Optimizing Transmission from Distant Wind Farms*

Sompop Pattanariyankool † and Lester B. Lave ‡

December 2009

* We thank Jay Apt for providing the wind farm data and comments. Ron Goettler gave useful comments.

† Corresponding author. PhD Candidate, Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue Pittsburgh PA 15213. Email: spattana@andrew.cmu.edu
Tel.: +1 412 268 4225; Fax: +1 412 268 7357

‡ Harry B. and James H. Higgins Professor of Economics and University Professor, Co-Director Carnegie Mellon Electricity Industry Center, Tepper School of Business, Carnegie Mellon University. Email: ll01@andrew.cmu.edu
Abstract

We explore the optimal size of the transmission line from distant wind farms, modeling the tradeoff between transmission cost and benefit from delivered wind power. We also examine the benefit of connecting a second wind farm, requiring additional transmission, in order to increase output smoothness. Since a wind farm has a low capacity factor, the transmission line would not be heavily loaded, on average; depending on the time profile of generation, for wind farms with capacity factor of 29-34%, profit is maximized for a line that is about ¾ of the nameplate capacity of the wind farm. Although wind generation is inexpensive at a good site, transmitting wind power over 1,000 miles (about the distance from Wyoming to Los Angeles) doubles the delivered cost of power. As the price for power rises, the optimal capacity of transmission increases. Connecting wind farms lowers delivered cost when the wind farms are close, despite the high correlation of output over time. Imposing a penalty for failing to deliver minimum contracted supply leads to connecting more distant wind farms.
1. Introduction

California and 29 other states have renewable portfolio standards (RPS) that will require importing electricity generated by wind from distant locations. A long transmission line increases cost significantly since its capacity factor is approximately the same as the wind farms it serves, unless storage or some fast ramping technology fills in the gaps left by wind generation. We explore issues surrounding importing wind electricity from distant wind farms, including: the delivered cost of power, considering both generation and transmission, the cost of the transmission line, when to pool the output of two wind farms to send over a single transmission line, and what additional distance would the owner be willing to go for a better wind site in order to minimize the cost of delivered power.

Wind energy is the cheapest available renewable at good wind sites. Since no fuel is required, generation cost depends largely on the investment in the wind farm and the wind characteristics (described by the capacity factor). Assuming $1,915/kW for the cost of the wind farm, annual operations and maintenance (O&M) costs of $11.50/kW-yr, variable O&M cost of $5.5/MWh, a blended capital cost of 10.4%, and a 20 year life time for the turbine, generation costs at the wind farm are around $76/MWh, $66/MWh, $59/MWh, $56/MWh, and $53/MWh, respectively, for the wind farms with capacity factors of 35%, 40%, 45%, 47.5%, and 50%, respectively.

Wind is the fastest growing renewable energy, adding 8,558 MW of capacity in 2008, 60% more than the amount added in 2007 (Wiser and Bolinger, 2009). Total wind capacity in 2008 was 25,369 MW, about 2.2% of U.S. total generation nameplate capacity.

Good wind sites (class 4-6 with average wind speed 7.0-8.0 m/s at 50 m height (AWEA, 2008)), accounting for 6% of the U.S. land, could supply 1.5 times current U.S.
electricity demand (DOE, 2007). However, transmission is a key barrier for wind power development, since good wind sites are generally remote from load centers (DOE, 2008a). The low capacity factor of a wind turbine, added to the remoteness of good wind sites, makes transmission a major cost component. Denholm and Sioshansi (2008) note that transmission costs can be lowered by operating the transmission line at capacity through storage or fast ramping generation at the wind farm. Whether this co-location lowers the delivered cost of electricity depends on the site characteristics and other factors.

Low utilization of a long transmission line could double the cost of delivered wind power, since a wind turbine’s capacity factor is only 20-50%. Using actual generation data from wind farms, we model the optimal capacity of a transmission line connecting a wind farm to a distant load. We show that the cost of delivered power is lowered by sizing the transmission line to less than the capacity of the wind farm.

We assume the wind farm is large enough to require its own transmission line without sharing the cost with another wind farm or load in a different location. While we know of no example of a wind farm building its own transmission, T. Boon Pickins proposed to do this. If 1,000 MW were to be sent 1,000 miles or more, available capacity in short, existing lines would not be helpful. We also extend the model to two wind farms, trading off the additional transmission needed to connect the farms against the less correlated output. The relationship between wind output correlation and distance is modeled using the wind data from UWIG (2007). Finally, we model the effect of charging wind farms for failing to provide the minimum supply requirement.
2. Model

2.1 One Wind Farm Model

In this model, a wind farm is large enough to require its own high voltage DC transmission line to the load. The project consists of the wind farm and transmission line. We formulate the model as the owner of the wind farm and transmission line seeking to maximize profit. However, in this case the objective function is equivalent to seeking to maximize social welfare, as explained below.

The general form of the objective function for optimizing the capacities of the transmission line is:

\[
MAX \sum_{0\leq i \leq 40} \sum_{j=1}^{N} \frac{p_{ji}}{(1+r)^i} \min[q_{ji}, sK] - aC(sK) - WC_1 - \frac{WC_2}{(1+r)^{20}}
\]

- \( K \) = capacity of the wind farms (MW)
- \( s \) = transmission capacity normalized by total capacity of the wind farm (called “transmission capacity factor”)
- \( a \) = length of the transmission line (mile)
- \( C(sK) \) = cost per mile of \( sK \) MW transmission line built in year 0
- \( i = i^{th} \) hour in a year
- \( j = j^{th} \) year (from 1\textsuperscript{st} - 40\textsuperscript{th})
- \( N \) = 8,760 hours in a year
- \( p_{ji} \) = the expected price of wind power ($/MWh) in year \( j \) at hour \( i \)
- \( q_{ji} \) = the expected delivered wind power (MWh) in year \( j \) at hour \( i \)
- \( r \) = the discount rate
- \( WC_1 \) = cost of the wind turbines built in year 0
- \( WC_2 \) = cost of the wind turbines built in year 20
The lifetimes of the transmission line and wind turbine are assumed to be 40 and 20 years respectively. Thus, the turbines must be replaced in year 20. We also assume that construction is instantaneous for both transmission and turbines.

