Winners and Losers from Vertical Integration Between Natural-Gas and Electricity Markets

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ABSTRACT

Electricity systems in many parts of the world are becoming more dependent upon natural gas as an electricity-generation fuel. As such, electricity and natural-gas markets are becoming more interconnected. Contemporaneously, some electricity and natural-gas markets are integrating vertically, through the merger of electricity and natural-gas suppliers. The market-efficiency impacts of such vertical integration are unclear. On one hand, vertical integration could exacerbate market power, whereas on another it could mitigate double marginalization. To study this question, this paper develops a Nash-Cournot model of the two interconnected markets. The model is converted into a linear complementarity problem, which allows deriving Nash equilibria readily. Some theoretical results are derived for the case of a merger involving symmetric firms. In addition, the model is applied to a stylized example with a range of parameter values. We find that integration is social-welfare enhancing—which implies that mitigating double marginalization outweighs the exercise of market power. In most cases, the effects of merger can give rise to a prisoner's-dilemma-type outcome. Merger is beneficial to the merging firms. However, profits of non-merging firms and total supplier profits decrease following a merger. Overall, our results suggest that vertical integration in energy markets may be socially beneficial.

Keywords: Energy market, industrial organization, vertical integration, market power, double marginalization

JEL: C61, C72, D43, L1, L94, L95, Q4

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1. INTRODUCTION

Electricity and natural-gas systems in many parts of the world are becoming more interdependent, due to the increasing use of natural gas as an electricity-generation fuel. For instance, data from U.S. Energy Information Administration show that during 1950 natural gas produced 45 TWh of U.S. electricity (13% of total production). This amount increased to 1627 TWh (41% of total production) during 2020. An overlooked issue that arises from this increased linkage between electricity and natural-gas systems is the impact of vertical integration between the two commodity markets. Tirole (1988) defines vertical integration as one firm owning multiple steps in the supply chain of a commodity or product. In the aforementioned case, vertical integration is a single firm owning natural-gas facilities, which are used to supply fuel to the electricity-generation sector. One could envision cases wherein vertical integration exacerbates market power. For instance, consider

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a vertically integrated firm that owns an electricity-generation technology that is not reliant upon natural gas as a fuel. If competing electricity suppliers use natural gas as an electricity-generation fuel, the vertically integrated firm could withhold natural-gas supply to foreclose electricity-market competition.

Vertical integration does present, however, a countervailing benefit in that it can mitigate double marginalization. The seminal work of Spengler (1950) examines two monopolies in a vertical supply chain. The upstream firm is a monopoly supplier of an input to the downstream firm, which is a monopoly supplier to end customers. Because both the upstream and downstream firms behave as monopolists (*i.e.*, each withholds supply to increase its profit), the market outcome is more efficient if the two firms merge into a single monopolist. Thus, vertical integration between a natural-gas and electricity supplier could yield a net efficiency benefit if mitigation of double marginalization outweighs increased exercise of market power by the merged firm.

This paper develops a Nash-Cournot model of interdependent natural-gas and electricity markets. We model an oligopoly that includes includes vertically integrated suppliers of both commodities as well as suppliers of one commodity only. Using necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions, which characterize a global optimum of each supplier's profit-maximization problem, we recast the Nash-Cournot problem as a linear complementarity model, which can be solved using off-the-shelf software to compute a Nash equilibrium. We use our model to derive theoretical results for the special case of merger between symmetric suppliers. We demonstrate our model and our theoretical findings with an illustrative case study and parametric analyses. We show that in many cases vertical merger is social- and consumer-welfare enhancing but yields producer-welfare losses. This result suggests that the mitigation of double marginalization outweighs any negative impact of the merged firm exercising market power. In all cases, the merged firm's profit increases whereas other firms' profits decrease post-merger. Thus, there are strong incentives for vertical integration, but firms can fall prey to a prisoners'-dilemma-type outcome in many cases. Altogether, our results suggest that vertical mergers between the natural-gas and electricity sectors can be economically beneficial.

Our work makes two contributions to the extant literature. First, we develop a flexible methodology to model and compute Nash-Cournot equilbria in interconnected natural-gas and electricity markets. Our model includes fairly basic cost structures and technical constraints on the two systems. However, our use of a complementarity approach to compute Nash equilibria is amenable to including more complex costs or constraints and can be applied to other vertical supply chains. There is a rich literature that applies Nash-Cournot models to study the effects of market structure on electricity and natural-gas markets. Bushnell et al. (2008) examine the effects of horizontal and vertical market structure within the electricity industry on the efficiency of wholesale electricity markets. Dukhanina et al. (2019) use a Nash-Cournot framework to examine the economic impacts of merging natural-gas-trading zones. More recent works examine the two markets together (Knittel, 2003; Chen et al., 2020a). However, much of the literature uses stylized models to allow for analytical solution. Our use of a complementarity approach allows for richer models of the supply chain compared to these analyses and the stylized model of Spengler (1950). Complementarity modeling is a fairly common approach to computing Nash-Cournot equilbria in energy markets (Hobbs, 2001). Although there are existing works that employ Nash-equilibrium concepts to study natural-gas and electricity markets (Chen et al., 2020c,a,b), these works focus on studying the extent of co-ordinated decision-making between the two physical systems. Importantly, these works do not consider the impacts of market and industry structure upon market and economic efficiency, as we do. To our knowledge, our work is the first to extend the work of Spengler (1950) to study the impacts of double marginalization between the natural-gas and electricity sectors. The second contribution of our work is to develop a better understanding of the countervailing impacts of vertical integration between natural-gas and electricity markets. Our finding of mixed welfare impacts of vertical integration is observed in other market analyses that show mixed effects of double marginalization (Pinopoulos, 2019). Some of our findings (*e.g.*, producer-welfare losses and the prisoners'-dilemma-type outcome) stray from those of the seminal work of Spengler (1950).

The remainder of this paper is organized as follows. Section 2 presents our model formulation, including its conversion into a linear complementarity model. Section 3 presents our theoretical results, with formal proofs given in the appendix. Section 4 presents case-study results. Section 5 concludes.

