Optimized Offers for Cascaded Hydroelectric Generators in a Market with Centralized Dispatch

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***Abstract*—We examine the problem of a generator offering generation and ancillary services from a set of cascaded hydroelectric units to a centrally dispatched market that does not account for watershed constraints. By modeling the least-cost dispatch problem and computing the resulting schedules and market prices, we formulate a stochastic bilevel optimization problem that maximizes the generator's expected profits under different demand and supply realizations. To account for potential infeasibilities in the resulting hydroelectric dispatch, we include penalties in the generator's objective function. We propose a simple technique that replaces the lower-level dispatch problem with its linearized Karush-Kuhn-Tucker optimality conditions to convert the problem to a single-level mixed-integer program. We use two numerical case studies, based on actual river systems, to demonstrate the benefits of the proposed model.**

***Index Terms*—Hydroelectric generation, offer optimization, economic dispatch, power system economics**

Nomenclature

A. Index Sets

T	time index set
Ω	set of scenarios
H	hydroelectric powerhouse index set
S	hydroelectric reservoir index set
B	set of steps allowed in offer curves
A	set of ancillary service (AS) products
A_u	set of upward AS products

B. Market Bids

ξ^ω	probability of scenario ω
$K^\omega_{b,t}$	price in step b of rival generators' hour-t scenario-ω energy offer
$\bar{P}^\omega_{b,t}$	quantity in step b of rival generators' hour-t scenario-ω energy offer
$W^\omega_{b,t}$	price in step b of hour-t scenario-ω market energy demand function
$\bar{D}^\omega_{b,t}$	quantity in step b of hour-t scenario-ω market energy demand function
$\kappa^\omega_{a,b,t}$	price in step b of rival generators' hour-t scenario-ω type-a AS offer
$\bar{\alpha}^\omega_{a,b,t}$	quantity in step b of rival generators' hour-t scenario-ω type-a AS offer
$\beta^\omega_{a,t}$	hour-t scenario-ω type-a AS market demand
m	deviation penalty, as a fraction of day-ahead price

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C. Hydroelectric Generator's Offer Parameters

$\hat{c}_{h,b,t}$	price in step b of powerhouse h's hour-t energy offer
$\hat{Q}_{h,b,t}$	quantity in step b of powerhouse h's hour-t energy offer
$\overline{Q}_h$	powerhouse h's daily energy limit
$\hat{\pi}_{h,a,t}$	powerhouse h's hour-t type-a AS offer price
$\hat{R}_{h,a,t}$	powerhouse h's hour-t type-a AS offer quantity

D. System Operator's Dispatch Variables

$P^\omega_{b,t}$	quantity purchased from step b of rival generators' hour-t energy offer in scenario ω
$D^\omega_{b,t}$	quantity sold to step b of hour-t energy demand function in scenario ω
$q^\omega_{h,b,t}$	quantity purchased from step b of powerhouse h's hour-t energy offer in scenario ω
$\alpha^\omega_{a,b,t}$	quantity purchased from step b of rival generators' hour-t type-a AS offer in scenario ω
$r^\omega_{h,a,t}$	type-a AS purchased from powerhouse h during hour t of scenario ω

E. Hydroelectric Generators' Cost and Constraint Parameters

Q_h	powerhouse h's maximum rated capacity
e_h	powerhouse h's generation efficiency
v_s	value of water in reservoir s
$R_{h,a}$	powerhouse h's actual type-a AS capacity
$n_{s,t}$	natural water inflows to reservoir s
L^-_s	reservoir s's minimum water level
L^+_s	reservoir s's maximum water level
L^0_s	reservoir s's starting water level
L^T_s	reservoir s's ending target water level
χ	allowable tolerance on target water level
$\rho(s)$	set of powerhouses that draw water from reservoir s
$\nu(s)$	set of powerhouses that deliver water to reservoir s
ι_h	travel time between powerhouse h and its immediate downstream reservoir

F. Hydroelectric Generator Operations Variables

$l^\omega_{s,t}$	reservoir s's ending hour-t scenario-ω water level
$z^\omega_{h,t}$	powerhouse h's hour-t scenario-ω water use
$g^\omega_{h,t}$	powerhouse h's hour-t scenario-ω actual generation
$\delta^{\omega,+}_{h,t}$	powerhouse h's hour-t scenario-ω positive energy deviation
$\delta^{\omega,-}_{h,t}$	powerhouse h's hour-t scenario-ω negative energy deviation
$\theta^\omega_{h,a,t}$	powerhouse h's hour-t scenario-ω actual type-a AS provision
$\zeta^\omega_{h,a,t}$	powerhouse h's hour-t scenario-ω type-a AS deviation

$\tilde{g}^{\omega,-}_{h,t}$ powerhouse h's hour-t scenario-ω generation if downward AS is provided in real-time

$\tilde{g}^{\omega,+}_{h,t}$ powerhouse h's hour-t scenario-ω generation if upward AS is provided in real-time

I. Introduction

MANY restructured electricity markets endow a system operator (SO) with operational control of the power system. The SO solicits offers and bids from generators and loads. These are input to a welfare-maximization problem, which determines day-ahead generator schedules, loads that are served, and settlement prices. Generators or loads that deviate from these schedules must typically remunerate the SO for replacement energy or ancillary services (AS), based on real-time prices.

This centralized market paradigm can be problematic for some generating resources. This is because the structure of the cost and operating constraint data that the SO accepts in an offer may not fully capture the capabilities of a plant. This is especially true of cascaded hydroelectric generators, *e.g.*, plants that are on a river catchment. In addition to generation limits on powerhouses, hydroelectric generators are also constrained by water availability. Consider, as an example, the simple watershed illustrated in Fig. 1, which consists of four reservoirs on a river connected by four powerhouses. How each powerhouse can be operated depends on the operation of the others, since the reservoirs have minimum- and maximum-water-levels.

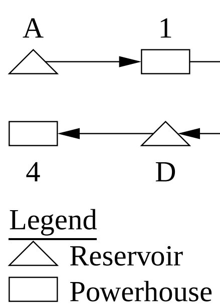

Fig. 1. Simple river system.

Thus, feasible operation of cascaded hydroelectric generators requires their dispatch to be optimized in an integrated fashion with constraints coupling their operation. Moreover, these constraints are dynamic in that the time-t availability of a powerhouse depends on how it and the other powerhouses are scheduled at other times. There are numerous models in the literature that explicitly capture such interdependencies in optimizing hydroelectric systems. These approach the problem from the perspective of a hydroelectric generator that minimizes the cost of supplying its load or maximizes profit against exogenous market prices [1]–[5]. Pousinho *et al.* [6] relax the exogenous price assumption by modeling a stepped inverse demand function, giving generation-sensitive prices.

Operating a cascaded hydroelectric system in a centralized market can be more difficult, since many markets do not consider the constraints that couple its operation. For instance, the California ISO (CAISO) market only allows hydroelectric generators to submit offers with constraints on each powerhouse individually, but without any constraints coupling their operations. If the powerhouses are offered into such a market with true cost and constraint parameters,[1] the resulting dispatch may be infeasible. This exposes the hydroelectric generator to the added financial risk of deviation penalties. Moreover, since the SO dispatches the overall system with incorrect hydroelectric operating constraints, resulting real-time operations may be inefficient compared to the true welfare maximum that accounts for watershed constraints.