Transmission investment has economies of scale over the relevant range; the cost per MW decreases as capacity of the line increases (Weiss and Spiewak, 1999). Line capacity is defined as the “thermal capacity” in megawatts (MW) (Baldick and Kahn (1993)). The transmission line cost is $C(q)$ per mile, where $q$ is the capacity of the line. $C(q)$ is increasing and concave, $C'(q) \geq 0$ and $C''(q) \leq 0$.

We assume no line loss (delivered power equals the injected amount) and the wind distribution (output) is the same in all years.

According to Barradale (2008), about 76% of wind power is purchased via a long term power purchasing agreement (PPA) that specifies a fixed price or price adjusted by inflation. Electricity price paid to the wind farm is assumed to be constant over time and unrelated to the quantity of wind power supplied. Thus, whether the owner seeks to maximize profit or a public authority seeks to maximize social welfare, the goal is to maximize the benefit of delivered wind power by optimizing the size of the transmission line.

Given these assumptions, the variables $q_i$ and $P$ are used instead of $q_{ij}$ and $p_{ij}$ to represent the constant annual output and fixed price. The optimization problem is simplified as follow.

\[
MAX_{0 \leq s \leq 1} \ NPV = P\left(\sum_{j=1}^{40} \frac{1}{(1+r)^j}\right) \sum_{i=1}^{N} \left(\min[q_i, sK]\right) - aC(sK) - WC_1 - \frac{WC_2}{(1+r)^{20}}
\]

Let $\sum_{j=1}^{40} \frac{1}{(1+r)^j} = \beta$, $\sum_{i=1}^{N} \left(\min[q_i, sK]\right) = Q(s, K)$ and $WC_1 + \frac{WC_2}{(1+r)^{20}} = WC$.

The above objective function can be written as;

\[
MAX_{0 \leq s \leq 1} \ NPV = \beta P Q(s, K) - aC(sK) - WC
\]
The optimal transmission capacity is determined by the tradeoff between the incremental revenue from delivering additional electricity and the incremental cost of the capacity increase.

The optimization problems is solved numerically by using the search algorithm to find the maximum point over the range of feasible transmission capacity; $0 \leq s \leq 1$.

2.2 Two Wind Farms Model

The model with two wind farms assumes a branch line of “$b$” miles connecting farm 2 to farm 1, which is connected to the customer with a main line of “$a$” miles. If the two farms are so distant that it is cheaper to connect each to the customer, the previous model applies.

![Network topology](image)

Figure 1: Simplified network topology of the model with 2 wind farms

This model explores the effect of output correlation on the optimal transmission capacity. The basic one-farm model assumptions are retained and the two farms have the same capacity. ($K$ MW). The investor chooses the optimal size of both transmission lines to maximize profit. The objective function is:

$$
MAX_{0 \leq s_2, \ s_1 \leq 1} {NPV} = \beta \sum_{i=1}^{N} q_i (s_1, s_2, K) - aC(s_1, K) - bC(s_2, K) - WC_{11} - WC_{12} - \frac{WC_{21} + WC_{22}}{(1 + r)^20}
$$

$s_1$ = the transmission capacity factor (main line)

$s_2$ = the transmission capacity factor (branch line)

$aC(s_1, K)$ = cost of $a$ miles main transmission line capacity $s_1 K$ MW built in year 0
\( bC(s_2, K) \) = cost of \( b \) miles branch transmission line capacity \( s_2K \) MW built in year 0

\( WC_{11} \) and \( WC_{12} \) = cost of 1\textsuperscript{st} and 2\textsuperscript{nd} wind farms built in year 0

\( WC_{21} \) and \( WC_{22} \) = cost of 1\textsuperscript{st} and 2\textsuperscript{nd} wind farms built in year 20

\( q_i(s_1, s_2, K) \) = the expected delivered wind power at hour \( i \) from both wind farms

Note that \( q_i(s_1, s_2, K) = q_i(s_1, K) + q_{2i}(s_2, K) \) where \( q_i(s_1, s_2, K) \leq s_1K \).\( q_i(s_1, K) \) is the power generated by the 1\textsuperscript{st} farm. \( q_{2i}(s_2, K) \) is the delivered power from the 2\textsuperscript{nd} farm such that \( q_{2i}(s_2, K) \leq s_2K \).

Let \( \sum_{i=1}^{N} q_i(s_1, s_2, K) = Q(s_1, s_2, K) \) and \( WC_{11} + WC_{12} + \frac{WC_{21} + WC_{22}}{(1+r)^{20}} = WC \). The objective function can be formulated as;

\[
\text{MAX}_{s_1, s_2} \ NPV = \beta BPQ(s_1, s_2, K) - aC(s_1, K) - bC(s_2, K) - WC
\]

The optimization problem is solved numerically by evaluating the objective function over a two-dimensional grid of \( s_1 \) and \( s_2 \) values.

3. Data and variables

1. Wind data

We use hourly wind power generation data from four Northeastern U.S. wind farms covering January-June and assume the July-December data are similar. The data are shown in figure 1. The data were normalized so that the maximum output (the nameplate capacity) was equal to 1. Descriptive statistics and output correlation & distance between farms are shown in the tables below.
Figure 2: Hourly distribution of wind power

Table 1: Descriptive statistics of the wind farm output

<table>
<thead>
<tr>
<th>Wind farm</th>
<th>Capacity factor (%)</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>32.73</td>
<td>0.0840</td>
</tr>
<tr>
<td>B</td>
<td>34.73</td>
<td>0.0871</td>
</tr>
<tr>
<td>C</td>
<td>29.92</td>
<td>0.0821</td>
</tr>
<tr>
<td>D</td>
<td>29.77</td>
<td>0.0738</td>
</tr>
</tbody>
</table>

Table 2: Output correlation and distance between farms (mile)

<table>
<thead>
<tr>
<th>Farm</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>0.77</td>
<td>0.69</td>
<td>0.35</td>
</tr>
<tr>
<td>B</td>
<td>56</td>
<td>1.0</td>
<td>0.71</td>
<td>0.46</td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>63</td>
<td>1.0</td>
<td>0.36</td>
</tr>
<tr>
<td>D</td>
<td>219</td>
<td>250</td>
<td>200</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: correlations are shown on and above the diagonal and distances are shown below the diagonal.

