy production in time period t of day d (kW)
$\delta_{d,t}$ power discharged from energy storage in time period t of day d (kW)
$\iota_{d,t}$ power charged into energy storage in time period t of day d (kW)
$\sigma_{d,t}$ SOE of energy storage at end of time period t of day d (kWh)
$\tau_{d,t}$ net power imported from the grid in time period t of day d (kW)
$\chi_{d,t}$ total power charged into PEVs in time period t of day d (kW)

PEV-Owners' Decision Variables

$\bar{\chi}_{d,t,v,n}$ charging demand of block n of PEV owner type v in time period t of day d (kW)

I. Introduction

TECHNOLOGICAL advances and policy decisions are prompting rapid growth in the adoption of plug-in electric vehicles (PEVs) [1]. Relative to conventional vehicles, PEVs have high operating efficiencies and no direct emissions. Thus, PEVs hold promise in stemming climate change [2]. However, the environmental impact of PEV adoption and use depends on the generation mix of the power system from which they are charged [3]. Renewable generation plays a key role in determining these impacts.

Co-ordinating the design and operation of PEV-charging infrastructure with renewable resources raises issues, because both technologies present operational uncertainties (*e.g.*, weather conditions and charging patterns) [4]. The technical literature takes different approaches to co-ordinate the planning of PEV-charging infrastructure and renewable energy resources.

One body of work [4]–[6] examines the configuration and location of charging stations and renewable resources at the grid side. Shojaabadi *et al*. [4] develop a multi-objective model to optimize the design of PEV-charging stations and the deployment of wind generation, taking into account the perspective of a distribution-system planner and the owners of the wind generators. Hung *et al*. [5] design, with the objective of minimizing energy losses, PEV-charging stations that have photovoltaic (PV) solar panels installed in them. Erdinc *et al*. [6] examine the sizing and siting of renewable generation units, PEV-charging stations, and energy storage within a distribution system.

A second body of work [7]–[11] aggregates the chargers and renewable generators in a single local charging station and determines the optimal size of these components. Gunter *et al*. [7] propose a methodology for designing grid-connected systems that consist of PEV chargers, distributed generation, and energy storage. They demonstrate that systems with PV panels can be most cost effective than those without. Fazelpour *et al*. [8] develop a model to design a parking facility with distributed generation to minimize power losses and improve voltage profiles. Chandra Mouli *et al*. [9] optimize the configuration of energy storage that is deployed in a PEV-charging station that uses PV panels. They show that properly sized energy storage can reduce grid dependency of the charging station by 25%. Other works examine this problem with consideration of lifecycle costs [10] and better characterization of PEV-charging demands using queuing models [11].

Although the literature tackles the problem of planning PEV-charging infrastructure, many existing works assume that the decision maker has full knowledge of the real-time operating state of the system. Thus, the uncertainties that complicate infrastructure planning are ignored [6], [7], [10] or represented using simple probabilistic models [4], [11]. Moreover, the existing literature neglects the strategic self-interested nature of PEV owners in deciding whether to use a public PEV-charging station. The use of public charging stations by PEV owners depends on multiple factors, including personal preferences and the tariff that is set for charging energy [12], [13]. Numerous works demonstrate that retail tariffs can be structured to drive PEV-charging loads to be co-incident with the availability of low-cost [14], [15] or renewable [16] energy.

This paper seeks to address these gaps in the extant literature by proposing a new modeling framework to optimize the design of a PEV-charging station with distributed energy resources. Our model assumes that the owner designs and operates the station while transacting in the wholesale electricity market and determining the retail tariff for PEV-charging energy. The owner's objective is to maximize the net revenue that is earned from building and operating the station. We embed within the station owner's upper-level problem lower-level problems that capture the behavior of PEV owners, who determine the use of the charging station to maximize their net utilities.

The bi-level model includes a robust formulation, which captures uncertainty in the real-time operation of the charging station (*e.g.*, weather conditions, wholesale prices, and the number of PEV drivers). Robust optimization can be used to incorporate uncertainty into planning and operational models involving PEVs [17]. A benefit of robust optimization is that it does not require detailed knowledge of the structure or distribution of the underlying random variables. Conversely, stochastic optimization does require such information. This feature of robust optimization is beneficial for a planning problem, as estimating distributions of wholesale prices and PEV ownership years into the future (which would be required, given the long-lived nature of the components in a charging station) may be challenging.

To solve the bi-level robust-optimization model, first we replace the lower-level problems with their necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions [18]–[20]. This use of KKT conditions is a common approach to converting a bi-level problem into a more tractable single-level problem [21]. The resulting single-level robust-optimization model is solved by applying a column-and-constraint-generation algorithm [22]. Wang *et al.* [23] apply this algorithm to solve a robust-optimization model that plans transmission and energy-storage expansion. The column-and-constraint-generation algorithm is based on the idea of Benders's decomposition [24], which is used widely to solve robust-optimization problems [25]. Other potential algorithms to solve robust-optimization models include cutting-plane methods [17] and alternating direction method of multipliers [26].

We demonstrate the effectiveness of the proposed model using a comprehensive numerical case study, from which we derive analytical insights. We show the benefits of the proposed model in capturing uncertainty and the self-interested behavior of PEV owners in deciding whether to use the public charging station and how to shape their demands for charging energy.

Overall, our work makes two contributions to the existing literature. First, we develop a novel bi-level robust formulation for optimizing the design of a public charging station. This model accounts for uncertainty and the self-interested behavior of PEV owners in deciding whether and to what extent to use the public charging station. Our model determines how to price charging energy to shape PEV-charging loads to follow the availability of low-cost and renewable energy. Second, we apply KKT conditions and the constraint-and-column-generation algorithm to solve the otherwise intractable model efficiently.

The remainder of this paper is organized as follows. Overviews of the problem and model structure are given in Section II. Section III provides the model formulation, while Section IV details the solution method. Section V summarizes the results of our case study. Section VI concludes.

II. Model Overview

We employ a bi-level robust-optimization framework to model the design and operation of the PEV-charging station. The charging station is assumed to consist of PEV-charging piles that are co-located with distributed renewable generators and energy storage. Thus, the design of the charging station that is optimized by the model is the capacity of the PEV-charging piles, distributed renewable generators, and energy storage that are built. The charging station is grid-connected, meaning that it can purchase deficit and sell surplus energy from and to the wholesale market.

We model the design of the charging station from the perspective of a private owner, as opposed to a distribution-system operator or PEV owners installing charging station(s). Thus, we do not model the broader power system (*e.g.*, unit commitment, optimal power flow, or power quality on the distribution feeder). Such considerations could be added to a model that optimizes the design of a charging station. However, it would assume a different modeling perspective than that which we we take.

