Investment Equilibria Involving Gas-Fired Power Units in Electricity and Gas Markets

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***Abstract*—We study investment equilibria in electricity and gas markets wherein electricity producers and natural gas suppliers behave strategically. We consider also hybrid producers that own both generating units and gas sources. Each strategic producer determines its investment decisions in gas-fired units, and its offering and bidding strategies to maximize its own profit, anticipating electricity and gas market-clearing outcomes. Producers owning gas-fired units submit bids to the gas market to procure fuel and offers to the electricity market to sell electricity. The resulting model is recast as an equilibrium problem with equilibrium constraints that we solve using a direct approach. Numerical results from two test systems illustrate the proposed methodology.**

***Index Terms*—Electricity market, gas market, investment, strategic offering and bidding, equilibria**

NOMENCLATURE

Indices and Sets

$\mathbb{C}_m$	set of gas compressors connected to node m
$d/\mathcal{D}$	index/set of electricity demands
$e/\mathcal{E}$	index/set of gas demands
$\mathbb{E}_i$	set of electric buses connected directly to bus i
$f/\mathcal{F}$	index/set of candidate gas-fired units
$\mathbb{G}_m$	set of gas nodes connected directly to node m
$i,j/\mathcal{I}$	indices/set of electric power system buses
$i(u/d)$	power system bus where power unit u/electricity demand d is located
$k/\mathcal{K}$	index/set of gas compressors
$l/\mathcal{L}$	index/set of producers
$m,n/\mathcal{M}$	indices/set of gas system nodes
$m(w/e)$	gas system node where gas source w/gas demand e is located
REF	reference bus of the power system
t/T	index/set of operating conditions
$v/\mathcal{V}$	index/set of existing power units
$w/\mathcal{W}$	index/set of gas sources
Ω_l^{GE}	set of existing gas-fired units owned by producer l
Ω_l^{GC}	set of candidate gas-fired units for producer l
Ω_l^{C}	set of non-gas-fired units owned by producer l
Ω_l^{S}	set of gas sources owned by producer l
Θ_i^{D}	set of electricity demands connected to bus i
Θ_i^{GC}	set of candidate gas-fired units at bus i
Θ_i^{GE}	set of existing power units connected to bus i
Ψ_m^{L}	set of gas demands connected to node m
Ψ_m^{GC}	set of candidate gas-fired units at node m
Ψ_m^{GE}	set of existing gas-fired units connected to node m
Ψ_m^{S}	set of gas sources connected to node m

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Parameters and Constants

$b_{i,j}$	susceptance of transmission line i,j (p.u.)
$C_{d,t}^{\mathrm{EL}}$	marginal utility of electricity demand d in operating condition t (\$/p.u.)
$C_{e,t}^{\mathrm{GL}}$	marginal utility of gas demand e in operating condition t (\$/Mm^3)
C_v^{G}	marginal cost of non-gas-fired unit v (\$/p.u.)
C_f^{OC}	operation and maintenance (O&M) cost of candidate gas-fired unit f (\$/p.u.)
C_v^{OE}	O&M cost of existing gas-fired unit v (\$/p.u.)
C_w^{S}	marginal production cost of gas source w (\$/Mm^3)
$F_k^{\mathrm{C,max}}$	gas-transportation limit of compressor k (Mm^3/h)
$F_e^{\mathrm{L,max}}$	maximum demand of gas demand e (Mm^3/h)
$F_w^{\mathrm{S,max}}$	capacity of gas source w (Mm^3/h)
$F_f^{\mathrm{GC,max}}$	maximum fuel consumption by candidate gas-fired unit f (Mm^3/h)
$F_v^{\mathrm{GE,max}}$	maximum fuel consumption by existing gas-fired unit v (Mm^3/h)
K_f	annualized capital cost of candidate unit f (\$/p.u.)
$K^{\max}$	investment budget (\$)
$P_v^{\mathrm{G,max}}$	capacity of power unit v (p.u.)
$P_{i,j}^{\max}$	capacity of power line i,j (p.u.)
$P_d^{\mathrm{L,max}}$	maximum load of electricity demand d (p.u.)
$W_{m,n}$	Weymouth constant of pipeline m,n ((Mm^3/h)/bar)
$X_f^{\max}$	maximum capacity of candidate gas-fired unit f (p.u.)
χ	planning-reserve margin (p.u.)
η_v	heat rate of existing gas-fired unit v (Mm^3/p.u.)
η_f	heat rate of candidate gas-fired unit f (Mm^3/p.u.)
σ_t	weight on operating condition t (h)
ϑ_k	conversion efficiency of gas compressor k (p.u.)
$\rho_k^{\mathrm{C,min}}$	minimum squared ratio of compressor k (p.u.)
$\rho_k^{\mathrm{C,max}}$	maximum squared ratio of compressor k (p.u.)
$\Pi_m^{\max}$	maximum squared gas pressure at node m (bar^2)
$\Pi_m^{\min}$	minimum squared gas pressure at node m (bar^2)

Variables

$F^{\mathrm{C}}_{k,t}$	fuel flow through compressor k in operating condition t (Mm^3/h)
$F^{\mathrm{GE}}_{v,t}$	fuel consumed by existing gas-fired unit v in operating condition t (Mm^3/h)
$F^{\mathrm{GC}}_{f,t}$	fuel consumed by candidate gas-fired unit f in operating condition t (Mm^3/h)
$F^{\mathrm{L}}_{e,t}$	served non-generation-related gas demand e in operating condition t (Mm^3/h)
$F_{m,n,t}$	gas flow through pipeline m,n in operating condition t (Mm^3/h)
$F^{\mathrm{S}}_{w,t}$	gas supplied in operating condition t by source w (Mm^3/h)
$P^{\mathrm{GC}}_{f,t}$	power output of candidate gas-fired unit f in operating condition t (p.u.)
$P^{\mathrm{GE}}_{v,t}$	power output of existing unit v in operating condition t (p.u.)
$P^{\mathrm{L}}_{d,t}$	amount of demand d served in operating condition t (p.u.)
X_f	capacity of candidate gas-fired unit f (p.u.)
$\theta_{i,t}$	phase angle of bus i in operating condition t (rad)
$\Pi^{\mathrm{in}}_{k,t}$	squared inlet gas pressure of compressor k in operating condition t (bar^2)
$\Pi^{\mathrm{out}}_{k,t}$	squared outlet gas pressure of compressor k in operating condition t (bar^2)
$\Pi_{m,t}$	squared gas pressure at node m in operating condition t (bar^2)
$\alpha_{f,t}$	strategic offer price of candidate gas-fired unit f in operating condition t in the electricity market ($/p.u.)
$\alpha_{v,t}$	strategic offer price of existing unit v in operating condition t in the electricity market ($/p.u.)
$\beta_{w,t}$	strategic offer price of gas source w in operating condition t in the gas market ($/Mm^3)
$\gamma^{\mathrm{GC}}_{f,t}$	strategic bid price (to procure gas) of candidate unit f in operating condition t in the gas market ($/Mm^3)
$\gamma^{\mathrm{GE}}_{v,t}$	strategic bid price (to procure gas) of existing unit v in operating condition t in the gas market ($/Mm^3)

I. Introduction

REPLACING coal-fired and other power units with gas-fired ones is increasingly attractive. On one hand, low gas prices make investment in gas-fired power units economically attractive. On the other hand, the net-load fluctuations caused by renewable energy sources call for the flexibility provided by gas-fired power units.

Because power-generation-investment decisions are made often within a market framework, investment models pertaining to gas-fired power units should represent the gas market and fuel-procurement cost. The electricity market also should be represented to capture revenues from electricity sales.

Thus, we propose an equilibrium model that captures strategic investment in gas-fired units and strategic offering and bidding in both electricity and gas markets.

We consider stand-alone power and gas producers and hybrid producers that own both power units and gas sources. The strategic investments in gas-fired units, and the strategic offering and bidding decisions made by each producer are represented by a bi-level problem. The upper-level subproblem seeks to maximize producers' profits, while the lower-level subproblems represent the electricity market clearing (EMC) and gas market clearing (GMC) in a set of operating conditions.

The bi-level problem of each producer is transformed into a mathematical program with equilibrium constraints (MPEC) by replacing the lower-level subproblems with their optimality conditions. Jointly considering the MPECs of all the producers yields an equilibrium problem with equilibrium constraints (EPEC). We use a direct solution approach [1]–[3] that replaces the MPECs with their KKT conditions to compute generalized Nash equilibria.