2. MODELING APPROACH

We present our modeling approach in two steps. First, we provide the assumptions that underlie the structure of the vertical supply chain and the profit-maximization problems of the suppliers. Second, we outline the steps to convert the resultant Nash-Cournot model into a linear complementarity model.

2.1 Model Assumptions and Formulation

Our model assumes a vertical supply chain that consists of two commodities—natural gas and electricity. There is a set, $\mathcal{N}$, of suppliers, the production technology of each of which is characterized by five parameters. For each $n \in \mathcal{N}$, $Q_n^G \geq 0$ and $Q_n^E \geq 0$ are firm n's natural-gas- and electricity-production capacities, respectively. A firm, $n \in \mathcal{N}$, that supplies only one of the two commodities is a special case, wherein one of Q_n^G or Q_n^E is equal to zero and the other is strictly positive. For each $n \in \mathcal{N}$, $c_n^G \geq 0$ represents firm n's per-unit variable cost of producing natural gas. Producing electricity incurs a two-part cost. For each $n \in \mathcal{N}$, $c_n^E \geq 0$ represents firm n's per-unit non-natural-gas-related variable cost of producing electricity. In addition, $\forall n \in \mathcal{N}$, $\eta_n \geq 0$ represents firm n's per-unit use of natural gas to produce electricity. For each $n \in \mathcal{N}$, firm n determines two production quantities—q_n^G and q_n^E represent its natural-gas and electricity outputs, respectively. These production decisions are made by the firms simultaneously.

Electricity demand is represented by the inverse demand function:

$$p^E(r^E) = A^E - B^E r^E,$$

where A^E and B^E are fixed parameters, the variable:

$$r^E = \sum_{n \in \mathcal{N}} q_n^E,$$

is defined as retail electricity demand by end consumers, and $p^E(r^E)$ gives the market-clearing price of electricity. There are two sources of natural-gas demand—retail demand by end consumers as well as demand by downstream electricity generators. Retail natural-gas demand by end consumers is represented by the inverse demand function:

$$p^G(r^G) = A^G - B^G r^G, \tag{1}$$

where A^G and B^G are fixed parameters, retail natural-gas demand by end consumers is defined as:

$$r^G = \sum_{n \in \mathcal{N}} \left(q_n^G - \eta_n q_n^E \right), \tag{2}$$

and $p^G(r^G)$ gives the market-clearing price of natural gas.

For each $n \in \mathcal{N}$, firm n's profit-maximization problem is:

$$\max_{q_n^G, q_n^E} \quad \left[A^G - B^G \sum_{n' \in \mathcal{N}} \left(q_{n'}^G - \eta_{n'} q_{n'}^E\right) - c_n^G\right] q_n^G \tag{3}$$

$$+ \left\{A^E - B^E \sum_{n' \in \mathcal{N}} q_{n'}^E - c_n^E - \left[A^G - B^G \sum_{n' \in \mathcal{N}} \left(q_{n'}^G - \eta_{n'} q_{n'}^E\right)\right] \eta_n \right\} q_n^E$$

$$\text{s.t.} \quad 0 \le q_n^G \le Q_n^G \tag{4}$$

$$0 \le q_n^E \le Q_n^E. \tag{5}$$

Objective function (3) consists of two terms, which give firm n's total profit from producing and selling natural gas and electricity, respectively. The first term in (3) uses the definitions in (1) and (2) to compute the market-clearing price of natural gas, based on retail natural-gas supply that is not used to supply downstream electricity producers. Along the same lines, the second term in (3) uses the market-clearing price of natural gas to compute the cost of natural gas that firm n uses for electricity production. Thus, we assume that the prices of natural gas that is used by end consumers and by downstream electricity producers are equal. Otherwise, there would be arbitrage opportunities (*i.e.*, end consumers could profitably buy natural gas from or sell natural gas to electricity producers). Constraints (4) and (5) impose non-negativity and capacity limits on firm n's production decisions.

There are two important distinctions between our model and that which Spengler (1950) uses to analyze double marginalization. In the framework that Spengler (1950) uses, demand for the upstream product is driven solely by production of the downstream firm. Conversely, our framework has two sources of demand for natural gas—retail demand by end consumers and natural gas that is used as an input fuel for downstream electricity production. Second, Spengler (1950) assumes one monopoly supplier of each of the upstream and downstream commodities and compares that to a case wherein the two monopoly suppliers merge. Our framework allows for multiple suppliers, some of which are vertically integrated between the two commodity markets.

2.2 Linear Complementarity Model of Nash-Cournot Equilibrium

A Nash equilibrium is a set of production levels, q_n^{G*} and q_n^{E*}, $\forall n \in \mathcal{N}$ with the property that $\forall n \in \mathcal{N}$, q_n^{G*} and q_n^{E*} solves firm n's profit-maximization problem, (3)–(5), while taking the production levels of firm n's rivals as fixed equal to $q_{n'}^{G*}$ and $q_{n'}^{E*}$, $\forall n' \in \mathcal{N}, n' \neq n$. Thus, computing a Nash equilibrium amounts to solving simultaneously (3)–(5), $\forall n \in \mathcal{N}$. Because we have that B^G, B^E, and η_n, $\forall n \in \mathcal{N}$ are non-negative, (3) is concave in q_n^G and q_n^E. Moreover, (4) and (5) are linear and satisfy Slater conditions (Bertsekas, 1995). Thus, $\forall n \in \mathcal{N}$, an optimal solution to (3)–(5) can be characterized by its necessary and sufficient KKT conditions.

