Shadow Price-Based Co-ordination of Natural Gas and Electric Power Systems

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***Abstract*—Increasing use of natural gas for electricity production places added strains on pipeline systems that are used for transporting fuel. Pipeline constraints require power system operators to account for natural gas-supply restrictions in their operational processes. This paper proposes separate optimization models for clearing day-ahead wholesale markets for scheduling power and natural gas systems. We then develop a market-based mechanism that allows for efficient co-ordination of the two systems. Importantly, the co-ordination mechanism only requires the exchange of fuel-price, -supply, and -demand information between the two markets. This can be contrasted with other co-ordination mechanisms that require operations of the two systems by a single entity. Thus, we provide a computationally tractable co-ordination mechanism that does not require the exchange of proprietary information between natural gas and electricity system operators. We demonstrate the effectiveness and scalability of the technique using a numerical example.**

***Index Terms*—Power system modeling, power system operation, natural gas, optimization, decomposition**

I. Introduction

NATURAL gas and electric power systems are becoming more interdependent because of increasing use of natural gas-fired generation units.[1][2][3] This trend is driven by investment in natural gas-fired units, which is due to their operational flexibility, high efficiency, and relatively low operating cost. Many natural gas-fired units currently utilize interruptible transportation services, as opposed to firm fuel-delivery contracts [1]. These natural gas-fired units must rely on purchasing fuel in the day-ahead spot market. As such, they may not necessarily know the fuel prices that they will ultimately face when submitting offers to the day-ahead electricity market. Moreover, the lack of firm fuel-delivery contracts means that the amount of fuel that is available to such units is uncertain and depends on the state of the natural gas system. As such, the interdependency between these two systems affects their operations and the wholesale electricity and natural gas prices.

This interdependency calls for greater co-ordination and co-optimization of the two systems [2]. A number of formative works develop different models to this end. Zhao *et al*. [3] propose a two-stage stochastic unit commitment problem that accounts for uncertain natural gas-supply constraints. By virtue of using a transportation-network model, their work neglects natural gas-flow physics. Martínez-Mares and Fuerte-Esquivel [4] use a steady-state natural gas-pipeline model to analyze flows within coupled natural gas and power networks. Other works [5], [6] employ linearizations of steady-state natural gas-network models to co-optimize natural gas and power flows. Correa-Posada and Sánchez-Martín [7] investigate the co-ordinated operation of the systems using approximations of the natural gas-network dynamics. Zlotnik *et al*. [8] develop a model of natural gas-system dynamics, which uses partial differential equations that are derived from first principles. The objective is to analyze the benefits of higher-fidelity natural gas modeling for system co-optimization.

Several works examine the effects of other system components. Liu *et al*. [9] develop a co-optimization problem for power and natural gas systems with high wind penetrations. Chen *et al*. [10] develop an integrated natural gas and power flow model that considers correlations between energy demand and wind production, electrically driven natural gas compressors, and power-to-gas units. Cui *et al*. [11] study the effect of demand response in the co-ordinated operation of power and natural gas systems. Zhou *et al*. [12] investigate the effects of variations in natural gas pressure across a pipeline network and the ramping of natural gas-fired generating units.

Another body of work studies different types of interdependencies between natural gas and power systems. Liu *et al*. [13] propose a security-constrained unit commitment model with consideration of natural gas-supply limits. Liu *et al*. [14] propose a bi-level model that minimizes electricity-production cost subject to natural gas-system feasibility. Liu *et al*. [15] study the co-ordinated scheduling of power and natural gas systems from the perspective of a joint operator that uses a steady-state natural gas flow model.

A final set of works investigate interactions between the two systems taking a market-based, as opposed to central-planning, perspective. Rubio *et al*. [2] provide a description of the structure of the natural gas market and price formation therein. Dueñas *et al*. [16] analyze the optimal operation of

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[1] https://www.eia.gov/todayinenergy/detail.php?id=34612

[2] https://www.eia.gov/electricity/monthly/update/archive/december2017/

[3] https://www.eia.gov/todayinenergy/detail.php?id=31172

natural gas-fired units in a competitive electricity market. Their model takes account of demand variations that are caused by renewable energy sources as well as natural gas purchases and capacity contracting. Morais and Lima [17] show the effects of natural gas- and electricity-pricing schemes on generation-planning decisions. Pepper *et al*. [18] propose a spot market model for natural gas that can be used for scheduling and price formation.

This paper builds upon this literature by proposing a market-based mechanism for co-ordinated scheduling of natural gas and electric power systems, which requires limited information exchange between the two systems. Our approach employs a first-principles physics-based dynamic natural gas-flow model [8], [19], which is extended to provide a market-clearing model that accurately reflects natural gas-flow transit. We employ such a model because a comparison of steady-state and dynamic gas flow models [20] shows that the latter is needed to adequately represent system behavior. A unit commitment model is used to represent the day-ahead scheduling of the electric power system [21]. Our work is motivated by increasingly frequent situations in which high demand for natural gas for electricity generation creates pipeline congestion. Specifically, we envision cases in which the natural gas system is not able to fully service natural gas-fired units at the levels that are specified by the unit commitment and economic dispatch solution. Our proposed model addresses such an occurrence by re-scheduling generating units so as to relieve pipeline constraints, allowing the natural gas system to be operated optimally within its physical bounds.

Our proposed work makes a number of contributions to the extant literature. First, the natural gas model that we propose is a highly non-linear and non-convex continuous-time optimal control problem. Thus, directly integrating the pipeline model into a unit commitment would yield a computationally intractable mixed-integer non-linear optimization problem [22]. Our work allows this intractable model to be decomposed into separate natural gas and power system models that are tractable. Thus, the integrated model can be efficiently solved by iterating between the two decomposed models. This is a contribution towards developing an efficient means to co-ordinate planning of natural gas and power systems. A test case with 25 natural gas nodes and 24 electric buses is solved within six iterations. Second, the co-ordination mechanism that we propose accounts for the salient physical properties of both systems. Third, full co-ordination by a single entity might be institutionally undesirable due to regulatory and confidentiality considerations.[4] Our proposed methodology allows for effective co-ordination by solely exchanging price and scheduling information between the two systems, while keeping network data and customer information for each system confidential. This reduces the potential barriers to co-ordinating the operations of the two systems. Finally, we provide physical and economic interpretations of the proposed co-ordination mechanism. Specifically, the mechanism provides intra-day natural gas prices that account for physical constraints. These prices, in turn, provide time-varying operating costs of the natural gas-fired units in the unit commitment problem. Intra-day natural gas pricing is not common practice today, meaning that our work can advance the economic efficiency of fuel use. Although other works [23], [24] examine price-based co-ordination of natural gas and power systems, ours advances these works by examining the convergence properties of such a mechanism to an optimum.

The remainder of this paper is organized as follows. An overview of the proposed co-ordination mechanism and the constituent models is given in Section II. Detailed model formulations are given in Sections III and IV. The co-ordination mechanism and solution algorithm are described in Section V. Section VI illustrates the proposed model and algorithm using an example 25-node natural gas system that is coupled with the 24-bus IEEE RTS. Finally, Section VII provides concluding remarks and suggestions for future work.

[4]http://energy.mit.edu/publication/growing-concerns-possible-solutions/

II. Overview of Co-ordination Mechanism

Our proposed co-ordination mechanism consists of a discrete-time approximation of a non-linear natural gas-system optimal control problem and a unit commitment model. The overall economic objective of co-ordinating the two systems is to maximize the combined social welfare that they produce. Social welfare is defined as the sum of consumer and producer welfare [25]. Natural gas-fired generating units play a unique role in the two systems, insomuch as they are consumers in the natural gas system and producers in the power system. The objective of the unit commitment model is to minimize the total cost of supplying the fixed electric loads. Conversely, the natural gas model is formulated to maximize the economic value of fuel that is supplied to natural gas-fired units, while satisfying other natural gas demands. The sum of these two objectives represents the combined social welfare that is generated by the two systems. We propose that these models can be solved iteratively to arrive at a solution that is simultaneously optimal for both systems. Details of the two models are given in Sections III and IV, respectively.

The natural gas model is formulated to determine natural gas injections, flows, and withdrawals to maximize social welfare of its use subject to physical system constraints. The value of natural gas that is used by each natural gas-fired unit is given by the electricity price at the bus where it is located. Because electricity prices can vary with time, the value of natural gas use can be time-varying as well. Natural gas-fired units also have upper bounds on their fuel use, which are determined by the dispatched production levels that are given by the unit commitment solution. The natural gas model provides time-varying (*e.g*., hourly or subhourly) natural gas prices at different supply nodes, which are used to model the cost of operating natural gas-fired units in the unit commitment.

