A Centrality Measure for Electrical Networks

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***Abstract*—We derive a measure of "electrical centrality" for AC power networks, which describes the structure of the network as a function of its electrical topology rather than its physical topology. We compare our centrality measure to conventional measures of network structure using the IEEE 300-bus network. We find that when measured electrically, power networks appear to have a scale-free network structure. Thus, unlike previous studies of the structure of power grids, we find that power networks have a number of highly-connected "hub" buses. This result, and the structure of power networks in general, is likely to have important implications for the reliability and security of power networks.**

***Index Terms*—Scale-Free Networks, Connectivity, Cascading Failures, Network Structure**

I. Introduction

In bulk electric power networks, wide-scale cascading failures happen more often than would be expected if failures were random and independent, and their sizes followed a normal distribution [1]. Despite reliability standards and significant technological advances, the frequency of large blackouts in the United States has not decreased since the creation of NERC in 1965 [2]. The US blackout of August 2003, along with the Italian blackout of September 2003, convinced even skeptics that fundamental weaknesses exist in transmission infrastructures. Many have sought to explain this weakness as a function of a shift in electricity industry structure from regulated, vertically integrated utilities with moderate interregional trade, to a diverse set of market participants using the transmission infrastructure to facilitate long distance energy transactions [3,4]. While the merits and shortcomings of electricity market restructuring are a contentious subject, at the very least it is fair to say that the number and size of blackouts on the North American electric grid has not decreased in recent years, as shown in Figure 1.

Recent advances in network and graph theory have drawn links between the topological structure of networks (particularly networks consisting of social ties and infrastructures) and the vulnerability of networks to certain types of failures. Many classifications of network structures have been studied in the field of complex systems, statistical mechanics, and social networking [5,6], as shown in Figure 2, but the two most fruitful and relevant have been the random network model of Erdös and Renyi [7] and the "small world" model inspired by the analyses in [8] and [9]. In the random network model, nodes and edges are connected randomly. The small-world network is defined largely by relatively short average path lengths between node pairs, even for very large networks. One particularly important class of small-world networks is the so-called "scale-free" network [10, 11], which is characterized by a more heterogeneous connectivity. In a scale-free network, most nodes are connected to only a few others, but a few nodes (known as hubs) are highly connected to the rest of the network. The signature property of a scale-free network is a power-law distribution (a very fat-tailed distribution) of node connectivity or degree.

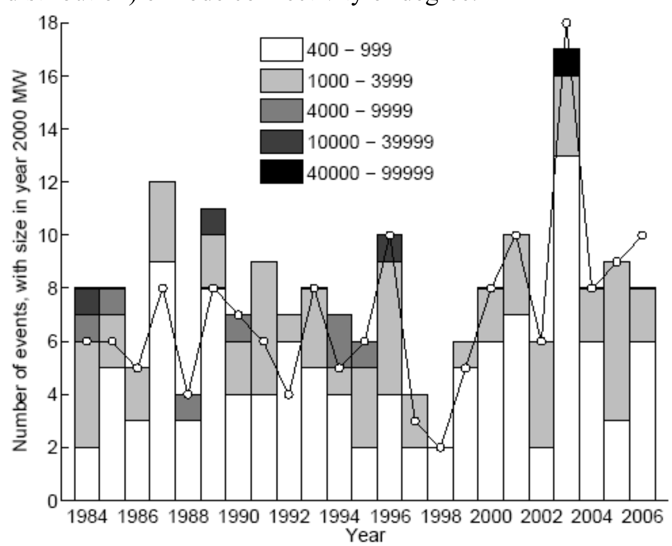

Figure 1: The number of large blackouts according to [NERC DAWG] between 1984 and 2006. The bars show the data after adjusting for demand growth (using year 2000 as the base year). The line shows the number of blackouts >400 MW before this adjustment. Clearly, the frequency of large cascading failures in the North American electric power system has not decreased during the 23 year period shown here. Assuming that large failures were not more common before this period, the overall frequency of large blackouts has not decreased since the formation of NERC in 1965.

The structural differences between the random and scale-free network models have important implications for network vulnerability. In particular, random networks tend to be fairly robust to targeted attacks but relatively vulnerable to a series of random failures or attacks. Scale-free networks, on the other hand, are highly vulnerable to targeted attacks or failures at one of the hubs, but are more robust to failures at randomly

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chosen nodes [34]. Similarly, a network's structure has important implications for strategies aimed at protecting the network against cascading failures or more deliberate attacks.[2] The world wide web is a good example of a scale-free network; the network as a whole can be made more robust by increasing the reliability of a fairly small set of critical hubs (such as google.com or yahoo.com).

