Equilibria in Investment and Spot Electricity Markets: A Conjectural-Variations Approach

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Abstract

We study generation-capacity planning in a restructured oligopolistic electricity market, taking a conjectural-variations approach. We do this through a two-stage model that captures an initial set of investments followed by equilibria in a series of spot electricity markets, in which firms make production decisions. Although we model the generating firms as being quantity-setters, we do not model a Nash-Cournot equilibrium. Instead, we assume that the firms endogenize the impacts of their production decisions on rivals through reaction parameters, giving a conjectural-variations model of the spot-market equilibrium. Equilibrium conditions in each spot market, as a function of the investment decisions, are derived. This allows characterizing an equilibrium at the investment stage. The proposed model allows the derivation of analytical expressions that characterize such multi-stage equilibria. This proposed model provides insights on the outcomes and characteristics of investment decisions in an imperfectly competitive market setting. Such insights may allow policymakers to understand the efficiency implications of oligopolistic market structures.

Keywords: OR in energy, generation-capacity planning, electricity spot markets, conjectural variations

1. Introduction

Generation-capacity planning is the problem of determining optimal investments in new generating capacity to supply future loads. Massé and Gibrat (1957) provide one of the first models to optimize generation-capacity planning. Pereira et al. (1985); Sanghvi and Shavel (1986) extend the work of Massé and Gibrat (1957) by developing decomposition techniques to solve more efficiently generation capacity-planning models. Murphy and Soyster (1983) study generation-capacity planning from the perspective of a regulated utility, while Murphy and Smeers (2005) study generation-capacity planning in a restructured electricity market. This latter work shows that if a market is perfectly competitive, the resulting investments are the same as those of the regulated utility. However, absent perfect competition market-based and regulated investments may differ. This finding calls for models that can capture generation-capacity investments in an imperfectly competitive market.

Numerous works, including those of Daxhelet and Smeers (2001); Hobbs (2001); Jing-Yuan and Smeers (1999); Oliveira and Costa (2018); Pineau et al. (2011); Ruiz et al. (2012), develop models to study power system operations in an oligopolistic market setting. However, the literature is more limited in terms of works that examine generation-capacity planning and investment within an oligopolistic setting. The

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work of Chuang et al. (2001) is one such example. Murphy and Smeers (2005) discuss the importance of studying generation investment and operations jointly in a market environment. They develop such a model within a duopoly-market setting with perfectly competitive and open- and closed-loop Nash-Cournot equilibria. Allaz and Vila (1993); Fudenberg and Tirole (1991) discuss the differences between open- and closed-loop Nash-Cournot equilibria. Within the context of capacity investment, the former considers building capacity and selling production through long-term contracts in the absence of a spot market. In the latter, investment decisions are made in the first stage, which then are followed by operating decisions in second-stage spot markets. Day et al. (2002) note that Nash-Cournot equilibria can be computed relatively easily. Thus, they are used extensively to represent competition in restructured electricity markets. This includes the works of Andersson and Bergman (1995); Borenstein and Bushnell (1999); Cardell et al. (1997); Cunningham et al. (2002); Jing-Yuan and Smeers (1999); Oren (1997); Ramos et al. (1999); Rivier et al. (2001); Schmalensee and Golub (1984); Siddiqui et al. (2019); Siriruk and Valenzuela (2011); Valenzuela and Mazumdar (2007); Younes and Ilic (1999).

Conjectural variations is an approach that is used widely to study oligopolistic markets, including electricity markets. Baltensperger et al. (2016) note that it is particularly useful if high capital costs or substantial infrastructure requirements create barriers to entry, thereby creating opportunities for the exercise of market power. Our work extends the existing literature that is related to generation-capacity planning within an oligopolistic market. Specifically, we take a conjectural-variations approach to modeling closed-loop Nash-Cournot (*i.e.*, quantity-setting) equilibria. A conjectural-variations approach models an oligopolistic market wherein firms react to price by conjecturing how changes in price impact the production decisions of their competitors. The firms use these conjectures to determine their own optimal production levels. Day et al. (2002) note that for representing imperfect competition, conjectural-variations models are more general, realistic, and practical than Nash-Cournot models. Nash-Cournot models can be viewed as special cases of conjectural-variations models wherein each firm assumes that its competitors do not change their production levels in reaction to price changes that are caused by *their* competitors. Moreover, unlike conjectural-variations models, Nash-Cournot models may yield unrealistic results if demand is sufficiently price-inelastic. It is normally unrealistic to assume that a supplier can increase the equilibrium price significantly without *any* reaction from its competitors. Furthermore, calculating equilibria under a conjectural-variations framework can be computationally tractable and efficient, insomuch as analytical expressions for market equilibria may be available more readily. This makes a conjectural-variations approach useful for markets that have complex systems underlying them.

Our model assumes that there is a collection of generation firms that make investment decisions simultaneously in the first stage. This is followed by production decisions in a series of second-stage spot markets. The capacity-expansion problem suffers from the curse of dimensionality if a sufficiently large number of operating periods is taken into account. The capacity-expansion literature shows that a granular (*e.g.*, hourly or sub-hourly) representation of the spot market yields computationally challenging and oftentimes intractable problems. Several works surmount the intractability and computational challenge by modeling a reduced number of operating periods that are selected in an *ad hoc* manner. The seasonal and diurnal variability of demand and resource (*e.g.*, wind and solar) profiles are represented using 17 time-slices in the deterministic capacity expansion model that is developed by Short et al. (2011). Similarly, Pina et al. (2011) divide a year into four seasons, each of which is represented by three days that are modeled at hourly temporal resolution. Several studies, including the works of Alvarez et al. (2017); Baringo and Conejo (2013); Ploussard et al. (2017); Wogrin et al. (2014), focus on selecting representative operating periods in a more methodical manner. These works aim to balance the tractability of the resulting planning model with the fidelity of the solutions that are obtained. Liu et al. (2018) propose two clustering techniques, which capture the relevant demand and resource patterns and their correlations, to select representative days. Their results suggest that 30 representative days can capture annual demand and resource variability.