The unit commitment model is formulated to minimize the cost of serving electric loads. In addition to time-varying locational marginal prices (LMPs) for natural gas, the optimal control problem also provides fuel-supply limits to natural gas-fired units. These limits are calculated based on the amounts of fuel that are supplied to natural gas-fired units in each time step, which are obtained from the solution of the optimal control problem. Feasibility of the natural gas flows is obtained by clearing a natural gas market using price bids that are formed from the electric LMPs.

The intended use of the two models is to plan and co-ordinate the operation of the two systems day-ahead. As such, most of the pertinent parameters (*e.g.*, non-generation-related natural gas and electricity demands) can be forecasted relatively accurately. Similarly, power flows are represented in the unit commitment using a linearized dc, as opposed to a nonlinear ac, model [14]. Day-ahead operational models typically employ dc power flow equations. As such, and to clearly show the interactions between the two systems, we do not include uncertainties or ac power flows in the models. Uncertainties or nonlinear power flows can be incorporated in the models, with commensurate complexity increases.

The two models are solved sequentially in an iterative fashion until convergence. The final solution for each system is optimal in its own problem given the information from the other system. While we do not provide a rigorous proof of optimality of total social welfare from the two systems, we argue that social welfare is improved compared to a case of no co-ordination between the two systems. The proposed co-ordination mechanism requires minimal information exchange between the natural gas and power systems. Indeed, the information exchanged can be likened to demand bids and supply offers by natural gas-fired units in the natural gas and electricity markets, respectively.

III. Natural Gas-System Model

We provide a detailed formulation of the natural gas-system model. The underlying optimal control problem, on which our model is based, is continuous in time and space for each pipeline and is continuous in time for the control functions [19]. We apply a discretization approach to obtain a finite-dimensional nonlinear optimization problem that can be solved using a standard software package (*e.g.*, IPOPT) to approximate the solution to the continuous problem [26]. We introduce the notation that is used, list the values used for conversions, and then provide the model formulation. Unless otherwise noted, the decision variables are all given in p.u.

A. Notation

Sets and Indices

g	index of natural gas-fired units in set, Φ^{G}
i, j	indices of natural gas-system nodes in set, Υ
t	index of time steps in set, T
Δ	set of natural gas pipelines
$\underline{\Delta}$	set of pipelines that have a compressor installed at their beginning node
$\overline{\Delta}$	set of pipelines that have a compressor installed at their end node
Υ_s	set of natural gas-supply nodes
$\Phi_i^{\mathrm{G,G}}$	set of natural gas-fired units that are connected to natural gas node i

Constants and Parameters

$A_{(i,j)}$	cross-sectional area of pipeline connecting nodes i and j [m^2]
a	speed of wave propagation in natural gas [m/s]
$D_{(i,j)}$	cross-sectional diameter of pipeline connecting nodes i and j [m]
$d_{i,t}^{\mathrm{G}}$	time step-t non-generation-related natural gas load at node i [kg/s]
$\hat{d}_{g,t}^{\mathrm{GV}}$	time step-t maximum fuel demand of natural gas-fired unit g [kg/s]
$f_{(i,j)}$	friction factor of pipeline connecting nodes i and j
h	time between successive time steps [s]
$L_{(i,j)}$	length of pipeline connecting nodes i and j [m]
l_b	pipeline-length scaling factor [m]
M	energy content of 1 kg of natural gas [MBTU]
$p_{i,t}$	time step-t cost of natural gas at node i [\$/MBTU]
$s_{i,t}^{\max}$	time step-t maximum natural gas supply at node i [kg/s]
t_b	time-scaling constant [s]
$V^{\mathrm{OLL,G}}$	value of lost natural gas load [\$/MBTU]
W_1	scaling factor on social welfare of natural gas use
W_2	scaling factor on regularization term
$\underline{\alpha}_{(i,j)}^{\max}$	maximum compression ratio of the compressor located at the beginning node of the pipeline connecting nodes i and j
$\overline{\alpha}_{(i,j)}^{\max}$	maximum compression ratio of the compressor located at the end node of the pipeline connecting nodes i and j
$\underline{\alpha}_{(i,j)}^{\min}$	minimum compression ratio of the compressor located at the beginning node of the pipeline connecting nodes i and j
$\overline{\alpha}_{(i,j)}^{\min}$	minimum compression ratio of the compressor located at the end node of the pipeline connecting nodes i and j
$\beta_{(i,j)}$	resistance constant for pipeline connecting nodes i and j
$\underline{\eta}_{(i,j)}$	fuel factor coefficient of compressor located at the beginning node of the pipeline connecting nodes i and j
$\overline{\eta}_{(i,j)}$	fuel factor coefficient of compressor located at the end node of the pipeline connecting nodes i and j
$\hat{\lambda}_{g,t}$	time step-t value of fuel for natural gas-fired unit g [\$/MBTU]
ρ_b	nominal natural gas density [kg/m^3]
$\rho_i^{\max}$	maximum natural gas density at node i [kg/m^3]
$\rho_i^{\min}$	minimum natural gas density at node i [kg/m^3]
$\rho_{(i,j)}^{\max}$	maximum natural gas density in pipeline connecting nodes i and j [kg/m^3]
$\rho_{(i,j)}^{\min}$	minimum natural gas density in pipeline connecting nodes i and j [kg/m^3]
ϕ_b	nominal natural gas flux [kg/(m^{2}s)]
φ_b	nominal natural gas flow [kg/s]

Decision Variables

$d_{g,t}^{\mathrm{GV}}$	time step-t fuel supplied to natural gas-fired unit g
$d_{i,t}^{\mathrm{G,shed}}$	time step-t unserved non-generation-related natural gas load at node i [kg/s]
$s_{i,t}$	time step-t natural gas supplied at node i

$\underline{\alpha}_{(i,j),t}$ time step-t compression ratio of compressor located at the beginning of the pipeline connecting nodes i and j

$\overline{\alpha}_{(i,j),t}$ time step-t compression ratio of compressor located at the end of the pipeline connecting nodes i and j

$\rho_{i,t}$ time step-t/node-i natural gas density

$\underline{\rho}_{(i,j),t}$ time step-t natural gas density at the beginning of the pipeline connecting nodes i and j

$\overline{\rho}_{(i,j),t}$ time step-t natural gas density at the end of the pipeline connecting nodes i and j

$\underline{\phi}_{(i,j),t}$ time step-t natural gas mass flux at the beginning of the pipeline connecting nodes i and j

$\overline{\phi}_{(i,j),t}$ time step-t natural gas mass flux at the end of the pipeline connecting nodes i and j

B. Definitions, Unit Conversions, and Normalizations

The elements of the set, Δ, are defined on pairs of nodes. Thus, the element, $(i,j) \in \Delta$, denotes a pipeline with i as its beginning node and j as its end node. This is distinguished from $(j,i) \in \Delta$, which denotes a pipeline with opposite beginning and end nodes. We use a value of 46 MJ/kg as the energy content of natural gas. Thus, using the relations that 1 MJ is equivalent to $1/3600$ MWh and that 1 MWh $\approx$ 3.412 MBTU, we use $M = 0.0436$ MBTU as the energy content of 1 kg of natural gas. We also, from basic geometry, have that $A_{(i,j)} = \pi \cdot (D_{(i,j)}/2)^2$. Finally, a number of constants are used to express the decision variables in dimension-free form. These are defined as $t_b = l_b/a$, $\beta_{(i,j)} = f_{(i,j)} l_b/(2D_{(i,j)})$, $\phi_b = a\rho_b$, and $\varphi_b = a\rho_b l_b^2$.