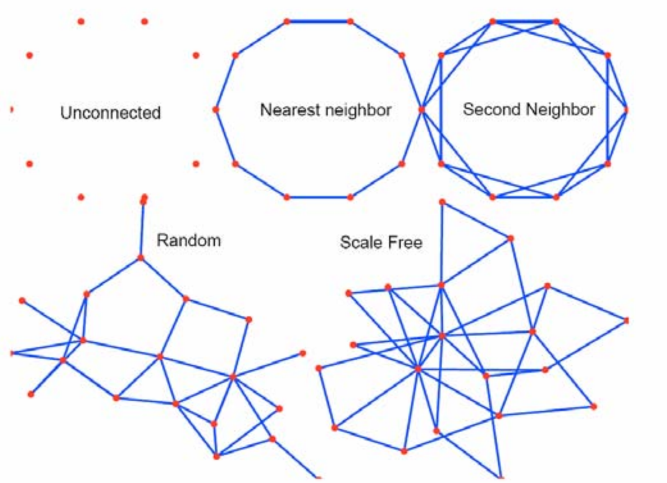

Figure 2: Small examples of the network structures used in this paper.

The principal shortcoming of existing attempts to measure the network structure of electrical grids lies in the failure to explicitly incorporate the physical laws governing the flow of electricity within these networks. While the topological (node-edge) structure of an electrical power network may suggest one set of behaviors and vulnerabilities, the electrical structure of the network may suggest something altogether different. The flow of power through the network is governed by Kirchoff's Laws and not simply by topology. To address this shortcoming we introduce an electrical connectivity metric which incorporates the information contained in the system impedance matrix. This metric is referred to here as "electrical centrality." Electrical centrality accommodates Kirchoff's Laws and is more accurately captures the properties of node centrality, relative to metrics based on node-edge connectivity.

This paper is organized in five sections. Section I is this introduction, and section II reviews the existing literature related to the subject matter. Section III reviews the classical network connectivity metrics, which provide information only on the topological structure of the network in question, and discusses some existing applications of these metrics to power networks. Section IV introduces the electrical centrality metric. In Section V we use the IEEE 300-bus network to illustrate the differences between our electrical connectivity metric and the classical topological connectivity metrics. Section VI offers some conclusions and extensions.

[2] It is important to realize that, particularly in complex systems such as power networks, building more is not always better. The North American blackout of August 2003 was followed by a number of calls for massive investments in transmission infrastructure; [12], for example, suggested a program of investment and grid modernization amounting to $100 billion. The analysis of [13] and [14] suggests that simply building more transmission lines is unlikely to meet reliability goals, and [15] suggests that more intelligent control strategies can improve reliability at lower costs.

II. Network Connectivity Metrics and Applications to Power Networks

Network connectivity is often measured using the *degree* of nodes in the network – the number of edges (and thereby other nodes) connected to a given node. The degree, *k*, distribution varies substantially from one network structure to another. In a regular lattice structure (such as the nearest neighbor and second neighbor graphs shown in Figure 2) *k* is constant for all nodes. Random networks have an exponential degree distribution, as do most small-world networks. When a small-world network shows a power-law degree distribution, rather than an exponential, it is known as a scale-free network [11]. If P(k) is the probability that a randomly chosen node has degree *k*, the degree distribution of a scale free network follows $P(k) \sim k^{-\gamma}$. In most real networks, γ falls in the range of 2 to 3. Many seemingly different types of networks have been found to possess a scale-free structure, including the world wide web, airline networks, and protein interactions in yeast [5].

Power networks represent a natural test case for complex systems and network theory. However, the lack of widely available data on the actual topology of the power grid, particularly in the United States, has dramatically limited the number of analyses. The few existing studies generally find that power networks exhibit some kind of nonrandom structure, but there is disagreement over the actual structure of the network. In particular the existing studies disagree with respect to whether the tail of the degree distribution follows a power law or exponential distribution. Portions of the North American grid have been discussed in [5, 6, 16, 18 – 20]. The North American grid as a whole is discussed in [18], in which the authors report that the degree distribution of the power grid has an exponential or "single-scale" [11] form with a fast-decaying tail, although the distribution of the number of transmission lines passing through a given node (the "betweenness" of a given node) does follow a power law. The authors of [16] find the same single-scale structure in the degree distribution for a portion of the electric network in California. The same result appears in [5] and [6] for the power grid of the Western United States. However, [10, 17] estimate a power-law relationship in the degree distribution for both the Western [10, 17] and Eastern [17] portions of the U.S. power grid.

Structural properties of the Italian, French, and Spanish power networks can be found in [19] and [20]. These European power networks are generally found to have the same single-scale topological structure as the North American grid, with an exponential tail in the degree distribution. The distribution of nodal demands (or loads), however, follows a power law: most buses in the grid serve lower-than average demand, but a small number of buses serve a relatively large amount of demand.