For each $n \in \mathcal{N}$, we let $\mu_n^{G,-}$ and $\mu_n^{G,+}$, respectively, denote Lagrange multipliers that are associated with the left- and right-hand sides of double-sided inequality (4) in firm n's profit-maximization problem. For each $n \in \mathcal{N}$, we define $\mu_n^{E,-}$ and $\mu_n^{E,+}$ analogously for (5). Then, $\forall n \in \mathcal{N}$, the KKT conditions for firm n's profit-maximization problem are:

$$-A^G + B^G \sum_{n' \in \mathcal{N}} \left(q^G_{n'} - \eta_{n'} q^E_{n'} \right) + c^G_n + B^G q^G_n - B^G \eta_n q^E_n - \mu^{G,-}_n + \mu^{G,+}_n = 0 \tag{6}$$

$$-B^G \eta_n q^G_n - A^E + B^E \sum_{n' \in \mathcal{N}} q^E_{n'} + c^E_n + \left[A^G - B^G \sum_{n' \in \mathcal{N}} \left(q^G_{n'} - \eta_{n'} q^E_{n'} \right) \right] \eta_n + B^E q^E_n \tag{7}$$
$$+B^G {\eta_n}^2 q^E_n - \mu^{E,-}_n + \mu^{E,+}_n = 0$$

$$0 \le q^G_n \perp \mu^{G,-}_n \ge 0 \tag{8}$$
$$q^G_n \le Q^G_n \perp \mu^{G,+}_n \ge 0 \tag{9}$$
$$0 \le q^E_n \perp \mu^{E,-}_n \ge 0 \tag{10}$$
$$q^E_n \le Q^E_n \perp \mu^{E,+}_n \ge 0, \tag{11}$$

where $\perp$ denotes complementary-slackness or orthogonality (Sioshansi and Conejo, 2017). Thus, a Nash equilibrium can be computed by solving simultaneously (6)–(11), $\forall n \in \mathcal{N}$.

Complementary-slackness conditions (8)–(11) are nonlinear. This nonlinearity arises because a generic complementary-slackness condition of the form:

$$0 \le x \perp y \ge 0,$$

is mathematically equivalent to:

$$0 \le x \tag{12}$$
$$y \ge 0 \tag{13}$$
$$xy = 0. \tag{14}$$

Fortuny-Amat and McCarl (1981) propose an approach to linearize complementary-slackness conditions, which requires adding one auxiliary binary variable for each condition, but can make the conditions more computationally tractable. Specifically, we introduce a binary variable, ξ, and replace (12)–(14) with:

$$0 \le x \le M\xi \tag{15}$$
$$M \cdot (1 - \xi) \ge y \ge 0 \tag{16}$$
$$\xi \in \{0, 1\}, \tag{17}$$

where M is a sufficiently large constant.

Thus, our approach to computing Nash-Cournot equilibria is to solve simultaneously (6)–(11), $\forall n \in \mathcal{N}$, where we use (15)–(17) to linearize (8)–(11). The resultant system of equations and inequalities have the same mathematical structure as the constraints of a mixed-integer linear optimization model and can be solved readily using standard off-the-shelf optimization software.

We note, finally, that (1) and (2) provide the linkage between the two commodity markets, specifically through the parameters, $\eta_1, \dots, \eta_{|\mathcal{N}|}$. Indeed, if $\eta_n = 0$, $\forall n \in \mathcal{N}$, our model devolves into two separate Nash-Cournot problems, one for each of the two commodities. This nature of the coupling of the two commodity markets is evident from KKT conditions (6)–(7), because they are the sole conditions that involve natural-gas- and electricity-production decisions together. However, if $\eta_n = 0$, $\forall n \in \mathcal{N}$, then (6) and (7) involve, respectively, only natural-gas- and electricity-production decisions, thereby decoupling the two commodity markets.

3. THEORETICAL RESULTS

We begin with a theoretical analysis of Nash equilibria that are given by (6)–(11), $\forall n \in \mathcal{N}$. To make the analysis tractable, we consider a stylized case with symmetric suppliers that have the same costs and per-unit fuel use to produce electricity and that have neither binding capacity nor non-negativity constraints. The following formalizes these assumptions.

Assumption 1 *For all $n \in \mathcal{N}$ we have that $c_n^G = c^G$, $c_n^E = c^E$, and $\eta_n = \eta$.*

Assumption 2 *The parameters, A^G, B^G, c^G, A^E, B^E, Q^G, c^E, η, and Q^E are chosen so that a solution to (6)–(11) is equivalent to a solution (6)–(7) with $\mu_n^{G,-} = \mu_n^{G,+} = \mu_n^{E,-} = \mu_n^{E,+} = 0$, $\forall n \in \mathcal{N}$.*

Assumption 1 states that the firms are symmetric with respect to cost and per-unit electricity-production fuel use. Assumption 2 allows us to focus our analysis on interior solutions of (6)–(11), thereby simplifying the analysis considerably. Under these assumptions, (6)–(11), $\forall n \in \mathcal{N}$ become:

$$-A^G + B^G \sum_{n' \in \mathcal{N}} \left(q_{n'}^G - \eta q_{n'}^E\right) + c^G + B^G q_n^G - B^G \eta q_n^E = 0 \tag{18}$$

$$-B^G \eta q_n^G - A^E + B^E \sum_{n' \in \mathcal{N}} q_{n'}^E + c^E + \left[A^G - B^G \sum_{n' \in \mathcal{N}} \left(q_{n'}^G - \eta q_{n'}^E\right)\right] \eta + B^E q_n^E + B^G \eta^2 q_n^E = 0, \tag{19}$$

for all $n \in \mathcal{N}$.

Contrasting (18) and (19) with (6)–(11) shows two ways in which the former are simplified by our assumptions. First, because of Assumption 1, c^G, c^E, and η are not indexed by firm in (18) and (19). Second, because of Assumption 2, the Lagrange multipliers that are in (6) and (7) and complementary-slackness requirements (8)–(11) do not appear in (18) and (19).

The following lemmata use (18) and (19) to examine and compare profit-maximizing production decisions of firms that produce only one of the two products to vertically integrated firms. To this end, we begin by characterizing each firm's optimal reaction function. For each $n \in \mathcal{N}$, we define:

$$q_{n-}^G = \sum_{n' \in \mathcal{N}, n' \neq n} q_{n'}^G,$$

and:

$$q_{n-}^E = \sum_{n' \in \mathcal{N}, n' \neq n} q_{n'}^E,$$

as aggregate natural-gas and electricity production, respectively, by firm n's competitors.

Lemma 1 *For all $n \in \mathcal{N}$, (18) and (19) yield the profit-maximizing reaction functions:*

$$q_n^G = \frac{1}{2B^G} \cdot \left[A^G - c^G + B^G \eta \cdot \left(2q_n^E + q_{n-}^E\right) - B^G q_{n-}^G\right], \tag{20}$$

and:

$$q_n^E = \frac{1}{2B^E} \cdot \left(A^E - c^E - c^G \eta - B^E q_{n-}^E\right). \tag{21}$$

Proof. See appendix. ■

Next, the following lemmata use (20) and (21) to examine marginal impacts of rivals' production levels on profit-maximizing production decisions of a firm that supplies only one of the two commodities.