C. Model Formulation

Our model considers dynamic constraints for transient natural gas flows [19], [26], thereby extending steady-state natural gas market-clearing problems that appear in the literature [22], [27]. The model formulation is as follows:

$$\max\ W_1 M h \sum_{t\in T}\left(\sum_{g\in\Phi^{\mathrm{G}}} \hat{\lambda}_{g,t} d^{\mathrm{GV}}_{g,t} - \sum_{i\in\Upsilon_s} p_{i,t} s_{i,t} - \sum_{i\in\Upsilon} V^{\mathrm{OLL,G}} d^{\mathrm{G,shed}}_{i,t}\right) - W_2 \sum_{t\in T}\sum_{g\in\Phi^{\mathrm{G}}} \frac{\left(d^{\mathrm{GV}}_{g,t+1} - d^{\mathrm{GV}}_{g,t}\right)^2}{h^2} \tag{1}$$

$$\text{s.t.}\quad t_b \cdot (\underline{\rho}_{(i,j),t+1} - \underline{\rho}_{(i,j),t} + \overline{\rho}_{(i,j),t+1} - \overline{\rho}_{(i,j),t})/h + 2l_b \cdot (\overline{\phi}_{(i,j),t} - \underline{\phi}_{(i,j),t})/L_{(i,j)} = 0, \quad \forall (i,j) \in \Delta, t \in T \tag{2}$$

$$t_b \cdot (\underline{\rho}_{(i,j),t} + \overline{\rho}_{(i,j),t})(\underline{\phi}_{(i,j),t+1} - \underline{\phi}_{(i,j),t} + \overline{\phi}_{(i,j),t+1} - \overline{\phi}_{(i,j),t})/h + 2l_b \cdot \left((\overline{\rho}_{(i,j),t})^2 - (\underline{\rho}_{(i,j),t})^2\right)/L_{(i,j)} = -\beta_{(i,j)} \left|\underline{\phi}_{(i,j),t} + \overline{\phi}_{(i,j),t}\right| (\underline{\phi}_{(i,j),t} + \overline{\phi}_{(i,j),t}), \quad \forall (i,j) \in \Delta, t \in T \tag{3}$$

$$\underline{\alpha}_{(i,j),0} = \underline{\alpha}_{(i,j),T}, \quad \forall (i,j) \in \underline{\Delta} \tag{4}$$

$$\overline{\alpha}_{(i,j),0} = \overline{\alpha}_{(i,j),T}, \quad \forall (i,j) \in \overline{\Delta} \tag{5}$$

$$\rho_{i,0} = \rho_{i,T}, \quad \forall i \in \Upsilon \tag{6}$$

$$\underline{\phi}_{(i,j),0} = \underline{\phi}_{(i,j),T}, \quad \forall (i,j) \in \Delta \tag{7}$$

$$\overline{\phi}_{(i,j),0} = \overline{\phi}_{(i,j),T}, \quad \forall (i,j) \in \Delta \tag{8}$$

$$\sum_{(i,j)\in\Delta} \underline{\phi}_{(i,j),t} A_{(i,j)} - \sum_{j,i\in\Delta} \overline{\phi}_{j,i,t} A_{j,i} = s_{i,t} - d^{\mathrm{G}}_{i,t} + d^{\mathrm{G,shed}}_{i,t} - \sum_{g\in\Phi^{\mathrm{G,G}}_i} d^{\mathrm{GV}}_{g,t}, \quad \forall i \in \Upsilon_s, t \in T \tag{9}$$

$$\sum_{(i,j)\in\Delta} \underline{\phi}_{(i,j),t} A_{(i,j)} - \sum_{j,i\in\Delta} \overline{\phi}_{j,i,t} A_{j,i} = -d^{\mathrm{G}}_{i,t} + d^{\mathrm{G,shed}}_{i,t} - \sum_{g\in\Phi^{\mathrm{G,G}}_i} d^{\mathrm{GV}}_{g,t}, \quad \forall i \in \Upsilon/\Upsilon_s, t \in T \tag{10}$$

$$0 \le d^{\mathrm{GV}}_{g,t} \le \hat{d}^{\mathrm{GV}}_{g,t}/\varphi_b, \quad \forall g \in \Phi^{\mathrm{G}}, t \in T \tag{11}$$

$$0 \le d^{\mathrm{G,shed}}_{i,t} \le d^{\mathrm{G}}_{i,t}, \quad \forall i \in \Upsilon, t \in T \tag{12}$$

$$0 \le s_{i,t} \le s^{\max}_{i,t}/\varphi_b, \quad \forall i \in \Upsilon_s, t \in T \tag{13}$$

$$\rho_{i,t} = \rho^{\min}_i/\rho_b, \quad \forall i \in \Upsilon_s, t \in T \tag{14}$$

$$\underline{\rho}_{(i,j),t} = \underline{\alpha}_{(i,j),t}\rho_{i,t}, \quad \forall (i,j) \in \underline{\Delta}, t \in T \tag{15}$$

$$\overline{\rho}_{(i,j),t} = \overline{\alpha}_{(i,j),t}\rho_{j,t}, \quad \forall (i,j) \in \overline{\Delta}, t \in T \tag{16}$$

$$\underline{\rho}_{(i,j),t} = \rho_{i,t}, \quad \forall (i,j) \in \Delta/\underline{\Delta}, t \in T \tag{17}$$

$$\overline{\rho}_{(i,j),t} = \rho_{j,t}, \quad \forall (i,j) \in \Delta/\overline{\Delta}, t \in T \tag{18}$$

$$\rho^{\min}_i/\rho_b \le \rho_{i,t} \le \rho^{\max}_i/\rho_b, \quad \forall i \in \Upsilon, t \in T \tag{19}$$

$$\underline{\rho}^{\min}_{(i,j)}/\rho_b \le \underline{\rho}_{(i,j),t} \le \underline{\rho}^{\max}_{(i,j)}/\rho_b, \quad \forall (i,j) \in \Delta, t \in T \tag{20}$$

$$\overline{\rho}^{\min}_{(i,j)}/\rho_b \le \overline{\rho}_{(i,j),t} \le \overline{\rho}^{\max}_{(i,j)}/\rho_b, \quad \forall (i,j) \in \Delta, t \in T \tag{21}$$

$$\underline{\alpha}^{\min}_{(i,j)} \le \underline{\alpha}_{(i,j),t} \le \underline{\alpha}^{\max}_{(i,j)}, \quad \forall (i,j) \in \underline{\Delta}, t \in T \tag{22}$$

$$\overline{\alpha}^{\min}_{(i,j)} \le \overline{\alpha}_{(i,j),t} \le \overline{\alpha}^{\max}_{(i,j)}, \quad \forall (i,j) \in \overline{\Delta}, t \in T. \tag{23}$$

Objective function (1) is defined as the sum of two terms. The first term computes the difference between the value of fuel that is delivered to natural gas-fired units and the cost of obtaining natural gas from supply nodes, to which the cost of unserved non-generation-related natural gas load is added. The value of lost natural gas load, $V^{\mathrm{OLL,G}}$, is assumed to be sufficiently high that natural gas load is only curtailed if the pipeline system cannot feasibly supply it. This term represents the social welfare or net value of fuel that is supplied to natural gas-fired units. The expression, Mh, that multiplies this term gives the per-unit energy content of natural gas that is delivered between two successive time steps.

The second term in (1) is a regularizer that imposes costs on deviations in the fuel supply to natural gas-fired units between successive time steps. We include this regularizer because the natural gas-system model is a space- and time-discretization of a numerically ill-conditioned continuous optimal control problem. As such, the resulting physical flows may exhibit significant fluctuations, making convergence between the natural gas and unit commitment models difficult. The regularizer helps to avoid such fluctuations, giving a smoother dispatch for the natural gas-fired units and speeding convergence. The scaling factors, W_1 and W_2, control the relative weight that is given to the two terms and scale the objective function relative to the constraint residuals.

The regularizer may distort the operation of the natural gas system and the resulting social welfare. Thus, W_2 is adjusted to keep the regularization term sufficiently small to minimize distortions. Our numerical testing shows that a regularizer that is less than 5% of the total objective-function value gives desirable convergence properties while minimizing solution distortion. Other choices, such as replacing the regularizer by a constraint, requires identifying appropriate bounds, which may be difficult. Mak *et al.* [26] propose a two-stage optimization approach to define such bounds.

Constraints (2) and (3) represent natural gas-mass-conservation and -flow-momentum balance in each pipeline, respectively. Constraints (4)–(8) enforce time-periodicity for the compressor ratio of each compressor, natural gas density at each node, and natural gas-mass flux in each pipeline, respectively. These time-periodicity constraints guarantee that the optimal control problem that the optimization model approximates is well posed. This is done by returning the state of the system at time step T to its initial state at time step 0. The rationale behind the time-periodicity requirement is that in the long-run the amount of natural gas that is injected into the system should balance the amount that is withdrawn. Time-periodicity conditions would yield such balance. In place of time-periodicity, one could impose mass balance over the optimization period on certain subsystems of the network [24].

Constraints (9) and (10) represent natural gas-flow balance at supply and non-supply nodes, respectively. Constraints (11) impose fuel limits on natural gas-fired units. As noted in Section II and further detailed in Section V, these limits are taken from the solution of the unit commitment problem. Constraints (12) restrict the amount of non-generation-related natural gas load that is shed to be no greater than the corresponding demand. Constraints (13) impose limits on the amount of natural gas that is supplied at supply nodes. Constraints (14) fix the natural gas density at supply nodes. Constraints (15) and (16) give the relationship between the natural gas density of pipelines that have compressors installed and the densities at the beginning and end nodes of such pipelines. Constraints (17) and (18) give the same relationship for pipelines without compressors. The remaining constraints provide lower and upper bounds for nodal natural gas density, natural gas densities at the beginning and end nodes of pipelines, and boost ratios of compressors.

