Equilibria in Electricity and Natural Gas Markets with Strategic Offers and Bids

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***Abstract*—We study market equilibria that are achieved by strategic firms that participate in electricity and natural gas markets. Strategic firms submit their offers and bids to both markets with the aim of maximizing profit or utility and we consider firms that can include a combination of electricity and natural gas supply and demand. The strategic actions of these firms are represented by upper-level problems that are optimized subject to shared lower-level problems that represent the clearing of electricity and natural gas markets. This market structure and our modeling approach yields a multiple-leader/two-follower complementarity problem. We develop a modeling approach that can find equilibria with different characteristics, *e.g.*, maximized social welfare, producer profits, or consumer welfare. We demonstrate numerically that producers aim typically to increase market prices while consumers seek to decrease them.**

***Index Terms*—Power system market, natural gas market, strategic offering, strategic bidding, complementarity modeling**

Nomenclature

Indices, Sets, and Functions

$\mathbb{C}(m)$	set of natural gas compressors that are connected to node m
d	index of electricity demands in set, Λ^E
e	index of natural gas demands in set, Λ^G
$\mathbb{E}(i)$	set of buses that are connected directly to bus i
$\mathbb{G}(m)$	set of nodes that are connected directly to node m
i, j	indices of electric buses in set, $\mathbb{B}$
k	index of natural gas compressors in set, $\mathbb{C}$
l	index of firms in set, $\mathbb{L}$
m, n	indices of natural gas nodes in set, $\mathbb{N}$
REF	reference bus
v	index of generating units in set, Ω^E
w	index of natural gas sources in set, Ψ^S
Θ_i^D	set of electricity demands that are connected to bus i
Θ_i^G	set of units that are connected to bus i
Λ_l^{EL}	set of strategic electricity demands of firm l
Λ^{EO}	set of non-strategic electricity demands
Λ_l^{GL}	set of strategic natural gas demands of firm l
Λ^{GO}	set of non-strategic natural gas demands
Ψ_m^L	set of natural gas demands that are connected to node m
Ψ_m^G	set of natural-gas-fired units that are connected to node m
Ψ_m^S	set of natural gas sources that are connected to node m
Ω_l^G	set of natural-gas-fired units that are owned by firm l
Ω_l^R	set of other generating units that are owned by firm l
Ω_l^S	set of natural gas sources that are owned by firm l

Parameters and Constants

$b_{i,j}$	susceptance of the line connecting buses i and j (p.u.)
C_d^{EL}	marginal utility of electricity demand d (\$/p.u.)
C_e^{GL}	marginal utility of natural gas demand e (\$/Mm^3)
C_v^G	marginal production cost of other generating unit v (\$/p.u.)
C_v^{O}	non-fuel operating cost of natural-gas-fired unit v (\$/p.u.)
C_w^S	marginal production cost of natural gas source w (\$/Mm^3)
$F_k^{C,\max}$	natural-gas-transportation limit of compressor k (Mm^3/h)
$F_e^{L,\max}$	quantity of natural gas demand e (Mm^3/h)
$F_w^{S,\max}$	capacity of natural gas source w (Mm^3/h)
$F_v^{G,\max}$	maximum fuel available to natural-gas-fired unit v (Mm^3/h)
$P_v^{G,\max}$	capacity of generating unit v (p.u.)
$P_{i,j}^{\max}$	capacity of the line connecting buses i and j (p.u.)
$P_d^{L,\max}$	quantity of electricity demand d (p.u.)
$W_{m,n}$	Weymouth constant of the pipeline connecting nodes m and n (Mm^3/h/bar)
η_v	heat rate of natural-gas-fired unit v (Mm^3/h/p.u.)
θ_k	conversion efficiency of natural gas compressor k (p.u.)
$\Pi_m^{\max}$	maximum squared natural gas pressure at node m (bar^2)
$\Pi_m^{\min}$	minimum squared natural gas pressure at node m (bar^2)
$\rho_{C,k}^{\max}$	maximum squared compression ratio of compressor k (p.u.)
$\rho_{C,k}^{\min}$	minimum squared compression ratio of compressor k (p.u.)

Variables of Upper-Level Problem

α_v	offer of generating unit v (\$/p.u.)
β_w	offer of natural gas source w (\$/Mm^3)

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γ_v^G bid of natural-gas-fired unit v in natural gas market (\$/Mm^3)
ε_e bid of natural gas demand e (\$/Mm^3)
ς_d bid of electric demand d (\$/p.u.)

Variables of Lower-Level Electricity-Market Problem

P_v^G active power output of generating unit v (p.u.)
P_d^L amount of electric demand d served (p.u.)
δ_i bus-i phase angle (rad)

Variables of Lower-Level Natural-Gas-Market Problem

$F_{m,n}$ natural gas flow through the pipeline connecting nodes m and n (Mm^3/h)
F_k^C natural gas flow through compressor k (Mm^3/h)
F_v^G fuel that is consumed by natural-gas-fired unit v (Mm^3/h)
F_e^L amount of natural gas demand e that is served (Mm^3/h)
F_w^S natural gas that is supplied by source w (Mm^3/h)
Π_m squared natural gas pressure at node m (bar^2)
Π_k^{in} squared inlet pressure of compressor k (bar^2)
Π_k^{out} squared outlet pressure of compressor k (bar^2)

I. Introduction

CURRENT practice sees many electricity and natural gas markets being cleared independently of one another. However, the two markets are coupled, insomuch as many electric power systems rely on an increasing amount of natural-gas-fired generation [1]–[4]. Thus, the independent clearing of these two markets may be inefficient [5], [6]. Both markets see some exercise of market power, which results in prices being manipulated. In the case of suppliers, output is restricted to increase prices [7], [8], whereas strategic consumers aim to decrease prices [9]–[11]. The case of an integrated strategic firm that owns electricity and natural gas supplies or demands raises the potential for the simultaneous exercise of market power on one or both sides of both markets. As such, it is beneficial to have a modeling framework that can capture such strategic interdependencies.

The technical literature provides a number of approaches to model interactions between electricity and natural gas markets. Diagoupis *et al*. [12] quantify the impact of failures in the natural gas system on the electricity market. This impact can be mitigated by using natural-gas-storage facilities. Ordoudis *et al*. [13] assess the value of co-ordinating electricity and natural gas markets by comparing the results that are obtained from independent and integrated market-clearing models. Chen *et al*. [14] develop a coupled market-clearing model for electricity and natural gas that considers the pricing of reserved natural-gas-supply capacity. Wang *et al*. [15] propose a best-response decomposition algorithm to identify an equilibrium between electricity and natural gas markets with bilateral energy trading.

Other works focus on methods to compute equilibria from the perspective of market participants. Spiecker [16] analyzes the exercise of market power by strategic natural gas producers and its impact on the electricity market. Gil *et al*. [17] study the co-ordination of electricity and natural gas markets, and shows that co-ordination can increase the profits of participants in the two markets. Khazeni *et al*. [18] develop an equilibrium model for strategic energy retailers in electricity and natural gas markets, while Wang *et al*. [8] develop an equilibrium model to determine strategic offering behavior in an integrated electricity and natural gas market. Ji and Huang [19] propose a bi-level model to maximize the profits of integrated firms participating in electricity and natural gas markets.