Lemma 2 *For any $n \in \mathcal{N}$ that produces natural gas only, we have:*

$$\frac{\partial}{\partial q_{n-}^{G}} q_n^G = -\frac{1}{2}, \tag{22}$$

and:

$$\frac{\partial}{\partial q_{n-}^{E}} q_n^G = \frac{\eta}{2}. \tag{23}$$

Proof. See appendix. ■

Lemma 3 *For any $n \in \mathcal{N}$ that produces electricity only, we have:*

$$\frac{\partial}{\partial q_{n-}^{G}} q_n^E = 0, \tag{24}$$

and:

$$\frac{\partial}{\partial q_{n-}^{E}} q_n^E = -\frac{1}{2}. \tag{25}$$

Proof. See appendix. ■

Lemmata 2 and 3 show differences in how upstream and downstream firms react to production decisions of the two commodities. By (22), a firm that produces natural gas only exhibits standard Nash-Cournot behavior with respect to its own commodity—it reduces its own output to maintain a higher price if its rivals increase natural-gas production. On the other hand, increased electricity production reduces retail natural-gas consumption (*i.e.*, downstream electricity generators consume more fuel). Reduced retail natural-gas consumption increases the natural-gas price, which incentivizes greater production from the natural-gas supplier to exploit the increased price, as is given by (23).

Partial derivative (25) shows that a firm that produces electricity only exhibits standard Nash-Cournot behavior with respect to its own commodity. Partial derivative (24) can be interpreted as such a firm behaving as a price-taker with respect to the cost of natural gas. This interpretation stems from an oligopsonist having:

$$\frac{\partial}{\partial q_{n-}^{G}} q_n^E > 0,$$

i.e., it increases electricity production if its rivals increase natural-gas production, due to the resultant decreased fuel price.

Next, the following lemma conducts a similar marginal analysis to those of Lemmata 2 and 3 for a vertically integrated firm that produces both natural gas and electricity.

Lemma 4 *For any $n \in \mathcal{N}$ that produces both natural gas and electricity, we have:*

$$\frac{\partial}{\partial q_{n}^{E}} q_n^G = \eta, \tag{26}$$

$$\frac{\partial}{\partial q_{n-}^{G}} q_n^G = -\frac{1}{2}, \tag{27}$$

$$\frac{\partial}{\partial q_{n-}^{E}} q_n^G = 0, \tag{28}$$

$$\frac{\partial}{\partial q_{n}^{G}} q_n^E = 0,$$

$$\frac{\partial}{\partial q_{n-}^{G}} q_n^E = 0,$$

and:

$$\frac{\partial}{\partial q_{n-}^E} q_n^E = -\frac{1}{2}. \tag{29}$$

Proof. See appendix. ■

Contrasting Lemma 4 with Lemmata 2 and 3 shows two key differences between the behavior of a vertically-integrated firm and single-commodity producers. Equality (26) can be interpreted as a vertically integrated firm self-supplying fuel on the margin. This interpretation stems from η being its marginal fuel consumption. Indeed, a vertically integrated firm produces natural gas efficiently *vis-à-vis* the downstream market, insomuch as it increases natural-gas supply to the retail market at the same rate that its electricity production consumes fuel. This result can be contrasted with (23), which shows that a firm that supplies natural gas only replaces only half of fuel that is consumed for electricity production on the margin (to maintain a higher natural-gas price). Thus, *ceteris paribus*, we conclude that due to the difference between (23) and (26), a vertical merger will yield higher natural-gas production from the merged firm as compared to pre-merger.

Furthermore, contrasting (23) and (28) shows that a vertically integrated firm does not increase natural-gas production to exploit the higher natural-gas price that is caused by increased electricity production. On the other hand, (27) implies that a vertically integrated firm does exhibit standard Nash-Cournot behavior with respect to how its natural-gas production reacts to the natural-gas production of its rivals. Taken together, a vertically integrated firm withholds supply to increase the natural-gas price, but is efficient in how its downstream fuel consumption impacts its natural-gas supply.

We conclude by noting that (26) implies increased self-supply of fuel and increased electricity supply (due to a lower fuel price) by a vertically integrated firm. Thus, following from (22) and (25), single-commodity producers should reduce natural-gas and electricity production.

4. CASE STUDY

We begin our case study by considering a base case with two natural-gas and two electricity suppliers in the market (*i.e.*, four firms total) that are not horizontally or vertically integrated. The per-unit costs of the two natural-gas suppliers are $c^G = 2$ \$/MMBtu[1] and the per-unit costs of the two electricity suppliers are computed from $c^E = 5$ \$/MWh and $\eta = 1$ MMBtu/MWh. The firms' capacities are set sufficiently high so as not to impose binding constraints. The inverse demand functions for the two commodity markets are $p^G(r^G) = 10 - 0.00005 r^G$ and $p^E(r^E) = 55 - .0025 r^E$. These values are selected to yield equilibria with production quantities and prices that are reasonable for wholesale natural-gas and electricity markets. Our model is programmed using GAMS 38.2.1 and solved using DICOPT 38.2.1 on a computer with an Apple M1 processor and 8 GB of memory. In all cases, equilibrium computation is nearly instantaneous.

