Clean air and affordable electricity?

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Abstract— Performance criteria of the power industry such as environmental impact, electricity prices, and quality/reliability of the service are functions of fuel market, government regulations, the state of the art of technology, and the combined actions of different industry participants. We argue that to prescribe optimal government interventions, it is imperative to understand as much as possible about the dynamics of the interaction between industry participants, as well as the interactions with fuel and technology markets. In this paper we propose a model that relates air emissions and electricity prices with government policies regarding allocation of emissions allowances, fines, subsidies and investments in R&D. We present results of the simulation of a simplified model.

Index Terms—Pollution control, market models, governmental factors, game theory.

I. INTRODUCTION

Coal-fired power units account for more than 50% of electricity generation and are the single biggest source of air pollution in the U.S. The reduction of emissions from the electricity generation sector implies either retrofitting existing coal-fired power plants or replacing them with newer and cleaner technologies. Retrofitting existing coal plants to reduce emissions of sulfur-oxides, nitrogen-oxides, mercury, or carbon-dioxide requires equipment that is expensive to install and operate. The retirement of existing power plants also implies intensive capital investments and therefore higher costs for electricity generation.

Since the availability of affordable electricity is a necessary condition for GDP growth, the goal of minimizing the level of toxic emissions in the atmosphere while keeping electricity prices low poses a very serious challenge to society.

A very relevant question arises: how should the government act over a finite period of time to achieve desired levels of atmospheric emissions and electricity prices?

In order to shed some light on this issue, we propose a model of the electricity generation system that (1) explicitly considers a number of control instruments that the government can use to balance atmospheric emissions and electricity prices, and (2) accounts for the fact that industry participants make strategic decisions to maximize their profits and face several sources of uncertainty. The model is constituted by four subsystems: the fuel market, the electricity market, the environmental or allowances market, and the market of equipment for control of emissions.

A computational implementation of a simplified model and a simulation over a number of years allow us to comment about how this problem needs to be studied.

II. BACKGROUND

There are at least three big efforts to understand the effects that environmental legislations for the electricity generation system may have on the U.S: 1) the National Energy Modeling System (NEMS)[2], 2) the Argonne National Laboratory’s AMIGA model [3], and 3) the EPA’s Integrated Planning Model (IPM)[4].

NEMS and AMIGA are general equilibrium models of the U.S. economy, while IPM is a bottom-up linear programming model of the electric power sector. All assume that decisions by industry participants are made with perfect foresight, and forecast electric power sector decisions for a given set of environmental regulations.

The NEMS and AMIGA models forecast capacity additions, fuel dispatching, and electricity prices based on different endogenous and exogenous inputs of the electric sector and the U.S. economy. IPM forecasts decisions made from the national to the plant level in response to legislative requirements seeking to minimize the net present value of the cost of compliance over the planning horizon.

A recent analysis using NEMS was prepared in response to a request by the U.S. Congress to examine the costs of imposing caps on power sector emissions of SO2, NOx, Hg and CO2. Some results of this analysis are contained in “Strategies for Reducing Multiple Emissions from Electric Power Plants”[5]. AMIGA and IPM models have also been recently used to assess the impacts of legislations to reduce emissions from the electricity sector.

1 Electricity supply from coal accounted for 54% in 2001. EIA forecasts that under reference case assumptions, this tendency will remain in the next 20 years.[1]

2 Also Johnson and Keith [6] developed a model that forecasts capacity additions, retirements, and retrofits for different prices of
We believe that it is necessary to analyze the problem with a model that differs from the existing ones in that the model (1) explicitly accounts for the fact that industry participants are making decisions under uncertainties about future regulations, fuel prices, and other participant’s actions, (2) accounts for different instruments of government intervention such as subsidies and investments in R&D, and for details of the regulation such as how and when allowances are allocated to generating units, and (3) frames the question of what the government should do as an optimization problem in which both electricity price and air-emissions are considered.

III. MODEL DESCRIPTION

We represent the electricity generation sector with a number of generating units that vary in fuel, size, and efficiency.

The generation cost for each unit is determined by some fixed costs, the cost of fuel, and the cost of complying with environmental regulations.