Despite this large body of work, we believe that there are some important gaps in the existing literature, which our work seeks to fill. First, the existing literature does not have works that consider firms that determine production and consumption decisions strategically in electricity and natural gas markets. Wang *et al*. [8] and Ji and Huang [19] investigate the exercise of market power by producers, while Khazeni *et al*. [18] consider strategic behavior on the part of consumers. Second, many existing works do not consider network constraints in the electricity and natural gas systems. Such constraints can yield important insights into the exercise of market power, as agents may have locational market power as a result of network congestion. Finally, many works rely on heuristics to find market equilibria [8], [18], [19]. Such techniques can be sensitive to the point that is used to initialize the heuristic algorithm and cannot find reliably equilibria with different properties (*i.e*., maximized social welfare, producer profits, or consumer welfare).

Given these gaps, this work proposes a multiple-leader/two-follower structure to modeling strategic behavior in coupled electricity and natural gas markets. This structure has multiple strategic firms determine simultaneously supply offers and demand bids into the two markets (depending on the types of assets that they own) in the upper level. The lower level represents the simultaneous clearing of the two markets, which depends on the offers and bids. Our work makes the following three main contributions to the extant literature.

1) Our model structure is unique and general, as it allows us to capture integrated firms that participate on the demand or supply sides of electricity or natural gas markets.
2) We extend the solution technique of Ruiz *et al*. [20] to capture the physical characteristics of electricity and natural gas systems. Doing so allows us to identify efficiently a range of market equilibria with different characteristics (*e.g*., most competitive, oligopolistic, or least competitive on the supply or demand side).
3) Computational results from two numerical examples are used to provide insights on the types of market equilibria that are achieved and the impacts of market and network structure therein.

The remainder of this paper is organized as follows. Section II details our market model and approach to finding market equilibria. Sections III and IV present and analyze, respectively, an illustrative example and two case studies. Section V concludes.

II. Market-Equilibrium Model

Our approach to modeling market equilibria employs a bi-level structure, which is illustrated in Fig. 1. The upper level consists of a set of firms, each of which owns some combination of electricity and natural gas supplies and demands. Each firm is strategic, insomuch as it can optimize its supply offers and demand bids to manipulate the market outcome, with the aim of maximizing its utility. The lower level consists of two interrelated markets—one for electricity and the other for natural gas—that clear independently of one another. The two markets have complete information interchange. The structure that we assume, whereby the two markets clear independently of one another, reflects well the real-world operation of electricity and natural gas markets. The interrelationship between the two markets is that natural gas is an input fuel for some generation units (*i.e.*, we model units that are and are not natural-gas-fired). Moreover, some firms participate directly in both markets.

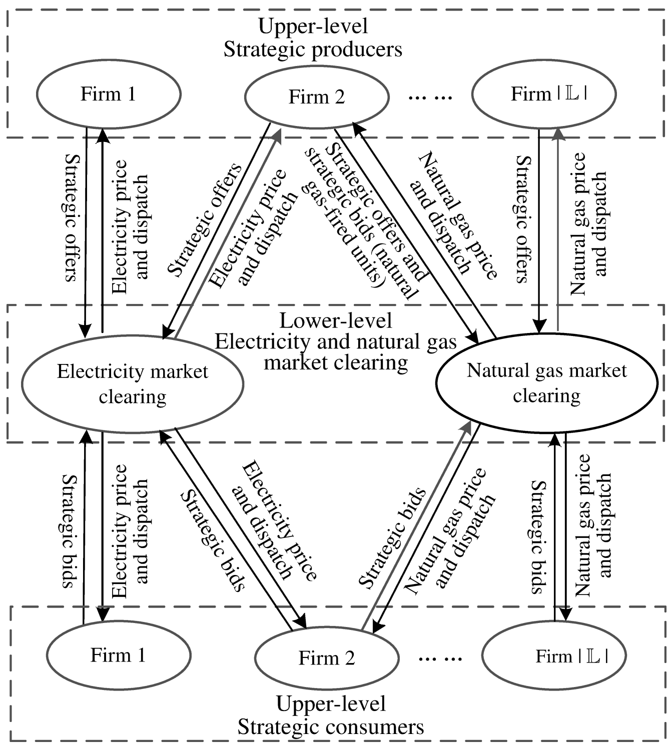

Fig. 1. Assumed structure of the multiple-leader/two-follower model.

We make seven key assumptions in our model. First, we consider a single-hour operating period for the two markets. This is to reduce the model size—multiple operating periods can be represented with increased computational complexity. Second, we employ a linearized power flow model for the electricity-market model, which is consistent with current practice in wholesale electricity markets. The natural gas system is represented using a second order cone (SOC)-based flow model. A linear natural-gas-flow model can be used, however, as an approximation that requires refinement to achieve accuracy [15]. Third, we assume that all electricity and natural gas supplies are strategic, whereas there are some electricity and natural gas demands that are not (*i.e.*, demands that do not correspond to the strategic firms). Fourth, we assume that each strategic supplier and demand has a single offer/bid price. The formulation can be extended easily to allow for multi-block offers and bids. Fifth, all natural-gas-fired generators participate as strategic suppliers in the electricity market and as strategic consumers in the natural gas market. Sixth, we assume no cross-price elasticity of demands between the two markets, meaning that there are no fuel-substitution options. Finally, we assume simultaneous clearing of the two markets. Sequential clearing of the two markets can result in efficiency losses compared to this assumption.

We proceed in this section by formulating and giving optimality conditions for the lower-level problems. Then, we formulate each firm's upper-level problem as a bi-level model and give an equivalent single-level formulation. Finally, we discuss our approach to computing Nash equilibria with different characteristics.

A. Lower-Level Models

1) Electricity-Market Model: The electricity market-clearing model is formulated as:

$$\max_{\Xi_{\mathrm{E}}^{\mathrm{P}}} \sum_{l\in\mathbb{L},d\in\Lambda_l^{EL}} \varsigma_d P_d^L + \sum_{d\in\Lambda^{EO}} C_d^{EL} P_d^L - \sum_{v\in\Omega^E} \alpha_v P_v^G \quad (1)$$

$$\text{s.t.} \sum_{d\in\Theta_i^D} P_d^L + \sum_{j\in\mathbb{E}(i)} b_{i,j}\cdot(\delta_i-\delta_j) = \sum_{v\in\Theta_i^G} P_v^G; \quad (2)$$

$$\forall i\in\mathbb{B} \quad (\lambda_i)$$

$$0\le P_d^L \le P_d^{L,\max}; \forall d\in\Lambda^E \quad (\rho_{1,d}^{\min},\rho_{1,d}^{\max}) \quad (3)$$

$$b_{i,j}\cdot(\delta_i-\delta_j)\le P_{i,j}^{\max}; \forall i\in\mathbb{B}, j\in\mathbb{E}(i) \quad (\rho_{2,i,j}^{\max}) \quad (4)$$

$$0\le P_v^G\le P_v^{G,\max}; \forall v\in\Omega^E \quad (\rho_{3,v}^{\min},\rho_{3,v}^{\max}) \quad (5)$$

$$\theta_{REF}=0; \quad (\rho_4) \quad (6)$$

where the dual variable that is associated with each constraint is in parentheses to its right. The primal-variable set of the model is $\Xi_{\mathrm{E}}^{\mathrm{P}} = \{P_v^G, P_d^L, \delta_i\}$ while its dual variable set is $\Xi_{\mathrm{E}}^{\mathrm{D}} = \{\lambda_i, \rho_{1,d}^{\min}, \rho_{1,d}^{\max}, \rho_{2,i,j}^{\max}, \rho_{3,v}^{\min}, \rho_{3,v}^{\max}, \rho_4\}$. Objective function (1) computes the social welfare that is engendered by transacting the sale of electricity between suppliers and consumers. Constraints (2) impose bus-level load balance. Constraints (3) bound electricity demand that is served. Constraints (4) impose the load-carrying limits of transmission lines. Constraints (5) impose production limits on generators. Constraint (6) fixes the phase angle of the reference bus to zero.