The technical literature provides a number of approaches to model the co-ordinated long-term planning of power and gas systems. Barati *et al.* [4] propose an integrated framework for expansion planning of generation and power and gas transmission. Qiu *et al.* [5] develop a power- and gas-expansion model that imposes carbon constraints. Chaudry *et al.* [6] propose a combined electricity- and gas-expansion model that considers investment in power units, power lines, pipelines, compressors, and gas-storage facilities. Shao *et al.* [7] develop a robust model for integrated electric- and gas-system planning that considers power system resilience. Zhao *et al.* [8] propose a two-stage stochastic optimization model for co-ordinated expansion planning of power and gas systems. Odetayo *et al.* [9] develop a chance-constrained joint-expansion model, where the role of gas storage is to manage short-term uncertainties in power and gas demands. Ding *et al.* [10] develop a multi-stage stochastic programming model for expansion planning of electricity and gas networks, where sequential investment decisions are made. Cheng *et al.* [11] develop a decentralized approach for integrated-energy-system-expansion planning, in which carbon-emission constraints are represented. Zhang *et al.* [12] and He *et al.* [13] develop a joint-expansion-planning model that satisfies the $N-1$ criterion. Bent *et al.* [14] develop a combined electricity- and gas-network-expansion model with endogenous gas-price feedback.

The models that are proposed in these works take the perspective of a central planner under perfectly competitive markets, which may be unrealistic. Moreover, these works do not represent strategic behavior in electricity and gas markets when modeling investment in gas-fired generation. Given this context, our work makes the following two formative contributions to the existing literature.

1) It develops an EPEC framework to represent the interactions between strategic investors and producers in electricity and gas markets.
2) It identifies a range of investment equilibria. This is done by converting the EPEC into a computationally tractable mixed-integer linear optimization problem.

The remainder of this paper is organized as follows. Section II provides the mathematical formulation of each producer's bi-level model. Section III details the MPECs, the EPEC, and the proposed solution methodology. Sections IV and V summarize numerical results of two test systems. Section VI concludes.

II. Model Formulation

Fig. 1 depicts the structure of the proposed problem. The upper level includes a set of power, gas, and hybrid producers. The lower level represents EMC and GMC under different operating conditions. Poncelet *et al.* [15] provide an approach to select representative operating conditions. Producers that own gas-fired units behave strategically in both markets through electricity-supply offers and fuel-procurement bids. The lower-level EMC and GMC are interrelated indirectly by producers that participate in both markets. We assume that the two markets clear simultaneously. Sequential market clearing can yield efficiency losses. The upper- and lower-level problems are interrelated in the following two ways.

1) Electricity locational marginal prices (ELMPs) and gas locational marginal prices (GLMPs), which are obtained from the lower-level EMC and GMC problems, respectively, affect producer profits in the upper-level problems.
2) Strategic investment and offering and bidding decisions, which are determined in the upper-level problem, affect the lower-level EMC and GMC problems.

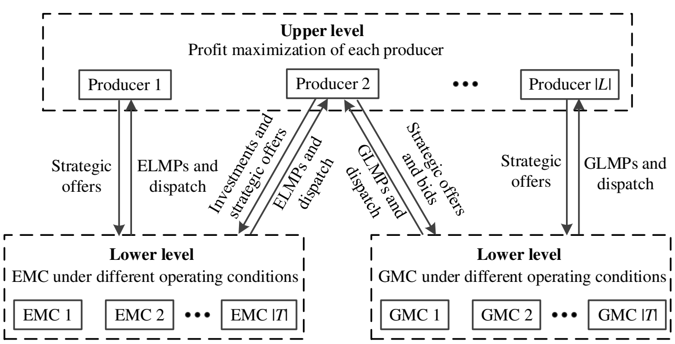

Fig. 1. Problem structure.

Formulations of the upper- and lower-level problems are provided below.

A. Upper-Level Problem

Upper-level objective function (1) represents the profit of strategic producer l. Specifically, the terms, $\eta_v u_{m(v),t}$ and $\eta_f u_{m(f),t}$, in (1) represent the variable fuel cost of existing gas-fired unit v and candidate gas-fired unit f in operating condition t, respectively. $i(v)$ and $i(f)$ denote the electric buses where existing and candidate units v and f, respectively, are located. $m(v)$ and $m(f)$ denote the gas nodes where gas-fired units v and f are located. $m(w)$ denotes the gas node where gas source w is located.

Objective function (1) is optimized subject to the constraints:

$$0 \leq X_f \leq X_f^{\max}, \forall f \in \Omega_l^{\mathrm{GC}} \tag{2a}$$

$$\sum_{f \in \mathcal{F}} K_f X_f \leq K^{\max} \tag{2b}$$

$$\sum_{f \in \mathcal{F}} X_f + \sum_{v \in \mathcal{V}} P_v^{\mathrm{G,max}} \geq (1+\chi) \sum_{d \in \mathcal{D}} P_{d,1}^{\mathrm{D,max}} \tag{2c}$$

$$\alpha_{v,t} \geq 0, \forall v \in \left\{\Omega_l^{\mathrm{C}}, \Omega_l^{\mathrm{GE}}\right\}, t \in T \tag{2d}$$

$$\alpha_{f,t} \geq 0, \forall f \in \Omega_l^{\mathrm{GC}}, t \in T \tag{2e}$$

$$\beta_{w,t} \geq 0, \forall w \in \Omega_l^{\mathrm{S}}, t \in T \tag{2f}$$

$$\gamma_{v,t}^{\mathrm{GE}} \geq 0, \forall v \in \Omega_l^{\mathrm{GE}}, t \in T \tag{2g}$$

$$\gamma_{f,t}^{\mathrm{GC}} \geq 0, \forall f \in \Omega_l^{\mathrm{GC}}, t \in T. \tag{2h}$$

Constraints (2a) limit the capacity of candidate gas-fired units that can be built by producer l. Regulatory constraint (2b) is a generic investment budget limit affecting all of the investors. Regulatory constraint (2c) imposes a planning-reserve margin, which is defined relative to the maximum demand [2], which is assumed to occur in operating condition $t = 1$. Constraints (2d) and (2e) require generating offers to be non-negative. Similarly, constraints (2f) require gas-supply offers to be non-negative. Finally, constraints (2g) and (2h) require fuel-procurement bids for gas-fired units to be non-negative.

B. Lower-Level EMC

The EMC for operating condition t is:

$$\min_{\Xi_t^{\mathrm{E_P}}} \sum_{v \in \mathcal{V}} \alpha_{v,t} P_{v,t}^{\mathrm{GE}} + \sum_{f \in \mathcal{F}} \alpha_{f,t} P_{f,t}^{\mathrm{GC}} - \sum_{d \in \mathcal{D}} C_{d,t}^{\mathrm{EL}} P_{d,t}^{\mathrm{L}} \tag{3}$$

subject to:

$$\sum_{d \in \Theta_i^{\mathrm{D}}} P_{d,t}^{\mathrm{L}} - \sum_{v \in \Theta_i^{\mathrm{GE}}} P_{v,t}^{\mathrm{GE}} - \sum_{f \in \Theta_i^{\mathrm{GC}}} P_{f,t}^{\mathrm{GC}} + \sum_{j \in \mathbb{E}_i} b_{i,j} \cdot (\theta_{i,t} - \theta_{j,t}) = 0 : \lambda_{i,t} \forall i \in \mathcal{I} \tag{4a}$$

$$b_{i,j} \cdot (\theta_{i,t} - \theta_{j,t}) \leq P_{i,j}^{\max} : \rho_{1,i,j,t}^{\max} \forall i \in \mathcal{I}, j \in \mathbb{E}_i \tag{4b}$$

$$0 \leq P_{d,t}^{\mathrm{L}} \leq P_{d,t}^{\mathrm{L,max}} : \rho_{2,d,t}^{\min}, \rho_{2,d,t}^{\max} \forall d \in \mathcal{D} \tag{4c}$$

$$0 \leq P_{v,t}^{\mathrm{GE}} \leq P_v^{\mathrm{G,max}} : \rho_{3,v,t}^{\min}, \rho_{3,v,t}^{\max} \forall v \in \mathcal{V} \tag{4d}$$

$$0 \leq P_{f,t}^{\mathrm{GC}} \leq X_f : \rho_{4,f,t}^{\min}, \rho_{4,f,t}^{\max} \forall f \in \mathcal{F} \tag{4e}$$

$$\theta_{\mathrm{REF},t} = 0 : \rho_{5,t} \forall t \in T. \tag{4f}$$

The dual variable that is associated with each constraint is indicated after the colon. The primal-variable set of the EMC problem of operating condition t is $\Xi_t^{\mathrm{E_P}} = \{P_{v,t}^{\mathrm{GE}}, P_{f,t}^{\mathrm{GC}}, P_{d,t}^{\mathrm{L}}, \theta_t\}$, while the dual-variable set is $\Xi_t^{\mathrm{E_D}} = \{\lambda_{i,t}, \rho_{1,i,j,t}^{\max}, \rho_{2,d,t}^{\max}, \rho_{2,d,t}^{\min}, \rho_{3,v,t}^{\max}, \rho_{3,v,t}^{\min}, \rho_{4,f,t}^{\max}, \rho_{4,f,t}^{\min}, \rho_{5,t}\}$.