While some of the topological studies of power networks simply sought to tease out information about the structure of the network (as in [5]), some papers focus on the relationship between network structure and system reliability. For the North American power grid, [16] introduces the concept of "connectivity loss," which measures the change in the ability of the network to deliver power following the disconnection of a substation from the network.[3] They find that the connectivity loss is significantly larger when nodes representing hubs are disconnected from the network, relative to the removal of random nodes. This suggests that the power grid has some kind of scale-free structure, even if it does not appear directly in the degree distribution. A similar attack vulnerability is shown in [18] for the Nordic power network and that of the Western U.S. In [17] the authors construct a probabilistic model of cascading failures as a function of the exponent γ in the power law distribution and the number of nodes with degree one. They find that the loss of load probability (LOLP) predicted by the model for the Eastern and Western Interconnects is similar to the LOLP from actual reliability studies performed by the Bonneville Power Administration.

III. An Alternative Connectivity Metric

The existing literature on the structure and vulnerability of electrical power networks largely takes a topological approach to measure distance and connectivity. That is, the power grid is described as a simple graph of nodes connected by edges. Vulnerability analyses focus on the physical islanding of portions of the grid through attacks or failures at multiple nodes. The topological approach paints a somewhat confusing picture of power networks. Referencing the IEEE 300-bus network shown in Figure 3, it is apparent that while there may be a few nodes in the network that fall outside of an exponential probability distribution (and thus some evidence of a power-law tail in the degree distribution), the power network lacks the strong hub structure that is apparent in scale-free networks such as the world wide web and some airline networks.

We argue that a fundamental weakness of the existing structural studies of power grids is that they focus on topological connectivity, while ignoring the electrical connectivity. In particular, a reasonably complete characterization of the network structure of the power grid would need to incorporate the following properties of electrical systems:

1. Flow in a power network is governed by Kirchoff's Laws and not by decisions made by individual actors at individual nodes. Power injections propagate through the network following a path of least resistance, apportioned by the relative complex impedance of each equivalent path.
2. Power networks are best described by undirected graphs. The model in [17] argues that the power network can be thought of as a directed graph associated with a particular state of the network, and small perturbations in the system are unlikely to change the directionality. That is, every state of the network can be mapped into a set of directional power flows. This does not hold true for many cases in a power network. Due to the highly non-linear and generally non-convex[4] nature of the equations that govern power flow in AC electrical systems, it is possible, and in some cases quite likely, that small changes in the state of the network can reverse the direction of power flow along a particular path.
3. Because flows are governed by Kirchoff's laws, to the extent that power networks are characterized by a hub structure, there is no *a priori* way to tell whether the hubs represent large inexpensive generators, load pockets, substation tie-points, or some other physical structure (this assumption is made in [16]).[5] That is, a simple model of preferential attachment, as in [10], is unlikely to be a good evolutionary model explaining the emergent structure of power networks.

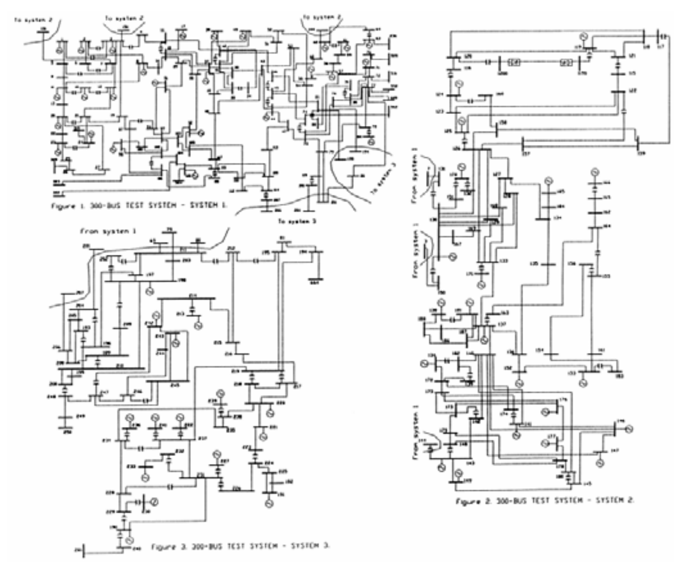

Figure 3: Topology of the IEEE 300-bus test system.

Of these, the most basic structural modeling issue is somehow incorporating Kirchoff's Laws into the network structure metrics. Power grids tend to have highly meshed structures, so the mere topological proximity between two nodes will, in many circumstances, have less of an impact on the performance of the network than electrical proximity, which may or may not correspond to topological distance. The weighted degree distributions discussed in [22] represent a step towards resolving this measurement issue, but in the analysis of [22] the weights are based on capacity constraints, not any kind of path admittance or law of motion.

[3] The data set used in [16] cannot identify with certainty which nodes represent generator substations, and which nodes represent connection points to the distribution network or tie-points within the high-voltage transmission network. The authors make some assumptions regarding which nodes represent each type of substation.

[4] [21] offers evidence that convexity of the set of feasible solutions to the AC power flow problem is rarely, if ever, achieved.

