Using Price-Based Signals to Control Plug-in Electric Vehicle Fleet Charging

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***Abstract*—We study decentralized plug-in electric vehicle (PEV) charging control, wherein the system operator (SO) sends price-based signals to a load aggregator (LA) that optimizes charging of a PEV fleet. We study a pricing scheme that conveys price and quantity information to the LA and compare it to a simpler price-only scheme. We prove that the price/quantity-based mechanism can yield a socially optimal solution. We also examine several numerical case studies to demonstrate the superior performance of the price/quantity-based scheme. The price/quantity scheme yields nearly identical PEV charging costs compared to the social optima, whereas the price-only scheme is highly sensitive to the choice of a regularization penalty term that is needed to ensure convergence. We also show that the time to compute an equilibrium with the price-only mechanism can be up to two orders of magnitude greater than with the price/quantity scheme and can involve 24 times more information exchange between the SO and LA.**

***Index Terms*—Plug-in electric vehicles, economic dispatch, charging control, market design**

NOMENCLATURE

A. Sets, Parameters, and Functions

T	set of hours in optimization horizon
I	set of conventional generators
Γ	set of plug-in electric vehicle (PEV) groups
$\mathcal{I}_{\gamma,t}$	indicator that PEVs in group $\gamma \in \Gamma$ are parked at the charging station during hour $t \in T$
N_γ	number of PEVs in group γ
X_γ	total energy demand of PEV group γ
R	power capacity of PEV charger
D_t	hour-t PEV energy demand
L_t	hour-t non-PEV energy demand
P_i^-	generator $i \in I$'s minimum power capacity
P_i^+	generator i's maximum power capacity
R_i^-	generator i's ramp-down limit
R_i^+	generator i's ramp-up limit
$F_i(\cdot)$	generator i's generation cost function
$C_t(\cdot)$	load aggregator's (LA's) hour-t PEV charging energy cost function
ξ	regularization penalty in LA's cost function
Δ	price-quantity function updating parameter
V	value of lost load

B. System Operator's Decision Variables

$q_{i,t}$	hour-t output of generator i
σ_t	hour-t PEV charging load served

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C. Load Aggregator's Decision Variables

b_t	total hour-t charging energy scheduled by the LA
$\delta_{\gamma,t}$	hour-t charging energy scheduled by the LA for PEV group γ
u_γ	charging energy for PEV group γ that is unscheduled by the LA

D. Multiple-Load Aggregator Case

$\mathcal{A}$	set of LAs
Γ_a	set of PEV groups that arrive at LA $a \in \mathcal{A}$'s charging station
$\mathcal{I}_{a,\gamma,t}$	indicator that PEVs in group γ of LA a are parked at the charging station during hour t
$N_{a,\gamma}$	number of PEVs in group γ of LA a
$X_{a,\gamma}$	total energy demand of PEV group γ of LA a
$D_{a,t}$	hour-t PEV energy demand of LA a
$C_{a,t}(\cdot)$	LA a's hour-t PEV charging energy cost function
$b_{a,t}$	total hour-t charging energy scheduled by LA a
$\delta_{a,\gamma,t}$	hour-t charging energy scheduled by LA a for PEV group γ
$u_{a,\gamma}$	charging energy for PEV group γ that is unscheduled by LA a

E. Random PEV Driving Pattern Case

S	set of future PEV arrival scenarios
π_s	probability that scenario $s \in S$ occurs
$\mathcal{I}_{s,\gamma,t}$	indicator that PEVs in group γ are parked at the charging station during hour t in scenario s
$N_{\gamma,s}$	number of PEVs in group γ in scenario s
$X_{\gamma,s}$	total energy demand of PEV group γ in scenario s
$D_{t,s}$	hour-t PEV energy demand in scenario s
$C_{t,s}(\cdot)$	LA's hour-t PEV charging energy cost function in scenario s
$q_{i,t,s}$	hour-t output of generator i in scenario s
$\sigma_{t,s}$	hour-t PEV charging load served in scenario s
$b_{t,s}$	total hour-t charging energy scheduled by the LA in scenario s
$\delta_{\gamma,t,s}$	hour-t charging energy scheduled by the LA for PEV group γ in scenario s
$u_{\gamma,s}$	charging energy for PEV group γ that is unscheduled by the LA in scenario s

I. INTRODUCTION

RECENT developments have increased interest in plug-in electric vehicles (PEVs). PEVs can introduce costs and benefits to electric power systems. The primary cost is that PEVs add new loads, which can increase strains on generation, transmission, and distribution assets [1]. A number of works model the effects of PEV charging on generation costs [2], [3], transmission [4], and distribution transformer loading [5], [6].

PEVs can offer benefits, however, since there is flexibility in when charging loads can be served. Sioshansi *et al*. [7]–[10] demonstrate that if PEV charging is properly coordinated with commitment and dispatch, system-wide generator efficiency can be improved. This is because the mix of generators that is committed can be changed to include more efficient units that, due to operational constraints, could not be used without the PEV charging loads. Wu *et al*. [11] model energy allocation among a fleet of PEVs between day-ahead and real-time timeframes. They model a day-ahead block energy purchase, based on price and PEV energy demand forecasts. They then optimize actual energy delivery to the fleet based on real-time conditions. Sortomme and El-Sharkawi [12], [13] demonstrate the benefits of what they term unidirectional vehicle-to-grid services, wherein PEVs adjust their real-time charging loads to provide regulation and other ancillary services. Clement-Nyns *et al*. [5] demonstrate the benefits of coordinated PEV charging in minimizing distribution feeder losses. Richardson *et al*. [14], [15] model distribution impacts of PEV charging with the objective of maximizing energy delivered to vehicles while respecting network constraints.

These benefits of PEVs presuppose some form of control to coordinate charging and power system operations. Most PEV analyses assume centralized control by the system operator (SO), which schedules PEV charging subject to minimum service requirements [2], [7], [16]. This is akin to solving a social planner's problem. While centralized control maximizes coordination between power system operations and PEV charging, it may be difficult to implement. Centralized control may require the SO to track the status of each PEV, raising data management and computational issues. Bashash and Fathy [17] suggest the use of a universal control signal that may overcome this limitation, however. Otherwise, decentralized control, for instance through price signals, is offered as an alternative [1], [18]. Under such a scheme, charging decisions are left to PEV owners or perhaps to a load aggregator (LA) that optimizes charging for a fleet of PEVs. Time-variant electricity tariffs are designed to encourage optimal charging behavior. For instance, a time-of-use rate levies a lower energy price during certain hours, encouraging charging at these times. Real-time pricing, which dynamically sets prices based on real-time marginal energy costs, provides PEV owners with even finer-grained price signals. Indeed, if prices are set equal to the marginal cost of energy from the social planner's solution, they should support a Nash equilibrium in which a self-interested SO and LAs follow the social planner's solution.

A difficulty in implementing such a scheme is that one must typically solve the social planner's problem to find equilibrium-supporting prices. Alternatively, one may use iterative price discovery [19], [20]. Such a framework assumes that the SO sends real-time prices to LAs after receiving charging demands, while LAs adjust their charging patterns in response to the prices. This is done repeatedly until settling at an equilibrium. A shortcoming of this approach, however, is that equilibria may be unstable or difficult to compute [21], without any convergence guarantee. Ma *et al*. [19] overcome these issues by adding a regularization term, which penalizes differences between each PEV's charging profile and the population average, to the LAs' objective functions. While providing convergence guarantees, there is no clear economic rationale for including such penalties. This is because the regularization term does not reflect the social cost of a good or service being consumed by the LA.

Following on these seminal works, this paper further explores the use of price-based techniques to coordinate PEV charging with power system operations. We examine a simple price-only control mechanism, in which LAs are given the most recent marginal prices at each iteration. We contrast this with a scheme that sends price and quantity information to the LAs. Specifically, the LA is given data specifying prices as a function of the quantity of charging load scheduled. This function is iteratively constructed by collecting marginal price data as the SO and LA interact. We prove that under mild conditions the price/quantity scheme induces an equilibrium that is an optimum of the social planner's problem, without the need for any regularization terms. We also use numerical case studies to compare the performance of the two control schemes. Our results demonstrate that the price/quantity mechanism can find an equilibrium that is close to a social optimum. While the price-only scheme can provide similar costs to a social optimum, its performance is very sensitive to the choice of regularization penalty term. Moreover, we show that the price-only mechanism typically requires much more computational effort to find an equilibrium than the price/quantity one does.

The remainder of this paper is organized into six sections. Section II describes the setting that we study, provides formulations of the SO's, LA's, and social planner's problems, and details the pricing schemes examined. Section III presents our theoretical results showing that the price/quantity mechanism can achieve social optimality. Sections IV–VI summarize our numerical case studies and their results. Section VII concludes.

II. Market Models

We model the market as consisting of two interacting players—an SO, which determines generator dispatch, and an LA, which schedules PEV charging. The SO solves an economic dispatch problem to minimize the cost of serving PEV and non-PEV loads. Based on the dispatch solution, the SO sends a price signal to the LA, which is used in the LA's charging scheduling problem to minimize fleet-wide costs. The LA is assumed to have complete discretion to schedule charging within the window of time that each PEV is parked. The optimized charging schedule is sent to the SO, which then reoptimizes the system dispatch and sends an updated signal to the LA. We compute an equilibrium by iterating between the SO and LA problems, until neither player changes its decisions between two successive iterations. We examine two different pricing signals, which are detailed in Section II-D.

We now provide detailed formulations of the SO and LA problems. We then describe the iterative technique used to compute equilibria between the SO and LA and the pricing schemes considered in our analysis. The model formulations and theoretical results in this section and the next assume that there is a single LA that has perfect foresight of future PEV charging demand. We examine case studies, in which these two assumptions are relaxed, in Sections V and VI.

A. System Operator's Economic Dispatch Problem

The system operator is assumed to solve a standard economic dispatch problem, the formulation of which is:

$$\min \sum_{t \in T} \left[\sum_{i \in I} F_i(q_{i,t}) + V \cdot (D_t - \sigma_t) \right]; \tag{1}$$

$$\text{s.t. } \sum_{i \in I} q_{i,t} = L_t + \sigma_t; \qquad \forall\ t \in T; \tag{2}$$

$$P_i^- \leq q_{i,t} \leq P_i^+; \quad \forall\ i \in I; t \in T; \tag{3}$$

$$R_i^- \leq q_{i,t} - q_{i,t-1} \leq R_i^+; \quad \forall\ i \in I; t \in T; \tag{4}$$

$$0 \leq \sigma_t \leq D_t; \quad \forall\ t \in T. \tag{5}$$

Objective function (1) minimizes cost and we assume convex subdifferentiable generator cost functions. The model allows the SO to not serve PEV charging loads scheduled by the LA. Doing so incurs a cost, however, which is given by an assumed value of lost load, V. The ability to curtail PEV charging is included to ensure that the SO's problem is feasible for any charging load scheduled by the LA. We assume throughout, however, that V is greater than the marginal cost of generation, meaning that if generating capacity is available it is optimal for the SO to serve PEV charging demand.