The upper-level model represents the profit-maximizing decisions of the PEV-station owner. First, the station owner determines the configuration of the station (*i.e.*, values of x^C, x^R, $x^{S,E}$, and $x^{S,P}$) and time-of-use tariffs to levy on PEV owners for charging energy (*i.e.*, values of $\pi_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}$). After these planning decisions are made, nature selects the worst-possible outcome of the uncertain variables (*i.e.*, values of $\gamma_{d,t}$ and $\pi^W_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}$ and $\Upsilon_{d,t,v}, \forall d \in \mathcal{D}, t \in \mathcal{T}, v \in \mathcal{V}$) within some uncertainty budget. Thus, nature represents the realization of random variables, which are the real-time availability of energy from the renewable generator that the station owner builds, wholesale energy prices, and the number of PEVs that arrive into the station. Due to our use of a robust modeling framework, nature is assumed to select the worst possible outcomes (from the perspective of the charging-station owner) for these random variables (subject to the uncertainty budget). Finally, the PEV-station owner makes recourse decisions regarding the real-time operation of the charging station (*i.e.*, determine values of $y_{d,t}$, $\delta_{d,t}$, $\iota_{d,t}$, $\sigma_{d,t}$, $\tau_{d,t}$, and $\chi_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}$). We assume that in doing so, the charging-station owner is able to curtail output of the renewable generator below real-time availability, if so desired.

The lower-level model represents the behavior of PEV owners, who determine how much charging energy to obtain from the charging station to maximize their net utilities. PEV owners are represented by types [16], [27]. All of the PEV owners corresponding to a type are assumed to have the same (or sufficiently similar) driving patterns, willingness-to-pay for charging energy, *etc*. Thus, we represent type-v PEV owners as determining $\bar{\chi}_{d,t,v,n}, \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N}$ to maximize their net utilities.

Fig. 1 provides a schematic timeline of the sequence of decisions that are made in the model. It shows that the station owner first determines the design of the charging station and the prices to levy for PEV-charging energy. Next, nature determines the worst-possible outcomes for the random variables. Finally, the station owner determines its recourse decisions and PEV owners determine their charging-energy demands.

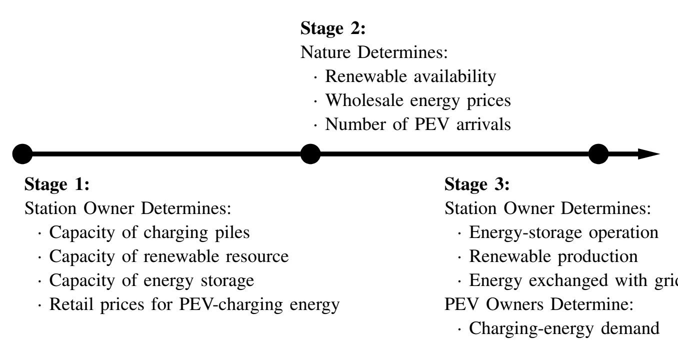

Fig. 1. Schematic timeline of decisions.

III. Model Formulation

We begin by providing the formulation of the PEV owners' utility-maximization problems. Type-v PEV owners determine $\bar{\chi}_{d,t,v,n}, \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N}$ by solving:

$$\max \sum_{d\in\mathcal{D},t\in\mathcal{T},n\in\mathcal{N}} \Delta \cdot (u_{v,n} - \pi_{d,t})\bar{\chi}_{d,t,v,n} \tag{1}$$

$$\text{s.t. } \epsilon^{\min}_{d,v} \leq \Delta \sum_{n\in\mathcal{N}} \bar{\chi}_{d,t,v,n} \leq \epsilon^{\max}_{d,v}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\mu^{1,\min}_{d,t,v}, \mu^{1,\max}_{d,t,v}) \tag{2}$$

$$0 \leq \bar{\chi}_{d,t,v,n} \leq \bar{\epsilon}_{v,n}; \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N}; \quad (\mu^{2,\min}_{d,t,v,n}, \mu^{2,\max}_{d,t,v,n}) \tag{3}$$

where the Lagrange multipliers that are associated with each constraint are indicated in parentheses at the end of the constraint. Objective function (1) maximizes the net utility of obtaining energy from the PEV-charging station. Constraints (2) ensure that the total amount of energy that is charged is within the appropriate bounds while (3) ensure that the amount of charging energy that is obtained in each block is less than the block size.

Our bi-level robust-optimization model is formulated as:

$$\max_{\Omega_P} - c^C x^C - c^R x^R - c^{S,E} x^{S,E} - c^{S,P} x^{S,P} + \min_{\Omega_N} \max_{\Omega_R} \sum_{d\in\mathcal{D},t\in\mathcal{T}} \theta_d \Delta \cdot \left(\pi_{d,t}\chi_{d,t} - \pi^W_{d,t}\tau_{d,t}\right) \tag{4}$$

$$\text{s.t. } 0 \leq x^C \leq \bar{X}^C \tag{5}$$

$$0 \leq x^R \leq \bar{X}^R \tag{6}$$

$$0 \leq x^{S,E} \leq \bar{X}^{S,E} \tag{7}$$

$$0 \leq x^{S,P} \leq \bar{X}^{S,P} \tag{8}$$

$$0 \leq \pi_{d,t} \leq \bar{\pi}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{9}$$

$$\gamma^{\min}_{d,t} \leq \gamma_{d,t} \leq \gamma^{\max}_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{10}$$

$$\beta^{\gamma,\min}_d \leq \frac{\sum_{t\in\mathcal{T}} \gamma_{d,t}}{\sum_{t\in\mathcal{T}} \tilde{\gamma}_{d,t}} \leq \beta^{\gamma,\max}_d; \forall d \in \mathcal{D} \tag{11}$$

$$\pi^{W,\min}_{d,t} \leq \pi^W_{d,t} \leq \pi^{W,\max}_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{12}$$

$$\beta^{\pi,\min}_d \leq \frac{\sum_{t\in\mathcal{T}} \pi^W_{d,t}}{\sum_{t\in\mathcal{T}} \tilde{\pi}^W_{d,t}} \leq \beta^{\pi,\max}_d; \forall d \in \mathcal{D} \tag{13}$$

$$\Upsilon^{\min}_{d,t,v} \leq \Upsilon_{d,t,v} \leq \Upsilon^{\max}_{d,t,v}; \forall d \in \mathcal{D}, t \in \mathcal{T}, v \in \mathcal{V} \tag{14}$$

$$\beta^{\Upsilon,\min}_{d,v} \leq \frac{\sum_{t\in\mathcal{T}} \Upsilon_{d,t,v}}{\sum_{t\in\mathcal{T}} \tilde{\Upsilon}_{d,t,v}} \leq \beta^{\Upsilon,\max}_{d,v}; \forall d \in \mathcal{D}, v \in \mathcal{V} \tag{15}$$

$$-\bar{\tau} \leq \tau_{d,t} \leq \bar{\tau}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^{1,\min}_{d,t}, \lambda^{1,\max}_{d,t}) \tag{16}$$

$$0 \leq \chi_{d,t} \leq x^C; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{17}$$

$$0 \leq y_{d,t} \leq \gamma_{d,t} x^R; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^{3,\max}_{d,t}) \tag{18}$$

$$\sigma^S_{\min} x^{S,E} \leq \sigma_{d,t} \leq \sigma^S_{\max} x^{S,E}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{19}$$

$$0 \leq \iota_{d,t} \leq x^{S,P}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^{5,\max}_{d,t}) \tag{20}$$

$$0 \leq \delta_{d,t} \leq x^{S,P}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^{6,\max}_{d,t}) \tag{21}$$