The cost of fuel for a particular unit depends on its heat rate and fuel price. Assuming that environmental regulations provide a Cap-and-Trade (CAT) System, the environmental cost for each plant will be a function of the number of allowances it has been allocated by the government, the price of allowances, and the price of add-on Emissions Control Devices (ECD).

We assume all generating units reserve all their capacity production for sale on the spot market. All generators submit their bids to the Independent System Operator (ISO), who runs an economic dispatch model to meet electricity demand.

The electricity price is the clearing price obtained in a uniform-price auction. Electricity demand is assumed to be inelastic and is treated as an exogenous input.

Figure 1 illustrates the interaction among different variables considered in the model.

![Diagram of system](Image)

Fig. 1. Overview of the system.

A. Electricity Market

Let $p^*_e$ represent the electricity price at time, and $\delta_t$ the demand of electricity. Let $s^*_i$ be the supply function bid by generator $i$ at time $T$ which specifies the quantity of power offered and the price demanded. If $p^*_e$ is equal to the market-clearing price of an uniform-price auction, then $p^*_e = p^{(k)}_e$ where $p^{(k)}_e$ is the electricity price bid by the last generator (most-expensive supplier) needed to meet demand. That is, $p^{(k)}_e$ satisfies:

$$p^{(k)}_e - \delta_t \leq s^{(1)}_i (p^{(k)}_e) + s^{(2)}_i (p^{(k)}_e) + \ldots + s^{(k)}_i (p^{(k)}_e)$$

In our model we will assume that the market is truly competitive and generators are profit-oriented so they adjust their power output to the level for which their marginal cost of electricity generation is equal to the expected spot price \[7\]. Therefore, a generator’s bid to the spot market corresponds to its marginal cost of electricity generation, which includes the cost of fuel and the cost of compliance with environmental regulations.

B. Market of Emissions Control Devices

We assume that the capital cost of ECDs decreases as companies and the government put money in to Research and Development (R&D). We assume that the amount that firms devote to R&D is proportional to allowance prices.

The different technology-based alternatives that allow the reduction of emissions in electric generating units can be split into two categories: (1) installation of add-on emissions control equipment, (2) modification of the firing process. The first category includes devices such as wet and dry scrubbers to remove SO$_2$, selective catalytic reduction devices (SCR) to remove NO$_x$, Carbon Injection Devices (CI) to remove Mercury, and Carbon Capture and Sequestration technologies (CCS) to reduce CO$_2$ emissions. In the second category of alternatives to reduce emissions we can include methods such as switching fuel (for example to low-sulfur coal), modifying the unit to cofire other fuels (for example, biomass or natural gas), and improving the heat rate of the unit. In this paper we use the term ECD to refer to all possible technologies of both categories.

For any ECD it is true that installation and operation costs, removal rates, and energy penalties are generator specific and cannot be forecasted with complete accuracy until they are operating. However, it is also true that the degree of uncertainty is somewhat higher for less tested technologies such as CCS.

Assuming that capital and OM (operation and maintenance) costs of ECDs evolve with time as new necessities to reduce emissions generate investments in R&D, we have chosen to model the capital cost of ECDs as a state variable related to government policies and private investments.
Let \( \tilde{r} \) represent the aggregated investment in R&D by all units in time \( t \). Let \( r_{g,i} \) be the amount invested in R&D by the government and \( m_{i,t} \) be the subsidy the government will give to unit \( i \) for installing ECD \( k \) at time \( t \).

Therefore, \( h_{i,t}^k \), the cost of installing ECD \( k \) to unit \( i \) at time \( t \) will be given by

\[
h_{i,t}^k = h_{i-1,k} - m_{i,t} - \tilde{\alpha}_i \left( \tilde{r}_{t-1}, \tilde{r}_{t-2} \right) - \tilde{\alpha}_2 \left( r_{g,t-1}, r_{g,t-2} \right)
\]

where \( \tilde{\alpha}_i(.) \) and \( \tilde{\alpha}_2(.) \) are random variables whose probability distributions have parameters that depend on the R&D investments of previous periods.

**C. Fuel market**

Prices of fuels are determined by the aggregated demand at a particular period of time. The quantities demanded of each fuel are a result of the amount of electricity generated by each generating unit. We assume here that the supply functions for each fuel are time invariant.

Let \( p^s_k(Y^s_t) \) be the inverse supply function of fuel \( k \) at time \( \tau \), which measures the price at which sellers are willing to supply a given amount of fuel.