One solution, which completely eliminates dispatch infeasibilities, is for the generator to forecast the optimal dispatch of its units and self-schedule its generation.[2] Gfrerer [7] models an energy self-scheduling problem using optimal control techniques and transforms it into an equivalent nonlinear optimization. Bauer *et al.* [8] expand upon this by using dynamic and nonlinear optimization techniques to solve numerical case studies. Fleten and Kristoffersen [9] introduce a stochastic model, in which real-time prices are uncertain, to optimize hydroelectric energy self-schedules. Faria and Fleten [10] expand this by allowing the hydroelectric generator to adjust its day-ahead schedule in the real-time market to correct infeasibilities or suboptimality caused by incorrect day-ahead forecasts. Kazempour *et al.* [11], [12], Ahmadi *et al.* [13] and Abgottspon and Andersson [14] expand upon these works by modeling self scheduling of energy and ancillary services.

Almeida and Conejo [15] study equilibrium medium-term generation decisions of competing hydroelectric firms. They model the interaction as an equilibrium problem subject to equilibrium constraints. The upper-level problems in their model represent the generation decisions made by the hydroelectric firms and the lower-level problems the resulting dispatch of thermal generators. This problem is akin to a self-scheduling problem, since the strategic variable that they focus on is generation quantity. Molina *et al.* [16] similarly model Nash-Cournot equilibria between competing firms in a hydrothermal system. As with the work of Almedia and Conejo [15], this analysis also focuses on generation quantity as the strategic variable of interest. Moreover, Molina *et al.* [16] do not explicitly model a centralized dispatch, which is the focus of our work.

A downside to relying on self-schedules is that it can introduce coordination losses that the SO's welfare maximization is designed to address [17]. Moreover, self-scheduling can foreclose on economic and efficiency gains if the generator incorrectly forecasts market conditions. Thus, an alternative is for the hydroelectric generator to submit more flexible offers to the SO. In doing so, it should 'tailor' the offer in such a way that balances gains from profitable dispatch against possible dispatch infeasibilities.

This paper proposes a stochastic bilevel modeling approach to optimize such hydroelectric offers. The lower level is the

[1] By 'constraint parameters,' we mean the types of constraints that the market accepts, such as an hourly generation capacity, but no coupling constraints.

[2] Many centralized markets allow explicit self-scheduling. If not, a generator can replicate a self-schedule by offering its desired dispatch at the price floor, guaranteeing that it is accepted by the market.

SO's welfare maximization, which determines the dispatch and market prices based on demand and offers from the hydroelectric and rival units. The SO's problem only includes individual unit constraints, and does not consider the watershed constraints coupling the powerhouses. The upper level maximizes the hydroelectric generator's expected profit, which consists of energy and AS payments less deviation penalties. The hydroelectric generator behaves as a price-maker, insomuch as it accounts for the effect of its offers on energy and AS prices. Bids and offers submitted by other market participants are assumed uncertain. Hydroelectric deviations are defined as the difference between the SO's dispatch and how the watershed can be feasibly operated.

Because the SO's problem is a convex linear program, we can replace it with its necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions. We use standard techniques to linearize the KKT complementary slackness conditions [18]–[20], allowing us to convert the bilevel offer-optimization problem into a single-level mixed-integer linear program (MILP). We also use two numerical case studies, based on actual river systems in the CAISO control area, to demonstrate the efficacy of our model in improving hydroelectric profit and system efficiency relative to the hydroelectric generator submitting true cost and constraint parameters.

Our work builds off of the model developed by Bakirtzis *et al*. [19], which optimizes stepped energy offers for thermal generators in a centrally dispatched market. Our work differs in a number of ways, however, making several important contributions to the literature. First, we study the more complex problem of structuring hydroelectric generation offers, which have complex dynamic constraints coupling their operation. Secondly, given the importance of using hydroelectric resources for AS in some systems, we model both energy and AS. The inclusion of AS adds more complicating relationships between the offer parameters in the SO's dispatch and in operating the powerhouses on the watershed. Third, we explicitly focus on the issue of the SO dispatching the watershed infeasibly, and how offers should be structured to tradeoff between higher profits in some scenarios against deviation penalties in others.

The remainder of this paper is organized as follows. Section II gives the formulation of the SO's welfare-maximization and the hydroelectric generator's offer-optimization models. Section III details the steps to convert the bilevel offer optimization into a MILP. Section IV describes our numerical case studies and results. Section V concludes.

II. Models

We restrict attention to a day-ahead market, which is modeled at hourly timesteps. The hydroelectric generator similarly optimizes its offers using a one-day planning horizon and operates its units at hourly timesteps. The hydroelectric generator's offer optimization is modeled as a two-stage stochastic program. In the first stage, the hydroelectric generator submits offers into the day-ahead market, without knowing the demand and supply bids of other market participants. In the second stage the rivals' bids are realized and the SO provides the hydroelectric generator with a provisional dispatch schedule and day-ahead prices. Using this schedule and constraints on how the watershed can be feasibly operated, the hydroelectric generator determines how to operate its plants and how much to deviate from the SO's schedule. We assume that the hydroelectric generator must purchase replacement energy and AS for any schedule deviations from the real-time market, and that this incurs a cost that is proportional to the day-ahead price.

A. System Operator's Welfare Maximization

Centralized electricity markets rely on generator-supplied cost and constraint data to determine the day-ahead dispatch. Although individual markets differ in terms of the offers that generators may submit, we assume a generic archetypal structure. Specifically, the hydroelectric generator may offer for each of its powerhouses a: (i) non-decreasing stepped marginal generation cost curve, (ii) a price/quantity pair for each traded AS product, and (iii) a daily generation limit. We assume that the marginal generation cost curves and AS offers can differ for each of the 24 hours. The daily generation limit is assumed to only apply to scheduled generation, without consideration of possible real-time AS deployments.

These assumptions closely match the structure of the CAISO market [21]. The CAISO requires most resources to submit offers covering their entire operating ranges in all hours. Hydroelectric generators are exempt from this requirement and may offer only the quantity determined to be feasible for delivery, exactly because of their complex operating constraints [22]. There are other possible extensions to the SO's model that could be included in our offer optimization. These are discussed in Section II-C. We exclude these from our analysis to simplify model notation.

For a given scenario, ω, which encompasses a realization of non-hydroelectric generation offers and energy and AS demand bids, the SO's dispatch model is formulated as:

$$\max \sum_{t\in T}\left[\sum_{b\in B}(W^{\omega}_{b,t}D^{\omega}_{b,t}-K^{\omega}_{b,t}P^{\omega}_{b,t})-\sum_{h\in H,b\in B}\hat{c}_{h,b,t}q^{\omega}_{h,b,t}\right.$$
$$\left.-\sum_{a\in A}\left(\sum_{b\in B}\kappa^{\omega}_{a,b,t}\alpha^{\omega}_{a,b,t}+\sum_{h\in H}\hat{\pi}_{h,a,t}r^{\omega}_{h,a,t}\right)\right]; \qquad (1)$$

$$\text{s.t.}\ \sum_{b\in B}P^{\omega}_{b,t}+\sum_{h\in H,b\in B}q^{\omega}_{h,b,t}=\sum_{b\in B}D^{\omega}_{b,t}; \qquad (2)$$
$$\forall\, t \qquad (\lambda^{\omega}_{t})$$

$$\sum_{b\in B}\alpha^{\omega}_{a,b,t}+\sum_{h\in H}r^{\omega}_{h,a,t}\geq\beta^{\omega}_{a,t};\quad \forall\, a,t \quad (\lambda^{\omega}_{a,t}) \qquad (3)$$

$$q^{\omega}_{h,b,t}\leq\hat{Q}_{h,b,t};\quad \forall\, h,b,t \quad (\eta^{\omega}_{h,b,t}) \qquad (4)$$

$$\sum_{t\in T,b\in B}q^{\omega}_{h,b,t}\leq\overline{Q}_{h};\quad \forall\, h \quad (\eta^{\omega}_{h}) \qquad (5)$$

$$r^{\omega}_{h,a,t}\leq\hat{R}_{h,a,t};\quad \forall\, h,a,t \quad (\eta^{\omega}_{h,a,t}) \qquad (6)$$