$$\sigma_{d,t} = \sigma_{d,t-1} + \eta^S \Delta\iota_{d,t} - \Delta\delta_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{22}$$

$$\sigma_{d,0} = \sigma_{d,|\mathcal{T}|}; \forall d \in \mathcal{D} \tag{23}$$

$$\tau_{d,t} + y_{d,t} + \delta_{d,t} = \iota_{d,t} + \chi_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{24}$$

$$\eta^C \Delta\chi_{d,t} = \Delta \sum_{v\in\mathcal{V},n\in\mathcal{N}} \Upsilon_{d,t,v}\bar{\chi}_{d,t,v,n}; \quad \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{25}$$

$$(1)\text{–}(3); \forall v \in \mathcal{V}; \tag{26}$$

where the dual variable that is associated with each constraint is denoted in parentheses to its right. We define:

$$\Omega_P = \left\{x^C, x^R, x^{S,E}, x^{S,P}\right\} \cup \{\pi_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}\};$$

as the set of the station owner's planning variables:

$$\Omega_N = \left\{\gamma_{d,t}, \pi^W_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}\right\} \cup \{\Upsilon_{d,t,v}, \forall d \in \mathcal{D}, t \in \mathcal{T}, v \in \mathcal{V}\};$$

as the set of nature's variables, and:

$$\Omega_R = \{y_{d,t}, \delta_{d,t}, \iota_{d,t}, \sigma_{d,t}, \tau_{d,t}, \chi_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}\};$$

as the set of the station owner's recourse variables.

Objective function (4) maximizes the net revenue that is earned by the station owner. The four terms in (4) represent the annualized cost of installing the station components, while the remaining terms represent the cost of operating the station. These latter terms appear after the 'min' and 'max' to represent nature's choice of the worst-possible outcome and the station owner's recourse decisions that react to nature. The model has three constraint sets. The first set of constraints, (5)–(9), pertain to the station owner's planning decisions, the second set, (10)–(15), pertain to nature's choice of the uncertain variables, and the remaining pertain to station operations.

Constraints (5)–(8) impose restrictions on the amount of charging-pile, distributed generation, and energy-storage capacity, respectively, that can be installed. These can represent budgetary, physical-space, or resource limits. Constraints (9) are regulatory restrictions on how high the retail price of charging energy can be.

Constraints (10), (12), and (14) impose confidence-interval bounds on the uncertain variables, whereas (11), (13), and (15) impose polyhedral uncertainty budgets on them [28].

Constraints (16) restrict the amount of power that is exchanged with the power system, based on the transformer capacity. Constraints (17) restrict the amount of charging power that can be provided to PEVs based on the installed capacity of the charging piles. Constraints (18) restrict renewable generation in each time period based on the installed capacity of the generator and the real-time capacity factor of the generator, which captures the impact of weather conditions on renewable output. These constraints allow renewable production to be curtailed below real-time availability if so desired by the charging-station owner.

Constraints (19)–(21) restrict the SOE and charging and discharging power of the energy storage, based on its associated capacities. Constraints (22) govern how the SOE of energy storage evolves from one time period to the next, whereas (23) require the ending SOE of the energy storage on each day to equal its starting SOE. This is a heuristic means of ensuring that stored energy has carryover value from one day to the next [29]. We use a single parameter, η^S, which is applied to energy that is charged into the energy storage, to represent energy losses from the energy-storage cycle. Alternatively, one could apply separate efficiency parameters to energy that is charged into and discharged from energy storage. We opt to use a single efficiency parameter to simplify the model notation. We do not include any direct cost on the use of energy storage in (4). Such a cost could be included, for instance to account for accelerated degradation of energy storage as a result of cycling [30], [31]. None of these changes would affect the structure of our model or the solution algorithm substantively.

Constraints (24) define the amount of energy that the charging station exchanges with the power system in terms of how the resources in the charging station are operated. Constraints (25) ensure that all PEV-charging demands are met. Constraints (26) embed the PEV owners' utility-maximization problems within the station owner's problem.

IV. Solution Technique

Model (4)–(26) is a bi-level optimization problem in which the upper-level has a three-level $\max$-$\min$-$\max$ structure. As such, this problem is intractable. We deal with this computational difficulty through two steps. First, we replace the PEV owners' lower-level problems with their necessary and sufficient KKT conditions, which yields a single-level problem with a $\max$-$\min$-$\max$ structure. Then, we employ a column-and-constraint-generation algorithm whereby the single-level problem is decomposed into a master problem and subproblem. These problems are solved iteratively and new variables and optimality cuts are added to the master problem until a solution satisfying a desired optimality criterion is obtained.

We proceed in this section by converting the bi-level problem into a single-level problem first. Then we provide the decomposed master problem and subproblem of the single-level problem. Finally, we provide an outline of the iterative solution algorithm.

A. Conversion of Bi-Level to Single-Level Optimization

Type-v PEV owners' problem (1)–(3) is a linear optimization model and satisfies the Slater condition. Thus, KKT conditions:

$$\Delta \cdot (u_{v,n} - \pi_{d,t}) - \mu^{1,\min}_{d,t,v} + \mu^{1,\max}_{d,t,v} - \mu^{2,\min}_{d,t,v,n} + \mu^{2,\max}_{d,t,v,n} = 0; \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N} \tag{27}$$

$$0 \le \mu^{1,\min}_{d,t,v} \perp \Delta \sum_{n \in \mathcal{N}} \bar{\chi}_{d,t,v,n} \ge \epsilon^{\min}_{d,v}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{28}$$

$$0 \le \mu^{1,\max}_{d,t,v} \perp \Delta \sum_{n \in \mathcal{N}} \bar{\chi}_{d,t,v,n} \le \epsilon^{\max}_{d,v}; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{29}$$

$$0 \le \mu^{2,\min}_{d,t,v,n} \perp \bar{\chi}_{d,t,v,n} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N} \tag{30}$$

$$0 \le \mu^{2,\max}_{d,t,v,n} \perp \bar{\chi}_{d,t,v,n} \le \bar{\epsilon}_{v,n}; \forall d \in \mathcal{D}, t \in \mathcal{T}, n \in \mathcal{N} \tag{31}$$

are necessary and sufficient for a global optimum [18]. As such, we can replace (26) in the bi-level problem with (27)–(31) $\forall v \in \mathcal{V}$, which gives an equivalent single-level problem. Henceforth, we refer to this single-level problem as $\mathcal{P}$.

B. Decomposition of $\mathcal{P}$

To apply the column-and-constraint-generation algorithm, we decompose $\mathcal{P}$ into a master problem and subproblem. The master problem includes the first-stage variables, which determine the design of the charging station and the prices that are levied for PEV-charging energy, and the primal and dual variables of (27)–(31). The subproblem corresponds to the remaining decisions.