The terms, $(\overline{\rho}_{i,j,t})^2$, $(\underline{\rho}_{i,j,t})^2$, and $|\underline{\phi}_{i,j,t} + \overline{\phi}_{i,j,t}|(\underline{\phi}_{i,j,t} + \overline{\phi}_{i,j,t})$ in constraints (3), $\underline{\alpha}_{i,j,t}\rho_{i,t}$ in constraints (15), and $\overline{\alpha}_{i,j,t}\rho_{j,t}$ in constraints (16), as well as the regularizer in objective function (1) are nonlinear. Thus, the natural gas model is nonlinear and non-convex.

IV. Power System Model

This section provides a detailed formulation of the unit commitment model that is used to optimize the operation of the power system. We proceed by introducing model notation and then giving the formulation. In defining the generating units, we label them as being either natural gas-fired or other thermal units. Natural gas-fired units are distinguished from other thermal units in that they rely on fuel that is supplied by the natural gas system to operate. Other thermal units use some other type of fuel, which creates no dependency on the natural gas system. Finally, some of the notation that is used (*i.e.*, g, t, T, and Φ^{G}) is common to the natural gas and power system models. This notation is not redefined here.

A. Notation

Sets and Indices

- m, n indices for power system buses in set, Λ
- o index for other thermal units in set, Φ^{O}
- ref index of reference bus
- Λ_n set of power system buses that are directly connected to bus n by a transmission line
- $\tau(t)$ set of time steps that are in the same hour as time step t
- $\Phi_n^{\mathrm{G,E}}$ set of natural gas-fired units that are connected to power system bus n
- Φ_n^{O} set of other thermal units that are connected to power system bus n

Constants and Parameters

- $B_{m,n}$ susceptance of the transmission line connecting buses m and n [S]
- b_g^{G} heat rate of natural gas-fired unit g [MBTU/MWh]
- c_g^{G} non-fuel variable generation cost of natural gas-fired unit g [\$/MWh]
- $c_g^{\mathrm{G,SU}}$ start-up cost of natural gas-fired unit g [\$]
- $c_o^{\mathrm{O,SU}}$ start-up cost of other thermal unit o [\$]
- c_o^{O} variable generation cost of other thermal unit o [\$/MWh]
- $d_{n,t}^{\mathrm{E}}$ bus-n electric demand in time step t [MW]
- $F_{m,n}^{\max}$ capacity of transmission line connecting buses m and n [MW]
- $P_g^{\mathrm{G,max}}$ generating capacity of natural gas-fired unit g [MW]
- $P_g^{\mathrm{G,min}}$ minimum power output of natural gas-fired unit g when it is online [MW]
- $\hat{P}_{g,t}^{\mathrm{G}}$ maximum available fuel for natural gas-fired unit g in time step t [MW]
- $P_o^{\mathrm{O,max}}$ generating capacity of other thermal unit o [MW]
- $P_o^{\mathrm{O,min}}$ minimum power output of other thermal unit o when it is online [MW]
- RD_g^{G} ramp-down limit of natural gas-fired unit g [MW/h]
- RD_o^{O} ramp-down limit of other thermal unit o [MW/h]
- RU_g^{G} ramp-up limit of natural gas-fired unit g [MW/h]
- RU_o^{O} ramp-up limit of other thermal unit o [MW/h]
- $V^{\mathrm{OLL,E}}$ value of lost electric load [\$/MW]
- $\hat{\zeta}_{g,t}$ time step-t marginal fuel price for natural gas-fired unit g [\$/MBTU]

Decision Variables

- $E_{g,t}^{\mathrm{G}}$ time step-t energy production of natural gas-fired unit g [MW-time step]
- $E_{o,t}^{\mathrm{O}}$ time step-t energy production of other thermal unit o [MW-time step]

$L^{\text{shed}}_{n,t}$	time step-t unserved energy at bus n [MW]
$P^{\text{G}}_{g,t}$	time step-t power output of natural gas-fired unit g [MW]
$P^{\text{O}}_{o,t}$	time step-t power output of other thermal unit o [MW]
$x^{\text{G}}_{g,t}$	binary variable that equals 1 if natural gas-fired unit g is on in time step-t and equals 0 otherwise
$x^{\text{O}}_{o,t}$	binary variable that equals 1 if other thermal unit o is on in time step-t and equals 0 otherwise
$y^{\text{G}}_{g,t}$	binary variable that equals 1 if natural gas-fired unit g is started up at the beginning of time step-t and equals 0 otherwise
$y^{\text{O}}_{o,t}$	binary variable that equals 1 if other thermal unit o is started up at the beginning of time step-t and equals 0 otherwise
$z^{\text{G}}_{g,t}$	binary variable that equals 1 if natural gas-fired unit g is shutdown at the beginning of time step-t and equals 0 otherwise
$z^{\text{O}}_{o,t}$	binary variable that equals 1 if other thermal unit o is shutdown at the beginning of time step-t and equals 0 otherwise
$\theta_{n,t}$	time step-t phase angle of bus n [rad]

B. Model Formulation

The unit commitment problem is formulated as:

$$\min \sum_{t\in T}\Bigg\{\sum_{g\in\Phi^{\text{G}}}\left[\frac{h}{3600}(c^{\text{G}}_g+\hat{\zeta}_{g,t}b^{\text{G}}_g)E^{\text{G}}_{g,t}+c^{\text{G,SU}}_g y^{\text{G}}_{g,t}\right] \quad (24)$$

$$+\sum_{o\in\Phi^{\text{O}}}\left(\frac{h}{3600}c^{\text{O}}_o E^{\text{O}}_{o,t}+c^{\text{O,SU}}_o y^{\text{O}}_{o,t}\right)$$

$$+\frac{h}{3600}\sum_{n\in\Lambda}V^{\text{OLL,E}}L^{\text{shed}}_{n,t}\Bigg\}$$

$$\text{s.t.} \sum_{m\in\Lambda_n}B_{m,n}\cdot(\theta_{n,t}-\theta_{m,t})+d^{\text{E}}_{n,t}-L^{\text{shed}}_{n,t} \quad (25)$$

$$=\sum_{g\in\Phi^{\text{G,E}}_n}P^{\text{G}}_{g,t}+\sum_{o\in\Phi^{\text{O}}_n}P^{\text{O}}_{o,t},\quad \forall t\in T,n\in\Lambda$$

$$-F^{\max}_{m,n}\le B_{m,n}\cdot(\theta_{m,t}-\theta_{n,t})\le F^{\max}_{m,n}, \quad (26)$$

$$\forall t\in T,n\in\Lambda,m\in\Lambda_n$$

$$\theta_{\text{ref},t}=0,\quad \forall t\in T \quad (27)$$

$$x^{\text{G}}_{g,t}P^{\text{G,min}}_g\le P^{\text{G}}_{g,t}\le x^{\text{G}}_{g,t}P^{\text{G,max}}_g, \quad (28)$$

$$\forall t\in T,g\in\Phi^{\text{G}}$$

$$x^{\text{O}}_{o,t}P^{\text{O,min}}_o\le P^{\text{O}}_{o,t}\le x^{\text{O}}_{o,t}P^{\text{O,max}}_o, \quad (29)$$

$$\forall t\in T,o\in\Phi^{\text{O}}$$

$$E^{\text{G}}_{g,t}=\left(P^{\text{G}}_{g,t}+P^{\text{G}}_{g,t+1}\right)/2,\quad \forall t\in T,g\in\Phi^{\text{G}} \quad (30)$$

$$E^{\text{O}}_{o,t}=\left(P^{\text{O}}_{o,t}+P^{\text{O}}_{o,t+1}\right)/2,\quad \forall t\in T,o\in\Phi^{\text{O}} \quad (31)$$

$$-RD^{\text{G}}_g\le P^{\text{G}}_{g,t+1}-P^{\text{G}}_{g,t}\le RU^{\text{G}}_g, \quad (32)$$

$$\forall t\in T,g\in\Phi^{\text{G}}$$

$$-RD^{\text{O}}_o\le P^{\text{O}}_{o,t+1}-P^{\text{O}}_{o,t}\le RU^{\text{O}}_o, \quad (33)$$

$$\forall t\in T,o\in\Phi^{\text{O}}$$

$$0\le L^{\text{shed}}_{n,t}\le d^{\text{E}}_{n,t},\qquad \forall t\in T,n\in\Lambda \quad (34)$$

$$x^{\text{G}}_{g,t}=x^{\text{G}}_{g,t'},\quad \forall t\in T,t'\in\tau(t),g\in\Phi^{\text{G}} \quad (35)$$

$$x^{\text{O}}_{o,t}=x^{\text{O}}_{o,t'},\quad \forall t\in T,t'\in\tau(t),o\in\Phi^{\text{O}} \quad (36)$$

$$y^{\text{G}}_{g,t}-z^{\text{G}}_{g,t}=x^{\text{G}}_{g,t}-x^{\text{G}}_{g,t-1},\quad \forall t\in T,g\in\Phi^{\text{G}} \quad (37)$$

$$y^{\text{O}}_{o,t}-z^{\text{O}}_{o,t}=x^{\text{O}}_{o,t}-x^{\text{O}}_{o,t-1},\quad \forall t\in T,o\in\Phi^{\text{O}} \quad (38)$$

$$x^{\text{G}}_{g,t},y^{\text{G}}_{g,t},z^{\text{G}}_{g,t}\in\{0,1\},\quad \forall t\in T,g\in\Phi^{\text{G}} \quad (39)$$

$$x^{\text{O}}_{o,t},y^{\text{O}}_{o,t},z^{\text{O}}_{o,t}\in\{0,1\},\quad \forall t\in T,o\in\Phi^{\text{O}} \quad (40)$$

$$E^{\text{G}}_{g,t}b^{\text{G}}_g\le\hat{P}^{\text{G}}_{g,t},\quad \forall t\in T,g\in\Phi^{G}. \quad (41)$$