Objective function (3) is the negative social welfare (SW) that is engendered by the electricity marketa. Its first two terms represent the production cost of existing and candidate power units, respectively. The last term represents the utility of power demands. We use single-block offers and bids for each production unit and demand, respectively. Offering and bidding quantities are not variables of our model. However, our model can be extended to include multiple quantity blocks to represent a desired offer or bid curve.

Constraints (4) pertain to power system operations. Specifically, (4a) represent active power-flow balance at each bus. Its dual variable, $\lambda_{i,t}$, is the ELMP of bus i in operating condition t. Constraints (4b) enforce the transmission capacity of each power line. Constraints (4c) bound electricity demands.

$$\min_{\Xi^{\mathrm{UL}}} \underbrace{\sum_{f\in\Omega_l^{\mathrm{GC}}} K_f X_f}_{\text{Investment cost}} - \underbrace{\sum_{t\in T}\sigma_t\cdot\left(\sum_{v\in\Omega_l^{\mathrm{C}}} P_{v,t}^{\mathrm{GE}}\cdot\left(\lambda_{i(v),t}-C_v^{\mathrm{G}}\right)+\sum_{v\in\Omega_l^{\mathrm{GE}}} P_{v,t}^{\mathrm{GE}}\cdot\left(\lambda_{i(v),t}-C_v^{\mathrm{OE}}-\eta_v u_{m(v),t}\right)\right)}_{\text{Profits from existing power units}}$$
$$-\underbrace{\sum_{t\in T}\sigma_t\sum_{f\in\Omega_l^{\mathrm{GC}}} P_{f,t}^{\mathrm{GC}}\cdot\left(\lambda_{i(f),t}-C_f^{\mathrm{OC}}-\eta_f u_{m(f),t}\right)}_{\text{Profits from candidate gas-fired power units}}-\underbrace{\sum_{t\in T}\sigma_t\sum_{w\in\Omega_l^{\mathrm{S}}} F_{w,t}^{\mathrm{S}}\cdot\left(u_{m(w),t}-C_w^{\mathrm{S}}\right)}_{\text{Profits from gas sources}} \tag{1}$$

Constraints (4d) and (4e) impose output bounds on existing power units and candidate gas-fired units, respectively. Constraint (4f) sets the phase angle at the reference bus to zero.

C. Lower-Level GMC

The GMC for operating condition t is:

$$\min_{\Xi_t^{\mathrm{G_P}}} \sum_{w\in\mathcal{W}} \beta_{w,t}F_{w,t}^{\mathrm{S}} - \sum_{e\in\mathcal{E}} C_{e,t}^{\mathrm{GL}}F_{e,t}^{\mathrm{L}} - \sum_{v\in\Omega^{\mathrm{GE}}} \gamma_{v,t}^{\mathrm{GE}}F_{v,t}^{\mathrm{GE}} - \sum_{f\in\Omega^{\mathrm{GC}}} \gamma_{f,t}^{\mathrm{GC}}F_{f,t}^{\mathrm{GC}} \tag{5}$$

subject to:

$$\sum_{e\in\Psi_m^{\mathrm{L}}} F_{e,t}^{\mathrm{L}} + \sum_{v\in\Psi_m^{\mathrm{GE}}} F_{v,t}^{\mathrm{GE}} + \sum_{f\in\Psi_m^{\mathrm{GC}}} F_{f,t}^{\mathrm{GC}} - \sum_{w\in\Psi_m^{\mathrm{S}}} F_{w,t}^{\mathrm{S}} + \sum_{n\in\mathbb{G}_m} F_{m,n,t} + \sum_{k\in\mathbb{C}_m} (1+\vartheta_k)F_{k,t}^{\mathrm{C}} = 0 : u_{m,t}\forall m\in\mathcal{M} \tag{6a}$$

$$F_{m,n,t}\left|F_{m,n,t}\right| = W_{m,n}^2\cdot(\Pi_{m,t}-\Pi_{n,t}) : \Phi_{2,m,n,t}\forall m\in\mathcal{M}, n\in\mathbb{G}_m \tag{6b}$$

$$0\le F_{k,t}^{\mathrm{C}}\le F_k^{\mathrm{C,max}} : \Phi_{3,k,t}^{\min},\Phi_{3,k,t}^{\max}\forall k\in\mathcal{K} \tag{6c}$$

$$0\le F_{w,t}^{\mathrm{S}}\le F_w^{\mathrm{S,max}} : \Phi_{4,w,t}^{\min},\Phi_{4,w,t}^{\max}\forall w\in\mathcal{W} \tag{6d}$$

$$0\le F_{e,t}^{\mathrm{L}}\le F_{e,t}^{\mathrm{L,max}} : \Phi_{5,e,t}^{\min},\Phi_{5,e,t}^{\max}\forall e\in\mathcal{E} \tag{6e}$$

$$\Pi_m^{\min}\le\Pi_{m,t}\le\Pi_m^{\max} : \Phi_{6,m,t}^{\min},\Phi_{6,m,t}^{\max}\forall m\in\mathcal{M} \tag{6f}$$

$$\Pi_{k,t}^{\mathrm{in}}\rho_k^{\mathrm{C,min}}\le\Pi_{k,t}^{\mathrm{out}}\le\Pi_{k,t}^{\mathrm{in}}\rho_k^{\mathrm{C,max}} : \Phi_{7,k,t}^{\min},\Phi_{7,k,t}^{\max}\forall k\in\mathcal{K} \tag{6g}$$

$$0\le F_{v,t}^{\mathrm{GE}}\le F_v^{\mathrm{GE,max}} : \Phi_{8,v,t}^{\min},\Phi_{8,v,t}^{\max}\forall v\in\Omega^{\mathrm{GE}} \tag{6h}$$

$$0\le F_{f,t}^{\mathrm{GC}}\le F_f^{\mathrm{GC,max}} : \Phi_{9,f,t}^{\min},\Phi_{9,f,t}^{\max}\forall f\in\Omega^{\mathrm{GC}}. \tag{6i}$$

The primal-variable set of the GMC problem for operating condition t is $\Xi_t^{\mathrm{G_P}} = \{F_{w,t}^{\mathrm{S}}, F_{e,t}^{\mathrm{L}}, F_{v,t}^{\mathrm{GE}}, F_{f,t}^{\mathrm{GC}}, F_{m,n,t}, F_t^{\mathrm{C}}, \Pi_{m,t}\}$, while its dual variable set is $\Xi_t^{\mathrm{G_D}} = \{u_{m,t}, \Phi_{2,m,n,t}, \Phi_{3,k,t}^{\max}, \Phi_{3,k,t}^{\min}, \Phi_{4,w,t}^{\max}, \Phi_{4,w,t}^{\min}, \Phi_{5,e,t}^{\max}, \Phi_{5,e,t}^{\min}, \Phi_{6,m,t}^{\max}, \Phi_{6,m,t}^{\min}, \Phi_{7,k,t}^{\max}, \Phi_{7,k,t}^{\min}, \Phi_{8,v,t}^{\max}, \Phi_{8,v,t}^{\min}, \Phi_{9,f,t}^{\max}, \Phi_{9,f,t}^{\min}\}$.

Objective function (5) is the negative SW derived from the gas market. The first term represents gas-production costs. The second term represents the utility of non-electricity-related gas demands, while the last two terms represent the utility of electricity-related gas demands.

Constraints (6) pertain to the operation of the gas system. Specifically, (6a) represent nodal gas-flow balance, which includes non-electricity-related gas demands, gas consumption from existing and candidate gas-fired units, gas-source production, and the gas flow through pipelines and compressors. The dual variable, $u_{m,t}$, that is associated with (6a) represents the GLMP of node m in operating condition t. Constraints (6b) relate the gas flow to the squared pressure drop at the two ends of each pipeline. Constraints (6c) represent the transportation capacity of compressors, which limit the power consumption of these compressors. Constraints (6d) represent the production capacity of gas sources. Constraints (6e) bound the non-electricity-related gas demands served. Constraints (6f) limit the nodal gas pressures. Constraints (6g) impose minimum and maximum compression ratios on compressors. The inlet and outlet pressures of gas compressors and gas nodal pressures are related as:

$$\Pi_{k,t}^{\mathrm{in}} = \Pi_{m,t}; \forall t\in T, k\in\mathbb{C}(m)^{\mathrm{in}}$$
$$\Pi_{k,t}^{\mathrm{out}} = \Pi_{m,t}; \forall t\in T, k\in\mathbb{C}(m)^{\mathrm{out}},$$

where $\mathbb{C}(m)^{\mathrm{in}}$ and $\mathbb{C}(m)^{\mathrm{out}}$ denote, respectively, the set of compressors which have their inflow to and outflow from node m.