$$\sum_{a\in A\setminus A_u}r^{\omega}_{h,a,t}\leq\sum_{b\in B}q^{\omega}_{h,b,t};\quad \forall\, h,t \quad (\mu^{\omega}_{h,d,t}) \qquad (7)$$

$$\sum_{b\in B}q^{\omega}_{h,b,t}+\sum_{a\in A_u}r^{\omega}_{h,a,t}\leq\sum_{b\in B}\hat{Q}_{h,b,t}; \qquad (8)$$

$$\forall\ h,t \qquad (\mu^{\omega}_{h,u,t})$$

$$P^{\omega}_{b,t} \le \bar{P}^{\omega}_{b,t}; \qquad \forall\ b,t \qquad (\sigma^{P,\omega}_{b,t}) \tag{9}$$

$$D^{\omega}_{b,t} \le \bar{D}^{\omega}_{b,t}; \qquad \forall\ b,t \qquad (\sigma^{D,\omega}_{b,t}) \tag{10}$$

$$\alpha^{\omega}_{a,b,t} \le \bar{\alpha}^{\omega}_{a,b,t}; \qquad \forall\ a,b,t \quad (\sigma^{\alpha,\omega}_{a,b,t}) \tag{11}$$

$$q^{\omega}_{h,b,t}, P^{\omega}_{b,t}, D^{\omega}_{b,t}, r^{\omega}_{h,a,t}, \alpha^{\omega}_{a,b,t} \ge 0; \tag{12}$$

$$\forall\ h,b,a,t \qquad (\gamma^{q,\omega}_{h,b,t}, \gamma^{P,\omega}_{b,t}, \gamma^{D,\omega}_{b,t}, \gamma^{r,\omega}_{h,a,t}, \gamma^{\alpha,\omega}_{a,b,t})$$

where the Lagrange multiplier associated with each constraint is given in the parentheses.

Objective (1) maximizes social welfare. This is defined as the energy sold to the market multiplied by the willingness to pay that is expressed through the demand bids, less the cost of energy and AS procurements. Load-balance constraints (2) require energy that is sold to exactly equal total supply from hydroelectric and non-hydroelectric generators. Although this constraint can be satisfied by setting the generation and demand variables all equal to zero, this is generally suboptimal since the willingness to pay for energy is typically higher than generation costs are. AS-balance constraints (3) similarly ensure that AS demand is satisfied by either hydroelectric or non-hydroelectric generators.

Constraints (4) limit hydroelectric generation dispatched from each block based on the generator's offer and constraints (5) enforce the daily energy limits. Constraints (6) similarly limit hydroelectric AS dispatch by the quantity offered. Constraints (7) ensure that the amount of downward AS reserved from each hydroelectric generator is less than its energy dispatch. Constraints (8) ensure that the sum of energy and upward AS procurements are less than the total quantity offered into the market. Constraints (9) through (11) limit market energy and AS sales and purchases based on the quantities bid. Constraints (12) enforce non-negativity.

We let $\mathcal{P}^{\omega}(\hat{c}, \hat{Q}, \overline{Q}, \hat{\pi}, \hat{R})$ denote the scenario-ω dispatch problem, which depends on the hydroelectric generator's offer parameters. More specifically, because we focus on hydroelectric generator dispatch and deviation penalties, we use the notation $(q^{\omega}, r^{\omega}, \lambda^{\omega}, \lambda^{\omega}_a) \in \arg\max \mathcal{P}^{\omega}(\hat{c}, \hat{Q}, \overline{Q}, \hat{\pi}, \hat{R})$ to indicate that (q^{ω}, r^{ω}) are optimal in $\mathcal{P}^{\omega}(\hat{c}, \hat{Q}, \overline{Q}, \hat{\pi}, \hat{R})$ and $(\lambda^{\omega}, \lambda^{\omega}_a)$ are corresponding Lagrange multiplier values on constraints (2) and (3) that satisfy the KKT conditions.

B. Hydroelectric Generator's Offer Optimization

Our analysis assumes a general river catchment topology, consisting of connected powerhouses and reservoirs. Figs. 1 and 2 show simple and more complex river systems, respectively. In addition to inflows from upstream powerhouses, reservoirs can also have natural water inflows, for instance from tributaries. Each reservoir has a fixed starting water level and constraints on the minimum and maximum amount of water that it can hold. There is also a target ending water level for each reservoir. This target water level can represent a physical constraint or could be a target given by a medium- or long-term hydroelectric planning model [15]. Each powerhouse is assumed to have a fixed generating capacity and efficiency. The efficiency is measured by the volume of water drawn from the forebay per MWh generated. This efficiency is combined with a fixed water value to compute the cost of generation. Again, this water value may represent a direct cost associated with water use. More often, however, this is an implicit opportunity cost, estimated by a medium- or long-term hydroelectric planning model. We include both reservoir targets and water values to allow greater flexibility in how medium- and long-term hydroelectric planning data are captured in the hydroelectric generator's offer optimization. In practice, a generator may opt to only use one of these in its short-term planning.

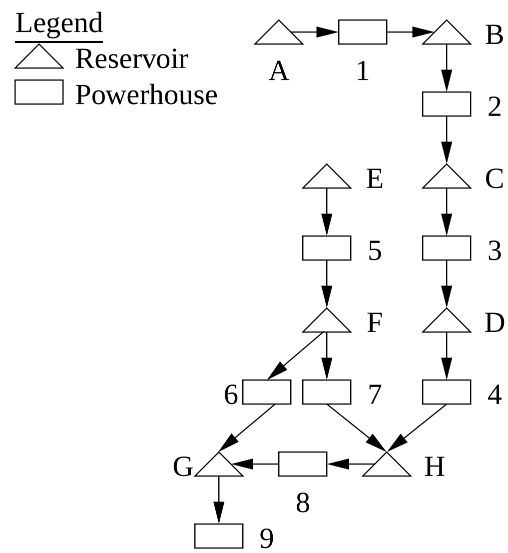

Fig. 2. Complex river system.

Given these assumptions, the generator's offer optimization is formulated as:

$$\max \sum_{\omega \in \Omega} \xi^{\omega} \sum_{t \in T, h \in H} \Bigg[\lambda^{\omega}_t m \cdot (\delta^{\omega,+}_{h,t} + \delta^{\omega,-}_{h,t}) - \sum_{b \in B} \lambda^{\omega}_t q^{\omega}_{h,b,t} + \sum_{a \in A} \lambda^{\omega}_{a,t} \cdot [r^{\omega}_{h,a,t} - \zeta^{\omega}_{h,a,t} m] - \sum_{s \in S : h \in \rho(s)} v_s z^{\omega}_{h,t} \Bigg]; \tag{13}$$

$$\text{s.t.}\ \delta^{\omega,+}_{h,t} - \delta^{\omega,-}_{h,t} + g^{\omega}_{h,t} = \sum_{b \in B} q^{\omega}_{h,b,t}; \qquad \forall\ h,t,\omega \tag{14}$$

$$\zeta^{\omega}_{h,a,t} + \theta^{\omega}_{h,a,t} \ge r^{\omega}_{h,a,t}; \qquad \forall\ h,a,t,\omega \tag{15}$$

$$z^{\omega}_{h,t} \ge e_h g^{\omega}_{h,t}; \qquad \forall h,t,\omega \tag{16}$$

$$l^{\omega}_{s,t} = l^{\omega}_{s,t-1} + n_{s,t} - \sum_{h \in \rho(s)} z^{\omega}_{h,t} + \sum_{h \in \nu(s)} z^{\omega}_{h,t-\iota_h}; \qquad \forall\ s,t,\omega \tag{17}$$

