Pricing in Centrally Committed Electricity Markets

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Abstract

A long-standing issue with centrally committed electricity markets is the design of non-confiscatory and equilibrium-supporting prices. This is because the social planner's problem in such a market is a non-convex unit commitment. O'Neill et al. (2005) propose a pricing scheme that overcomes these issues for any market that can be formulated as a mixed-integer linear program. Moreover, claims appear in the literature implying that such a payment mechanism is currently in use in a number of organized electricity markets. Using an illustrative example, we demonstrate that this is not the case. We further demonstrate that the pricing scheme proposed by O'Neill et al. (2005) has some important implications for long-run capacity investment.

Keywords: Pricing, unit commitment, equilibrium

1. Introduction

Centrally committed electricity markets rely on a system operator (SO) to make binding generator operation decisions. Interested readers are referred to the work of Baldick et al. (2005), which provides an excellent overview of such markets. The SO does this by soliciting multi-part offers from generators, specifying their cost structure and operating constraints, and bids for energy and reserves from consumers. These offers and bids are input to a unit commitment problem, which the SO solves to determine the socially optimal commitment and dispatch of generators and the allocation of energy and reserves to consumers. Sheble and Fahd (1994) provide a detailed survey of the unit commitment literature. The SO also uses the unit commitment solution to generate a set of prices for the commodities (*e.g.*, energy and reserves) traded in the market. A difficulty in generating such prices is that linear commodity payments can be confiscatory in a market with indivisibilities or other non-convexities. In the context of unit commitment, non-convexities arise due to binary generator commitment decisions (*i.e.*, a generator either has to be in an offline or online state in each time interval). Moreover, as Hobbs et al. (2001); Johnson et al. (1997) note, non-convexities in unit commitment problems imply that there are no linear commodity prices that 'support' the socially optimal solution. This means that if generators and consumers are left to make production and consumption decisions on their own, based on a set of linear prices, they typically have incentives to deviate from the SO's optimal solution.

O'Neill et al. (2005) propose an alternate pricing scheme, which we hereafter refer to as **T**. They show that **T** supports the central planner's solution for any market that can be formulated as a mixed-integer linear program (MILP), by finding a set of prices that yield zero profits for all activities in the optimal solution. In the context of electricity markets, **T** works by first solving the MILP of the unit commitment problem to determine a socially optimal solution. They then solve an augmented linear program (LP), in which the integrality restrictions are relaxed and equality constraints fixing the integer variables to their optimal values are added. Since this LP has a well-defined dual, the corresponding optimal dual variable

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values can be used as prices. Specifically, they propose making linear payments for each of the commodities traded, and a supplemental payment associated with each 'integer' activity. In an electricity market, the first part of their proposal is to make payments for energy and reserves based on the dual variables associated with the system energy- and reserve-balance constraints. The second part makes discriminatory payments to each generator based on the dual variable associated with the constraints fixing the commitment decisions to their optimal values. O'Neill et al. (2005) note that the prices on these commitment activities may be negative, for instance to incent a generator to remain offline if it is suboptimal to commit it. In such a case, the generator receives this negative payment only if it commits itself.

In practice, most centrally committed electricity markets overcome the confiscation issue using linear payments for energy and reserves and supplemental 'make-whole' payments. These make-whole payments compensate each generator for incurred cost, which is calculated using the bids submitted to the SO, that is not recovered through energy and reserve payments. Thus, the make-whole scheme ensures that no generator has a net profit loss (on the basis of its bids) at the end of the unit commitment period. One might be tempted to conjecture that these make-whole payments are equivalent to **T**. Indeed, claims either explicitly stating or implicitly suggesting this can be found in the literature. For instance, Araoz-Castillo and Jörnsten (2010) state:

> Reference [7] [the work of O'Neill et al. (2005)] proposes an augmented pricing problem, where the MILP is solved with integrality constraints (6) set to their optimal value, that is $z_i = Z_i^*$. The dual prices obtained from constraints (3), (4), and (6) will represent the price for three commodities, namely the price for electricity, for capacity price and for start-up or construction respectively.
>
> Under this approach, the generators will receive a price for the commodity amount produced and an additional amount to compensate them for any loss incurred using these commodity prices. However, there are some instances where these prices are negative.

Baldick et al. (2005); O'Neill (2010) make similar statements that could be interpreted as suggesting that **T** is equivalent to the make-whole mechanism. However, we provide an example below showing that **T** can exhibit properties that are quite different from make-whole payments. We further show that **T** has some potentially important implications for long-run generation capacity investment in an electricity system.

