A Quantitative Analysis of the Relationship Between Congestion and Reliability in Electric Power Networks

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Abstract: *Restructuring efforts in the U.S. electric power sector have tried to encourage transmission investment by independent (non-utility) transmission companies, and have promoted various levels of market-based transmission investment. Underlying this shift to "merchant" transmission investment is an assumption that new transmission infrastructure can be classified as providing a congestion-relief benefit or a reliability benefit. In this paper, we demonstrate that this assumption is largely incorrect for meshed interconnections such as electric power networks. We focus on a particular network topology known as the Wheatstone network to show how congestion and reliability can represent tradeoffs. Lines that cause congestion may be justified on reliability grounds. We decompose the congestion and reliability effects of a given network alteration, and demonstrate their dependence through simulations on a 118-bus test network. The true relationship between congestion and reliability depends critically on identifying the relevant range of demand for evaluating any network externalities.*

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1. Introduction: Transmission Investment in the Old and New Electricity Industry

One distinct difference between restructured and traditionally-organized electric power systems is the way in which congestion is managed. Grid operators in many areas where restructuring has not occurred use a command-and-control procedure known as transmission loading relief (TLR), which physically manages constraints by rationing access to portions of the transmission network. Independent System Operators (ISO) or Regional Transmission Organizations (RTO) on the other hand, generally use market-based methods to manage congestion. Of particular concern to policymakers and system planners is that both metrics indicate that stress on the system has risen dramatically, particularly since the beginning of restructuring. Congestion costs have risen on both a gross and average level in the PJM market (Joskow 2005a, Blumsack 2006a, Ch. 1), while the incidence of TLRs has risen by a factor of five (Joskow 2005a, Blumsack, Apt, and Lave 2006).

In the regulated electricity industry, the transmission system is essentially a vehicle for the delivery of bulk power. The transmission network could also act as a physical hedge against unplanned generator outages. Either way, the main purpose of utility transmission planning was to support system reliability. Reliability reflects the goal that the system should be redundant enough to avoid service interruptions even in the face of contingencies. Examples of some common reliability metrics are:

1. The $N - k$ criterion; whether the system can continue to provide uninterrupted service to customers in the face of a contingency in which k out of N pieces of equipment are lost, damaged, or otherwise disconnected from the network;
2. the Loss of Load Probability (LOLP), defined as the probability over some period of time that the network will fail to provide uninterrupted service to customers;
3. the Loss of Energy Expectation (LOEE) and Loss of Energy Probability (LOEP), defined as the expected amount and proportion of customer demand not served over some time frame. These are also known as the Unserved Energy Expectation or Probability.

Transmission planning in the regulated industry represented one step in an overall integrated resource plan, in which utilities ensure resource adequacy in generation, transmission, and distribution for a given set of peak and seasonal demand scenarios (Coxe and Ilić 1998). Under industry restructuring, an increasing number of investment decisions have been made by decentralized market participants (such as independent power producers), and not the vertically-integrated utility. Baldick and Kahn (1993) outline how issues of economics and externalities, formerly handled within the utility planning process, may yield different or conflicting solutions in the context of markets and open transmission access. Transmission companies must also accommodate energy-market outcomes into their investment decisions. Thus, whereas prices in the regulated environment were an output of the planning process (and were determined largely by the decisions of regulators), prices in the restructured era have become inputs to the planning process (Coxe and Ilić 1998). Prices in the energy market must solve both the short-run operations problem (clearing the hourly or daily market for electric energy) and the long-run investment problem (Yu, Leotard, and Ilić 1999, Blumsack 2006a, Ch. 1).

Just as a non-utility or "merchant" generation sector emerged following the passage of the Energy Policy Act of 1992, restructuring brought with it the potential for a merchant transmission sector. These transmission-only companies would build lines in RTO territories in response to congestion pricing signals (Joskow and Tirole 2005), and would be compensated with financial or physical congestion rights associated with any incremental capacity created by their investments. Hogan (1992) proposed a system of point-to-point financial transmission rights (FTR), which would grant the holder the right to the difference in locational prices between any two points on the grid. Thus, the value of an FTR can be determined as a by-product of the energy market; this allows market participants to hedge locational price risk (Aruqui et. al. 2005, Patiño-Echeverri and Morel 2006). The analysis of Chao and Peck (1996) introduced the concept of tradable "flowgate" rights, which would associate congestion payments with physical congestion on specified paths. More recently, Gribik et. al. (2005) have considered augmenting the flowgate model to separate payments into a capacity component and a component reflecting the physical aspects of transmission lines (such as admittance). Apt and Lave (2003) propose a two-part tariff for transmission investment, which would combine locational pricing with the megawatt-mile charge described in Yu and David (1997) for long-run marginal cost pricing. Vogelsang (2001, 2004) has advocated performance-based regulation for non-utility transmission, while Lecinq and Ilić (1997) describe a possible peak-load pricing formulation for transmission, based on the work of Crew and Kleindorfer (1979) for the regulation of public utilities.

Proponents of both FTRs and flowgates have argued that congestion rights can be used to promote merchant transmission investment. Bushnell and Stoft (1996, 1997) show that if transmission investors are compensated with incremental FTRs, merchant transmission investment will be economically efficient (in the sense that profitable investments will also be beneficial to the network), as long as the FTRs are allocated according to the "feasibility rule."[1] Oren (1997) argues that using congestion contracts tied to market prices favors generators with market power, and that the use of tradeable flowgate rights removes localized market power associated with network constraints. The dominant merchant transmission model, based on FTRs, has been criticized for failing to account for system dynamics driven by uncertainty in demand, as in Yu, Leotard, and Ilić (1999). Joskow and Tirole (2005) find that efficiency of the merchant transmission model is not robust to deviations from the underlying economic assumptions.

Underlying the merchant transmission model, as well as variations like the "participant funding" model (Hebert 2004, Roark 2006), is an implicit assumption that a distinction can be drawn between "economic" investments (which primarily relieve congestion along a readily-identifiable portion of the network) and "system" investments (which promote reliability). A related distinction divides projects into those affecting a reasonably small number of identifiable network participants and those affecting the entire network.

[1] The feasibility rule allocates FTRs in such a way as to respect all the network constraints; it was originally devised by Hogan (1992) to ensure that the RTO's FTR obligations did not exceed its congestion revenues.

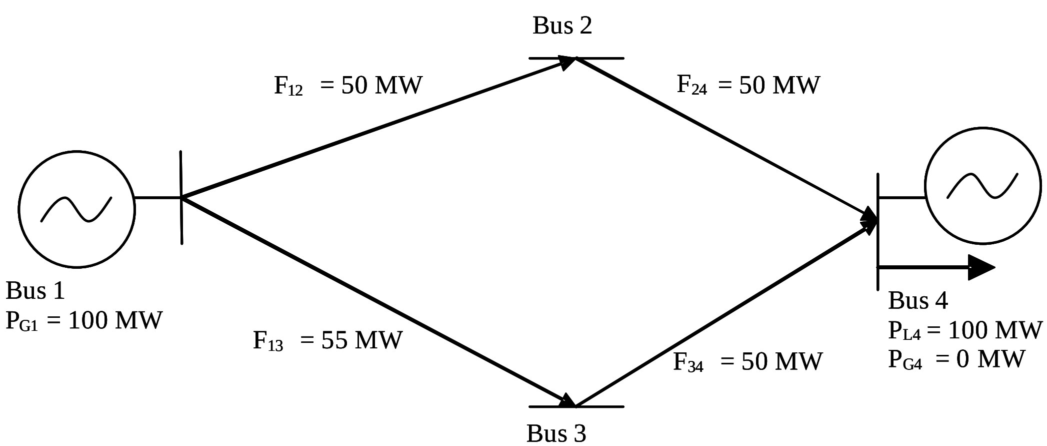
Transmission policy, both at the RTO level and at FERC, has encouraged planners to make either or both of these distinctions, and to emphasize the congestion and competitive effects of new power lines (Awad et. al. 2004, Sauma and Oren 2006).[2]

Joskow (2005b) has argued that attempts to divide transmission investments into congestion-relief and reliability categories, as in Hogan (2003) and Shanker (2003), amounts to a meaningless dichotomy, since most investments have been made (and will continue to be made) by regulated utilities driven by reliability criteria. This paper goes one step further and provides a quantitative analysis of the degree to which congestion and reliability are independent system attributes. We find that the dichotomy is not simply meaningless. In many cases, it is wrong. Increasing network reliability by adding new AC transmission lines can actually increase congestion (though in some instances the two are complementary – congestion can be reduced and reliability enhanced with a single line). Further, this relationship is a function of both the network topology and the level of demand. The degree to which congestion and reliability represent tradeoffs faced by system planners and investors is more pronounced for some network topologies than others. Assessing whether a given transmission project has significant externalities (positive or negative) depends on identifying the relevant range of demand.

