Phase Transitions in the Probability of Cascading Failures*

Huaiwei Liao[1], Jay Apt[2], Sarosh Talukdar[3]

Carnegie Mellon University, Pittsburgh, PA 15213

***Abstract*—A cascading failure can be thought of as an alternating sequence of equipment-outages and threshold-crossings. This paper studies the probability of such failures in two simple models of electric power networks. The experimental results display phase transitions--large and abrupt changes in the probability of a cascading failure with only small changes in network stress. We conjecture that such phase transitions also occur in actual power networks. If this conjecture is true, on-line techniques for assessing the risk of cascading failures could be based on searching the neighborhood of the current operating point for the nearest phase transition.**

I. Introduction

NERC data indicate that the cause of most cascading failures is a multiple contingency—a random disturbance, such as a short circuit, accompanied by a number of pre-existing but hidden failures in the protection system. One might think that multiple contingencies are extremely rare. But NERC data show they happen often enough to give the distribution of cascading failures a fat tail (in contrast to an exponentially falling tail, as in a normal distribution). In other words, the probabilities of large cascading failures are substantial; even failures big enough to blackout an entire network, such as the Eastern Interconnect, could happen.

This paper deals with a) the probability that cascading failures will develop from multiple contingencies, and b) the variation of this probability with the stress of the network. Experiments are conducted on two types of networks—a regular, square, 50X50 grid, and a simplified model of a 3357-node power network. Both types demonstrate pronounced phase transitions. In what follows, the experiments are described, and some conclusions are drawn from their results.

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[1] Huaiwei Liao is a PhD student in the ECE Department of CMU.

[2] Jay Apt is a Distinguished Service Professor in Engineering and Public Policy and executive director of the CMU Electricity Industry Center.

[3] Sarosh Talukdar (talukdar@cmu.edu) is a Professor of Electrical and Computer Engineering in CMU.

II. Models

The outage of a component, such as a transmission line or generator, is a discrete event that suddenly changes the network's configuration. The dynamic response of the network to such changes can over-stress some of its remaining components. Thousands of relays spread over the network, seek to keep the over-stresses from lasting long enough to cause harm. Each relay measures a few state variables (for example, v_k and i_k, the voltage and current at node-k), and checks a built-in threshold (for example, $|v_k/i_k| \geq a$, where a is a pre-set constant). When the threshold is crossed, the relay instructs one or more breakers to operate, taking the components in the relay's care out of danger (and out of service). Symbolically:

$$T_0 = f\,(E_0, X_0, Z_0) \qquad (1)$$

where E_0 is a set of simultaneous outages at time t_0, X_0 is the state of the network at t_0, Z_0 is the configuration of the network at t_0, and f calculates the network's dynamic response to outages as well as predicting T_0, the set of threshold-crossings caused by this response.

Unless T_0 is empty, it will produce E_1, a set of further outages. If all the relays and breakers work correctly, E_1 will be just as the network's designers intended. But relays and breakers can fail, and often, these failures go undetected till the relays and breakers are required to act. Therefore:

$$E_1 = g\,(T_0, H) \qquad (2)$$

where H is the set of hidden failures in the network and g represents the mechanism by which hidden failures and threshold-crossings interact.

The value of H is uncertain, and is perhaps the largest source of uncertainty in attempts to predict E_1. Other sources are in the values of the thresholds and the models of network-dynamics.

Just as E_0 leads to T_0 and E_1, so to, E_1 can lead to still further crossings and outages in a long cascade. In other words, $C(S)$, a cascading failure of length L and size |S|, can be thought of as an alternating sequence of equipment-outages and threshold-crossings:

$$C(S) = \{E_0, T_0, E_1, T_1, \ldots, E_L, T_L\} \quad (3)$$

where:

E_0 is a set of one or more outages that starts the cascade; E_0 is usually caused by a multiple-contingency.

T_n is the *n*-th set of threshold-crossings (constraint violations) in the sequence

E_n is the *n*-th set of equipment-outages in the sequence

T_L is the only empty set in the sequence, and signals its end

$S = E_0 \vee E_1 \vee \ldots \vee E_L$ is the set of all the components lost during the cascade.