1) Master Problem: The master problem is:

$$\max_{\Omega_M} - c^C x^C - c^R x^R - c^{S,E} x^{S,E} - c^{S,P} x^{S,P} + \Psi \tag{32}$$

$$\text{s.t. (5)–(9), (27)–(31)} \tag{33}$$

$$\Psi \ge \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \theta_d \Delta \cdot \left(\pi_{d,t} \chi^{(i)}_{d,t} - \pi^{W,(i)}_{d,t} \tau^{(i)}_{d,t} \right); \quad \forall i = 1, \ldots, I \tag{34}$$

where:

$$\Omega_M = \Omega_P \cup \Psi \cup \left\{ \mu^{1,\min}_{d,t,v}, \mu^{1,\max}_{d,t,v}, \forall d \in \mathcal{D}, t \in \mathcal{T}, v \in \mathcal{V} \right\} \cup \left\{ \bar{\chi}_{d,t,v,n}, \mu^{2,\min}_{d,t,v,n}, \mu^{2,\max}_{d,t,v,n}, \forall d \in \mathcal{D}, t \in \mathcal{T}, v \in \mathcal{V}, n \in \mathcal{N} \right\};$$

i is a counter that corresponds to the iteration number of the column-and-constraint-generation algorithm, and I indicates the number of iterations of the algorithm that have been conducted thus far.

The first four terms in (32) represent the capital cost of installing the components of the charging station. Ψ is an auxiliary variable that approximates the resulting operating cost of the station. Constraint set (33) imposes the first-stage constraints and the KKT conditions that characterize an optimal solution to the PEV owners' problems. Constraints (34) are the optimality cuts that are added iteratively in the course of the algorithm. These cuts are generated based on the optimized values of nature's, the station owner's recourse, and the PEV owners' decisions that are obtained from the subproblem in each iteration. Thus, for instance, $\chi^{(i)}_{d,t}$, $\pi^{W,(i)}_{d,t}$, and $\tau^{(i)}_{d,t}$ represent optimized values of $\chi_{d,t}$, $\pi^W_{d,t}$, and $\tau_{d,t}$ that are obtained from solving the subproblem in the ith iteration of the algorithm.

2) Subproblem: The subproblem represents the remainder of the problem (*i.e.*, nature's and the station owner's resource decisions). As such, it has a $\min$-$\max$ structure, which is simplified by converting it into a $\min$-only structure by replacing the station owner's recourse problem with necessary and sufficient optimality conditions.

a) Linear Dual of Recourse Problem: We begin by examining the following simplified recourse problem, which is obtained by fixing the first-stage, nature's, and the PEV owners' decisions:

$$\max_{\Omega_R} \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \theta_d \Delta \cdot \left(\pi_{d,t} \chi_{d,t} - \pi^W_{d,t} \tau_{d,t} \right) \tag{35}$$

s.t. (16)–(25).

By substituting (22) into (19) and (23) and (24) into (17), (25), and (35), we obtain:

$$\max_{\Omega_{R'}} \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \theta_d \Delta \cdot \left[\pi_{d,t} \cdot (\tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t}) \right. \tag{36}$$

$$-\pi^W_{d,t}\tau_{d,t}\Big]$$

$$\text{s.t. (16), (18), (20), (21)} \tag{37}$$

$$0 \le \tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t} \le x^C; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{38}$$
$$(\lambda^{2,\min}_{d,t}, \lambda^{2,\max}_{d,t})$$

$$\sigma^S_{\min} x^{S,E} \le \sigma_{d,0} + \sum_{\zeta \le t}\left(\eta^S \Delta\iota_{d,\zeta} - \Delta\delta_{d,\zeta}\right) \tag{39}$$
$$\le \sigma^S_{\max} x^{S,E}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^{4,\min}_{d,t}, \lambda^{4,\max}_{d,t})$$

$$\sum_{t \in \mathcal{T}}\left(\eta^S \Delta\iota_{d,t} - \Delta\delta_{d,t}\right) = 0; \forall d \in \mathcal{D} \quad (\lambda^7_d) \tag{40}$$

$$\eta^C \cdot (\tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t}) \tag{41}$$
$$= \sum_{v \in \mathcal{V}, n \in \mathcal{N}} \Upsilon_{d,t,v}\bar{\chi}_{d,t,v,n}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\lambda^8_{d,t})$$

where:

$$\Omega_{R'} = \{y_{d,t}, \delta_{d,t}, \iota_{d,t}, \tau_{d,t}, \forall d \in \mathcal{D}, t \in \mathcal{T}\};$$

and the dual variable that is associated with each constraint is indicated to its right. Problem (36)–(41) is a simplified version of the recourse problem in which $\chi_{d,t}$ and $\sigma_{d,t}$ have been projected out.

The linear dual of (36)–(41) is given by:

$$\min_{\Omega^D_R} \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \Bigg[\bar{\tau} \cdot \left(\lambda^{1,\max}_{d,t} - \lambda^{1,\min}_{d,t}\right) + x^C \lambda^{2,\max}_{d,t}$$
$$+ \gamma_{d,t} x^R \lambda^{3,\max}_{d,t} + x^{S,E} \cdot \left(\sigma^S_{\min}\lambda^{4,\min}_{d,t} + \sigma^S_{\max}\lambda^{4,\max}_{d,t}\right)$$
$$+ x^{S,P}\lambda^{5,\max}_{d,t} + x^{S,P}\lambda^{6,\max}_{d,t}$$
$$+ \lambda^8_{d,t} \sum_{v \in \mathcal{V}, n \in \mathcal{N}} \Upsilon_{d,t,v}\bar{\chi}_{d,t,v,n}\Bigg]$$

$$\text{s.t. } \lambda^{2,\min}_{d,t} + \lambda^{2,\max}_{d,t} + \lambda^{3,\max}_{d,t} + \eta^C \lambda^8_{d,t} \ge \theta_d \Delta\pi_{d,t};$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T} \quad (y_{d,t})$$

$$\lambda^{2,\min}_{d,t} + \lambda^{2,\max}_{d,t} - \Delta \sum_{\zeta \ge t}\left(\lambda^{4,\min}_{d,\zeta} + \lambda^{4,\max}_{d,\zeta}\right) + \lambda^{6,\max}_{d,t}$$
$$- \Delta\lambda^7_d + \eta^C \lambda^8_{d,t} \ge \theta_d \Delta\pi_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\delta_{d,t})$$

$$-\lambda^{2,\min}_{d,t} - \lambda^{2,\max}_{d,t} + \eta^S \Delta \sum_{\zeta \ge t}\left(\lambda^{4,\min}_{d,\zeta} + \lambda^{4,\max}_{d,\zeta}\right)$$
$$+ \lambda^{5,\max}_{d,t} + \eta^S \Delta\lambda^7_d - \lambda^8_{d,t} \ge -\theta_d \Delta\pi_{d,t};$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\iota_{d,t})$$

$$\lambda^{1,\min}_{d,t} + \lambda^{1,\max}_{d,t} + \lambda^{2,\min}_{d,t} + \lambda^{2,\max}_{d,t} + \eta^C \lambda^8_{d,t} \tag{42}$$
$$= \theta_d \Delta \cdot (\pi_{d,t} - \pi^W_{d,t}); \forall d \in \mathcal{D}, t \in \mathcal{T} \quad (\tau_{d,t})$$
$$\lambda^{1,\min}_{d,t}, \lambda^{2,\min}_{d,t}, \lambda^{4,\max}_{d,t} \le 0; \forall d \in \mathcal{D}, t \in \mathcal{T}$$
$$\lambda^{1,\max}_{d,t}, \lambda^{2,\max}_{d,t}, \lambda^{3,\max}_{d,t}, \lambda^{4,\max}_{d,t}, \lambda^{5,\max}_{d,t}, \lambda^{6,\max}_{d,t} \ge 0;$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T}$$

where:

$$\Omega^D_R = \Big\{\lambda^{1,\min}_{d,t}, \lambda^{1,\max}_{d,t}, \lambda^{2,\min}_{d,t}, \lambda^{2,\max}_{d,t}, \lambda^{3,\max}_{d,t}, \lambda^{4,\min}_{d,t},$$
$$\lambda^{4,\max}_{d,t}, \lambda^{5,\max}_{d,t}, \lambda^{6,\max}_{d,t}, \lambda^7_d, \lambda^8_{d,t}; \forall d \in \mathcal{D}, t \in \mathcal{T}\Big\};$$

and the primal variable that is associated with each dual constraint is indicated in parentheses to its right.