[2] Tariffs of the northeastern RTOs still contain language promoting the merchant AC transmission model. A more public expression of support for the model can be found in New York ISO (2005). Section 1221 of the Energy Policy Act of 2005 allows for the designation of "national interest transmission corridors," the locations of which will be determined based on the results of periodic congestion studies by the U.S. Department of Energy. See FERC Docket No. RM-06-12-000, "Regulations for Filing Applications for Permits to Site Interstate Electric Transmission Corridors."

2. Wheatstone Networks and the Braess Paradox

Consider the four-bus test network shown in Figure 1. There are two generators in the system, at buses 1 and 4; each is assumed to have a capacity of 100 MW. The load at bus 4 has a constant real power demand (denoted P_{Li}) of 100 MW. There is no other net demand in the system. Buses 2 and 3 are merely tie-points, with neither net generation nor load. The four transmission lines in the network are all rated to 55 MW.[3] Lines S_{12} and S_{34} have identical admittances, and lines S_{13} and S_{24} have identical admittances.



Network Data

	Line S_{12}	Line S_{13}	Line S_{24}	Line S_{34}
Susceptance B_{ij} (p.u.)	0.06	0.03	0.06	0.03
Capacity $F_{ij,\ max}$ (MW)	55	55	55	55

Figure 1: Four-bus parallel test network

The cost curves of the generators are parameterized as:

$$C_1(P_{G1}) = 200 + 10.3P_{G1} + 0.008P_{G1}^2 \quad (1)$$

$$C_4(P_{G4}) = 300 + 50P_{G4} + 0.1P_{G4}^2, \quad (2)$$

where P_{Gi} is the real power output of the ith generator.

[3] We express the line limits in MW since we are concerned with real power flows. Thermal and stability limits in actual networks are given in MVA, and are rated for both real and reactive power.

Figure 1 shows the flow of power through the network. The locational marginal prices (LMP) derived from a lossless DC optimal power flow run on the network are all equal to \$12.11/MWh. The optimal power flow (OPF) algorithm dispatches generation assets in merit (least-cost) order subject to the physical constraints of the electric network. The problem is formulated as (Schweppe, et. al. 1988, Wood and Wollenberg 1996):

$$\min_{P_{Gi},\theta_i,F_{ij}} C_{G1}(P_{G1}) + C_{G4}(P_{G4}) \tag{3}$$

subject to the following constraints, for all *i* and *j* except the slack (or reference) bus:

$$\begin{aligned}
&\sum_i P_{Gi} = \sum_i P_{Li} \\
&F_{ij} = B_{ij}\left(\theta_i - \theta_j\right) \\
&P_i = \sum_j F_{ij} \\
&\left|F_{ij}\right| \leq F_{ij}^{\max} \\
&0 \leq P_{Gi} \leq P_{Gi}^{\max} \\
&F_{ij}^{\max} \geq 0.
\end{aligned} \tag{4}$$

where B_{ij} is the susceptance of the transmission line connecting buses *i* and *j*, F_{ij} is the flow of real power from bus *i* to bus *j*, P_{Li} is the real power demanded at bus *i*, θ_i is the voltage angle at the *i*th bus, and $P_i = P_{Gi} - P_{Li}$ is the net real power demanded at the *i*th bus.

Schweppe, et. al. (1988) derive optimal locational prices (also known as locational marginal prices or LMP) from the OPF formulation in equations (3) and (4). In the DC OPF with no resistive losses, the LMP at the *i*th bus is given by (Yu, Leotard, and Ilić 1999):

$$\pi_i = \lambda + \sum_k H_{ik}\mu_k \,, \tag{5}$$

where λ is the system marginal cost of generation, μ_k is the shadow price of transmission along line *k* (derived from the solution of the OPF in equation 4), and H_{ik} is the *ik*th entry of the power transfer distribution matrix:

$$[H]_{ik} = \frac{\partial F_k}{\partial P_i} \,. \tag{6}$$

Sauer (1981) and Baldick (2003) discuss calculating the entries of the distribution factor matrix under the DC power flow assumptions.

Equations (5) and (6) show that the LMP at bus *i* represents the social marginal cost of supplying an additional unit of power at bus *i* (Wu et. al. 1996). Thus, in the absence of generation or transmission constraints (or resistive losses), LMPs should be equal throughout the network. This is the case for the network in Figure 1. The total cost of serving the 100 MW of demand at bus 4 is equal to $1,620 per hour.

Next, suppose that a fifth transmission line was added to the network in Figure 1, connecting buses 2 and 3. This transforms the network from a simple parallel network to a meshed network consisting of two back-to-back triangles, as shown in Figure 2. The results of the DC OPF in equation (3) run on this modified network are also shown in Figure 2. The generator cost curves and other network parameters are identical to the network in Figure 1.

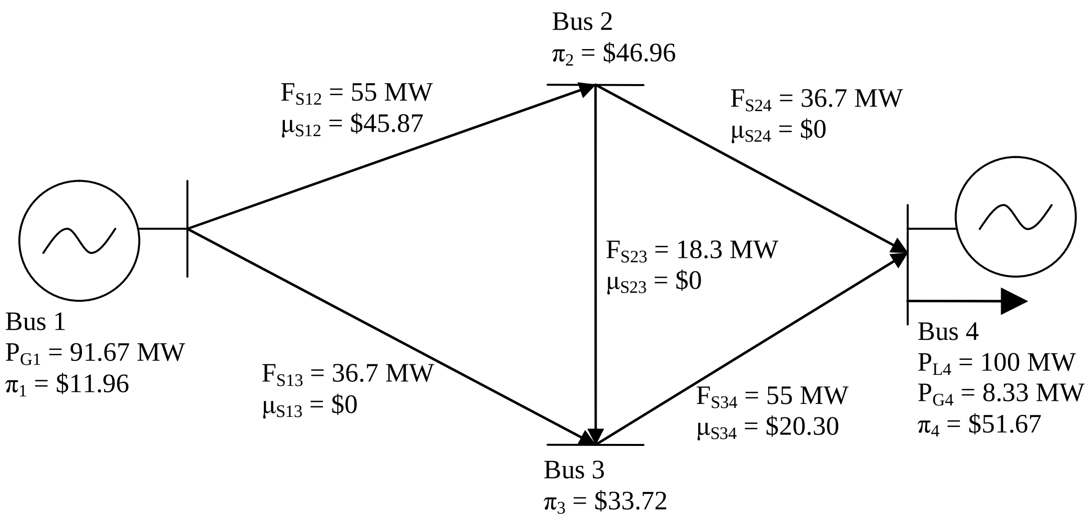


Figure 2: Four-bus Wheatstone test network.

As can be seen from Figure 2, adding the link between buses 2 and 3 causes congestion in the network. Lines S_{12} and S_{34} hit their rated limit of 55 MW. The remaining two lines on the boundary of the network (lines S_{13} and S_{24}) each carry 36.7 MW. The congestion in the network implies that the generator at bus 1 can no longer supply all 100 MW demanded by the load at bus 4; generator 1 only produces 91.67 MW. The remainder is made up with the more-expensive generator at bus 4, which produces 8.33 MW of power (in the absence of a generator at bus 4, blackouts totaling 8.33 MW would occur in the network). This increases the total cost of serving the load to $1,945 per hour.

The network topology shown in Figure 2 is known as the Wheatstone network, and the link between buses 2 and 3 is known as the Wheatstone bridge.[4] The phenomenon that adding links to a network can actually cause congestion is known as the Braess Paradox. This behavior was first formalized outside of electric circuits by Braess (1968) in the context of automotive highway networks, but has been discovered to occur in a variety of other type of networks. Korilis, Lazar, and Orda (1997, 1999) examine the Braess Paradox in the context of internet routing, while Calvert and Keady (1993) and Bean, Kelly, and Taylor (1997) discuss the Paradox for more general networks, including telecommunications and pipes. Arnott and Small (1994) discuss an actual example of the Braess Paradox in a German road network.

3. Decomposing Congestion and Reliability

Our focus on the Wheatstone configuration is motivated in part by Duffin (1965) and Milchtaich (2005), who demonstrate that any network topology can be decomposed into purely series-parallel components and components containing an embedded Wheatstone sub-network. Although it is possible for a simple series-parallel power network to exhibit

[4] The earliest known reference, given in Ekelöf (2001), is Charles Wheatstone's 1843 article, "An Account of Several New Instruments and Processes for Determining the Constants of a Voltaic Circuit," *Philos. Trans. R. Soc.* Pt. II, pp. 303 – 327. Ekelöf's interesting history notes that Wheatstone did not actually invent the circuit configuration.

