Carnegie Mellon Electricity Industry Center
CEIC Working Paper 02-02
www.cmu.edu/ceic

Residual Risk and the Valuation of Leases under Uncertainty and Limited Information

David C. Rode[*†‡]
drode@daimc.com

Paul S. Fischbeck[*+‡]
pf12@andrew.cmu.edu

Steve R. Dean[†]
sdean@daimc.com

* Carnegie Mellon Electricity Industry Center
Carnegie Mellon University

† DAI Management Consultants, Inc.
1370 Washington Pike
Bridgeville, PA 15017
Voice (412) 220-8920
Fax (412) 220-8925

‡ Department of Social and Decision Sciences
Carnegie Mellon University
Pittsburgh, PA 15213
Voice (412) 268-3240

\+ Department of Engineering and Public Policy
Carnegie Mellon University
Pittsburgh, PA 15213

Abstract:

For a variety of tax, accounting, and economic reasons, leasing has become an enormously popular method of financing the acquisition of capital assets. In particular, for power generation assets, the use of leasing products (including synthetic and leveraged leases) has grown substantially in recent years. However, the long-lived nature of most power generation assets, together with their often unique physical and economic characteristics, makes estimation of their residual values challenging. In an environment where uncertainties are commonplace and in which capital structures are often highly leveraged, even small changes in residual values can have a significant impact on the profitability of financial institutions and other investors. This article outlines a framework for analyzing the uncertainty in residual values for assets, such as power generation facilities, for which few data points exist.

Keywords: Leasing, Project Finance, Residual Value, Risk Management, Valuation

Appears as Rode, D., P. Fischbeck, and S. Dean. Residual Risk and the Valuation of Leases under Uncertainty and Limited Information. *Journal of Structured and Project Finance* **7:4** (2002): 37-49.

INTRODUCTION[1]

The leasing of capital assets has several significant advantages for corporations and is often attractive for individuals as well. The total value of assets financed by leases has grown considerably over the last decade. The biggest risk to financial institutions in this explosion of lease finance is uncertainty over the residual value. In a conventional leasing transaction, the financing institution pre-commits to purchase the asset from the lessee at a specific future point in time at a pre-determined price. Making such a commitment requires the financing institution (or lessor) to have a reasonably accurate estimate of the future value of the asset, as any difference between the actual value and the pre-determined residual value represents either a gain or a loss to the lessor.

The ability to estimate future asset values accurately depends on a number of factors, but particularly on an active aftermarket in the asset under consideration. It should be noted, however, that even the presence of such an active aftermarket does not guarantee a riskless transaction for lessors. Just in 2000, the automobile leasing industry alone lost about $11 billion on overestimated residual values [*Arizona Republic*, April 7, 2001]. This works out to approximately $1,756 on each car returned at the end of a lease [McReynolds, 1999]. Stated yet another way, of the cars returned, approximately three quarters of them were sold at a loss.[2] The largest auto leasing institutions have suffered staggering losses as a result of misestimating residual values: DaimlerChrysler AG wrote down $442 million in losses on its leasing portfolio [*Wall Street Journal*, January 26, 2001] and Bank One took a $535 million loss, largely because of falling residual values [*Arizona Republic*, April 7, 2001].

With banks losing such large amounts on relatively conventional assets such as cars, there is perhaps even greater cause for concern when one considers the increasing tendency towards "exotic" leases (*e.g.*, leveraged transactions) and leases on highly illiquid assets. For example, the growing popularity of fractional jet ownership has prompted a surge in aircraft leases and an escalation in residual value estimates as lenders compete with each other for market share [Marray, 2000]. However, even aircraft must resemble reasonably liquid assets in comparison to the many industrial facilities financed with leases.

The trend in the leasing industry has been towards the financing of larger, more complex, longer-lived assets. Each of these factors, however, contributes to a significant amount of uncertainty facing the financing institutions. Further, to the extent that these assets have not been financed by leases in the past, little information tends to exist on their values over time. Thus, in many cases, lenders and risk managers face an unenviable situation: highly consequential exposures, virtually no information, and risks that could potential lay dormant for several years, if not decades.[3]

In this paper, we will outline a framework for confronting this uncertainty. In the absence of extensive data, we must rely on two premises: (1) a plausible, internally consistent model, and (2) a judiciously constructed simulation of possible future states of the world. Although most analysts use "only IRR and net-advantage-to-leasing statistics" [Mukherjee, 1991], simulation has been used to analyze leases. Simulation is also a commonly used tool in appraisal and valuation studies (*e.g.*, see Hertz [1979], Gain [1990], Li [2000]). In fact, Balcombe and Smith [1999] refer to the Monte Carlo simulation approach as a "refinement" to traditional methods that "minimizes demands on an appraiser's prior knowledge."

In addition to incorporating uncertainty, we would want our framework to make optimal use of available information, as well as new information in the future. While some of the risk in business decisions arises from fundamental environmental uncertainty (things that *cannot be known* in advance), much of the risk results from a simple lack of information (things the *modeler does not know* in advance). Valuation information is often treated as a proprietary asset and not made publicly available. Transfers of assets, when they occur, are often conducted privately between the parties involved. Indeed, much of the value managers contribute lies in their experience and ability to provide informed estimates in the absence of hard data. If a model is to be useful to managers, it must not only incorporate their existing knowledge, but also allow them to adapt to new information as it becomes available.

Models that fail to recognize the multiplicity of possible future outcomes are bound to result in unpleasant surprises.[4] This paper develops a framework for addressing residual risk in lease transactions. This framework explicitly recognizes the impact of uncertainty on value and illustrates how the resultant uncertainties are related to specific influences on asset values. Further, this framework allows new data to be incorporated as it becomes available, tightly integrating expert judgment and empirical experience.