Constraints (6h) and (6i) limit the fuel consumption of existing and candidate gas-fired units, respectively.

Constraints (6b) are nonlinear, which complicates the solution of problem (5)–(6). For simplicity and tractability, we linearize (6b) using the first-order Taylor expansion [16] as:

$$\operatorname{sgn}\left(F_{m,n,t}^0\right)\left(2F_{m,n,t}^0F_{m,n,t}-\left(F_{m,n,t}^0\right)^2\right) = W_{m,n}^2 \times(\Pi_{m,t}-\Pi_{n,t}) : \Phi_{2,m,n,t}\forall m\in\mathcal{M}, n\in\mathbb{G}_m, t\in T. \tag{7}$$

The nonlinear term, $F_{m,n,t}\left|F_{m,n,t}\right|$, on the left-hand side of (6b) is linearized around a given operating condition, $F_{m,n,t}^0$. First, we solve a bi-level model that neglects (6b), and the solution obtained (*i.e.*, the value of $F_{m,n,t}^0$ for each operating scenario, t) is used as the linearization point in (7).

Both EMC (3)–(4) and GMC (5), (6a), (6c)–(7) are linear programming (LP) problems, for which the strong-duality theorem holds.

The variable set of upper-level problem (1)–(2) is $\Xi^{\mathrm{UL}} = \{X_{f,t}, \alpha_{v,t}, \alpha_{f,t}, \beta_{w,t}, \gamma_{v,t}^{\mathrm{GE}}, \gamma_{f,t}^{\mathrm{GC}}, \Xi_t^{\mathrm{E_P}}, \Xi_t^{\mathrm{G_P}}\}$, which includes the decision variables of producers and the primal variables of the EMC and GMC problems.

Our model allows one producer simultaneously to own gas sources and gas-fired power units. Depending on where a producer's gas sources and gas-fired power units are located, its participation in the markets differs.

1) If the gas source and gas-fired power unit are not located at the same gas node, the producer uses the gas-pipeline network to transfer the gas from gas production nodes

to its gas-fired unit. In this case, the gas-fired power unit buys fuel from the gas market.

2) If a producer's gas source and gas-fired power unit are located at the same gas node, the solution obtained from our model should have the strategic bid (to procure fuel) that is provided by the gas-fired power unit equal to the strategic offer provided (to supply gas) by the gas source. This is because our model maximizes the producer's profit.

III. Solution Methodology

For each strategic producer, bi-level model (3)–(5), (6a), (6c)–(7) can be transformed into an MPEC by replacing the lower-level EMC and GMC problems with their optimality conditions (primal constraints, dual constraints, and the strong-duality equality). The resulting MPEC for producer l is:

$$\text{objective:}(3) \tag{8}$$

subject to:

1) upper-level constraints:

$$0 \leq X_f \leq X_f^{\max} \forall f \in \Omega_l^{\text{GC}} \tag{9a}$$

$$\sum_{f \in \mathcal{F}} K_f X_f \leq K^{\max} \tag{9b}$$

$$\sum_{f \in \mathcal{F}} X_f + \sum_{v \in \mathcal{V}} P_v^{\text{G,max}} \geq (1+\chi) \sum_{d \in \mathcal{D}} P_{d,1}^{\text{D,max}} \tag{9c}$$

$$\alpha_{v,t} \geq 0 \forall v \in \left\{\Omega_l^{\text{C}}, \Omega_l^{\text{GE}}\right\}, t \in T \tag{9d}$$

$$\alpha_{f,t} \geq 0 \forall f \in \Omega_l^{\text{GC}}, t \in T \tag{9e}$$

$$\beta_{w,t} \geq 0 \forall w \in \Omega_l^{\text{S}}, t \in T \tag{9f}$$

$$\gamma_{v,t}^{\text{GE}} \geq 0 \forall v \in \Omega_l^{\text{GE}}, t \in T \tag{9g}$$

$$\gamma_{f,t}^{\text{GC}} \geq 0 \forall f \in \Omega_l^{\text{GC}}, t \in T \tag{9h}$$

2) primal constraints of the EMC problems (one set for each operating condition, t):

$$\sum_{d \in \Theta_i^{\text{D}}} P_{d,t}^{\text{L}} - \sum_{v \in \Theta_i^{\text{GE}}} P_{v,t}^{\text{GE}} - \sum_{f \in \Theta_i^{\text{GC}}} P_{f,t}^{\text{GC}} + \sum_{j \in \mathbb{E}_i} b_{i,j} \cdot (\theta_{i,t} - \theta_{j,t}) = 0 \forall i \in \mathcal{I}, t \in T \tag{10a}$$

$$b_{i,j} \cdot (\theta_{i,t} - \theta_{j,t}) \leq P_{i,j}^{\max} \forall i \in \mathcal{I}, j \in \mathbb{E}_i, t \in T \tag{10b}$$

$$0 \leq P_{d,t}^{\text{L}} \leq P_{d,t}^{\text{L,max}} \forall d \in \mathcal{D}, t \in T \tag{10c}$$

$$0 \leq P_{v,t}^{\text{GE}} \leq P_v^{\text{G,max}} \forall v \in \mathcal{V}, t \in T \tag{10d}$$

$$0 \leq P_{f,t}^{\text{GC}} \leq X_f \forall f \in \mathcal{F}, t \in T \tag{10e}$$

$$\theta_{\text{REF},t} = 0 \tag{10f}$$

3) dual constraints of the EMC problems (one set for each operating condition, t):

$$\alpha_{v,t} - \lambda_{i(v),t} + \rho_{3,v,t}^{\max} - \rho_{3,v,t}^{\min} = 0 \quad \forall v \in \mathcal{V}, t \in T \tag{11a}$$

$$\alpha_{f,t} - \lambda_{i(f),t} + \rho_{4,f,t}^{\max} - \rho_{4,f,t}^{\min} = 0 \quad \forall f \in \mathcal{F}, t \in T \tag{11b}$$

$$\lambda_{i(d),t} - C_{d,t}^{\text{EL}} + \rho_{2,d,t}^{\max} - \rho_{2,d,t}^{\min} = 0 \quad \forall d \in \mathcal{D}, t \in T \tag{11c}$$

$$\sum_{j \in \mathbb{E}_i} b_{i,j} \cdot (\lambda_{i,t} - \lambda_{j,t}) + \sum_{j \in \mathbb{E}_i} b_{i,j} \cdot \left(\rho_{1,i,j,t}^{\max} - \rho_{1,j,i,t}^{\max}\right) = 0 \forall i \in \mathcal{I}, i \neq \text{REF}, t \in T \tag{11d}$$

$$\sum_{j \in \mathbb{E}_{\text{REF}}} b_{\text{REF},j} \cdot (\lambda_{\text{REF},t} - \lambda_{j,t}) + \sum_{j \in \mathbb{E}_{\text{REF}}} b_{\text{REF},j} \cdot \left(\rho_{1,\text{REF},j,t}^{\max} - \rho_{1,j,\text{REF},t}^{\max}\right) + \rho_{5,t} = 0 \forall t \in T \tag{11e}$$

$$\rho_{1,i,j,t}^{\max} \geq 0 \forall i \in \mathcal{I}, j \in \mathbb{E}_i, t \in T \tag{11f}$$

$$\rho_{2,d,t}^{\min}, \rho_{2,d,t}^{\max} \geq 0 \forall d \in \mathcal{D}, t \in T \tag{11g}$$

$$\rho_{3,v,t}^{\min}, \rho_{3,v,t}^{\max} \geq 0 \forall v \in \mathcal{V}, t \in T \tag{11h}$$

$$\rho_{4,f,t}^{\min}, \rho_{4,f,t}^{\max} \geq 0 \forall f \in \mathcal{F}, t \in T \tag{11i}$$

4) strong duality for the EMC problems (one for each operating condition, t):

$$\sum_{v \in \mathcal{V}} \alpha_{v,t} P_{v,t}^{\text{GE}} + \sum_{f \in \mathcal{F}} \alpha_{f,t} P_{f,t}^{\text{GC}} - \sum_{d \in \mathcal{D}} C_{d,t}^{\text{EL}} P_{d,t}^{\text{L}} = - \sum_{i \in \mathcal{I}, j \in \mathbb{E}_i} \rho_{1,i,j,t}^{\max} P_{i,j}^{\max} - \sum_{d \in \mathcal{D}} \rho_{2,d,t}^{\max} P_d^{\text{L,max}} - \sum_{v \in \mathcal{V}} \rho_{3,v,t}^{\max} P_v^{\text{G,max}} - \sum_{f \in \mathcal{F}} \rho_{4,f,t}^{\max} X_f : \Upsilon_{l,t} \forall t \in T \tag{12}$$