Braess' Paradox,[5] congested Wheatstone networks have a number of other properties of interest, as described in Blumsack (2006a Ch. 3 and 4):

1. The Wheatstone bridge causes congestion in other parts of the network, but the bridge itself does not become congested. Relieving congestion in the Wheatstone network requires either upgrading both congested lines, or removing the Wheatstone bridge from the system.
2. Both congested lines sport non-negative shadow prices in the DC OPF model.[6] Financially, each of these shadow prices individually does not represent the social value of incremental transmission upgrades. The sum of the two shadow prices does represent the social value of incremental upgrades on both congested lines.
3. The use of shadow prices or LMPs to guide network expansion would lead transmission companies or system planners to upgrade both congested lines. However, removing the Wheatstone bridge would accomplish the same goals, perhaps at a lower cost.
4. In a system with non-utility or "merchant" transmission, the Wheatstone network represents a case where investors could earn profits by building lines that cause congestion, even if congestion rights are allocated according to the "feasibility allocation rule" described in Hogan (1992) and Bushnell and Stoft (1996).

[5] Milchtaich's paper builds on Duffin's result by proving that for a large class of undirected networks, the existence of an embedded Wheatstone sub-network is a necessary and sufficient condition for the network to display the Braess Paradox. This result does not apply to power networks (Blumsack 2006, Ch. 3).
[6] This may not be true in the AC optimal power flow model.

Despite the negative effects on the system, the Wheatstone bridge may be justified on reliability grounds. Consider the Wheatstone network shown in Figure 2, except suppose that lines S_{24} and S_{34} can carry 100 MW. Without the Wheatstone bridge, an outage on either line S_{24} or S_{34} will restrict the power transfer to 55 MW between buses 1 and 4. Thus, for a load of 100 MW at bus 4, the expensive generator at bus 4 must generate 45 MW in order to avoid shedding any load. This increases the cost of operating the network according to the differences in marginal costs of the two generators. If the Wheatstone bridge is installed in the network, the power transfer between buses 1 and 4 in event of an outage across line S_{24} or S_{34}, is 100 MW, and the load can continue to be served with the inexpensive generator at bus 1.

During normal system operations, the Wheatstone bridge imposes a cost on the system in the form of congestion. In the case of a contingency on line S_{24} or S_{34} the Wheatstone bridge offers a reliability benefit to the system, in the sense that the unserved energy expectation and probability decrease. Thus, increased reliability is earned at the cost of increased congestion. The only exception to this tradeoff occurs at very low levels of demand – less than 55 MW in the case of the Wheatstone network in Figure 2. The value of the Wheatstone bridge is the difference between the reliability benefit that it offers to the system over some period of time and the congestion costs it imposes on the system.

3.1. Congestion and Reliability Metrics

Consider a system with *NB* buses and *NL* links. During normal operations, the system congestion cost (or benefit) associated with a given line *k* is measured by the difference in total cost of serving a given demand profile $\{P^*_{L1},...,P^*_{L,NB}\}$ with line *k* in the network, and the total cost of serving an identical demand profile without line *k* in the network. The congestion cost associated with line *k* can be written as:

$$CC_k = \sum_{i=1}^{NB}\left(C_i(P^k_{Gi}) - C_i(P^{-k}_{Gi})\right), \tag{7}$$

where P^k_{Gi} is generation at the *i*th bus with line *k* in the network, and P^{-k}_{Gi} represents generation at the *i*th bus without line *k* in the network. In the case of the Wheatstone network in Figure 2, we have $CC \geq 0$, but in theory this need not necessarily hold in more general networks.

Based on the discussion of the four-bus Wheatstone test network in Section 2, we examine the reliability benefit associated with line *k* in the event of an outage on line $m \neq k$. The reliability metric we use is the expected cost of unserved energy (ECUE). In the event of an outage on line $m \neq k$, line *k* provides a reliability benefit to the system if the cost of unserved energy is lower with line *k* in the network than without line *k* in the network. Quantifying the cost of unserved energy requires describing end-use consumption in terms of a demand or value function. Following Bohn, Caramanis, and Schweppe (1984) and Joskow and Tirole (2006), end-use consumption at the *i*th bus is assumed to follow a demand function $\pi_i(P_{Li})$, where π_i is the price faced by consumers at bus *i*. Consumer surplus at the *i*th bus is therefore given by the value function:

$$v_i(\overline{P_{Li}}) = \int_0^{\overline{P_{Li}}} \pi_i(P_{Li})dP_{Li}\,. \tag{8}$$

Let U be a Bernoulli random variable equal to one (with probability u) in the case of an outage on line $m \neq k$, and equal to zero (with probability 1- u) when there is no outage on line $m \neq k$. In the event of an outage, suppose that consumption at the ith bus is given by $P_{Li}^{k,-m}$ if line k is connected to the network, and $P_{Li}^{-k,-m}$ if line k is not connected to the network. Then we can write the value of unserved energy as:

$$VUE_i = U \times \left(v(P_{Li}^{k,-m}) - v(P_{Li}^{-k,-m})\right). \tag{9}$$

If there is no outage on the line, then $UE_i = 0$ and thus $VUE_i = 0$. Also, if $P_{Li}^{k,-m} = P_{Li}^{-k,-m}$ (that is, real power transfer into bus i is not affected by line outages or the presence of line k), then the cost of unserved energy is also zero. Assuming that the outage probability is independent of the level of demand, the expected value of unserved energy is:

$$EVUE_i = u \times \left(v(P_{Li}^{k,-m}) - v(P_{Li}^{-k,-m})\right). \tag{10}$$

From the point of view of the utility performing reliability assessments, equation (10) represents the expected cost of unserved energy, as in Zerriffi, et. al. (2005). If equations (5) and (6) hold for all buses in the network, then we can say that line k provides a reliability benefit to the entire network.

From equations (5) and (6), we can express the expected net benefit of a given line *k* as:

$$E(NB) = \underbrace{u \times \left(v(P_{Li}^{k,-m}) - v(P_{Li}^{-k,-m})\right)}_{\text{reliability}} - \underbrace{(1-u)\sum_{i=1}^{NB}\left(C_i(P_{Gi}^{k}) - C_i(P_{Gi}^{-k})\right)}_{\text{congestion}} . \tag{11}$$

Equations (7) through (11) decompose the congestion attribute of a given transmission upgrade from its effect on reliability. Blumsack, Ilić, and Lave (2006) derive explicit topological conditions under which the addition of a new transmission line will decrease congestion, but will not improve reliability (and vice versa). In particular, they show that reliability and congestion can represent tradeoffs only if the network has an embedded Wheatstone sub-network. Blumsack (2006b) provides a graph-theoretic algorithm for identifying embedded Wheatstone structures in larger networks. Application of the cost-benefit calculus in equations (7) through (11) to the four-bus Wheatstone test network yields the following insights:

1. For a level of demand P_{L4} = 100 MW, the Wheatstone bridge congests the network, but also provides a reliability benefit in the case of an outage on lines S_{24} or S_{34}. The Wheatstone bridge will pass the cost-benefit test if the marginal value of power consumption to the consumer at bus 4 is at least as large at the difference in marginal costs of the two generating units.
2. Whether the Wheatstone bridge provides a net benefit or imposes a net cost on the system is highly sensitive to the network topology and the level of demand at bus 4.

3. In a simple series-parallel network, a transmission upgrade cannot simultaneously provide a reliability benefit while imposing a congestion cost. Thus, congestion and reliability can only represent tradeoffs in systems with an embedded Wheatstone sub-network (Blumsack, Ilić, and Lave 2006a).

4. Application to the IEEE 118-bus Network

The four-bus test network from Section 3 is useful for illustrating the concepts behind equations (7) through (11), but is not a very descriptive model of an actual system. A Wheatstone network embedded in a larger system might have net generation or load at all four buses; generation or load at one end of the Wheatstone bridge could sufficiently alter the pattern of flows such that congestion is avoided. The reliability benefit of the Wheatstone bridge also depends on how it interacts with the rest of the network; the loss of one of the boundary links might simply be made up by increased flows from other portions of the network, with the Wheatstone bridge not adding any benefit to the system at all.