Constraint set (2) enforces the load-balance requirement that generation and demand be exactly equal in each hour. Constraint sets (3) and (4) are output and ramping limits on conventional generators. We assume that the non-PEV loads are such that there is always some excess generating capacity available to serve PEV charging needs in every hour—although the amount of generation capacity may be insufficient to serve all of the PEV charging demand in any given hour. Constraint set (5) limits the amount of PEV charging energy served to be no greater than the energy scheduled by the LA. We let $V^*_{SO}(D)$ denote the optimal objective function value when the LA submits the PEV charging schedule, D.

B. Load Aggregator's Charging Scheduling Problem

We assume that the LA classifies the PEVs by groups and that all of the PEVs within a group arrive and depart the charging station at the same time. We further assume that all of the PEVs within a group have the same demand for charging energy upon arrival. We make this assumption to simplify notation and it can be relaxed without losing any of our theoretical results. The LA schedules PEV charging to minimize costs, with these costs based on a price signal sent by the SO. We define $C_t(\cdot)$ as the total cost of PEV charging energy levied on the LA, as a function of the total hour-t charging load. To provide a general formulation of the LA's problem, we do not assume that these pricing functions have any specific structure. We detail the pricing schemes that we examine, and the corresponding form of these functions, in Section II-D. The formulation of the LA's problem is:

$$\min \sum_{t \in T} C_t(b_t) + V \cdot \sum_{\gamma \in \Gamma} u_\gamma; \tag{6}$$

$$\text{s.t. } b_t = \sum_{\gamma \in \Gamma} \delta_{\gamma,t}; \quad \forall\ t \in T; \tag{7}$$

$$0 \leq \delta_{\gamma,t} \leq R \cdot N_\gamma \cdot \mathcal{I}_{\gamma,t}; \quad \forall\ \gamma \in \Gamma; t \in T; \tag{8}$$

$$\sum_{t \in T} \delta_{\gamma,t} + u_\gamma = X_\gamma; \quad \forall\ \gamma \in \Gamma; \tag{9}$$

$$u_\gamma \geq 0; \quad \forall\ \gamma \in \Gamma. \tag{10}$$

Objective function (6) minimizes PEV charging costs. The model allows the LA to leave PEV charging demand unserved, which incurs a penalty given by the value of lost load. Constraint set (7) defines the total amount of charging energy scheduled in each hour as the sum of charging energy allocated to the different PEV groups. Constraint set (8) limits the amount of charging energy scheduled to each group in each hour to be less than the capacity of the chargers—which are assumed to be homogeneous. It further restricts PEVs within each group to only be charged during hours in which they are parked at the charging station. Constraint set (9) defines the amount of unserved charging demand for each PEV group as the difference between the charging demand and scheduled load while constraint set (10) requires unserved charging demand to be non-negative.

C. Equilibrium Computation

We compute a Nash equilibrium between the SO and LA using an iterative technique, which is outlined in Algorithm 1 [19], [20]. When $k = 0$ the algorithm initializes by fixing the PEV demands equal to zero (Step 4), solving the SO's problem (Step 9), and determining the initial price signals to send the LA (Step 10). In subsequent iterations, the LA solves its PEV scheduling problem based on the most recent price information (Step 6) and these updated charging schedules are used in the SO's problem (Steps 7 and 9) and price updating (Step 10). This iterative process repeats until reaching a Nash equilibrium (*i.e.*, neither the SO nor LA has an incentive to unilaterally deviate from its optimal decision in the previous iteration) or we conduct more than $\bar{k}$ iterations. This iteration limit is needed for some pricing schemes, which can yield unstable Nash equilibria that are difficult to compute.

D. Price Signals

Commodity pricing is typically based on the marginal cost of an incremental unit of demand. In the context of our problem, this would be given by the values of the Lagrange multipliers associated with constraint set (2) in the SO's problem. We let $\lambda_t(D)$ denote the Lagrange multipliers of these constraints when the LA submits the PEV charging schedule, D, and specifically define λ_t^k as the Lagrange multipliers of these constraints when solving the SO's problem in the kth iteration of Algorithm 1. We now detail the two pricing schemes that we study.

Algorithm 1 Equilibrium Computation

1: $k \leftarrow 0$ ▷ Set iteration count to zero
2: **repeat**
3: **if** $k = 0$ **then**
4: $D_t \leftarrow 0$ for all $t \in T$
5: **else**
6: $(b^k, \delta^k, u^k) \leftarrow \arg\min$ (6) s.t. (7) − (10)
7: $D_t \leftarrow b_t^k$ for all $t \in T$
8: **end if**
9: $(q^k, \sigma^k) \leftarrow \arg\min$ (1) s.t. (2) − (5)
10: Update $C_t(\cdot)$ based on $\{q^\kappa, \sigma^\kappa\}_{\kappa=0}^k$
11: $k \leftarrow k + 1$
12: **until** $k \geq \bar{k}$ **or** $\left[(b^{k-2}, \delta^{k-2}, u^{k-2}) = (b^{k-1}, \delta^{k-1}, u^{k-1})\right.$ **and** $\left.(q^{k-2}, \sigma^{k-2}) = (q^{k-1}, \sigma^{k-1})\right]$

1) Marginal Pricing: The marginal pricing scheme assumes that the price of charging energy in each hour is equal to the value of the corresponding Lagrange multiplier from the most recent solution of the SO's problem. Thus, the updated marginal pricing function after the kth iteration of Algorithm 1 is defined as:

$$C_t(b_t) = \lambda_t^{k-1} \cdot b_t + \xi \cdot (b_t - b_t^{k-1})^2. \tag{11}$$

The $\xi \cdot (b_t - b_t^{k-1})^2$ term, where $\xi \geq 0$, is a regularization term that penalizes the LA for changing its charging schedule from the schedule submitted in the previous iteration. We include this term based on the work of Ma *et al*. [19], who show that it ensures convergence of Algorithm 1, so long as ξ is properly chosen. When $k = 0$, the regularization term is excluded.

2) Price/Quantity-Based Signal: This pricing scheme uses all of the marginal price data collected in all of the previous iterations of Algorithm 1 to construct a non-decreasing stepped price function. This price function is constructed iteratively, since each updated vector of PEV charging demands results in different optimal solution/Lagrange multiplier pairs of the SO's problem. We let $\Lambda_t^k(b_t)$ denote the updated price function constructed in Step 10 of the kth iteration of Algorithm 1 and describe how Λ^k is iteratively constructed.

When $k = 0$, we fix $\Lambda_t^0(b_t) = \lambda_t^0, \forall\ t$. For any $k \geq 1$ we define $\dot{b}_t^{k,0}, \dot{b}_t^{k,1}, \ldots, \dot{b}_t^{k,k}$, to be the set of breakpoints in $\Lambda_t^{k-1}(b_t)$, where by convention we take $\dot{b}_t^{k,0} = 0$ and $\dot{b}_t^{k,k} = +\infty$. We then define:

$$D_t^{k,-} = \max_i \left\{ \dot{b}_t^{k-1,i} : \Lambda_t^{k-1}(\dot{b}_t^{k-1,i}) < \lambda_t^k, \dot{b}_t^{k-1,i} < D_t \right\}, \tag{12}$$

as the largest breakpoint in $\Lambda_t^{k-1}(b_t)$ which is smaller than D_t and gives a smaller price than the corresponding Lagrange multiplier. If no such breakpoint exists, by convention we let $D_t^{k,-} = 0$. Similarly, we define:

$$D_t^{k,+} = \min_i \left\{ \dot{b}_t^{k-1,i} : \lim_{\nu \to 0^+} \Lambda_t^{k-1}(\dot{b}_t^{k-1,i} + \nu) \geq \lambda_t^k \right\}, \tag{13}$$

as the smallest breakpoint in $\Lambda_t^{k-1}(b_t)$ that has a larger price than the Lagrange multiplier found in the current iteration. If no such breakpoint exists, we let $D_t^{k,+} = +\infty$. The new prices are then defined as:

$$\Lambda_t^k(b_t) = \begin{cases} \lambda_t^k, & \text{if } \beta^{k,-} < b_t \leq \beta^{k,+}, \\ \Lambda_t^{k-1}(b_t), & \text{otherwise}, \end{cases} \tag{14}$$

where:

$$\beta^{k,-} = D_t - \min\left\{ \Delta, \frac{D_t - D_t^{k,-}}{2} \right\}, \tag{15}$$

$$\beta^{k,+} = \max\left\{ D_t, D_t^{k,+} \right\}, \tag{16}$$

and $\Delta > 0$ is a fixed parameter. The total cost of energy levied on the LA in the kth iteration of Algorithm 1 is defined as:

$$C_t(b_t) = \int_0^{b_t} \Lambda_t^{k-1}(\psi) d\psi. \tag{17}$$

In essence, our method begins with an underestimate of the true cost of PEV charging, since we fix $\Lambda_t^0(b_t)$ equal to the marginal cost of energy if $D = 0$. At each iteration, we observe a new Lagrange multiplier value associated with the latest PEV charging load and assume that this marginal cost applies in a small neighborhood of the current charging bid, D_t.

Fig. 1 illustrates our proposed price-updating scheme for a particular hour through two iterations of Algorithm 1. When $k = 0$, $\Lambda_t^0(b_t)$ is initially fixed equal to λ_t^0. When $k = 1$, the LA solves its charging control problem and schedules b_t^1 MW, resulting in a Lagrange multiplier value of λ_t^1 in the SO's problem. To update the price function, the SO determines that $D_t^{1,-} = 0$ and $D_t^{1,+} = +\infty$. The updated price function is then:

$$\Lambda_t^1(b_t) = \begin{cases} \lambda_t^0, & \text{if } b_t \leq D_t^{2,-}, \\ \lambda_t^1, & \text{if } D_t^{2,-} < b_t. \end{cases} \tag{18}$$

Note that in this example, $\Delta < (b_t^1 - D_t^{1,-})/2$, since the second segment of the $\Lambda_t^1(b_t)$ function does not extend down to the midpoint between b_t^1 and $D_t^{1,-}$.

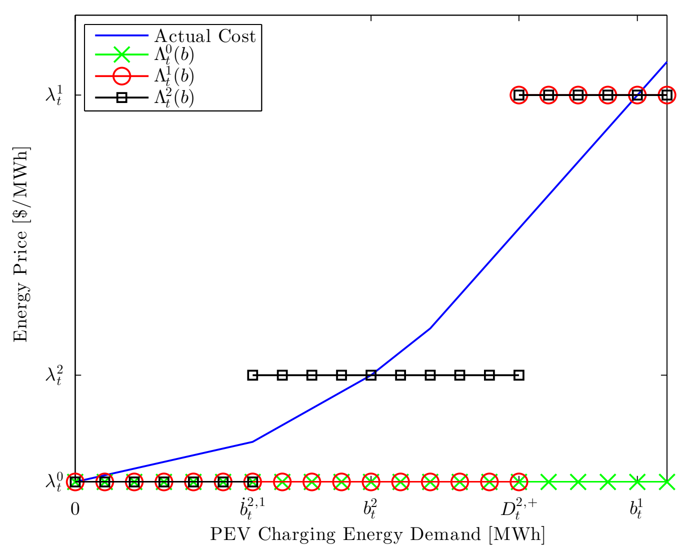

Fig. 1. Updated price function after three iterations of Algorithm 1 used with price/quantity-based signal.