Table 2: Output correlation and distance between farms (mile)

2. Financial variable

The discount rate in this model is 10.4% (20% equity at 20% and 80% debt at 8%).
3. Transmission cost data and estimation

The cost of a transmission line varies with distance and terrain, but the greatest uncertainty concerns regulatory delay and the cost of acquiring the land. To reflect this uncertainty, we perform a sensitivity analysis with cost varying between 20% and 180% of the base cost in the next section. We use DOE (2002) data to estimate the transmission line cost function. The data are adjusted to reflect the current cost of DC transmission construction\(^1\).

The functional form of the cost function is; \(\text{cost per mile} = e^{a MW^\beta} \). The coefficient \(\beta\) indicates how much cost increases as the line capacity increases by 1%; elasticity with respect to line capacity. By using a log-log transformation, the transmission line cost function is estimated as a log-linear function of transmission capacity \((MW)\). The transmission line cost, as a function of capacity is estimated using ordinary least square (OLS); see Appendix A. As expected, the estimated result displays economies of scale of transmission line investment.

\[
\text{cost per mile} = e^{0.55415 MW^{0.5759}}
\]

The regression equation has \(R^2 = 0.94\) and all parameters are statistically significant. The t-statistics for the constant and the parameter of \(MW\) are 35.31 and 10.24 respectively.

4. Wind turbine cost

Costs of new wind turbines have risen and then fallen in the past few years; we use $1,915/kW as the installed cost of a wind farm (Wiser and Bolinger, 2009), $11.5/KW-year fixed O&M cost and $5.5/MWh variable O&M cost (Wind Deployment System (WinDS) model (DOE, 2008b)).

\(^1\) The reported cost for high voltage transmission line covers a wide range (ISO-NE, 2007). If the cost of the transmission line were half or twice the cost we assume, the cost of the transmission would be halved or doubled assuming the capacity of the line is fixed.
5. Electricity price

The electricity price in this study is the real hourly electricity price paid to the owner of the wind farm and transmission. Since the wind farm operator has little control over when the turbines generate electricity, we assume that she receives the average price for the year for each MWh. We assume that the delivered price paid to the wind farm investor is $160/MWh included all federal and state subsidies. In addition, for simplicity, we assume the electricity price over the next forty years is constant, after adjusting for inflation.

4. Results

4.1 Results for One Wind Farm

We focus on delivering wind power over a distance of 1,000 miles, about the distance from Wyoming to Los Angeles; California’s renewable portfolio standard will require large amounts of wind energy from distant sites. For a 1,000 mile long transmission line, the optimal transmission capacity, utilization rate, profit and delivered output for the four wind farms are shown in Appendix C, Table C1. The optimal capacity is 74 - 79% of the wind farms’ capacity. As expected, among the four wind farms, those with higher capacity factors have higher optimal transmission capacity, profit and delivered output, with lower delivered power cost.

![Figure 3: Transmission capacity and delivered output](image-url)
Figure 3 shows the relationship between the capacity of the transmission line and the delivered wind power of Farm A. The slope of the curve represents the marginal benefit of transmission capacity. As transmission capacity increases, marginal benefit decreases since the turbine’s output is at full capacity for only a few hours per year. Farm A’s optimal transmission capacity is 79% of the farm’s capacity, but the transmission line delivers 97% of the wind power generated. Adding 21 percentage points to transmission capacity increases delivered output by only 3 percentage points.

Figure 4: Price vs. transmission capacity factor (s)

Figure 4 shows the relationship between price and transmission capacity (s) of farm A derived from the first order condition. The first order condition shows the optimal capacity decision, even when profit is negative, although the investor would not build the wind farm and transmission for a negative return. At price below $55/MWh, the optimal capacity is zero. Optimal transmission capacity and delivered power rise rapidly as price goes from $55 to 200/MWh due to economies of scale in transmission investment and the initial high marginal benefit of the transmission line (as seen in Figure 3). The scale economies are essentially exhausted and virtually almost all of the generated power is being delivered by the time a $300 price is reached; higher prices would increase transmission capacity little.
As price rises, the value of the delivered electricity rises, increasing the value of transmission capacity; almost all of the generated power is being delivered by the time a $300 price is reached. The supply curve is shown in Figure 5.

The delivered cost of wind power (transmission cost included) ranges from $144 to 169/MWh for these 4 wind farms. Since the generation cost is $78 to 92/MWh, transmission is 44-46% of the total cost; see Appendix C, Table C1. Note that the price paid to the investor is $160/MWh.

![Figure 5: The supply curve](image)

The output distribution of a wind farm also affects the optimal transmission capacity. In order to test the effect of the distribution, all 4 wind farms’ output data are modified to have a 50% capacity factor. As shown in Appendix C Table C2, the optimal transmission capacity, profit and delivered output of farms A, B and C are about the same. However, Farm D has the lowest transmission capacity and the highest profit, since it has less output distributed in the 80-100% of nameplate capacity range. As a result, farm D faces less trade-off between transmission capacity and loss of high level output. In

---

2 Consider two wind farms with 30% capacity factor, if the turbine produced at 100% of capacity 30% of the time and zero capacity the rest, the optimal transmission capacity would be 100%; if it produced at 30% of capacity 100% of the time, the optimal transmission capacity would be 30%.
addition, the farm with lower output standard deviation tends to have lower optimal transmission capacity.