The base case compares having four independent suppliers to a case wherein one natural-gas and one electricity supplier merge, which yields a market with a total of three firms. Figures 1 and 2 summarize pre- and post-merger natural-gas and electricity production levels, respectively. Vertical integration causes increased natural-gas and electricity production by the merged firm. Following from (26), the rationale for this behavior is that increasing natural-gas production decreases the natural-gas price, which reduces the cost of electricity production (for the integrated firm as well as the unintegrated electricity supplier). This reduced electricity-production cost allows the merged firm to increase its electricity production profitably. Figures 1 and 2 show that the unmerged natural-gas and electricity suppliers decrease their production levels post-merger, which follows from (22) and (25). This behavior is to mitigate the price-suppressing effect of increased natural-gas and electricity production from the merged firm. Despite this effort by the unintegrated firms, prices

[1] Despite its not being an SI unit, we measure natural-gas production using MMBtu, as that is the common unit of measure in North American wholesale natural-gas markets.

of both commodities decrease. The natural-gas price decreases from \$6.88/MMBtu pre-merger to \$5.64/MMBtu post-merger. The electricity price sees a similar decrease from \$28.46/MWh pre-merger to \$25.18/MWh post-merger. Before merger, each natural-gas supplier earns a profit of \$475 000 and each electricity supplier earns a profit of \$1 100 000. Post-merger, the vertically integrated firm earns a total profit of \$1 917 000, which is a 22% increase over the combined pre-merger profits of the two merging firms. Profits of the unintegrated natural-gas and electricity suppliers decrease to \$264 000 and \$846 000, respectively.

Figure 1: Pre- and post-merger natural-gas production by merging and unintegrated natural-gas firms in base case.

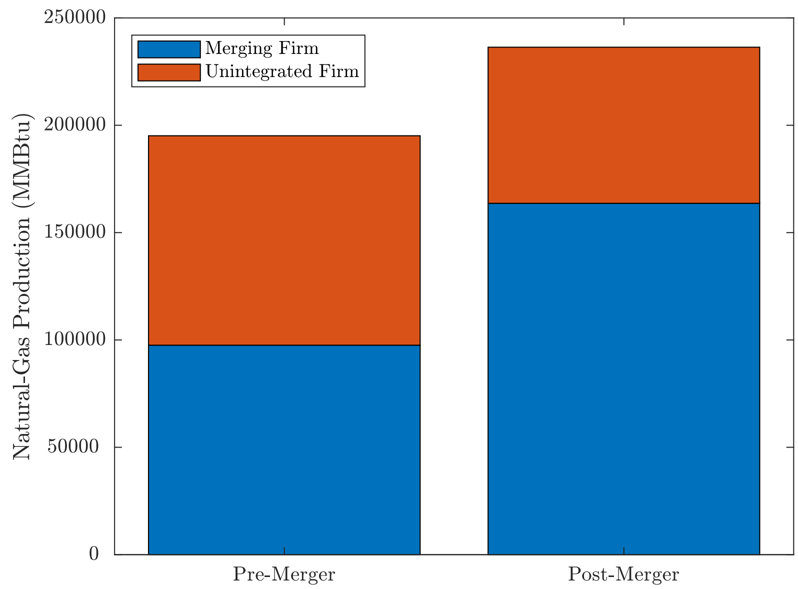


Figure 2: Pre- and post-merger electricity production by merging and unintegrated electricity firms in base case.

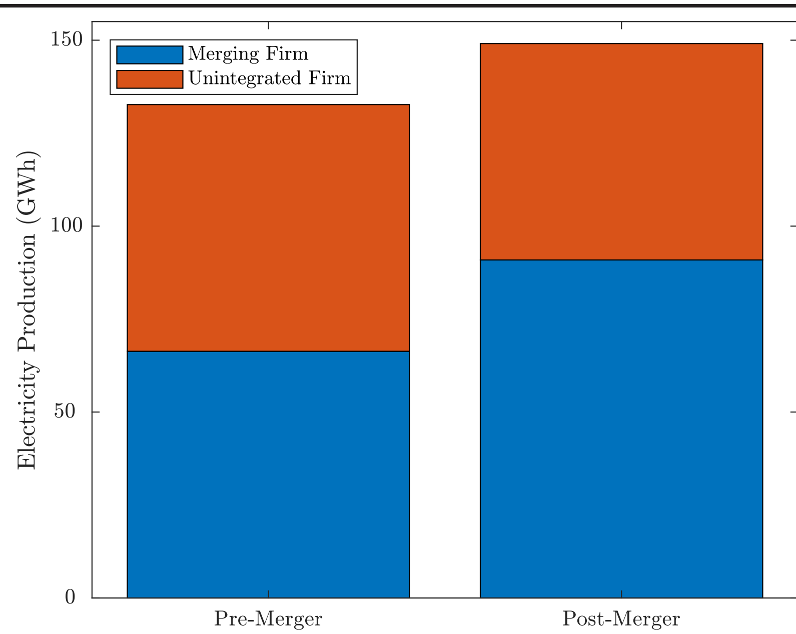


Table 1 summarizes the breakdown of pre- and post-merger social welfare between producers and consumers and between the two commodity markets. There is an overall 9% social-welfare increase from vertical integration, which is driven by a substantive consumer-surplus increase and some producer-welfare losses. Moreover, most of the social-welfare gains are in the downstream electricity market. The welfare impacts of merger upon the natural-gas market are mostly wealth transfers from producers to consumers.

Table 1: Breakdown of Pre- and Post-Merger Social Welfare ($ Thousand) in Base Case

	Pre-Merger	Post-Merger
Natural Gas		
Producer Welfare	951	860
Consumer Welfare	97	190
Social Welfare	1 049	1 050
Electricity		
Producer Welfare	2 201	2 169
Consumer Welfare	1 760	2 223
Social Welfare	3 961	4 391
Total		
Producer Welfare	3 152	3 028
Consumer Welfare	1 858	2 413
Social Welfare	5 010	5 441

We summarize now four parametric analyses, which demonstrate the sensitivity of our findings to successive mergers, the intensity of natural-gas use for electricity production, supply costs, and demand elasticity.

4.1 Successive Merger

The first sensitivity analysis that we conduct considers successive vertical mergers. All of the assumptions of this case are identical to those of the base case, except that the market has initially three unintegrated natural-gas and three unintegrated electricity suppliers. We compare the equilibrium with these six unintegrated suppliers to equilibria with successive vertical mergers, *i.e.*, first one natural-gas and electricity supplier merge, yielding five firms, followed by another vertical merger, which yields four firms, and finally a third merger yields three vertically integrated firms. Table 2 summarizes equilibrium pre-merger natural-gas and electricity prices, as well as prices under each successive vertical merger. As expected from the theoretical analysis in Section 3 and our base case, each of the three successive mergers results in successive price decreases. These price decreases have the same underlying cause as under the base case—the newly merged firm increases natural-gas production to decrease the cost of electricity production. In doing so, the merged firm is able to increase profitably its electricity production. The other firms—even the previously merged firm in the cases of the second and third merger—reduce their natural-gas and electricity production to mitigate the price suppression.