2) Natural-Gas-Market Model: The natural-gas-market model is formulated as:

$$\max_{\Xi_{\mathrm{G}}^{\mathrm{P}}} \sum_{e\in\Lambda^{GO}} C_e^{GL} F_e^L + \sum_{l\in\mathbb{L},e\in\Lambda_l^{GL}} \varepsilon_e F_e^L \quad (7)$$

$$+\sum_{m\in\mathbb{N}}\left(\sum_{v\in\Psi_m^G}\gamma_v^G F_v^G - \sum_{w\in\Psi_m^S}\beta_w F_w^S\right)$$

$$\text{s.t.} \sum_{w\in\Psi_m^S} F_w^S = \sum_{k\in\mathbb{C}(m)}(1+\theta_k)F_k^C + \sum_{e\in\Psi_m^L} F_e^L \quad (8)$$

$$+\sum_{v\in\Psi_m^G} F_v^G + \sum_{n\in\mathbb{G}(m)} F_{m,n}; \forall m\in\mathbb{N} \quad (u_m)$$

$$(F_{m,n}/W_{m,n})^2 = \Pi_m - \Pi_n; \quad (9)$$

$$\forall m\in\mathbb{N}, n\in\mathbb{G}(m) \quad (\Phi_{1,m,n})$$

$$\Pi_k^{in}\rho_{C,k}^{\min} \le \Pi_k^{out} \le \Pi_k^{in}\rho_{C,k}^{\max}; \quad \forall k \in \mathbb{C} \quad (\Phi_{2,k}^{\min}, \Phi_{2,k}^{\max}) \tag{10}$$

$$0 \le F_k^C \le F_k^{C,\max}; \forall k \in \mathbb{C} \quad (\Phi_{3,k}^{\min}, \Phi_{3,k}^{\max}) \tag{11}$$

$$0 \le F_w^S \le F_w^{S,\max}; \quad \forall m \in \mathbb{N}, w \in \Psi^S \quad (\Phi_{4,w}^{\min}, \Phi_{4,w}^{\max}) \tag{12}$$

$$0 \le F_e^L \le F_e^{L,\max}; \forall e \in \Lambda^G \quad (\Phi_{5,e}^{\min}, \Phi_{5,e}^{\max}) \tag{13}$$

$$\Pi_m^{\min} \le \Pi_m \le \Pi_m^{\max}; \forall m \in \mathbb{N} \quad (\Phi_{6,m}^{\min}, \Phi_{6,m}^{\max}) \tag{14}$$

$$F_{m,n} \ge 0; \forall m \in \mathbb{N}, n \in \mathbb{G}(m) \quad (\Phi_{7,m,n}) \tag{15}$$

$$0 \le F_v^G \le F_v^{G,\max}; \quad \forall m \in \mathbb{N}, v \in \Psi_m^G; \quad (\Phi_{8,v}^{\min}, \Phi_{8,v}^{\max}) \tag{16}$$

where the Lagrange multiplier that is associated with each constraint is in parentheses to its right. The primal-variable set of the model is $\Xi_G^P = \{F_{m,n}, F_k^C, F_v^G, F_e^L, F_w^S, \Pi_m, \Pi_k^{\text{in}}, \Pi_k^{\text{out}}\}$ while the Lagrange-multiplier set is $\Xi_G^D = \{u_m, \Phi_{1,m,n}, \Phi_{2,k}^{\min}, \Phi_{2,k}^{\max}, \Phi_{3,k}^{\min}, \Phi_{3,k}^{\max}, \Phi_{4,w}^{\min}, \Phi_{4,w}^{\max}, \Phi_{5,e}^{\min}, \Phi_{5,e}^{\max}, \Phi_{6,m}^{\min}, \Phi_{6,m}^{\max}, \Phi_{7,m,n}, \Phi_{8,v}^{\min}, \Phi_{8,v}^{\max}\}$.

Objective function (7) maximizes the social welfare that is attained from transacting natural gas between supplies and consumers. Natural gas can be consumed as natural gas demands (*i.e.*, $e \in \Lambda^G$) or as fuel for natural-gas-fired units. Constraints (8) impose nodal flow balance. Constraints (9) relate the natural gas flow on each pipeline to the change in the squared pressure between its two ends. Constraints (10) and (11) impose the compression-ratio limits and natural-gas-transportation limits of the compressors, respectively. Constraints (12) impose output limits on natural gas sources. Constraints (13) bound natural gas demands that are served. Constraints (14) limit the natural gas pressure at each node. Constraints (15) specify the direction of natural gas flows on each pipeline, by restricting the flows to be non-negative. Knowing the direction of natural gas flows *a priori* is a reasonable assumption in modeling short-term operations of natural gas systems [21], [22]. Modeling a natural gas system with bi-directional flows through pipelines and compressors offers greater flexibility but at increased computational cost [23]. Constraints (16) impose limits on the amount of fuel that is supplied to natural-gas-fired units. The inlet and outlet pressures of natural gas compressors and natural gas nodal pressures are interrelated by:

$$\Pi_k^{\text{in}} = \Pi_m; \forall k \in \mathbb{C}(m)^{\text{in}}$$
$$\Pi_k^{\text{out}} = \Pi_m; \forall k \in \mathbb{C}(m)^{\text{out}}$$

where $\mathbb{C}(m)^{\text{out}}$ and $\mathbb{C}(m)^{\text{in}}$ denote, respectively, the sets of compressors that have their outflow from and their inflow to node m.

The natural-gas-market model is non-convex due to (9). However, (9) can be convexified by replacing it with the SOC constraints [24]:

$$(F_{m,n}/W_{m,n})^2 \le \Pi_m - \Pi_n; \forall m \in \mathbb{N}, n \in \mathbb{G}(m)$$

which can be written more compactly as:

$$\left\| \begin{matrix} 2F_{m,n}C_{m,n} \\ \Pi_m - \Pi_n - 1 \end{matrix} \right\|_2 \le \Pi_m - \Pi_n + 1; \quad \forall m \in \mathbb{N}, n \in \mathbb{G}(m) \quad (\Lambda_{m,n}) \tag{17}$$

where $\Lambda_{m,n} = \left(\Lambda_{m,n}^1, \Lambda_{m,n}^2, \Lambda_{m,n}^3\right)^\top \in \mathcal{K}$ denotes the dual cones that are associated with Constraints (17) and $\mathcal{K}$ denotes a cone. Thus, $\Lambda_{m,n} \in \mathcal{K}$ means $\left(\Lambda_{m,n}^1\right)^2 + \left(\Lambda_{m,n}^2\right)^2 \le \left(\Lambda_{m,n}^3\right)^2$.

B. Optimality Conditions for Lower-Level Models

ς and α are upper-level variables. Thus, (1)–(6) is a linear optimization problem, an optimal solution of which can be characterized from the problem's primal and dual constraints and the strong-duality equality. Similarly, because ε, γ^G, and β are upper-level variables, natural-gas-market model (7), (8), (10)–(17) is an SOC problem. Strong duality applies to such problems under mild conditions [25], which we assume to hold. Thus, we can characterize an optimal solution to this model using its primal and dual constraints and the strong-duality equality.