The magnitude of the relationship between congestion and reliability is ultimately an empirical question. We examine the congestion and reliability effects of four Wheatstone sub-networks in the IEEE 118-bus test network, using the same simulation procedure as in Section 3. The four Wheatstone sub-networks are indicated in Figure 3, which also shows the topology of the 118-bus network.[7] The networks are labeled A, B, C, and D. The system topologies for the four networks and illustrative base case power flows are

[7] Data for the test network were downloaded from the IEEE Power Systems Test Case Archive at http://www.ee.washington.edu/research/pstca/.

shown in Table 1, for comparison with the four-bus Wheatstone test network of Figure 2.[8] The base-case power flows are used to determine "upstream" and "downstream" portions of each embedded Wheatstone sub-network, as described below.[9]

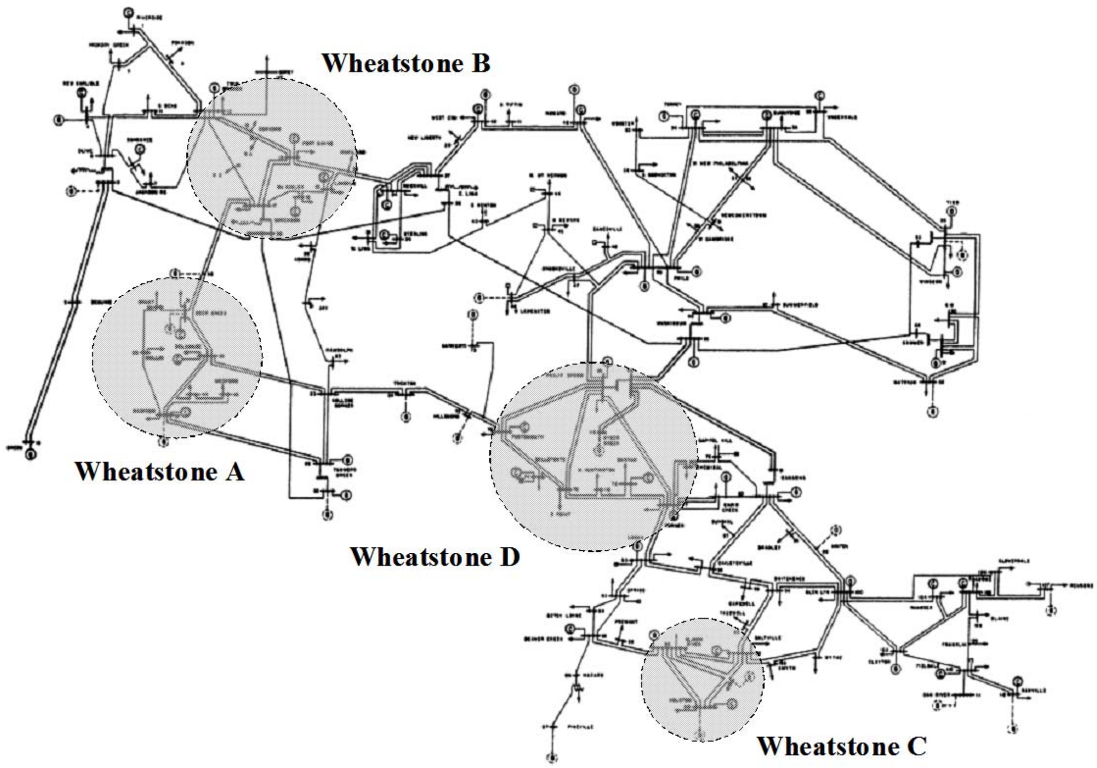


Figure 3: Four Wheatstone sub-networks of the IEEE 118-bus test network.

For each level of demand in each Wheatstone sub-network, the associated congestion cost is measured using equation (7). Thus, the congestion cost is defined to be the difference in total system cost to serve identical demand profiles in a system with the Wheatstone bridge and without the Wheatstone bridge.

[8] One-line diagrams illustrating the base-case power flow results can be found in Blumsack (2006a, Ch.6)

[9] Depending on supply and demand conditions throughout the network, it is theoretically possible that the "upstream" and "downstream" portions could change. We did not encounter this phenomenon in the simulations performed for this paper.

		Wheatstone Sub-Network A	B	C	D
S_{12}	Reactance (p.u.)	0.09	0.02	0.12	0.13
	Base-Case Flow (MW)	27	13	220	104
S_{13}	Reactance	0.12	0.02	0.05	0.14
	Base-Case Flow	23	2	186	33
S_{24}	Reactance	0.09	0.05	0.05	0.1
	Base-Case Flow	32	92	121	210
S_{34}	Reactance	0.07	0.01	0.05	0.19
	Base-Case Flow	15	25	257	19
S_{23}	Reactance	0.09	0.04	0.05	0.12
	Base-Case Flow	35	118	151	140

Table 1: Topology and base-case power flows in four Wheatstone sub-networks embedded in the IEEE 118-bus test system.

Measuring the cost of unserved energy is more subtle, and requires differentiating involuntary load-shedding from voluntary load curtailment. Assuming that the LMPs contain the proper components, as in Bohn, Caramanis and Schweppe (1984), end-use customers seeing market prices will voluntarily curtail demand at a level P_{Li}*, where the marginal social cost of an additional unit of power exceeds their marginal value function; that is, when $\text{LMP}_i(P_{Li}^*) > v_i'(P_{Li}^*)$. Involuntary load-shedding is a control action on the part of a centralized grid operator, arising from physical scarcity of generation resources or transfer capacity into a given location in the network. Joskow and Tirole (2006) argue that in systems consisting of a mixture of price-sensitive and price-insensitive loads, socially optimal grid management requires that price-sensitive load never be involuntarily curtailed.

Following Joskow and Tirole's argument, we make two simplifying assumptions that abstract from the possibility of voluntary load curtailment in our simulations, and ensure that any unserved energy arises due to physical limitations in the network. Our first assumption is that all consumers in the network have an identical value function for electricity. We will call this marginal value the value of lost load (VOLL).[10] The second assumption is that the VOLL is constant for any level of electricity use, and is equal to $1000/MW-interrupted. We also assume that all generators in the network have a constant marginal cost. Our chosen VOLL is well above the marginal cost of generation in the 118-bus network. Figure 4 shows the system marginal cost curve.

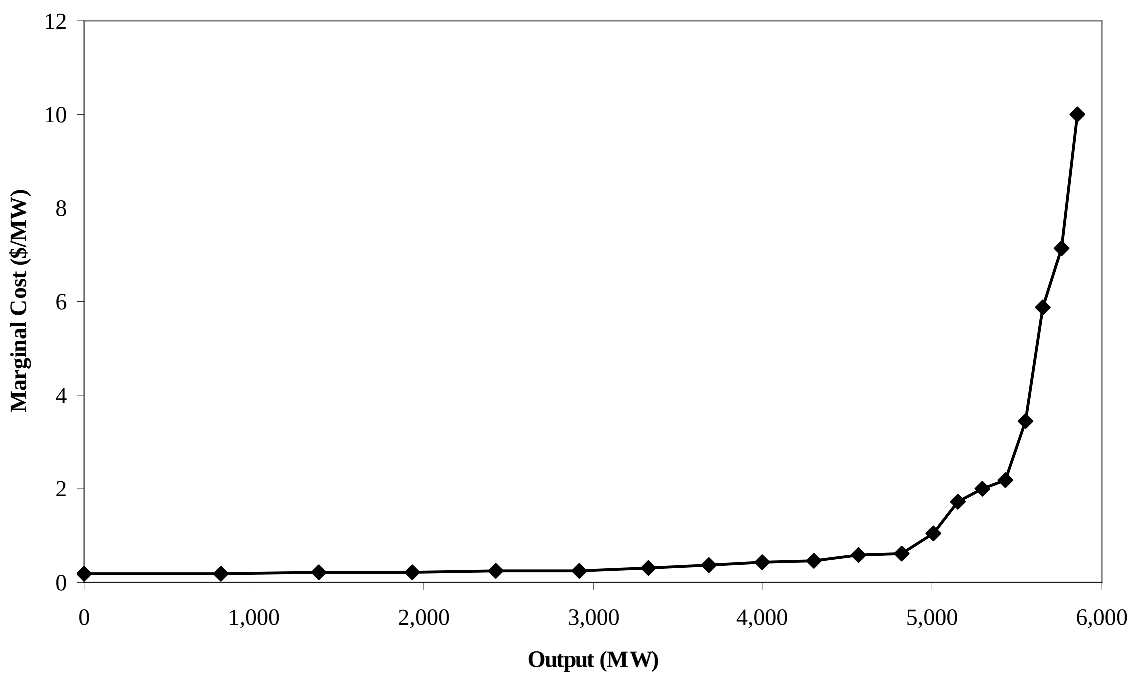


Figure 4. System marginal cost curve for the IEEE 118-bus network.