<table>
<thead>
<tr>
<th></th>
<th>Transmission capacity factor (s)</th>
<th>Transmission cost $</th>
<th>Additional transmission cost $</th>
<th>Additional revenue $</th>
<th>Decrease in profit $</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.7880</td>
<td>1.785 x 10⁹</td>
<td>2.626 x 10⁸</td>
<td>1.248 x 10⁸</td>
<td>1.378 x 10⁸</td>
</tr>
<tr>
<td>B</td>
<td>0.8075</td>
<td>1.810 x 10⁹</td>
<td>2.373 x 10⁸</td>
<td>0.946 x 10⁸</td>
<td>1.427 x 10⁸</td>
</tr>
<tr>
<td>C</td>
<td>0.7747</td>
<td>1.767 x 10⁹</td>
<td>2.799 x 10⁸</td>
<td>1.534 x 10⁸</td>
<td>1.265 x 10⁸</td>
</tr>
<tr>
<td>D</td>
<td>0.7388</td>
<td>1.720 x 10⁹</td>
<td>3.276 x 10⁸</td>
<td>1.049 x 10⁸</td>
<td>2.227 x 10⁸</td>
</tr>
</tbody>
</table>

Table 3: Implication of increasing transmission to 100% of wind farm capacity

Table 3 shows profit reduction in present value term when the transmission line is expanded from the profit maximizing capacity to the wind farm’s nameplate capacity. Profit reduction is calculated as the difference between the additional cost of building the line at full capacity and revenue from additional delivered wind power (in present value); expanding the line to full capacity costs $127 to $223 millions. Although it may seem wasteful to spill some of the power generated by the wind farm, beyond the optimal transmission capacity, the incremental cost of increasing transmission capacity is greater than the value of the additional power delivered. For example, building a line at full capacity for Farm A increases the cost of the line by 15% while delivering only 3% of additional power.

The best wind sites are distant from load and so there is a tradeoff between lower generation cost and lower transmission cost. Figure 6 shows the delivered cost of power from wind farms with capacity factors of 35% and 50% as a function of the distance of the wind farm from load. For a 50% capacity factor wind farm, the delivered cost of power doubles when it is just over 1,063 miles away. For a 35% capacity wind farm, the delivered cost of power is doubled when the wind farm is just under 1,000 miles away.
A more interesting interpretation of the figure is to see how much further you would be willing to go to get power from a 50% capacity factor wind farm compared to a 35% capacity factor wind farm. At a cost of delivered power of $100/MWh, a 35% capacity wind farm could be 300 miles away while a 50% capacity wind farm could be 1,000 miles away. Thus, if a 35% capacity factor wind farm were located 300 miles away, the customer would be willing to go up to an additional 700 miles. For any delivered cost of electricity, the horizontal difference between the two lines is the additional distance a customer would be willing to go to get to a wind farm with capacity factor 50% rather than 35%. Since much of the USA has a minimally acceptable wind site within 300 miles, they would not find it attractive to go to the best continental wind sites in the upper Midwest.

To ensure reliability, power systems must satisfy an N – 1 criterion. The variability of wind output puts an additional burden on the generation system. The cheapest way of meeting the N – 1 criterion is by using spinning reserve; this reserve can also ramp up and down to fill the gaps in wind generation. As reported by CAISO (2008), the total cost of ancillary services per MWh in 2008 (monthly average) is from $0.42-1.92/MWh. This
cost includes spinning reserve, non spinning reserve and regulation. The cost of spinning reserve alone is about $0.15-0.67$/MWh. Thus, the cost of purchasing spinning reserve would add less than 1% to the cost of delivered power.

**Sensitivity analysis**

We perform 4 sensitivity analyses: transmission cost vs. optimal capacity, the optimal transmission capacity vs. length of the line, transmission capacity vs. the discount rate, and profit vs. the discount rate. Other parameters are assumed to stay at former levels.

**Farm A: 1,000 miles at price $160/MWh**

![Figure 7: Transmission cost and optimal line capacity of farm A](image)

**Transmission cost and optimal capacity**

The optimal transmission capacity is solved for transmission cost varying plus or minus 80% of the base line. When the transmission line costs 80% less than the base case, the optimal size of the transmission line is 96% of the wind farm capacity and the cost of delivered power is $96/MWh. When the transmission line costs 80% more than the base case, the optimal capacity of the transmission line is 62% of the wind farm capacity and the cost of delivered power is $210/MWh. The base case values have the transmission line at 79% of the capacity of the wind farm with a delivered cost of $149/MWh.
**Figure 8: Optimal transmission capacity and transmission length of farm A**

**Transmission capacity and length:** Transmission cost increases with the length of the line. Figure 7 can be interpreted as showing the effects of shortening the line to 200 miles or lengthening it to 1,800 miles. As transmission cost increases, the optimal capacity of the transmission line relative to the wind farm capacity decreases for a given power price, as shown in Figure 8. A longer transmission line results in lower delivered output.

**Figure 9: Profit and discount rate of farm A**

**Profit and a discount rate:** Profit steadily decreases as the discount rate increases. As shown in Figure 9, the IRR (Internal Rate of Return) of this project is around 14% at a $160/MWh price (the discount rate giving zero NPV).
Transmission capacity and discount rate: Increasing the cost of capital (equity and loans) increases the cost of the transmission line. Figure 10 shows that the capacity of the line declines as the discount rate increases.

4.2 Results for Two Wind Farms

The data from 4 wind farms is used to maximize profit when two wind farms share the same central transmission line. By bundling 2 wind farms, the capacity of the main transmission line is almost double compared with the one wind farm case and so the transmission line can take advantage of economies of scale at this level (see Appendix C Table C3 for detail). The correlation between outputs of wind farms generally decreases as the farms are more distant. Here we investigate connecting wind farms that are more distant, trading off the cost of the additional transmission against the lower correlation of output. We examine each of the 12 possible pairs. Note that the pair AB means that the main transmission line goes to A, with a secondary line to B. The results from AB and BA are similar. The model is solved under 3 scenarios.

