The Spectrum of Power from Utility-Scale Wind Farms and Solar Photovoltaic Arrays

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Abstract—**The power spectral density of the output of utility-scale wind farms and solar photovoltaic (PV) arrays is examined to provide information on the character of fluctuations in real power output; the power spectrum constrains the character of fill-in power. Both one second and one hour samples from several wind farms and ten second and one minute resolution data from four solar PV arrays are analyzed. The measured output power for wind follows a Kolmogorov spectrum over more than four orders of magnitude, from 30 seconds to 2.6 days. That for PV is significantly flatter; thus fluctuations at short time scales are larger relative to those at long time scales for PV than for wind. While wind's capacity factor varies from 32 % at the sites examined to 40% at excellent sites, the capacity factor for a 4.6 MW PV array in Arizona is determined to be 19% over two years.**

Index Terms—**solar power generation, spectral analysis, wind, wind energy, wind power generation.**

I. Introduction

WIND power produced by turbines and solar photovoltaic power both vary with time. Wind data with one second time resolution for one 1.5 MW nameplate capacity wind turbine are shown in Fig. 1.

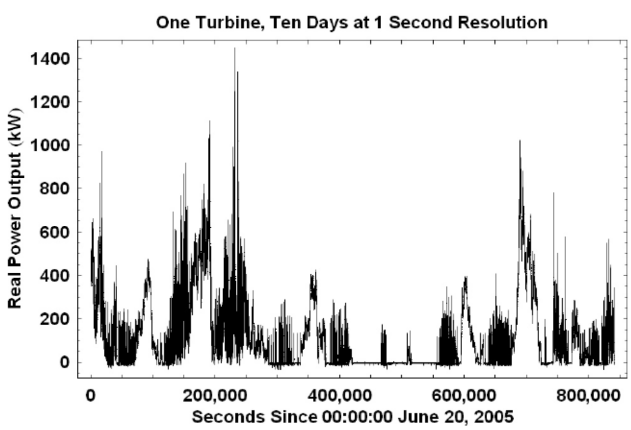

Fig. 1. Real power output (kW) sampled with one second resolution for one 1.5 MW turbine for 10 days. Negative values are due to turbine system electrical loads.

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Solar photovoltaic power from the largest existing U.S. array, a 4.6 MW system in Arizona, also exhibits intermittency. Solar PV data at 10 second time resolution for one day are shown in Fig. 2.

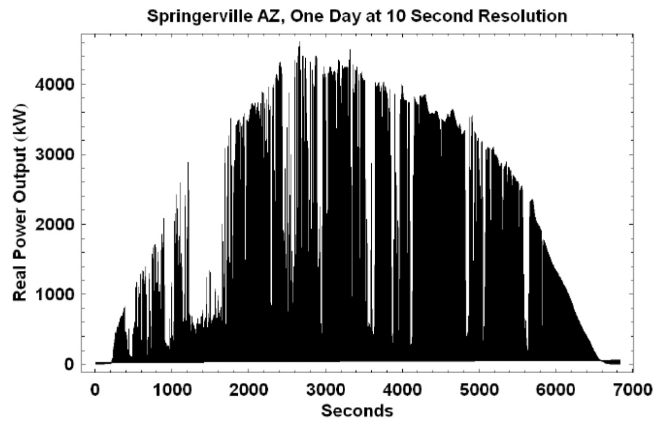

Fig. 2. Real power output (kW) sampled with ten second resolution for a 4.6 MW solar PV array in northeastern Arizona over the daylight period of one day.

Even in Arizona, the majority of days do not exhibit steady solar PV power. Fig. 3 shows a week's data.

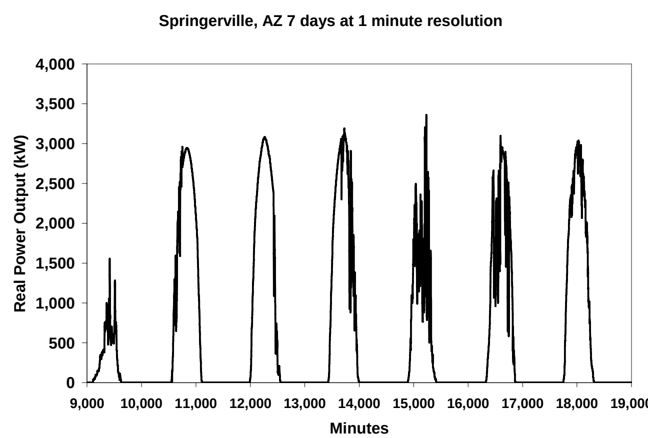

Fig. 3. Real power output (kW) sampled with one minute resolution for a 4.6 MW solar PV array in northeastern Arizona for one week.

The output of the wind turbines and solar PV arrays is not random. The character of the variations can be examined in several ways. One method [1] is to construct a histogram of the step size in output over time. Here we estimate the power spectrum for both wind and solar PV, since this method provides additional insights into integration issues.

II. Data

Wind real power output data with 1 second time resolution for sixteen 1.5 MW turbines at two wind farms were obtained for a continuous 10-day period. In addition, hourly time resolution data for 104 turbines at four wind farms separated by 30 – 400 km were obtained for six months, and data for two of these wind farms were obtained for one year.

Solar PV real power output data from a 4.59 MW fixed latitude array at 10 second time resolution were obtained for 2 months. Data with one minute resolution were obtained from the same array for two continuous years (Fig. 4). The observed capacity factor over the two years was 19.1%, accounting for the increase in array capacity during the period examined.