STRUCTURE AND RISKS OF TRADITIONAL RESIDUAL VALUATION MODELS

The valuation of assets under uncertainty is a long-studied problem in financial economics. Beginning with the standard assumption that current value is simply the present worth of net future benefits, the appraisal literature recognizes three primary valuation methods [Appraisal Institute, 1996]: comparable sales, replacement/reproduction cost, and income capitalization. For competitive, liquid, and efficient markets, these three approaches should yield identical values (or, rather, sales and cost approaches should converge on the income capitalization approach). They exist as different models, however, because they impose different informational requirements on the appraiser.[5]

The typical form that we will be dealing with in analyzing leases is a *valuation curve*. This curve depicts the percentage of value remaining in an asset over time (Figure 1a gives an example of simple straight-line depreciation and Figure 1b illustrates an empirically-derived valuation curve). Such curves are useful for lessors who have not yet determined the lease duration or, having determined the duration, are attempting to assign a price to the lease. For example, in Figure 1a the asset is estimated to lose half of its value over 20 years and continue to diminish in value until the salvage value is obtained in year 30. The asset in Figure 1b loses a greater percentage of its value in the first few years and then stabilizes.[6]

[Figure 1a]

[Figure 1b]

These curves contain a significant amount of information – and information of substantial value to leasing institutions. However, they also have several weaknesses in their standard construction. First and foremost, there is no recognition of uncertainty in the depreciation of value over time. Failures in the estimation of residual values are often attributed to insufficient data, incorrectly specified models, and the like. There is little recognition, however, of any *fundamental* level of uncertainty. That is, the idea that no matter how much information one has, the future cannot always be predicted with perfect accuracy. Of course, most analysts probably believe this to be the case, but the absence of such recognition in the models used to evaluate leases leads to inconsistent recognition of the true risks involved. To put it succinctly, the possibility of errors is typically recognized, but the question of *how wrong could I be* often remains unasked.

The second weakness of the traditional valuation curve is related to the first and involves the omission of probabilistic data. It is valuable, for example, not only to know how wrong an estimate *can* be, but also to know how likely errors of *any* size might be. For example, if the valuation curve at a particular point indicates that the residual value is 40% of the original value, it is important to know not only that the value could range from 25% to 55% (possibility), but also that 95% of the time it is between 35% and 45% (likelihood). More formally, what is desired is a probabilistic distribution of values for each point on the traditional valuation curve. This effectively adds a third dimension to the charts in Figure 1.

Without this information, and the model used to produce it, it becomes very difficult to logically analyze the sensitivity of any estimate to any of the factors influencing value. No information is conveyed concerning departures from the single-line, static estimate. Any manager with sufficient real-world experience would be quick to note that it would be highly unusual for the depreciation of asset values to trace out the estimated valuation curve *exactly*. And yet, such a curve provides no guide to value should reality deviate from the estimates even slightly!

In the next section, we develop a simple model of the evolution of asset values that is firmly based on the well-established principles of the appraisal literature (*e.g.*, Alico [1989], Appraisal Institute [1996]). We formally extend traditional models, however, to incorporate the effects of uncertainty. In doing this, we agree with Marston, Winfrey, and Hempstead [1968: p. 7] insofar as they view uncertainty in value as an inevitability:

> "All values are of the nature of forecasts of events and are subject to the uncertainties of all prophecies. Values fluctuate with changes in prevailing opinions of what the future is likely to bring. They can never be determined by formulas or computations alone."

We, however, take a substantially different position in reaction to such a claim. Specifically, we take issue with the last statement that they can "never be determined by formulas or computation alone." We must exercise caution in determining their intent from a single phrase, but such a statement could encourage surrendering to subjectivity in the determination of value. We are far more optimistic about the usefulness of models and "computations" to produce useful information. Surely this is a better approach than such subjective and questionable methods as those described in the review of Kahn, Case, and Schimmel, [1963] or the "six-tenths factor" of Chilton [American Association of Cost Engineers, 1978].[7] Uncertainties must be identified, estimated, and modeled. Only then can meaningful values be determined.

AN APPROACH TO MODELING VALUATION CURVES WITH UNCERTAINTY

The technique developed here, in Balcombe and Smith's [1999] words "minimizes demands on an appraiser's prior knowledge." This is not meant to suggest that data should not be pursued or its importance marginalized. *If you have data, use it!* We argue, however, that for many tasks there is insufficient data, if any data at all. In these cases, rather than simply have an appraiser throw up his or her hands in defeat and resort to subjectivity, we advocate the development and use of formal models, used in a simulation environment. When the truth is that much blurrier, all the more effort should be devoted to understanding the "blur" – modeling the uncertainty.

Our approach begins with the three components of depreciation: useful life (deterioration), economic obsolescence, and technological obsolescence. Once put into service, many assets begin to deteriorate as they are used. This deterioration continues until the point at which they are no longer usable for their intended purpose. In many cases, the IRS [Internal Revenue Service, 1995; 2000] sets guidelines for the length of an asset's life. These lengths are used to calculate depreciation expenses for accounting statements and tax reporting.

For many assets – especially long-lived, technically unique assets – actual useful life is uncertain. A steel plant, for example, may be expected to have a useful life of 30 years, although it could be as short as 25 years or as long as 45 years. Any number of environmental and operational factors could influence this value, contributing to its uncertainty. Thus, for the purposes of an appraisal valuation, this uncertainty must be considered. Any misestimation of the useful life of the asset could significantly influence the present value of its ability to generate revenue. For example, if the average useful life was claimed to be 32 years, but there was a 50% chance that the useful life would less than 32 years, then there would be a 50% chance that a revenue shortfall would occur.

Graphically, we could conceptualize this (using straight-line deprecation for the sake of clarity in this example) as in Figure 2. A distribution of valuation curves is produced, centered at the most likely value estimated, but also recognizing explicitly that the value was uncertain.