[5] Some information may be contained in the network admittance matrix, as discussed below. For example, transformers tend to have much larger admittances than other pieces of network equipment.

The degree distribution is but one centrality measure ranking the importance of given nodes in the network. Another metric, more popular in the analysis of social networks than physical or technological networks, is betweenness [23, 24], defined in [25] as the fraction of shortest paths between pairs of nodes that happen to pass through a given node. In the most basic sense, betweenness in and of itself suffers from the same modeling issue as the degree distribution – it considers the topological structure of the network, but not the pattern of network flow propagation. This shortcoming has been recognized in [25] through the use of an electrical circuit as an example of how the shortest path (which would include nodes with a high betweenness) may not carry the largest amount of information (or current, in this particular case). The circuit example is then generalized to discuss a broader class of betweenness measures based on random walks through networks.

Social network analysts have introduced a measure known as information centrality to describe how information is passed through a network of associates [26, 27]. Operationally, this is similar to gossip coming through the grapevine. The information centrality associated with a given node (or actor or agent, in the specific case of social networks), describes how much of the network flow travels along each path beginning or ending with that node. Expanding this metric to physical networks, information centrality is (at least conceptually) very closely related to the power transfer distribution factor (PTDF) matrix [28], in which the *i*,*j*th element indicates the change in power flow on the *j*th transmission line from a marginal change in net power injection at the *i*th bus.

Flow-based betweenness metrics were originally considered in [29], with the circuit analysis of [25] an interesting variation. The authors of [30] demonstrate that if information propagates through the network following Kirchoff's Laws, and if the network can be described as a single-commodity network (that is, there are single points of injection and withdrawal), then the resulting betweenness metric based on electrical flow is identical to the information centrality.[6]

IV. ARE POWER NETWORKS SCALE-FREE?

That betweenness is equivalent to information centrality in certain circumstances is an interesting result. However, real power networks have multiple generators and loads, and most likely experience significant loop flows. Thus, the use of standard network metrics to compare power systems to standard network models is likely to be misleading. We examine the differences between power networks and other generic networks models using two novel methods. First, we use the information contained in the network impedance matrix to calculate equivalent electrical distances between pairs of nodes, and examine the properties of the graph based on this distance metric compared to the usual topological distance metrics. Second, we construct electrical networks to conform to the properties of three common network models, and compare the electrical properties of these networks with those of an actual network of a similar size.

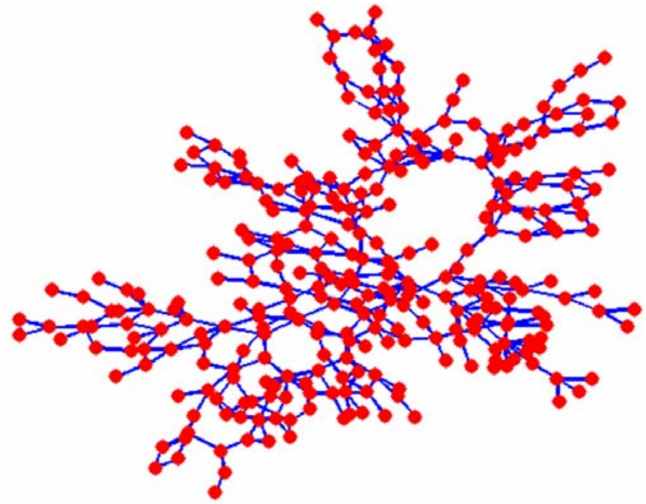

Figure 4: A node-branch representation of the IEEE 300-bus test system.

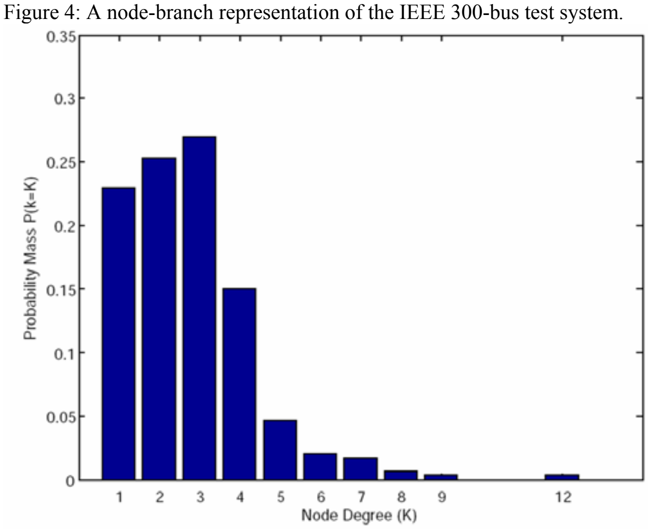

Figure 5: The probability mass function (histogram) for node degree in the IEEE 300 bus network.