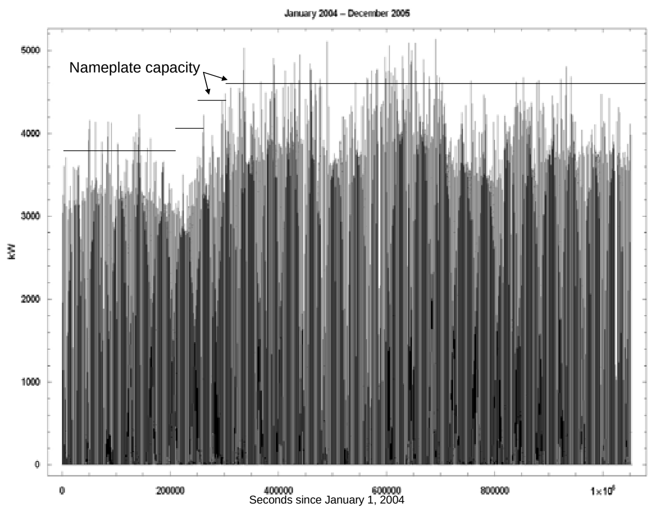

Fig. 4. Real power output (kW) sampled with one minute resolution for a 4.6 MW solar PV array in northeastern Arizona for two years, during which time the size of the array was increased.

It should be noted that the measured solar PV array output power exceeded the nameplate capacity on frequent occasions (Fig. 4). The operator, Tucson Electric Power, attributes this to sunlight focusing near the edge of clouds. This effect has important implication for circuit protection specifications.

III. Power Spectral density

We have described previously the estimation of power spectrum (sometimes termed the power spectral density or PSD) for such data [2]. Briefly, after taking the Fourier transform of the time series, we follow [3] in constructing the periodogram estimate of the power spectrum. In order to take advantage of the large number of data points in a data set to reduce the variance at any given frequency, the data set may be partitioned into several time segments. The Fourier transform of each segment is then taken and a periodogram estimate constructed. The periodograms are then averaged at each frequency, reducing the variance of the final estimate by the number of segments (and reducing the standard deviation by the reciprocal of the square root of the number of segments). In this work, eight segments are used.

For the wind data, the 10-day, 1-second resolution data and the 1-year, hourly data were combined as described in [2]. For the solar PV data, the power spectrum was estimated for the 2 years of 1 minute resolution data and separately for 2 months of 10 second resolution data [4]. As one would expect, the solar data exhibits strong peaks at the frequency corresponding to 24 hours (1.16 x 10^{-5} Hz) and its harmonics.

Fig. 5 is the power spectrum of wind from the summed power output of 16 turbines at two wind farms taken with 1 second time resolution along with the power spectrum of power from a 4.6 MW solar photovoltaic array taken at 10 second resolution; the spectra have been scaled to overlay at a frequency corresponding to 18 hours. For clarity, the low-pass filter effects in each system [2, 4] have been removed by not displaying the data at frequencies high enough to be so affected.

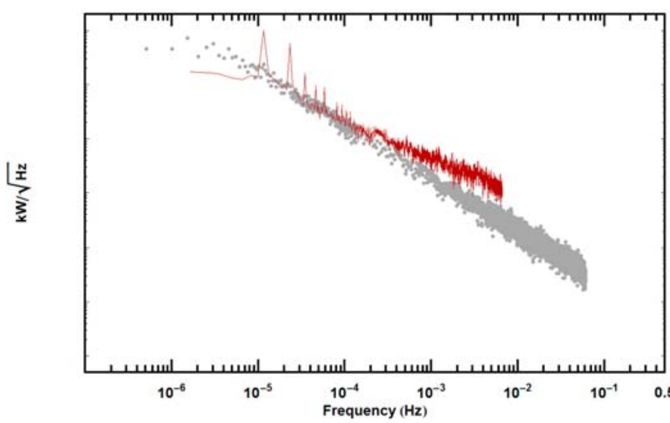

Fig. 5. Power spectrum of real power output from 16 wind turbines at 2 wind farms (light grey points) and from a 4.6 MW solar PV array (red line). The largest peak in the solar PV power spectrum is at the frequency corresponding to 24 hours; higher harmonics are also present.

IV. Discussion

As we have previously reported [2], the power spectrum of real power output from wind turbines follows a Kolmogorov spectrum ($f^{-5/3}$) over at least four orders of magnitude in frequency. This result contradicts that in a power spectrum of wind speed from 1.9 x 10^{-7} to 0.25 Hz published in 1957 [5] that has been reprinted in two recent handbooks [6, 7] and a review paper [8]. This widely-cited 50 year old spectrum has a pronounced “spectral gap” between about 3 x 10^{-5} and 7 x 10^{-3} Hz with very little energy. Modern data do not support this conclusion.

In contrast, the power spectrum of solar PV power output has a significantly flatter spectrum, approximated by $f^{-1.3}$ (Fig. 5 and [4]). Although the data reported here are from a single PV array (albeit the largest in the United States), we have also analyzed the power spectrum of the combined output of three PV tracking arrays separated by several hundred km [4], finding the same slope.

As previously described [2], at large penetration of renewables, it is more efficient to match the fluctuations of intermittent renewable sources with an ensemble of firm power sources than with a single such source.

The present work represents the first comparison of the power spectra of utility-scale wind an solar PV installations. Due to the larger magnitude of solar PV power output fluctuations relative to those of wind at time scales shorter than approximately 3½ hours, the costs of large scale solar PV integration are likely to be larger than those of wind.

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