Let \( y_{i,\tau}^s(s_t^i) \) be the amount of fuel \( k \) consumed by generating unit \( i \) at time \( \tau \), when producing an electricity output of \( s_t^i \). Let \( Y^s_t = \sum y_{i,\tau}^s(s_t^i) \), where \( s_t^i \) is determined by the economic dispatch performed in the electricity market as in (2).

In this paper we will assume that supply functions for coal and gas are time invariant and exogenously determined. This means that conditions for the extraction and transportation of coal and gas for the time scope analyzed here are not affected by other variables of the model and remain constant.

**D. Environmental Market**

1) Cap-and-Trade System

In this model we will assume that there is a CAT system to control emissions of different pollutants. Under a CAT system, the government sets a cap and issues an equivalent number of emission permits or allowances which can be traded in the market. The government can allocate allowances in two different ways, grandfathering allowances or auctioning them. Under the grandfathering approach, government allocates to existing generating units a number of emission allowances for free. The number of allowances allocated to each plant might be determined by some historical data (as in the case of the EPA Acid Rain Program) and remain that way for a number of years, or can be periodically adjusted. The unit is authorized to emit as many tons of pollutant as allowances has been allocated. Since the number of allowances allocated covers only a portion of its emissions, to comply with regulations the unit has three alternatives: (a) to install emissions control equipment, (b) to buy “emissions allowances” in the market, (c) to reduce its electricity output.

Often, units that install Environmental Control Devices (ECD) end up emitting less than what they have been allowed. The emissions allowances not used can be sold in the market or if allowed, be banked for use in future years. Those generating units that have emissions over the number of allowances held must pay a fine to the government for each ton. In this sense, the government can be seen as a seller of an unlimited number of allowances at a price equal to the fine.

To make decisions regarding whether and when to install emissions-control equipment, decision makers have to consider the expected cost of compliance for each of the possible alternatives. The expected cost of compliance depends on the capital and operating cost of ECDs, number of allowances allocated, level of emissions, and price of allowances.

2) The decision to install an ECD

Modeling the decision that power plants’ owners make in regard to ECDs poses several difficulties.

We can assume that at each period \( t \), generators will design a strategy or a plan regarding which ECDs should they install and use every future period to minimize the expected cost of compliance over a planning horizon \( T \), and will act accordingly. Such strategy will be reviewed next period as new information allows a more accurate calculation of the expected cost of compliance.

Let \( L^i \) be an indicator matrix of the ECDs installed by generator \( i \) at each period, so \( L^i_{t,k} = 1 \) if generator \( i \) decides to install ECD \( k \) in period \( t \) and \( L^i_{t,k} = 0 \) otherwise. Similarly let \( U^i \) be an indicator matrix of the ECD the generator uses in each period. Therefore \( U^i_{t,k} = 1 \) if generator \( i \) uses ECD \( k \) in period \( t \) and \( U^i_{t,k} = 0 \) otherwise.

Let \( \tilde{e}_{i,t} \) represent the air emissions of generator \( i \) at time \( t \), and \( \tilde{a}_{i,t} \) represent the number of allowances (for each pollutant) allocated by the government, to generator \( i \) for period \( t \). If \( \tilde{w}_{i,t} \) represents the price of allowances at time \( t \), \( h^i_{t,k} \) represents the capital cost for generator \( i \) to install ECD \( k \) at time \( t \), and \( O^i_{t,k} \) represents the Operation and Maintenance Cost, then the expected value of the cost of compliance for unit \( i \) over a planning horizon \( T \), calculated at time \( t \), \( E[c^i_{t,T}] \), is given by:

\[
e[c^i_{t,T}] = \sum_{t} \left[ E \left[ \tilde{e}_{i,t}^* \right] \tilde{w}_{i,t}^* + h^i_{t,t} \tilde{a}_{i,t} + O^i_{t,t} \tilde{a}_{i,t} \right] \left(1 + p_r \right)
\]

We assume that in order to use a particular ECD, a unit has to install it \( \Pi \) periods in advance. Note that every installment
decision that plant makes in previous periods is a constraint for future periods. In other words, at time \( t \), columns 1, 2,..,\( t-1 \) of \( L' \) and \( U' \) are not decision variables, but initial conditions, because they represent decisions that were already executed in previous periods. These path dependencies in the solution make this problem one of Dynamic Programming. Also, since \( \tilde{e}^t_i \), \( \tilde{a}^t_i \) and \( \tilde{w}^t_i \) are random variables, the problem is a Stochastic Dynamic Program or a Multi-stage Stochastic Program \(^3[8] \).