Table 2: Equilibrium Natural-Gas and Electricity Prices Under Successive Vertical Mergers

Commodity	Pre-Merger	First Merger	Second Merger	Third Merger
Natural Gas ($/MMBtu)	5.94	5.16	4.53	4.00
Electricity ($/MWh)	23.90	21.74	20.17	19.00

Table 3 demonstrates this phenomenon by summarizing firm profits before and after the successive mergers. Total industry profit is highest before any mergers occur. Following the first merger, profit of the newly integrated firm is 31% higher than the total pre-merger profits of the natural-gas and electricity suppliers that merge. Profits of each remaining unintegrated natural-gas and electricity supplier decrease by 36% and 20%, respectively. If a second vertical merger occurs, profit of the newly merged firm is 35% higher following mergerer as compared to the total pre-merger profits of the two merging firms. This second merger causes the profits of the remaining unintegrated natural-gas and electricity suppliers to decrease by a further 36% and 16% relative to their profit levels following the first merger. In addition, the profit of the first vertically integrated firm decreases by 27% compared to after the first merger. We do not endogenize the dynamics of merger decisions (*cf*. the work of Hart and Tirole (1990) for one such analysis). Nevertheless, the results of this analysis and those in Table 3 in particular, suggests that firms in vertical supply chains may face prisoners'-dilemma-type incentives. Each firm has a strong incentive to seek a vertical merger, because by doing so its profit increases whereas by not merging it is susceptible to a profit loss if a rival merges. However, as each vertical merger occurs, total industry profit decreases. Figure 3, which summarizes total pre- and post-merger consumer and producer surplus, shows that vertical mergers are beneficial to consumers and social-welfare enhancing.

Table 3: Suppliers' Profits ($ Thousand) Under Successive Vertical Mergers

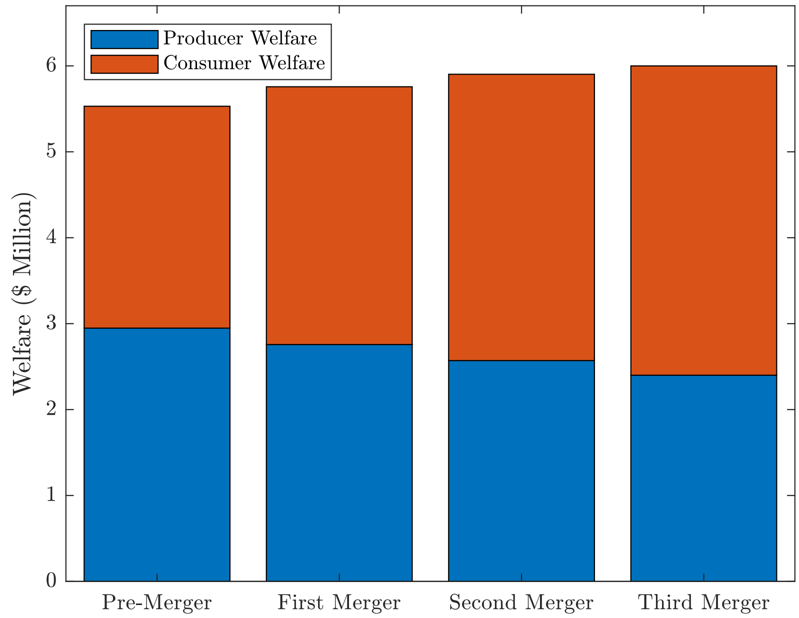
Supplier	**Pre-Merger**	**First Merger**	**Second Merger**	**Third Merger**
Natural Gas				
Firm 1	311	n/a	n/a	n/a
Firm 2	311	199	n/a	n/a
Firm 3	311	199	128	n/a
Electricity				
Firm 1	672	n/a	n/a	n/a
Firm 2	672	536	n/a	n/a
Firm 3	672	536	452	n/a
Vertically Integrated				
Firm 1	n/a	1 286	995	800
Firm 2	n/a	n/a	995	800
Firm 3	n/a	n/a	n/a	800
Total	2 948	2 757	2 570	2 400

4.2 Natural-Gas Use to Produce Electricity

Our second sensitivity analysis considers the values of η_n, $\forall n \in \mathcal{N}$. This analysis begins with the same setting as the base case—the market consists initially of two unintegrated natural-gas and two unintegrated electricity suppliers and each electricity supplier has $\eta = 1$. We contrast market equilibria before and after a single vertical merger (*i.e.*, a single natural-gas and electricity supplier merge) under this base case to cases with different values of η_n, $\forall n \in \mathcal{N}$. Specifically, we consider six sensitivity cases. The first two, which we term 'Low-η Merged' and 'High-η Merged', assume that $\eta_n = 1$ for the electricity supplier that does not merge and $\eta_n = 0.25$ and $\eta_n = 5$, respectively, for the electricity supplier that does merge. The next two cases are termed 'Low-η Unmerged' and 'High-η Unmerged' and assume that $\eta_n = 1$ for the electricity supplier that merges and $\eta_n = 0.25$ and $\eta_n = 5$, respectively, for the electricity supplier that does not merge. The final two cases are termed 'Low-η All' and 'High-η All' and assume that $\eta_n = 0.25$ and $\eta_n = 5$, respectively, for all electricity suppliers.

Table 4 summarizes percentage changes in production and prices of the two commodities and welfare as a result of the single vertical merger. With the exception of High-η Unmerged and High-η All, the directions of the changes under these sensitivity cases is consistent with the base case. Vertical merger increases natural-gas production by the merged firm, which decreases electricity-production cost and increases the merged firm's electricity production. The unmerged firms reduce

Figure 3: Producer and consumer welfare under successive vertical mergers.



their production to mitigate the price-suppressing effect of the merger. These changes increase the profit of the merged firm, decrease profit of the unmerged firms, and yield producer-welfare losses which are offset by consumer- and social-welfare gains. The magnitudes of the differences are sensitive to the values of $\eta_n, \forall n \in \mathcal{N}$.