1) Electricity-Market Model: A set of necessary and sufficient conditions for a global optimum of (1)–(6) is (2)–(6) and:

$$\alpha_v - \lambda_{i(v)} + \rho_{3,v}^{\max} - \rho_{3,v}^{\min} = 0; \forall v \in \Omega^E \tag{18}$$

$$\sum_{j \in \mathbb{E}(i)} b_{i,j} \cdot (\lambda_i - \lambda_j + \rho_{2,i,j}^{\max} - \rho_{2,j,i}^{\max}) = 0; \quad \forall i \in \mathbb{B}, i \ne \text{REF} \tag{19}$$

$$\sum_{j \in \mathbb{E}(i)} b_{\text{REF},j} \cdot (\lambda_{\text{REF}} - \lambda_j + \rho_{2,\text{REF},j}^{\max} - \rho_{2,j,\text{REF}}^{\max}) + \rho_4 = 0 \tag{20}$$

$$-\varsigma_d + \lambda_{i(d)} + \rho_{1,d}^{\max} - \rho_{1,d}^{\min} = 0; \forall l \in \mathbb{L}, d \in \Lambda_l^{EL} \tag{21}$$

$$-C_d^{EL} + \lambda_{i(d)} + \rho_{1,d}^{\max} - \rho_{1,d}^{\min} = 0; \forall d \in \Lambda^{EO} \tag{22}$$

$$\rho_{1,d}^{\min}, \rho_{1,d}^{\max} \ge 0; \forall d \in \Lambda^E \tag{23}$$

$$\rho_{2,i,j}^{\max} \ge 0; \forall i \in \mathbb{B}, j \in \mathbb{E}(i) \tag{24}$$

$$\rho_{3,v}^{\min}, \rho_{3,v}^{\max} \ge 0; \forall v \in \Omega^E \tag{25}$$

$$\sum_{l \in \mathbb{L}, d \in \Lambda_l^{EL}} \varsigma_d P_d^L + \sum_{d \in \Lambda^{EO}} C_d^{EL} P_d^L - \sum_{v \in \Omega^E} \alpha_v P_v^G = \sum_{d \in \Lambda^E} P_d^{L,\max} \rho_{1,d}^{\max} + \sum_{i \in \mathbb{B}, j \in \mathbb{E}(i)} P_{i,j}^{\max} \rho_{2,i,j}^{\max} + \sum_{v \in \Omega^E} P_v^{G,\max} \rho_{3,v}^{\max}; \quad (\Upsilon_l) \tag{26}$$

where $i(v)$ and $i(d)$ denote the buses at which unit v and demand d are located, respectively. Conditions (18)–(25) are constraints of the dual problem of (1)–(6) while (26) is the strong-duality condition. Υ_l is the Lagrange multiplier that we associate with (26), which we discuss when we embed these conditions within each firm's upper-level problem.

2) Natural-Gas-Market Model: The optimality conditions of the natural-gas-market model are given by (8), (10)–(17)

and:

$$\beta_w - u_{m(w)} - \Phi_{4,w}^{\min} + \Phi_{4,w}^{\max} = 0; \forall w \in \Psi^S \quad (27)$$

$$-\varepsilon_e + u_{m(e)} - \Phi_{5,e}^{\min} + \Phi_{5,e}^{\max} = 0; \forall l \in \mathbb{L}, e \in \Lambda_l^{GL} \quad (28)$$

$$-C_e^{GL} + u_{m(e)} - \Phi_{5,e}^{\min} + \Phi_{5,e}^{\max} = 0; \forall e \in \Lambda^{GO} \quad (29)$$

$$u_m - u_n - 2\Lambda_{m,n}^1/C_{m,n} - \Phi_{7,m,n} = 0; \quad \forall m \in \mathbb{N}, n \in \mathbb{G}(m) \quad (30)$$

$$-\sum_{n\in\mathbb{G}(m)} (\Lambda_{m,n}^2 + \Lambda_{m,n}^3 - \Lambda_{n,m}^2 - \Lambda_{n,m}^3) + \sum_{k\in\mathbb{C}(m)^{\text{out}}} (\Phi_{2,k}^{\max} - \Phi_{2,k}^{\min}) + \Phi_{6,m}^{\max} - \Phi_{6,m}^{\min} + \sum_{k\in\mathbb{C}(m)^{\text{in}}} (\Phi_{2,k}^{\min}\rho_k^{\min} - \Phi_{2,k}^{\max}\rho_k^{\max}) = 0; \forall m \in \mathbb{N} \quad (31)$$

$$(1+\theta_k)u_{m_k^{\text{in}}} - u_{m_k^{\text{out}}} - \Phi_{3,k}^{\min} + \Phi_{3,k}^{\max} = 0; \forall k \in \mathbb{C} \quad (32)$$

$$u_m - \gamma_v^G - \Phi_{8,v}^{\min} + \Phi_{8,v}^{\max} = 0; \forall m \in \mathbb{N}, v \in \Psi_m^G \quad (33)$$

$$\Phi_{2,k}^{\min}, \Phi_{2,k}^{\max} \geq 0; \forall k \in \mathbb{C} \quad (34)$$

$$\Phi_{3,k}^{\min}, \Phi_{3,k}^{\max} \geq 0; \forall k \in \mathbb{C} \quad (35)$$

$$\Phi_{4,w}^{\min}, \Phi_{4,w}^{\max} \geq 0; \forall w \in \Psi^S \quad (36)$$

$$\Phi_{5,e}^{\min}, \Phi_{5,e}^{\max} \geq 0; \forall e \in \Lambda^G \quad (37)$$

$$\Phi_{6,m}^{\min}, \Phi_{6,m}^{\max} \geq 0; \forall m \in \mathbb{N} \quad (38)$$

$$\Phi_{7,m,n} \geq 0; \forall m \in \mathbb{N}, n \in \mathbb{G}(m) \quad (39)$$

$$\Phi_{8,v}^{\min}, \Phi_{8,v}^{\max} \geq 0; \forall m \in \mathbb{N}, v \in \Psi_m^G \quad (\varpi_{13,l,v}^{\min}, \varpi_{13,l,v}^{\max}) \quad (40)$$

$$\sum_{e\in\Lambda^{GO}} C_e^{GL}F_e^L + \sum_{l\in\mathbb{L},e\in\Lambda_l^{GL}} \varepsilon_e F_e^L + \sum_{v\in\Psi_m^G} \gamma_v^G F_v^G - \sum_{w\in\Psi_m^S} \beta_w F_w^S - \sum_{m\in\mathbb{N}} \left(\Pi_m^{\max}\Phi_{6,m}^{\max} - \Pi_m^{\min}\Phi_{6,m}^{\min}\right) - \sum_{m\in\mathbb{N}}\sum_{n\in\mathbb{G}(m)} {\Lambda_{m,n}}^\top \Delta_{m,n} - \sum_{w\in\Psi_m^S} F_w^{S,\max}\Phi_{4,w}^{\max} - \sum_{v\in\Psi_m^G} F_v^{G,\max}\Phi_{8,v}^{\max} - \sum_{k\in\mathbb{C}} F_k^{C,\max}\Phi_{3,k}^{\max} - \sum_{e\in\Lambda^G} F_e^{L,\max}\Phi_{5,e}^{\max} = 0; \quad (\kappa_l) \quad (41)$$

where $\Delta_{m,n} = (2F_{m,n}/C_{m,n}, \Pi_m - \Pi_n - 1, \Pi_m - \Pi_n + 1)^\top \in \mathcal{K}$ denotes the primal cones that are associated with (17). $m(w)$ and $m(e)$ denote the nodes at which natural gas source w and demand e, respectively, are located, and m_k^{out} and m_k^{in} denote outflow and inflow nodes, respectively, of compressor k. Conditions (27)–(40) are constraints of the dual problem of (7), (8), (10)–(17), while (41) is the strong-duality condition. κ_l is the Lagrange multiplier that we associate with (41), which we discuss when we embed these conditions within each firm's upper-level problem.