At each period the installation and use of ECDs is given by the first column of matrices \( L' \) and \( U' \) that solve:

\[
\begin{align*}
\text{Min} & \quad E \left[ C^t_i \left(L^t_i, U^t_i \right) \right] \\
\text{s.t.} & \quad \tilde{l}_{ij}^t: j \in [0, t-1] \\
& \quad u_{i,t}^t \leq \sum_{k=1}^{n} \tilde{l}_{ik}^t \quad \forall i, j, t \in [0, T] 
\end{align*}
\]

Note that (4) implies that compliance is mandatory and that banking of allowances is not possible. This means that when emissions are higher than the number of allowances held, the unit has to buy the difference \( \tilde{e}^t_i - \tilde{a}^t_i \) at a price \( \tilde{w}^t_i \). Similarly, when emissions are lower than the number of allocated allowances, the unit has to sell the difference at the same price \( \tilde{w}^t_i \). In practice, under the Acid Rain Program units are allowed to bank all the SO\(_2\) allowances they have left and use them in future years. Units are also allowed to bank NO\(_x\) allowances but these banked allowances are discounted.

3) Calculating the expected cost of compliance

If we accept that units make their compliance decisions by trying to minimize the expected cost, a natural question is how they calculate it. Given that the number of allowances allocated, price of allowances, and costs of ECDs in coming years are all random variables, how do units calculate the expected cost of compliance for a certain strategy?

In the following subsections we comment on the uncertainty inherent to key variables and the way they affect the decision making process of power plant makers in regard to a compliance strategy.

3.1) Uncertainty in future regulations.

The current number of allowances allocated for each pollutant is known by generators, but the number of allowances that will be allocated in future years is highly uncertain. It is expected that significant changes in the future air emissions regulations will heavily affect electricity generators that burn fossil fuels. However, neither the timing nor the stringency of these future regulations is known. This uncertainty in future regulations has a cost. Waiting to decide

\[ \text{until all legislative, regulatory, and judicial uncertainty is resolved could prove costly; however, “locking in” an emission-control technology too soon could prove equally expensive. On one hand, plants might face stringent regulations without being prepared for that, and could be forced to buy expensive emission allowances. On the other hand, the installment of a particular control technology can preclude or make more expensive the option to install newer technologies more efficient or better suited for updated regulations. In [9] a method to compute the expected cost of regulatory uncertainty for coal-fired plants is proposed for a set of plausible regulatory scenarios with attached probabilities. Under this approach, each plausible regulatory scenario determines plants’ emissions, price of allowances, and fuel prices, so when a regulatory scenario is realized there are no more uncertainties. In this paper, we take a different approach and assume that even in the absence of regulatory uncertainty, the plant has still to consider that other important variables such as emissions, allowances and fuel prices are unknown and partially determined by its own decisions and its interaction with other market participants.}

3.2) Uncertainty in unit emissions

Unit emissions are proportional to the power output and therefore cannot be predicted with accuracy without knowing future utilization capacity factors for the unit. If the plant participates in the spot market, then its electricity output is also a random variable that depends upon electricity demand, market structure, and the generator’s bid. If the bid is affected by the cost of compliance with environmental regulations, then the compliance strategy and unit emissions are related in a closed loop fashion.

3.3) Uncertainty in emissions-allowances prices

The price of allowances depends on the compliance decisions made by all the units participating in the market of allowances. For example, if all generating units decide to install ECDs and reduce emissions to a level below the legal requirements, then the price of allowances will drop to zero. On the other hand, if allowances are scarce, prices can approach the fine’s price.