2. Example

We begin with a simple numerical example consisting of a single-hour with 1010 MW of load, no reserves, and two generators. The operating costs and capacities of the generators are summarized in Table 1. We verify that linear commodity payments for energy only is confiscatory and that **T** overcomes this issue. We also demonstrate that the prices generated by **T** are different from the make-whole mechanism used in most centrally committed electricity markets. Indeed, while make-whole payments allow generator 1 to earn inframarginal rents, generator 1's discriminatory payment under **T** is negative, eliminating all of these rents.

Table 1: Generator characteristics

Generator	Startup Cost [$]	Variable Cost [$/MWh]	Capacity [MW]
1	0	10	1000
2	1000	15	100

In this example, the SO must determine whether to startup the two generators, which are binary decisions, which we denote u_1 and u_2. It must also decide how much energy, q_1 and q_2, each should produce. Using the generator characteristics in Table 1, we have the following MILP:

$$\begin{aligned}
\min_{q,u} \quad & 1000 \cdot u_2 + 10 \cdot q_1 + 15 \cdot q_2 \\
\text{s.t.} \quad & q_1 + q_2 = 1010 \\
& 0 \leq q_1 \leq 1000 \cdot u_1 \\
& 0 \leq q_2 \leq 100 \cdot u_2 \\
& u_1, u_2 \in \{0, 1\},
\end{aligned}$$

the unique optimal solution of which is $(q_1^*, q_2^*, u_1^*, u_2^*) = (1000, 10, 1, 1)$. The standard energy-pricing scheme used in electricity markets is to replace the integer variables with their optimal values, giving the following LP:

$$\begin{aligned}
\min_{q} \quad & 10 \cdot q_1 + 15 \cdot q_2 \\
\text{s.t.} \quad & q_1 + q_2 = 1010 && (\eta) \\
& 0 \leq q_1 \leq 1000 \\
& 0 \leq q_2 \leq 100,
\end{aligned}$$

where η is the dual variable associated with the load-balance constraint. This dual variable value is used to remunerate generators for energy provided. Thus, generator i's net profit under this scheme is:

$$(\eta^* - c_i) \cdot q_i^* - s_i \cdot u_i^*,$$

where c_i and s_i are its variable and startup costs, respectively. In our example, the value of this dual variable is \$15/MWh, giving the two generators net profits of \$5000 and $-$\$1000, respectively. This pricing scheme is confiscatory, because generator 2 sets the marginal price of energy and cannot recover its fixed startup cost. With the standard make-whole payment scheme, generator 2 receives a supplemental payment of \$1000 to recover this lost profit while generator 1 receives no supplemental payment and keeps \$5000 of inframarginal rent.

Under **T**, the SO solves the following augmented LP:

$$\begin{aligned}
\min_{q,u} \quad & 1000 \cdot u_2 + 10 \cdot q_1 + 15 \cdot q_2 \\
\text{s.t.} \quad & q_1 + q_2 = 1010 && (\eta) \\
& q_1 \leq 1000 \cdot u_1 \\
& q_2 \leq 100 \cdot u_2 \\
& u_1 = 1 && (\mu_1) \\
& u_2 = 1 && (\mu_2) \\
& q_1, q_2 \geq 0,
\end{aligned}$$

where the variables in the parentheses indicate the dual variable associated with each primal constraint. Since the u_i's are fixed at their MILP-optimal values, this LP yields the same solution as the original MILP. Moreover, we now have prices, μ_1 and μ_2, which are used for supplemental payments to each generator associated with its integer startup activity. Thus, generator i's net profit under **T** is:

$$(\eta^* - c_i) \cdot q_i^* + (\mu_i^* - s_i) \cdot u_i^*.$$

The unique dual-optimal solution gives $\mu_2^* = 1000$, indicating that generator 2 receives the same supplemental payment as with the make-whole mechanism. On the other hand, we have $\mu_1^* = -5000$, meaning that under **T** generator 1 foregoes all inframarginal rents and makes zero profit in net.

Indeed, it is straightforward to show that in a more general single-period electricity market with N generators, every generator receives exactly zero net profit under $\mathbf{T}$. To see this, we define the more general version of the augmented LP as:

$$
\begin{aligned}
\min_{q,u} \quad & \sum_{i=1}^{N} s_i \cdot u_i + c_i \cdot q_i && && \\
\text{s.t.} \quad & \sum_{i=1}^{N} q_i = D && && (\eta) \\
& q_i \leq K_i \cdot u_i && \forall\, i = 1, \ldots, N && (\lambda_i) \\
& u_i = u_i^* && \forall\, i = 1, \ldots, N && (\mu_i) \\
& q_i \geq 0 && \forall\, i = 1, \ldots, N, &&
\end{aligned}
$$

where K_i is generator i's capacity and D is the demand. The dual of this LP is:

$$
\begin{aligned}
\max_{\eta,\lambda,\mu} \quad & D \cdot \eta + \sum_i u_i^* \cdot \mu_i && \\
\text{s.t.} \quad & \eta + \lambda_i \leq c_i && \forall\, i = 1, \ldots, N \qquad (1) \\
& -K_i \cdot \lambda_i + \mu_i = s_i && \forall\, i = 1, \ldots, N \qquad (2) \\
& \lambda_i \leq 0 && \forall\, i = 1, \ldots, N.
\end{aligned}
$$