In our work, we assume that the representative spot markets are selected *a priori*. We take a conjectural-variations approach, wherein each firm represents the spot-market production decisions of its rivals through reaction parameters. This allows us to derive closed-form analytical expressions for the spot-market equilibria. With such expressions we can conduct comparative statics or other analyses of market equilibria easily. These types of analyses cannot be undertaken normally if a Nash-Cournot model is employed. This is because spot-market equilibria under a Nash-Cournot framework are found typically using a mixed complementarity model. This complexity of using Nash-Cournot models is seen, for instance, in the works of García-Alcalde et al. (2002); Hobbs (2001); Pineda et al. (2018); Ramos et al. (1999); Rivier et al. (2001). Because these works consider many operating periods over long investment periods, they yield computationally intense problems that cannot be analyzed as easily as our proposed conjectural-variations approach allows.

The remainder of this paper is organized as follows. Section 2 presents our proposed market model and also shows some of its structural properties. The model is demonstrated *via* stylized analysis and numerical examples in Section 3. Section 4 provides some remarks regarding the use of our proposed model. Section 5 concludes.

2. Market Model

We model the market as consisting of two decision stages for a set, I, of generating firms. In the first stage, the generating firms determine simultaneously how much generating capacity to build. We let O_i denote Firm i's starting capacity and N_i denote additional capacity that it invests in, both of which are measured in MW. The cost to Firm i of adding N_i MW of generating capacity is:

$$d_i + c_i^N N_i, \tag{1}$$

where d_i and c_i^N are non-negative constants.

In the second stage, after the investment decisions are made, the generating firms compete in a set, T, of hour-long spot markets. The firms are modeled as being quantity-setting competitors. This means that simultaneously the firms determine their production levels in each spot market, after which the market price adjusts to clear the market (*i.e.*, for demand to exactly equal supply). We let $q_{i,t}$ denote the production level, which is measured in MW, of generating Firm i in the hour-t spot market. We assume that the firms have linear generation costs. Thus, the cost to Firm i of producing $q_{i,t}$ MW in hour t is:

$$c_i(q_{i,t}) = a_i + b_i q_{i,t}, \tag{2}$$

where a_i and b_i are non-negative constants. We assume that the demand in each hour changes linearly with the price. Thus, the hour-t spot electricity price is given by:

$$P_t(q_t) = \gamma_t - \beta_t \sum_{i \in I} q_{i,t}, \tag{3}$$

where γ_t and β_t are non-negative constants and $q_t = (q_{1,t}, q_{2,t}, \ldots, q_{|I|,t})$ is a vector of hour-t production levels of the generating firms. We assume hereafter that $P_t(q_t)$ is sufficiently large compared to $c_i'(q_{i,t})$ for all firms to produce a strictly positive amount of energy in each spot market.

Although the generating firms are assumed to be quantity-setters in the spot markets, we do not model the firms as being Cournot competitors. Rather, we take a conjectural-variations approach, wherein each firm conjectures the impact of its production decisions on those of its rivals. We let $\sigma_{i,j}$ be Firm i's conjecture of how Firm j adjusts its production level in response to a change in Firm i's production ($\sigma_{i,j}$ is referred to commonly as a reaction parameter). Thus, from Firm i's perspective:

$$\frac{d}{dq_{i,t}} q_{j,t}(q_{i,t}) = \sigma_{i,j}. \tag{4}$$

As discussed by Ruiz et al. (2010, 2012), the reaction parameter can be interpreted as characterizing the level of market competition. For instance, $\sigma_{i,j} = -1, \forall i,j \in I, i \neq j$ yields perfect competition, $\sigma_{i,j} = 0, \forall i,j \in I, i \neq j$ yields Cournot competition, and $\sigma_{i,j} = |I| - 1, \forall i,j \in I, i \neq j$ can arise if all of the firms are symmetric and participating in a symmetric cartel equilibrium. Other values of $\sigma_{i,j}$ between these extreme cases correspond to intermediate levels of competition between a cartel and perfect competition.

Because a case in which $\sigma_{i,j} = 0, \forall i, j \in I, i \neq j$ yields Cournot competition, our proposed model is more general than assuming Nash-Cournot behavior. Although positive values of reaction parameters are possible (*e.g.*, in a cartel), such behavior is unlikely in an electricity market due to market monitoring and mitigation. Nevertheless, we consider cases with positive reaction parameters in our results for analytical and theoretical purposes, and to demonstrate the flexibility of our modeling framework.

A complication that arises with the use of a conjectural-variations framework is that a rational firm should understand that:

$$\frac{d}{dq_{i,t}} q_{j,t}(q_{i,t}) = 0, \tag{5}$$

for any firm that is producing at capacity. Thus, we define Φ_t as the set of firms that are producing at their capacities in hour t (*i.e.*, firms with $q_{i,t} = O_i + N_i$). Then, we define:

$$\hat{\sigma}^t_{i,j} = \begin{cases} \sigma_{i,j}, & \text{if } j \notin \Phi_t; \\ 0, & \text{otherwise}; \end{cases} \tag{6}$$

as the hour-t *effective* reaction parameter. Also, we define:

$$\hat{\sigma}^t_i = \sum_{j \in I, j \neq i} \hat{\sigma}^t_{i,j}. \tag{7}$$

Due to the sequential nature of the market interaction, we seek a subgame-perfect Nash equilibrium. Thus, we proceed with our analysis of the market in two steps in the following subsections. First, we examine the second-stage spot markets, in which generating firms make their production decisions with the investment levels fixed. Then, we examine the first stage, in which the firms make their investment decisions.

2.1. Stage 2: Spot Market

The second stage consists of the firms determining their production levels in each hour to maximize their profits. The individual spot markets are not linked together explicitly (*i.e.*, the only link between them is that the same capacity constraint, which is determined by each firm's first-stage investment decision, applies). As such, we can analyze each spot market independently of the others.

Firm i's hour-t production level is determined to maximize its profit, meaning that it is derived from the following profit-maximization problem:

$$\max_{q_{i,t}} \left[\gamma_t - \beta_t \sum_{j \in I} q_{j,t}(q_{i,t}) - b_i \right] q_{i,t} - a_i \tag{8}$$

$$\text{s.t.} \quad q_{i,t} \leq O_i + N_i, \tag{9}$$

where we write $q_{j,t}$ as a function of $q_{i,t}$ to emphasize that Firm i accounts for the impacts of its production level on those of its rivals in making its own production decision. We also exclude the non-negativity constraint, because we assume that the market price is sufficiently high for all generators to produce a strictly positive quantity.