Forecasting allowances prices has proven to be a very difficult task in the recent past. For instance, when the Clean Air Amendment was enacted, the cost of compliance with the Acid Rain Program standards for SO\(_2\) was estimated to be $400-$1000/ton, but by 2000, allowances ranged in price from $130 to $155 and have remained close to $140. The NO\(_x\) budget offers another example; although forecasts of marginal control costs ranged from $500/ton to about $2,500/ton and in very few cases close to $5,000/ton, some trades in early 1999 occurred about $7000/ton but prices later fell to less that $1000/ton,[10] In 2003, trades for NO\(_x\) allowances for vintage 2004 and 2005 occurred at prices between $3,000/ton and $4,000/ton.[11]

In [12] it is argued that in phase II of the CAAA90, a high supply of SO\(_2\) allowances and prices much lower than expected

\[ \text{\footnote{Given that variables } L' \text{ and } U' \text{are binary, the problem is linear mixed-integer.}} \]
occurred in part because owners of big power plants preferred
to invest in expensive scrubbers rather than incur the risk of an
allowances shortage. Later, the situation with NOx was the
opposite. Many plants preferred to wait to see what would
happen in the market of NOx allowances before installing any
expensive control technology. This “wait and see” approach
of many plants has been at least one of the causes of the high
prices of NOx allowances seen in recent years. Estimation of
allowance prices under multi-pollutant regulation poses
additional difficulties, due mainly to synergies between the
control of SO2, NOx and CO2. In our model we will assume that
the forecasts that power plant owners use to make their
compliance decisions are based on the allowance prices they
observe, which are clearing prices in a uniform price auction.

4) Clearing allowances prices

Allowances suppliers (for a certain pollutant) are the units
that, as a result of having installed an ECD, have lower
emissions than the number of allowances allocated by the
government. In contrast, allowances buyers are the units
whose emissions exceed the government allocated allowances.

Let \( B^j_t \) represent the set of buyers of allowances for
pollutant \( j \) in time \( t \). Since ECDs reduce the emissions of
different pollutants at different rate, a particular unit can be a
supplier of allowances of one pollutant and a buyer of
allowances of other pollutant. For example a unit that installs
a dry scrubber can supply SO2 allowances, but could need to
buy NOx allowances. This situation might change over time as
the unit changes its emissions because of a change in the
electricity output or because of the use or non-use of certain
ECD.

Let \( \delta a^j_t \) represent the demand of allowances for pollutant \( j \)
at time \( t \) and let \( e^j_t \) and \( a^j_t \) represent the \( j \)-th component of
\( e^i_t \) and \( a^i_t \) respectively. Therefore,

\[
\delta a^j_t = \sum_{i \in B^j_t} (e^j_t - a^j_t)
\]  

(8)

As with electricity, we assume that the price of allowances
for pollutant \( j \) at time \( t \), \( w^j_{t,j} \) is the market clearing price of a
uniform-price auction. If the demand of allowances is higher
than the number of allowances supplied by units, then price
will be equal to the fine, otherwise it will be equal to the bid of
the last unit called to sell allowances to meet demand.

\[
w^j_{t,j} = \begin{cases} 
  \text{\( pa^j_t \) if } \delta a^j_t > \sum_{i \in B^j_t} (a^j_t - e^j_t) \\
  F^j_t \text{ otherwise}
\end{cases}
\]  

(9)

where \( F^j_t \) is the fine for no compliance with regulations for
pollutant \( j \), and \( pa^j_t \) is the bid of the last unit called to sell
allowances to meet demand, when suppliers of allowances
have been stacked in ascendant order according to their bids.

5) Simulating compliance decisions by power plants

If we accept that the outcome of every compliance decision
made by a power plant is strongly dependent on the decisions
made by other plants, then the interaction among different
plants has to be modeled as a strategic game.

In our model, we assume that decisions made by unit are
those that result in a Nash Equilibrium (NE).

To illustrate our thought, consider the example of a system
with only three units making a decision regarding whether to
install an SO2-scrubber. To simplify the analysis assume:

a) The only pollutant capped is SO2 and there is a CAT
system in which the three units are partially grandfathered so
they receive a fixed amount of SO2 emissions allowances.

b) There is no uncertainty regarding future environmental
regulations, so participant units know for certain how many
allowances they will receive from the government for their
entire planning horizon.

c) There is no uncertainty about future emissions. That is,
participants can forecast with accuracy their future emissions
because their future electricity output and fuel composition are
known.

d) The allowances market clears as described in 3.

e) The choice to install a scrubber is a “now or never”
option. That is, if the unit chooses not to install a scrubber, it
can not install it later.

f) Once installed, the scrubber has to be used. For all
plants, the scrubber reduces 90% of the unit SO2-emissions.

g) Each seller of allowances (units that choose to control
emissions) bids its variable cost for reducing emissions.

h) The payoffs of each plant under every circumstance are
“common knowledge.”

i) All plants make their decision simultaneously.