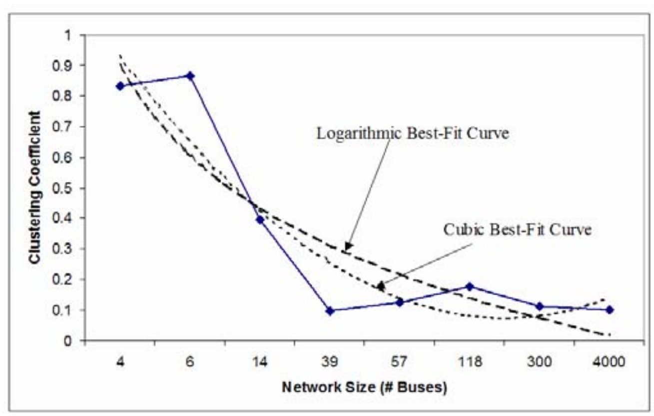

Figure 6: Based on the Watts-Strogatz clustering coefficient, the IEEE 300-bus test system shares the structural complexity of much larger electric power networks.

[6] This would seem to confirm that the information centrality is equivalent to the power transfer distribution matrix for the special case of a power network with one (net) generator and one (net) load (a single-commodity network). As shown in [31], it is possible to create an equivalent single-commodity power network out of a multi-commodity network only if there are no loop flows. Thus, this special case is probably not all that interesting.

The test case for this work is the IEEE 300-bus test network, which has 300 buses (nodes) and 411 branches (transmission lines or transformers). Figure 4 shows the topology of this network, while Figure 5 shows its degree distribution. The 300-bus network is much smaller than most real power grids, which may have tens, if not hundreds of thousands of buses and transmission lines. However, the 300-bus system is sufficiently large so as to share several structural properties with actual power grids. An example is shown in Figure 6, which shows the Watts-Strogatz [9] clustering coefficient for standard IEEE test systems of various sizes, as well as the clustering coefficient for the power grid of the Western U.S., as reported in [9, 15]. While the 300-bus system is smaller than actual power networks, it is sufficiently interesting for our analysis.

A. Structural Properties of the 300-Bus Network

The topological representation of the IEEE 300-bus network in Figures 4 and 5 can be visually compared with similarly-sized small-world and scale-free networks, as shown in Figure 7. The representation of the IEEE 300-bus network shown in Figure 4 does not immediately reveal any highly-connected hubs that would suggest a scale-free network structure. Similarly, the degree distribution of the 300-bus network (Figure 5) does not fit well with a power law statistic. Further, the geodesic paths between random node pairs are longer than would be suggested by a Watts-Strogatz small-world structure, where the average geodesic path length is less than six, even for very large networks. The topological structure of the 300-bus network would lead us to agree with the analysis in [5, 16, 18] that power grids are not characterized by a scale-free structure.

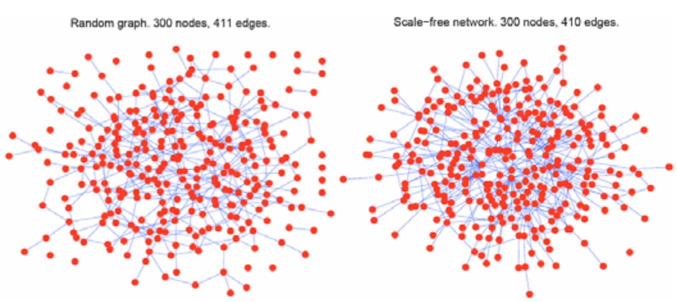

Figure 7: Topological representation of the IEEE 300-bus network (bottom center) as compared with a similarly-sized random network (upper right) and scale-free network (upper left).

However, as noted above and in [25, 30], flow in electrical networks is governed by Kirchoff's Laws, which result in unique patterns of interaction between nodes in a network. The topological structure of the power network may therefore not say very much regarding the behavior of the network, and particularly its vulnerabilities. Kirchoff's Laws are captured in the system bus-bus admittance matrix, defined by:

$$Y_{kl}^{bus} = \begin{cases} G_{kl} + jB_{kl} & k \neq l \\ -\sum_{k \neq l} \left(G_{kl} + jB_{kl} \right) & k = l \end{cases} \tag{1}$$

The definition of the Y^{bus} matrix used here captures both the real and reactive (imaginary) portions of the line admittances. The Y^{bus} matrix tends to be sparse, since $Y_{kl}^{bus} = 0$ for pairs of nodes *k* and *l* that do not share a direct physical connection. In our analysis we actually use the inverse of the Y^{bus} matrix, which is traditionally denoted the Z^{bus} (or impedance) matrix. Inverting the sparse Y^{bus} matrix for a fully connected electrical power network, yields a non-sparse (dense) matrix, denoted Z^{bus}. The equivalent electrical distance between nodes *k* and *l* is thus given by the magnitude of the relevant entry of the Z^{bus} matrix. Smaller $|Z_{kl}^{bus}|$ correspond to shorter electrical distances (and a larger propensity for power to flow between these nodes, subject to capacity constraints along any of the topological paths).