$$L^-_s \le l^{\omega}_{s,t} \le L^+_s; \qquad \forall\ s,t,\omega \tag{18}$$

$$(1-\chi) L^T_s \le l^{\omega}_{s,T} \le (1+\chi) L^T_s; \qquad \forall\ s,\omega \tag{19}$$

$$0 \le g^{\omega}_{h,t} \le Q_h; \qquad \forall\ h,t,\omega \tag{20}$$

$$0 \le \theta^{\omega}_{h,a,t} \le R_{h,a}; \qquad \forall\ h,a,t,\omega \tag{21}$$

$$\tilde{g}^{\omega,-}_{h,t} = g^{\omega}_{h,t} - \sum_{a \in A \setminus A_u} \theta^{\omega}_{h,a,t}; \qquad \forall\ h,t,\omega \tag{22}$$

$$\tilde{g}^{\omega,+}_{h,t} = g^{\omega}_{h,t} + \sum_{a \in A_u} \theta^{\omega}_{h,a,t}; \qquad \forall\ h,t,\omega \qquad (23)$$

$$\tilde{g}^{\omega,-}_{h,t} \geq 0; \qquad \forall\ h,t,\omega \qquad (24)$$

$$\tilde{g}^{\omega,+}_{h,t} \leq Q_h; \qquad \forall\ h,t,\omega \qquad (25)$$

$$l^{\omega}_{s,t-1} - \sum_{h \in \rho(s)} e_h \tilde{g}^{\omega,-}_{h,t} + \sum_{h \in \nu(s)} e_h \tilde{g}^{\omega,+}_{h,t-\iota_h} \leq L^{+}_{s}; \qquad (26)$$
$$\forall\ s,t,\omega$$

$$L^{-}_{s} \leq l^{\omega}_{s,t-1} - \sum_{h \in \rho(s)} e_h \tilde{g}^{\omega,+}_{h,t} + \sum_{h \in \nu(s)} e_h \tilde{g}^{\omega,-}_{h,t-\iota_h}; \qquad (27)$$
$$\forall\ s,t,\omega$$

$$(q^{\omega}, r^{\omega}, \lambda^{\omega}, \lambda^{\omega}_{a}) \in \arg\max \mathcal{P}^{\omega}(\hat{c}, \hat{Q}, \overline{Q}, \hat{\pi}, \hat{R}); \qquad (28)$$
$$\forall\ \omega$$

$$\hat{c}_{h,b,t} \geq \hat{c}_{h,b-1,t}; \qquad \forall\ h,b,t \qquad (29)$$

$$\hat{Q}_{h,b,t}, \overline{Q}_h, \hat{R}_{h,a,t}, \delta^{\omega,+}_{h,t}, \delta^{\omega,-}_{h,t}, \zeta^{\omega}_{h,a,t} \geq 0. \qquad (30)$$
$$\forall\ h,b,a,t,\omega$$

This model treats the hydroelectric operation variables $(l, g, \delta, \theta, \zeta, \tilde{g})$ and offers $(\hat{c}, \hat{Q}, \overline{Q}, \hat{\pi}, \hat{R})$ and the SO's hydroelectric dispatch (q, r) and energy and AS prices (λ, λ_a) as decision variables.

Objective function (13) maximizes expected profit. The $-\lambda^{\omega}_{t} q^{\omega}_{h,b,t}$ and $\lambda^{\omega}_{a,t} r^{\omega}_{h,a,t}$ terms are revenues earned from day-ahead energy and AS dispatch. Due to the sign conventions used in the SO's dispatch problem, energy is priced at $-\lambda^{\omega}_{t}$ whereas AS at $\lambda^{\omega}_{a,t}$. The $\lambda^{\omega}_{t} m \cdot (\delta^{\omega,+}_{i,t} + \delta^{\omega,-}_{i,t})$ and $-\lambda^{\omega}_{a,t} \zeta^{\omega}_{i,a,t} m$ terms represent penalties on deviations from the day-ahead schedule. The penalties are assumed to be proportional to the day-ahead price. We assume that these penalties apply to both over- and under-generated energy but only to AS shortfalls. The $-v_s z^{\omega}_{h,t}$ term represents the opportunity cost of water used to generate energy.

Constraints (14) and (15) define energy and AS deviations, respectively, as the difference between the actual amounts supplied by the hydroelectric generator and the SO's dispatch. Constraints (16) define powerhouse water use in terms of their generation and efficiency. We enforce these constraints as inequalities (as opposed to equalities) to allow for water spillage. While typically undesirable (due to the associated opportunity cost) water spillage may be necessary to meet reservoir water level targets and constraints. If some of the reservoirs in the system are not designed to allow for spillage, the associated constraints could be enforced as equalities. Similarly, if there is an upper-bound on spillage, this could be enforced by placing explicit upper-bounds on the $z^{\omega}_{h,t}$ variables. We allow for spillages in the model to ensure that the watershed can be feasibly dispatched without violating upper reservoir constraints. This may run contrary to actual hydroelectric operations, however, since spillages may not be decided upon on an hourly basis.

Water-balance constraints (17) define each reservoir's hour-t water level as its hour-$(t-1)$ water level less what is used to generate electricity plus inflows from tributaries and upstream powerhouses. The inflows from the upstream powerhouses are indexed by $t - \iota_h$ to account for the travel time between each powerhouse and the downstream reservoir. Constraints (18) enforce the reservoir water limits and constraints (19) force the ending water level to be within a band around the target.

Constraints (20) and (21) enforce generation and AS limits on each powerhouse, respectively. Constraints (22) through (27) further ensure that each powerhouse can feasibly supply energy in real-time if the AS provided is called. These constraints assume that the generator must be able to sustain AS production for a minimum of one hour. Constraints (22) define each generator's resulting generation level if all of the downward AS provided are called in real-time. Constraints (23) define the generation levels if upward AS are called. Constraints (24) and (25) ensure that these resulting generation levels satisfy each powerhouse's capacity limits. Constraints (26) and (27) further ensure that the water level of each reservoir remains within its bounds if AS are called.

Constraint (28) requires the dispatch and energy and AS prices to be an optimal solution/Lagrange multiplier pair in the SO's welfare maximization problem, given the hydroelectric generator's offers. Constraints (29) force the variable energy cost offers to be monotone. Constraints (30) enforce non-negativity. Since this model allows the hydroelectric generator full flexibility to deviate from the SO's prescribed dispatch, this problem is guaranteed to be feasible (so long as the water levels in the river system do not violate any minimum reservoir constraints).

C. *Extensions of SO and Offer-Optimization Models*

Our model assumes a relatively simple SO model and hydroelectric system. Some of these simplifications are made to ease notation. Others are made for technical reasons related to our solution methodology. The method used to solve the hydroelectric offer-optimization model is reliant on the SO's dispatch problem being a convex program that satisfies some constraint qualification conditions [23]. This assumption assures that the KKT conditions are both necessary and sufficient for a global optimum to the SO's problem.

We can, however, relax some of the simplifications assumed. We now discuss some possible extensions of our model.

1) Self-Schedules: As noted before, many SOs allow generators to self-schedule energy and AS. We do not explicitly model self-schedules, but could by adding additional self-schedule offer parameters that the hydroelectric generator can determine. These self-schedules would be included in the SO's load-balance constraint. Market rules typically allow SOs to curtail self-schedules for non-economic reasons (*e.g.*, if the self-schedule causes a load imbalance, infeasible power flows, or other threats to system security). We could model self-schedule curtailment by adding such variables to the SO's dispatch problem, with a high penalty cost in the objective function. Indeed, modeling self-schedule curtailment is necessary to ensure that the SO's dispatch problem is always feasible for any set of hydroelectric offers.

Alternatively, a generator can replicate a self-schedule by submitting a piece of the stepped generation cost curve at the price floor, which our model does allow. We could model AS self-schedules similarly, although we assume that each powerhouse offers a single block of AS capacity at a single price. This assumption is made to simplify notation, but the model can be easily generalized to include stepped AS offers.