5) primal constraints of the GMC problems (one set for each operating condition, t):

$$\sum_{e \in \Psi_m} F_{e,t}^{\text{L}} + \sum_{v \in \Psi_m^{\text{GE}}} F_{v,t}^{\text{GE}} + \sum_{f \in \Psi_m^{\text{GC}}} F_{f,t}^{\text{GC}} - \sum_{w \in \Psi_m} F_{w,t}^{\text{S}} + \sum_{n \in \mathbb{G}_m} F_{m,n,t} + \sum_{k \in \mathbb{C}_m} (1+\vartheta_k) F_{k,t}^{\text{C}} = 0 \forall m \in \mathcal{M}, t \in T \tag{13a}$$

$$\text{sgn}\left(F_{m,n,t}^0\right)\left(2F_{m,n,t}^0 F_{m,n,t} - \left(F_{m,n,t}^0\right)^2\right) = W_{m,n}^2 \cdot (\Pi_{m,t} - \Pi_{n,t}) \forall m \in \mathcal{M}, n \in \mathbb{G}_m, t \in T \tag{13b}$$

$$0 \leq F_{k,t}^{\text{C}} \leq F_k^{\text{C,max}} \forall k \in \mathcal{K}, t \in T \tag{13c}$$

$$0 \leq F_{w,t}^{\text{S}} \leq F_w^{\text{S,max}} \forall w \in \mathcal{W}, t \in T \tag{13d}$$

$$0 \leq F_{e,t}^{\text{L}} \leq F_{e,t}^{\text{L,max}} \forall e \in \mathcal{E}, t \in T \tag{13e}$$

$$\Pi_m^{\min} \leq \Pi_{m,t} \leq \Pi_m^{\max} \forall m \in \mathcal{M}, t \in T \tag{13f}$$

$$\Pi_{k,t}^{\text{in}} \rho_k^{\text{C,min}} \leq \Pi_{k,t}^{\text{out}} \leq \Pi_{k,t}^{\text{in}} \rho_k^{\text{C,max}} \forall k \in \mathcal{K}, t \in T \tag{13g}$$

$$0 \leq F_{v,t}^{\text{GE}} \leq F_v^{\text{GE,max}} \forall v \in \Omega^{\text{GE}}, t \in T \tag{13h}$$

$$0 \leq F_{f,t}^{\text{GC}} \leq F_f^{\text{GC,max}} \forall f \in \Omega^{\text{GC}}, t \in T \tag{13i}$$

6) dual constraints for the GMC problems (one set for each operating condition, t):

$$\beta_{w,t} - u_{m(w),t} + \Phi_{4,w,t}^{\max} - \Phi_{4,w,t}^{\min} = 0 \quad \forall w \in \mathcal{W}, t \in T \tag{14a}$$

$$u_{m(e),t} - C_{e,t}^{\text{GL}} + \Phi_{5,e,t}^{\max} - \Phi_{5,e,t}^{\min} = 0 \quad \forall e \in \mathcal{E}, t \in T \tag{14b}$$

$$u_{m,t} - u_{n,t} + 2\text{sgn}\left(F_{m,n,t}^0\right) F_{m,n,t}^0 \Phi_{2,m,n,t} = 0 \quad \forall m \in \mathcal{M}, n \in \mathbb{G}_m, t \in T \tag{14c}$$

$$- \sum_{n \in \mathbb{G}_m} W_{m,n}^2 \cdot (\Phi_{2,m,n,t} - \Phi_{2,n,m,t}) + \Phi_{6,m,t}^{\max} \tag{14d}$$

$$- \Phi^{\min}_{6,m,t} + \sum_{k\in\mathbb{C}(m)^{\text{in}}} \left(\Phi^{\min}_{7,k,t}\rho^{\min}_k - \Phi^{\max}_{7,k,t}\rho^{\max}_k\right)$$
$$+ \sum_{k\in\mathbb{C}(m)^{\text{out}}} \left(\Phi^{\max}_{7,k,t} - \Phi^{\min}_{7,k,t}\right) = 0 \forall m \in \mathcal{M}, t \in T$$

$$(1+\vartheta_k)u_{m^{\text{in}}_k,t} - u_{m^{\text{out}}_k,t} + \Phi^{\max}_{3,k,t} - \Phi^{\min}_{3,k,t} = 0 \quad \forall k \in \mathcal{K}, t \in T \tag{14e}$$

$$-\gamma^{\text{GE}}_{v,t} + u_{m(v),t} + \Phi^{\max}_{8,v,t} - \Phi^{\min}_{8,v,t} = 0 \quad \forall v \in \Omega^{\text{GE}}, t \in T \tag{14f}$$

$$-\gamma^{\text{GE}}_{f,t} + u_{m(f),t} + \Phi^{\max}_{9,f,t} - \Phi^{\min}_{9,f,t} = 0 \quad \forall f \in \Omega^{\text{GC}}, t \in T \tag{14g}$$

$$\Phi^{\min}_{3,k,t}, \Phi^{\max}_{3,k,t} \geq 0 \forall k \in \mathcal{K}, t \in T \tag{14h}$$

$$\Phi^{\min}_{4,w,t}, \Phi^{\max}_{4,w,t} \geq 0 \forall w \in \mathcal{W}, t \in T \tag{14i}$$

$$\Phi^{\min}_{5,e,t}, \Phi^{\max}_{5,e,t} \geq 0 \forall e \in \mathcal{E}, t \in T \tag{14j}$$

$$\Phi^{\min}_{6,m,t}, \Phi^{\max}_{6,m,t} \geq 0 \forall m \in \mathcal{M}, t \in T \tag{14k}$$

$$\Phi^{\min}_{7,k,t}, \Phi^{\max}_{7,k,t} \geq 0 \forall k \in \mathcal{K}, t \in T \tag{14l}$$

$$\Phi^{\min}_{8,v,t}, \Phi^{\max}_{8,v,t} \geq 0 \forall v \in \Omega^{\text{GE}}, t \in T \tag{14m}$$

$$\Phi^{\min}_{9,f,t}, \Phi^{\max}_{9,f,t} \geq 0 \forall f \in \Omega^{\text{GC}}, t \in T \tag{14n}$$

7) and strong duality for the GMC problems (one for each operating condition, t):

$$\sum_{w\in\mathcal{W}} \beta_{w,t}F^{\text{S}}_{w,t} - \sum_{e\in\mathcal{E}} C^{\text{GL}}_{e,t}F^{\text{L}}_{e,t} - \sum_{v\in\Omega^{\text{GE}}} \gamma^{\text{GE}}_{v,t}F^{\text{GE}}_{v,t} \tag{15}$$
$$- \sum_{f\in\Omega^{\text{GC}}} \gamma^{\text{GC}}_{f,t}F^{\text{GC}}_{f,t} =$$
$$- \sum_{m\in\mathcal{M},n\in\mathbb{G}_m} \operatorname{sgn}\left(F^0_{m,n,t}\right)\left(F^0_{m,n,t}\right)^2 \Phi_{2,m,n,t}$$
$$- \sum_{k\in\mathcal{K}} F^{\text{C,max}}_k \Phi^{\max}_{3,k,t} - \sum_{w\in\mathcal{W}} F^{\text{S,max}}_w \Phi^{\max}_{4,w,t}$$
$$- \sum_{e\in\mathcal{E}} F^{\text{L,max}}_e \Phi^{\max}_{5,e,t}$$
$$- \sum_{m\in\mathcal{M}} \left(\Pi^{\max}_m \Phi^{\max}_{6,m,t} - \Pi^{\min}_m \Phi^{\min}_{6,m,t}\right)$$
$$- \sum_{v\in\Omega^{\text{GE}}} F^{\text{GE,max}}_v \Phi^{\max}_{8,v,t} - \sum_{f\in\Omega^{\text{GC}}} F^{\text{GC,max}}_f \Phi^{\max}_{9,f,t} :$$
$$\kappa_{t,l} \forall t \in T,$$

where m^{in}_k and m^{out}_k denote inflow and outflow nodes of compressor k, respectively.

Constraints (10)–(12) represent the optimality conditions of EMC problems for all of the operating conditions, while (13)–(15) represent the optimality conditions of GMC problems for all of the operating conditions. Thus, producer l's MPEC is (8)–(15).

Generalized Nash equilibria can be computed by solving simultaneously all of the producers' MPECs. This can be done efficiently by combining the KKT conditions for each MPEC, which gives an EPEC [3]. The KKT conditions of producer l's MPEC, which we denote KKT_l, consist of the following three sets of conditions.