C. Upper-Level Model

To formulate the upper-level firms' optimization problems, we note that the dual variables, λ, that are associated with (2) represent the electric locational marginal prices (LMPs), which are given in \$/p.u. Similarly, the dual variables, u, that are associated with (8) represent the natural gas LMPs and are given in \$/Mm^3/h. We assume that these LMPs are used for settlement of the two markets. With this assumption, firm l's optimization problem is given by:

$$\max_{\Xi_{\text{UL}}} \sum_{d\in\Lambda_l^{EL}} (C_d^{EL} - \lambda_{i(d)})P_d^L + \sum_{e\in\Lambda_l^{GL}} (C_e^{GL} - u_{m(e)})F_e^L + \sum_{v\in\Omega_l^G\cup\Omega_l^R} \lambda_{i(v)}P_v^G - \sum_{v\in\Omega_l^R} C_v^G P_v^G - \sum_{v\in\Omega_l^G} (C_v^{\text{O}} + \eta_v u_{m(v)})P_v^G + \sum_{w\in\Omega_l^S} (C_w^S - u_{m(w)})F_w^S \quad (42)$$

$$\text{s.t.} \quad \alpha_v \geq 0; \forall v \in \Omega_l^C \cup \Omega_l^G \quad (43)$$

$$\beta_w \geq 0; \forall w \in \Omega_l^S \quad (44)$$

$$\gamma_v^G \geq 0; \forall v \in \Omega_l^G \quad (45)$$

$$\varepsilon_e \geq 0; \forall e \in \Lambda_l^{GL} \quad (46)$$

$$\varsigma_d \geq 0; \forall d \in \Lambda_l^{EL} \quad (47)$$

$$\text{(1)–(6), (7), (8), (10)–(17);} \quad (48)$$

where $m(v)$ denotes the node at which natural-gas-fired unit v is located. The variable set of this model is $\Xi_{\text{UL}} = \{\alpha_v, \beta_w, \gamma_v^G, \varepsilon_e, \varsigma_d, \Xi_{\text{E}}^{\text{P}}, \Xi_{\text{E}}^{\text{D}}, \Xi_{\text{G}}^{\text{P}}, \Xi_{\text{G}}^{\text{D}}\}$. Objective function (42) computes the total utility and profit that the firm earns from market transactions. The first two terms in the sums in the objective function give the total utility that the firm derives from electricity and natural gas, respectively, from the market for direct consumption (*i.e.*, the second term does not account for fuel purchased for any natural-gas-fired generating units that the firm may own). The remaining terms represent the profit that the firm earns from selling electricity and natural gas. All of these market transactions are settled in the objective function using the LMPs, λ and u.

Constraints (43)–(47) force all of the offers and bids that are submitted by the firm to be non-negative. Constraint (48) embeds the two market-clearing models within the firm's optimization. This is because the market clearing models determine the quantities that are transacted and the LMPs, both of which appear in (42). Firm l has values of α, β, γ^G, ε, and ς that correspond to supplies and demands that it owns as direct decision variables. Moreover, all of the primal and dual variables and Lagrange multipliers of the lower-level models are 'indirect' decision variables in firm l's problem, as firm l determines the values of these in modeling the markets clearing.

Bi-level problem (42)–(48) can be converted to an equivalent single-level mathematical program with equilibrium constraints (MPEC) by replacing (48) with:

$$\text{(2)–(6), (8), (10)–(41).} \quad (49)$$

D. Market-Equilibrium Model

Solving firm l's MPEC gives an optimal set of offers and bids, given a fixed set of offers and bids for its rival firms. This is because the supply offers and demand bids that correspond to assets that are owned by firm l's rivals are held fixed while firm l optimizes its own strategy. As such, firm l's

MPEC gives a partial equilibrium or firm l's best response to a fixed set of its rivals' offers and bids. Our goal is to find a Nash equilibrium, wherein all of the firms follow a strategy profile with the property that no firm has an individually beneficial unilateral deviation [26], [27]. One way to find a Nash equilibrium is to solve simultaneously all of the firms' MPECs. We take a related approach, which is to combine the KKT conditions that are associated with each firm's MPEC into a system of equations and inequalities [20]. The KKT conditions of firm l's MPEC consist of the following three set of conditions.

1) Primal constraints of firm l's MPEC, which consist of (2), (6), (8), (18)–(22), (26), (27)–(33), and (41).
2) Stationarity conditions that are obtained from differentiating the Lagrangian of firm l's MPEC with respect to the MPEC's primal variables.
3) Complementarity constraints that pertain to the inequality constraints in firm l's MPEC.

Because there are numerous complicated expressions in these KKT conditions, we do not list them here. Rather, we refer interested readers to examples of other works [20], [28], [29] that list explicitly all of the KKT conditions of other illustrative market-equilibrium models.

To solve efficiently the system of equations and inequalities that are obtained from the KKT conditions of the MPECs, we impose them as constraints of an optimization problem. Doing so yields an equilibrium problem with equilibrium constraints (EPEC), which is a nonlinear optimization problem. In theory the EPEC can have any arbitrary objective function, as the purpose of the EPEC is to find a set of offers and bids for all of the firms and market-clearing solutions for the lower-level problems that are simultaneously optimal in all of the MPECs. In practice, however, it is beneficial to choose judiciously the objective function of the EPEC. This is because a strategic game may have many Nash equilibria [20]. As such, we impose three different objective functions on the EPEC, with the aim of finding a bounding range of Nash equilibria.

The first objective function:

$$\max_{\Xi_{\text{UL}}} \sum_{d\in\Lambda^E} C_d^{EL} P_d^L + \sum_{e\in\Lambda^G} C_e^{GL} F_e^L - \sum_{l\in\mathbb{L}} \Bigg[\sum_{v\in\Omega_l^R} C_v^G P_v^G + \sum_{v\in\Omega_l^G} C_v^{\text{O}} P_v^G + \sum_{w\in\Omega_l^S} C_w^S F_w^S \Bigg], \quad (50)$$

maximizes total social welfare. As such, Nash equilibria that are found with (50) as the objective function of the EPEC tend to be highly competitive. The fourth term in (50), which is:

$$\sum_{l\in\mathbb{L}} \sum_{v\in\Omega_l^G} C_v^{\text{O}} P_v^G,$$

computes the non-fuel operating cost of the natural-gas-fired units. However, the fuel cost does not appear directly in this term. This is because the fuel cost is captured implicitly in the final term in (50), which computes the total cost of supplying natural gas. The second objective function:

$$\max_{\Xi_{\text{UL}}} \sum_{l\in\mathbb{L}} \Bigg[\sum_{v\in\Omega_l^G\cup\Omega_l^R} \lambda_{i(v)} P_v^G - \sum_{v\in\Omega_l^R} C_v^G P_v^G - \sum_{v\in\Omega_l^G} (C_v^{\text{O}} + \eta_v u_{m(v)}) P_v^G + \sum_{w\in\Omega_l^S} (u_{m(w)} - C_w^S) F_w^S \Bigg], \quad (51)$$

maximizes the total profits of all of the firms from selling electricity and natural gas. Thus, equilibria that are obtained with this objective function tend to see the exercise of market power by suppliers. The third objective function:

$$\max_{\Xi_{\text{UL}}} \sum_{l\in\mathbb{L}} \Bigg[\sum_{d\in\Lambda_l^{EL}} (C_d^{EL} - \lambda_{i(d)}) P_d^L + \sum_{e\in\Lambda_l^{GL}} (C_e^{GL} - u_{m(e)}) F_e^L \Bigg], \quad (52)$$

maximizes the total utility that all of the firms gain from purchasing electricity and natural gas. Thus, equilibria that are obtained with this objective function tend to see the exercise of market power by consumers. This objective function includes only utility from natural gas that is purchased for direct consumption (*i.e.*, it excludes the value of fuel that is procured for natural-gas-fired units that are owned by integrated firms).