2) Load Flow: We do not model power flows or transmission constraints in the SO's dispatch problem. A linearized DC load flow model could be easily incorporated into the SO's dispatch problem, since the problem would retain a linear structure. In this case, hydroelectric profit function (13) would be changed to reflect the fact that energy prices and deviation penalties are location-specific. Depending on the specific power system and market being evaluated the inclusion of transmission constraints may be important as they may be a major source of infeasible hydroelectric dispatches from the SO.

3) Head-Dependent Powerhouse Efficiencies: A complication of modeling hydroelectric generation is that powerhouse efficiency is a nonlinear function of turbine efficiency, net head, and plant discharge [24]. At the same time, net head is a nonlinear function of the reservoir's water level and released flow. Following a number of other works [9], [15], [16], we ignore these effects and assume that each reservoir's net head remains relatively constant over the course of the one-day planning horizon. Other works capture these nonlinearities using piecewise-linear approximations of powerhouse efficiency [1]–[4], [6], [10], [25]–[27] or by using a nonlinear model [5], [7], [8], [24], [28].

Either of these modeling methods could be used to represent actual hydroelectric operations in the offer-optimization problem. As discussed in Section III, we are able to simplify the bilevel optimization given by (13)–(30) into a MILP. If the water-use terms in constraint sets (16), (26), and (27) are replaced with piecewise-linear approximations, the bilevel model can still be converted to a MILP, which can be solved using commercial software packages. Including piecewise-linear approximations will result in a larger MILP, however, which may introduce computational complexity problems, especially since the model we propose is a daily offer optimization. Otherwise, if a nonlinear model is used, the bilevel optimization is instead converted to a mixed-integer nonlinear program, which may be significantly more difficult to solve.

4) Powerhouse Non-Convexities: Powerhouse operations may entail other non-convexities, including turbine startup and shutdown costs and forbidden operating zones [4], [6]. Such non-convexities can be easily included in the offer-optimization model, since they can be captured by adding binary decision variables to the constraints representing actual powerhouse operations. Doing so results in the bilevel offer-optimization problem being converted to a MILP. As with modeling powerhouse efficiencies using piecewise-linear approximations, including such non-convexities will result in a larger MILP, which may introduce computational complexity issues.

5) Multiple Ownership: Our model assumes that all of the reservoirs and powerhouses in the river system are owned by a single profit-driven entity. In some river systems the resources are owned by competing firms [26]. Incorporating a multi-firm structure into our offer-optimization model could be done if the aim is to optimize the offers of a single firm. For instance, suppose that the firm of interest operates the two lower reservoirs and powerhouses in Fig. 1. The behavior of the other firm that operates the two upper reservoirs and powerhouses could be modeled based on exogenous water inflows to reservoir 'C' from the rival firm. Indeed, these inflows could be modeled randomly to account for uncertainty in the rival's behavior.

This approach assumes that the rival firm(s) make dispatch decisions that result in a feasible reservoir dispatch problem for the firm of interest. If, for instance, the firm of instance operates the two lower reservoirs and powerhouses in Fig. 1 and the other firm does not release sufficient water to satisfy the minimum water level constraint of reservoir 'C,' then the firm of interest would have an infeasible offer optimization problem. Such a situation could not be meaningfully analyzed using our model.

III. Linearization of Hydroelectric Offer-Optimization Problem

The hydroelectric offer-optimization is a bilevel problem. We use standard techniques, which are outlined below, to convert this problem into a single-level MILP [18]–[20].

A. Bilevel Optimization

Inclusion of constraints (28) makes the offer optimization a bilevel problem. Since the SO problem is a linear program, we know that any global maximum must satisfy the KKT conditions [23]. Moreover, since the SO problem is convex, these conditions are also sufficient for a global maximum. Thus, we can replace constraints (28) in the offer optimization with the following KKT conditions for each possible realization of the SO's problem:

$$\hat{c}_{h,b,t} + \lambda_t^\omega + \eta_{h,b,t}^\omega + \eta_h^\omega - \mu_{h,d,t}^\omega + \mu_{h,u,t}^\omega - \gamma_{h,b,t}^{q,\omega} = 0 \quad \forall\ h,b,t,\omega \tag{31}$$

$$K_{b,t}^\omega + \lambda_t^\omega + \sigma_{b,t}^{P,\omega} - \gamma_{b,t}^{P,\omega} = 0; \quad \forall\ b,t,\omega \tag{32}$$

$$-W_{b,t}^\omega - \lambda_t^\omega + \sigma_{b,t}^{D,\omega} - \gamma_{b,t}^{D,\omega} = 0; \quad \forall\ b,t,\omega \tag{33}$$

$$\hat{\pi}_{h,a,t} - \lambda_{a,t}^\omega + \eta_{h,a,t}^\omega + \mu_{h,d,t}^\omega - \gamma_{h,a,t}^{r,\omega} = 0; \quad \forall\ h,a \in A \setminus A_u,t,\omega \tag{34}$$

$$\hat{\pi}_{h,a,t} - \lambda_{a,t}^\omega + \eta_{h,a,t}^\omega + \mu_{h,u,t}^\omega - \gamma_{h,a,t}^{r,\omega} = 0; \quad \forall\ h,a \in A_u,t,\omega \tag{35}$$

$$\kappa_{a,b,t}^\omega - \lambda_{a,t}^\omega + \sigma_{a,b,t}^{\alpha,\omega} - \gamma_{a,b,t}^{\alpha,\omega} = 0; \quad \forall\ a,b,t,\omega \tag{36}$$

$$\sum_{b\in B} P_{b,t}^\omega + \sum_{h\in H, b\in B} q_{h,b,t}^\omega = \sum_{b\in B} D_{b,t}^\omega; \quad \forall\ t,\omega \tag{37}$$

$$\sum_{b\in B} \alpha_{a,b,t}^\omega + \sum_{h\in H} r_{h,a,t}^\omega \geq \beta_{a,t}^\omega \perp \lambda_{a,t}^\omega \geq 0; \quad \forall\ a,t,\omega \tag{38}$$

$$q_{h,b,t}^\omega \leq \hat{Q}_{h,b,t} \perp \eta_{h,b,t}^\omega \geq 0; \quad \forall\ h,b,t,\omega \tag{39}$$

$$\sum_{t\in T, b\in B} q_{h,b,t}^\omega \leq \overline{Q}_h \perp \eta_h^\omega \geq 0; \quad \forall\ h,\omega \tag{40}$$

$$r_{h,a,t}^\omega \leq \hat{R}_{h,a,t} \perp \eta_{h,a,t}^\omega \geq 0; \quad \forall\ h,a,t,\omega \tag{41}$$

$$\sum_{a\in A\setminus A_u} r_{h,a,t}^\omega \leq \sum_{b\in B} q_{h,b,t}^\omega \perp \mu_{h,d,t}^\omega \geq 0; \tag{42}$$

$$\forall\ h,t,\omega$$

$$\sum_{b\in B} q^{\omega}_{h,b,t} + \sum_{a\in A_u} r^{\omega}_{h,a,t} \leq \sum_{b\in B} \hat{Q}_{h,b,t} \perp \mu^{\omega}_{h,u,t} \geq 0; \qquad \forall\ h,t,\omega \tag{43}$$

$$P^{\omega}_{b,t} \leq \bar{P}^{\omega}_{b,t} \perp \sigma^{P,\omega}_{b,t} \geq 0; \qquad \forall\ b,t,\omega \tag{44}$$

$$D^{\omega}_{b,t} \leq \bar{D}^{\omega}_{b,t} \perp \sigma^{D,\omega}_{b,t} \geq 0; \qquad \forall\ b,t,\omega \tag{45}$$

$$\alpha^{\omega}_{a,b,t} \leq \bar{\alpha}^{\omega}_{a,b,t} \perp \sigma^{\alpha,\omega}_{a,b,t} \geq 0; \qquad \forall\ a,b,t,\omega \tag{46}$$

$$\begin{aligned} &q^{\omega}_{h,b,t}, P^{\omega}_{b,t}, D^{\omega}_{b,t}, r^{\omega}_{h,a,t}, \alpha^{\omega}_{a,b,t} \geq 0 \perp \\ &\quad \gamma^{q,\omega}_{h,b,t}, \gamma^{P,\omega}_{b,t}, \gamma^{D,\omega}_{b,t}, \gamma^{r,\omega}_{h,a,t}, \gamma^{\alpha,\omega}_{a,b,t} \geq 0. \qquad \forall\ h,b,a,t,\omega \end{aligned} \tag{47}$$

The $\perp$ symbol in conditions (38) through (47) denote complementary slackness between each constraint of the SO problem and its associated Lagrange multiplier.