b) $\min$*-Only Subproblem:* Because the station owner's recourse problem is linear, an optimal solution can be characterized by its primal, dual, and complementary-slackness conditions. Thus, the simplified $\min$-only subproblem is given by:

$$\min_{\Omega_S} \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \theta_d \Delta \cdot \big[\pi_{d,t} \cdot (\tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t}) \tag{43}$$
$$-\pi^W_{d,t}\tau_{d,t}\big]$$

$$\text{s.t. (10)–(15), (40), (41), (42)} \tag{44}$$

$$-\bar{\tau} \le \tau_{d,t} \perp \lambda^{1,\min}_{d,t} \le 0; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{45}$$

$$\tau_{d,t} \le \bar{\tau} \perp \lambda^{1,\max}_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{46}$$

$$0 \le \tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t} \perp \lambda^{2,\min}_{d,t} \le 0; \tag{47}$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$\tau_{d,t} + y_{d,t} + \delta_{d,t} - \iota_{d,t} \le x^C \perp \lambda^{2,\max}_{d,t} \ge 0; \tag{48}$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$y_{d,t} \le \gamma_{d,t} x^R \perp \lambda^{3,\max}_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{49}$$

$$\sigma^S_{\min} x^{S,E} \le \sigma_{d,0} + \sum_{\zeta \le t}\left(\eta^S \Delta\iota_{d,\zeta} - \Delta\delta_{d,\zeta}\right) \tag{50}$$
$$\perp \lambda^{4,\min}_{d,t} \le 0; \forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$\sigma_{d,0} + \sum_{\zeta \le t}\left(\eta^S \Delta\iota_{d,\zeta} - \Delta\delta_{d,\zeta}\right) \le \sigma^S_{\max} x^{S,E} \tag{51}$$
$$\perp \lambda^{4,\max}_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$\iota_{d,t} \le x^{S,P} \perp \lambda^{5,\max}_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{52}$$

$$\delta_{d,t} \le x^{S,P} \perp \lambda^{6,\max}_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T} \tag{53}$$

$$\lambda^{2,\min}_{d,t} + \lambda^{2,\max}_{d,t} + \lambda^{3,\max}_{d,t} + \eta^C \lambda^8_{d,t} \ge \theta_d \Delta\pi_{d,t} \tag{54}$$
$$\perp y_{d,t} \ge 0; \forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$\lambda^{2,\min}_{d,t} + \lambda^{2,\max}_{d,t} - \Delta \sum_{\zeta \ge t}\left(\lambda^{4,\min}_{d,\zeta} + \lambda^{4,\max}_{d,\zeta}\right) \tag{55}$$
$$+ \lambda^{6,\max}_{d,t} - \Delta\lambda^7_d + \eta^C \lambda^8_{d,t} \ge \theta_d \Delta\pi_{d,t} \perp \delta_{d,t} \ge 0;$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T}$$

$$-\lambda^{2,\min}_{d,t} - \lambda^{2,\max}_{d,t} + \eta^S \Delta \sum_{\zeta \ge t}\left(\lambda^{4,\min}_{d,\zeta} + \lambda^{4,\max}_{d,\zeta}\right) \tag{56}$$
$$+ \lambda^{5,\max}_{d,t} + \eta^S \Delta\lambda^7_d - \lambda^8_{d,t} \ge -\theta_d \Delta\pi_{d,t} \perp \iota_{d,t} \ge 0;$$
$$\forall d \in \mathcal{D}, t \in \mathcal{T}$$

where:

$$\Omega_S = \Omega_N \cup \Omega_{R'} \cup \Omega^D_R.$$

Objective function (43) is written with $\chi_{d,t}$ and $\sigma_{d,t}$ having been projected out of the problem. Constraint set (44) imposes the uncertainty budget, optimality conditions of the PEV owners' problems, and the primal and dual equality constraints. Constraints (45)–(53) and (54)–(56) are, respectively, the primal and dual inequality constraints and the associated complementary-slackness conditions.

c) Linearization of Complementarity Conditions: The complementary-slackness conditions in (28)–(31) and (45)–(56) are nonlinear. We apply the technique that is proposed by Fortuny-Amat and McCarl [32] to linearize these conditions.

This technique requires the use of additional auxiliary binary variables.

C. Column-and-Constraint-Generation Algorithm

Algorithm 1 provides pseudocode that details the steps of the column-and-constraint-generation algorithm. Line 1 initializes the algorithm by setting starting values for the lower and upper bounds of the objective-function value, the iteration counter, and the number of iterations that have been conducted thus far, which are denoted by z_{LB}, z_{UB}, k, and I, respectively.

Algorithm 1 Column and Constraint Generation

1: **initialize:** $z_{\text{LB}} \leftarrow -\infty$, $z_{\text{UB}} \leftarrow +\infty$, $k \leftarrow 1$, $I \leftarrow 0$
2: **repeat**
3: $\quad \omega_M^{(k)} \leftarrow \arg\max_{\Omega_M}$ (32) s.t. (33)–(34)
4: $\quad z_{\text{UB}} \leftarrow \min\{z_{\text{UB}}, z_M^{(k)}\}$
5: $\quad \omega_S^{(k)} \leftarrow \arg\min_{\Omega_S}$ (43) s.t. (44)–(56) with $\omega_M^{(k)}$ fixed
6: $\quad z_{\text{LB}} \leftarrow \max\{z_{\text{LB}}, z_S^{(k)}\}$
7: $\quad$ **if** $z_{\text{UB}} - z_{\text{LB}} > \Gamma$ **then**
8: $\qquad$ create (34) for $i = k$ with $\chi_{d,t}^{(i)} \leftarrow \chi_{d,t}^{(k)}$, $\pi_{d,t}^{W,(i)} \leftarrow \pi_{d,t}^{W,(k)}$, and $\tau_{d,t}^{(i)} \leftarrow \tau_{d,t}^{(k)}; \forall d \in \mathcal{D}, t \in \mathcal{T}$
9: $\qquad I \leftarrow I + 1$
10: $\quad$ **end if**
11: **until** $z_{\text{UB}} - z_{\text{LB}} \leq \Gamma$