Finally, we add the rational-transaction constraints:

$$F_v^G = \eta_v^G P_v^G; \forall l \in \mathbb{L}, v \in \Omega_l^G. \quad (53)$$

Intuitively, each of these constraints requires that each natural-gas-fired unit have an electricity-supply offer (in the electricity market) that is consistent with the fuel-purchase bid that it submits in the natural gas market. Hence, (53) couple equilibrium behavior in the two markets.

E. Confirmation of Nash Equilibria

A solution to an EPEC is not necessarily a Nash equilibrium of the original equilibrium problem. Rather, the only structural property of an EPEC solution that can be guaranteed is that it satisfies simultaneously the KKT conditions of all of the MPECs. An EPEC solution could, for instance, be a saddle point [30]. Hence, after an EPEC solution is found, we employ an additional step to verify that it satisfies the Nash equilibrium condition that no firm has a profitable unilateral deviation.

To outline this step, we let $\{Z_l^*\}_{l\in\mathbb{L}}$ denote the values of (42) for each of the firms, as computed using the EPEC solution. Next, we solve sequentially each firm's MPEC, while holding the offers of all of its rivals fixed equal to the EPEC solution. We let $\tilde{Z}_l$ denote the optimal value of (42) that is obtained from solving firm l's MPEC. If $\tilde{Z}_l > Z_l^*$ for any firm, then the EPEC solution is not a Nash equilibrium. Indeed, this means that solving firm l's MPEC yields a profitable deviation for it. Otherwise, if $\tilde{Z}_l \le Z_l^*$ for all $l \in \mathbb{L}$, then no firm has a unilaterally profitable deviation from the EPEC solution, meaning that the EPEC solution is indeed a Nash equilibrium.

III. EXAMPLE

This section summarizes the results of a simple three-firm, two-bus, three-node example, the topology of which is shown in Fig. 2. Natural-gas-fired unit 2 provides the point of coupling between the two systems. For simplicity, we do not consider strategic demand-side bids in this example, focusing instead on strategic supply-side offers only. Firms 1 and 3 own unit 1 (which is not natural-gas-fired) and natural gas source 2, respectively. Firm 2 owns both natural-gas-fired unit 2 and natural gas source 1, meaning that it is an integrated firm that participates in both markets. Case study data are provided in an online supplement.[1] The EPEC and the MPEC (the latter is used for equilibrium confirmation) are programmed in GAMS 24.7 and solved using BARON 16.3.4.

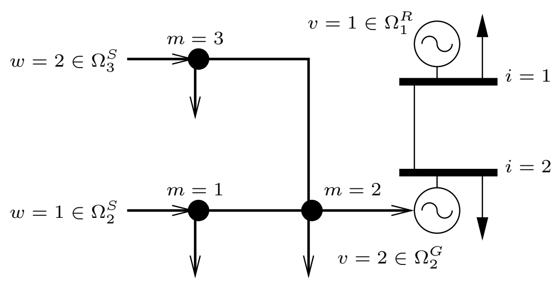

Fig. 2. Topology of the three-firm, two-bus, three-node example that is used in Section III.

Because there are no strategic demands, we compute EPEC equilibria with Objective functions (50) and (51), which correspond to social-welfare and supplier-profit maximization, respectively. Objective function (52), which maximizes the exercise of demand-side market power, is meaningless in the absence of strategic demands. We compare the equilibria that are obtained using these two objective functions to two other extreme market-structure cases. One is a perfectly competitive market, where all of the suppliers offer into the market at their true marginal costs and natural-gas-fired unit 2 submits fuel-demand bids according to (53). This case is modeled by solving (49) to obtain the outcomes of the two markets. The second extreme case is a monopoly, which we model by assuming that a single firm owns all of the generating units and natural gas sources. This case is modeled by solving the MPEC of that one firm.

Table I summarizes the profits that are earned by the strategic suppliers and the total social welfare that is engendered under the four market equilibria that we model. As expected, the perfectly competitive and monopoly equilibria yield the most and least social welfare, respectively. Interestingly, we find that the two extreme market equilibria (when the supply assets are owned by independent utility-maximizing firms) can yield the same overall market efficiency as perfect competition and a monopoly, depending on what equilibrium the market settles at. This suggests that (at least within the context of our simple example) if regulatory authorities provide adequate oversight of the market, it could deliver the same overall outcome to society that perfect competition can. However, the welfare-maximizing EPEC equilibrium results in differences in the distribution of gains to market participants. Indeed, although the welfare-maximizing EPEC equilibrium yields the same social welfare as perfect competition, more than half of the welfare accrues to suppliers in the EPEC equilibrium. Conversely, consumers retain nearly two-thirds of the social welfare under perfect competition. These distributional differences do have important political-economy implications.

[1]https://doi.org/10.6084/m9.figshare.7144448.v1

TABLE I
PROFITS AND SOCIAL WELFARE ($) UNDER DIFFERENT MARKET EQUILIBRIA IN EXAMPLE IN SECTION III

Equilibrium	Profit				Social Welfare
	Firm 1	Firm 2	Firm 3	Total	
Competitive	0	250	0	250	744
EPEC (50)	100	310	56	467	744
EPEC (51)	105	388	200	693	708
Monopoly	105	388	200	693	708

The EPEC has an excess degree of freedom, insomuch as the dual variables and Lagrange multipliers, respectively, that are associated with strong-duality conditions (26) and (41) can be fixed to different values when solving the EPEC. Doing so makes solution of the EPEC less computationally challenging [28], [29]. Fixing these dual variables and Lagrange multipliers to different values can yield different market equilibria. Table II demonstrates this by showing three different equilibria (the first one is the same equilibrium that is summarized in Table I) that are obtained from the EPEC with Objective function (51), which maximizes total supplier profits. Interestingly, these equilibria vary in terms of competitiveness. Indeed, the third equilibrium that is summarized in Table II engenders the same amount of social welfare as the competitive equilibrium that is summarized in Table I. Although even more of the social welfare in the third equilibrium that is summarized in Table II accrues to suppliers compared to the EPEC equilibrium that is summarized in Table I with (50) as the objective function.

TABLE II
PROFITS AND SOCIAL WELFARE ($) UNDER DIFFERENT MARKET EQUILIBRIA THAT ARE OBTAINED FROM THE EPEC WITH OBJECTIVE FUNCTION (51) IN EXAMPLE IN SECTION III

Equilibrium	Profit				Social Welfare
	Firm 1	Firm 2	Firm 3	Total	
1	105	388	200	693	708
2	83	398	200	681	711
3	105	327	57	489	744

Table III summarizes the impact of transmission-network congestion on market equilibria. It shows the impact of reducing the capacity of the transmission line connecting the two buses in the network from 15 MW, which is the baseline value, on generator profits and social welfare. It summarizes these results for EPEC equilibria with (51) as the objective function. As expected, transmission congestion reduces social welfare, as congestion can restrict the use of lower-cost resources to serve demands. However, transmission congestion can be beneficial to suppliers. The table shows that firm 2, in particular, benefits from transmission congestion. This is due to congestion increasing the amount that generator 2 produces as well as the electric LMP which its output receives.