Assumptions e) and f) are needed to justify the framework
of a “one move” game. Otherwise, we would have to consider
a repeated game in which units that have not installed a
scrubber can decide to do it in future moves, and those that
already installed it can decide not to use it.

Assume plant emissions, scrubber cost and allowances
allocation as presented in Table 1. (Scrubber (Wet Flue Gas
Desulfurization System WFGD costs retrieved from IECM
model using default values. [13])

<table>
<thead>
<tr>
<th>TABLE 1. WFGD COSTS AND ALLOWANCES ALLOCATION</th>
</tr>
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<tbody>
<tr>
<td>Unit</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1 (Big)</td>
</tr>
<tr>
<td>2 (Small)</td>
</tr>
<tr>
<td>3 (Small)</td>
</tr>
</tbody>
</table>
The payoff for each plant is its expected cost of compliance for the entire planning horizon. To represent all possible combinations of decisions of the three plants with their correspondent payoffs we can draw a “game-tree” [14] with three players as in Figure 2.

![Game tree](image)

Each terminal of the three in Figure 2 corresponds to the vector of payoffs. The vector labeled as v corresponds to the payoff when the three plants chose to control their emissions, and the vector labeled as z corresponds to the payoff when all the plants choose to comply with regulation buying allowances. The first component of each vector of payoffs corresponds to the payoff of unit 1, the second component corresponds to the payoffs for unit 2, and so on.

If we assume a fine for no compliance of $2,000, and an allowances market as the one described in section 3, then the price of SO2 allowances will be $0, for outcome s, $250 for outcomes u and v, $400 for outcome w, and $2000 (fine price) for outcomes x, y and z.

Note that the best outcome for plant 1, w, occurs when this plant chooses to buy allowances and the other two plants choose to control their emissions. In this case, plants 2 and 3 supply all the allowances unit 1 needs to buy to comply, and the price of each allowance is $400/ton. For plant 2 (or plant 3) the best outcome occurs when plant 2 (and/or plant 3) chooses to buy allowances and plant 1 chooses to control its emissions. In this case plant 1 can sell all the allowances that plant 2 and/or plant 3 needs to comply with regulations. The worst outcome for every unit occurs when it decides to buy allowances and these are sold at the fine price. There are two Nash equilibria in pure strategy, v and w. By finding the NE in mixed strategies we find that unit 1 chooses to control emission with probability P1=0.269, and units 2 and 3 choose to control emissions with probabilities P2=P3=0.661 (The units choose to control at “random” according to those probabilities).

If we remove assumption j) and force plant 1 to make the first move, then the only Nash equilibrium obtained is node v. In the simplified simulation that we describe before, a similar game is modeled, but in this case the decision of whether or not to install an ECD is not restricted to the previous period, but allowed to be made at any time in the planning horizon.

**E. Control Instruments**

In this model we consider three different ways in which the government can intervene in the system:

1) Through allowances allocation: The government has a direct way to control the market of allowances by setting the acceptable emissions level at a certain point and allocating allowances to utilities accordingly.

2) Through the level of the fine for non-compliance: The government can also set the value of the fines for each ton of emissions that a unit has over its level of allowances.

3) Affecting the “installation threshold” for ECD: The government can make more attractive the option of installing ECD to power units in two ways:

a) Lowering the cost of ECD: The government could provide subsidies to buy ECD and have short-term impact in the level of installed ECD. The government can also support R&D activities to increase the likelihood of substantial improvements in the cost of ECD.

b) Even if the cost of ECD is the same, the government can make the option of installing ECDs seem more attractive by making utilities expect to use them for a longer time.