[Figure 2]

In addition to simple physical deterioration, economic and technological factors may influence the *practical* useful life of an asset. It is entirely reasonable that assets physically capable of operation may nevertheless not operate for economic reasons. For example, the passing of time increases the likelihood that newer (more efficient) assets may have lower variable costs. Assets with higher variable costs may be kept out of the market unless there is sufficient demand to support higher-cost production. A similar case could be made for technological influences as well.

In addition to these three distinct influences, we wish to emphasize that a key component in our modeling efforts involves the *correlation* between various factors influencing value. For example, economic events may impact the useful life of a plant (*e.g.*, by changing the aggressiveness of its operating schedule), technological events may impact the economic competitiveness of a plant, accelerating or decelerating economic obsolescence, and so forth. It is important to maintain distinctiveness between components when estimating their influence, but it is equally important to model the relationships *between* those components when they are combined in an overall model. We wish to include relevant relationships, but not double count.

Having framed the modeling problem that we face, we will now address the modeling of each component in turn: useful life, economic obsolescence, technological obsolescence, and correlation. Our general approach will be to develop, component by component, a valuation equation for assets that is amenable to simulation. For the sake of simplicity, we will work strictly with percentages of value and ignore any salvage values.

Incorporation of these factors is a trivial adjustment and does not change the basic modeling results.

Useful Life

We begin with useful life because there is usually a well-known data point from which to start. Any particular asset is given a useful life for tax and accounting purposes. Generally, this amount represents the amortized initial cost of the asset. This approach assumes that the asset will be replaced once it is fully deteriorated. The estimated values for such time periods are based on historical experience and manufacturers' claims regarding the expected useful life of their products.

As one might expect, however, the *actual* useful life of a *single* particular asset may vary substantially from the *average* useful life for an entire *class* of assets. Accordingly, it is important to incorporate the impact of uncertainty over the useful life of a plant. Suppose, for example, that we wish to model the useful life process for straight-line depreciation. [1] represents the equation for such a process, where V_t is the percentage of value remaining at time *t*, V_0 is the percentage of initial value (in this paper, we shall always assume, without loss of generality, that this is 100%), *t* is the current period, and $\tilde{n}$ is the expected useful life of the asset. The tilde over any variable will denote its expectation under some probability distribution.

$$V_t = V_0\left(1 - \frac{t}{\tilde{n}}\right) \quad [1]$$

This portion, which accounts solely for recovery (amortization) of replacement cost, has only one uncertain variable: *n*, the number of periods of useful life.

It should be noted that we are not limited to straight-line depreciation for this component. Any other depreciation method could be incorporated without significant complication. For example, the equation for the double-declining balance method is given by [2].[8]

$$V_t = V_0\left(1 - \frac{2}{\tilde{n}}\right)^t \quad [2]$$

The sum-of-the-year's-digits method is slightly more complicated, but can also be included [8]. Let $SYD(t)$ be the sum-of-the-year's-digits depreciation charged during period *t*.

$$SYD(t) = \frac{2V_0(\tilde{n} - t + 1)}{\tilde{n}(\tilde{n} - 1)} \quad [3]$$

$$V_t = V_0 - \sum_{i=1}^{t} SYD(i) \quad [4]$$

$$V_t = V_0 - \sum_{i=1}^{t} \frac{2V_0(\tilde{n} - i + 1)}{\tilde{n}(\tilde{n} + 1)} \quad [5]$$

In [5], the coefficient $\frac{2V_0}{\tilde{n}(\tilde{n}+1)}$ is a constant and can be moved outside the summation.

$$V_t = V_0 - \frac{2V_0}{\tilde{n}(\tilde{n}+1)} \sum_{i=1}^{t} (\tilde{n} - i + 1) \qquad [6]$$

Further, note that $\sum_{i=1}^{t} (\tilde{n} - i + 1)$ has a closed form of $t\tilde{n} - \frac{t(t-1)}{2}$. Therefore,

$$V_t = V_0 - \frac{2V_0}{\tilde{n}(\tilde{n}+1)} \left[t\tilde{n} - \frac{t(t-1)}{2} \right] \qquad [7]$$

$$\therefore \qquad V_t = V_0 \left[1 - \frac{t}{\tilde{n}+1} \left(2 - \frac{t-1}{\tilde{n}} \right) \right] \qquad [8]$$

Regardless of which method is used, a distribution of useful life curves is produced. Suppose, for example, that an analyst's best estimate (the most likely or modal value) of the useful life of a facility is 30 years, but that the actual useful life could be as low as 25 or as high as 40. In such a case, we may wish to model *n* as a triangular distribution with parameters (25, 30, 40).[9] If more data is available, a more precise distributional specification can be obtained. If extensive data is available, empirical distributions can be used (*e.g.*, a bootstrap estimator [Chernick, 1999] of *n*). Figure 3 illustrates the three-dimensional nature of the valuation curve in a two-dimensional space as percentiles, with two selected distributional "slices" broken out for illustration. This curve provides the "backbone" of our analysis. However, our valuation cannot stop at this level. It is commonly recognized that asset values may fluctuate over time and depart substantially from their capitalized income values as macroeconomic, industry-specific, and technological factors influence the scarcity and perceived value of assets.

[Figure 3]

Consider, for example, an existing steel plant. Future regulation, such as that recently initiated by the United States protecting the domestic steel industry, may raise the value of the plant above its initial income capitalization. Likewise, the threat of future entry by efficient competitors may depress the market and reduce its values over the remainder of its expected life. A future technological innovation that reduces the variable costs of new steel plants relative to current plants may also detract from its original value. Looking ahead to correlations, possible scenarios could include interaction between economic and technological factors. For example, domestic government action in the face of potential foreign competition (which would depress current values) may take the form of R&D subsidization to stimulate technological innovation in domestic production (which would increase current values of domestic plants).