TABLE III
PROFITS AND SOCIAL WELFARE ($) UNDER MARKET EQUILIBRIA THAT ARE OBTAINED FROM THE EPEC WITH OBJECTIVE FUNCTION (51) WITH DIFFERENT AMOUNTS OF TRANSMISSION CAPACITY IN EXAMPLE IN SECTION III

$P_{1,2}^{\max}$	Profit				Social Welfare
	Firm 1	Firm 2	Firm 3	Total	
15	105	388	200	693	744
10	90	410	200	700	735
7	81	421	200	702	730

IV. CASE STUDY

We present here the results of two case studies. The first is based on the Belgian electric and natural gas systems whereas the second considers the IEEE 57-bus system, which is coupled with a 134-node natural gas system.

A. Belgian Electric and Natural Gas Systems

First, we consider a case study that is based on a three-firm, 24-bus,[2] 20-node [31] representation of the Belgian electric and natural gas systems. Fig. 3 shows the network topology. The natural-gas-fired units that are located at buses 2, 3, 6, 8, 16, 15 and 22 are connected to nodes 4, 3, 4, 4, 6, 11, and 13, respectively. There is 13.95 GW of installed generating capacity, of which 30.2% is provided by natural-gas-fired units. Table IV summarizes the buses and nodes at which the three firms own generating units and natural gas sources. The table shows that firm 2 is an integrated firm while the other two participate in one of the two markets only.

TABLE IV
BUSES AND NODES AT WHICH THE THREE FIRMS OWN GENERATING UNITS AND NATURAL GAS SOURCES IN THE BELGIUM-BASED CASE STUDY IN SECTION IV-A

Firm	Buses with Generating Units	Nodes with Natural Gas Sources
1	2, 3, 5, 6, 8, 12	n/a
2	11, 16, 15, 18, 22	8, 13, 14
3	n/a	1, 2, 5

We begin by analyzing a base case, in which none of the demands are strategic and contrast this base case to three sensitivity cases. The first two sensitivity cases have 20% higher marginal utilities for natural gas demands and 30% higher marginal utilities for electricity demands compared to the base case, respectively. The third sensitivity case has 20% higher operating costs (relative to the base case) for the generating units that are not natural-gas-fired.

Table V summarizes the market equilibria that are obtained from the EPEC with (51) as the objective function in the four cases. Contrasting the results shows how the two markets interact with one another. Increasing the utilities of the natural gas demands results in higher profits for natural gas suppliers but lower electricity-supplier profits. This is because increased natural-gas-demand utilities lead to higher natural gas production and prices, which increases supply costs of natural-gas-fired generators. Conversely, higher electricity-demand utilities yield higher profits to *both* electricity and natural gas producers. This is because electricity prices rise and higher electricity production yields higher natural gas production and prices as well. Increasing the cost of non-natural-gas-fired units decreases the profits of electricity suppliers while increasing the profits of natural gas suppliers. These profit impacts are because the electricity sector relies on more natural-gas-fired generation (increasing fuel-supplier profits), as a result of natural gas consumption increasing from 4.98 Mm^3/h in the base case to 9.48 Mm^3/h in the increased-cost case.

TABLE V
PROFITS ($ THOUSAND) UNDER MARKET EQUILIBRIA THAT ARE OBTAINED IN THE DIFFERENT CASES FROM THE EPEC WITH OBJECTIVE FUNCTION (51) IN BELGIUM-BASED CASE STUDY IN SECTION IV-A

Case	Electricity Profits		Natural Gas Profits		Total Profits
	Firm 1	Firm 2	Firm 2	Firm 3	
Base	120	155	55	85	414
Natural Gas Utility	113	143	76	139	470
Electricity Utility	174	195	70	92	531
Electricity Cost	104	139	55	90	388

To show the impacts of demand-side market power, we consider a case in which a fourth strategic firms has electricity demands at buses 7, 9, 23, and 24 and natural gas demands at nodes 10, 12, 19, and 20. Table VI summarizes the properties of equilibria that are obtained under the four market-structure cases that are considered in Table I, in addition to one other case in which (52) is used as the EPEC objective function.

TABLE VI
PROFITS, UTILITY, AND SOCIAL WELFARE ($ THOUSAND) UNDER DIFFERENT MARKET OUTCOMES WITH STRATEGIC DEMAND IN BELGIUM-BASED CASE STUDY IN SECTION IV-A

Equilibrium	Profit				Firm-4 Utility	Social Welfare
	Firm 1	Firm 2	Firm 3	Total		
Competitive	74	129	31	234	84	493
EPEC (50)	94	152	31	277	74	493
EPEC (51)	113	217	78	408	31	462
EPEC (52)	80	136	31	247	81	493
Monopoly	119	211	78	408	34	465

The competitive market and EPEC equilibria using (50) or (52) as the objective function yield the highest social welfare among the cases that we examine. The competitive market yields also the highest consumer utility and lower total supplier profit, because it gives the lowest load-weighted natural gas and electric LMPs among the equilibria that we model. Although using (52) as the EPEC objective function maximizes demand utility, the equilibrium that is obtained in