We know that any inframarginal generator, i, will have $(q_i^*, u_i^*) = (K_i, 1)$. Thus, the complementary slackness conditions and constraint ($1i$) imply that:

$$\lambda_i^* = c_i - \eta^*.$$

Substituting this into ($2i$) gives:

$$
\begin{aligned}
(\eta^* - c_i) \cdot K_i + \mu_i^* - s_i &= 0 \\
(\eta^* - c_i) \cdot q_i^* + (\mu_i^* - s_i) \cdot u_i^* &= 0,
\end{aligned}
$$

implying zero net profit. In the case of the marginal generator, j, we have that $0 \leq q_j^* \leq K_j$ and $u_i^* = 1$. Constraint ($2j$) implies that:

$$-K_j \cdot \lambda_j^* + \mu_j^* - s_j = 0,$$

and combining this with the non-positivity of λ_j gives:

$$-q_j^* \cdot \lambda_j^* + \mu_j^* - s_j \leq 0. \tag{3}$$

The complementary slackness conditions and constraint ($1j$) further imply that:

$$-\lambda_j^* \geq \eta^* - c_j. \tag{4}$$

Combining (3) and (4) gives:

$$(\eta^* - c_j) \cdot q_j^* + (\mu_j^* - s_j) \cdot u_j^* \leq 0.$$

Since these prices support the SO's solution and generator j could always choose to remain offline (earning zero profit in net), we know the prices must yield exactly zero profit. Finally, any generator, l, that is not committed receives zero profit, since $(q_l^*, u_l^*) = (0, 0)$.

3. Conclusion

This counterexample demonstrates that the pricing scheme used by most SOs that operate centrally committed markets can, under certain circumstances, provide generators and consumers with incentives to deviate from the SO's socially optimal solution. This is important to stress, since claims appear in the literature that can be construed as implying that the make-whole payments used by SOs are equivalent to $\mathbf{T}$. More specifically, there are four possible cases for the optimal values of u_i^* and μ_i^* to consider. If $u_i^* = 1$ and $\mu_i^* > 0$, then $\mathbf{T}$ and the make-whole mechanism provide the same incentives, since it is efficient for generator i to operate and a positive supplemental payment must be provided to guarantee this. If $u_i^* = 0$ and $\mu_i^* > 0$, $\mathbf{T}$ and the make-whole mechanism similarly provide the same incentives for generator i to remain offline. If $u_i^* = 0$ and $\mu_i^* < 0$, it is suboptimal for generator i to operate. $\mathbf{T}$ provides a negative supplemental payment to guarantee this, whereas the make-whole provision does not—implying that generator i may have different incentives under such a scheme. Finally, if $u_i^* = 1$ and $\mu_i^* < 0$, $\mathbf{T}$ imposes a 'penalty' on generator i for operating, which eliminates its inframarginal rents, whereas the make-whole provision does not.

This final case and our example suggests that $\mathbf{T}$ may have important implications for long-term capacity expansion. This is because restructured electricity markets may rely on spot market prices and associated inframarginal rents to signal the need for capacity to be added to the system. Indeed, one can explicitly model load curtailment as a 'generation technology' with zero startup cost and variable cost equal to a high value of lost load (as an example, Kariuki and Allan (1996) estimate a value of lost load of between \$4600/MWh and \$18500/MWh). The resulting high price of energy when generation capacity is exhausted and load is curtailed is intended to provide a strong signal for capacity investment. Our example demonstrates, however, that in such a case $\mathbf{T}$ would result in zero net profit for each generator, assuming no internal constraints other than simple capacity constraints. This is consistent with a goal of Scarf (1994) (noted by O'Neill et al. (2005)), which is to find a set of prices that yield zero profits for all activities in an optimal solution in the presence of non-convexities. It should be stressed, however, that our example is highly stylized. More complex settings with binding internal constraints (*e.g.*, ramp limits or stepped marginal generation costs) on the activities can result in positive generator profits. These examples, nevertheless, point to the need for further research on commodity pricing in markets with non-convexities and centrally committed electricity markets in particular. Herrero et al. (2014) provide a formative attempt to address these long-term investment issues.

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