Objective function (24) minimizes total electricity-generation costs. The $h/3600$ term multiplies electricity generation and unserved load to scale these values to be measured in MWh. This is necessary because the set, T, may represent sub-hourly time steps to provide sufficient time granularity for the natural gas-system model. As such, $L^{\text{shed}}_{n,t}$, $P^{\text{G}}_{g,t}$, and $P^{\text{O}}_{o,t}$ measure load curtailment and generation in 'MW-time step.' The $h/3600$ term rescales these to be measured in MWh. Scaling of the start-up cost is not necessary, because $y^{\text{G}}_{g,t}$ and $y^{\text{O}}_{o,t}$ only take on a value of 1 at the time step at which the corresponding unit is switched on.

Constraints (25) enforce load balance at each bus. Constraints (26) impose transmission limits and constraints (27) set the phase angle of the reference bus equal to zero. Constraints (28) and (29) impose minimum and maximum generation levels on the units during time steps in which they are online. These constraints also enforce zero production from units during time steps in which they are offline. Constraints (30) and (31) compute the amount of electric energy that is produced by each natural gas-fired and other thermal unit in each time step. In doing so, these constraints assume that the power output of each unit changes linearly from one time step to the next. Constraints (32) and (33) impose ramping limits on the generating units. Constraints (34) restrict the amount of load shed to be no greater than the demand. Constraints (35) and (36) require that the commitment status of the generating units not change during an hour. This is a typical 'implicit' restriction in unit commitment problems, insomuch as such problems are often modeled at hourly time steps. We allow generation levels to change during an hour if sub-hourly time steps are modeled. Indeed, the 'jumps' in the power output of the units can be reduced if a finer time resolution is modeled. Otherwise, ramp-based unit commitment [28] can also be employed to reduce such jumps, if so desired. By using sub-hourly time steps, our model can better represent the real-time dispatch of the system, which varies on a sub-hourly basis. Day-ahead dispatch models typically assume constant demand and dispatch levels over the course of each hour, which is divorced from actual real-time operations. Constraints (37) and (38) define the values of the start-up and shutdown binary variables for the generating units in terms of changes in the commitment variables (*i.e.*, changes in the value of x). Constraints (39) and (40) impose integrality on the commitment variables. Constraints (41) impose fuel limits on natural gas-fired units. As noted before, these limits are taken from the solution of the natural gas-system problem.

Because our focus is on the performance of the proposed coordination mechanism, we exclude a number of complicating constraints, such as minimum up and down times and network losses. Incorporating these constraints into the unit commitment model is straightforward. We expect that the behavior of the proposed co-ordination mechanism would be qualitatively similar with such constraints included in the model.

V. ITERATIVE SOLUTION ALGORITHM

Sections III and IV detail the models that are used to optimize the operations of the natural gas and power systems, respectively, in isolation. This section outlines the proposed mechanism for co-ordinating the operation of these two systems. We proceed by first detailing the iterative solution and then discussing the economic properties of a convergent solution that is given by the algorithm.

A. Algorithm

The proposed algorithm is implemented by iteratively solving problems (1)–(23) and (24)–(41) and 'communicating' quantities of natural gas demanded and supplied (with corresponding prices at which these quantities are demanded and supplied) between the two problems. This information exchange is within the scope of what is currently shared between the natural gas and wholesale electricity markets. Thus, an important property of our proposed co-ordination mechanism is the minimal requirement for information exchange. This means that system models and potentially sensitive operational information do not need to be shared between entities that own or operate the two systems.

To describe our proposed co-ordination algorithm, we first define $\zeta_{i,t}$ as the Lagrange multiplier that is associated with the node-i/time step-t natural gas-balance constraint. If node i is a supply node, then $\zeta_{i,t}$ is the Lagrange multiplier that is associated with constraint (9) for node i and time step t. Otherwise, $\zeta_{i,t}$ is the Lagrange multiplier that is associated with constraint (10) for node i and time step t. We also define $\lambda_{n,t}$ as the dual variable that is associated with constraint (25) for bus n and time step t. These dual variables are obtained by re-solving a linear relaxation of the unit commitment problem in which the binary variables are fixed equal to their optimal values.

The $\lambda_{n,t}$ dual variables are used to obtain values of $\hat{\lambda}_{g,t}$, after accounting for each unit's heat rate, which are inputs to the natural gas-system model. Specifically, after obtaining values for $\lambda_{n,t}$ from the unit commitment problem, we update $\hat{\lambda}_{g,t}$ as:

$$\hat{\lambda}_{g,t} \leftarrow \frac{3600\lambda_{n,t}}{hb_g^{\mathrm{G}}}, \tag{42}$$

where n is the power system bus that natural gas-fired unit g is located at. We analogously use the Lagrange multipliers, $\zeta_{i,t}$, that are obtained from solving the natural gas-system model to obtain values of $\hat{\zeta}_{g,t}$, which are inputs to the unit commitment model. Specifically, we update $\hat{\zeta}_{g,t}$ as:

$$\hat{\zeta}_{g,t} \leftarrow \frac{\zeta_{i,t}}{W_1 Mh}, \tag{43}$$

where i is the natural gas node that natural gas-fired unit g is connected to.

In addition to this dual information, the two models exchange primal information, which represents the maximum amount of fuel that is demanded by and supplied to natural gas-fired units in the natural gas-system and unit commitment models, respectively. Specifically, after obtaining a solution to the unit commitment problem we update $\hat{d}_{g,t}^{\mathrm{GV}}$ as:

$$\hat{d}_{g,t}^{\mathrm{GV}} \leftarrow \frac{b_g^{\mathrm{G}} E_{g,t}^{\mathrm{G}}}{3600M}. \tag{44}$$

Similarly, after obtaining a solution to the natural gas-system model we update $\hat{P}_{g,t}^{\mathrm{G}}$ as:

$$\hat{P}_{g,t}^{\mathrm{G}} \leftarrow 3600M d_{g,t}^{\mathrm{GV}} \varphi_b. \tag{45}$$

Algorithm 1 provides pseudocode outlining the proposed co-ordination mechanism. The algorithm takes as an input, on line 1, a convergence tolerance, ϵ. Line 2 initializes the iteration counter and sets starting values for $\hat{\zeta}_{g,t}$ and $\hat{P}_{g,t}^{\mathrm{G}}$. Lines 3–12 are the main iterative loop. Line 4 solves the unit commitment model and saves the incumbent energy-production levels of the natural gas-fired units to $E_{g,t}^{\mathrm{G},(k)}$ (where k is a superscript denoting the iteration number). Lines 5 and 6 update $\hat{\lambda}_{g,t}$ and $\hat{d}_{g,t}^{\mathrm{GV}}$ as outlined in (42) and (44), respectively.