[10] Joskow and Tirole (2006) demonstrate how the value of lost load can be derived directly from the value function.

Thus, the value of unserved energy can be rewritten as:

$$VUE = U \times \left(VOLL \times P_{Li}^{k,-m} - VOLL \times P_{Li}^{-k,-m} \right). \tag{9'}$$

Note that in equation (9') we are implicitly defining line *k* as the Wheatstone bridge. The expected net benefit of a given embedded Wheatstone bridge is thus:

$$E(NB) = \underbrace{u \times \left(VOLL \times P_{Li}^{k,-m} - VOLL \times P_{Li}^{-k,-m} \right)}_{\text{reliability}} - \underbrace{(1-u)\sum_{i=1}^{NB}\left(C_i(P_{Gi}^{k}) - C_i(P_{Gi}^{-k}) \right)}_{\text{congestion}}. \tag{11'}$$

We evaluate the costs and benefits of each embedded Wheatstone sub-network through simulations. Our simulations focus on the effects of variation in demand at the downstream end of the Wheatstone network, and on the network topology. We consider levels of demand at the downstream node between 0 and 500 MW, while holding demand constant at other locations in the network.[11] In the simulations, we focus on line outages that occur on the boundary of each Wheatstone sub-network; we consider outage probabilities between 10^{-7} and 10^{-1} (thus, the $(1-u)$ term in equation (11') is never large).[12] We also assume that each line outage lasts only one period (since we assume a constant value of lost load in our analysis, the length of line outages should not materially affect our results).

[11] The case of demand varying simultaneously at all buses is considered in Blumsack, Ilić, and Lave (2006b).
[12] Larger outage probabilities were examined but are not included here. Once the outage probability becomes much larger than 10% (u = 0.1), at larger levels of demand, both the congestion cost and reliability benefit get large enough so as to obscure the behavior of the net benefit function at lower levels of demand.

We assume that there is a system operator who dispatches generating resources to solve the following optimization problem:

$$\min_{P_{Li}, P_{Gi}} \sum_i \left(C_i (P_{Gi}) + VOLL_i \times UE_i \right) \tag{12}$$

subject to:

$$\sum_j F_{ij}^{transfer} = P_{Gi} - P_{Li} \qquad \forall i \tag{12a}$$

$$\mathbf{g}(\mathbf{x}) = \mathbf{0} \tag{12b}$$

$$\mathbf{h}(\mathbf{x}) \leq \mathbf{0} \tag{12c}$$

where $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ represent the equality and inequality constraints in the optimal power flow problem, such as capacity limits on generators and transmission lines. We use the DC optimal power flow formulation in our simulations, as in equations (3) and (4), so the constraints in equations (12b) and (12c) are all linear. The term $\sum_j F_{ij}^{transfer}$ represents the total amount of power than can be transferred into or out of the *i*th bus.[13] Thus, constraint (12a) represents the power balance condition.

[13] Transfer capacity out of a given bus is constrained by the amount of available generation and transmission. Transfer capacity into a given bus (or other area of a power network) depends on the supply/demand balance in other portions of the network. We take the total transfer capacity as exogenous. Wood and Wollenberg (1996) discuss the calculation of marginal and total transfer capacity.

Calculating the net benefit of a given Wheatstone bridge in equation (11') requires solving the optimal power flow problem in equations (12) for four different topological cases:

Case I: The "base case" set of DC optimal power flows, where the sub-network has the Wheatstone bridge, and there is no assumed contingency on any of the transmission lines.

Case II: Same as Case I, but the DC optimal power flows are run on the sub-network without the Wheatstone bridge.

Case III: This case assumes an outage on one of the boundary links in the Wheatstone sub-network, but assumes the sub-network has a Wheatstone bridge.

Case IV: An outage is assumed on one of the links, and there is no Wheatstone bridge in the sub-network.

The power-flow results from Cases I and II are used to calculate the congestion effect of the Wheatstone bridge, and the results from Cases III and IV are used to calculate the reliability effect of the Wheatstone bridge. Wheatstone sub-networks C and D are representative of our simulation results, so we discuss those in greater detail than Wheatstone sub-networks A and B. The full set of results from simulations run on all four sub-networks are in Blumsack (2006a, Ch.6).

4.1. Cost-Benefit Analysis of Wheatstone C

Wheatstone sub-network C is located in the southeastern portion of the IEEE 118-bus network, as shown in Figure 3. This Wheatstone has two of its four component buses connected to the external network. From Table 1, we see that power flows from the external network through the Wheatstone network towards bus 90. Thus, bus 90 is designated as the downstream node for this sub-network.

Figures 5 and 6 show the congestion cost and reliability benefit of the bridge in Wheatstone C. The real power demand at bus 90 is assumed to vary between 0 and 500 MW; we hold demand constant at all other nodes in the network. The positive value for the congestion cost indicates congestion charges associated with the Wheatstone bridge. At lower levels of demand, Figure 5 shows that the congestion caused by the Wheatstone bridge increases with the level of demand, just as in the four-bus test network shown in Figure 2. At demand levels larger than 450 MW, generation from elsewhere in the network is dispatched to meet the increased load at bus 90, and the congestion cost associated with the Wheatstone bridge declines. The expected congestion cost does not vary widely with the outage probability because we only consider small outage probabilities (less than a 10% chance of an outage).

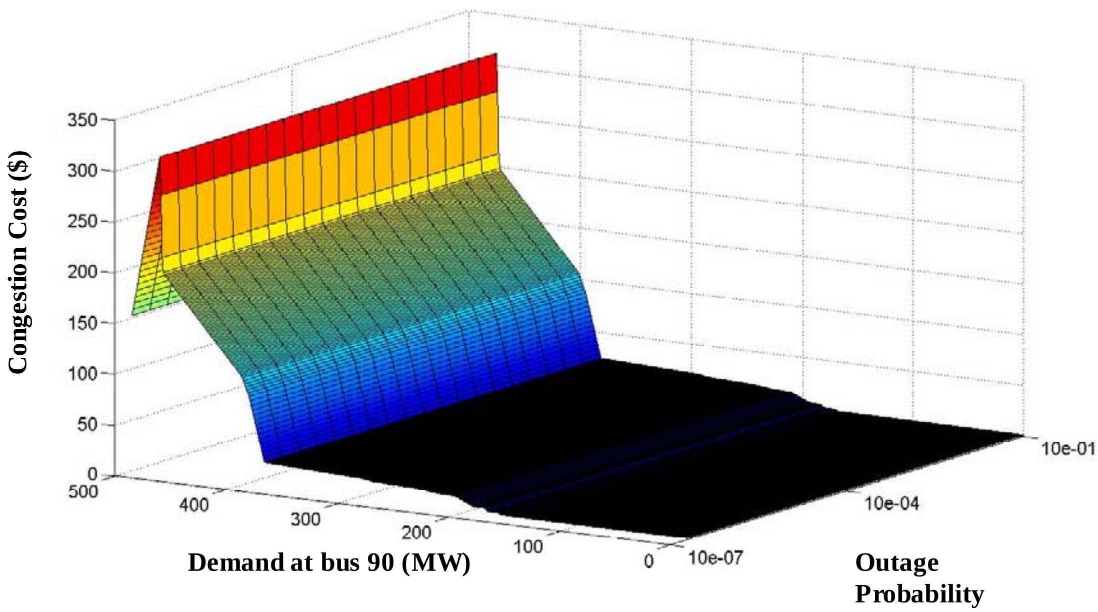


Figure 5: Expected congestion cost associated with the bridge in Wheatstone C.