1) Primal equality constraints of producer l's MPEC, which consist of (10a), (10f), (11a)–(11d), (12), (13a), (13b), (14a)–(14g), and (15).
2) Stationarity conditions, which are obtained by setting the gradient of the Lagrangian of producer l's MPEC equal to zero.
3) Complementarity conditions that are associated with the inequality constraints that are in producer l's MPEC.

For sake of simplicity, we do not list the KKT conditions here. Deriving KKT conditions is a relatively simple exercise. For example, the solver EMP,[1] which is available in GAMS, derives KKT conditions automatically.

In addition to these KKT conditions, a generalized Nash equilibrium should satisfy the following sets of equations:

$$F^{\text{GE}}_{v,t} = \eta_v P^{\text{GE}}_{v,t} \forall v \in \Omega^{\text{GE}}, t \in T \tag{16a}$$

$$F^{\text{GC}}_{f,t} = \eta_f P^{\text{GC}}_{f,t} \forall f \in \Omega^{\text{GC}}, t \in T, \tag{16b}$$

which ensure that fuel that is consumed by each gas-fired unit in the EMC solution equals fuel that is supplied in the GMC solution. Constraints (16) assume that the gas consumption of each gas-fired unit is linear in its active-power output. Thus, the resulting EPEC is:

$$\text{KKT}_l \forall l \in \mathcal{L} \text{ and (16)}. \tag{17}$$

Because system of equalities and inequalities (17) is nonlinear, we linearize it using the following three steps [3].

1) Strong-duality equalities (12) and (15) are replaced by the equivalent complementarity conditions, (4b)–(4e) and (6c)–(6i), of the EMC and GMC problems, respectively.
2) The complementary-slackness conditions in KKT_l are linearized using the technique that is proposed by Fortuny-Amat and McCarl [17], which requires binary variables.
3) Bilinear terms involving $\Upsilon_{l,t}$ and $\kappa_{l,t}$, *i.e.*, the dual variables that are associated with (12) and (15), are linearized using binary expansion (which is an approximation) or by fixing them to values that are obtained using trial-and-error.

The big-M values that are used in linearization step 2 are obtained using trial-and-error. We denote the linearized version of (17) as LKKT_{all}.

Because EPEC (17) may have multiple solutions [2], we use the following auxiliary optimization problem:

$$\min \sum_{f\in\mathcal{F}} K_f X_f - \sum_{t\in T,v\in\Omega^{\text{C}}} \sigma_t P^{\text{GE}}_{v,t} \cdot \left(\lambda_{i(v),t} - C^{\text{G}}_v\right) \tag{18}$$
$$- \sum_{t\in T,v\in\Omega^{\text{GE}}} \sigma_t P^{\text{GE}}_{v,t} \cdot \left(\lambda_{i(v),t} - C^{\text{OE}}_v - \eta_v u_{m(v),t}\right)$$
$$- \sum_{t\in T,f\in\mathcal{F}} \sigma_t P^{\text{GC}}_{f,t} \cdot \left(\lambda_{i(f),t} - C^{\text{OC}}_f - \eta_f u_{m(f),t}\right)$$
$$- \sum_{t\in T,w\in\mathcal{W}} \sigma_t F^{\text{S}}_{w,t} \cdot \left(u_{m(w),t} - C^{\text{S}}_w\right)$$
$$\text{s.t. } \text{LKKT}_{\text{all}},$$

which maximizes the total profit (TP) of all producers, to search for equilibria in which producers maximize the joint exercise of market power. Objective function (18) can be

[1] https://www.gams.com/latest/docs/UG_EMP.html

linearized [1], [2]. Alternative objectives, such as maximizing social welfare or the profit of an individual producer, can be used to search for other equilibria.

The resulting EPEC model is a mixed-integer LP (MILP) problem, which can be solved using branch-and-cut solvers, such as CPLEX or GUROBI.

We use a diagonalization algorithm [18] to check whether or not an EPEC solution is a generalized Nash equilibrium.

IV. Illustrative Example

To illustrate the proposed model, this section presents results from a simple example. The assumed topologies of the networks are shown in Fig. 2. The coupling between the gas and power systems includes an existing gas-fired unit at bus 3 (node 3) and two candidate gas-fired units at bus 1 (node 1) and bus 3 (node 3). Producer 1 owns existing power unit 1 and candidate gas-fired unit 1, while producer 3 owns gas source 1. Producer 2 owns existing gas-fired unit 2, candidate gas-fired unit 2, and gas source 2. The two candidate gas-fired units have maximum capacities of 200 MW each. Their annualized capital costs are \$7600/MW and \$9000/MW, respectively, while their heat rates are 0.005 Mm^3/MWh and 0.0045 Mm^3/MWh, respectively. We consider three operating conditions with weights of 1095 h, 4380 h, and 3285 h, during which the total electricity demands are 300 MW, 225 MW, and 150 MW, respectively, and the total gas demands are 2.0 Mm^3/h, 1.5 Mm^3/h, and 1.0 Mm^3/h, respectively. Table I summarizes the marginal utilities of electricity and natural gas demands in the three operating conditions.

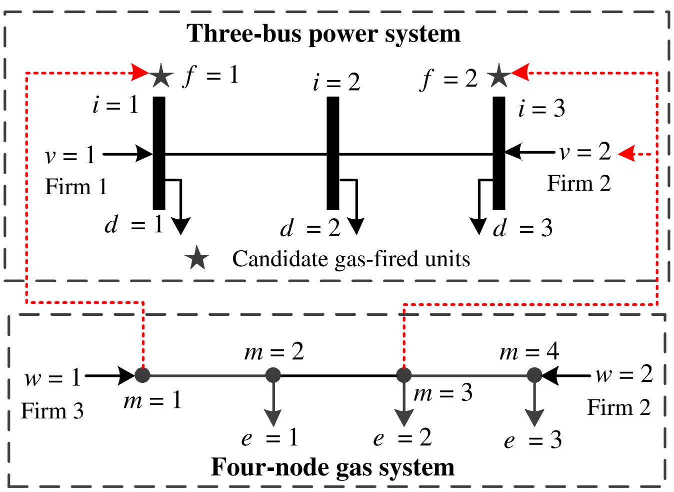

Fig. 2. Example: Coupled three-bus power system and four-node gas system.

EPEC model (18) is solved using CPLEX and GAMS on the NEOS Server [19].

To investigate the impact of power system congestion on investment equilibria, we consider cases in which the transmission capacity of the line connecting buses 2 and 3 is 200 MW, 160 MW, 140 MW, and 100 MW. Tables II–IV summarize results for the example. These tables demonstrate the following three findings.

1) Reducing the capacity of the line connecting buses 2 and 3 results in less capacity of the candidate gas-fired unit at bus 3 being built. This increases the profits of producers 1 and 3 while reducing that of producer 2.
2) There is no congestion in the case with 200 MW of transmission capacity between buses 2 and 3. Congestion surpluses with 160 MW, 140 MW and 100 MW of transmission capacity between buses 2 and 3 are \$0.88 million, \$0.77 million, and \$0.55 million, respectively. The reduced transmission surplus is due to reduced ELMP differences and less flow between buses 2 and 3, as shown in Table III (the ELMP during $t = 3$ is always \$20/MWh, which is why it is not shown in the table).
3) Table IV summarizes GLMPs in the peak-demand operating condition. It shows that as transmission capacity between buses 2 and 3 is reduced, nodes 3 and 4 (which fuel primarily the gas-fired units at bus 3) becomes less stressed, with a commensurate drop in their GLMPs. Conversely, the GLMP at node 1 increases, due to greater fuel demand from gas-fired unit 1.

TABLE I
Example: Marginal Utilities of Electricity and Natural Gas Demands in the Three Operating Conditions

	Electricity Demand Utility (\$/MWh)			Gas Demand Utility (\$/Mm^3)		
t	$i=1$	$i=2$	$i=3$	$m=1$	$m=2$	$m=3$
1	25	26	24	2800	3000	2900
2	23	24	22	2600	2800	2700
3	21	22	20	2400	2600	2500

TABLE IV
Example: GLMPs in Operating Condition 1 (Peak Demand) (\$/Mm^3)

$P_{2,3}^{\max}$	$m=1$	$m=2$	$m=3$	$m=4$
200	2678	2800	2900	2900
160	2795	2800	2803	2803
140	2800	2800	2800	2800
100	2800	2800	2800	2800

These types of findings can be used by a market regulator or policymaker to promote investments that increase SW.