Economic Obsolescence

To account for the fact that economic and technological factors may influence values beyond what was indicated by an analysis of an asset's useful life, we will subject our valuation curve to influence from a sequence of *adjustment factors*. These factors, related to the economic obsolescence, may be either positive or negative. Because of this, the commonly used term "obsolescence" in this context can be misleading. It presumes that economic forces will serve only to degrade the value of an asset over time. Over a sufficiently long time interval, this is almost surely the case. However, there can be intermediate periods of time in which asset values could increase as well (*e.g.*, operational lags and transaction costs may impede convergence to a long-run equilibrium and lead to premiums or discounts in asset values).

We will model these economic adjustment factors by their impact on the useful life value of the asset. These factors can influence value at any point on the curve. Thus, they constitute a time series of influence from economic sources [9]. Let κ be a sequence of adjustments to the useful life valuation curve because of economic factors over time.[10] For example, a value of –0.05 would indicate that economic factors reduced the value of the asset 5% below the value indicated by the useful life function.

$$\boldsymbol{\kappa} = \{\kappa_1, \kappa_2, \cdots, \kappa_{\tilde{n}}\} \qquad [9]$$

Since economic factors may have long durations or unfold across several periods (depending on the time scale used), we may wish to incorporate sequential dependence in the series (autoregression) [10].

$$\kappa_t = \sum_{i=1}^{m} \phi_i^{\kappa} \kappa_{t-i} + \varepsilon_t \qquad \text{s.t. } m \leq \tilde{n}, \quad \varepsilon \sim N(0, \sigma) \qquad [10]$$

In [10], the ϕ represent the autoregression coefficients and ε is the uncorrelated, white-noise error. In practice, however, we often do not have sufficient information to conduct a formal time series analysis (but see the discussion in the next section on incorporation of new data). In practice, the adjustment factors must often be estimated with expert judgment. To minimize subjectivity, we suggest the following process:

1. Determine the required detail for the time window (do the factors need to be estimated for every period? Every five periods? Are they constant for all periods? Are there prominent dates on which to expect valuation-relevant activity?).
2. Determine the signs for each adjustment factor estimated (are they all positive? All negative? Do they switch from one to the other?).
3. Determine the magnitude of the adjustment.

It is important to keep in mind the purpose of the economic adjustment factors: *incremental* adjustment of value not already captured by deterioration or technological factors. Accordingly, many of the factors may be zero unless there is evidence of a *continual* change in value from economic forces. In reasonably competitive markets, economic theory would suggest that any effects would be incorporated relatively quickly. Thus, one might expect isolated adjustments for economic factors.

As usual, the past can be the prologue to the future. In the absence of other information, a manager may wish to approximate the frequency with which significant economic events occur. For example, suppose that in the last 10 years foreign competition has engaged in anti-competitive “dumping” four times; thus, we might say that there is approximately a 0.4 chance that a negative economic impact will be realized in any year. The manager must then estimate the relative strength and appropriateness of past influences and make any adjustments necessary in order to calibrate the current model to historical experience (*i.e.*, has the competitive environment changed appreciably since the most recent episode?).

A different approach would use the average overall economic impact calculated for other projects and incorporate such an adjustment on a per-period basis for this model. For example, if it was estimated that an asset generally loses 25% of its value (beyond simple depreciation) due *solely* to economic factors and that the asset has a 5-year life, it may be reasonable to simply assume that the economic obsolescence adjustment factor is –0.05 for each period. Again, if more data is available, more sophisticated modeling approaches should clearly be used. However, in the absence of better data, this approach serves to preserve consistency and clarity of procedure.

Technological Obsolescence

Technological obsolescence can be incorporated in exactly the same fashion as economic obsolescence, and thus, we do not dwell on the details. The basic idea again involves a sequence of technological adjustment factors [11]. This sequence may have autoregressive structure as well [12]. Finally, just as with the economic adjustment factors, technological factors may be positive, as well as negative.

$$\boldsymbol{\tau} = \{\tau_1, \tau_2, \cdots, \tau_{\tilde{n}}\} \qquad [11]$$

$$\tau_t = \sum_{i=1}^{m} \phi_i^{\tau} \tau_{t-i} + \varepsilon_t \qquad \text{s.t. } m \le \tilde{n}, \quad \varepsilon \sim N(0, \sigma) \qquad [12]$$

Technological factors could include such events as stricter environmental regulation (*e.g.*, new noise regulations at airports that mandated “hush kits” on jet engines made older, unmodified aircraft much less valauble), tighter standards for fuel efficiency (*e.g.*, in the 1970s, new pollution controls on automobiles made older, pre-control vehicles more desirable and valuable), or other such impacts. We mention the technological impact of environmental factors because such factors may contribute to an *increase* in the value of older, pre-existing projects. For example, an increase in the cost of gasoline makes fuel efficient cars more valuable. Likewise, significantly higher gas prices could decrease the residual values of gas-guzzling SUVs.

Correlation

For the three components estimated above, there are three unknown variables that must be estimated: n, $\boldsymbol{\kappa}$, and $\boldsymbol{\tau}$. In addition, subcomponents of the economic and technological adjustment factors must be estimated, depending on how they are specified. As we suggested earlier, however, correlation between these factors is a critically important aspect of our modeling approach and thus, must be estimated.

Naturally, given sufficient data, very precise estimates of correlation could be determined. In the absence of such data, however, we may instead approximate a value by determining first the direction of the relationship (positive or negative) and then the strength of the relationship. Ascertaining the direction should be relative straightforward. For example, a negative technological adjustment factor would most likely reduce the expected life of an asset. This would be indicative of a positive correlation: changes in one variable were in the same direction as changes in the other variable. It is important to keep in mind, however, that each adjustment factor could have its own correlation with the other variables. For example, some economic influences could be positively correlated with expected useful life while others could be negatively correlated.