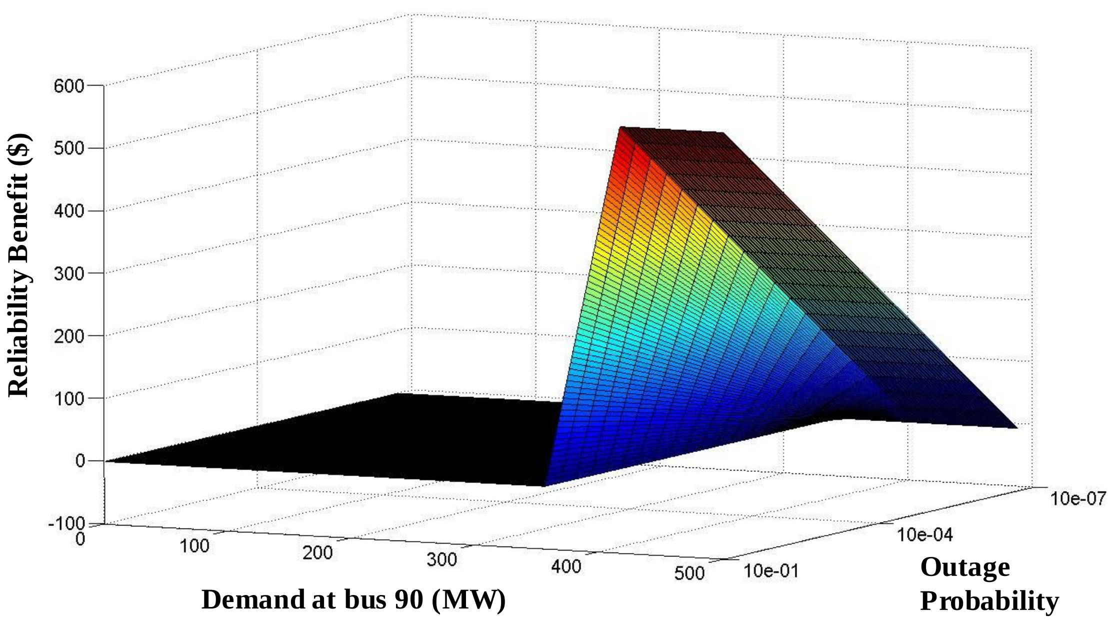


Figure 6: Expected reliability benefit associated with the bridge in Wheatstone C.

The reliability benefit associated with the Wheatstone bridge is shown in Figure 6. At low levels of demand, the capacity in Wheatstone sub-network C is large relative to demand, so a single line outage makes little difference in the ability of power to be transferred across the network towards bus 27. At larger levels of demand, the expected reliability benefit is highly sensitive to both the level of demand and the outage probability.

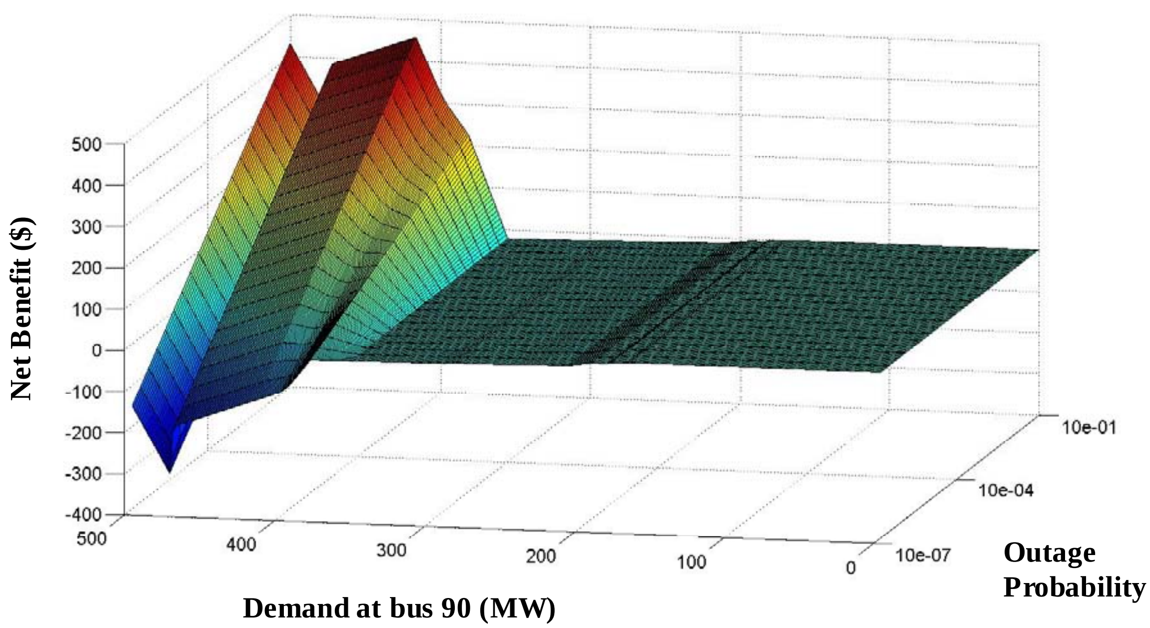


Figure 7: Expected net benefit associated with the bridge in Wheatstone C.

The net benefit of the bridge in Wheatstone sub-network C is shown in Figure 7. An instructive comparison can be made between the behavior of Wheatstone C and the four-bus test network discussed in Section 2. In Wheatstone C, congestion and reliability are only independent for low levels of demand (150 MW or less). For this range of demand, the Wheatstone imposes a congestion cost while the reliability benefit is zero. Only at higher levels of demand does the net benefit function indicate the tradeoff between the congestion cost imposed by the Wheatstone bridge and its reliability benefit. Once the reliability benefit kicks in, the net benefit function will rise more sharply if the probability of an outage is larger; for low outage probabilities, the congestion component of the net benefit function dominates.

4.2. Cost-Benefit Analysis of Wheatstone D

The second Wheatstone sub-network discussed here is located in the middle of the IEEE 118-bus network, just northwest of Wheatstone C. It is located near some of the system's larger and less expensive generating units located at buses 80 and 65. The congestion and reliability properties of this sub-network should be different than the other three Wheatstone sub-networks. The base-case power flow run on this Wheatstone sub-network indicates that bus 77 should be considered the downstream bus; power flows from the external network through the Wheatstone towards bus 77.

Figures 8 and 9 show the congestion cost and reliability benefit associated with the Wheatstone bridge in sub-network D. The tradeoff between congestion and reliability evident in Wheatstone sub-networks A, B, and C is not as evident. In the other three sub-networks discussed here, the congestion cost rises (more or less) monotonically with demand. However, Figure 8 shows the congestion cost rising and falling in a roller-coaster pattern. For the most part, the Wheatstone bridge in sub-network D has negative congestion costs, meaning that the presence of the bridge reduces congestion rather than causes congestion. The reliability benefit associated with the Wheatstone bridge in sub-network D, as a function of demand and the outage probability, behaves similarly to the other three Wheatstone sub-networks.

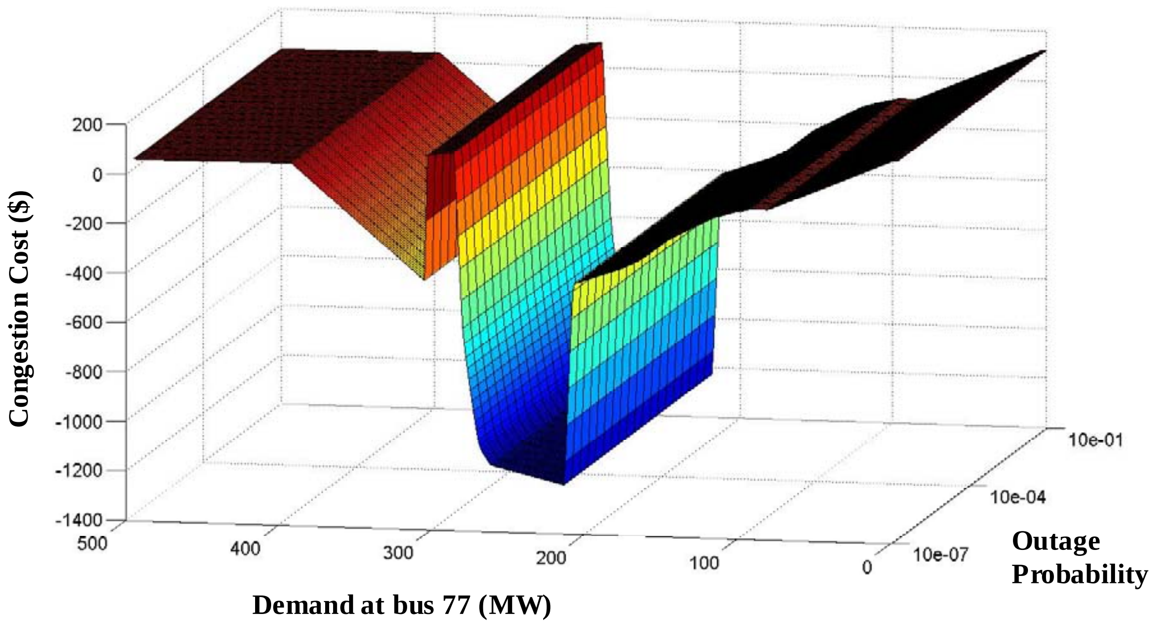


Figure 8: Expected congestion cost associated with the bridge in Wheatstone D.