When $k = 2$, the LA reoptimizes PEV charging using the updated price function and schedules b_t^2 MW, resulting in

Lagrange multiplier value λ_t^2. The SO also determines that $D_t^{2,-} = 0$ and $D_t^{2,+}$ is the value noted in Fig. 1. This gives the updated price function:

$$\Lambda_t^2(b_t) = \begin{cases} \lambda_t^0, & \text{if } b_t \le \dot{b}_t^{2,1}, \\ \lambda_t^2, & \text{if } \dot{b}_t^{2,1} < b_t \le D_t^{2,+}, \\ \lambda_t^1, & \text{if } D_t^{2,+} < b_t. \end{cases} \quad (19)$$

E. Convergence of Algorithm 1

Convergence of Algorithm 1 can be guaranteed with both pricing schemes. In the case of the price-only scheme, convergence is governed by the choice of ξ, insomuch as the algorithm is guaranteed to terminate if this parameter is chosen properly [19]. We prove (*cf.* Lemma 2) that the price/quantity-based scheme is guaranteed to converge, with the convergence rate governed by the choice of Δ.

F. Social Planner's Problem

The social planner's problem assumes that a single entity optimizes both power system dispatch and PEV charging. This is the counterfactual 'ideal' case against which we benchmark our price-based charging control schemes. This problem is formulated as:

$$\min \sum_{t \in T} \left[\sum_{i \in I} F_i(q_{i,t}) + V \cdot (D_t - \sigma_t) \right] + \sum_{\gamma \in \Gamma} V \cdot u_\gamma; \quad (20)$$

$$\text{s.t. (2)–(5) and (7)–(10);} \quad (21)$$

$$b_t = D_t; \quad \forall\, t \in T. \quad (22)$$

The social planner's problem minimizes the cost of generation and unserved PEV charging demand subject to the same set of power system and PEV charging constraints in the SO and LA problems. Constraint set (22) defines the PEV charging demand parameters in the SO's problem in terms of the charging decision variables in the LA's problem.

III. Social Optimality of Pricing Schemes

We now turn to proving that the price/quantity-based control scheme proposed in Section II-D2 can yield a socially optimal PEV charging schedule. This discussion is divided into two parts. We first show that a socially optimal PEV charging schedule is optimal for the LA, if the LA is given the true price/quantity functions. We then show that the iterative technique used to construct the $\Lambda_t^k(b_t)$ functions causes the LA's optimal charging pattern to converge to a near-social optimum. Before proceeding, we explicitly state an assumption underlying our theoretical results.

Assumption 1: The non-PEV loads are such that there is generating capacity available to provide some PEV charging energy in each hour. Moreover, the value of lost load, V, is sufficiently large compared to the marginal cost of generation that it is optimal for the SO to provide some PEV charging energy in each hour, if it is requested by the LA. □

We next note that we can rewrite the social planner's problem as:

$$\min \sum_{\gamma \in \Gamma} V \cdot u_\gamma + \left\{ \begin{array}{ll} \min & \sum_{t \in T} \left[\sum_{i \in I} F_i(q_{i,t}) \right. \\ & \qquad \left. + V \cdot (b_t - \sigma_t) \right] \\ \text{s.t.} & \text{(2)–(5)} \end{array} \right\}; \quad (23)$$

$$\text{s.t. (7)–(10),} \quad (24)$$

where constraint (22) is explicitly substituted into the objective. By the principal of optimality, this is equivalent to:

$$\min V_{SO}^*(b) + \sum_{\gamma \in \Gamma} V \cdot u_\gamma; \quad (25)$$

$$\text{s.t. (7)–(10),} \quad (26)$$

Thus, the Karush-Kuhn Tucker (KKT) conditions of the social planner's problem are:

$$\frac{\partial}{\partial b_t} V_{SO}^*(b_t) + \mu_t^7 = 0 \quad (27)$$

$$-\mu_t^7 - \mu_{\gamma,t}^{8,-} + \mu_{\gamma,t}^{8,+} + \mu_\gamma^9 = 0 \quad (28)$$

$$V + \mu_\gamma^9 - \mu_\gamma^{10} = 0, \quad (29)$$

where μ_t^7, $\mu_{\gamma,t}^8$, and μ_γ^9 are Lagrange multipliers on constraint sets (7), (8), and (9), respectively. We also prove in Lemma 3 (*cf.* the appendix) that:

$$\frac{\partial}{\partial b_t} V_{SO}^*(b_t) = \lambda_t(b). \quad (30)$$

Hence, condition (27) becomes:

$$\lambda_t(b) + \mu_t^7 = 0. \quad (31)$$

We let $\Lambda_t^*(b_t) = \lambda_t(b)$ denote the true function that gives the Lagrange multiplier value associated with constraint (2) in hour t when $D = b$ in the SO's problem. We show in Lemma 4 (*cf.* the appendix) that $\Lambda_t^*(b_t)$ is non-decreasing in b_t. Based on these two lemmas, we now show that if the price of PEV charging energy levied on the LA is given by (17) using the true price functions, an optimum of the social planner's problem is also an optimum of the LA's problem.

Lemma 1: Suppose that Assumption 1 is true and that the price levied on the LA for PEV charging energy is given by (17) using the $\Lambda^*(\cdot)$ price functions and all of the $\Lambda_t^*(b_t)$ functions are non-decreasing. Then an optimum of the social planner's problem is also an optimum of the LA's problem.

Proof: The KKT conditions[1] for the LA's problem are:

$$C_t'(b_t) + \mu_t^7 = 0 \quad (32)$$

$$-\mu_t^7 - \mu_{\gamma,t}^{8,-} + \mu_{\gamma,t}^{8,+} + \mu_\gamma^9 = 0 \quad (33)$$

$$V + \mu_\gamma^9 - \mu_\gamma^{10} = 0. \quad (34)$$

[1] Since the $\Lambda_t^*(b_t)$ functions may be discontinuous (*e.g.*, step functions), the LA's objective function is not necessarily continuously differentiable. We can, however, appeal to a generalized form of the KKT conditions for a convex subdifferentiable objective, which the LA's problem is guaranteed to have, that gives the same result.

From (17) we have that:

$$\begin{aligned} C'_t(b_t) &= \frac{d}{db_t}\int_0^{b_t} \Lambda^*_t(\psi)d\psi \\ &= \Lambda^*_t(b_t) \\ &= \lambda_t(b_t). \end{aligned} \tag{35}$$

Since we assume that the $F_i(\cdot)$ functions are convex and $\Lambda^*_t(b_t)$ functions non-decreasing, the LA's and social planner's problems are both convex, and the KKT conditions are also sufficient for optima. Since optima of the LA's and social planner's problems have the same necessary conditions, an optimum of the social planner's problem must be optimal in the LA's problem as well. ■

We hereafter let b^* denote a PEV charging schedule that is optimal in the LA's problem, if the price levied on the LA is given by (17) using the $\Lambda^*(\cdot)$ price functions. Suppose that b^k, the solution to the LA's problem in the kth iteration of Algorithm 1, is not equal to b^*. Given the high cost of unscheduled PEV charging in the LA's problem, it must be true that for some pairs of hours, t and t', we have $b^k_t > b^*_t$ and $b^k_{t'} < b^*_{t'}$. We refer to t as the hour in which PEV charging is overscheduled and t' as the hour in which it is underscheduled. Thus, for $\{b^k\}$ to get close to b^* we must have PEV charging shifted from over- to under-scheduled hours. We show in the following lemma that our price function updating scheme achieves this.

Lemma 2: Suppose that Assumption 1 is true, the price levied on the LA for PEV charging energy is given by (17) using the $\Lambda^k(\cdot)$ price functions, and that all of the $\Lambda^*_t(b_t)$ functions are non-decreasing. Then there exists a function, $d(\Delta)$, with:

$$d(\Delta) \geq 0, \quad \lim_{\Delta\to 0^+} d(\Delta) = 0; \tag{36}$$

such that after receiving a new approximation of the price functions in each iteration, the LA always prefers to reduce (increase) scheduled charging energy in hours in which it is initially overscheduled (underscheduled), unless $|b^k_t - b^*_t| \leq d(\Delta)$ for all t, in which case $b^k = b^{k+1}$.

Proof: Note that our pricing mechanism starts with $\Lambda^0_t(b_t) = \Lambda^*_t(0)$ for all t, which is an underestimate of the true marginal price, since the $\Lambda^*_t(b_t)$ functions are non-decreasing. Suppose that $b^1 \neq b^*$, meaning that there must be t and t' such that $b^1_t > b^*_t$ and $b^1_{t'} < b^*_{t'}$. Since PEV charging loads can be feasibly shifted from hour t to t', it must be true that $\Lambda^*_t(0) \leq \Lambda^*_{t'}(0)$. Moreover, since $\Lambda^*_{t'}(\cdot)$ is assumed to be non-decreasing, it must be true that $\Lambda^*_t(0) \leq \Lambda^*_{t'}(b_{t'})$ for all $b_{t'} \geq 0$.

We now prove the desired result using an inductive argument. When $k = 1$ our updated price functions have:

$$\Lambda^1_t(b_t) = \Lambda^*_t(0), \text{ if } b_t \leq b^1_t - \eta^1_t, \tag{37}$$

and:

$$\Lambda^1_{t'}(b_{t'}) = \lambda^1_{t'}, \text{ if } b_{t'} \geq b^1_{t'}, \tag{38}$$

where $\eta^1_t = \min\{\Delta, b^1_t/2\}$. We now define the PEV charging schedule, $\hat{b}(\eta^1_t)$, as:

$$\hat{b}_r(\eta^1_t) = \begin{cases} b^1_t - \eta^1_t, & \text{if } r = t, \\ b^1_{t'} + \eta^1_t, & \text{if } r = t', \\ b^1_r, & \text{otherwise.} \end{cases} \tag{39}$$

Note that with the updated price functions, the difference between the LA's cost of charging schedule b^1 and $\hat{b}(\eta^1_t)$ is $\eta^1_t \cdot (\lambda^1_t - \lambda^1_{t'})$. If $\lambda^1_t > \lambda^1_{t'}$, the cost difference is positive, and the LA prefers $\hat{b}(\eta^1_t)$ and would update its charging schedule accordingly in the following iteration. Otherwise, the LA prefers b^1 and does not update its schedule in the following iteration. Moreover, the LA would be unwilling to change its schedule to any $\hat{b}(\eta)$ with $\eta \geq \eta^1_t$ with the Λ^1 price functions, since doing so further substitutes lower-priced energy in hour t with higher-priced energy in hour t'. Note, however, that by construction, $\Lambda^1_t(\cdot)$ and $\Lambda^1_{t'}(\cdot)$ underestimate the true cost of PEV charging schedules of the form $\hat{b}(\eta)$ with $\eta \geq \eta^1_t$. Since with these underestimated prices the LA has no desire to adjust its charging schedule, there is no need to determine the true marginal cost of energy for $b_t < b^1_t$ and $b_{t'} > b^1_{t'}$ to find a near-socially optimal charging schedule. Thus, the incumbent solution is near-socially optimal.