Determining the strength of association can be done roughly by analogy to the R^2 statistic in regression. The traditional Pearson correlation coefficient serves as the basis of this statistic. The R^2 statistic determines the percent of variation in the dependent variable that is explained by the independent variables. This definition allows a rough qualitative estimation of the correlation coefficient.

Suppose, for example, that we have a model in two variables *A* and *B*. If we can determine that the relationships between *A* and *B* is positive, we can proceed to the next step. Suppose that in addition to a positive relationship, we estimate that *A* "explains" approximately 50% of the variance in *B*. By *explain*, we mean here only that when *B* exhibits some behavior, 50% of the time *A* also exhibits that behavior. According to our method, we may then estimate that the correlation between *A* and *B* is positive and equal to $\sqrt{0.50}$ or roughly 0.71.[11]

The Complete Model

The complete model is given by combining the four components discussed above. Upon such a combination, the equation describing the evolution of an asset's value as it depreciates across time is given in [13].

$$V_t = V_0\left(1 - \frac{t}{\tilde{n}}\right)\prod_{i=1}^{t}\left(1+\kappa_i\right)\left(1+\tau_i\right) \qquad [13]$$

This value equation can easily be used in a simulation context. Each iteration of the simulation would draw a new state *n*-tuple containing the correlated random draws for the estimated useful life and adjustment factors. These variables would then be used to calculate a single value for the estimated depreciated value of the asset.

Conducting a simulation analysis would involve thousands of such iterations at each point $t \in [1, \tilde{n}]$ of interest along the valuation curve. To illustrate this process, suppose that we estimated the following parameters according to the methods described above. Let the estimated useful life of the asset be distributed as a triangular distribution with minimum, modal, and maximum values of 25, 30, and 40, respectively. Let the technological adjustment factors be estimated annually and distributed uniformly over the range of –2% to 1%. Let the economic adjustment factors be represented by the discrete sequence in Table 1 corresponding to anticipated significant economic events over the next 40 years. These may be based on, for example, long-run forecasts of industry supply and demand, standard competitive analysis of the asset's position in the variable cost hierarchy, or macroeconomic business cycles. These values will vary in a range around the modal values indicated according to a modified beta distribution.

[Table 1]

[Table 2]

Finally, suppose that the correlation structure is determined by expert judgment according to the process described in the previous section. This matrix, given in Table 2, is then transformed into the closest positive definite matrix. Together, the model can produce simulated valuation curves, which can be represented as in Figure 4 below with their percentile values.

[Figure 4]

The incremental information obtained from our approach over a simple single-scenario approach, can be seen by asking the following questions:

(1) How many periods pass before the asset has lost (at least) half its value?
(2) What is the probability that the asset has any value after 30 years?
(3) If the financing institution has committed to repurchase the asset in year 15 at 19% of its initial value, what is the probability that it will lose money on the transaction (*i.e.*, what is the residual risk)?

The answers to these questions can be obtained easily from the simulation data, but are prohibitively difficult to obtain otherwise.

For example, the number of periods that have passed before the asset has lost at least half its value becomes, given the simulation data, a simple counting exercise. Using a cumulative distribution function (CDF), Figure 5 illustrates the probability that the condition (residual value is less than half initial value) is obtained. As is apparent, this likelihood varies between years 6 and 9.

[Figure 5]

The probability that the value is greater than zero after 30 years can be answered with a similar calculation. Figure 6 illustrates the solution to this question. This value is near 60% in the 30th year, but eventually falls to zero by the 40th year.

[Figure 6]

The third question is one that most members of credit committees will recognize as of primary interest. Ultimately, any financing institution – indeed, any business institution – must reduce any complex transaction to the bottom line: *will this generate a profit or a loss for the company?* Given the parameters of some hypothetical financing arrangement, how likely is it that the deal will end poorly for the financing institution (*i.e.*, with a loss)? In this case, we are interested in only year 15 where the financing institution has committed to a residual value of 19%. The question, then, is how likely is it that the actual value in year 15 will be less than 19%. If we plot the data for year 15 as a CDF (Figure 7), the answer is obvious: approximately 76% (*i.e.*, over three-quarters of the time, the financing institution could expect to lose money on such a transaction). To have at least a 50% chance of breaking-even (or making money), the residual value would have to be set lower, around 16%.

[Figure 7]

The point of these demonstrations is to illustrate that some important questions can be answered with our relatively simple model. Naturally, any answer is only as good as the model that produces it. However, a properly constructed and executed simulation model can reward users with an enormous variety of data for relatively little incremental effort. This is not simply an academic exercise; these questions involve core assumptions of many business transactions and are deserving of the benefits of rigorous management thought. A small investment in modeling effort can pay large dividends.

CALIBRATION AND THE INCORPORATION OF NEW DATA

The model that we have developed above has been developed on the presumption that very little, if any, relevant data exists about the asset in question. For some assets, this is likely to be the case (*e.g.*, new, highly unique, complex, or infrequently traded assets). For other assets, however, there is at least *some* data available, and such data should be brought to bear in search of a solution. Moreover, even if no data is available initially, it is plausible that, over time, data will become available. This section will briefly examine how such data might be incorporated into managerial decision making concerning asset valuation, both before an asset is placed in service and after.

Estimation of Adjustment Factors

Suppose that some data is available on asset values of the type under consideration. The availability of data certainly does not preclude our model from being used. In fact, we can use pre-existing data quite easily to enhance the probabilistic forecasts generated by our model. To do this, we will use a "factor model" approach similar to Ross's [1976] arbitrage pricing theory (APT).

Ideally, given the existence of historical asset values, we could estimate an equation such as [14], a *k*-factor model. This equation reveals directly, by way of the regression coefficients, the impact of various (economic and technological) factors on the valuation of particular assets.

$$V_{t+1} = \alpha D\left(V_t, V_0\right) + \beta_1 F_1 + \cdots + \beta_k F_k \qquad [14]$$

In [14], α and β_i represent coefficients for the factors. The first factor, represented by the function *D*, accounts for the depreciation adjustment in the current period.[12] The remaining *k* factors, then, measure the sensitivity of residual (non-depreciation) changes to various economic and technological factors (*e.g.*, inflation, tariff rates, competitors' R&D expenditures).