Let A represent the allowances allocated by the government, F represent the fines charged for non-compliance, M represent the subsidies given to units that install ECDs and RG be the money invested by the government in R&D for ECDs. To answer the question of which policy instruments are better to achieve low emissions and fair electricity prices, an optimization framework seems appropriate. The constraints of such an optimization problem are given by the characteristics of the units that constitute the sector, and the structure of the markets for fuel, electricity and allowances. At least three different ways to formulate this problem can be considered:

1.) A multi-objective optimization problem, where the goal is to minimize at the same time the amount of emissions over a planning horizon, the amount of money invested by the government in R&D and subsidies for ECD, and the electricity prices. All the goals are combined in a single objective function as a weighted sum of the money the government has put into the system (subsidies and R&D investments, minus fines collected), emissions and electricity prices.

\[
\min_{A,F,M,\alpha,\beta} \left[ \frac{1}{2} \left( \sum_i \sum_j m_{i,j} + \sum r_{g,j} - \sum \sum F_{i,j} \right) + \sum \sum \beta \left( \sum_{i,j} e_{i,j} \right) + \sum \gamma_i p_{e_i} \right]
\]

subject to: (1),(2),...,(11)

In (10), \(\alpha, \beta, \gamma\) are weights which have to be chosen carefully so the tradeoffs between the three key variables (government expenditure, emissions and electricity prices) are well represented. In the last term in (10), \(\gamma\) must be chosen related to the expected electricity demand \(\delta_e\) because the...
demand level captures the relative importance of electricity prices at each period.

2.) An optimization problem where the goal is to minimize the amount of emissions while keeping electricity prices and money spent by the government under a certain level.

$$\min_{y, \lambda, b, c} z = \sum_{i} \beta \left( \sum_{t} e_{i,t}^{r} \right)$$

s.t. 

$$\sum_{i} y_{i} \leq \psi_{1}$$

$$\sum_{i} \sum_{j} m_{i,j} + \sum_{i} r_{i} - \sum_{i} F_{i,j} \leq \psi_{2}$$

(1), (2), ..., (11)

\(\psi_{1}\) and \(\psi_{2}\) represent the maximum allowed levels for electricity prices and government expenditure in Environmental Policy.

3.) To minimize the electricity prices and amount of money spent by the government while keeping emissions under a certain level. So far this has been the approach adopted by the U.S. Acid Rain Program for controlling emissions of \(SO_{2}\) which began in year 2000, setting a permanent annual cap of 8.95 million tons from all affected utilities [6].

$$\min_{y, b, c} z = \sum_{i} y_{i}$$

s.t. 

$$\sum_{i} \beta \left( \sum_{t} e_{i,t}^{r} \right) \leq \psi_{1}$$

$$\sum_{i} \sum_{j} m_{i,j} + \sum_{i} r_{i} - \sum_{i} F_{i,j} \leq \psi_{2}$$

(1), (2), ..., (11)

IV. SIMULATION OF A SIMPLIFIED MODEL

A. Description of simplified model

In order to illustrate the model described, we implemented a simplified model, in which we assume the only pollutant regulated is \(SO_{2}\) and the only ECD available is a Wet Flue Gas Desulphurization System (WFGD). We also disregard here all the effects that money in R&D might have in the evolution of ECD technologies and assume there are no changes in the capital cost of the WFGD. We also disregard the effects that uncertainty in future regulations has in the optimality of decisions made by plants and assume they all have perfect foresight of government policies.

In order to keep running times short, we chose to represent the U.S. electricity market with a reduced number of plants. We looked at all the coal-fired units in the U.S. and divided them into four equally numerous groups according to its generation capacity. Then we calculated the average of key variables for each group, and defined four hypothetical plants as being the average plant of each group. These four hypothetical units correspond to what we could call the “very small,” “small,” “medium” and “large” prototypical units. Similarly we characterized the prototypical gas units and chose to include in the model only 6 units – the ones that represent the small, medium and large units of each type. To estimate the cost of installing WFGD we used the IECM[13]\(^5\). These prototypical coal and gas units are included in the model with attributes as those in the table 2.

In order to explore the optimal values for the variables controlled by the government (number of allowances allocated and price of fine for no compliance), we simulate the electricity and allowance markets for 11 years, for different combinations of allowances allocated and fines. The algorithm followed can be summarized as follows:

- For all the plausible government policies, for each year of the planning horizon,
  1. Simulate how units make their decision as to whether or not to install a WFGD.
  2. For each day
     a. Simulate the electricity market. (Find schedules for each plant and clearing price.)
     b. Record \(SO_{2}\) emissions and expenses in electricity.
  4. Calculate emissions and dollars spent in for each policy, as well as the value of the objective function as in (10).