The Karush-Kuhn-Tucker (KKT) conditions for (8) and (9) are:

$$-\gamma_t + \beta_t \sum_{j \in I} q_{j,t}(q_{i,t}) + b_i + \beta_t q_{i,t} \sum_{j \in I} \frac{d}{dq_{i,t}} q_{j,t}(q_{i,t}) + \eta_{i,t} = 0 \tag{10}$$

$$q_{i,t} \leq O_i + N_i \perp \eta_{i,t} \geq 0, \tag{11}$$

where $\eta_{i,t}$ is the Lagrange multiplier that is associated with the capacity constraint. Because (8) is concave and (9) is linear, the KKT conditions are both necessary and sufficient for a global optimum. Using (3) and (6) and noting that:

$$\frac{d}{dq_{i,t}} q_{i,t}(q_{i,t}) = 1, \tag{12}$$

Equation (10) simplifies to:

$$P_t(q_t) - b_i - \beta_t q_{i,t} \cdot (1 + \hat{\sigma}_i^t) - \eta_{i,t} = 0. \tag{13}$$

Combining (11) and (13) we have:

$$q_{i,t}^*(N) = \min\left\{\frac{P_t(q_t^*(N)) - b_i}{\beta_t \cdot (1 + \hat{\sigma}_i^t)}, O_i + N_i\right\}, \tag{14}$$

as Firm i's profit-maximizing hour-t production level, where $q_t^*(N)$ is a vector of all of the firms' hour-t profit-maximizing production levels. We write $q_{i,t}^*$ (and q_t^*) as depending on $N = (N_1, N_2, \ldots, N_{|I|})$, which is a vector of investment decisions, to show explicitly the impact of stage-1 decisions on the outcomes of the stage-2 spot markets.

Substituting (14) into (3) gives:

$$P_t(q_t^*(N)) = \gamma_t - \beta_t \sum_{i \in I} q_{i,t}^*(N) = \gamma_t - \beta_t \cdot \left(\sum_{i \in \Phi_t}(O_i + N_i) + \sum_{i \notin \Phi_t} \frac{P_t(q_t^*(N)) - b_i}{\beta_t \cdot (1 + \hat{\sigma}_i^t)}\right), \tag{15}$$

which becomes:

$$P_t^*(N) = \frac{\gamma_t - \beta_t \sum\limits_{i \in \Phi_t}(O_i + N_i) + \sum\limits_{i \notin \Phi_t} \frac{b_i}{1 + \hat{\sigma}_i^t}}{1 + \sum\limits_{i \notin \Phi_t} \frac{1}{1 + \hat{\sigma}_i^t}}, \tag{16}$$

as the hour-t spot price, when the price is isolated on the left-hand side of the equation. We write the price as a function of N—as opposed to q_t^*—in (16) because we have a closed-form expression that relates the spot-market prices to the stage-1 investment levels (*i.e.*, substituting for q_t^*).

2.2. Stage 1: Investment

The first stage consists of the firms determining simultaneously their investment levels to maximize their profits over the two stages. Thus, Firm i's investment decision is formulated as:

$$\max_{N_i} \sum_{t \in T} \left\{[P_t^*(N) - b_i] q_{i,t}^*(N) - a_i\right\} - d_i - c_i^N N_i \tag{17}$$

$$\text{s.t.} \quad N_i \geq 0. \tag{18}$$

The KKT conditions for Firm i's profit-maximization problem are:

$$-\sum_{t \in T}\left\{q_{i,t}^*(N) \frac{\partial}{\partial N_i} P_t^*(N) + [P_t^*(N) - b_i] \frac{\partial}{\partial N_i} q_{i,t}^*(N)\right\} + c_i^N - \alpha_i = 0 \tag{19}$$

$$N_i \geq 0 \perp \alpha_i \geq 0. \tag{20}$$

Objective function (17) is concave and (18) is linear, thus the KKT conditions are necessary and sufficient for a global maximum.

Solving the KKT conditions requires expressions for two partial derivatives. From (14) we have that:

$$\frac{\partial}{\partial N_i} q_{i,t}^*(N) = \begin{cases} 1, & \text{if } i \in \Phi_t; \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Similarly, from (16) we have that:

$$\frac{\partial}{\partial N_i} P_t^*(N) = \begin{cases} -\beta_t / [1 + \sum_{j \notin \Phi_t} 1/(1 + \hat{\sigma}_j^t)], & \text{if } i \in \Phi_t; \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

Combining these two partial derivatives with (19) and (20) allows us to obtain closed-form investment equilibria. As noted in Section 1, a benefit of our conjectural-variations approach to modeling the market (as opposed to a computational Nash-Cournot approach) is that it allows us to obtain closed-form expressions for the investment and spot-market equilibria.

We conclude our analysis of the proposed model by proving the following proposition, which shows how each firm's investment cost, C_i^N, impacts its equilibrium investment level, N_i.

Proposition 1. *Firm i's equilibrium investment level, N_i, is decreasing in its marginal investment cost, C_i^N.*

Proof. Without loss of generality, we consider a case in which $N_i > 0$. The case in which $N_i = 0$ is uninteresting, because increasing C_i^N results clearly in $N_i = 0$ still being the equilibrium investment level (*i.e.*, Firm i does not increase its investment from zero if, *ceteris paribus*, its marginal investment cost increases). Firm i's investment decision is governed by (19) and (20). If C_i^N increases, (19) can be satisfied by either an increase in α_i or an increase in:

$$\sum_{t\in T}\left\{q_{i,t}^*(N)\frac{\partial}{\partial N_i}P_t^*(N)+[P_t^*(N)-b_i]\frac{\partial}{\partial N_i}q_{i,t}^*(N)\right\}. \tag{23}$$

An increase in α_i violates (20), thus we focus on an increase in (23). Substituting (21) and (22) into (23) gives:

$$\sum_{t\in T}\left\{q_{i,t}^*(N)\frac{\partial}{\partial N_i}P_t^*(N)+[P_t^*(N)-b_i]\frac{\partial}{\partial N_i}q_{i,t}^*(N)\right\} = \sum_{t\in T:i\in\Phi_t}\left\{\frac{-q_{i,t}^*(N)\beta_t}{1+\sum_{j\notin\Phi_t}1/(1+\hat{\sigma}_j^t)}+[P_t^*(N)-b_i]\right\}. \tag{24}$$

Equation (24) can increase only if $q_{i,t}^*(N)$ decreases or $P_t^*(N)$ increases. From (21) and (22) we see that $q_{i,t}^*(N)$ decreases and $P_t^*(N)$ increases only if N decreases, proving the desired result. □

2.3. Equilibrium Consistency

A difficulty in applying the model that is developed in Section 2 is that the spot-market and investment equilibria depend on which firms are operating at their capacities in each spot-market period. This is exemplified by the definition of the spot-market quantities and prices, which are given by (14) and (16), respectively, and by partial derivatives (21) and (22). Thus, finding spot-market and investment equilibria becomes a combinatorial search, in which one determines which firms are producing at their capacities in each spot-market period. The goal of this search is to find 'consistent' equilibria, wherein the 'correct set' of generating firms are conjectured as being capacitated in the correct set of spot-market periods. Normally, this requires evaluating $|T|^{|I|}$ cases. If the goal is to find a single consistent equilibrium, one can do so by rank-ordering the spot-market periods based on the level of demand. Capacity constraints are more likely to be binding during high-demand periods. Thus, one can 'work down' through the rank-ordering of spot-market periods to find a consistent equilibrium.