Suppose that in the kth iteration we have $b^k \neq b^*$ and that $b^k_t > b^*_t$ and $b^k_{t'} < b^*_{t'}$. According to the price updating scheme we have:

$$\Lambda^k_t(b_t) = \Lambda^*_t(0), \text{ if } b_t \leq b^k_t - \eta^k_t, \tag{40}$$

and:

$$\Lambda^k_{t'}(b_{t'}) = \lambda^k_{t'}, \text{ if } b_{t'} \geq b^k_{t'}, \tag{41}$$

where $\eta^k_t = \min\{\Delta, b^k_t/2\}$. We can define the PEV charging schedule, $\hat{b}(\eta^k_t)$, as in (39). The difference between the LA's cost of charging schedule b^k and $\hat{b}(\eta^k_t)$ is $\eta^k_t \cdot (\lambda^k_t - \lambda^k_{t'})$. If $\lambda^k_t > \lambda^k_{t'}$ the LA updates it charging schedule to $\hat{b}(\eta^k_t)$ in the next iteration. Otherwise, it does not. Repeating the same argument as when $k = 1$, the $\Lambda^k_t(\cdot)$ and $\Lambda^k_{t'}(\cdot)$ function underestimate the true cost of PEV charging schedules of the form $\hat{b}(\eta)$ with $\eta \geq \eta^k_t$. Since charging schedules of this form are suboptimal with underestimated costs, there is again no need to determine the true marginal cost of energy for $b_t < b^k_t$ and $b_{t'} > b^k_{t'}$ to find a near-socially optimal charging schedule. Thus, the incumbent solution is again near-socially optimal.

We finally note that adjusting the value of Δ gives Λ^k functions and the resulting b^k that can be made arbitrarily close to Λ^* and b^*. ■

The crux of this result is that if PEV charging is over- and under-scheduled in a pair of hours, t and t', respectively, the updating scheme makes the price of charging energy in hour t relatively less attractive than that in hour t'. This incentivizes the LA to shift some of its charging load from hour t to t'. This process repeats iteratively until the approximated price functions, $\Lambda^k_t(\cdot)$ and $\Lambda^k_{t'}(\cdot)$, give reasonable approximations of the true price functions in a neighborhood of b^*_t and $b^*_{t'}$, at which point b^k_t and $b^k_{t'}$ are near-socially optimal and the LA does not update its schedule. Note that depending on how far b^0_r is from b^*_r, it may take a different number of iterations for the price functions and schedules in different hours to converge. The parameter, Δ, controls the rate of convergence and final solution quality. Larger values result in a more coarse approximation of the $\Lambda_r^*(\cdot)$ functions, but also means that fewer iterations must be conducted before convergence.

IV. Case Study

We examine three numerical case studies to demonstrate the benefits of directly controlling PEV charging and the relative performance of the two price-based control schemes. We begin, in this section, with a case that conforms to the assumptions underlying our theoretical results. Namely, there is a single LA that knows PEV arrival and departure times and charging demands with certainty *a priori*. We then examine two sensitivity cases with multiple LAs and random PEV driving patterns in Sections V and VI, respectively. These are intended to study the effectiveness of the control schemes when these assumptions are relaxed. We measure the performance of the control schemes by system cost and the number of iterations, amount of CPU time, and amount of data exchanged between the SO and LA to compute equilibria.

A. Case Study Data

We examine power system operations and PEV charging over a one-day period. The modeled power system consists of three thermal generators. Generator characteristics are summarized in Table I. We assume each conventional generator, i, has a convex quadratic cost function, identical ramp-up and -down limits given by R_i, and starting generation levels given by $q_{i,0}$. We assume 24500 PEV arrivals, with times parked at the charging station and charging demands that are summarized in Table II (the 24500 PEV arrivals corresponds to the sum of the N_γ values reported). This gives a total PEV charging demand of 200 MWh, which amounts to a 4% increase in electricity demand over the one-day period. Each PEV charging station is assumed to have a 4 kW power limit. Fig. 2 shows the system's non-PEV load and the net load when the social planner schedules PEV charging or in an uncontrolled case in which PEVs charge immediately upon arrival at a charging station.

TABLE I
CONVENTIONAL GENERATOR CHARACTERISTICS

Unit	P_i^-	P_i^+	R_i	$q_{i,0}$	$F_i(q_i)$
1	10	120	15	70	$110.85 + 5.36 \cdot q_1 + 0.0050 \cdot q_1^2$
2	10	80	10	55	$80.87 + 10.72 \cdot q_2 + 0.0137 \cdot q_2^2$
3	20	150	100	60	$85.24 + 36.51 \cdot q_3 + 0.0087 \cdot q_3^2$

When computing equilibria using the marginal pricing scheme, we use different values ranging between 0.05 and 20 for the regularization penalty term, ξ. We assume $\Delta = 0.5$ kWh when defining $\beta^{k,-}$ in the price/quantity-based signal. We terminate Algorithm 1 if the maximum absolute difference between the charging profiles of two consecutive iterations is less than 0.1% and set an iteration limit of $\bar{k} = 2000$. This high iteration limit is needed for the price-only control scheme, the convergence properties of which is highly sensitive to the choice of ξ. All of the models are formulated using version 12.1.0 of the `AMPL` modeling package and solved with version 12.1.0 of the `CPLEX` optimization package on a quad-core 2.7 GHz Linux system with 4 GB of RAM.

TABLE II
PEV ARRIVAL AND DEPARTURE TIMES AND CHARGING ENERGY DEMANDS

γ	Hours Parked	N_γ	X_γ	γ	Hours Parked	N_γ	X_γ
1	1–7	500	3.75	22	8–17	500	5.5
2	1–8	1000	7.5	23	9–15	250	2.5
3	1–9	500	3.5	24	9–16	1250	13
4	1–10	500	4	25	9–17	2500	17
5	1–11	250	2	26	9–18	1000	10
6	1–12	250	1.75	27	10–16	1000	9.5
7	2–7	250	2	28	10–17	1500	14
8	2–8	1000	7.5	29	10–18	1000	11
9	2–9	1000	6	30	11–17	1000	8
10	2–10	250	3	31	11–18	1000	7.5
11	2–11	250	2	32	12–17	500	3
12	2–12	250	1.75	33	12–18	500	5
13	3–9	250	2	34	12–19	500	4
14	3–10	250	2.5	35	13–18	250	3
15	3–11	250	2	36	13–19	500	2
16	3–12	250	1.75	37	14–19	500	2
17	4–11	250	1.5	38	14–20	500	5
18	4–12	250	1.5	39	15–20	500	5
19	5–11	250	1.25	40	15–21	250	1
20	8–15	500	5	41	16–20	500	3
21	8–16	500	5	42	16–21	250	2

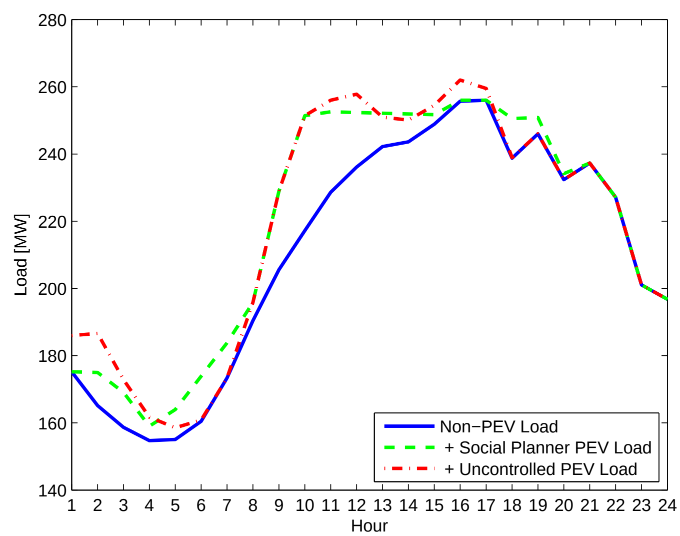

Fig. 2. System load.

B. Case Study Results

We compare system performance between the cases listed in Table III, which rely on different forms of PEV charging scheduling. Case 1 assumes that there are no PEVs whereas Case 5 assumes uncontrolled charging. Case 2 assumes that charging is controlled by a social planner and cases 3a through 3d assume price-only control with different regularization penalty terms. Case 3c, which uses $\xi = 1.025$, has the best performance in terms of cost and equilibrium computation time. Case 4 uses the proposed price/quantity-based control scheme. Table III lists total generation and PEV charging curtailment costs. Since all PEV charging demands are met in all of the cases, curtailment costs are zero.

The cost differences between each of Cases 2 through 5 and Case 1 represent incremental PEV charging costs with different control schemes. Table IV reports these charging

TABLE III
CASES EXAMINED AND TOTAL COSTS

Case	PEV Charging	Total Cost [$]
1	None	68589
2	Social Planner	73972
3a	Price-Only ($\xi = 0.1$)	74195
3b	Price-Only ($\xi = 1$)	73974
3c	Price-Only ($\xi = 1.025$)*	73973
3d	Price-Only ($\xi = 20$)	74098
4	Price/Quantity	73973
5	Uncontrolled	74708

costs in the different cases examined. These cost are given as absolute dollar amounts and on a per-MWh (of PEV charging load) basis. As expected, the social planner minimizes PEV charging costs whereas uncontrolled charging is the most costly, resulting in 14% higher driving costs than the social optimum.

TABLE IV
PEV CHARGING COSTS

Case	[$]	[$/MWh]
Social Planner	5383	26.88
Price-Only ($\xi = 0.1$)	5606	28.00
Price-Only ($\xi = 1$)	5385	26.89
Price-Only ($\xi = 1.025$)*	5384	26.89
Price-Only ($\xi = 20$)	5509	27.51
Price/Quantity	5384	26.89
Uncontrolled	6119	30.56

The price/quantity-based mechanism results in very slight cost increases compared to the social planner. This increase reflects the incremental cost of relying on price-based charging control. The increase is due to the iterative price updating process terminating before reaching the true social optimum (*cf*. Lemma 2) and is a consequence of the approximate nature of the price function (*i.e.*, the choice of Δ). The cost difference between the price/quantity and social planner cases decreases as $\Delta \rightarrow 0$, although with more computation needed. Our choice of $\Delta = 0.5$ kWh requires 25 iterations of Algorithm 1 and 3.23 s of CPU time to compute an equilibrium, with trivial efficiency losses.