- Scenario 1: \( a = 1,000 \) miles and \( b = \) the actual distance between farms
Scenario 2: $a = 1,000$ miles and $b =$ the distance calculated from the estimated relationship between correlation and distance (Appendix B). Farm A is paired with a fictitious wind farm whose capacity factor is the same as A where the correlation between the outputs of the two wind farms is calculated from the estimated correlation-distance relationship.

Scenario 3: This scenario has the same configuration as scenario 2 but imposes a penalty per MWh when the delivered power from the wind farms falls below a minimum requirement level.

In scenario 1, the total cost of wind power (both generation and transmission cost included) ranges from $134$ to $153$/MWh, taking advantage of the economies of scale in transmission. The cost of generation ranges from $80$ to $92$/MWh, approximately the same as the one wind farm model. Transmission cost still accounts for more than one third of the delivered wind power cost.

When the second wind farm is close to the first, the output from the two wind farms are highly correlated. The second wind farm would help lower the delivered cost of electricity through economies of scale of transmission but this cost saving must be traded off against the length of the connecting transmission line.

In addition, when the length of the second transmission line is shorter, the capacity of the line ($s_2$) is higher. A shorter line translates to lower cost, which makes a slightly higher capacity more profitable. In addition, like the one wind farm model, capacity factor is the key factor that determines profit from the project.

Given the correlation-distance relationship, in scenario 2, we vary the correlation over the relevant range, calculate the implied distance, and then optimize the capacity of the transmission line to maximize profit. Farm A is paired with a wind farm of the same capacity, but we vary the distance (and thus the output correlation) between the two farms.
The simulated data used in scenario 2 are random numbers generated with the specific correlation with farm A and have capacity factor 30%.

While the correlation of output from two wind farms decreases with the distance between them, the correlation also varies with terrain and wind direction. The pair of wind farms with lower correlation tends to have higher utilization rate of the main transmission line. This can be considered as the effect of output smoothing by aggregating wind farms with low output correlation. The transmission line is used more efficiently when output is smoother (see Appendix C Table C4 for detail). If the system needs the smoother wind power output, more money is needed to invest in longer transmission.

Lower output correlation implies lower transmission capacity and higher transmission (main line) utilization rate. Without a price premium for smoothed output, the shorter distance between farms is more profitable than a low correlation. Thus, for this distance-correlation relationship, investors would want to build wind farms close together, despite the high correlation of their outputs. Thus, the optimal distance between wind farms is zero, as long as the second farm has the same capacity factor as the first.

Figure 11: Profit at penalty $160/MWh with minimum delivery requirement 400 MW
Scenario 3 analyzes the effect of imposing a minimum output requirement of 400 MW (20% of the total nameplate capacity) by the buyer. If the wind farm cannot fulfill this requirement, it has to buy power from other generators or pay the buyer the financial penalty. This cost is defined as an imbalance price. In addition, this imbalance price is assumed to be higher than or equal to the price paid to the wind farm.

As expected, the pair with lower correlation has lower imbalance output. As shown in Appendix C Table C5, the imbalance output (the amount in MWh that cannot meet the requirement) increases steadily as the correlation between wind farms’ output increases. In addition, the result from this scenario shows the different investment decision from scenario 2. In scenario 2 without the minimum output requirement penalty, the wind farm projects with high output correlation and short transmission line are more profitable. Imposing the minimum requirement increases the optimal distance between wind farms, resulting in an optimal output correlation in the range 0.4 – 0.6, as shown in Figure 11.

5. Conclusion

This analysis illustrates the complications with deciding where to site wind farms, trading off the lower cost of better wind potential against transmission cost, how large a transmission line to build, and where to locate a second wind farm if it is to be connected to the load with the same transmission line. The results are based on actual data with some extensions. The wind farm capacity factors, costs of transmission, and correlation among wind farms are unique to each location; an analysis of a specific location could be optimized using the methods presented here.

Since a 1,000 mile transmission line roughly doubles the delivered cost of power, decreasing the variability of generation at the wind farm lowers power costs. However, two connected wind farms only raise the capacity factor slightly. Imposing a minimum
requirement penalty leads to changes that increase firm output, although raising the
generation cost.

For a wind farm with a 1,000 mile transmission line, about the distance from
Wyoming to Southern California, the intermittency and low capacity factor of wind farms
increases the cost of transmission significantly. We find that the delivered cost of power
and optimal capacity of the transmission line increase with the price paid for the power,
and decrease with the wind farm’s capacity factor, the distance from load, and the
discount rate.

For a delivered price of $160/MWh, the optimal capacity of the transmission line
is 74-79%; only 3% of generated power is wasted for this transmission line. When two
wind farms are bundled, economies of scale in transmission increase the optimal capacity
of the transmission line and lower the cost of delivered power. When we examine the
distance between wind farms, trading off lower output correlation with greater distance
between farms, we find that closer farms have the lowest cost of delivered power.
However, when there is a penalty for failing to deliver a minimum amount of power, the
distant wind farms become more profitable.
Appendix A: Transmission cost function estimation

The data used for transmission cost function estimation are from DOE (2002) which is the cost data from 1995 study. The data are adjusted by the factor of 3. We need to adjust the data that makes the approximation close to the current cost range of transmission line. The factor of 3 gives the estimated transmission within the range of the observed transmission project cost. The transmission cost data is the limitation in this model. The cost data at different thermal capacity from the same source is necessary for estimating the cost function. In the future, when more appropriate cost data is available, the approach here can be applied. In addition, the actual cost of the DC line, if available, is an alternative for the study.

\[
\ln(\text{cost}) = 10.55415 + 0.5759 \ln(MW)
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>10.55415</td>
<td>0.298873</td>
<td>35.31313</td>
<td>0.0000</td>
</tr>
<tr>
<td>\ln(MW)</td>
<td>0.575873</td>
<td>0.056237</td>
<td>10.24006</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.937421     F-statistic 104.8589
Adjusted R-squared 0.928481     Prob(F-statistic) 0.000018
S.E. of regression 0.194477     Log likelihood 3.097459
Sum squared resid 0.264748     Durbin-Watson stat 1.814451

Table 1: Transmission cost function estimation result

![Graph showing the estimated transmission cost vs. actual cost (log-linear)](image_url)
Appendix B: Relationship between distance and wind-output correlation

We use the wind speed data from 9 wind speed observation sites in Colorado (UWIG, 2007) to estimate the relationship between distance and correlation. There are 3,909 hourly wind speed observations used for correlation coefficient estimation.