[2]https://doi.org/10.5281/zenodo.999150

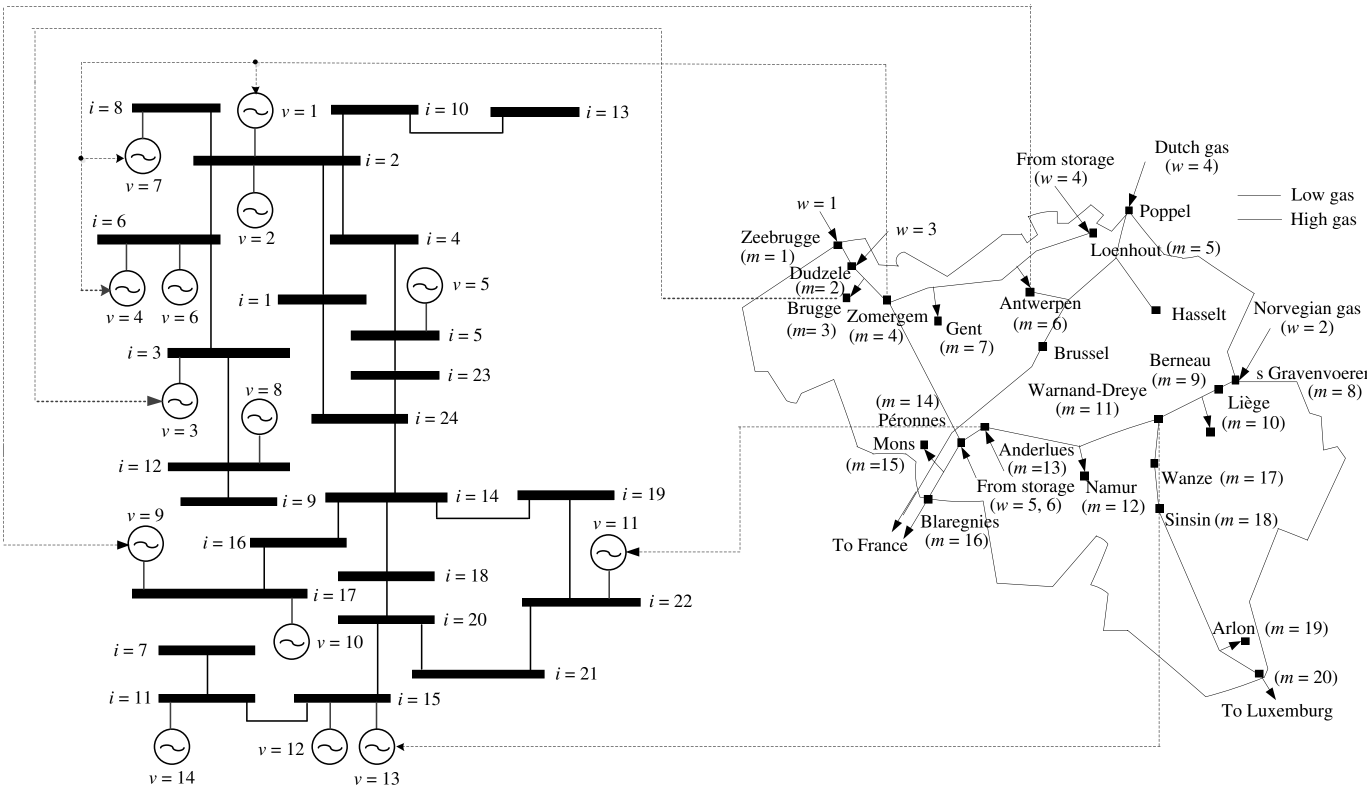

Fig. 3. Belgian-based 24-bus power system and 20-node natural gas system used in the case study in Section IV-A.

this case yields less demand-side utility than the competitive equilibrium does. This is because the EPEC equilibrium allows suppliers to exercise market power, which results in some losses to the demand side (which translate into higher supplier profits).

Both the EPEC equilibrium using (51) as the objective function and the monopoly case yield social welfare losses compared to the three other cases. Interestingly, this EPEC equilibrium yields lower social welfare compared to the monopoly case. This means that there are additional dead-weight losses that arise from the exercise of market power, primarily by firm 2 (the integrated firms that owns natural-gas- and electricity-supply assets). This is consistent with other works that demonstrate that vertical integration between upstream and downstream markets can yield efficiency losses under certain circumstances [32].

B. IEEE 57*-Bus Electric and* 134*-Node Natural Gas Systems*

This section examines a second case study that couples the IEEE 57-bus system [33] to a 134-node model of the tree-like Greek natural gas system.[3] The natural gas system consists of three natural gas sources, 45 demand nodes, 132 pipelines, and one compressor. There are natural-gas-fired units located at buses 1–3, which are connected to nodes 2, 8, and 15, respectively. We consider three strategic suppliers—firm 1 owns power units at buses 1, 3, and 6; firm 3 owns natural gas sources at nodes 1 and 80; and firm 2 owns generating units at buses 2, 8, 9, and 12 and a natural gas source at node 20. A fourth strategic firm owns electricity demands at 10 buses and natural gas demands at 18 nodes.

We use this case study to investigate the impacts of natural-gas-pressure limits on market equilibria. We consider a case in which the minimum natural gas pressures are increased by 10% as compared to the base case. Table VII summarizes the market equilibria that are obtained from the EPEC with (51) as its objective function. The table shows that restricted natural-gas-pressure limits result in higher overall supplier profits, at the cost of lower social welfare. This is because the strategic natural gas suppliers are able to use the more limited operating range of the natural gas system to exercise market power. Indeed, firms 2 and 3, both of which own natural gas supplies, have higher profits with the restricted natural-gas-pressure limits. Conversely, firm 1, which owns generating units only, has lower profits if the natural gas system is more constrained. Similarly, firm 4, which owns strategic demands only, has lower utility with more restricted natural-gas-pressure limits. As shown in Fig. 4, which summarizes natural gas LMPs at a selected subset of the nodes in the equilibria with the two sets of pressure limits, natural gas prices are higher under the more-constrained case. The supply-weighted average of natural gas LMPs in the base case is \$8648/Mm^3. This increases to \$8959/Mm^3 when the pressure limits are restricted.

C. Computational Complexity

We conclude this section with a discussion of the computational complexity of our proposed model. All of the models that are presented in Sections III and IV are solved on a

[3]http://gaslib.zib.de/

TABLE VII
PROFITS, UTILITY, AND SOCIAL WELFARE ($ THOUSAND) UNDER MARKET OUTCOMES WITH DIFFERENT NATURAL-GAS-PRESSURE LIMITS IN CASE STUDY BASED ON IEEE 57-BUS SYSTEM IN SECTION IV-B

Pressure Limit	Profit				Firm-4 Utility	Social Welfare
	Firm 1	Firm 2	Firm 3	Total		
Base Case	5.4	24.1	8.7	38.2	9.8	66.3
Restricted	5.2	24.2	11.5	40.9	7.8	64.1

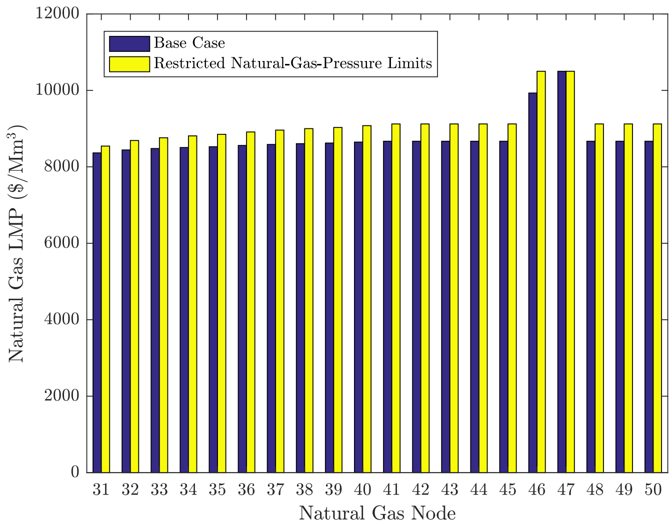

Fig. 4. Natural gas LMPs at a selected subset of nodes in the two equilibria that are summarized in Table VII.

computer with a 1.9-GHz Intel Core processor with 4 GB of memory. The EPECs in the most complex instances of the Belgium-based case study with strategic suppliers and consumers take 1515 s, 3832 s, and 2793 s of wall-clock time to solve with (50), (51), and (52), respectively, as the objective function. The case study in Section IV-B takes 4710 s and 5232 s of wall clock time to solve with the baseline and restricted natural-gas-pressure limits, respectively. This latter case study demonstrates the tractability of the model when it is applied to a large realistic case study.

V. CONCLUSION

This paper proposes a multiple-leader two-follower structure to model the interactions between wholesale electricity and natural gas markets. Importantly, our model structure allows us to account for network congestion, integration between the natural gas and electricity markets, and firms that exercise supply- and demand-side market power. Using a numerical example and two case studies we explore the power of our model in examining how network congestion and integration can impact the efficiency and distributional effects of the market. Hence, our work can help market operators, market participants and regulators to understand: 1) the coupling between electricity and natural gas markets, 2) how market power is exercised by integrated strategic firms that own both natural gas and electricity assets, and 3) market outcomes that can be attained in the two markets. Moreover, our model may help regulators refine the design of electricity or natural gas markets.

Our modeling approach assumes that electric and natural gas LMPs are used for market settlement. Not all wholesale markets employ the level of spatial granularity that we assume in market settlement. Our modeling framework could be used to study market outcomes under such restrictions, by changing how natural gas is priced in (42). The operational model that we use to represent the natural gas system is a simplification, as many natural gas systems are decentralized and involve more than one operator. Nevertheless, our modeling framework provides useful insights into how the coupling of the two markets can impact firm behavior.

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