Performance of the price-only mechanism, both in terms of charging cost and equilibrium computation, is highly sensitive to the choice of ξ. If this term is chosen correctly (*i.e.*, $\xi = 1.025$), the resulting charging patterns have costs that are nearly identical to the price/quantity and social planner cases. Poorly-chosen values result in Algorithm 1 not converging and much higher costs. This is illustrated in Fig. 3, which shows charging costs and equilibrium computation time with price-only charging control as a function of ξ. It also shows charging costs with social planning and uncontrolled charging, indicating how price-only control performs compared to these extreme cases. It should be noted that with price-only control, Algorithm 1 only converges with ξ between 0.95 and 2.5. However, the amount of time it takes to compute an equilibrium ranges between 448 iterations, which takes 41.8 s of CPU time, up to 1107 iterations taking 106.3 s.

There is also the related issue of information exchange between the SO and LA. Each iteration of Algorithm 1 requires the exchange of 48 numbers between the SO and

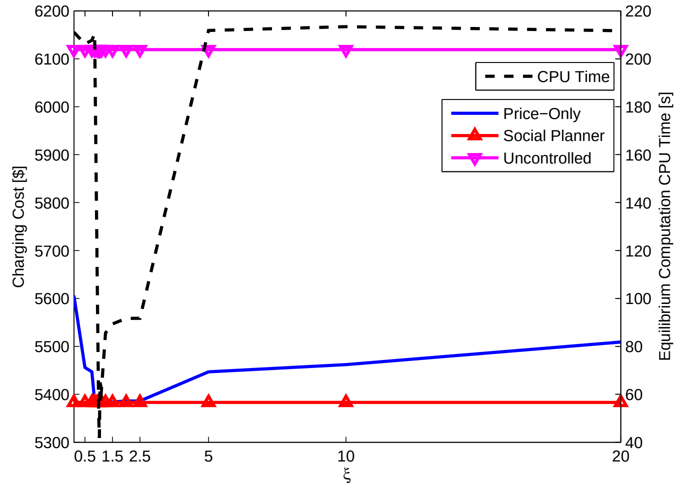

Fig. 3. PEV charging costs (solid line) and equilibrium computation time (dashed line) with price-only mechanism as a function of ξ.

LA when the price-only mechanism is used—the SO must communicate the updated hourly prices to the LA and the LA must send hourly charging loads back to the SO. By contrast, each iteration of Algorithm 1 requires the exchange of up to 96 numbers between the SO and LA with the price/quantity mechanism. This is because each iteration updates the price function by imposing a new price in each hour between two breakpoints (the price outside of these breakpoints remain the same as in the previous iteration, as illustrated in Fig. 1). Thus, the SO sends 24 hourly prices and 48 corresponding breakpoints to the LA, and the LA sends 24 hourly charging loads back. In the 25 iterations it takes for Algorithm 1 to converge with the price/quantity mechanism, 2400 numbers are exchanged. With the price-only mechanism, a minimum of 21504 number are exchanged (*i.e.*, with the best-case choice of $\xi = 1.025$).

C. Sensitivity of Optimal ξ to Problem Parameters

A further limitation of the price-only mechanism is that the optimal choice of ξ is highly sensitive to the underlying problem parameters. To demonstrate this, we examine three additional variations of the base case described in Section IV-A. The first assumes different costs for the three generators, given by:

$$F_1(q_1) = 110.85 + 5.36 \cdot q_1 + 0.0424 \cdot q_1^2,$$
$$F_2(q_2) = 80.87 + 10.72 \cdot q_2 + 0.0312 \cdot q_2^2,$$

and:

$$F_3(q_3) = 85.24 + 12.59 \cdot q_3 + 0.0137 \cdot q_3^2.$$

The second assumes the same generation costs as in the base case, but that generator 1's ramp-up and generator 2's ramp-down capabilities change to 30 and 20 MW, respectively. The third case assumes the same generation costs and ramping capabilities as in the base case, but that there are twice as many PEV arrivals before hour 12 and 70% fewer after hour 12, compared to the base case.

Table V shows the optimal choice of ξ, which gives the lowest PEV charging cost and equilibrium computation time, in the base and sensitivity cases. It also gives the range of ξ values for which Algorithm 1 converges in 2000 iterations or less. The table shows that the price-only mechanism does not converge in 2000 iterations in the last sensitivity case for any value of ξ tested.

TABLE V
OPTIMAL ξ IN DIFFERENT CASES

Case	Optimal ξ	Range of ξ for which Algorithm 1 Converges in 2000 Iterations
Base Case	1.025	0.95–2.5
Generation Cost	0.1	0.07–1.25
Ramp Limits	0.475	0.45–1.5
PEV Numbers	0.67	none

The discussion in Section IV-B shows that the price-only mechanism faces the added difficulty that its performance is highly sensitive to choosing a good regularization penalty parameter, ξ. Otherwise, equilibrium computation time and resulting costs can be significantly higher than the social optimum. The results in Table V further show that the optimal choice of ξ is very sensitive to the underlying problem parameters. This suggests that one may have to rely on real-time tuning to determine an optimal choice of ξ, which can be cumbersome and difficult. Moreover, this tuning process would have to be repeated as the generation mix, number of PEVs, driving patterns, and other system characteristics change over time.

V. MULTIPLE-LOAD AGGREGATOR CASE

In this case we assume that PEV charging is managed by multiple LAs.

A. Model Refinement

Before discussing the case study data and results, we first refine the model formulations and pricing schemes outlined in Section II to account for multiple LAs.

1) SO's Problem: The SO's problem is given by:

$$\min \sum_{t\in T}\left[\sum_{i\in I} F_i(q_{i,t}) + V\cdot\left(\sum_{a\in\mathcal{A}} D_{a,t} - \sigma_t\right)\right]; \qquad (42)$$

$$\text{s.t. } \sum_{i\in I} q_{i,t} = L_t + \sigma_t; \quad \forall\, t\in T; \qquad (43)$$

$$P_i^- \le q_{i,t} \le P_i^+; \quad \forall\, i\in I; t\in T; \qquad (44)$$

$$R_i^- \le q_{i,t} - q_{i,t-1} \le R_i^+; \quad \forall\, i\in I; t\in T; \qquad (45)$$

$$0 \le \sigma_t \le \sum_{a\in\mathcal{A}} D_{a,t}; \quad \forall\, t\in T. \qquad (46)$$

2) LAs' Problems: LA a's problem is formulated as:

$$\min \sum_{t\in T} C_{a,t}(b_{a,t}) + V\cdot \sum_{\gamma\in\Gamma_a} u_{a,\gamma}; \qquad (47)$$

$$\text{s.t. } b_{a,t} = \sum_{\gamma\in\Gamma_a} \delta_{a,\gamma,t}; \quad \forall\, t\in T; \qquad (48)$$

$$0 \le \delta_{a,\gamma,t} \le R\cdot N_{a,\gamma}\cdot \mathcal{I}_{a,\gamma,t}; \quad \forall\, \gamma\in\Gamma_a; t\in T; \qquad (49)$$

$$\sum_{t\in T}\delta_{a,\gamma,t} + u_{a,\gamma} = X_{a,\gamma}; \quad \forall\, \gamma\in\Gamma_a; \qquad (50)$$

$$u_{a,\gamma}\ge 0; \quad \forall\, \gamma\in\Gamma_a. \qquad (51)$$

3) Equilibrium Computation: We compute a Nash equilibrium between the SO and LAs using Algorithm 1, with several refinements that account for having multiple LAs. Specifically, Steps 4, 6, 7, 10, and 12 are updated to reflect there being multiple LAs. In Step 6 the optimal charging schedule is determined as $(b_a^k, \delta_a^k, u_a^k) \leftarrow \arg\min\ (47)$ s.t. $(48) - (51)$ and in Step 9 the optimal dispatch schedule is determined as $(q^k, \sigma^k) \leftarrow \arg\min\ (42)$ s.t. $(43) - (46)$.

4) Price Signals: We study the same two price-based signals as before. The marginal pricing scheme updates the pricing function after the kth iteration of Algorithm 1 as:

$$C_{a,t}(b_{a,t}) = \lambda_t^{k-1}\cdot b_{a,t} + \xi\cdot(b_{a,t} - b_{a,t}^{k-1})^2, \qquad (52)$$

for all a and t.

To define the pricing function with the price/quantity-based scheme, we define $\Lambda_{a,t}^k(b_{a,t})$ as the updated price function constructed in the kth iteration of Algorithm 1. As before, we initialize these function as $\Lambda_{a,t}^0(b_{a,t}) = \lambda_t^0, \forall\, a, t$. For $k\ge 1$ we define $\dot{b}_{a,t}^{k,0}, \dot{b}_{a,t}^{k,1}, \ldots, \dot{b}_{a,t}^{k,k}$, to be the set of breakpoints in $\Lambda_{a,t}^{k-1}(b_{a,t})$, where by convention we take $\dot{b}_{a,t}^{k,0} = 0$ and $\dot{b}_{a,t}^{k,k} = +\infty$. We then define:

$$D_{a,t}^{k,-} = \max_i \left\{\dot{b}_{a,t}^{k-1,i} : \Lambda_{a,t}^{k-1}(\dot{b}_{a,t}^{k-1,i}) < \lambda_t^k, \dot{b}_{a,t}^{k-1,i} < D_{a,t}\right\}, \qquad (53)$$

and:

$$D_{a,t}^{k,+} = \min_i \left\{\dot{b}_{a,t}^{k-1,i} : \lim_{\nu\to 0^+} \Lambda_{a,t}^{k-1}(\dot{b}_{a,t}^{k-1,i} + \nu) \ge \lambda_t^k\right\}. \qquad (54)$$

If no such points exists, we let $D_{a,t}^{k,-} = 0$ and $D_{a,t}^{k,+} = +\infty$, as appropriate. The new prices are then defined as:

$$\Lambda_{a,t}^k(b_{a,t}) = \begin{cases} \lambda_{a,t}^k, & \text{if } \beta_a^{k,-} < b_{a,t} \le \beta_a^{k,+}, \\ \Lambda_{a,t}^{k-1}(b_{a,t}), & \text{otherwise,} \end{cases} \qquad (55)$$

where:

$$\beta_a^{k,-} = D_{a,t} - \min\left\{\Delta, \frac{D_{a,t} - D_{a,t}^{k,-}}{2}\right\}, \qquad (56)$$

$$\beta_a^{k,+} = \max\left\{D_{a,t}, D_{a,t}^{k,+}\right\}. \qquad (57)$$

The total cost of energy levied on LA a in the kth iteration of Algorithm 1 is defined as:

$$C_{a,t}(b_{a,t}) = \int_0^{b_{a,t}} \Lambda_{a,t}^{k-1}(\psi)d\psi. \qquad (58)$$

5) Social Planner's Problem: As in the single-LA case, the social planner is assumed to control all SO and LA decisions simultaneously. Thus, the social planner's problem is given by:

$$\min \sum_{t \in T} \left[\sum_{i \in I} F_i(q_{i,t}) + V \cdot \left(\sum_{a \in \mathcal{A}} D_t - \sigma_t \right) \right] + \sum_{a \in \mathcal{A}, \gamma \in \Gamma_a} V \cdot u_{a,\gamma}; \quad (59)$$

$$\text{s.t. } (43)\text{–}(46) \quad (60)$$

$$(48)\text{–}(51); \quad \forall\, a \in \mathcal{A}; \quad (61)$$

$$b_{a,t} = D_{a,t}; \quad \forall\, a \in \mathcal{A}, t \in T. \quad (62)$$

B. Case Study Data

This case assumes the same underlying system, consisting of the three conventional generators and non-PEV loads as shown in Table I and Fig. 2. We assume that there are 26270 PEV arrivals over the course of the day and that the vehicles arrive at charging stations operated by three independent LAs. The first LA is assumed to serve exactly half of the PEVs in the base case. Thus, we assume that $N_{1,\gamma}$ and $X_{1,\gamma}$ are equal to exactly half of the values reported in Table II. The remaining PEVs are divided among the other two LAs based on the values reported in Tables VI and VII.