V. Case Study

This section summarizes the results from a case study that is based on a Belgian 24-node power system[2] and 20-node gas system [20], which are shown in Fig. 3. The power system includes 7 candidate gas-fired units at buses 2, 6, 8, 14, 15, 21, and 22. We consider three producers including producer 1, which owns the power units in area 1, producer 3, which owns the gas sources in area A, and producer 2, which owns the power units in area 2 and the gas sources in area B. To illustrate the proposed model, eight cases are considered.

[2]https://doi.org/10.5281/zenodo.999150

TABLE II
EXAMPLE: INVESTMENT RESULTS

$P_{2,3}^{\max}$	Investment Cost ($ million)	Added Capacity (MW) Unit 1	Unit 2	Profit ($ million) Firm 1	Firm 2	Firm 3	Total Profit ($ million)	Social Welfare ($ million)
200	2.12	42	200	0.71	16.37	3.74	20.82	26.95
160	1.97	37	188	1.42	15.67	3.78	20.87	26.90
140	2.10	40	200	1.78	14.96	4.34	21.08	26.85
100	2.05	80	160	3.49	13.25	4.41	21.15	26.58

TABLE III
EXAMPLE: ELMPS AND POWER FLOWS

	ELMPs ($/MWh)											
	$t=1$			$t=2$			$P_{1,2}$ (MW)			$P_{3,2}$ (MW)		
$P_{2,3}^{\max}$	$i=1$	$i=2$	$i=3$	$i=1$	$i=2$	$i=3$	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
200	24	24	24	22	22	22	15	−45	−30	165	180	120
160	25	25	24	23	23	22	20	−25	−30	160	160	120
140	25	25	24	23	23	22	40	−5	−20	140	140	100
100	25	25	24	23	23	22	80	35	21	100	100	69

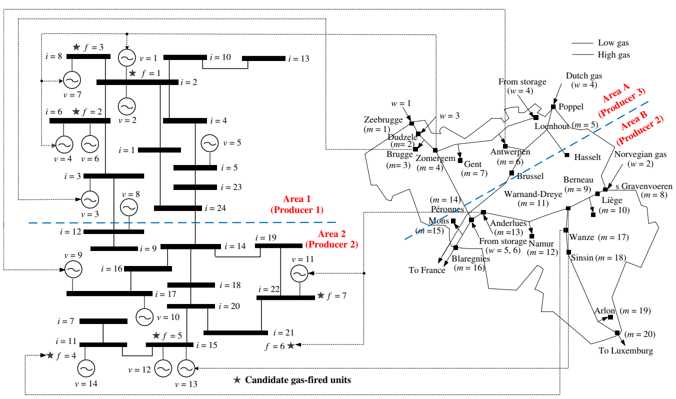

Fig. 3. Case Study: Belgian 24-node power system and 20-node gas system.

A. Tie-Line Transmission Capacity

We investigate the impact of the capacity of the line connecting bus 24, which is in area 1, to bus 14, which is in area 2. We consider cases in which the capacity of this line is 1720 MW and 3000 MW. Tables V and VI provide results for these cases. Table V shows that greater transmission capacity results in higher generation investment in area 2 and higher profit for producer 2 from the electricity market. Additionally, the power flow from area 2 to area 1 increases, and, consequently, for each operating condition, the ELMPs in area 2 increase, while the ELMPs in area 1 decrease (*cf.* Table VI).

TABLE VI
CASE STUDY: LOAD-WEIGHTED ELMPS ($/MWH) OF AREAS 1 AND 2 FOR TWO TIE-LINE CAPACITIES

	Area 1			Area 2		
$P_{14,24}^{\max}$	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
1720	60.0	50.0	45.0	49.9	40.0	36.0
3000	60.0	47.2	42.4	54.4	41.6	36.9

B. Gas-Pressure Limits

We consider two cases, in which the operation ranges of nodal gas pressures are between 30 bar and 70 bar and between 35 bar and 65 bar, respectively. Table VII and Fig. 4 summarize the results for these cases. These results indicate that stricter gas-pressure limits result in a) lower TP and SW (*cf.* Table VII), b) higher GLMPs (*cf.* Fig. 4), and c) lower profits for producers 1 and 2 from the electricity market due to higher fuel costs for gas-fired units (*cf.* Table VII).

TABLE VII
CASE STUDY: PRODUCERS' PROFITS, TP, AND SW FOR TWO GAS-PRESSURE RANGES

	Profit ($ million)					Social
		$l=2$				Welfare
Range	$l=1$	(Electricity)	(Gas)	$l=3$	Total	($ million)
30–70	1083	1044	151	984	3262	3723
35–65	1066	1019	195	941	3221	3654

These results could translate into policy action to reinforce the gas network, which may stimulate investments in gas-fired units and increase SW.

C. Error of the Gas-Flow Model

To measure the accuracy of linearized natural gas flow model (7), we solve an exact gas-flow model, consisting of nonlinear constraints (6a) and (6b), using Newton's method to obtain a gas-flow solution that satisfies all the equality constraints pertaining to the gas system. We set a slack gas node, the gas pressure of which is fixed while its gas supply is unknown prior to solving the exact gas-flow model. Specifically, we fix the variables that pertain to natural gas injections and demands for all nodes except the slack one, and the gas pressures for the slack node to the corresponding values that are obtained from the linearized EPEC model.

TABLE V
CASE STUDY: INVESTMENT RESULTS FOR TWO TIE-LINE CAPACITIES

$P_{14,24}^{\max}$	Investment Cost ($ million)	Added Capacity (GW) Area 1	Area 2	Profit ($ million) $l=1$	$l=2$ (Electricity)	$l=2$ (Gas)	$l=3$	Total	Social Welfare ($ million)
1720	35.9	2.40	2.39	1083	1044	151	984	3262	3723
3000	36.6	1.74	3.05	959	1074	130	961	3123	3716

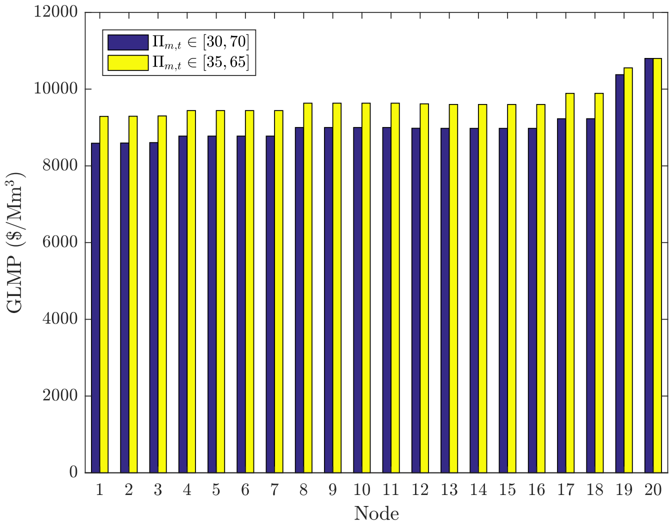

Fig. 4. Example: GLMPs for two sets of gas-pressure limits.

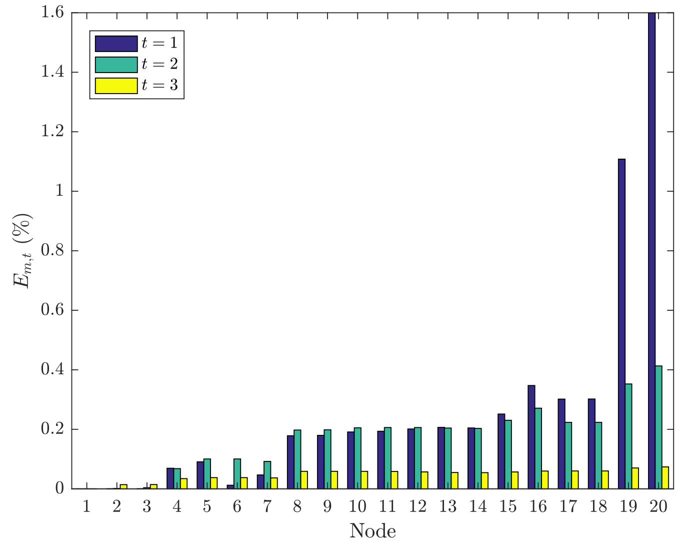

Fig. 5. Case Study: Linearization error of gas nodal pressures under three operating conditions.

Then, we define the following index:

$$\mathrm{E}_{m,t} = \frac{\sqrt{\Pi_{m,t}^{\mathrm{L}}} - \sqrt{\Pi_{m,t}^{\mathrm{E}}}}{\sqrt{\Pi_{m,t}^{\mathrm{E}}}} \cdot 100\% \forall m \in \mathcal{M}, t \in T, \quad (19)$$

where $\Pi_{m,t}^{\mathrm{L}}$ and $\Pi_{m,t}^{\mathrm{E}}$ denote the squared pressure of node m during operating condition t that is obtained from the linearized and exact natural gas flow models, respectively.