The same roller-coaster pattern of the net benefit function can be seen in Figure 10, which shows the total net benefit function as both demand at bus 77 and the outage probability vary. The shape of the total net benefit function is nearly identical to the shape of the congestion cost curve in Figure 8. We conclude from Figures 8 through 10 that congestion and reliability are not independent in Wheatstone D, but neither do they represent tradeoffs. In this case, congestion and reliability are complementary. The Wheatstone bridge could be justified for reliability reasons, but (over a large range of demand) congestion would decrease as well.

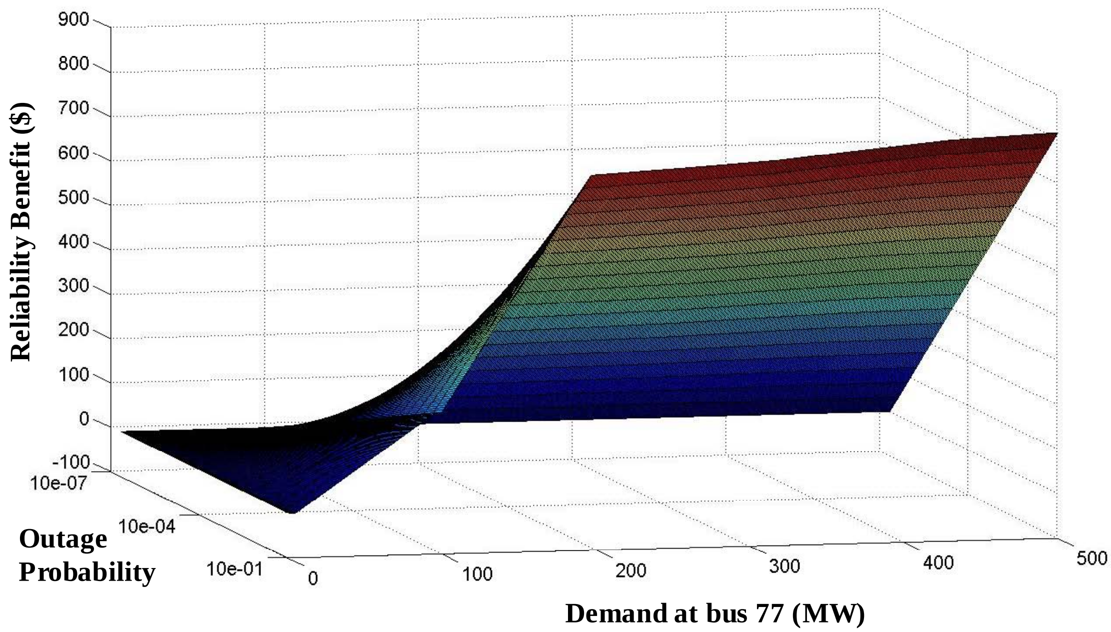


Figure 9: Expected reliability benefit associated with the bridge in Wheatstone D.

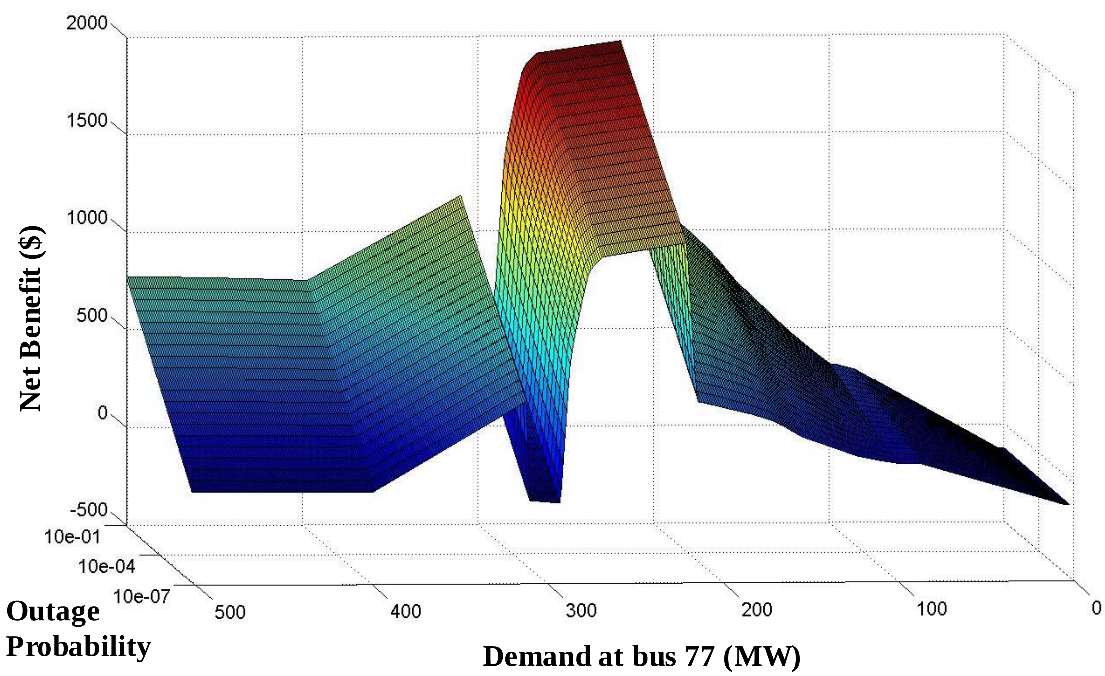


Figure 10: Expected net benefit associated with the bridge in Wheatstone D.

For a given level of the outage probability, the net benefit of the Wheatstone bridge should be an increasing function of the level of demand. Wheatstones A and C both behave this way, but the relationship is somewhat less clear for Wheatstone B and is virtually nonexistent for Wheatstone D. This highlights the influence of the system, and the importance of location, on a given individual Wheatstone sub-network. Wheatstone sub-networks A and C are located topologically further away from the center of the 118-bus network. More importantly, Wheatstones A and C have fewer connections to the external network. Thus, the external network has less influence over the behavior of Wheatstones A and C than over the behavior of Wheatstones B and D.

The most significant portion of the external network in explaining the behavior of Wheatstones D is the location of large and inexpensive generation in close proximity. Generators at buses 80 and 65 are directly connected to Wheatstone sub-network D; the generator at bus 80 is directly connected to the downstream load bus of Wheatstone D. Empirically, we found that changes in the dispatch of generator 65 had the greatest influence on the congestion-cost function shown in Figure 11. To illustrate the influence of these generators on the congestion cost and reliability benefit associated with the Wheatstone bridge in sub-network D, we artificially increased the marginal cost of the generator at bus 65 by a factor of ten, from $2.5/MWh to $25/MWh. The new net benefit, after the cost increase, is shown in Figure 11. After increasing the marginal costs of the generator at bus 65 to the point where it no longer changes dispatch in response to changes in demand at bus 77, the Wheatstone net benefit function looks much like the net benefit functions from Wheatstone C.

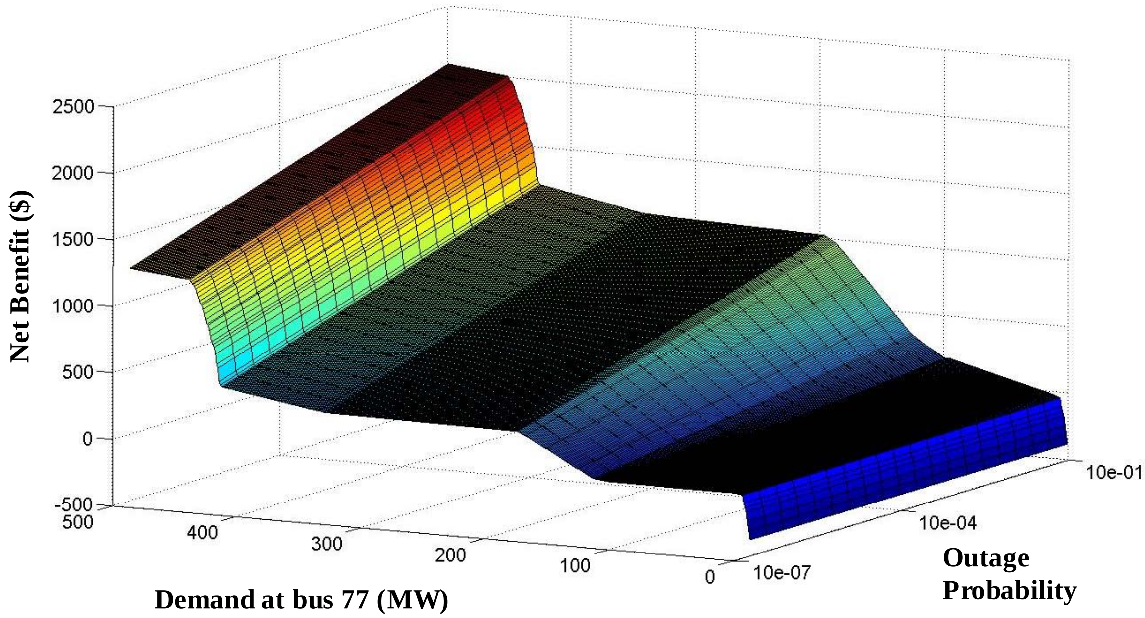


Figure 11: Expected net benefit associated with the bridge in Wheatstone D, after the marginal cost of generation has been increased at bus 65.