TABLE VI
PEV ARRIVAL AND DEPARTURE TIMES AND CHARGING ENERGY DEMANDS OF LOAD AGGREGATOR 2

γ	Hours Parked	$N_{2,\gamma}$	$X_{2,\gamma}$	γ	Hours Parked	$N_{2,\gamma}$	$X_{2,\gamma}$
1	7–12	50	0.4	24	10–16	100	0.78
2	7–13	100	0.9	25	10–19	300	2.4
3	7–15	100	0.8	26	10–20	200	1.75
4	7–16	300	2.4	27	10–21	50	0.4
5	7–17	200	1.6	28	11–17	50	0.4
6	7–18	100	0.8	29	11–18	50	0.4
7	7–19	50	0.4	30	11–19	100	0.8
8	8–13	100	0.8	31	12–16	150	1.2
9	8–14	100	1	32	12–17	250	2
10	8–16	300	2.75	33	12–18	200	1.6
11	8–17	600	4.8	34	12–19	100	0.82
12	8–18	400	3.45	35	12–20	50	0.4
13	8–19	200	1.6	36	13–18	150	1.2
14	8–20	50	0.6	37	13–19	100	0.8
15	9–14	200	1.6	38	13–20	100	0.8
16	9–15	50	0.4	39	13–21	50	0.4
17	9–16	200	1.6	40	14–19	300	2.5
18	9–17	900	7.9	41	14–20	100	0.9
19	9–18	500	4.3	42	14–22	200	1.85
20	9–19	100	0.8	43	14–23	100	0.8
21	9–20	100	0.8	44	15–22	100	0.8
22	9–21	50	0.45	45	15–23	400	3.7
23	10–15	100	0.84				

C. Case Study Results

Table VIII summarizes total system costs with multiple LAs, considering the same set of PEV charging cases as in the base case. As before, all of the PEV loads are satisfied, thus only generation costs are reported in the table (as there is zero curtailment cost). As expected, the social planner minimizes cost and the price/quantity mechanism yields the same cost-minimizing charging patterns. A value of $\xi = 3.5$ is optimal for the price-only mechanism, yielding the smallest charging cost that is comparable to the social planner's solution.

TABLE VII
PEV ARRIVAL AND DEPARTURE TIMES AND CHARGING ENERGY DEMANDS OF LOAD AGGREGATOR 3

γ	Hours Parked	$N_{3,\gamma}$	$X_{3,\gamma}$	γ	Hours Parked	$N_{3,\gamma}$	$X_{3,\gamma}$
1	1–7	200	1.55	17	3–11	100	0.78
2	1–8	350	2.82	18	3–14	100	0.78
3	1–9	700	5.5	19	3–15	60	0.6
4	1–10	300	2.3	20	3–16	150	1.2
5	1–11	100	0.88	21	3–17	50	0.38
6	2–8	100	0.78	22	15–20	50	0.39
7	2–9	200	1.56	23	15–21	150	1.2
8	2–10	400	3.6	24	15–22	100	0.78
9	2–11	200	1.56	25	15–23	100	0.78
10	2–12	50	0.4	26	16–20	30	0.24
11	2–14	80	0.65	27	16–22	200	1.56
12	2–15	180	1.45	28	16–23	500	3.9
13	2–16	280	2.2	29	16–24	100	0.78
14	2–17	120	0.95	30	17–22	100	0.74
15	2–18	60	0.5	31	17–23	260	2.1
16	3–10	200	1.56	32	17–24	400	3.7

TABLE VIII
CASES EXAMINED AND TOTAL COSTS WITH MULTIPLE LOAD AGGREGATORS

Case	PEV Charging	Total Cost [$]
1	None	68589
2	Social Planner	73850
3a	Price-Only ($\xi = 0.1$)	74187
3b	Price-Only ($\xi = 1$)	73881
3c	Price-Only ($\xi = 3.5$)*	73852
3d	Price-Only ($\xi = 20$)	73906
4	Price/Quantity	73850
5	Uncontrolled	74725

The performance of the price-only mechanisms is further illustrated in Fig. 4, which shows charging costs and equilibrium computation times as a function of ξ. Contrasting this with Fig. 3 shows that the range of ξ's that result in Algorithm 1 converging is markedly different in the multiple-LA case. Whereas values between 0.95 and 2.5 give a convergent solution in the single-LA case, values between 3 and 10 do so in the multiple-LA case. As before, the price/quantity mechanism takes considerably less computational effort—23 iterations, taking 7.3 s of CPU time and the exchange of 2208 numbers between the SO and LA—to find an equilibrium. The price-only mechanism takes a minimum of 91 iterations, 39.1 s of CPU time, and the exchange of 4368 numbers with the optimal choice of $\xi = 3.5$. If $\xi = 3$ is chosen instead, Algorithm 1 converges in 1090 iterations, taking 452.3 s of CPU and the exchange of 52320 numbers.

VI. RANDOM PEV DRIVING PATTERN CASE

We now consider a case in which there is a single LA but future PEV arrivals, departures, numbers, and charging demands are unknown when making current-hour charging and dispatch decisions.

A. Model Refinement

All of the optimization problems are formulated as two-stage stochastic programs, wherein the first stage is the first

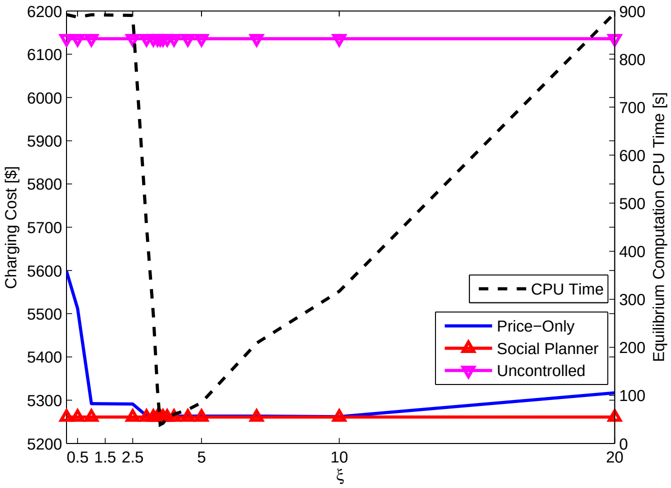

Fig. 4. PEV charging costs (solid line) and equilibrium computation time (dashed line) with price-only mechanism and multiple load aggregators as a function of ξ.

hour and the second stage the remaining hours of the optimization horizon. We assume that PEV arrivals are perfectly known in the first stage and scenarios represent uncertain PEV arrivals during the second stage. PEV uncertainty affects the SO problem since it may be optimal to adjust the current-hour dispatch so future charging demands can be met at minimum expected cost while respecting generator constraints. To the extent that the driving patterns allow, the SO may also want to intertemporally shift PEV charging to help manage this uncertainty. To achieve this, we allow the SO to provide the LA with scenario-dependent price signals.

1) SO's Problem: Since the SO provides the LA with scenario-dependent price signals, the PEV charging schedule is modeled as being scenario-dependent. The SO's stochastic economic dispatch problem is formulated as:

$$\min \sum_{t\in T; s\in S} \pi_s \cdot \left[\sum_{i\in I} F_i(q_{i,t,s}) + V\cdot(D_{t,s}-\sigma_{t,s})\right]; \quad (63)$$

$$\text{s.t.} \sum_{i\in I} q_{i,t,s} = L_t + \sigma_{t,s}; \qquad \forall\, t\in T; s\in S; \quad (64)$$

$$P_i^- \le q_{i,t,s} \le P_i^+; \quad \forall\, i\in I; t\in T; s\in S; \quad (65)$$

$$R_i^- \le q_{i,t,s} - q_{i,t-1,s} \le R_i^+; \quad \forall\, i\in I; t\in T; s\in S; \quad (66)$$

$$0 \le \sigma_{t,s} \le D_{t,s}; \quad \forall\, t\in T; s\in S; \quad (67)$$

$$q_{i,1,s} = q_{i,1,s'}; \quad \forall\, s,s'\in S; \quad (68)$$

$$\sigma_{1,s} = \sigma_{1,s'}; \quad \forall\, s,s'\in S. \quad (69)$$

This is nearly identical to the base-case formulation in Section II-A. The objective minimizes expected costs and the generation, charging limit, and ramping constraints are imposed in each scenario. Non-anticipativity constraints (68) and (69) encode the two-stage structure by forcing the first-stage decisions to be the same across all scenarios. The second-stage decisions can be scenario-dependent, however.

2) LA's Problem: The LA's problem is formulated as:

$$\min \sum_{t\in T; s\in S} \pi_s \cdot C_{t,s}(b_{t,s}) + V\cdot \sum_{\gamma\in\Gamma; s\in S} \pi_s \cdot u_{\gamma,s}; \quad (70)$$

$$\text{s.t. } b_{t,s} = \sum_{\gamma\in\Gamma} \delta_{\gamma,t,s}; \quad \forall\, t\in T; s\in S; \quad (71)$$

$$0 \le \delta_{\gamma,t,s} \le R\cdot N_{\gamma,s}\cdot \mathcal{I}_{s,\gamma,t}; \quad \forall\, \gamma\in\Gamma; t\in T; s\in S; \quad (72)$$

$$\sum_{t\in T} \delta_{\gamma,t,s} + u_{\gamma,s} = X_{\gamma,s}; \quad \forall\, \gamma\in\Gamma; s\in S; \quad (73)$$

$$u_{\gamma,s} \ge 0; \quad \forall\, \gamma\in\Gamma; s\in S; \quad (74)$$

$$b_{1,s} = b_{1,s'}; \quad \forall\, s,s'\in S; \quad (75)$$

$$\delta_{\gamma,1,s} = \delta_{\gamma,1,s'}; \quad \forall\, \gamma\in\Gamma; s,s'\in S. \quad (76)$$

The objective minimizes expected cost and the same constraints as in the base-case formulation are included but enforced scenario-wise. Non-anticipativity constraints are also included to force the first-stage decisions to be the same among all of the scenarios.