There are two principal difficulties in accurately predicting $C(S)$, given E_0:

Complexity: The relays in power networks set thresholds on several types of variables, including impedance, voltage, current, and frequency. The more detailed models for calculating the dynamics of these variables require large computational and data-gathering efforts.

Uncertainty: The responses of power networks to sudden changes are profoundly uncertain. Unknown load dynamics, imprecise parameter values, and hidden failures are among the reasons.

The procedures in the literature for simulating cascading failures invariably use DC load flows—the replacement of dynamic responses by a steady state approximation—to reduce network complexity [9,10,11]. In what follows, we will use DC load flows for the same reason, and probabilities to deal with the uncertainties.

III. Phase Transitions

Consider the probability, P(y), of a cascading failure, where:

$$P(y) = \text{Probability}\,[\, C\,(|S| \geq y) \;\big|\; E_0 \wedge H \;\wedge x \,] \quad (5)$$

In words, $P(y)$ is the probability that a random set of outages, E_0, will produce a cascading failure of size y or greater, given H and x, where H is a set of hidden failures and x is a measure of network-stress, such as average line-loading.

For the purposes of this work, a *phase transition* occurs when a small change in the stress on the network produces a large and abrupt change in P(y).

Three experiments for estimating $P(y)$ with simplified network models are described below.

Ω1: The network in this experiment is a regular, square, 50X50 grid whose branches are equal resistors (Figure 1). Generators and loads are distributed over many of the 2500 nodes. The generators and loads vary randomly in rating and location. The net generation is adjusted to be roughly the same as the net load. The difference is made-up by a slack generator at one corner of the grid. The power output of the *j*-th generator is set to $xG_{,j}$, and the power consumed by the *j*-th load is set to $xH_{,j}$, where the parameter, x, serves as a measure of the stress on the network, $G_{,j}$ is the rating of the *j*-th generator, and $H_{,j}$ is the rating of the *j*-th load. The same current-threshold is assigned to all the branches. Hidden failures are neglected. Each experiment is begun by choosing a value for x and randomly removing from 1 to 4 branches. The currents through the remaining branches are calculated using a DC-load-flow. If any current exceeds its threshold, the branch is removed. Calculations are continued till no further thresholds are exceeded.

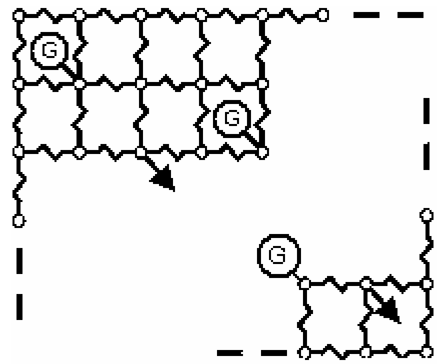

Figure 1: A 50X50 grid with equal resistors, randomly placed generators (G's), and randomly placed loads (arrows)

Ω2: The network for this experiment is obtained by removing 4 branches from the network of the previous experiment. Otherwise, this experiment is the same as Ω1.

Ω3: The network for this experiment is based on an actual power system with 3357 nodes (Figure 2). Branch-impedances are those of the transmission lines in the system, as are generator and load ratings. The branch-thresholds are taken to be the ratings of the lines. Otherwise this experiment is the same as Ω1.

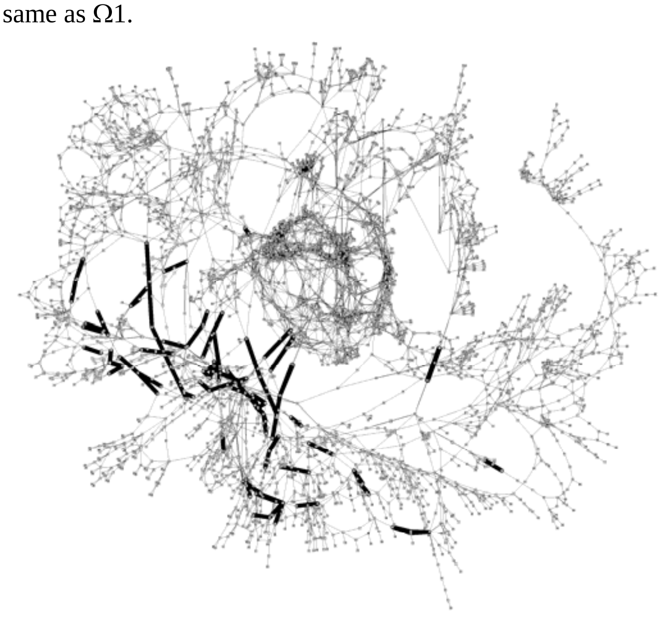

Figure 2: A 3357-node network. The darker arcs represent the lines lost in one of many cascading failures that were

studied.