4.3. Cost-Benefit Analyses of Wheatstones A and B

The net benefit functions for Wheatstone sub-networks A and B are shown in Figures 12 and 13. Both of these sub-networks are located in the northwestern portion of the IEEE 18-bus network. Like Wheatstone C, they are located closer to the boundary of the network; this differentiates them from Wheatstone D, which has a more interior location. Figure 12 shows that Wheatstones A and B display the same sort of tradeoff between congestion and reliability as Wheatstone C.

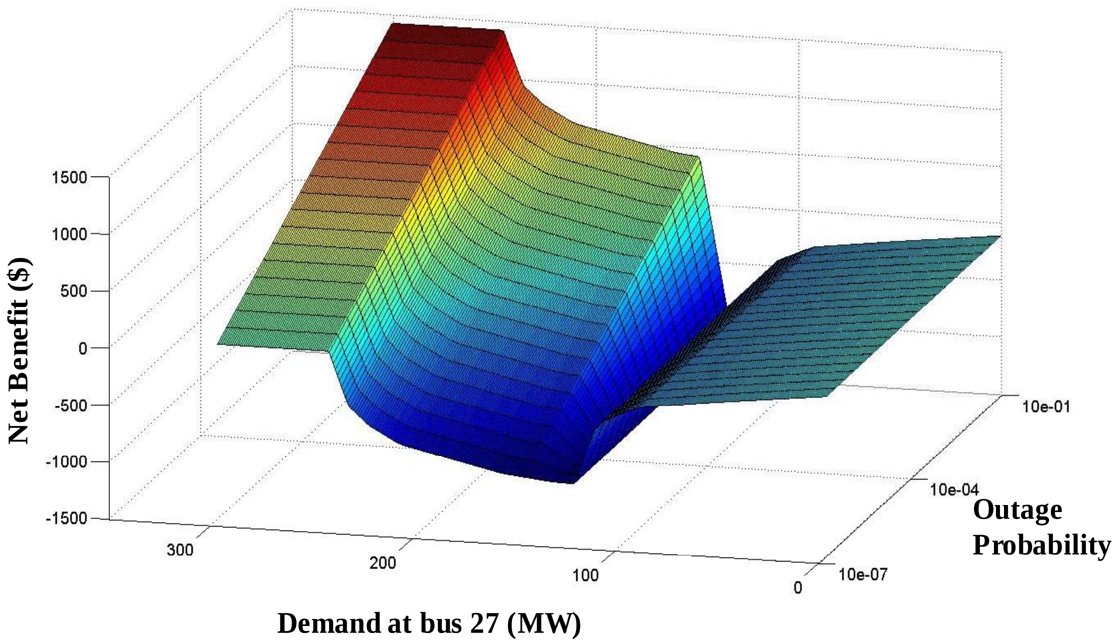


Figure 12: Expected net benefit associated with the bridge in Wheatstone A.

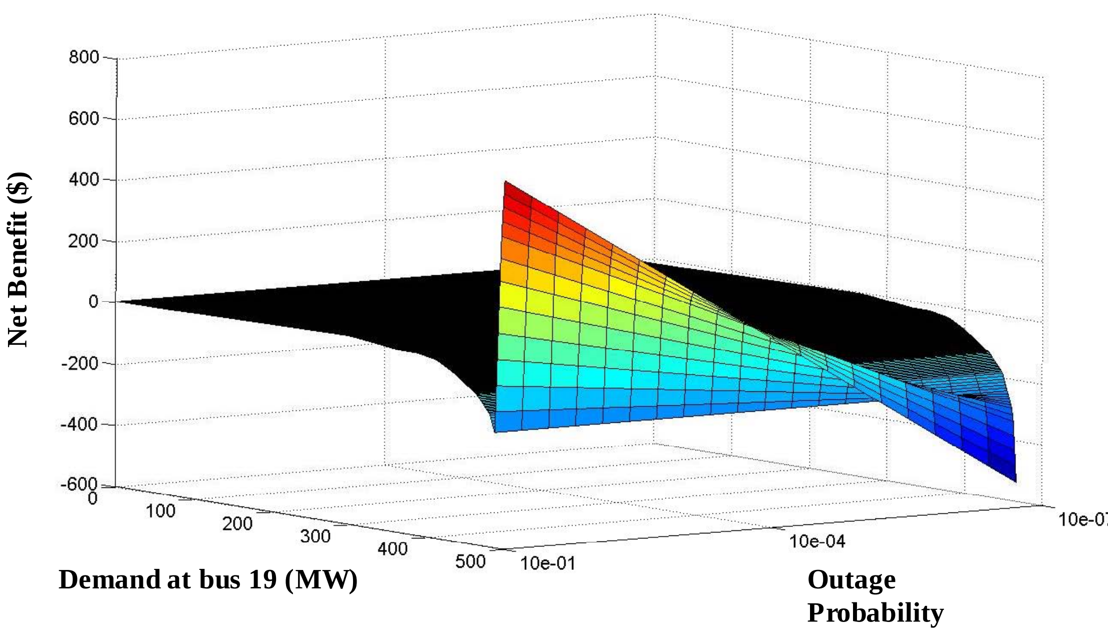


Figure 13: Expected net benefit associated with the bridge in Wheatstone B.

5. Conclusion

AC transmission investment is a systems problem. The nature of the problem has not changed with electric-industry restructuring, but some aspects of practice and policy have changed. In the past, most transmission investments were made with reliability and resource adequacy in mind. With the move to congestion pricing and market-based congestion management in many operating areas, and the policy focus on developing robust spot markets to support the congestion-management function of the RTO, the focus of transmission policy has changed. The job of the transmission network is not just to deliver power to customers, but is also to facilitate competition among generators. As such, the criteria for evaluating and deciding among transmission investments has shifted to emphasize the effects on competition and congestion.

Underlying this shift is an assumption that congestion and reliability are separable and independent attributes in electric networks. In this paper, we have shown that while the network attributes can be decomposed, they are rarely independent, except at levels of demand that are small relative to the capacity of the network to transfer power. Certain directed network investments, such as HVDC lines, might also be justified on economics alone, since these investments are effectively separable from the larger grid (Coxe and Ilić 1998). Whether congestion and reliability represent tradeoffs or complementary system attributes, the nature of the relationship is also dependent on the network topology and the degree to which the network is meshed. Transmission links built in simple series-parallel networks are more likely to affect a single system attribute than are links that create Wheatstone sub-networks or other meshed structures. This observation not only clarifies the nature of the externality involved, but also suggests that academics and policymakers need to work with more sophisticated network models.

We conclude that the behavioral attributes of the four-bus standalone Wheatstone network are not universally generalizable to Wheatstone structures within larger systems. The behavior of the embedded Wheatstone networks in the 118-bus system is influenced by their location, electrical proximity to nearby network resources (that is, relative impedances), and by the initial topology of the network as a whole. We find that Wheatstone sub-networks located in more densely-connected portions of the network are more likely to be influenced by variations in adjacent load and generation. One possible implication for the planning process is that locating a Wheatstone in the middle of a meshed network and near large sources of inexpensive generation is unlikely to cause major congestion problems that would exist if the same Wheatstone were placed at the boundary of the network.

The key to analyzing general network modifications is recognizing the nature of the relationship between congestion and reliability. As with any economic externality, the tradeoff needs to be defined over the relevant range of demand. Current transmission policy endorsed by some ISOs and RTOs, and even the U.S. Congress errs in failing to realize these distinctions. Current policy treats the transmission system as if individual lines could be divided into those that benefit the system through added redundancy, and those that harm the system by causing congestion. Transmission lines are also treated as if their contribution to the system is independent of the state of the system. The RTO and

FERC rationale for market-based merchant transmission is largely based on this false congestion-reliability dichotomy.

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