3) Equilibrium Computation: A Nash equilibrium is computed using a rolling-horizon iterative technique. The gist of the algorithm is that we roll through each hour of the entire optimization horizon, computing an equilibrium between the SO and LA in that hour. This equilibrium accounts for possible future PEV arrivals and charging demands. Algorithm 2 summarizes the steps involved in determining hour-r Nash equilibrium generator dispatch and PEV charging decisions. Algorithms 1 and 2 are nearly identical, except that the optimizations in Steps 6 and 9 are only carried out for hours $r,\ldots,|T|$. Once Algorithm 2 terminates, only hour-r generator dispatch and PEV charging decisions are fixed based on the final solution (*i.e.*, $q^k,\sigma^k,b^k,\delta^k$, and u^k). We then roll forward, setting $r \leftarrow r+1$ and repeat Algorithm 2 with updated PEV data and forecasts to determine the next hour's equilibrium generator dispatch and PEV charging decisions, again taking into account all future scenarios over the full optimization horizon, T.

Algorithm 2 Hour-r Equilibrium Computation with Random PEV Driving

1: $k \leftarrow 0$ ▷ Set iteration count to zero
2: **repeat**
3: **if** $k=0$ **then**
4: $D_{t,s} \leftarrow 0$ for all $t=r,\ldots,|T|; s\in S$
5: **else**
6: $(b^k,\delta^k,u^k) \leftarrow \arg\min$ (70) s.t. (71) – (76)
7: $D_{t,s} \leftarrow b^k_{t,s}$ for all $t=r,\ldots,|T|; s\in S$
8: **end if**
9: $(q^k,\sigma^k) \leftarrow \arg\min$ (63) s.t. (64) – (69)
10: Update $C_{t,s}(\cdot)$ based on $\{q^\kappa,\sigma^\kappa\}_{\kappa=0}^k$
11: $k \leftarrow k+1$
12: **until** $k \ge \bar{k}$ **or** $\left[(b^{k-2},\delta^{k-2},u^{k-2}) = (b^{k-1},\delta^{k-1},u^{k-1})\right.$ **and** $\left.(q^{k-2},\sigma^{k-2}) = (q^{k-1},\sigma^{k-1})\right]$

4) Price Signals: We examine the same two price-based signals that are considered in the base case. The only modification to the price updating procedures detailed in Section II-D is that the Lagrange multipliers found in solving the SO's problem are scenario-dependent, since load-balance constraints (64) are enforced for each scenario. We thus update the price functions, $C_{t,s}(\cdot)$, using the corresponding scenario's Lagrange multiplier.

5) Social Planner's Problem: The social planner's problem is formulated as:

$$\min \sum_{t\in T; s\in S} \pi_s \cdot \left[\sum_{i\in I} F_i(q_{i,t,s}) + V\cdot(D_{t,s}-\sigma_{t,s})\right] \quad (77)$$

$$+V\cdot \sum_{\gamma\in\Gamma; s\in S} \pi_s \cdot u_{\gamma,s};$$

$$\text{s.t. (64)–(69) and (71)–(76);} \quad (78)$$

$$b_{t,s} = D_{t,s}; \quad \forall\ t\in T; s\in S. \quad (79)$$

The social planner's problem is solved using a similar iterative rolling-horizon procedure to that used in Algorithm 2. Specifically, we start with $r \leftarrow 1$ and solve the social planner's problem using the two-stage scenario tree to determine hour-1 dispatch and PEV charging, which are fixed. We then roll forward and let $r \leftarrow r+1$, update the scenario tree, and resolve to determine the next hour's decisions. This is repeated iteratively until we determine optimal decisions over the entire planning horizon, T.

B. Case Study Data

This case assumes the same underlying generation mix and non-PEV loads as in the base case. We further assume that the *actual* PEV arrival and departure times, vehicle numbers, and charging demands are the same as in the base case (*i.e.*, the values given in Table II). We generate the scenario tree for the PEV charging problem by assuming that the LA relies on forecasts of future PEV arrivals, departures, and demands, which are denoted as $\tilde{\tau}_{\gamma,s}$, $\tilde{\tau}'_{\gamma,s}$, $\tilde{N}_{\gamma,s}$, and $\tilde{X}_{\gamma,s}$. We assume the arrival and departure time forecasts have the following unbiased distribution:

$$(\tilde{\tau}_{\gamma,s}, \tilde{\tau}'_{\gamma,s}) = \begin{cases} (\tau_\gamma+1, \tau'_\gamma), & \text{with probability } 0.2,\\ (\tau_\gamma, \tau'_\gamma), & \text{with probability } 0.6,\\ (\tau_\gamma, \tau'_\gamma-1), & \text{with probability } 0.2; \end{cases} \quad (80)$$

where τ_γ and τ'_γ are the actual arrival and departure times, given in Table II. The distribution of the number of PEVs and charging demand forecasts are given by:

$$(\tilde{N}_{\gamma,s}, \tilde{X}_{\gamma,s}) = \begin{cases} 0.85\cdot(N_\gamma, X_\gamma), & \text{with probability } 0.2,\\ (N_\gamma, X_\gamma), & \text{with probability } 0.6,\\ 1.15\cdot(N_\gamma, X_\gamma), & \text{with probability } 0.2. \end{cases} \quad (81)$$

These distributions are assumed to be jointly independent. We randomly generate 100 sample paths of $\tilde{\tau}_{\gamma,s}$, $\tilde{\tau}'_{\gamma,s}$, $\tilde{N}_{\gamma,s}$, and $\tilde{X}_{\gamma,s}$ to populate the scenario tree.

C. Case Study Results

Table IX summarizes total system costs with random PEV driving patterns in the different charging control cases. The values shown are *realized* costs, meaning that they are the costs incurred at the end of the day after vehicles arrive according to the data in Table II. Since all PEV loads are satisfied, only generation costs are reported in the table. Although the social planner still yields the most efficient charging schedule, there is a slight cost increase compared to the base case. This is because the uncertainty in future PEV charging demands results in different generator dispatch decisions, giving the cost increase. The cost difference between these two cases is the value of perfect PEV driving pattern information. Both the price-only and price/quantity mechanisms give costs that are comparable to the social planner's solution, if the correct value of ξ is chosen. Indeed, the optimal choice of $\xi = 0.7$ gives PEV charging costs that are slightly lower than the price/quantity mechanism does. As noted before, this is attributable to our choice of $\Delta = 0.5$ in defining $\beta^{k,-}$. If a value of $\Delta \le 0.325$ is used, the price/quantity mechanism outperforms the price-only one on the basis of cost.

TABLE IX
CASES EXAMINED AND TOTAL REALIZED COSTS WITH RANDOM PEV DRIVING PATTERNS

Case	PEV Charging	Total Cost [$]
1	None	68589
2	Social Planner	74015
3a	Price-Only ($\xi = 0.1$)	74105
3b	Price-Only ($\xi = 0.7$)*	74016
3b	Price-Only ($\xi = 1$)	74018
3c	Price-Only ($\xi = 20$)	74045
4	Price/Quantity	74019
5	Uncontrolled	74746

As before, the price-only mechanism takes considerably more effort to compute a Nash equilibrium. This is shown in Table X, which summarizes the computational performance of the two pricing schemes. The first column reports the range of iterations required to compute an equilibrium in each hour. This range reflects Algorithm 2 computing an equilibrium one hour at a time in a rolling fashion. Thus, it may take more iterations to find an equilibrium in one hour than another. The second column reports the number of hours of the day for which Algorithm 2 does not converge within 2000 iterations. The third column reports the total CPU time required to simulate the one-day period studied while the fourth shows the total amount of information exchange between the SO and LA to compute the one-day equilibrium. There is considerably more information exchanged in the random driving pattern case compared to the others due to the scenario-dependent pricing assumption and the rolling equilibrium computation. Specifically, each iteration of Algorithm 2 requires the exchange of $1+(|T|-r-1)\cdot|S|$ scenario-dependent hourly prices from the SO to the LA and the same number of charging loads from the LA to the SO when computing an hour-r equilibrium with the price-only mechanism. The price/quantity mechanism requires the SO to send the LA $3\cdot(1+(|T|-r-1)\cdot|S|)$ numbers in each iteration, corresponding to the new prices and breakpoints in each pricing function.

VII. CONCLUSIONS

We study decentralized price-based PEV charging control. Although centralized control by the SO or a social planner

TABLE X
COMPUTATIONAL PERFORMANCE OF PRICE-BASED CONTROL SCHEMES WITH RANDOM PEV DRIVING PATTERNS

Case	Iterations	Non-Conv. Hours	CPU Time [s]	Numbers Exchanged [m]
Price/Quantity				
$\Delta = 0.5$	10–25	0	436	0.3
$\Delta = 0.325$	14–45	0	614	0.4
Price-Only				
$\xi = 0.1$	163–2000	20	32382	387.9
$\xi = 0.5$	207–2000	4	13032	127.5
$\xi = 0.65$	188–2000	1	11029	88.6
$\xi = 0.7^*$	101–498	0	7532	66.7
$\xi = 0.75$	173–786	0	8934	73.5
$\xi = 1$	595–1267	0	20146	150.4
$\xi = 2.5$	310–2000	21	38490	405.4
$\xi = 5$	258–2000	15	33253	290.9
$\xi = 10$	175–2000	3	29488	87.6
$\xi = 20$	104–2000	17	29898	331.9

maximizes coordination benefits, it may have implementation issues. We propose a decentralized pricing scheme that conveys both price and quantity information gathered by the SO as it and the LA iteratively reoptimize system dispatch and PEV charging. We demonstrate that this method has theoretically attractive properties. Furthermore, our method does not rely on any regularization terms to guarantee convergence.

We use numerical case studies to demonstrate the benefits of our proposed pricing scheme, showing that it can find a near-socially optimal equilibrium. While the price-only mechanism can as well, its performance is highly sensitive to the choice of the regularization penalty term, ξ. If ξ is not chosen properly, PEV charging costs in the price-only case can be considerably higher than with the price/quantity mechanism. Regardless of the choice of ξ, the price-only mechanism requires considerably more computational effort and data exchange between the SO and LA to find an equilibrium. We also examine numerical case studies, in which the single-LA and deterministic driving pattern assumptions are relaxed. Our proposed method performs well in these cases as well.