In Figure 8 we redraw the 300-bus network to show the electrical connections rather than the topological connections. There are $[(300^2)/2 - 300] = 44{,}700$ distinct node-to-node connections in the Z^{bus} matrix for the 300-bus network; the links shown in Figure 8 represent the 411 node pairs that have the largest electrical connections. The threshold value for the entries of the Z^{bus} matrix is 0.00255, as shown in Figure 9. Thus, Figure 8 displays a graph approximately the same size as the original network topology shown in Figure 3 (300 nodes and 411 edges). The atomistic nodes show buses in the network that have small electrical connections to the network as a whole.

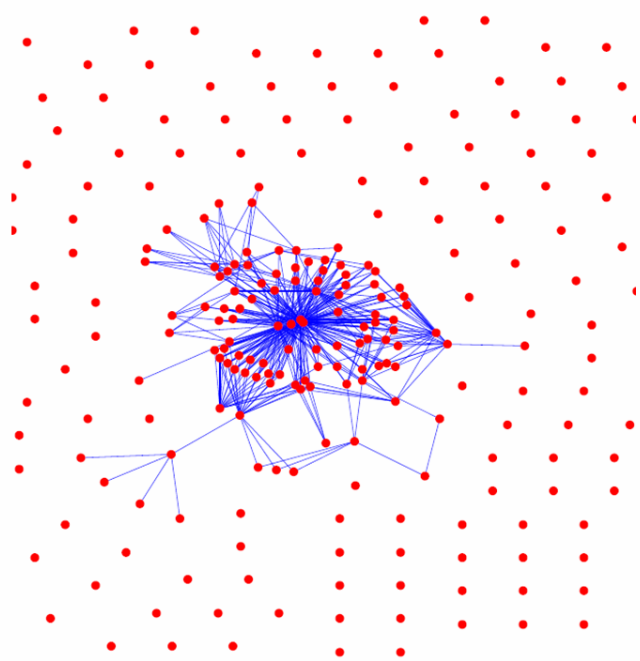

Figure 8: The IEEE 300-bus network has been redrawn here to highlight the structure of the electrical connections represented in the Z^{bus} matrix. To make this representation size-compatible with the topological representation in Figure 3, only the 411 strongest electrical connections (out of 44,700 total electrical connections) are shown.

The two representations of the 300-bus network suggest very different structures. From a topological perspective, the 300-bus power network looks neither like a random network nor a scale-free network. But from an electrical perspective (which captures the behavior of the network, not simply the physical structure), the 300-bus network looks to have a distinct group of nodes that are "electrical hubs" – that is, buses that are have a high electrical connectivity to the rest of the network. Power flowing through the network is, due to Kirchoff's Laws, much more likely to pass through these nodes than other nodes. In the language of social networks, these would correspond to nodes with a high betweenness or information centrality. Figure 10 redraws the topological representation of the 300-bus network, but with the node sizes adjusted according to the electrical centrality (nodes with a higher electrical centrality are shown larger in the figure).

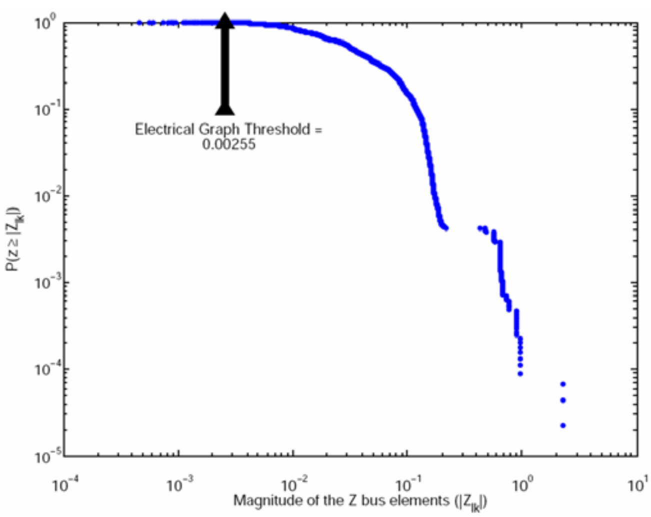

Figure 9: The inverse cumulative probability distribution function for all node-to-node electrical connections. The threshold at which only 411 stronger (smaller $|Z_{lk}^{bus}|$) connections exist is 0.00255, as indicated.