B. KKT Complementary Slackness Conditions

Adding complementary slackness conditions (38) through (47) to the offer-optimization problem introduces nonlinearities. This is because a complementary slackness condition of the form:

$$f(x) \leq 0 \perp \phi \geq 0, \tag{48}$$

is equivalent to:

$$f(x) \leq 0; \tag{49}$$

$$\phi \geq 0; \tag{50}$$

$$f(x)\phi = 0. \tag{51}$$

One can linearize this condition by introducing a binary auxiliary variable, ψ, which is equal to 1 if $f(x) < 0$ and 0 otherwise. We then replace conditions (49) through (51) with:

$$-M \cdot \psi \leq f(x) \leq 0; \tag{52}$$

$$0 \leq \phi \leq M \cdot (1-\psi); \tag{53}$$

$$\psi \in \{0,1\}; \tag{54}$$

where M is a sufficiently large constant [18].

We linearize complementary slackness conditions (38) through (47) by introducing one auxiliary binary variable for each. We can also use problem data to determine the highest value that each constraint and associated Lagrange multiplier can feasibly take, allowing us to determine reasonable values for M's used to linearize each condition. This gives a tighter MILP, reducing computational complexity.

C. Energy and Ancillary Service Revenues

Objective (13) has $-\lambda^{\omega}_t q^{\omega}_{h,b,t}$ and $\lambda^{\omega}_{a,t} r^{\omega}_{h,a,t}$ terms, which are bilinear in the decision variables. Following the work of Ruiz and Conejo [20], we linearize them using strong duality and complementary slackness conditions. Since the SO problem is a convex linear program, strong duality implies that:

$$\begin{aligned} &\sum_{t\in T}\Bigg\{\sum_{b\in B}(K^{\omega}_{b,t}P^{\omega}_{b,t} - W^{\omega}_{b,t}D^{\omega}_{b,t} + \bar{P}^{\omega}_{b,t}\sigma^{P,\omega}_{b,t} + \bar{D}^{\omega}_{b,t}\sigma^{D,\omega}_{b,t}) \\ &\quad + \sum_{a\in A}\Bigg[\sum_{b\in B}(\kappa^{\omega}_{a,b,t}\alpha^{\omega}_{a,b,t} + \bar{\alpha}^{\omega}_{a,b,t}\sigma^{\alpha,\omega}_{a,b,t}) - \beta^{\omega}_{a,t}\lambda^{\omega}_{a,t}\Bigg]\Bigg\} \\ &= -\sum_{h\in H, b\in B, t\in T}(\hat{c}_{h,b,t}q^{\omega}_{h,b,t} + \hat{Q}_{h,b,t}\eta^{\omega}_{h,b,t}) - \sum_{h\in H}\bar{Q}_h\eta^{\omega}_h \\ &\quad - \sum_{h\in H, a\in A, t\in T}(\hat{\pi}_{h,a,t}r^{\omega}_{h,a,t} + \hat{R}_{h,a,t}\eta^{\omega}_{h,a,t}) \\ &\quad - \sum_{h\in H, t\in T}\mu^{\omega}_{h,u,t}\sum_{b\in B}\hat{Q}_{h,b,t}. \end{aligned} \tag{55}$$

We next note that by summing conditions (31) over h, b, and t we have that for each ω:

$$\begin{aligned} \sum_{h\in H, b\in B, t\in T}\lambda^{\omega}_t q^{\omega}_{h,b,t} = &\sum_{h\in H, b\in B, t\in T}(-\hat{c}_{h,b,t}q^{\omega}_{h,b,t} \\ &- \eta^{\omega}_{h,b,t}q^{\omega}_{h,b,t} - \eta^{\omega}_h q^{\omega}_{h,b,t} + \mu^{\omega}_{h,d,t}q^{\omega}_{h,b,t} - \mu^{\omega}_{h,u,t}q^{\omega}_{h,b,t} \\ &+ \gamma^{q,\omega}_{h,b,t}q^{\omega}_{h,b,t}). \end{aligned} \tag{56}$$

We next note that conditions (39), (43), (40), (42), and (47) imply, respectively, that:

$$\eta^{\omega}_{h,b,t}q^{\omega}_{h,b,t} = \hat{Q}_{h,b,t}\eta^{\omega}_{h,b,t}; \tag{57}$$

$$\sum_{t\in T, b\in B}\eta^{\omega}_h q^{\omega}_{h,b,t} = \overline{Q}_h\eta^{\omega}_h; \tag{58}$$

$$\sum_{b\in B}\mu^{\omega}_{h,d,t}q^{\omega}_{h,b,t} = \mu^{\omega}_{h,d,t}\sum_{a\in A\setminus A_u} r^{\omega}_{h,a,t}; \tag{59}$$

$$\sum_{b\in B}\mu^{\omega}_{h,u,t}q^{\omega}_{h,b,t} = \left(\sum_{b\in B}\hat{Q}_{h,b,t} - \sum_{a\in A_u} r^{\omega}_{h,a,t}\right)\mu^{\omega}_{h,u,t}; \tag{60}$$

and:

$$\gamma^{q,\omega}_{h,b,t}q^{\omega}_{h,b,t} = 0. \tag{61}$$

Substituting these equalities into (56) gives:

$$\begin{aligned} \sum_{h\in H, b\in B, t\in T}\lambda^{\omega}_t q^{\omega}_{h,b,t} = &-\sum_{h\in H, b\in B, t\in T}(\hat{c}_{h,b,t}q^{\omega}_{h,b,t} \\ &+ \hat{Q}_{h,b,t}\eta^{\omega}_{h,b,t}) - \sum_{h\in H}\overline{Q}_h\eta^{\omega}_h \\ &- \sum_{h\in H, t\in T}\mu^{\omega}_{h,u,t}\sum_{b\in B}\hat{Q}_{h,b,t} + \sum_{h\in H, t\in T}\mu^{\omega}_{h,d,t}\sum_{a\in A\setminus A_u} r^{\omega}_{h,a,t} \\ &+ \sum_{h\in H, t\in T}\mu^{\omega}_{h,u,t}\sum_{a\in A_u} r^{\omega}_{h,a,t}. \end{aligned} \tag{62}$$