We will not dwell here on the details of analyzing such a model. Future work will present a detailed implementation and the statistical techniques are standard. We should indicate, however, that this is a somewhat data intensive method. Given this data, the simulation model developed would probably be very well specified. However, the amount of data required is often unreasonable in practice. We turn, then, to a simpler approach to estimating the adjustment factors, with the *caveat* that simpler, less data-intensive approaches are necessarily less accurate. Because this next method is similar to the method that we propose to re-calibrate the model after an asset has been placed in service, we shall discuss it in that context.

Mid-Course Correction

However well specified a model is at its inception, the world is sufficiently uncertain that adjustments to the model should be considered over time. This is particularly true as new data arrives concerning asset values. This new information, according to Bayesian decision theory, must be incorporated into the model to “update” the modeler’s prior beliefs. For example, suppose our model initially predicted that one year from now an asset would be worth 85% of its initial value. But, as transactions occur in the market, assets are being valued at only 75% of their value, our *prior* belief should be revised to account for this new information and return our so-called *posterior* or revised estimate of the asset’s value.

Several recalibration adjustments are possible for such mid-course corrections. The easiest, perhaps, is simply to redefine the current value as 100% (renormalization) and re-run the simulation for the remaining years. If there is reason to suspect that the original error was “unusual” and that all future technological and economic factors are still valid, then this simple recalibration will suffice.

The second type of recalibration is more involved. If the new information that arrives leads to doubt about the initial assumptions made to parameterize the model, then a greater degree of revision is in order. Specifically, not only would we want to redefine the current value as 100%, but we must also reconsider future economic and technological adjustment factor values. Recognizing that data tends to be substantially limited, it may be reasonable to adjust the future expected adjustment factors by an amount proportional to the incurred error. For example, suppose that we are three years into an asset's life and had estimated that it would lose 5% of its value each year. In actuality, we have learned that it lost 10% in year 1, 8% in year 2, and 11% in year 3 – on average 9.7% per year. This is 9.7/5.0 = 1.94 times the estimated amount, so a simple mid-course adjustment would be to multiply the remaining adjustment factors by this "calibration factor."[13]

The last type of adjustment that we will outline is formal Bayesian revision. Although there is insufficient space to provide a full account (the interested reader is directed to the excellent book by Gelman *et al.* [1995]), the general idea is illustrated rather easily. Put simply, our prior beliefs must be revised in the presence of new information to determine our posterior beliefs. The well-known Bayes formula specifies the form this revision must take in order to produce rational posterior beliefs. Suppose that we have estimated a distribution for some parameter (*e.g.*, the expected asset value in t periods) and now observe new information in the form of an actual value in period t. The specific value observed will influence our confidence in the initial estimate of the parameter's value (and perhaps our confidence in the model as well), either by strengthening our initial belief or by causing us to make an adjustment away from an (possibly erroneous) initial forecast. Figure 8 illustrates this process conceptually for the model output (percentage of value remaining).[14] The same process, however, could also be used to evaluate the adjustment factors and even the structural form of the model itself.

[Figure 8]

Whichever method is most suitable to the question at hand, the overarching concept is that new information can and should influence the course of the model over an asset's life. As old uncertainties are resolved and new uncertainties arise, the model should be adjusted to reflect such changes. The modular simulation approach developed here allows such changes to be made quickly and transparently.

CONCLUSIONS

In this paper, we have illustrated the need for the use of probabilistic valuation models, even when little or no information exists. In fact, one might argue that it is *precisely* when little information exists that logically constructed and internally consistent models are most useful. We have developed such a model and illustrated the potential insights to be gained from its output. In addition, we have suggested methods for the incorporation of new data into the valuation process over time.

As asset leasing gains in popularity and new and more exotic leasing transactions are structured, the importance of modeling the uncertainty in valuations becomes critically important. As margins are squeezed in an increasingly competitive leasing market and terms for leases become longer, the likelihood of uncertain events affecting the valuation process greatly increases. Firms that do not properly account for the significant uncertainty that they face in determining asset values are exposing themselves to significant risk – risk that is not always transparent and that presents organizational challenges of its own. Relatively straightforward simulation models can illuminate the impact of uncertainty across a broad cross-section of transactions and provide a common modeling platform for addressing firm-wide risks. Investing some time and effort in developing such modeling capabilities may go a long way towards mitigating asset-valuation risks.