According to Manwell et al (2002), wind power \( P \) per area \( A \) is the function of the wind speed \( V \) and air density \( \rho \).

\[
\frac{P}{A} = \frac{1}{2} \rho V^3
\]

Note that we used the same data source as DOE (2005) but we set some wind speed observations that are lower than the cut-in speed or higher than the cut-out speed to be 0. The cut-in speed and the cut-out speed \(^3\) of the wind turbine is 4.5 m/s and 30 m/s respectively (Gipe, 2004). We calculate the correlations coefficients of the cubic wind speed \( V^3 \) among the wind speed observation sites. Given that other variables in the formula \( A \) and \( \rho \) held constant, the correlation coefficients calculated \( V^3 \) are the estimated correlation coefficients of wind power among the wind sites.

Various models of distance and correlation are estimated including linear, quadratic and linear-log (correlation is a function of \( \ln(distance) \)). Ordinary Least Square (OLS) is used for the estimation. The linear-log model used by DOE (2005) is more suitable than the linear and quadratic models.

Note that in Figure B1 (right) some observations are deviate far from the estimated line, for example, close wind stations with low correlation. This could be due to terrain such as a ridge between the nearby locations.

---

\(^3\) From Gipe (2004), cut-in wind speed is the wind speed that a wind turbine starts to generate power. The wind turbine cannot generate power if the wind speed is lower than the cut-in level. Cut-out wind speed is the wind speed at which the wind turbine stops generating electricity in order to protect the equipment from an excessive wind speed. The wind turbine cannot generate power if the wind speed is higher than the cut-out level.
The linear-log model, \( \text{correlation} = a + b \ln(\text{distance}) \), seems best for this study.

The shape of the curve is similar to the curve from NREL (2007).

\[ \text{Correlation} = 1.557018 - 0.231544 \ln(\text{distance}) \]

![Figure B1: Linear and quadratic models (left) and linear-log model (right)](image)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>1.557018</td>
<td>0.145054</td>
<td>10.73408</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \ln(\text{distance}) )</td>
<td>-0.231544</td>
<td>0.029868</td>
<td>-7.752362</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.638679</td>
<td></td>
<td></td>
<td>60.0991</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.628052</td>
<td>Prob(F-statistic)</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.123475</td>
<td>Durbin-Watson stat</td>
<td>1.446861</td>
<td></td>
</tr>
<tr>
<td>Sum squared residual</td>
<td>0.518366</td>
<td>Log likelihood</td>
<td>25.24887</td>
<td></td>
</tr>
</tbody>
</table>

Table B1: Correlation and distance estimation result
Appendix C: Computation results

<table>
<thead>
<tr>
<th>Farm</th>
<th>Capacity factor</th>
<th>Trans. factor (s)</th>
<th>Trans. utilization</th>
<th>Profit</th>
<th>Cost per MWh ($M)</th>
<th>Cost per MWh (turbine)</th>
<th>Delivery (MWh)</th>
<th>Delivery/Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>32.73%</td>
<td>0.7880</td>
<td>40.22%</td>
<td>1.75 x 10^8</td>
<td>153.31</td>
<td>82.24</td>
<td>1.11 x 10^8</td>
<td>97.10%</td>
</tr>
<tr>
<td>B</td>
<td>34.73%</td>
<td>0.8075</td>
<td>42.00%</td>
<td>4.43 x 10^8</td>
<td>144.20</td>
<td>77.78</td>
<td>1.19 x 10^8</td>
<td>97.93%</td>
</tr>
<tr>
<td>C</td>
<td>29.92%</td>
<td>0.7747</td>
<td>37.02%</td>
<td>-2.05 x 10^8</td>
<td>168.68</td>
<td>91.99</td>
<td>1.01 x 10^8</td>
<td>96.11%</td>
</tr>
<tr>
<td>D</td>
<td>29.77%</td>
<td>0.7388</td>
<td>39.11%</td>
<td>-1.28 x 10^8</td>
<td>165.35</td>
<td>91.31</td>
<td>1.01 x 10^8</td>
<td>97.32%</td>
</tr>
</tbody>
</table>

Table C1: 1,000 mile transmission line at price $160/MWh

<table>
<thead>
<tr>
<th>Farm</th>
<th>Standard deviation</th>
<th>Trans. factor (s)</th>
<th>Trans. utilization</th>
<th>Profit</th>
<th>Cost per MWh ($M)</th>
<th>Cost per MWh (turbine)</th>
<th>Delivery (MWh)</th>
<th>Delivery/Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2472</td>
<td>0.9135</td>
<td>54.39%</td>
<td>3.39 x 10^8</td>
<td>77.45</td>
<td>53.10</td>
<td>1.74 x 10^8</td>
<td>99.64%</td>
</tr>
<tr>
<td>B</td>
<td>0.2594</td>
<td>0.9117</td>
<td>54.56%</td>
<td>3.40 x 10^8</td>
<td>77.33</td>
<td>53.04</td>
<td>1.74 x 10^8</td>
<td>99.77%</td>
</tr>
<tr>
<td>C</td>
<td>0.2337</td>
<td>0.9212</td>
<td>53.95%</td>
<td>3.39 x 10^8</td>
<td>77.55</td>
<td>53.09</td>
<td>1.74 x 10^8</td>
<td>99.66%</td>
</tr>
<tr>
<td>D</td>
<td>0.2231</td>
<td>0.8817</td>
<td>56.44%</td>
<td>3.42 x 10^8</td>
<td>76.84</td>
<td>53.02</td>
<td>1.74 x 10^8</td>
<td>99.79%</td>
</tr>
</tbody>
</table>

Table C2: 1,000 mile transmission line with 50% adjusted capacity factor farm at price $160/MWh
Table C3: 1,000 mile main transmission line and actual distance between farms at price $160/MWh