We next note that conditions (34), (41), and (47) imply that for all $a \in A\setminus A_u$ and ω:

$$\begin{aligned} \mu^{\omega}_{h,d,t}r^{\omega}_{h,a,t} = &-\hat{\pi}_{h,a,t}r^{\omega}_{h,a,t} + \lambda^{\omega}_{a,t}r^{\omega}_{h,a,t} - \eta^{\omega}_{h,a,t}r^{\omega}_{h,a,t} \\ &+ \gamma^{r,\omega}_{h,a,t}r^{\omega}_{h,a,t}; \end{aligned} \tag{63}$$

$$\eta^{\omega}_{h,a,t}r^{\omega}_{h,a,t} = \hat{R}_{h,a,t}\eta^{\omega}_{h,a,t}; \tag{64}$$

and:

$$\gamma^{r,\omega}_{h,a,t}r^{\omega}_{h,a,t} = 0. \tag{65}$$

We can use conditions (35), (41), and (47) to arrive at analogous equalities involving the upward AS products. Substituting all of these equalities into (55) and (62) gives:

$$\sum_{h\in H,b\in B,t\in T} \lambda_t^\omega q_{h,b,t}^\omega - \sum_{h\in H,a\in A,t\in T} \lambda_{a,t}^\omega r_{h,a,t}^\omega \tag{66}$$
$$= \sum_{t\in T}\left\{\sum_{b\in B}(K_{b,t}^\omega P_{b,t}^\omega - W_{b,t}^\omega D_{b,t}^\omega + \bar{P}_{b,t}^\omega \sigma_{b,t}^{P,\omega} + \bar{D}_{b,t}^\omega \sigma_{b,t}^{D,\omega})\right.$$
$$\left. + \sum_{a\in A}\left[\sum_{b\in B}(\kappa_{a,b,t}^\omega \alpha_{a,b,t}^\omega + \bar{\alpha}_{a,b,t}^\omega \sigma_{a,b,t}^{\alpha,\omega}) - \beta_{a,t}^\omega \lambda_{a,t}^\omega\right]\right\}.$$

Thus, we can linearize the bilinear revenue terms in objective (13) by replacing them with (66).

D. Deviation Penalties

Objective (13) also has $\lambda_t^\omega m \cdot (\delta_{h,t}^{\omega,+} + \delta_{h,t}^{\omega,-})$ and $-\lambda_{a,t}^\omega \zeta_{h,a,t}^\omega m$ terms, which are bilinear and cannot be linearized. We instead replace these terms in the objective with $\tau_t^\omega m \cdot (\delta_{h,t}^{\omega,+} + \delta_{h,t}^{\omega,-})$ and $-\tau_{a,t}^\omega \zeta_{h,a,t}^\omega m$, where τ_t^ω and $\tau_{a,t}^\omega$ are fixed parameters. We use the technique, outlined in Algorithm 1, to iteratively update the values of τ_t^ω and $\tau_{a,t}^\omega$ until arriving at an optimal set of offers. We use the notational convention, $\mathcal{N}(\tau, \tau_a)$, to represent the hydroelectric offer-optimization problem (including all of the linearizations described thus far), which depends on the values of the τ and τ_a parameters in the objective. We also use the notation $x \in \arg\max \mathcal{N}(\tau, \tau_a)$ to denote that x is an optimal vector of hydroelectric operation and offer, SO dispatch, and energy and AS price variables in the offer-optimization problem.

Algorithm 1 Equilibrium Computation

```
1: k ← 0                                         ▷ Initialize iteration count
2: τ_t^ω ← τ̄_t^ω and τ_{a,t}^ω ← τ̄_{a,t}^ω for all a, t, ω    ▷ Initialize penalty coefficients
3: repeat
4:     k ← k + 1
5:     x^k ← arg max N(τ, τ_a)
6:     τ_t^ω ← λ_t^{ω,k} and τ_{a,t}^ω ← λ_{a,t}^{ω,k} for all a, t, ω
7: until ||x^k − x^{k−1}|| ≤ ε or k ≥ k̄
```

Algorithm 1 begins by initializing the iteration counter and the deviation penalty terms (Steps 1–2). It then solves the offer-optimization problem with the incumbent penalty terms and updates the penalty terms based on the new solution (Steps 5–6). The superscript k indicates the solution found in the kth iteration of the algorithm. It continues resolving the problem with updated penalty terms until the same solution is found in two successive iterations, meaning that the hydroelectric offers and operations are optimal against the correct deviation penalties, or the iteration limit, $\bar{k}$, is exhausted (Step 7). The iteration limit is included because we cannot guarantee that Algorithm 1 converges to such an equilibrium in a finite number of iterations. In our numerical case studies (discussed in Section IV) the algorithm converges within four iterations.

IV. Case Study

We use two numerical case studies to examine the benefits of the proposed offer-optimization model. We first discuss the case study assumptions and data, then describe the two hydroelectric offer strategies considered, and finally summarize the resulting hydroelectric dispatch and profits and system operation costs.

A. Case Study Data

We examine the two hydroelectric watersheds shown in Figs. 1 and 2, which are based on actual rivers in the CAISO system, over a one-day period. Tables I and II summarize the characteristics of the reservoirs and powerhouses, respectively, for the river system in Fig. 1 and Tables III and IV summarize the characteristics for the other river. We assume that the natural water inflows to each reservoir are the same in each hour and that $\chi = 0.1$ in determining the ending target reservoir levels.

TABLE I
Reservoir Characteristics of River System in Figure 1

s	Reservoir Levels [acre-feet] L_s^-	L_s^+	L_s^0	L_s^T	$n_{s,t}$	v_s [$/acre-ft]
A	6000	117000	47000	47000	40	340
B	700	1300	900	900	50	0
C	33000	129000	46700	46700	170	250
D	180	300	300	300	5	0

TABLE II
Powerhouse Characteristics of River System in Figure 1

h	Q_h [MW]	e_h [acre-ft/MWh]	ι_h [hr]	$R_{h,a}$ [MW] Reg. Down	Reg. Up	Spin	Non Spin
1	2000	0.7	0	1000	1000	1000	1000
2	150	0.5	0	0	0	50	50
3	150	0.5	0	0	0	144	144
4	50	1.7	0	0	0	50	50

TABLE III
Reservoir Characteristics of River System in Figure 2

s	Reservoir Levels [acre-feet] L_s^-	L_s^+	L_s^0	L_s^T	$n_{s,t}$	v_s [$/acre-ft]
A	750000	1020000	820000	820000	0	85
B	25000	49500	40000	40000	90	70
C	2000	2500	2000	2000	30	0
D	3500	4000	4000	4000	500	0
E	3600	5500	4500	4500	3	0
F	700	1000	1000	1000	30	0
G	3200	4000	3700	3700	80	0
H	700	1000	800	800	400	0

The hydroelectric generator forecasts three equally-likely scenarios with different demand bids and offers from the rival generators. Fig. 3 shows the fixed price-inelastic demand in each scenario.[3] There is an additional 7–13% price-elastic demand in each hour, with willingness to pay of $33–68/MWh. The bottom half of Fig. 3 shows the resulting hourly energy prices when the price-inelastic and -elastic demands are

[3]This demand is bid into a market with a willingness to pay of $9999/MWh, implying that the SO only curtails it in exigent circumstances.

TABLE IV
POWERHOUSE CHARACTERISTICS OF RIVER SYSTEM IN FIGURE 1

h	Q_h [MW]	e_h [acre-ft/ MWh]	ι_h [hr]	$R_{h,a}$ [MW] Reg. Down	Reg. Up	Spin	Non Spin
1	40	3.7	1	0	0	0	0
2	170	1.1	0	120	120	120	120
3	115	1.7	0	114	114	114	114
4	110	2.4	1	112	112	112	112
5	20	1.6	1	0	0	0	0
6	40	0.7	2	40	40	40	40
7	60	0.5	2	60	60	60	60
8	35	5	0	35	35	35	35
9	120	2.6	0	120	120	120	120

cleared against the non-hydroelectric resources offered into the market under the three scenarios (*i.e.*, we assume that the hydroelectric generator offers no energy or AS and solve the dispatch problem considering non-hydroelectric generation offers only).