To illustrate this more clearly, we define:

$$\sigma_i = \sum_{j\in I, j\neq i}\sigma_{i,j}. \tag{25}$$

Next, we note that if none of the generation-capacity constraints are binding in hour-t, then we have from (14) and (3) that Firm i's hour-t production level is:

$$q_{i,t}^U = \frac{P_t(q_t^U)-b_i}{\beta_t\cdot(1+\sigma_i)}, \tag{26}$$

and the hour-t price is:

$$P_t(q_t^U) = \gamma_t - \beta_t\sum_{i\in I}q_{i,t}^U. \tag{27}$$

Simultaneously solving these two equations gives:

$$P_t^U = \frac{\gamma_t + \sum\limits_{i \in I} \frac{b_i}{1+\sigma_i}}{1 + \sum\limits_{i \in I} \frac{1}{1+\sigma_i}}, \tag{28}$$

as the hour-t price and:

$$q_{i,t}^U = \frac{P_t^U - b_i}{\beta_t \cdot (1 + \sigma_i)}, \tag{29}$$

as Firm i's hour-t production level, if none of the firms are capacitated in hour t. Then, we can define:

$$q_t^U = \sum_{i \in I} q_{i,t}^U, \tag{30}$$

as the aggregate hour-t production level if none of the firms are capacitated in hour t. We then re-order the set of spot-market periods in descending order based of the values of q_t^U. Let T' denote the re-ordered set of hours (*i.e.*, q_t^U is greatest for the first element of T' and smallest for the last element of T').

Algorithm 1 provides pseudocode of a proposed method to find all consistent equilibria by working down through the rank ordering of the spot-market periods, which is given by T'. Line 1 initializes the algorithm by assuming that none of the firms are capacitated in any of the hours. Lines 2–8 are the main iterative loop, wherein the algorithm works down through the rank-ordered spot-market periods to conjecture which firms are capacitated in which hours. The set of counters, $\tau_1, \tau_2, \ldots, \tau_{|I|}$, denote the spot-market periods in which each of Firms $1, 2, \ldots, |I|$ are conjectured as being capacitated (*i.e.*, Firm i is conjectured as being capacitated in all of hours $1, 2, \ldots, \tau_i$, when the hours are rank-ordered). Once a set of conjectures for which firms are capacitated in which hours is selected, (19)–(22) are used in Line 9 to determine the optimal investment levels. Lines 10 and 11 then compute the corresponding spot-market equilibria. Finally, Line 12 verifies whether the equilibrium is consistent, by ensuring that the prices are sufficiently high in each spot-market period for each firm to be producing and for the investment levels to be sufficiently high to support the production levels in all of the spot-market periods. If the conditions in Line 12 are satisfied, then the algorithm outputs the consistent equilibrium that is found.

Algorithm 1 Consistent-Equilibrium Search

1: $\Phi_t \leftarrow \emptyset, \forall t \in T'$
2: **for** $\tau_1 \leftarrow 1$ **to** $|T'|$ **do**
3: $\quad \Phi_{\tau_1} \leftarrow \Phi_{\tau_1} \cup 1$
4: $\quad$ **for** $\tau_2 \leftarrow 1$ **to** $|T'|$ **do**
5: $\quad\quad \Phi_{\tau_2} \leftarrow \Phi_{\tau_2} \cup 2$
6: $\quad\quad \ddots$
7: $\quad\quad$ **for** $\tau_{|I|} \leftarrow 1$ **to** $|T'|$ **do**
8: $\quad\quad\quad \Phi_{\tau_{|I|}} \leftarrow \Phi_{\tau_{|I|}} \cup |I|$
9: $\quad\quad\quad$ Solve (19) and (20) using (21) and (22) for N
10: $\quad\quad\quad$ Solve (16) for $P_t^*(N)$
11: $\quad\quad\quad$ Solve (14) for $q_{i,t}^*$
12: $\quad\quad\quad$ **if** $\min\limits_{t \in T'} P_t^*(N) \geq b_i$ **and** $\max\limits_{t \in T', t \geq \tau_i} q_{i,t}^* \leq O_i + N_i \leq q_{\tau_i,i}^U$ **then**
13: $\quad\quad\quad\quad$ **Output:** Consistent equilibrium
14: $\quad\quad\quad$ **end if**
15: $\quad\quad$ **end for**
16: $\quad$ **end for**
17: **end for**

3. Numerical Case Studies

We demonstrate Algorithm 1 and our proposed model with a stylized analysis and two numerical case studies. The stylized analysis considers a simple symmetric duopoly. This case is illustrative because we can derive closed-form expressions for the spot-market and investment equilibria. Then, we consider a duopoly with three representative spot-market periods. This case serves as a proof-of-concept for our modeling approach. Finally, we examine an asymmetric triopoly with 20 representative spot markets, which is used to demonstrate further the efficiency of our proposed model in finding investment equilibria. We also use the triopoly case to examine the tractability and computational requirements of increasing the case study to include 500 representative spot-market periods.