Lines 2–11 is the main iterative loop. Line 3 solves the master problem. We define:

$$\omega_M = \Big(x^C, x^R, x^{S,E}, x^{S,P}, \pi_{1,1}, \ldots, \pi_{|\mathcal{D}|,|\mathcal{T}|}, \Psi, \mu_{1,1,1}^{1,\min}, \mu_{1,1,1}^{1,\max}, \ldots, \mu_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|}^{1,\min}, \mu_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|}^{1,\max}, \bar{\chi}_{1,1,1,1}, \mu_{1,1,1,1}^{2,\min}, \mu_{1,1,1,1}^{2,\max}, \ldots, \bar{\chi}_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|,|\mathcal{N}|}, \mu_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|,|\mathcal{N}|}^{2,\min}, \mu_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|,|\mathcal{N}|}^{2,\max}\Big);$$

as the decision-variable vector of the master problem. The superscript, (k), in Line 3 denotes the values of the variables that are obtained in the kth iteration of the algorithm. Line 4 updates the upper bound based on the most recent master-problem solution. We let:

$$z_M^{(k)} = -c^C x^{C,(k)} - c^R x^{R,(k)} - c^{S,E} x^{S,E,(k)} - c^{S,P} x^{S,P,(k)} + \Psi^{(k)};$$

denote the optimal objective-function value from solving the master problem in the current iteration. Line 5 solves the subproblem where the first-stage and PEV owners' variables are fixed equal to the values that are in $\omega_M^{(k)}$. We define:

$$\omega_S = \Big(\lambda_1^7, \ldots, \lambda_{|\mathcal{D}|}^7, \gamma_{1,1}, \pi_{1,1}^W, y_{1,1}, \delta_{1,1}, \iota_{1,1}, \tau_{1,1}, \lambda_{1,1}^{1,\min}, \lambda_{1,1}^{1,\max}, \lambda_{1,1}^{2,\min}, \lambda_{1,1}^{2,\max}, \lambda_{1,1}^{3,\max}, \lambda_{1,1}^{4,\min}, \lambda_{1,1}^{4,\max}, \lambda_{1,1}^{5,\max}, \lambda_{1,1}^{6,\max}, \lambda_{1,1}^{8}, \ldots, \gamma_{|\mathcal{D}|,|\mathcal{T}|}, \pi_{|\mathcal{D}|,|\mathcal{T}|}^W, y_{|\mathcal{D}|,|\mathcal{T}|}, \delta_{|\mathcal{D}|,|\mathcal{T}|}, \iota_{|\mathcal{D}|,|\mathcal{T}|}, \tau_{|\mathcal{D}|,|\mathcal{T}|}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{1,\min}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{1,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{2,\min}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{2,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{3,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{4,\min}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{4,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{5,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{6,\max}, \lambda_{|\mathcal{D}|,|\mathcal{T}|}^{8}, \Upsilon_{1,1,1}, \ldots, \Upsilon_{|\mathcal{D}|,|\mathcal{T}|,|\mathcal{V}|}\Big);$$

as the decision-variable vector of the subproblem. Line 6 updates the lower bound, where we define

$$z_S^{(k)} = \sum_{d \in \mathcal{D}, t \in \mathcal{T}} \theta_d \Delta \cdot \Big[\pi_{d,t}^{(k)} \cdot \Big(\tau_{d,t}^{(k)} + y_{d,t}^{(k)} + \delta_{d,t}^{(k)} - \iota_{d,t}^{(k)}\Big) - \pi_{d,t}^{W,(k)} \tau_{d,t}^{(k)}\Big];$$

as the optimal objective-function value from solving the subproblem in the current iteration.

Line 7 checks if the algorithm has converged. If not, a new constraint is added in Line 8. If so, the algorithm terminates (*cf.* Line 11).

V. Case Study

This section summarizes a numerical case study that demonstrates the benefits of our proposed model.

A. Case-Study Data

Our case study considers a 2000-m^2 public charging station that can install up to 500 kW of solar PV panels, 600 kW of lithium-ion (Li-ion) energy storage with up to 1800 kWh of energy-carrying capability, and 2500 kW of charging piles. The charging station is connected to the grid with a 4000-kVA transformer.

The PV is assumed to have, an \$870/kW capital cost, an annual maintenance cost of \$12/kW-year, and a 25-year usable life.[1] Asset lifetimes are used with an assumed 6% discount rate to annualize the capital cost of the components, due to our modeling a representative year [6]. The Li-ion energy storage is assumed to have a 15-year usable life, 86.49% roundtrip efficiency, capital costs of \$200/kW and \$143/kWh, and an annual maintenance cost of \$0.80/kWh-year. The energy storage is assumed to have minimum and maximum SOEs of 30% and 90% respectively. The charging piles are assumed to be 95% efficient, have 20-year usable lives, a capital cost of \$100/kW, and an annual maintenance cost of \$6/kW-year.

We use four days (one for each season), which are modeled at half-hourly time steps, to represent the year. Figs. 2–4 summarize the assumed PEV, energy-price, and solar characteristics [33]. Fig. 2 shows the reference numbers of PEV arrivals to the station in each time period, which are the forecasted number of PEV arrivals to the station on each representative day. We assume three different types of PEV-usage profiles: one set of PEVs will be driven for long-range 100-km trips after departing the station and their batteries have a 50% SOE upon arrival to the station, a second set will be driven for medium-range 50-km trips after departing the station and their batteries have a 50% SOE upon arrival, and the final set will be driven for short-range 20-km trips and their batteries have a 30% SOE upon arrival. The duration of the trips after departing the station and the starting SOE are used to calibrate the charging-energy demands of the drivers. We assume that the number of PEV arrivals can deviate by up to 10% relative to the reference levels that are shown in

[1]https://www.nrel.gov/analysis/tech-cost-om-dg.html

Fig. 2. Each PEV is assumed to be a Nissan Leaf, with a 40-kWh battery with an allowable SOE range of between 20% and 90% and a power-consumption rate of 0.18 kWh/km.[2] One could generalize this case study to consider different PEV types. However, because the purpose of our case study is to serve as a proof-of-concept of our proposed model, assuming the same type of PEVs is acceptable. We assume that the retail tariff that can be levied by the charging station is capped to be no more than 50% above the wholesale price.

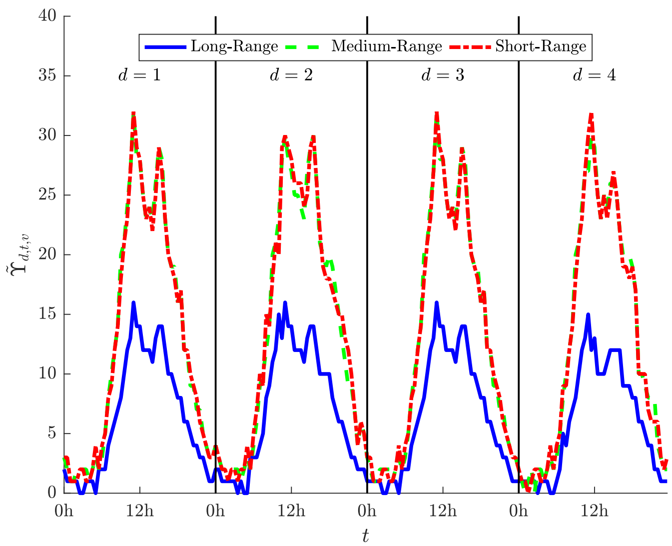

Fig. 2. Reference number of PEVs arriving to the station in each time period.