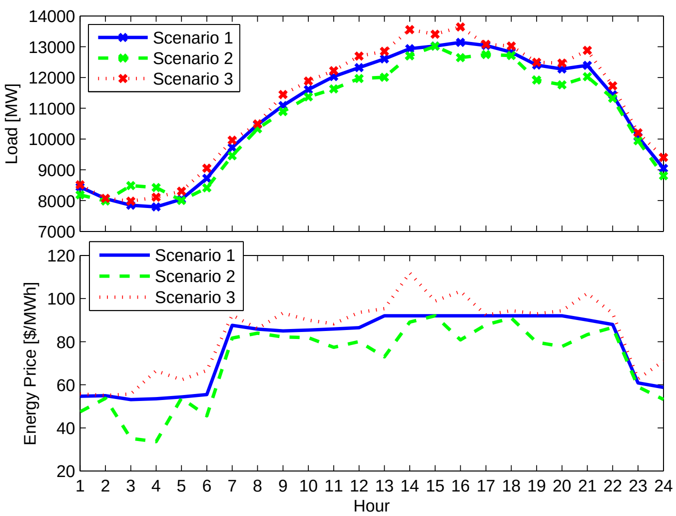

Fig. 3. Fixed price-inelastic loads and energy prices without any hydroelectric supply in three scenarios.

We assume that $m = 1.15$, meaning that the penalty on hydroelectric deviations is 115% of the energy or AS price. The models are formulated using version 12.1.0 of the `AMPL` mathematical programming language and solved using the branch and cut algorithm in `CPLEX` version 12.2.1. The models are solved on a quad-core 2.7 GHz Linux workstation with 4 GB of RAM.

B. Hydroelectric Offer Strategies

We contrast hydroelectric dispatch, actual operations, and profits in two cases. The first assumes that the hydroelectric generator provides the SO with 'true' cost and constraint data. In this case each powerhouse's generating and AS capacities and a generation cost, based on generating efficiency and water value, are submitted to the SO. The SO uses these costs and constraints in its dispatch, given by (1)–(12), to determine the hydroelectric generator's dispatch and energy and AS prices in each of the three scenarios modeled. Based on the dispatch given by the SO and the resulting market prices, the hydroelectric generator then determines the actual feasible operation of its units, considering the true watershed constraints. This is done using the following operation model, which is defined for each scenario, ω, as:

$$\min \sum_{t\in T, h\in H} \Bigg[-\lambda^{\omega}_t m \cdot (\delta^{\omega,+}_{h,t} + \delta^{\omega,-}_{h,t}) \tag{67}$$

$$+ \sum_{a\in A} \lambda^{\omega}_{a,t}\zeta^{\omega}_{h,a,t} m + \sum_{s\in S: h\in\rho(s)} v_s z^{\omega}_{h,t} \Bigg];$$

$$\text{s.t. } (14)\text{–}(27); \tag{68}$$

$$\delta^{\omega,+}_{i,t}, \delta^{\omega,-}_{i,t}, \zeta^{\omega}_{i,a,t} \geq 0; \qquad \forall\ i, a, t, \omega \tag{69}$$

where the SO's hydroelectric dispatch (q, r) is held equal to the solution from the SO's problem. Objective (67) minimizes the sum of energy and AS deviation costs, given by the $-\lambda^{\omega}_t m \cdot (\delta^{\omega,+}_{h,t} + \delta^{\omega,-}_{h,t})$ and $\lambda^{\omega}_{a,t}\zeta^{\omega}_{h,a,t} m$ terms, respectively, and hydroelectric generation costs, given by the $v_s z^{\omega}_{h,t}$ terms. Constraints (14)–(27) from the offer-optimization problem represent the watershed's true capabilities and constraints (69) impose non-negativity.

The second offer strategy that we consider uses the offer optimization model outlined in Section II-B with the linearizations discussed in Section III.

C. Hydroelectric Dispatch, Deviations, and Profits

Table V summarizes the expected financial performance of the two watersheds using the two offer strategies. The financial performance is broken down between revenues earned and water costs and deviation penalties borne. The optimized offers perform considerably better than making offers according to the true capabilities of the powerhouses, increasing expected profits of the watershed in Fig. 1 by close to a factor of three. The table shows that the offer-optimization model significantly increases hydroelectric generator revenues and reduces deviation penalties. With the simple river system deviation penalties are completely eliminated whereas with the other some dispatch deviations are allowed by the optimal offers. In the latter case the offers are structured in such a way to capture higher profits in one of the three scenarios, that result in an infeasible dispatch in another scenario.

TABLE V
EXPECTED HYDROELECTRIC GENERATOR PROFITS [$ THOUSAND]

	River 1 'True'	River 1 Optimized	River 2 'True'	River 2 Optimized
Revenues	252	400	818	1209
Water Costs	130	175	80	269
Deviation Penalties	45	0	18	11
Net Profits	77	225	721	929

Our offer-optimization model is also solved very efficiently using `CPLEX`. Optimizing the offers for the river system shown in Fig. 1 requires three iterations of Algorithm 1 and 20.3 s of wall clock time. The more complex river system requires four iterations of Algorithm 1 and 443.7 s of wall clock time.

Another benefit of the hydroelectric generator optimizing its offers is that it reduces infeasibilities in the SO's dispatch.

This means that the SO is better optimizing the use of the hydroelectric and other generating resources, giving a more efficient dispatch. We can measure this efficiency gain by comparing the optimal value of objective function (1), after the system is redispatched around the hydroelectric generators' actual operations. To do this, we first determine the true operations of the hydroelectric generators by solving (67)–(69). *Actual* hydroelectric energy and AS supply, as given by this model, are then fixed in the SO's dispatch problem, given by (1)–(12), which is solved to determine how the non-hydroelectric assets are operated. Table VI summarizes the increase in the one-day system surplus with the use of the proposed offer-optimization model, as opposed to submitting 'true' offers based on actual powerhouse capabilities only.

TABLE VI
INCREASE IN SOCIAL SURPLUS WITH USE OF OFFER-OPTIMIZATION MODEL AS OPPOSED TO 'TRUE' OFFERS

Watershed	Surplus Increase [$ Thousand]
1	392
2	620

V. CONCLUSIONS

This paper proposes a stochastic bilevel modeling approach to optimize the offers of a cascaded hydroelectric system into a centrally dispatched market. The lower level represents the SO's economic dispatch model, which gives the hydroelectric dispatch schedule and market prices for a set of hydroelectric offers. The upper level includes actual constraints on the watershed, which the SO does not account for in its dispatch. These constraints are included to model how the hydroelectric plants are actually operated to maximize profits less penalties for deviating from the SO's dispatch. The combined model is used to determine how hydroelectric offers should be structured. The hydroelectric generator faces a fundamental tradeoff in structuring these offers, which our model captures. Less flexible offers (*e.g.*, a self-schedule only) foreclose on potential gains if the generator incorrectly forecasts market conditions. More flexible offers allow for such gains, but may result in the SO dispatching the powerhouses infeasibly.

We show how the bilevel problem can be converted into an equivalent single-level MILP and propose a simple iterative algorithm to efficiently solve for an optimum. We use two numerical case studies, based on actual watersheds in the CAISO control area, to demonstrate the efficacy of our model. Our numerical case study shows that the optimized offers increase hydroelectric profits and reduce SO dispatch infeasibilities compared to submitting 'true' offers based on actual powerhouse capabilities. We also show that the optimized offers improve overall system efficiency. This is because the SO dispatches the other non-hydroelectric resources 'correctly,' since it does not expect hydroelectric output that is infeasible.

We model a simplified system without power flows in the SO dispatch, head-dependent powerhouse efficiencies, or other powerhouse non-convexities. We discuss how these complexities can be incorporated into our proposed model. We restrict our attention to this simplified model structure to ease the notation, derivations, and exposition.

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