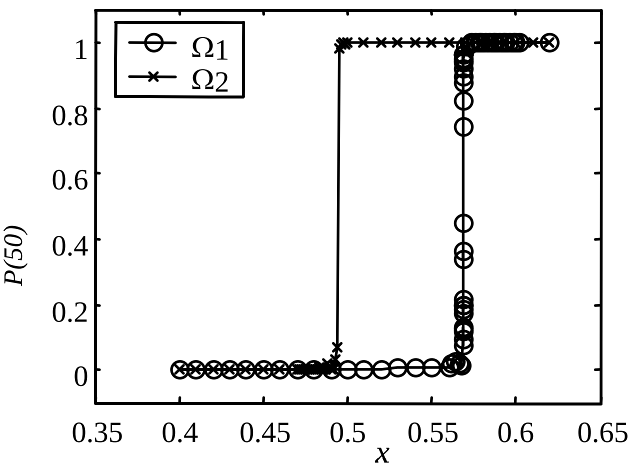

Figure 3: *P(50)*, the probability of a cascading failure of size 50 or greater, plotted against x, the base load multiplier, for the network of experiment Ω1, and for the weakened network of experiment Ω2. As expected, the weakened network has an earlier phase transition.

Typical results from many repetitions of Ω1, Ω2 and Ω3 are shown in Figure 3 and Figure 4.

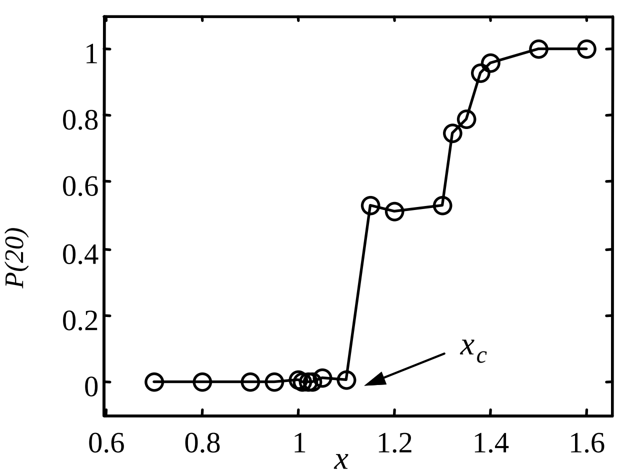

Figure 4: *P(20)*, the conditional probability of cascading failures of size 20 or greater, plotted against x, the base load multiplier, for the 3357-node network of Fig. 2. x_c marks the critical point at the beginning of a phase transition.

IV. CONCLUSIONS

We have repeated the experiments, Ω1, Ω2 and Ω3, with many different values of their network parameters. Every repetition displayed phase transitions of the sort shown in Figs 3 and 4.

Actual power networks are different from the networks we studied in at least three respects. Actual networks have multiple threshold-types, complex dynamics and hidden failures; our networks had only one threshold-type, were always in a steady state, and had no hidden failures. Nevertheless, we believe that our results extend to actual networks. We conjecture that the probability of a cascading failure, given a multiple contingency, is subject to phase transitions in actual power networks, just as it is in the simpler networks we studied.

If this conjecture is true, on-line techniques for assessing the risk of cascading failures could be based on searching the neighborhood of the current operating point for phase transitions in the cascading-failure-probability.

Questions raised by our work include:

1. Is our conjecture true? If so:
2. What set of multiple contingencies should be considered in assessing the probability of cascading failures?
3. What measures of stress, x, should be considered? (What measures span the neighborhood of the current operating point?)
4. How should hidden failures be modeled?
5. How should the dynamics of actual power networks be modeled?
6. How should the neighborhood of the current operating point be searched for phase transitions?

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