<table>
<thead>
<tr>
<th>Pair</th>
<th>Corr. (mile)</th>
<th>Trans (s₁) (utilization,%)</th>
<th>Trans (s₂)</th>
<th>Profit</th>
<th>Cost per MWh (turbine)</th>
<th>Cost per MWh (turbine)</th>
<th>Delivery (MWh)</th>
<th>Delivery/Generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.7665 (56)</td>
<td>1.6330 (40.61)</td>
<td>0.8952</td>
<td>1.50 x 10⁹</td>
<td>132.69</td>
<td>79.57</td>
<td>2.32 x 10⁸</td>
<td>98.78 %</td>
</tr>
<tr>
<td>BA</td>
<td>1.6326 (40.61)</td>
<td>0.8988</td>
<td>1.50 x 10⁹</td>
<td>132.71</td>
<td>79.58</td>
<td>2.32 x 10⁸</td>
<td>98.85 %</td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>0.6919 (19)</td>
<td>1.5586 (39.22)</td>
<td>0.9864</td>
<td>0.95 x 10⁹</td>
<td>141.08</td>
<td>86.33</td>
<td>2.14 x 10⁸</td>
<td>97.83 %</td>
</tr>
<tr>
<td>CA</td>
<td>1.5587 (41.32)</td>
<td>0.9925</td>
<td>0.95 x 10⁹</td>
<td>141.09</td>
<td>86.33</td>
<td>2.14 x 10⁸</td>
<td>97.83 %</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>0.3471 (219)</td>
<td>1.4013 (43.32)</td>
<td>0.8124</td>
<td>0.69 x 10⁹</td>
<td>146.18</td>
<td>86.93</td>
<td>2.13 x 10⁸</td>
<td>97.74 %</td>
</tr>
<tr>
<td>DA</td>
<td>1.3984 (43.32)</td>
<td>0.8829</td>
<td>0.69 x 10⁹</td>
<td>146.65</td>
<td>87.01</td>
<td>2.13 x 10⁸</td>
<td>97.71 %</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>0.7074 (63)</td>
<td>1.5641 (40.36)</td>
<td>0.9382</td>
<td>1.12 x 10⁹</td>
<td>138.40</td>
<td>83.59</td>
<td>2.11 x 10⁸</td>
<td>98.05 %</td>
</tr>
<tr>
<td>CB</td>
<td>1.5678 (40.29)</td>
<td>0.9196</td>
<td>1.13 x 10⁹</td>
<td>138.37</td>
<td>83.55</td>
<td>2.21 x 10⁸</td>
<td>98.06 %</td>
<td></td>
</tr>
<tr>
<td>BD</td>
<td>0.3552 (250)</td>
<td>1.4073 (44.50)</td>
<td>0.7929</td>
<td>0.89 x 10⁹</td>
<td>142.79</td>
<td>84.25</td>
<td>2.19 x 10⁸</td>
<td>97.88 %</td>
</tr>
<tr>
<td>DB</td>
<td>1.4141 (44.33)</td>
<td>0.8649</td>
<td>0.87 x 10⁹</td>
<td>143.27</td>
<td>84.18</td>
<td>2.20 x 10⁸</td>
<td>97.87 %</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>0.4572 (200)</td>
<td>1.4159 (40.80)</td>
<td>0.8038</td>
<td>0.33 x 10⁹</td>
<td>153.12</td>
<td>91.34</td>
<td>2.02 x 10⁸</td>
<td>97.49 %</td>
</tr>
<tr>
<td>DC</td>
<td>1.3981 (41.21)</td>
<td>0.8980</td>
<td>0.30 x 10⁹</td>
<td>153.66</td>
<td>91.59</td>
<td>2.02 x 10⁸</td>
<td>97.21 %</td>
<td></td>
</tr>
<tr>
<td>Pair</td>
<td>Corr. (mile)</td>
<td>Trans ($s_1$) (utilization, %)</td>
<td>Trans ($s_2$)</td>
<td>Profit</td>
<td>Cost per MWh</td>
<td>Cost per MWh (turbine)</td>
<td>Delivery (MWh)</td>
<td>Delivery/Generation</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
<td>-------------------------------</td>
<td>---------------</td>
<td>--------</td>
<td>--------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>A, A30</td>
<td>0.30 (227)</td>
<td>1.1865 (51.70)</td>
<td>0.5799</td>
<td>1.07 x 10⁹</td>
<td>138.81</td>
<td>86.03</td>
<td>2.15 x 10⁸</td>
<td>98.28 %</td>
</tr>
<tr>
<td>A, A35</td>
<td>0.35 (184)</td>
<td>1.1948 (51.13)</td>
<td>0.5769</td>
<td>1.10 x 10⁹</td>
<td>138.25</td>
<td>86.38</td>
<td>2.14 x 10⁸</td>
<td>98.26 %</td>
</tr>
<tr>
<td>A, A40</td>
<td>0.40 (147)</td>
<td>1.2105 (50.64)</td>
<td>0.5771</td>
<td>1.16 x 10⁹</td>
<td>137.07</td>
<td>86.07</td>
<td>2.15 x 10⁸</td>
<td>98.36 %</td>
</tr>
<tr>
<td>A, A45</td>
<td>0.45 (119)</td>
<td>1.2180 (50.34)</td>
<td>0.5980</td>
<td>1.20 x 10⁹</td>
<td>136.37</td>
<td>86.07</td>
<td>2.15 x 10⁸</td>
<td>98.32 %</td>
</tr>
<tr>
<td>A, A50</td>
<td>0.50 (96)</td>
<td>1.2493 (49.53)</td>
<td>0.6459</td>
<td>1.27 x 10⁹</td>
<td>135.23</td>
<td>85.27</td>
<td>2.17 x 10⁸</td>
<td>98.28 %</td>
</tr>
<tr>
<td>A, A55</td>
<td>0.55 (77)</td>
<td>1.2503 (49.13)</td>
<td>0.6186</td>
<td>1.25 x 10⁹</td>
<td>135.57</td>
<td>85.90</td>
<td>2.15 x 10⁸</td>
<td>98.43 %</td>
</tr>
<tr>
<td>A, A60</td>
<td>0.60 (62)</td>
<td>1.2585 (48.75)</td>
<td>0.6224</td>
<td>1.24 x 10⁹</td>
<td>135.45</td>
<td>86.00</td>
<td>2.15 x 10⁸</td>
<td>98.35 %</td>
</tr>
<tr>
<td>A, A65</td>
<td>0.65 (50)</td>
<td>1.2716 (48.22)</td>
<td>0.6382</td>
<td>124 x 10⁹</td>
<td>135.47</td>
<td>86.06</td>
<td>2.15 x 10⁸</td>
<td>98.30 %</td>
</tr>
<tr>
<td>A, A70</td>
<td>0.70 (40)</td>
<td>1.2952 (47.49)</td>
<td>0.6323</td>
<td>1.26 x 10⁹</td>
<td>135.21</td>
<td>85.78</td>
<td>2.16 x 10⁸</td>
<td>98.44 %</td>
</tr>
<tr>
<td>A, A75</td>
<td>0.75 (33)</td>
<td>1.3240 (48.04)</td>
<td>0.6689</td>
<td>1.23 x 10⁹</td>
<td>135.82</td>
<td>85.89</td>
<td>2.15 x 10⁸</td>
<td>98.49 %</td>
</tr>
<tr>
<td>A, A80</td>
<td>0.80 (26)</td>
<td>1.3314 (46.07)</td>
<td>0.6769</td>
<td>1.22 x 10⁹</td>
<td>135.95</td>
<td>86.03</td>
<td>2.15 x 10⁸</td>
<td>98.32 %</td>
</tr>
</tbody>
</table>