3.1. Symmetric-Duopoly Case

We begin with a detailed analysis of a symmetric duopoly, in which both firms have the same starting capacities, O, production-cost parameters, a and b, and marginal investment costs, C^N. To derive an investment equilibrium, we can use (21) and (22) to write (19) as:

$$-\sum_{t\in T:i\in\Phi_t}\left\{\frac{-q^*_{i,t}(N)\beta_t}{1+\sum_{j\notin\Phi_t}1/(1+\hat{\sigma}^t_j)}+[P^*_t(N)-b]\right\}+c^N-\alpha_i=0, \tag{31}$$

which characterizes Firm i's equilibrium investment level. Given the symmetric nature of the problem, a symmetric equilibrium is more likely to be consistent and we restrict attention to symmetric equilibria. Under this restriction, any spot-market period, t, in which Firm i is capacitated results in its rival being capacitated as well. We also know that if the firms are capacitated in spot-market Period t, then their outputs are given by $O+N$, where N is the investment level in a symmetric equilibrium. In light of these observations, (31) can be simplified further to:

$$-\sum_{t\in T:i\in\Phi_t}\{-(O+N)\beta_t+[P^*_t(N)-b]\}+c^N-\alpha_i=0. \tag{32}$$

We also have, from (3), that the price in spot-market periods during which the firms are capacitated is given by:

$$P^*_t(N)=\gamma_t-2\beta_t\cdot(O+N). \tag{33}$$

Combining (32) and (33) with (20) yields:

$$N=\frac{\sum\limits_{t\in T}\gamma_t-3O\sum\limits_{t\in T}\beta_t-b|T|-C^N}{3\sum\limits_{t\in T}\beta_t}, \tag{34}$$

as the symmetric equilibrium investment level.

Although the reaction parameters do not appear in (34), their values do play a role in determining whether an equilibrium is consistent or not (*cf.* Section 2.3). To demonstrate this, we consider a case in which $O=100$, $b=30$, $C^N=30$, and the demand data for three spot-market periods are given in Table 1. With these data, (34) yields four possible investment equilibria, which are summarized in Figure 1, and depend on the values of the reaction parameters. This is because the reaction parameters determine ultimately the spot-market periods in which the firms are capacitated. The four cases that are shown in Figure 1 result in either (i) no consistent investment equilibrium, which occurs if $\sigma_{1,2}=\sigma_{2,1}\in(0.1,0.3)$, (ii) $N=203$, (iii) $N=223$, or (iv) $N=166$. Figure 1 also shows the symmetric investment level that occurs in a Nash-Cournot equilibrium, as this corresponds to a case in which $\sigma_{1,2}=\sigma_{2,1}=0$. A reaction-parameter value for which no consistent equilibrium exists suggests that only mixed-strategy equilibria (which our model does not find and which is beyond the scope of our work) may exist. A mixed-strategy equilibrium can arise because any fixed sets of investments raise a unilateral deviation for at least one firm. This is because investment decisions affect spot-market prices and the effects can result in cases in which firms 'cycle' between high and low investment levels.

Table 1: Demand Data for Symmetric-Duopoly Case

t	γ_t	β_t
1	60	0.050
2	80	0.022
3	70	0.040

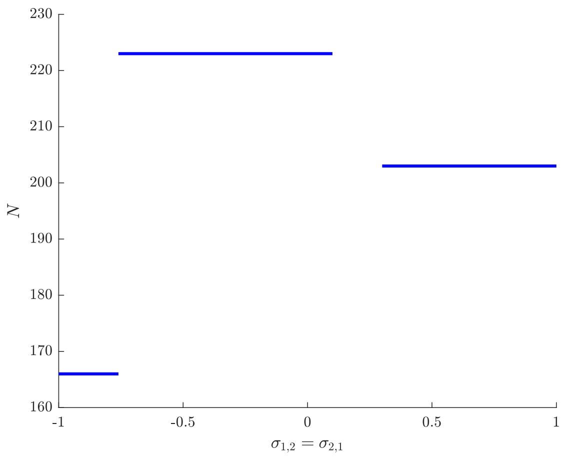

Figure 1: Equilibrium Investment Levels as a Function of Reaction Parameter in Symmetric-Duopoly Case

3.2. Asymmetric-Duopoly Case

Tables 2 and 3 summarize generating-firm and demand data, respectively, for an asymmetric-duopoly case with three representative spot-market periods. The two firms are assumed to have reaction parameters $\sigma_{1,2} = \sigma_{2,1} = -0.5$. Table 4 summarizes equilibrium spot-market production levels, $q^U_{i,t}$, and prices, p^U_t, in the three periods if none of the firms are capacitated.

Table 2: Generating-Firm Data for Asymmetric-Duopoly Case

i	a_i	b_i	O_i	c^N_i
1	200	30	100	30
2	500	25	50	30

Table 3: Demand Data for Asymmetric-Duopoly Case

t	γ_t	β_t
1	60	0.050
2	80	0.020
3	70	0.040

Table 4 provides two insights into the possible investment equilibria, which illustrates the rationale behind Algorithm 1. First, it is clear that Firm 1 will not invest in more than 700 MW of additional capacity while Firm 2's investment will be no greater than 1250 MW. This is because the two firms produce at most 800 MW and 1300 MW between the three spot-market periods and have starting generating capacities of 100 MW and 50 MW, respectively. Second, we can divide the potential investment levels of the two firms

Table 4: Equilibrium Spot-Market Production Levels and Prices in Asymmetric-Duopoly Case Assuming Firms are Uncapacitated

t	$q^U_{1,t}$	$q^U_{2,t}$	P^U_t
1	160	360	34
2	800	1300	38
3	300	550	36

into three intervals, which correspond to the spot-market periods in which each firm is capacitated. For instance, if $N_1 \in [0, 60)$ then Firm 1 produces at capacity in all three spot-market periods, if $N_1 \in [60, 200)$ then Firm 1 produces at capacity in spot-market Periods 2 and 3 *only*, while if $N_1 \in [200, 700)$ then Firm 1 produces at capacity in spot-market Period 2 *only*. The corresponding intervals of investment levels for Firm 2 are $N_2 \in [0, 310)$, $N_2 \in [310, 500)$ and $N_2 \in [500, 1250)$.

On the basis of this observation that each firm's investment level determines the spot-market periods in which it operates at capacity, we can search for market equilibria, using Algorithm 1. Thus, essentially we examine cases in which capacity constraints are binding in spot-market Period 2 *only*, followed by cases in which they are binding in Periods 2 and 3 *only*, and finally cases in which they are binding in all three periods.

As an example, consider a case in which we conjecture that both firms have binding capacity constraints in spot-market Period 2 *only*. In this case, we have that $\Phi_1 = \Phi_3 = \emptyset$ and $\Phi_2 = \{1, 2\}$. Using (19) and (20) we obtain $N_1 = 150$ and $N_2 = 450$. These investment levels are inconsistent with the conjecture that the two firms are capacitated in spot-market Period 2 *only*, because that only occurs if $N_1 \in [200, 700)$ and $N_2 \in [500, 1250)$. As such, there is no consistent equilibrium in which both firms are capacitated in spot-market Period 2 *only*.