Figure 1a: Sample Straight-Line Valuation Curve

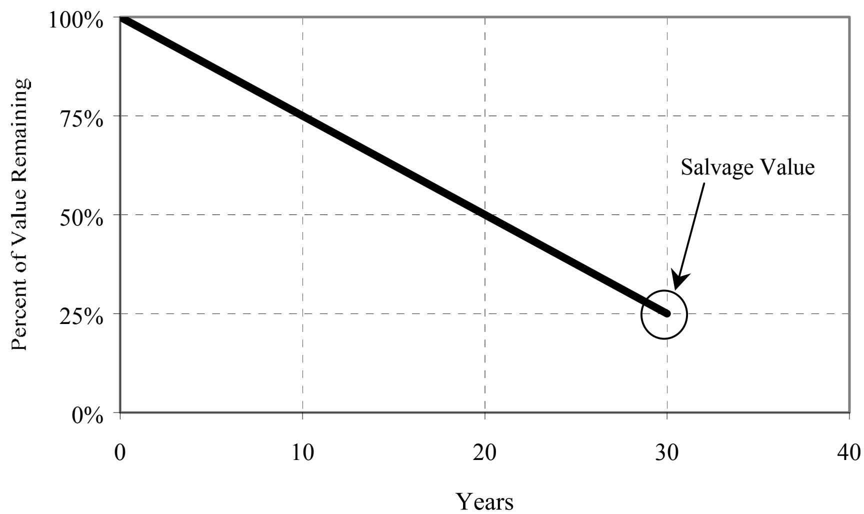

Figure 1b: Sample Empirical Valuation Curve

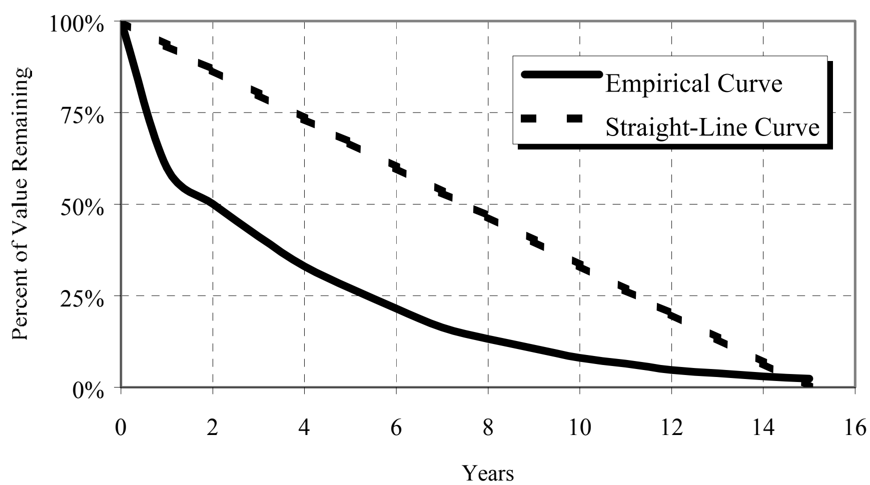

Figure 2: Valuation Curve with Useful Life Uncertainty

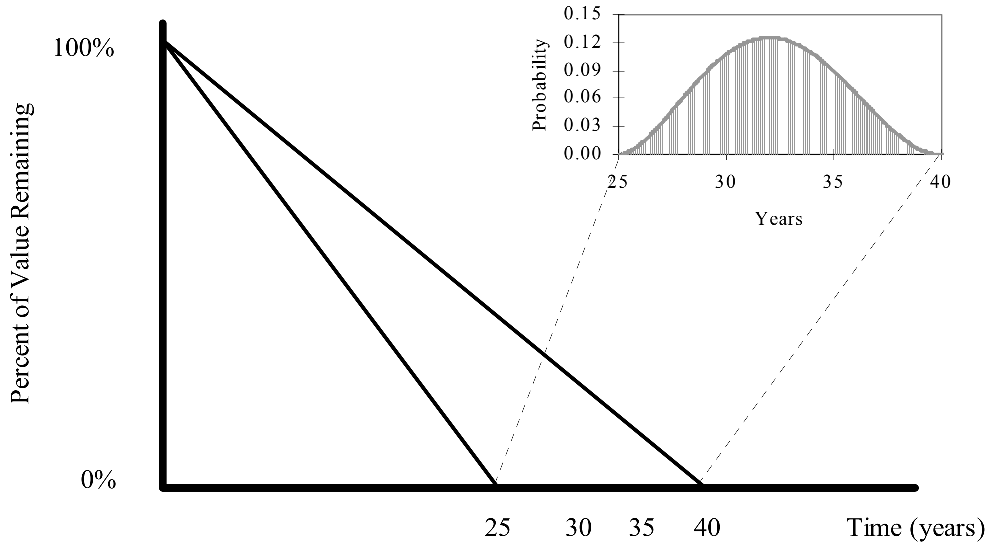

**Figure 3: Valuation Curve with Percentiles
Assuming Uncertain Life**