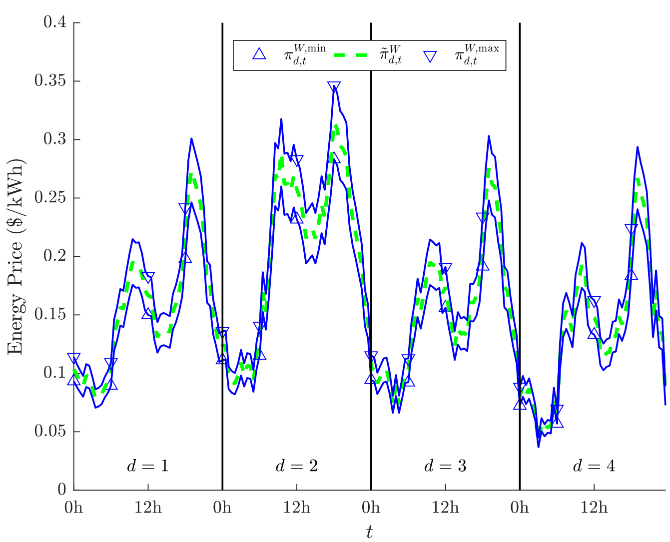

Fig. 3. Reference value and range of possible prices in each time period.

The model is programmed using version 25.0.1 of GAMS and solved using version 12.8.0 of CPLEX with default settings. The model is implemented on a computer with a 1.7-GHz Intel Core i7 processor, and 8 GB of memory.

[2]https://fueleconomy.gov/

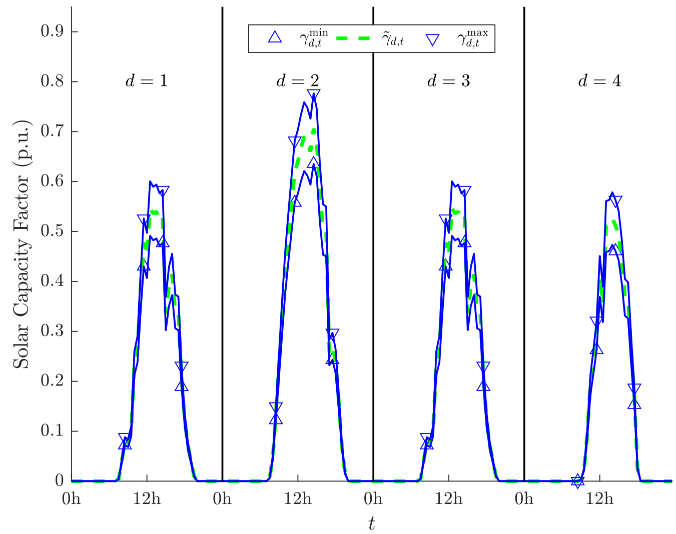

Fig. 4. Reference value and range of possible solar capacity factors in each time period.

B. *Benefit of Bi-Level Modeling Approach*

We begin by examining the benefits of using a bi-level modeling approach, wherein the decisions of the PEV owners are represented in the lower level. We do this by comparing our bi-level model to a single-level model where the retail prices and demands are assumed to be fixed. Specifically, we examine two cases using our bi-level model with the retail price of charging energy fixed equal to \$0.35/kWh and \$0.50/kWh, respectively. We contrast this with two cases in which charging demands are assumed to be fixed and the same two retail prices (*i.e.*, \$0.35/kWh and \$0.50/kWh) are used to determine revenues from selling charging energy.

Table I summarizes the results of using the two modeling approaches. The table shows that if the strategic behavior of the PEV owners is not taken into account (*i.e.*, if a single-level modeling approach is taken), the station owner over-estimates the demand for charging energy. This leads to over-sizing the charging station relative to the optimal level.

TABLE I
DESIGN OF CHARGING STATION WITH AND WITHOUT BI-LEVEL MODELING APPROACH

	Bi-Level Model		Single-Level Model	
	Retail Price ($/kWh)		Retail Price ($/kWh)	
	0.35	0.50	0.35	0.50
x^C	1543	660	2273	2273
x^R	500	500	500	500
$x^{S,E}$	1738	1217	1868	1868
$x^{S,P}$	482	375	565	565
Profit ($)	897538	844132	569039	286161

The profits that are reported in Table I are computed differently for the two modeling approaches. For the two cases in which the bi-level model is employed, the profit is given by the optimized value of (4). For the cases in which the single-level model is used, the sizes of the station components (*i.e.*, the values of x^C, x^R, $x^{S,E}$, and $x^{S,P}$) are determined using the single-level model in which PEV-charging demands are fixed. Then, these component sizes are fixed in the bi-level model, which is optimized to determine the *true* demand for PEV charging, given the fixed retail tariffs. The exceedingly low profits that are reported in Table I highlight the cost to the station owner of not accounting for driver behavior in sizing the station's components.

Table I shows that the same amount of PV capacity is built in all four cases. This suggests that PV is a low-cost source of charging energy. PV is able to deliver between 7% and 8% of PEV-charging energy (depending upon the retail price that is levied by the charging station). Indeed, 500 kW is the assumed limit on PV installation. This implies that if the charging station has a relaxed constraint, more PV panels would be installed, reducing the environmental impact of PEV charging.

Table I shows also that the profit of the charging station is lower when using the bi-level model with a fixed retail price of \$0.50/kWh, compared to the \$0.35/kWh case. This suggests that a price of \$0.50/kWh is too high, in the sense that it drives away PEV-charging demand that could be served economically by the charging station by increasing the size of its installed components. Fig. 5 demonstrates this by showing the optimized retail price of charging energy over the four representative days that are modeled. This optimized price is always below \$0.50/kWh, showing that such a price is too high and drives away too much PEV-charging demand. Although \$0.35/kWh is within the range of optimized prices, setting a 'flat' price of \$0.35/kWh across the entire year is suboptimal.

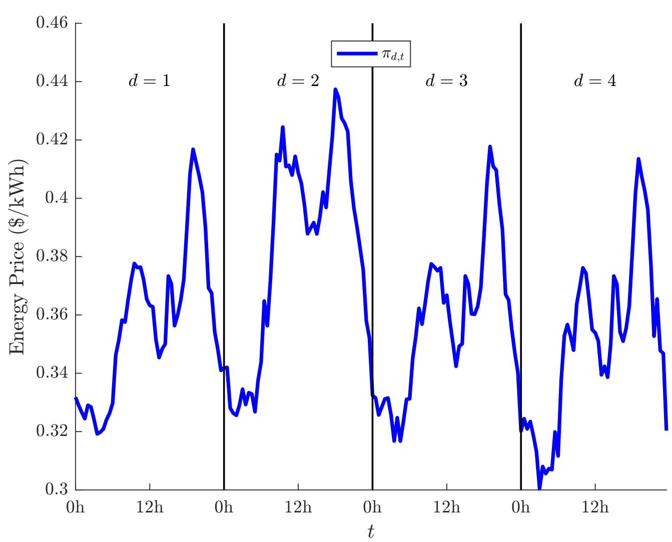

Fig. 5. Optimized retail price of charging energy.