The first row of Table 5 summarizes the analysis of the case in which we conjecture that the two firms are capacitated in spot-market Period 2 *only*. The table also shows the results of the same computations for the other cases that correspond to different spot-market periods during which the two firms are capacitated. The table shows that there is only a single consistent investment equilibrium, in which $N_1 = 178$, $N_2 = 394$, and both firms are capacitated in spot-market Periods 2 and 3 *only*. Indeed, if one is concerned solely with finding an equilibrium, Algorithm 1 can be terminated after evaluating the fourth case that is listed in the table (as opposed to examining all nine cases). Table 6 summarizes the spot-market outcomes, investment costs, and profits that are earned by the two firms in the consistent equilibrium that is identified in Table 5.

Table 5: Equilibrium Investment Levels in Asymmetric-Duopoly Case with Different Conjectures of Binding Capacity Constraints in Spot-Market Periods

Capacitated Periods		Investment Levels		
Firm 1	Firm 2	N_1^*	N_2^*	Consistent Equilibrium
2	2	150	450	No
2	2, 3	84	581	No
2, 3	2	209	421	No
2, 3	2, 3	178	394	Yes
2	1, 2, 3	150	450	No
1, 2, 3	2	213	419	No
2, 3	1, 2, 3	186	379	No
1, 2, 3	2, 3	168	399	No
1, 2, 3	1, 2, 3	127	314	No

We conclude our analysis of the duopoly case with three sensitivity analyses. First, we examine the

Table 6: Spot-Market Production Levels and Prices in Consistent Equilibrium in Asymmetric-Duopoly Case

		Firm 1		Firm 2	
t	$P_t^*(N)$	$q_{1,t}$	Spot-Market Profit [$]	$q_{2,t}$	Spot-Market Profit [$]
1	34	160	440	360	2740
2	66	278	9679	444	17525
3	41	278	2886	444	6659
Investment Cost [$]			5334		11832
Total Profit [$]			7671		15092

impact of changes in the investment-cost parameters, c_1^N and c_2^N, on the equilibrium investment levels of the two firms and the resulting profits that they earn. Figures 2 and 3 summarize each firm's equilibrium investment level and the resultant profit that it earns as a function of its own investment-cost parameter (*i.e.*, it does not show how a change in c_i^N affects the investment levels and profits of Firm i's rival). As expected from Proposition 1, Figure 2 shows that increasing a firm's investment cost reduces its investment level. Interestingly, a firm's profits are not necessarily decreasing in its investment cost. For instance, Firm 1 does not invest in any additional capacity if $c_1^N \geq 72$. However, this lack of investment by Firm 1 also results in Firm 2 investing in less capacity, which drives up spot prices and increases Firm 1's profits.

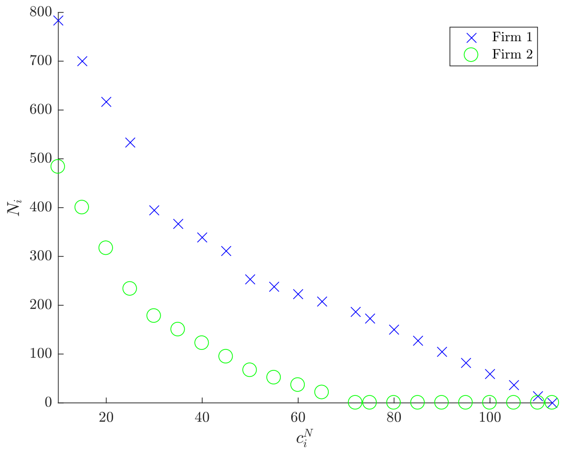

Figure 2: Equilibrium Investment Levels as a Function of Investment-Cost Parameters in Asymmetric-Duopoly Case

Table 7 summarizes the results of another sensitivity analysis, in which the two firms' production costs are reduced to $b_1 = 20$ and $b_2 = 15$. Interestingly, this case yields two investment equilibria. The first sees greater investment levels by both firms, and as such they are capacitated in spot-market Period 1 *only*. The second equilibrium sees less investment and the firms are capacitated in both spot-market Periods 1 and 2. The table shows that the first equilibrium is preferable from the perspective of Firm 2, consumers, and society as a whole. However, Firm 1 prefers the second equilibrium as it earns slightly higher profit (at the expense of Firm 2's profits and consumer and social welfare).

Figure 4 summarizes the results of a third sensitivity analysis, in which the two firms' reaction parameters are changed. It shows each firm's equilibrium investment level for different values of the reaction parameters, which are assumed to be equal for the two firms. Interestingly, there are no consistent equilibria for a case in which $\sigma_{1,2} = \sigma_{2,1} = 0$, meaning that there is no pure-strategy Nash-Cournot equilibrium for our asymmetric-duopoly case. This finding demonstrates a further benefit of our proposed conjectural-variations modeling approach relative to a Nash-Cournot model. There may be instances in which a Nash-Cournot equilibrium

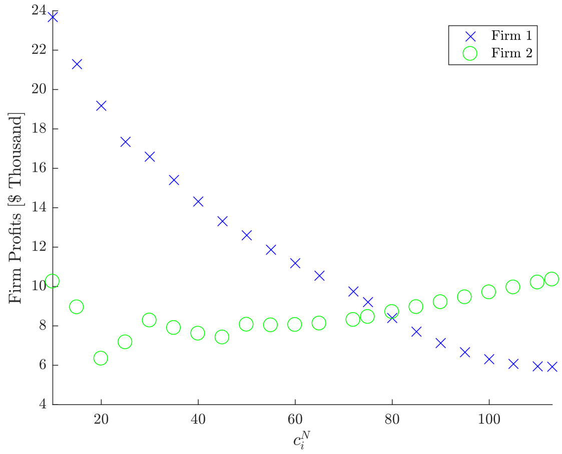

Figure 3: Equilibrium Firm Profits as a Function of Investment-Cost Parameters in Asymmetric-Duopoly Case

Table 7: Equilibrium Investment, Production, Profits, and Welfare Levels in Two Equilibria with $b_1 = 20$ and $b_2 = 15$ in Asymmetric-Duopoly Case

	Equilibrium	
	1	2
Firm 1		
N_1	317	289
$q_{1,1}$	417	389
$q_{1,2}$	400	389
$q_{1,3}$	240	240
Firm 2		
N_2	617	505
$q_{2,1}$	667	555
$q_{2,2}$	650	555
$q_{2,3}$	440	440
Generator Profits		
Firm 1	20024	21585
Firm 2	40691	38488
Total	60715	60073
Consumer Welfare	45355	38308
Social Welfare	106070	98382

does not exist but in which a conjectural-variations equilibrium does exist (with non-zero reaction parameters). Conversely, it is not possible for a set of problem parameters to yield a Nash-Cournot equilibrium without having at least one conjectural-variations equilibrium (because the Nash-Cournot equilibrium can be recovered by setting the reaction parameters equal to zero). This means that so long as the reaction parameters are chosen appropriately, our conjectural-variations approach may yield a consistent investment equilibrium in settings in which no Nash-Cournot equilibria exist.