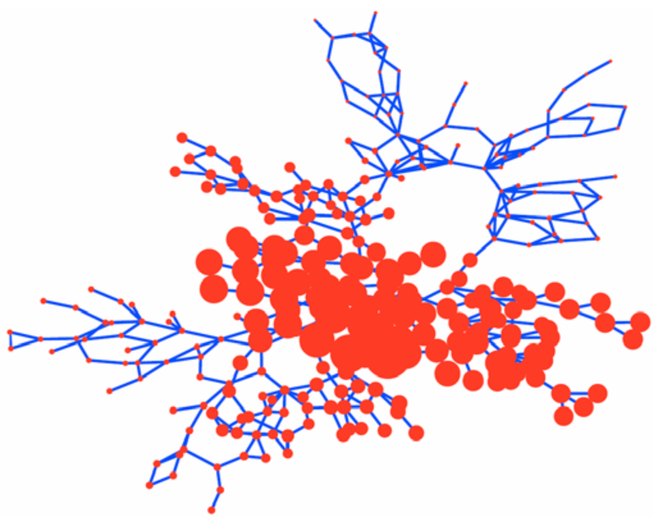

Figure 10: A representation of the IEEE 300-bus network, with the node-sizes adjusted to represent the relative magnitudes of the electrical betweenness measure. The nodes in the center with high electrical connectivity (critical nodes in the network) can clearly be seen.

Based on the electrical information contained in the Z^{bus} matrix, we re-interpret the degree distribution for the 300-bus network to show the electrical betweenness of each bus in the system. The electrical degree distribution is shown in Figure 11.

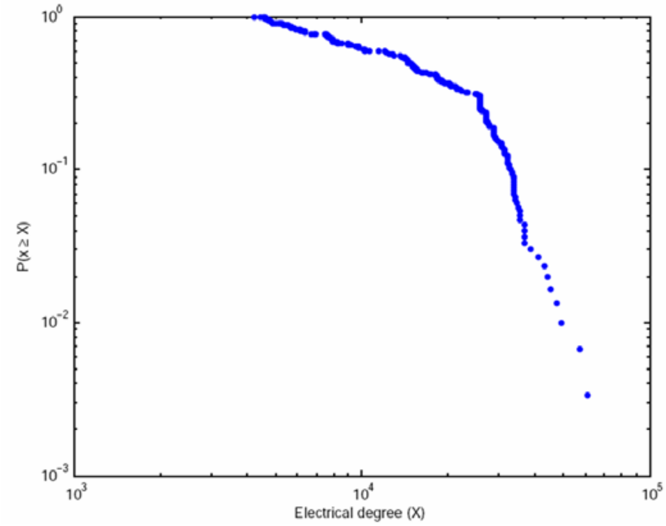

Figure 11: The inverse cumulative probability density function for the electrical betweenness (electrical degree) of the nodes in the IEEE 300-bus network.

B. Comparison of the IEEE 300 Bus Network with Other Network Structures.

Topologically, the 300 bus power network does not have the properties of a scale-free network. Electrically, however, there appear to be a number of highly-connected nodes similar to what would be expected from a scale-free network. We now extend this analysis by creating simple electrical networks from the network topologies shown in figures 2, 4 and 7. Specifically, we construct simple network of resistors corresponding to the topologies of the nearest-neighbor, random, and scale-free networks in addition to the 300-bus power network.

To build the resistor networks, at each node we place a 1 Ω resistor and represent each link by a 1 Ω resistor. For each network, we calculate a resistance matrix, which is essentially equivalent to the Y^{bus} matrix for an equivalent power network. The resistance matrix **R**, defines the sensitivity relationship between voltages and currents, as shown in eq. (2).

$$\mathbf{\Delta V} = \mathbf{R \Delta I}\,, \qquad (2)$$

Given this relationship, we can calculate a sensitivity matrix **S** according to:

$$S_{kl} = R_{kl} / R_{kk} \text{ for all } k \text{ and } l. \qquad (3)$$

The elements of **S**, describe the extent to which a state change at location *i* will affect a similar change at location *j*. Essentially this matrix tells the extent to which information (or whatever it is that is flowing through the system) will propagate through each network, or conversely, the extent to which information can be contained within a small area. In all five networks, sensitivity decays approximately linearly with the distance between nodes *i* and *j* (see Figure 12), but because of the high connectivity of the hubs in the scale free networks, and the relatively short distances between nodes in the random network, the number of nodes that would be affected by a given change is dramatically different for the different network structures (see Figure 13). In other words, if a scale-free or random network has properties similar to Kirchoff's Laws, it will be very difficult to contain the propagation of information to a small area. The power network has substantially more information propagation than that shown by the second neighbor lattice structure, but much less than the random and scale free test cases.