Fig. 5 shows $E_{m,t}$ for all nodes under three operating conditions. We observe from this figure that the linearization errors under operating condition $t = 1$ are larger than under the other two operating conditions. However, these linearization errors ($E_{m,t} \leq 1.6\% \forall m \in \mathcal{M}, t \in T$) are acceptable for practical applications. These results indicate that the linearized gas-flow model is sufficiently accurate.

D. Investment Equilibria under Perfect Competition

We compare investment equilibria under perfect and imperfect competition. Specifically, we consider the case in which all producers are non-strategic and offer at their marginal production costs, but remain strategic in their investment decisions. Tables VIII and IXsummarize the results with a tie-line capacity of 3000 MW (*i.e.*, $P_{14,24}^{\max} = 3000$ MW). The outcomes indicate that perfect competition results in 1) lower ELMPs and GLMPs and 2) equal total newly built capacity, but higher capacity built in Area 2. In addition, the SW under imperfect and perfect competition are similar. However, the producers' profits under imperfect and perfect competition differ significantly. This is because the perfect-competition case allows firms to exercise market through their investment decisions only. Conversely, the firms have greater purview to exercise market power through their investment, offering, and bidding strategies under imperfect competition. Fig. 6 summarizes the producers' profit in each operating condition under perfect and imperfect competition. The results in Table IXand Fig. 6 indicate that the market outcomes under perfect and imperfect competition are relatively close during operating condition $t = 1$, but are largely different under the other two operating conditions.

TABLE IX
CASE STUDY: LOAD-WEIGHTED ELMPS ($/MWH) AND GLMPS ($/MM^3) UNDER PERFECT AND IMPERFECT COMPETITION

Type of Competition	ELMP $t=1$	$t=2$	$t=3$	GLMP $t=1$	$t=2$	$t=3$
Imperfect	56.9	45.5	40.5	9002	7000	5600
Perfect	56.6	34.1	30.0	8416	5200	4500

E. Computation of Multiple Equilibria

We search for multiple equilibria by selecting different values of $\Upsilon_{l,t}$ and $\kappa_{l,t}$. Three equilibria, which are summarized in Table X, are found. The third equilibrium, in which the fixed values of $\Upsilon_{l,t}$ and $\kappa_{l,t}$ are larger than those in the other two equilibria, results in the highest TP and SW. The EPEC does

TABLE VIII
CASE STUDY: INVESTMENT RESULTS UNDER PERFECT AND IMPERFECT COMPETITION WITH $P^{\max}_{14,24} = 3000$ MW

Type of Competition	Investment Cost ($ million)	Added Capacity (GW)		Profit ($ million)					Social Welfare ($ million)
		Area 1	Area 2	$l=1$	$l=2$ (Electricity)	$l=2$ (Gas)	$l=3$	Total	
Imperfect	36.6	1.74	3.05	959	1074	130	961	3123	3716
Perfect	37.4	1.05	3.74	683	812	69	411	1975	3741

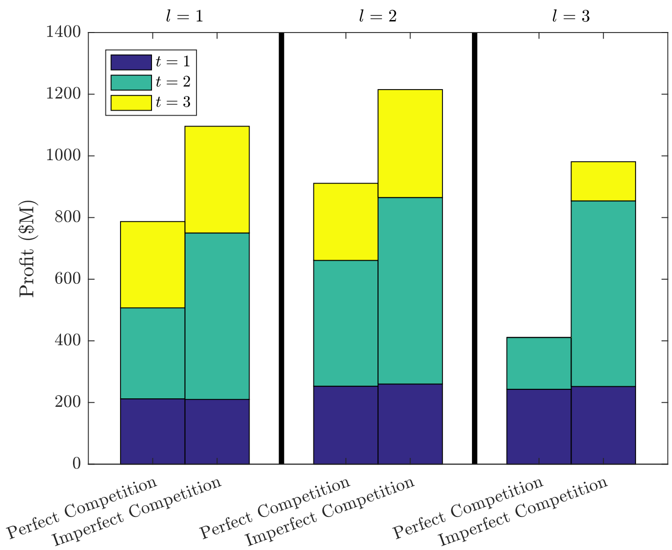

Fig. 6. Case Study: Producers' profit in each operating condition under perfect and imperfect competition.

not identify any equilibria if $\Upsilon_{l,t}$ and $\kappa_{l,t}$ are smaller than 3000 or larger than 150000.

TABLE X
CASE STUDY: PRODUCERS' PROFITS, TP, AND SW ($ MILLION) UNDER DIFFERENT EQUILIBRIA

$\Upsilon_{l,t} = \kappa_{l,t}$	Profit				Social
$\forall l \in \mathcal{L}, t \in T$	$l=1$	$l=2$	$l=3$	Total	Welfare
6000	1082	1027	981	3090	3524
9000	1082	1050	972	3104	3559
10000	1083	1195	984	3262	3723

F. Electricity and Gas Demands

We consider three cases. The first two cases have 5% higher electricity demands and 5% higher gas demands, respectively, relative to the base case. The third case has 5% higher marginal utility for gas demands relative to the base case. Table XI summarizes the equilibria that are obtained from the base case and cases with different demands. These results show the interaction between the electricity and gas markets. Increasing the electricity demands results in higher profits for both electricity and gas producers. However, increasing the gas demands or the utilities of the gas demands results in higher gas-producer profits but lower electricity-producer profits. This is because these two cases yield higher gas prices, which increases the fuel cost of gas-fired units. Higher electricity demand leads to increased investment in area 2, which is required to supply the added electricity demand.

G. Number of Producers

Table XII reports the impact of the number of producers on the computational complexity of the EPEC. Clearly, a larger number of producers results in higher computational burden. However, the EPEC model is solved in a reasonable amount of time.

TABLE XII
CASE STUDY: COMPUTATIONAL COMPLEXITY OF EPEC WITH DIFFERENT NUMBERS OF FIRMS

$\lvert\mathcal{L}\rvert$	Binary Variables	Columns	Rows	Solution Time (minutes)
3	2912	6054	7074	19
6	5160	10195	12128	24
8	7236	14225	17157	40
10	9373	17989	22026	52

H. Number of Operating Conditions

The number of operating conditions is increased to six, nine, and 12 in the case of 3000 MW of transmission capacity between buses 14 and 24. Equilibria that are obtained with different numbers of operating conditions are provided in Table XIII. This table shows that TP decreases gradually with the number of operating conditions. On the other hand, the EPEC remains computationally tractable, even with 12 operating conditions.

VI. CONCLUSION

This paper develops an EPEC to characterize investment equilibria that are reached by power, gas, and hybrid strategic producers. Our results show that transmission-capacity constraints in the power system and gas-pressure limits in the gas system impact the equilibria that are obtained, the profits of the competing producers, and both ELMPs and GLMPs. On the computational side, the MILP problem representing the EPEC is tractable for realistic power and gas systems. Our work highlights the importance of representing the gas-market and its associated network constraints in generation-investment problems that include gas-fired units. Our work can help a regulator (and other policymakers) to gain insight into 1) the coupling between electricity and gas markets, 2) the investment behaviors of strategic producers, and 3) how supply-side market power impacts the investment decisions and profits of each producer. Our model also may help a

TABLE XI
CASE STUDY: INVESTMENT RESULTS WITH DIFFERENT ELECTRICITY AND GAS DEMANDS

	Added Capacity (GW)		Profit ($ million)					Social Welfare
Case	Area 1	Area 2	$l = 1$	$l = 2$ (Electricity)	$l = 2$ (Gas)	$l = 3$	Total	($ million)
Base	2.40	2.39	1083	1044	151	984	3263	3723
High Electricity Demand	2.40	3.32	1096	1046	176	984	3302	3776
High Gas Demand	2.40	2.39	1071	1042	183	984	3280	3728
High Gas Utility	2.40	2.39	1053	1035	171	1110	3369	3848

TABLE XIII
CASE STUDY: EQUILIBRIUM RESULTS WITH DIFFERENT NUMBERS OF OPERATING CONDITIONS

	Profit ($ million)				Social Welfare	Binary			Solution
$\|T\|$	$l = 1$	$l = 2$	$l = 3$	Total	($ million)	Variables	Columns	Rows	Time (minutes)
3	1083	1195	984	3262	3723	2912	6054	7074	19
6	1068	1181	973	3222	3650	6275	12825	15292	41
9	1076	1166	970	3212	3683	8892	18492	21647	181
12	1078	1125	991	3194	3672	11573	24206	28206	347

regulator to design better rules for both electricity and gas markets.

Our modeling framework can be extended to consider uncertainty in renewable generation by introducing a larger number of operating conditions. This might result in intractability, which can be addressed by decomposition [21].

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