The compliance decisions made by participant units are those that correspond to the NE of a strategic game in which each of the coal units has the option to install the WFGD at any time during the 11 years. Since we are modeling 3 coal-fired units there are \(11^3=1,331\) cases that each plant has to consider. If there is a non-unique NE we assume that units choose the one that optimizes the government objective function.

We also assume that all units estimate their payoffs in the game based in a forecast of the environmental costs of the future. Environmental costs are forecasted based on an estimation of emission levels and allowances prices. Estimations of future emission levels are made assuming the utilization factor of the plant will be similar to previous years.

\(^5\) For plants 2 and 3 (medium and large), heat rate and capacity were specified in the IECM. Other parameters were left as the default in the model. The cost of WFGD for plant 1 was extrapolated, accounting for economies of scale.
The electricity market clears as described in IIIA. The electricity demand profile corresponds to the annual demand profile of ISO NE, scaled so it can be met with the capacity of the units modeled. Units bid their marginal cost which includes the cost of fuel and their environmental cost. The cost of coal is assumed to be $1.2/mmBtu for 11 years, and the cost of gas is assumed to increase from $4.18/mmBtu in year 1 to $4.64/mmBtu in year 11.

The environmental cost for those units that do not have WFGD is the cost of the allowances they will have to buy to cover their emissions. For those units that are sellers of allowances the environmental cost is the annualized capital cost of the WFGD minus the revenue for allowances sold each year.

**B. Results**

If the government allocation required each plant to reduce its SO\textsubscript{2} emissions by 30% and fixed the fine for no compliance in $2000/Ton, then the NE is obtained when the large plant installs an WFGD in year 1, and the other two plants do not install WFGD. In this case total SO\textsubscript{2} emissions during the 11 years considered are 228,000 tons and total expenditures in electricity are 1.11 billion. How good is this policy? Fig. 3 shows the minimum level of emissions that can be obtained for each level of electricity expenditure - “Pareto Frontier”- and the point of NE. The frontier is found by evaluating emissions levels and electricity expenditures for each of the 1,331 possible combinations of WFGD installation decisions (We omitted in the graph a very extreme point in which all the plants install WFGD). The point labeled as “optimal” corresponds to the ideal outcome if government assigned to emissions a value of $1,000/ton, to make the objective of reducing emissions commensurable with the objective of reducing the cost of electricity. The NE would be far from this point and clearly, the policy of requiring reductions of 30% and fixing a fine of $2,000 would not be good enough.

![Pareto Frontier and NE](image)

**Fig. 3.** Pareto Frontier and NE for a policy with 30% emissions reductions and $2000/ton fines.

In order to find the best policy, we can compare Pareto frontiers. Figure 4 shows the Pareto frontiers for two policies in which the fine is still $2,000/ton and the reductions required are 40% and 80%. The Pareto frontier when reductions required are 80% lies below the frontier when reductions required are 40%, so for the same electricity expenditures emissions can be much lower if the government allocates fewer allowances. A comparison of the Pareto frontiers of different levels of allowances allocation indicates that the best policy is to enforce reductions of more than 80%. However if the government wanted to cap the total expenditures in electricity at a level lower than $1.05 billion, then the number of allowances allocated should be higher.

![Pareto frontiers of two policies with fine = $2000.](image)

**Fig. 4.** Pareto frontiers of two policies with fine = $2000.
A similar analysis of the effect of the fines shows that results do not change when fines are between $1,000 and $3,000/ton.

**V. CONCLUSIONS**

We have described and illustrated a model that explicitly relates government policies with emissions and electricity prices. The accuracy of the analysis to design optimal policies depends upon the assumptions made regarding the behavior of industry participants. Such behavior is determined by the information the participants have available, the methods used to forecast future values of key variables, and participants’ approach to the risk and uncertainty.

A detailed and accurate model of the behavior of industry participants when facing different policies can enlighten any analysis the government makes to choose policies that maximize social welfare.

**VI. REFERENCES**


\[ c = e^{-rT} \left[ SN(d_1) e^{rT} - XN(d_2) \right] \]

\[ d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)(T)}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]