Table C4: Farm A paired with farms at different correlation (capacity factor 30%) with 1,000 mile main transmission line and the distance between farms calculated from relationship in Appendix B at price = $160/MWh
<table>
<thead>
<tr>
<th>Pair</th>
<th>Corr. (mile)</th>
<th>Profit @ penalty $160/MWh</th>
<th>Profit @ penalty $180/MWh</th>
<th>Profit @ penalty $200/MWh</th>
<th>Imbalance (MWh)</th>
<th>Delivery (MWh)</th>
<th>Imbalance/Delivery (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, A30</td>
<td>0.30 (227)</td>
<td>$4.71 \times 10^8$</td>
<td>$3.95 \times 10^8$</td>
<td>$3.20 \times 10^8$</td>
<td>$1.60 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>7.44%</td>
</tr>
<tr>
<td>A, A35</td>
<td>0.35 (184)</td>
<td>$4.84 \times 10^8$</td>
<td>$4.08 \times 10^8$</td>
<td>$3.31 \times 10^8$</td>
<td>$1.63 \times 10^7$</td>
<td>$2.14 \times 10^8$</td>
<td>7.61%</td>
</tr>
<tr>
<td>A, A40</td>
<td>0.40 (147)</td>
<td>$5.28 \times 10^8$</td>
<td>$4.49 \times 10^8$</td>
<td>$3.70 \times 10^8$</td>
<td>$1.68 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>7.81%</td>
</tr>
<tr>
<td>A, A45</td>
<td>0.45 (119)</td>
<td>$5.44 \times 10^8$</td>
<td>$4.62 \times 10^8$</td>
<td>$3.81 \times 10^8$</td>
<td>$1.73 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>8.06%</td>
</tr>
<tr>
<td>A, A50</td>
<td>0.50 (96)</td>
<td>$5.89 \times 10^8$</td>
<td>$5.05 \times 10^8$</td>
<td>$4.20 \times 10^8$</td>
<td>$1.79 \times 10^7$</td>
<td>$2.17 \times 10^8$</td>
<td>8.28%</td>
</tr>
<tr>
<td>A, A55</td>
<td>0.55 (77)</td>
<td>$5.39 \times 10^8$</td>
<td>$4.52 \times 10^8$</td>
<td>$3.64 \times 10^8$</td>
<td>$1.86 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>8.63%</td>
</tr>
<tr>
<td>A, A60</td>
<td>0.60 (62)</td>
<td>$4.97 \times 10^8$</td>
<td>$4.04 \times 10^8$</td>
<td>$3.10 \times 10^8$</td>
<td>$1.98 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>9.21%</td>
</tr>
<tr>
<td>A, A65</td>
<td>0.65 (50)</td>
<td>$4.58 \times 10^8$</td>
<td>$3.60 \times 10^8$</td>
<td>$2.62 \times 10^8$</td>
<td>$2.08 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>9.68%</td>
</tr>
<tr>
<td>A, A70</td>
<td>0.70 (40)</td>
<td>$4.35 \times 10^8$</td>
<td>$3.52 \times 10^8$</td>
<td>$2.51 \times 10^8$</td>
<td>$2.14 \times 10^7$</td>
<td>$2.16 \times 10^8$</td>
<td>9.92%</td>
</tr>
<tr>
<td>A, A75</td>
<td>0.75 (33)</td>
<td>$3.50 \times 10^8$</td>
<td>$2.40 \times 10^8$</td>
<td>$1.30 \times 10^8$</td>
<td>$2.33 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>10.81%</td>
</tr>
<tr>
<td>A, A80</td>
<td>0.80 (26)</td>
<td>$3.09 \times 10^8$</td>
<td>$1.96 \times 10^8$</td>
<td>$0.83 \times 10^8$</td>
<td>$2.41 \times 10^7$</td>
<td>$2.15 \times 10^8$</td>
<td>11.20%</td>
</tr>
</tbody>
</table>

Table C5: Farm A paired with farms at different correlation (capacity factor 30%) with 1,000 mile main transmission line and the distance between farms calculated from relationship in Appendix B at price = $160/MWh with 400 MW delivery requirement
Reference


CAISO (2008), Monthly Market Performance Report (January - December 2008), California ISO.