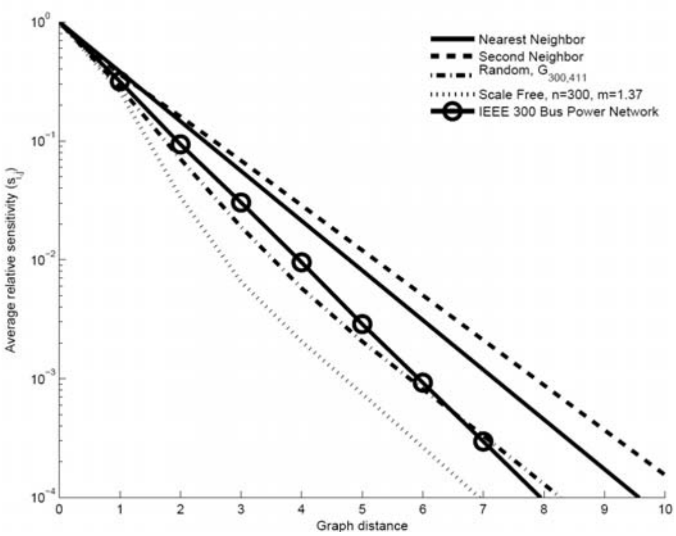

Figure 12: Average relative sensitivity $S_{i,j}$ plotted against the distance between *i* and *j*, for networks of resistors with different structures. For a given physical distance, the sensitivity is generally larger in the IEEE 300-bus network than in random or scale-free networks of approximately the same size. Information thus propagates more slowly in the 300-bus network than it would if the network had a scale-free or random structure.

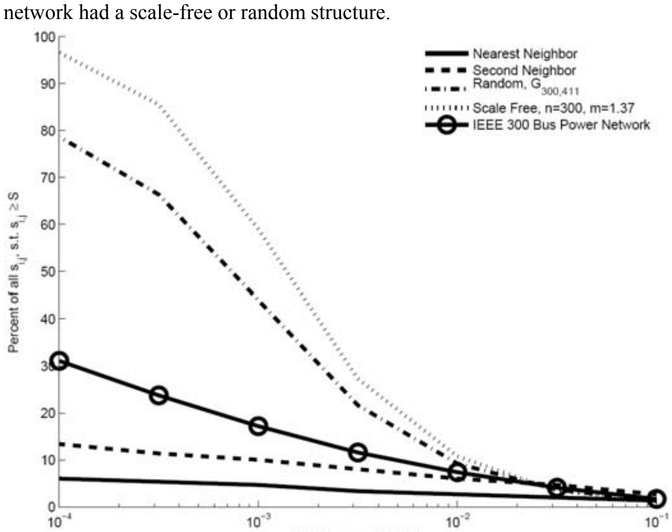

Figure 13: Distribution of the sensitivities for the networks of resistors. The quantity of a network affected by some change or disturbance depends on (or varies with) the structure of the network.

C. *Extensions: Implications for Vulnerability*

Network structure is closely related to network vulnerability [16]. Thus, a deeper understanding of the structure of electric power grids can yield important insights for decreasing vulnerability and improving reliability. Presently, the authors are looking at the relationship between the proposed electrical centrality metric and the vulnerability of the system to failure and attack at particular locations. While the outcome of this research is pending, it is clear that a complex-networks perspective can provide some insight into the vulnerability of electrical power networks.

V. Discussion and Extensions

A clear link exists between the topological structure of many networks and types of failures to which they are vulnerable or robust. Scale-free networks are particularly vulnerable to failures at their highly connected hubs. This makes them particularly susceptible to deliberate attacks on these hubs, but less vulnerable to failures at random locations. A series of denial-of-service attacks targeting a major hub in the world wide web (such as Google or Yahoo) could cause ripple effects crippling much of the physical internet [35]. The winter of 2006/2007 proved particularly difficult for airlines and their passengers, as snow and ice storms hitting hub airports caused major delays in areas where the weather was perfect. This suggests that resources aimed at protecting scale-free networks should primarily be directed towards strengthening the hubs.

The North American power blackout in August of 2003 has focused attention on network reliability and measures which could limit or prevent cascading failures. Many of the suggested measures, such as [12] involve large capital expenditures on new infrastructure. However, the nature of power networks is such that simply building lots of new transmission lines may not yield the desired performance improvements [13]. Further, it is not obvious that building more centrally-controlled infrastructure is necessarily a more cost-effective solution than implementing other novel control strategies [15].

Most analyses of the power grid have found a single-scale topological structure, which can arise if linking to high-connectivity nodes is sufficiently costly to impair preferential attachment in the growth of the network [11]. Due the cost of building new links (transmission lines) simply increasing the connectivity of the grid (as in [12]) is prohibitively expensive. Further, siting of new transmission lines is becoming increasingly difficult [32]; even with the possibility of federal intervention [33] it simply may not be feasible to expand the current network in a fashion which optimizes its resilience.

Our analysis shows that, measured properly, power networks bear some resemblance to scale-free networks. The hubs do not show up in a simple analysis of a power network's topology, but a more detailed look at the network's electrical structure reveals a network that shares many properties with other scale-free networks. The hubs in power networks themselves appear highly connected with other hubs, indicating the presence of a highly vulnerable "core." Further, disturbances in one part of an electrical network affect local areas substantially more than they affect remote regions. The effects are spread less widely in the power network relative to some other network structures.

VI. Acknowledgment

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