3.3. Triopoly Case

Tables 8 and 9 summarize generating-firm and demand data, respectively, for a triopoly case with 20-representative spot-market periods. The investment costs that are reported in Table 8 are annualized

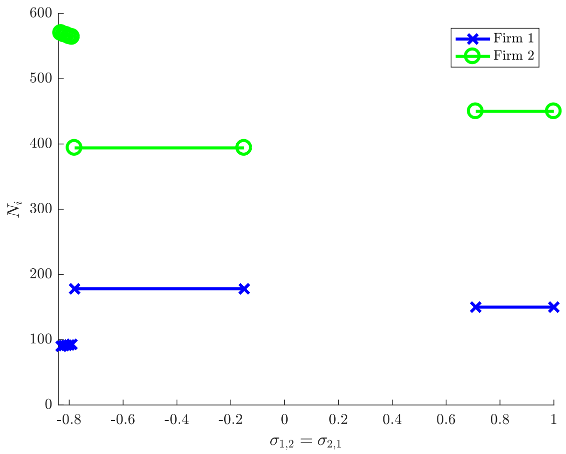

Figure 4: Equilibrium Investment Levels as a Function of Reaction Parameter in Asymmetric-Duopoly Case

over a 20-year investment horizon. Each of the 20 spot-market periods is assumed to represent 5% of the hourly operating periods in a typical year. The firms are all assumed to have the same reaction parameters $\sigma_{i,j} = -0.25, \forall i, j \in I, i \neq j$.

Table 8: Generating-Firm Data for Triopoly Case

i	a_i	b_i	O_i	c_i^N
1	200	26	100	100
2	500	24	50	100
3	400	25	75	100

Table 9: Demand Data for Triopoly Case

t	γ_t	β_t	t	γ_t	β_t
1	100	0.049	11	100	0.020
2	110	0.021	12	102	0.015
3	96	0.045	13	103	0.027
4	95	0.033	14	105	0.028
5	105	0.023	15	100	0.025
6	104	0.022	16	90	0.050
7	98	0.048	17	91	0.024
8	92	0.044	18	95	0.042
9	106	0.032	19	85	0.030
10	102	0.041	20	80	0.018

This set of parameter values yields a single unique equilibrium, in which $N_1 = 563$, $N_2 = 749$, and $N_3 = 677$. These investment levels lead to Firm 1 being capacitated in nine of the spot-market periods and the remaining two firms being capacitated in 10 periods. The firms' total annualized profits are \$144822, \$190533, and \$168940, respectively.

We conclude this case study by showing the results of two sensitivity analyses as well as an examination of the computational complexity of our proposed model. First, Figures 5 and 6 show the impacts of changing the investment-cost parameters, c_i^N, on equilibrium investment levels and the resultant profits for the three firms. Unlike Figures 2 and 3, these two figures show the impact of changing the investment costs of the three firms simultaneously (*i.e.*, we assume that $c_1^N = c_2^N = c_3^N$ in all of the cases that are shown in the figures). Some values of the investment-cost parameter yield multiple equilibria, which are shown in the figures by multiple points corresponding to the value of c_i^N. Firm 2's lower production cost (relative to the other two firms) allows it to invest in more capacity, regardless of the level of the investment-cost parameter.

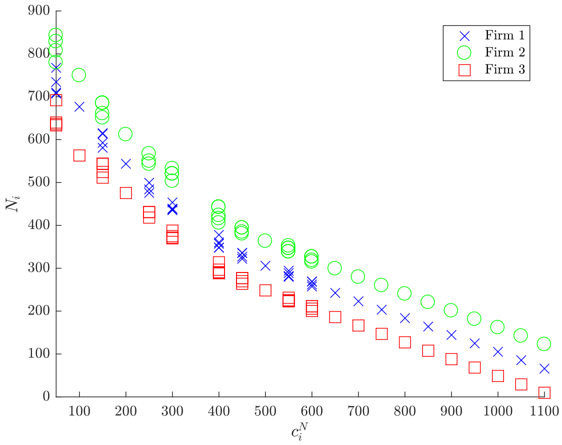

Figure 5: Equilibrium Investment Levels as a Function of Investment-Cost Parameters in Triopoly Case

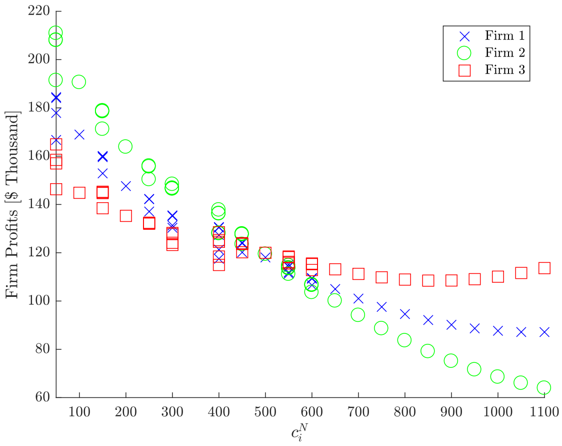

Figure 6: Equilibrium Firm Profits as a Function of Investment-Cost Parameters in Triopoly Case

For low values of the investment-cost parameter, Firm 2 earns relatively high profits, due to its lower production cost and the relatively low cost of added investment. However, at higher values of the investment-cost parameter, Firm 1 earns the highest profits despite its having the highest operating cost. This is because the higher investment cost results in relatively little capacity being added by any of the firms and Firm 1 having a relatively high starting capacity. For instance, if $c_1^N = c_2^N = c_3^N = 500$, Firm 2 only adds $N_2 = 115$ MW of capacity at a cost of $57367. This investment yields an additional $56791 in spot-market profits, meaning that Firm 1 still earns $576 more in total profits than Firm 2 does.