Table 4: Percentage Change in Production Levels, Prices, and Welfare Pre- and Post-Merger for Different Values of η_n, $\forall n \in \mathcal{N}$

	Base Case	Low-η Merged	High-η Merged	Low-η Unmerged	High-η Unmerged	Low-η All	High-η All
Production							
Natural Gas	+21.1	+5.1	+180.0	+24.7	+104.9	+5.5	+252.5
Electricity	+12.4	+1.9	+38.5	+8.0	+24.2	+1.2	+411.7
Price							
Natural Gas	−18.1	−6.6	−7.4	−16.5	−24.0	−6.4	−21.7
Electricity	−11.5	−2.3	−21.0	−9.4	−13.2	−1.7	−32.1
Welfare							
Producer	−3.9	−1.0	−14.9	−6.3	+2.1	−1.3	+122.2
Consumer	+29.9	+5.0	+83.5	+19.2	+59.8	+3.4	+638.0
Social	+8.6	+1.6	+10.6	+4.6	+17.1	+0.9	+196.7

An electricity supplier with $\eta_n = 0.25$ uses relatively little natural gas for electricity production. Thus, such a firm's electricity-production decisions are relatively insensitive to upstream natural-gas-production decisions and resultant natural-gas prices. As such, vertical merger in the Low-η cases tends to yield smaller differences between pre- and post-merger equilibria as compared to the High-η cases. Amongst the Low-η cases, vertical merger has the greatest impact if the unmerged firm has a low value of η_n. This result follows from the intuition from comparing (22) and (25). A benefit of merger is that a vertically integrated firm efficiently self supplies electricity-production fuel on the margin. This benefit is greater if the vertically integrated firm has a more natural-gas-intensive electricity-generation technology (*i.e.*, a higher value of η_n). Conversely, if the merged firm has a relatively low value of η_n, the benefits of vertical integration are less pronounced. We do not model the extreme case with $\eta_n = 0$, because such a case decouples the two commodity markets and our model reverts to two standard Nash-Cournot equilibria for each.

Some of the High-η cases yield results that are contrary to what is observed under the base case. High-η All yields a post-merger equilibrium that is most akin to the standard double-marginalization case that Spengler (1950) analyzes. Under High-η All, vertical merger yields producer-, consumer-, and social-welfare increases. The merged firm having $\eta_n = 5$ provides it with a strong incentive to increase natural-gas production and decrease its electricity-production cost. Although the two unmerged firms suffer profit losses relative to their pre-merger levels, the profit gain of the merged firm outweighs these losses, which gives the producer-welfare gain. High-η Merged and High-η Unmerged yield interesting equilibria in that the electricity producer with a high value of η_n has zero production pre-merger (*i.e.*, it is unable to compete profitably against the other electricity producer). Post-merger, the electricity producer with a high value of η_n has non-zero production. Under High-η Unmerged, the vertical merger increases profit of the merged firm (through the aforementioned benefit of increasing natural-gas production to reduce its own electricity-production cost) as well as the unmerged electricity producer, which is able to compete profitably due to the reduced fuel price. This profit increase to the unmerged electricity producer yields the small 2.1% producer-welfare increase from vertical merger that is reported in Table 4.

4.3 Production Cost

Our third sensitivity analysis considers changes to the cost of natural-gas and electricity production, through the values of c_n^G and $c_n^E, \forall n \in \mathcal{N}$. As in Section 4.2, we consider the same starting base case and contrast that with cases that have relatively high and low values of $c_n^G, \forall n \in \mathcal{N}$ and $c_n^E, \forall n \in \mathcal{N}$. Table 5 summarizes the results of our four sensitivity cases, wherein the values of one of c_n^G for all natural-gas suppliers or c_n^E for all electricity suppliers are changed together.

Table 5: Post-Merger Percentage Change in Production Levels, Prices, and Welfare for Different Values of c_n^G or $c_n^E, \forall n \in \mathcal{N}$

	$c_n^G, \forall n \in \mathcal{N}$		$c_n^E, \forall n \in \mathcal{N}$	
	0.01	8.00	0.01	20.00
Production				
Natural Gas	+21.4	+20.2	+23.9	+12.1
Electricity	+14.1	+6.8	+13.0	+10.0
Price				
Natural Gas	−20.6	−12.5	−21.1	−8.2
Electricity	−13.5	−5.8	−13.5	−6.8
Welfare				
Producer	−1.9	−8.7	−1.8	−8.8
Consumer	+33.8	+16.3	+31.8	+22.1
Social	+11.2	+0.2	+10.6	+1.3

Unlike the case of changing natural-gas use for electricity production, changing the marginal-cost parameters does not yield mixed effects on market equilibria. Vertical merger does not provide the merging firm with any mechanism to change the marginal-cost parameter. Nevertheless, Table 5 shows that the social-welfare benefit of vertical merger is muted if either of c_n^G or c_n^E is relatively high, $\forall n \in \mathcal{N}$. In the case of $c_n^G = 8.00, \forall n \in \mathcal{N}$, vertical merger reduces the extent to which the merged firm can reduce its electricity-production cost, because its cost of natural-gas supply is high. In the case of $c_n^E = 20.00, \forall n \in \mathcal{N}$, the fuel cost of electricity production is small relative to non-fuel costs, meaning that the merged firm has limited recourse to increase profitably its electricity supply.

4.4 Price-Responsiveness of Demand

Our final sensitivity case considers different values of B^E and B^G, which represent the direct relationship between price and retail consumption of electricity and natural-gas, respectively. We consider the same starting base case as in Section 4.2 with $B^G = 0.00005$ and $B^E = 0.0025$ and contrast that with cases with higher or lower values for these parameters. Table 6 summarizes the results of our four sensitivity cases regarding these parameters. The table shows that vertical merger has the same overall impact with different price-responsiveness of demand as it does under the base case. Namely, vertical merger increases profit of the merged firm, decreases profits of the remaining firms, increases production, decreases prices, and yields a consumer-welfare increase that outweighs the resultant producer-welfare loss. These effects of vertical merger are much greater in magnitude if B^G is relatively large or B^E is relatively small.