Algorithm 1 Natural gas/electricity co-ordination

```
1:  input: fix ϵ
2:  initialize: k ← 0, set values for ζ̂_{g,t}, P̂^G_{g,t}
3:  repeat
4:      min (24) s.t. (25)–(41)
5:      E^{G,(k)}_{g,t} ← E^G_{g,t}, ∀t ∈ T, g ∈ Φ^G
6:      λ̂_{g,t} ← 3600λ_{n,t}/(h b^G_g), ∀t ∈ T, g ∈ Φ^G
7:      d̂^{GV}_{g,t} ← b^G_g P^G_{g,t}/(3600M), ∀t ∈ T, g ∈ Φ^G
8:      max (1) s.t. (2)–(23)
9:      ζ̂_{g,t} ← ζ_{i,t}/(W_1 M h), ∀t ∈ T, g ∈ Φ^G
10:     P̂^G_{g,t} ← 3600M d^{GV}_{g,t} φ_b, ∀t ∈ T, g ∈ Φ^G
11:     k ← k + 1
12: until ||E^{G,(k−1)} − E^{G,(k−2)}||/||E^{G,(k−1)} + E^{G,(k−2)}|| ≤ ϵ
```

Line 8 then solves the natural gas-system model with the updated values of $\hat{\lambda}_{g,t}$ and $\hat{d}_{g,t}^{\mathrm{GV}}$. The solution to the natural gas-system model is used in lines 9 and 10 to update $\hat{\zeta}_{g,t}$ and $\hat{P}_{g,t}^{\mathrm{G}}$, respectively, as outlined in (43) and (45). This process repeats until achieving the convergence criterion in line 12, which requires that the change in the energy-production levels of all of the natural gas-fired units between two successive iterations be sufficiently small. The term, $E^{\mathrm{G},(k)}$, denotes a $||T|| \cdot ||\Phi^{\mathrm{G}}|| \times 1$ vector of $E_{g,t}^{\mathrm{G},(k)}$-variable values that are obtained in the kth iteration of the algorithm.

B. Economic Interpretation of Solution of Algorithm 1

If the power and natural gas systems are at a competitive equilibrium, the consumer surplus that a natural gas-fired unit earns in the natural gas system is identical to the producer surplus that unit earns in the power system. We can express the producer surplus that is earned by natural gas-fired unit g in time step t as:

$$\frac{h}{3600}\left[\lambda_{n,t} E_{g,t}^{G} - (c_g^{\mathrm{G}} + \hat{\zeta}_{g,t} b_g^{\mathrm{G}}) E_{g,t}^{G}\right], \tag{46}$$

where n is the bus at which generator g is located [24]. The time step-t consumer surplus that this generator earns in the natural gas system is given by:

$$\frac{h}{3600}(\omega_{g,t} - \hat{\zeta}_{g,t}) b_g^{\mathrm{G}} E_{g,t}^{G}, \tag{47}$$

where $\omega_{g,t}$ is the generator's time step-t willingness to pay for natural gas. Assuming that the two markets clear simultaneously, equality of (46) and (47) requires that:

$$\omega_{g,t} = \frac{\lambda_{n,t} - c_g^{\mathrm{G}}}{b_g^{\mathrm{G}}}.$$

Our proposed co-ordination mechanism acknowledges that the assumptions of perfect competition and simultaneous price formation in the two systems do not hold in practice [24]. As such, our mechanism has natural gas-fired units submit their estimates of locational trade values of natural gas (*i.e.*, values of $\hat{\zeta}_{g,t}$) to the electricity market. Once the electricity market clears, natural gas-fired units develop their estimates of willingness to pay for fuel, based on cleared LMPs. Clearing of the natural gas market results in new locational trade values, *ad infinitum*. If this process converges to a consistent set of LMPs, willingness to pay, and locational trade values, natural gas-fired units earn the same welfare in both markets, meaning that a competitive equilibrium is obtained.

VI. Case Study

This section demonstrates the proposed co-ordination mechanism using a case study that is based on a 25-node natural gas system that is coupled with the IEEE 24-bus RTS. We begin by describing the case study data and then summarize the results.

A. Data

Fig. 1 shows the topology of the 25-node natural gas system, which has one supply node ($i = 1$) and 24 pipelines, which are represented by the arcs in the figure. There are five natural gas-fired units, which are connected to natural gas nodes 6, 9, 12, 19, and 24, and are indicated in Fig. 1. To accurately represent their dynamic behavior, the pipelines are discretized into segments with lengths no greater than 10 km. The allowable pressure ranges for both nodes and pipelines are 3.447 MPa to 5.516 MPa and the allowable density ranges are 14.55 kg/m^3 to 48.51 kg/m^3. All of the pipelines are assumed to have friction factors of 0.01. Table I summarizes the length and diameter of each pipeline. There are five compressors installed in the natural gas system, which are denoted by the arrows in Fig. 1. Each arrow is pointing toward the end node of the pipeline on which the compressor is installed. The boost ratios of all of the compressors are limited to be between 1.0 and 1.6.

Two cases with different-length time steps are used to examine the proposed co-ordination scheme. The temporal physical dynamics of the natural gas system are represented using piecewise-linear (as opposed to piecewise-constant) functions on hourly (*i.e.*, $h = 3600$) and 15-minute (*i.e.*, $h = 900$) time segments. Electric demands are represented using hourly and 15-minute time steps in the two cases, respectively. As discussed in Section IV, constraints (30) and (31) assume that electric demands change linearly between successive time steps. A node with highly variable natural gas withdrawals, for instance because of a natural gas-fired unit starting up or shutting down, can experience non-trivial hourly dynamics. Thus, the hourly dynamics of natural gas flows may be sufficient for day-ahead optimization. On the other hand, increasing the time resolution from hourly to 15-minute periods can reduce the jumps in electricity demands. This may yield more accurate power system dispatch and natural gas-system dynamics.

Table II summarizes the characteristics of the five natural gas-fired and eight other thermal units. The other thermal units are all assumed to have the same 200-MW/h upward and downward ramping limits. Fig. 2 shows the aggregate electric load profile. We assume that the only natural gas loads are those that are associated with operating the natural gas-fired units, meaning that $d_{i,t}^{\mathrm{G}} = 0, \forall t \in T, i \in \Upsilon$.

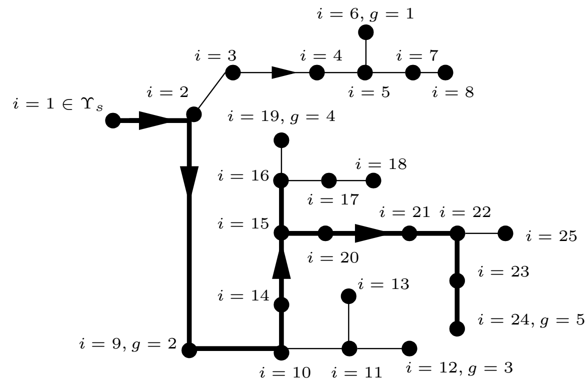

Fig. 1. Topology of natural gas system. Pipeline lengths are not to scale.

TABLE I
Natural Gas Pipeline Data

Starting Node	Ending Node	Length [km]	Diameter [m]
1	2	100	0.9144
2	3	30	0.6350
3	4	5	0.6350
4	5	15	0.6350
5	6	10	0.6350
6	7	5	0.6350
7	8	10	0.6350
2	9	5	0.9144
9	10	60	0.9144
10	11	5	0.6350
11	12	8	0.6350
11	13	6	0.6350
10	14	80	0.9144
14	15	10	0.9144
15	16	20	0.9144
16	17	3	0.6350
17	18	6	0.6350
16	19	5	0.6350
15	20	40	0.9144
20	21	5	0.9144
21	22	20	0.9144
22	23	5	0.9144
23	24	16	0.9144
22	25	8	0.6350

TABLE II
THERMAL- AND NATURAL GAS-FIRED-UNIT DATA

Other Thermal Unit			Natural Gas-Fired Unit			
o	c_o^{O}	$P_o^{\mathrm{O,max}}$	g	b_g^{G}	RD_g^{G},RU_g^{G}	$P_g^{\mathrm{G,max}}$
1	70	480	1	10.0	2400	2000
2	65	1200	2	9.0	900	800
3	72	480	3	9.5	900	800
4	62	1200	4	8.0	1400	1200
5	52	800	5	8.5	1400	1200
6	58	480				
7	53	1200				
8	54	1200				

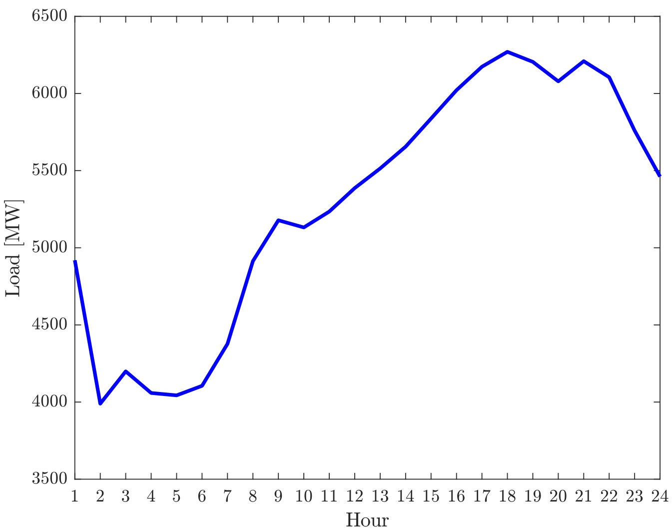

Fig. 2. Electric load profile.