Our pricing scheme assumes that the LA is minimizing charging costs on behalf of PEV owners. If charging scheduling is carried out by another agent with a different objective (*e.g.*, a profit-maximizing utility), our pricing scheme may not have the beneficial properties that we demonstrate. In such a case, one would have to cast this as a broader mechanism design problem, which aligns the LA's incentives with those of the social planner.

APPENDIX
PROPERTIES OF THE MARGINAL COST OF PEV CHARGING LOAD IN THE SO'S PROBLEM

Lemma 3: Suppose that Assumption 1 is true. Then:

$$\frac{\partial}{\partial b_t} V^*_{SO}(b_t) = \lambda_t(b). \tag{82}$$

Proof: Let $(q^*, \sigma^*) \in \arg\min V^*_{SO}(b)$ and $\mu_t^{5,+}$ denote the Lagrange multiplier associated with the upper-bound in hour-t constraint (5). We now consider two possible cases for the values that this Lagrange multiplier can take. We show that in both cases we have $\frac{\partial}{\partial b_t} V^*_{SO}(b_t) = \lambda_t(b)$.

1) $\mu_t^{5,+} = 0$: This indicates that $\sigma_t^* < b_t$. The Karush-Kuhn-Tucker (KKT) stationarity condition of the SO's problem associated with the variable σ_t is:

$$\lambda_t(b) = V + \mu_t^{5,-}, \tag{83}$$

where $\mu_t^{5,-}$ is the Lagrange multiplier associated with the lower-bounds of constraint (5). Since there is sufficient generating capacity to provide some PEV charging, and since V is assumed to be higher than the marginal cost of generation, we must have $\sigma_t^* > 0$ and $\mu_t^{5,-} = 0$, which implies that $\lambda_t(b) = V$.

We can also show that in this case $\frac{\partial}{\partial b_t} V^*_{SO}(b_t) = V$. To see this, define $\tilde{b}^\epsilon$ as:

$$\tilde{b}^\epsilon_r = \begin{cases} b_r + \epsilon, & \text{if } r = t, \\ b_r, & \text{otherwise,} \end{cases} \tag{84}$$

where ϵ satisfies $\sigma_t^* + \epsilon \leq b_t$. Let $(\tilde{q}, \tilde{\sigma}) \in \arg\min V^*_{SO}(\tilde{b}^\epsilon)$. We now argue that $\sigma^* = \tilde{\sigma}$. To see this, note that (q^*, σ^*) is feasible in the SO's problem when $D = \tilde{b}^\epsilon$. Moreover, we must have $\tilde{\sigma} \geq \sigma^*$, otherwise the SO can feasibly increase $\tilde{\sigma}$, giving a lower cost (due to V being assumed higher than the marginal cost of generation). Furthermore, we cannot have $\tilde{\sigma} > \sigma^*$. To see this, note that if so, then constraints (2) give:

$$\sum_{i \in I} \tilde{q}_{i,t} > \sum_{i \in I} q^*_{i,t}. \tag{85}$$

This means that $\tilde{q}$ must not be feasible when $D = b$, since q^* would then not be optimal (as the unserved load cost associated with σ^* is larger than that with $\tilde{\sigma}$). However, the technical generator constraints are identical in the two cases (*i.e.*, with $D = b$ and $D = \tilde{b}^\epsilon$), giving a contradiction. Hence, we must have $\sigma^* = \tilde{\sigma}$.

We finally note that:

$$\begin{aligned} \frac{\partial}{\partial b_t} V^*_{SO}(b_t) &= \lim_{\epsilon \to 0} \frac{V^*_{SO}(\tilde{b}^\epsilon) - V^*_{SO}(b)}{\epsilon} \\ &= V \\ &= \lambda_t(b). \end{aligned} \tag{86}$$

2) $\mu_t^{5,+} \neq 0$: In this case we have $\sigma_t^* = b_t$. Thus, the hour-t constraint (2) can be rewritten as:

$$\sum_{i \in I} q_{i,t} = L_t + b_t. \tag{87}$$

Since b_t is the parameter on the right-hand side of (87), we have that:

$$\frac{\partial}{\partial b_t} V^*_{SO}(b_t) = \lambda_t(b). \tag{88}$$

■

Lemma 4: Suppose that Assumption 1 is true and that the generation cost functions, $F_i(\cdot)$, are convex. Then $\lambda_t(b)$ is non-decreasing in b.

Proof: We define $\lambda_t(b)$ as the Lagrange multiplier of constraint (2) when $D = b$ in the SO's problem. From constraint (2) we have that:

$$\frac{\partial}{\partial \sigma_t}\lambda_t(b) = \frac{\partial}{\partial q_{i,t}}\lambda_t(b). \tag{89}$$

Note that if, for a particular t, at least one of the $q_{i,t}$ is not bounded by its generating capacity or ramping limit, the system has sufficient capacity to satisfy all of the hour-t PEV demand. Thus, we must have $\sigma_t = b_t$, otherwise the SO could feasibly serve more load and incur less cost, due to the assumed high value of lost load. If there is an unconstrained $q_{i,t}$, the KKT conditions of the SO's problem give:

$$\lambda_t(b) = \frac{\partial}{\partial q_{i,t}} F_i(q_{i,t}), \tag{90}$$

where i is the index of an unconstrained generator. We then have:

$$\frac{\partial}{\partial b_t}\lambda_t(b) = \frac{\partial^2}{\partial q_{i,t}^2} F_i(q_{i,t}) \geq 0. \tag{91}$$

If, conversely, all of the $q_{i,t}$ are capacity- or ramp-constrained for a particular t, this means there is insufficient generation capacity to satisfy all of the hour-t PEV demand. Thus, we must have $\delta_t < b_t$. The KKT conditions then give $\lambda_t(b) = V > 0$ and by assumption we know that:

$$\lambda_t(b) > \frac{\partial}{\partial q_{i,t}} F_i(q_{i,t}), \tag{92}$$

for all i. Thus, we have that:

$$\frac{\partial}{\partial b_t}\lambda_t(b) \geq 0, \tag{93}$$

indicating that $\lambda_t(b)$ is non-decreasing in b. ■

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REFERENCES

[1] P. Mohseni and R. G. Stevie, "Electric vehicles: Holy grail or Fool's gold," in *Power & Energy Society General Meeting, 2009*. Calgary, AB: Institute of Electrical and Electronics Engineers, 26-30 July 2009, pp. 1–5.

[2] P. Denholm and W. Short, "An Evaluation of Utility System Impacts and Benefits of Optimally Dispatched Plug-In Hybrid Electric Vehicles," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-620-40293, October 2006.

[3] M. Kintner-Meyer, K. Schneider, and R. Pratt, "Impacts Assessment of Plug-in Hybrid Vehicles on Electric Utilities and Regional US Power Grids: Part 1: Technical Analysis," *Journal of EUEC*, vol. 1, 2007.

[4] L. Wang, A. Lin, and Y. Chen, "Potential impacts of recharging plug-in hybrid electric vehicles on locational marginal prices," *Naval Research Logistics*, vol. 57, December 2010.

[5] K. Clement-Nyns, E. Haesen, and J. Driesen, "The Impact of Charging Plug-In Hybrid Electric Vehicles on a Residential Distribution Grid," *IEEE Transactions on Power Systems*, vol. 25, pp. 371–380, February 2010.

[6] Q. Gong, S. Midlam-Mohler, V. Marano, and G. Rizzoni, "PEV Charging Impact on Residential Distribution Transformer Life," in *2011 IEEE Energytech*. Cleveland, OH, United States: Institute of Electrical and Electronics Engineers, 25-26 May 2011.

[7] R. Sioshansi and P. Denholm, "The value of plug-in hybrid electric vehicles as grid resources," *The Energy Journal*, vol. 31, pp. 1–23, 2010.

[8] R. Sioshansi, R. Fagiani, and V. Marano, "Cost and emissions impacts of plug-in hybrid vehicles on the Ohio power system," *Energy Policy*, vol. 38, November 2010.

[9] R. Sioshansi and J. Miller, "Plug-in hybrid electric vehicles can be clean and economical in dirty power systems," *Energy Policy*, vol. 39, pp. 6151–6161, October 2011.

[10] R. Sioshansi, "Modeling the Impacts of Electricity Tariffs on Plug-in Hybrid Electric Vehicle Charging, Costs, and Emissions," *Operations Research*, vol. 60, pp. 506–516, May-June 2012.

[11] D. Wu, D. C. Aliprantis, and L. Ying, "Load Scheduling and Dispatch for Aggregators of Plug-In Electric Vehicles," *IEEE Transactions on Smart Grid*, vol. 3, pp. 368–376, March 2012.

[12] E. Sortomme and M. A. El-Sharkawi, "Optimal Charging Strategies for Unidirectional Vehicle-to-Grid," *IEEE Transactions on Smart Grid*, vol. 2, pp. 131–138, March 2011.

[13] ——, "Optimal Scheduling of Vehicle-to-Grid Energy and Ancillary Services," *IEEE Transactions on Smart Grid*, vol. 3, pp. 351–359, March 2012.

[14] P. Richardson, D. Flynn, and A. Keane, "Optimal Charging of Electric Vehicles in Low-Voltage Distribution Systems," *IEEE Transactions on Power Systems*, vol. 27, pp. 268–279, February 2012.

[15] ——, "Local Versus Centralized Charging Strategies for Electric Vehicles in Low Voltage Distribution Systems," *IEEE Transactions on Smart Grid*, vol. 3, pp. 1020–1028, June 2012.

[16] K. Parks, P. Denholm, and T. Markel, "Costs and emissions associated with plug-in hybrid electric vehicle charging in the xcel energy colorado service territory," National Renewable Energy Laboratory, Tech. Rep. NREL/TP-640-41410, May 2007.

[17] S. Bashash and H. K. Fathy, "Transport-Based Load Modeling and Sliding Mode Control of Plug-In Electric Vehicles for Robust Renewable Power Tracking," *IEEE Transactions on Smart Grid*, vol. 3, pp. 526–534, March 2012.

[18] D. M. Lemoine, D. M. Kammen, and A. E. Farrell, "An innovation and policy agenda for commercially competitive plug-in hybrid electric vehicles," *Environmental Research Letters*, vol. 3, pp. 1–10, January-March 2008.

[19] Z. Ma, D. Callaway, and I. Hiskens, "Optimal Charging Control for Plug-In Electric Vehicles," in *Control and Optimization Methods for Electric Smart Grids*, ser. Power Electronics and Power Systems, A. Chakrabortty and M. D. Ilić, Eds. Springer, 2012, vol. 3, pp. 259–273.

[20] L. Gan, U. Topcu, and S. H. Low, "Optimal Decentralized Protocol for Electric Vehicle Charging," *IEEE Transactions on Power Systems*, vol. 28, pp. 940–951, May 2013.

[21] C. Daskalakis, P. W. Goldberg, and C. H. Papadimitriou, "The Complexity of Computing a Nash Equilibrium," *SIAM Journal on Computing*, vol. 39, pp. 195–259, 2009.

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