The purpose of levying time-variant prices is to shape electricity demand around periods of low wholesale prices or high PV availability. Fig. 6 shows the resulting operation of the charging station under the optimized retail tariff. In addition to shifting PEV-charging demand around PV availability and wholesale prices, it allows for the use of energy storage to engage in some arbitraging of wholesale prices.

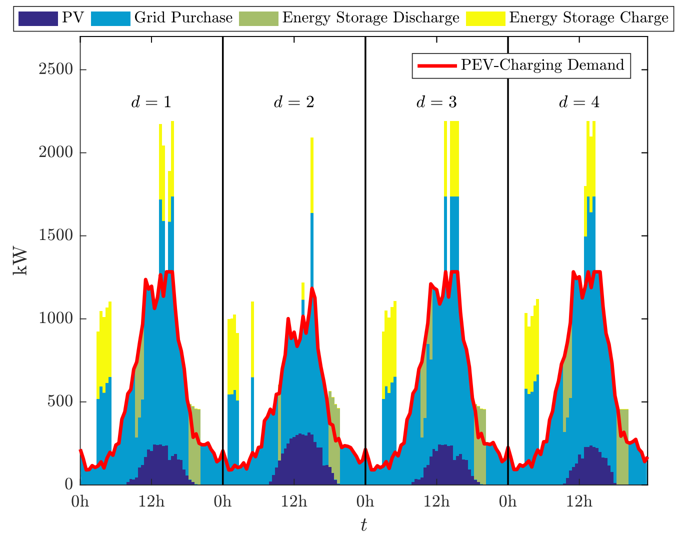

Fig. 6. Optimized operation of charging station.

C. Benefit of Uncertainty Characterization

We examine the benefits of modeling uncertainty by comparing a case of deterministic planning, in which the values of $\gamma_{d,t}$, $\pi^W_{d,t}$, and $\Upsilon_{d,t,v}$ are all fixed equal to their reference values, to using our robust-optimization model. Table II shows the results with three different uncertainty budgets for the robust model. In each case, the uncertainty budgets of the three robust variables are set equal to one another. That is, we examine cases in which $\beta_d^{\gamma,\min}$, $\beta_d^{\pi,\min}$, and $\beta_{d,v}^{\Upsilon,\min}$ are set equal to the same value of either 0.9, 0.8, or 0.7 while the values of $\beta_d^{\gamma,\max}$, $\beta_d^{\pi,\max}$, and $\beta_{d,v}^{\Upsilon,\max}$ are set equal to the same value of either 1.1, 1.2, or 1.3, respectively.

TABLE II
DESIGN OF CHARGING STATION WITH DETERMINISTIC AND ROBUST MODELING APPROACHES

	Robust Model Uncertainty Budget (p.u.)			Deterministic Model
	$[0.9, 1.1]$	$[0.8, 1.2]$	$[0.7, 1.3]$	
x^C	1283	958	809	1386
x^R	500	500	500	500
$x^{S,E}$	1667	1745	1866	1269
$x^{S,P}$	455	538	556	386
Expected Profit (\$)	1134982	959671	764973	1057759

Table II shows that the robust model tends to be more conservative in sizing the charging piles (compared to the deterministic model), while building larger energy-storage systems. This conservatism with respect to sizing the charging piles is due to uncertainty in PEV-charging demand. The larger energy-storage size is due to greater price volatility with a higher uncertainty budget. Energy storage helps the station owner hedge against price uncertainty by being able to use stored energy to manage price spikes.

The expected profits that are reported in Table II are obtained by fixing the sizes of the charging-station components based on the solutions that are given in each of the four cases that are reported. Then, 100000 replications of the uncertain variables (*e.g.*, $\gamma_{d,t}$, $\pi^{W}_{d,t}$, and $\Upsilon_{d,t,v}$) are sampled randomly, assuming that they have Gaussian distributions with mean equal to their reference values and standard deviations equal to 20% of their assumed means. The profit of operating the charging station under each sample is computed to estimate the expected profits that are reported in Table II. The table shows that the greater conservatism that comes with using robust optimization results in reduced profits for the charging-station owner, except in the case with a relatively small uncertainty budget (*i.e.*, when all values of $\beta^{\gamma,\min}_{d}$, $\beta^{\pi,\min}_{d}$, and $\beta^{\Upsilon,\min}_{d,v}$ are set equal to 0.9 and all values of $\beta^{\gamma,\max}_{d}$, $\beta^{\pi,\max}_{d}$, and $\beta^{\Upsilon,\max}_{d,v}$ are set equal to 1.1. Thus, choosing the uncertainty budget appropriately results in a reasonable tradeoff between conservatism and optimality. Uncertainty budgets that are 'too high' sacrifice optimality to hedge against uncertainty.

D. Impact of Energy-Storage Costs

As a final case, we examine the impacts of reducing the capital cost of the energy storage by up to 20% relative to the baseline values that we assume. Table III summarizes the value of (4) in cases with different energy-storage costs. As expected, the table shows that decreasing energy-storage costs increases the profit of the charging-station owner. This is because energy storage can be installed at a lower cost, meaning that more capacity is built. This increased energy-storage capacity results in the station owner having added flexibility in managing volatility in wholesale energy prices, the availability of PV-generated energy, and demands for PEV-charging energy. Further analyses could be conducted to determine the sensitivity of the optimized value of (4), the optimized configuration of the charging station and tariffs for PEV-charging energy, and the real-time use of the charging station.

TABLE III
OPTIMIZED VALUE OF (4) WITH DIFFERENT ENERGY-STORAGE COSTS

Storage-Cost Reduction Relative to Baseline (%)	Value of (4) ($)
0	974545
5	976156
10	980716
15	987504
20	996168

VI. CONCLUSIONS

This paper presents a novel methodology for optimizing the design of a public PEV-charging station. Our model takes account explicitly of uncertainty and the strategic behavior of PEV owners. As such, the model is formulated as a computationally intractable bi-level robust-optimization model, which has the optimized decisions of PEV owners at the lower level. Moreover, due to its robust nature, the upper-level problem has a $\max$-$\min$-$\max$ structure. We apply complementary theory to convert the problem into a single-level robust-optimization model. We solve the resulting model efficiently using a column-and-constraint-generation algorithm.

We demonstrate the model using a case study. In particular, we show how the explicit treatment of the strategic behavior of PEV owners and uncertainty impacts the optimal design of the charging station. We demonstrate also that varying over time the retail price that is levied for charging energy benefits the station owner by having PEV-charging demands follow the availability of low-cost and renewable energy. We show that by varying the uncertainty budget in the robust-optimization model, we can achieve a suitable balance between conservatism and optimality of the solution that is obtained.

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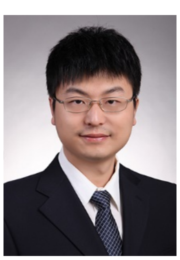

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