B. Results

The results of the two cases with hourly and 15-minute time steps are presented in this section. In both cases, the co-ordination problem is solved using Algorithm 1 both without and with regularization in the natural gas-system model. We use $\hat{\zeta}_{g,t} \leftarrow 3$ and $\hat{P}_{g,t}^{\mathrm{G}} \leftarrow +\infty, \forall t \in T, g \in \Phi^{\mathrm{G}}$ as initial values in line 2 of the algorithm and set $\epsilon = 10^{-3}$.

1) Hourly Time Steps: Fig. 3 summarizes total daily fuel consumption of the natural gas-fired units in each iteration of the algorithm, without and with regularization. The main iterative loop of Algorithm 1 must only be executed twice to attain convergence. As Fig. 3 shows, regularization results in slightly more fuel consumption by unit 2 but slightly less consumption by units 3 and 4.

Fig. 4 summarizes the dispatch of the 13 units in the first and final iterations without and with regularization. As expected, introducing fuel-supply and demand limits in the unit commitment and natural gas-system models results in lower dispatch of the natural gas-fired units. Contrasting the dispatch solutions that are computed with and without regularization shows that the regularizer yields dispatch levels that are less variable over time, which is the purpose of regularization. Specifically, the dispatch of natural gas-fired unit 2 in hours 8–24 exhibits reduced variability with regularization. Moreover, non-natural-gas-fired thermal units 1, 2, and 6 are not ramp-constrained in hours 2 and 8 with regularization.

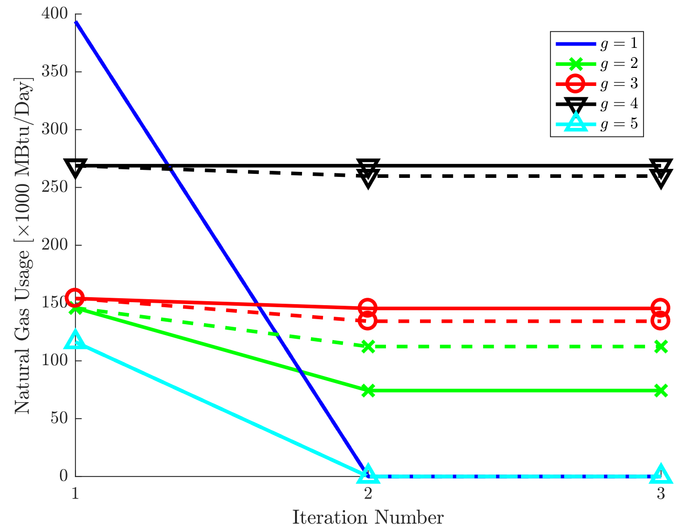

Fig. 3. Total daily fuel use of natural gas-fired units in each iteration of Algorithm 1 without (solid lines) and with (dashed lines) regularization with hourly time steps.

Fig. 5 shows the electric-load-weighted LMPs that are obtained in the initial and final iterations of the algorithm without and with regularization. The final LMPs are higher than those that are obtained in the initial iteration. This is because other thermal units, which have higher operating costs than the natural gas-fired units, are dispatched due to fuel-supply constraints on the latter. Regularization results in smaller LMP variability over time and elimination of price spikes in hours 8 and 24. The reduced LMP variability stems from the reduced variability in the units' dispatch. Regularization yields a smoother dispatch. As a result, ramping limits of the other thermal units are not binding and price spikes are mitigated [3], [29].

This case study is solved using a computer with a 2.6-GHz Intel Core $i7$ processor and 8 GB of RAM. The solution time for each single problem is less than 30 s.

2) 15-Minute Time Steps: Fig. 6 summarizes total daily fuel use of the natural gas-fired units in each iteration of Algorithm 1 without and with regularization with 15-minute time steps. Algorithm 1 must be executed four and six times, respectively, for convergence with and without regularization. Comparing Figs. 6 and 3 shows that 15-minute time steps yields similar results to those that are obtained from hourly time steps.

Fig. 7 summarizes the dispatch of the generating units in the first and final iterations without and with regularization. As in the case with hourly time steps, introducing the regularizer yields dispatch levels that are less variable over time. Moreover, contrasting the cases with hourly and 15-minute time steps without regularization shows that the use of 15-minute time steps results in more smoothly changing dispatch levels, despite the regularizer not being used.

Fig. 8 shows the hourly-averaged electric-load-weighted LMPs that are obtained in the initial and final iterations of the algorithm without and with regularization. As in the case with

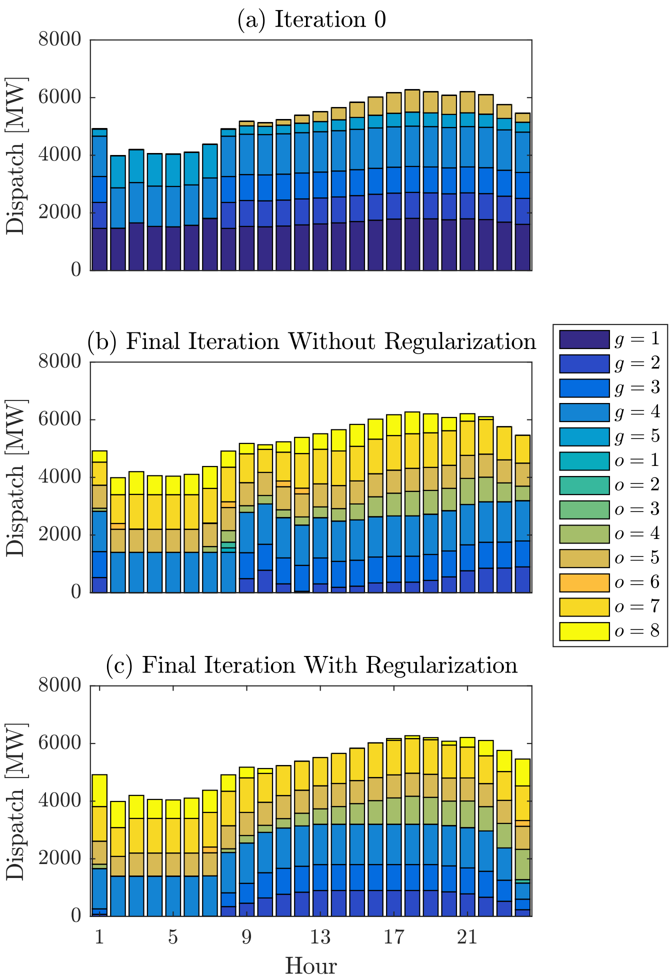

Fig. 4. Dispatch of units in (a) initial iteration, (b) final iteration without regularization, and (c) final iteration with regularization with hourly time steps.

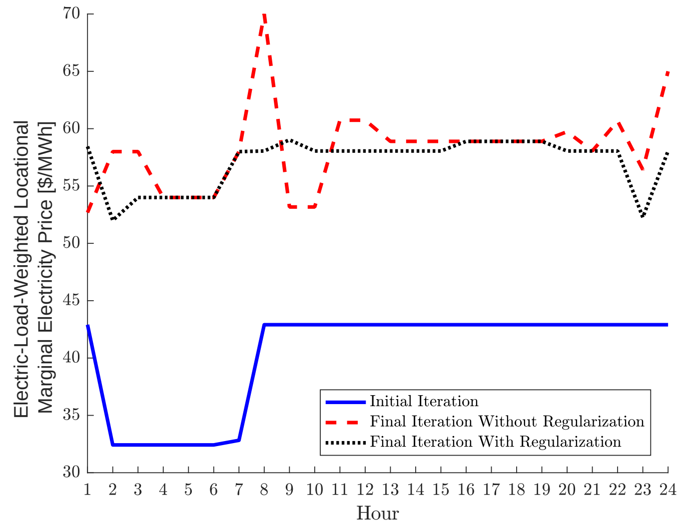

Fig. 5. Electric load-weighted LMPs in initial and final iterations of Algorithm 1 without and with regularization with hourly time steps.