Table 6: Post-Merger Percentage Change in Production Levels, Prices, and Welfare for Different Values of B^G or B^E

	B^G		B^E	
	0.0000001	0.005	0.000001	0.05
Production				
Natural Gas	+0.1	+1 452.1	+1 195.2	+0.1
Electricity	+3.0	+1 097.4	+901.2	+3.0
Price				
Natural Gas	−0.1	−54.7	−40.3	−0.1
Electricity	−3.6	−42.1	−17.5	−3.6
Welfare				
Producer	−0.1	+583.7	+786.1	−0.2
Consumer	+0.2	+6 128.0	+2 624.9	+0.4
Social	+0.1	+879.3	+866.6	+0.1

If $B^G = 0.005$, retail natural-gas consumers are relatively sensitive to quantity changes. As such, a vertically merged firm can cause a relatively large decrease in the price of natural gas that it uses for downstream electricity production from a relatively small increase in natural-gas production. Conversely, if $B^G = 0.0000001$, the vertically merged firm would need to increase natural-gas production by two orders of magnitude relative to what would be needed under the base case to achieve a given decrease of the natural-gas price. As a result, vertical merger has relatively muted impacts on the behavior of the merged firm in the case with $B^G = 0.0000001$. This impact of B^G on the behavior of the merged firm explains the relatively large production, price, and welfare changes that arise from vertical merger with $B^G = 0.005$, as compared to the cases with $B^G = 0.0000001$ or $B^G = 0.00005$.

Changing the value of B^E has the opposite effect on the behavior of a vertically merged firm compared to changing B^G. If $B^E = 0.05$, electricity consumers are relatively price sensitive to changes in electricity consumption. As such, a vertically merged firm has relatively muted incentives to increase electricity production, because doing so causes a sharp decrease in the electricity price and electricity revenue. Thus, the merged firm has limited incentives to increase natural-gas production, which is beneficial only insomuch as doing so decreases electricity-production cost to drive higher electricity output. Conversely, if $B^E = 0.000001$, electricity consumers are relatively price-insensitive to increased electricity production. This price-insensitivity increases a merged firm's incentive to increase natural-gas production, which reduces the cost of electricity production, thereby allowing the merged firm to increase electricity output.

5. CONCLUSIONS

This paper proposes a Nash-Cournot model to study the implications of vertical merger between an upstream natural-gas and downstream electricity industry. Using standard techniques, a Nash-Cournot equilibrium can be computed using an equivalent linear complementarity model, which can be solved using off-the-shelf software. Through comparative statics, a case study, and parametric analyses, we find that vertical merger is beneficial to consumers and from a societal perspective. These benefits arise from merger alleviating the inefficiency that is caused by double marginalization, *cf*. the difference between (23) and (26), which arises from the exercise of market power at the upstream and downstream of a vertical supply chain. Our finding is a natural extension of the seminal work of Spengler (1950) that studies double marginalization.

Our work extends that of Spengler (1950) in some important ways. For one, we show that double marginalization can be alleviated by vertical merger between commodity markets with oligopolies (as opposed to monopolies). In addition, we show that double marginalization remains a concern if there is retail demand for the upstream product. We show also that our findings are robust to the model parameters and market structure (number of firms). One key difference between our findings and those of Spengler (1950) is that we find producer-welfare losses under almost all cases. This finding gives rise to potential prisoners'-dilemma-type incentives, whereby there are strong incentives for firms to merge, but all are worse-off as a result of merger. The model of Spengler (1950) does not have such an outcome, demonstrating that the case that we model has some important differences with the work of Spengler (1950).

We take a computational approach to computing Nash-Cournot, by solving a linear complementarity problem. This approach provides flexibility to include more complex constraints than what could be considered in a stylized Nash-Cournot model that is solved analytically. Problem (3)–(5) includes relatively simple costs and constraints, to ease model exposition and notation. Nonetheless, electricity-transmission or natural-gas-pipeline constraints, more complex cost structures, or firms owning multi-technology production portfolios could be modeled using our computation approach (Sioshansi et al., 2008; Chen et al., 2019). Such models could prove useful to conduct more detailed analysis of a specific merger, which may raise case-specific concerns that do not appear in our analysis here.

6. APPENDIX

Proof of Lemma 1. For all $n \in \mathcal{N}$, (18) can be written as:

$$-A^G + B^G \cdot \left(q_n^G + q_{n-}^G - \eta q_n^E - \eta q_{n-}^E\right) + c^G + B^G q_n^G - B^G \eta q_n^E = 0, \tag{30}$$

which can be rewritten as (20).

To derive (21), we begin by writing (19) as:

$$-B^G \eta q_n^G - A^E + B^E \cdot \left(q_n^E + q_{n-}^E\right) + c^E + \left[A^G - B^G \cdot \left(q_n^G + q_{n-}^G - \eta q_n^E - \eta q_{n-}^E\right)\right] \eta + B^E q_n^E$$
$$+ B^G \eta^2 q_n^E = 0.$$

Substituting (30) into this equation gives:

$$-B^G \eta q_n^G - A^E + B^E \cdot \left(q_n^E + q_{n-}^E\right) + c^E + \left(c^G + B^G q_n^G - B^G \eta q_n^E\right) \eta + B^E q_n^E + B^G \eta^2 q_n^E = 0,$$

which simplifies to:

$$-A^E + B^E \cdot \left(q_n^E + q_{n-}^E\right) + c^E + c^G \eta + B^E q_n^E = 0,$$

and which can be rewritten as (21). ■

Proof of Lemma 2. For any $n \in \mathcal{N}$ that produces natural gas only, reaction function (21) does not apply and (20) simplifies to:

$$q_n^G = \frac{1}{2B^G} \cdot \left(A^G - c^G + B^G \eta q_{n-}^E - B^G q_{n-}^G \right). \tag{31}$$

The partial derivatives of this equation give the desired equalities. ■

Proof of Lemma 3. Reaction function (20) does not apply for any $n \in \mathcal{N}$ that produces electricity only. The partial derivatives of (21) of any such n gives the desired equalities. ■

Proof of Lemma 4. Reaction functions (20) and (21) apply for any $n \in \mathcal{N}$ that produces both natural gas and electricity. The desired equalities are obtained from the partial derivatives of these functions. Equality (28) requires (29) and application of chain rule. ■

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