For our third sensitivity analysis, Figure 7 summarizes the impact of changing all of the reaction parameters for all of the firms on the three firms' equilibrium investment levels. The figure shows that small changes in the reaction parameters result in relatively small or no changes in the equilibrium investment levels. This robustness is desirable, given that it may be difficult to anticipate accurately the reaction of a rival firm to production decisions. Reaction-parameter values that are close to zero yield equilibria that are relatively similar to those that are obtained from a Nash-Cournot model. Reaction-parameter values that are larger in magnitude yield investment levels that are more dissimilar to a Nash-Cournot equilibrium. This illustrates the flexibility of the conjectural-variations approach that we develop in modeling the impacts of imperfect competition on investment decisions.

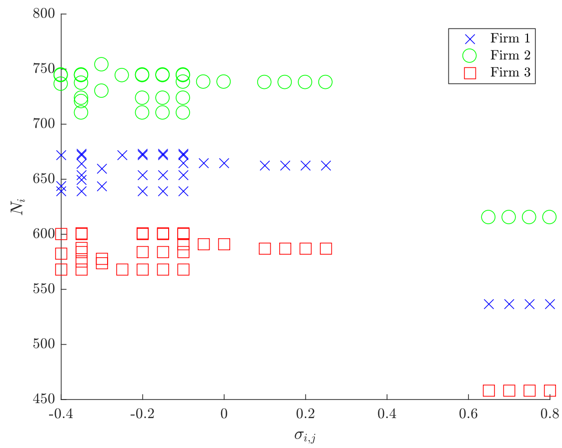

Figure 7: Equilibrium Investment Levels as a Function of Reaction Parameter for Triopoly Case

Table 10 compares the six equilibria that are obtained for the case in which $\sigma_{i,j} = -0.35, \forall i, j \in I, i \neq j$. The equilibria are fairly similar in terms of total investments and welfare levels. However, Equilibrium 1 maximizes consumer and social welfare whereas Equilibrium 6 is the best for total producer welfare. In terms of individual firm profits, Equilibrium 2 maximizes the profits of Firms 2 and 3 individually while Equilibrium 3 is optimal for Firm 1. The first two equilibria are relatively similar (compared to the others), which suggests that there may be 'clusters' of equilibria that are similar in terms of welfare and investment levels. This may be particularly true in larger problems, which may have relatively larger equilibrium sets.

Table 10: Equilibrium Investment, Production, Profits, and Welfare Levels for Six Equilibria with $\sigma_{i,j} = 0.35 \forall i, j \in I, i \neq j$ in Triopoly Case

	Equilibrium					
	1	2	3	4	5	6
N_1	563	562	605	605	584	579
N_2	749	751	705	750	724	730
N_3	677	676	678	634	654	657
Generator Profits						
Firm 1	184137	185898	195296	194990	193496	194448
Firm 2	249708	258476	237452	250045	247005	256660
Firm 3	219887	228281	220530	208500	217971	227225
Total	653732	672655	653278	653535	658472	678333
Consumer Welfare	1065115	1043290	1064408	1064761	1053173	1031026
Social Welfare	1718847	1725945	1717685	1718296	1711645	1709359

As a final sensitivity analysis, we examine the computational complexity of our proposed conjectural-variations model to compute investment equilibria. We do this by applying Algorithm 1 to a variety of triopoly cases with different numbers of representative spot-market periods. These computations are all implemented using `MATLAB` version R2015b on a 64-bit computer with a 3.2-GHz Intel Core i5 processor and 8 GB of memory. Table 11 summarizes the average wall clock time that is required for applying Algorithm 1 to obtain all of the equilibria of different-sized problems. For each case (*i.e.*, with different numbers of spot-market periods) the computation is repeated using either 10 or five replications of the problem parameters. The computation times demonstrate the tractability of our model, even if a large number of spot-market periods is considered.

Table 11: Equilibrium-Computation Time for Triopoly Case

$\|T\|$	Number of Replications	Average Time
20	10	0.4 s
50	10	8.4 s
100	10	2.4 min
150	10	13.5 min
200	10	1.1 h
300	10	5.2 h
500	5	77.6 h

4. Limitations and Remarks

Conjectural variations is a well established methodology for modeling oligopolistic markets. It does require, however, the specification or estimation of reaction parameters, which is beyond the scope of our work. Conrad (1989); Gollop and Roberts (1979); Kamien and Schwartz (1983); López de Haro et al. (2007) provide methods that can be used to estimate reaction parameters. Our sensitivity analyses (*cf.* Figures 1, 4, and 7) examine the impacts of varying reaction parameters on investment equilibria. Importantly, our results show that small changes in the reaction parameters result in relatively small or no changes in the equilibria themselves. This robustness is desirable, given difficulties that may arise in calibrating reaction parameters exactly for a real-world market.

We study numerical examples with either two or three firms. Our modeling approach is completely flexible, however, in the number of firms and spot-market periods that can be represented. Indeed, a strength of using our proposed conjectural-variations approach is its computational efficiency. Policymakers or regulators may wish to examine, for instance, what impact a merger or forced divestiture of assets would have on market competitiveness and efficiency. By making appropriate adjustments to the underlying problem parameters, equilibrium investment levels and the resulting spot-market equilibria can be examined easily. This demonstrates the potential value of our proposed model for policy and regulatory analysis (in addition to providing decision support for market participants themselves).

5. Conclusions

This paper proposes a conjectural-variations based approach to modeling investment and spot-market equilibria in imperfectly competitive oligopolistic electricity markets. Thus, this work contributes to the relatively scant extant literature that is related to investment in oligopolistic electricity markets. This can be contrasted with generation-capacity planning within a perfectly competitive market or a centralized-planning framework, which is relatively well examined in the literature. Although our modeling approach requires a combinatorial search of which firms are capacitated in each spot-market period, we develop a methodology to find an equilibrium efficiently by 'working down' the rank-ordering of the spot-market periods.

We demonstrate our modeling approach in duopoly and triopoly settings and conduct some sensitivity analyses of the market equilibria. We prove and demonstrate that higher investment costs translate into lower investment levels. We do find cases, however, in which a firm with a competitive advantage (*e.g.*, high starting capacity) benefits from higher investment costs as more costly investments can foreclose on potential competition. We also find cases in which a set of parameter values may yield multiple equilibria. As is common in non-co-operative games, these equilibria introduce trade-offs in terms of which market agents (*i.e.*, which firms and consumers) benefit from one equilibrium relative to another.

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