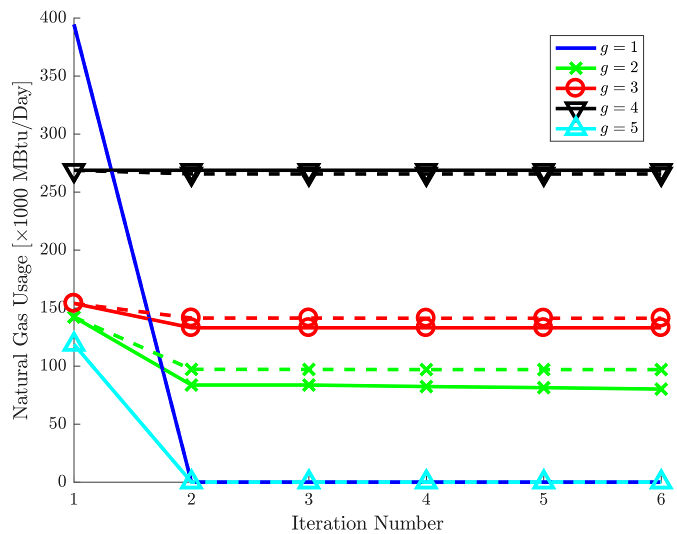

Fig. 6. Total daily fuel use of natural gas-fired units in each iteration of Algorithm 1 without (solid lines) and with (dashed lines) regularization with 15-minute time steps.

hourly time steps, the final LMPs are higher than those that are obtained in the initial iteration. Moreover, regularization yields less LMP variability over time and eliminates price spikes in hours 8 and 24. Comparing Figs. 5 and 8 shows that the price spikes that are in Fig. 5 without regularization are reduced when using 15-minute time steps. This shows that more granular temporal modeling of the system reduces both variability in the unit dispatch and price spikes.

This case is solved using the same computational environment that is applied to the case with hourly time steps. The solution time for each single problem is less than two minutes.

VII. Conclusions

This paper presents a market-based mechanism for co-ordinated scheduling of interdependent natural gas and electric power systems. The only types of information that must be exchanged between the systems are fuel prices and flow quantities for natural gas-fired units. The proposed co-ordination mechanism is of particular interest if high or varying fuel demands from natural gas-fired units create constrained conditions in the pipeline system. In such cases our mechanism takes corrective actions through reduced dispatch of natural gas-fired units and natural gas LMPs that reflect scarce supply.

The purpose of our proposed co-ordination mechanism is to ensure that natural gas that is 'demanded' by the power system to supply electric loads can be feasibly supplied without requiring the curtailment of any non-generation-related natural gas demand. If so, the co-ordination mechanism should not alter the operation of either system because the LMPs in the electric systems are typically higher than the marginal costs of dispatched generation units. We do not consider the rare cases in which electric LMPs are smaller than the marginal costs of dispatched generation units, which may occur with 'fast-ramping' conditions [3], and leave such cases for future research.

If, conversely, the natural gas that is needed to serve electric loads cannot feasibly be supplied, the mechanism aims to curtail natural gas supply based on the relative value of fuel at different points in the natural gas system (which are dependent

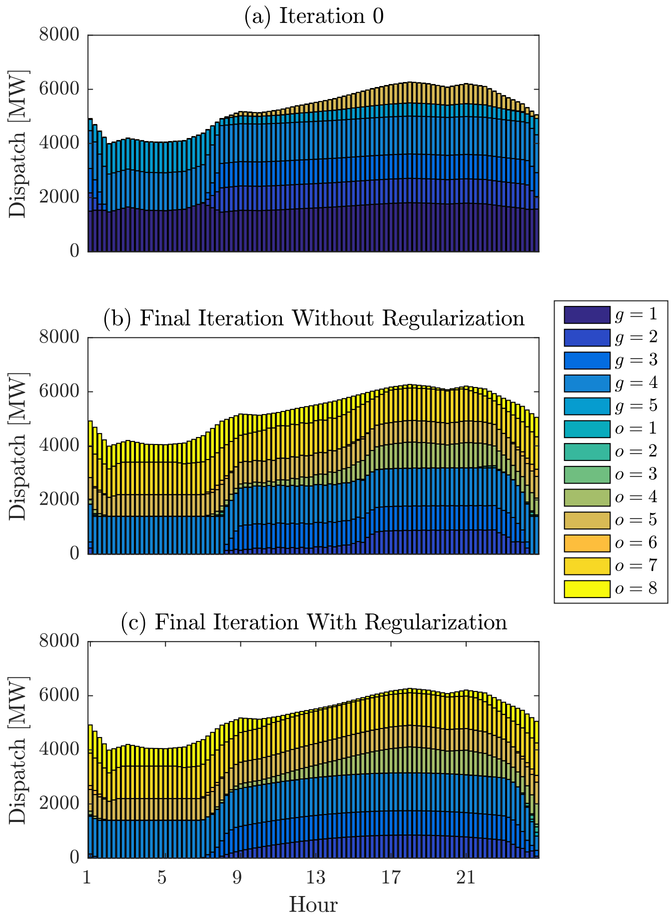

Fig. 7. Dispatch of units in (a) initial iteration, (b) final iteration without regularization, and (c) final iteration with regularization with 15-minute time steps.

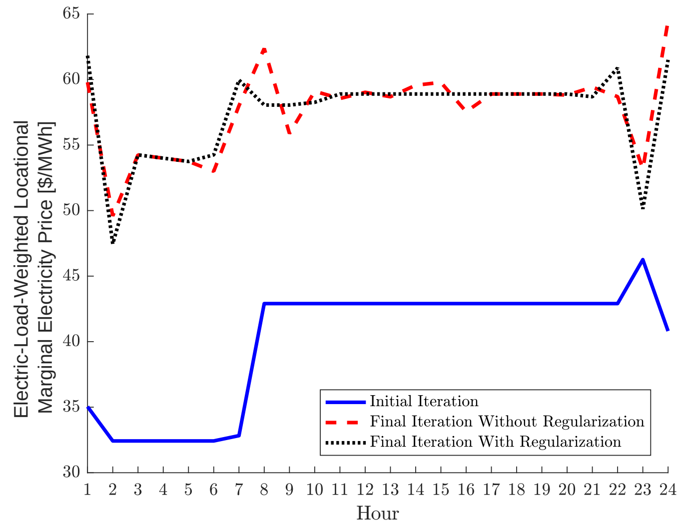

Fig. 8. Hourly-averaged electric load-weighted LMPs in initial and final iterations of Algorithm 1 without and with regularization with 15-minute time steps.

on the electric LMPs). We would expect decreases in welfare and increases in natural gas prices in cases in which the natural gas system is constrained. This should, in turn, result in natural gas-fired units having higher production costs.

It is difficult to contrast the performance of our proposed model to a baseline case without any co-ordination scheme. This is because without a co-ordination scheme, there is no 'rule' as to how natural gas deliveries are curtailed. Thus, it is difficult to quantify the benefits of our co-ordination scheme relative to a case of no co-ordination. If co-optimization of the two systems is infeasible or impractical, our scheme optimizes scheduling under congested conditions (*i.e.*, maximizes welfare). The curtailment that occurs when applying the co-ordination scheme can be observed by comparing Figs. 4.a and 7.a (which show the power system dispatch without co-ordination or considering the natural gas system) to Figs. 4.b-c and 7.b-c (which show the co-ordinated dispatch solution). The LMPs that are shown in Figs. 5 and 8 provide (approximately) the minimum price increase from this co-ordination. Consequently, any natural gas curtailments that occur may cause unserved energy in the power system, which will greatly decrease social welfare and increase electricity prices.

Our case studies show that only two to six iterations of the algorithm are needed for convergence. To implement the mechanism within the time constraints of the relevant decision cycles, each optimization model would need to be solved in under two hours of wall-clock time [24]. We expect that the proposed mechanism can converge within this required time frame. This is because the two operational problems can be solved on a high-performance computing platform if the mechanism is implemented in practice. Moreover, computationally efficient implementations of unit commitment and natural gas scheduling models can reduce the burden of solving the two problems. Thus, we believe that the proposed mechanism could be implemented in practice after further model and solution-algorithm development.

Our proposed co-ordination method is a heuristic that is based on Lagrangian Relaxation. Although there may be conditions under which convergence can be guaranteed, rigorous proofs of optimality and convergence are beyond the scope of this paper. These are potential topics for future work.

Implementing our proposed co-ordination mechanism has a number of challenges beyond the need for modeling and solution-algorithm improvements. First, implementing the proposed co-ordination scheme requires changing the current timing of natural gas and electricity markets. The current misalignment of the two markets is often reported as a barrier to co-ordination. Second, implementing this mechanism requires substantial advancement in automation and control of natural gas systems. However, preliminary studies indicate that such advances are feasible [23]. Our model and algorithm can be used to analyze the operation of these systems and quantify the benefit of such improvements.

The regularizer in the natural gas-system model can impact natural gas prices. This impact is small, due to scaling of the regularization term. Nevertheless, future work examining its impact on the incentive properties of the LMPs would be valuable. Similarly, the convergence properties of the co-ordination mechanism if $W_2 \to 0$ could address any incentive compatibility issues arising from use of the regularizer.

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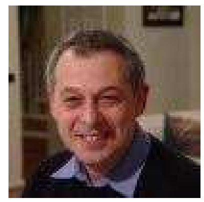

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