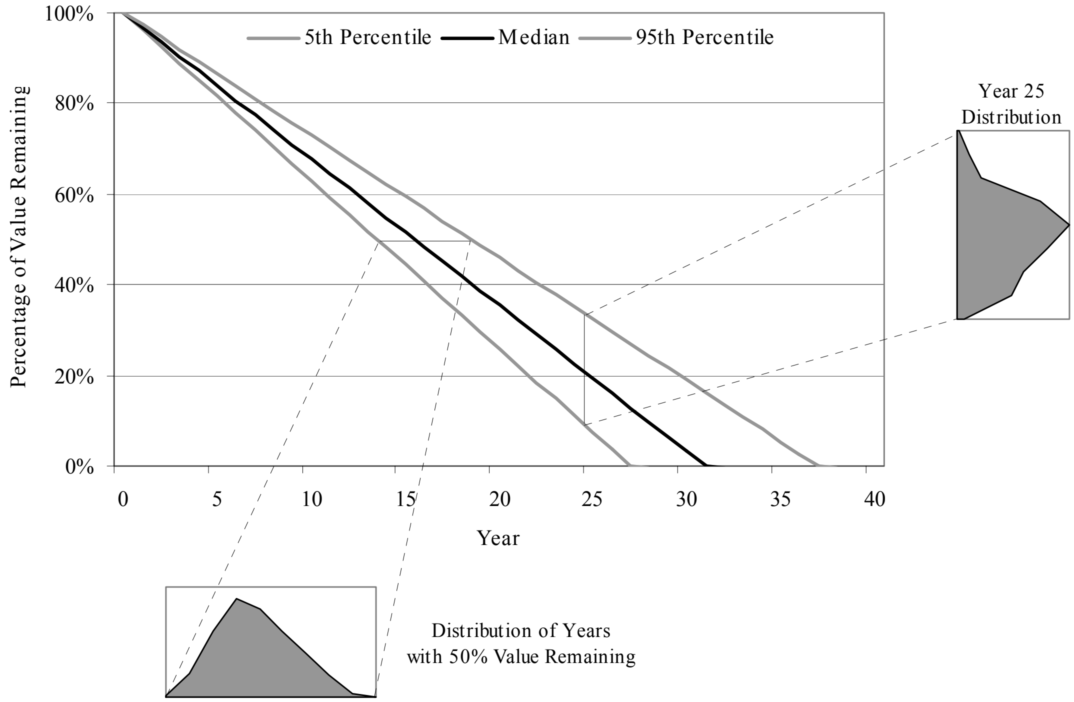

Figure 4: Valuation Curve under Uncertainty

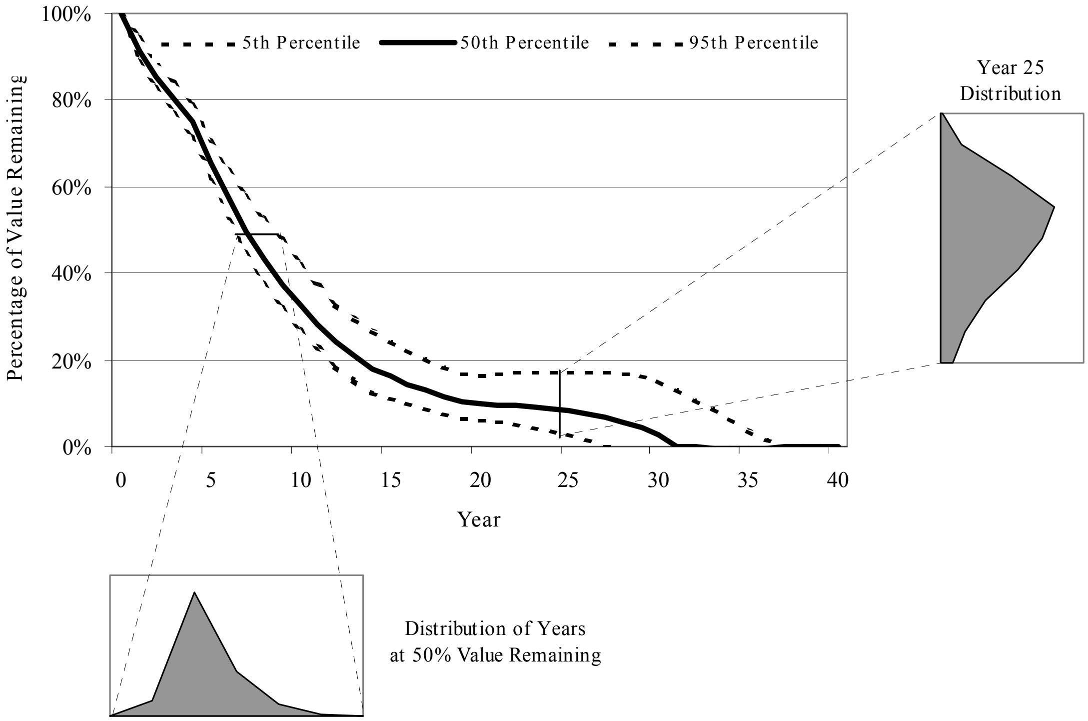

Figure 5: Probability that Value is Less Than 50% in a Given Year

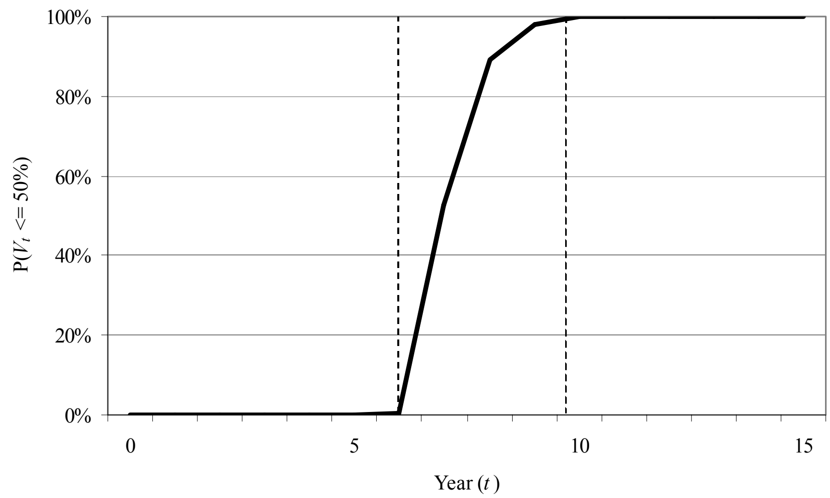

Figure 6: Probability of Non-Zero Value After 30 Years

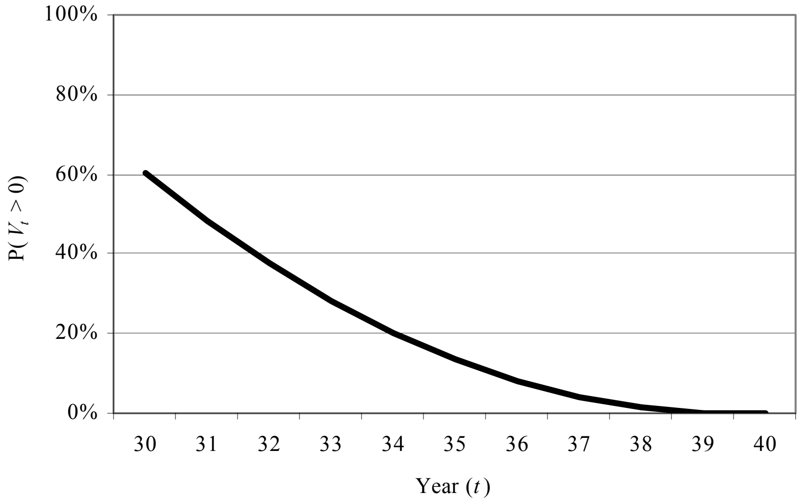

Figure 7: CDF of Year 15 Residual Values

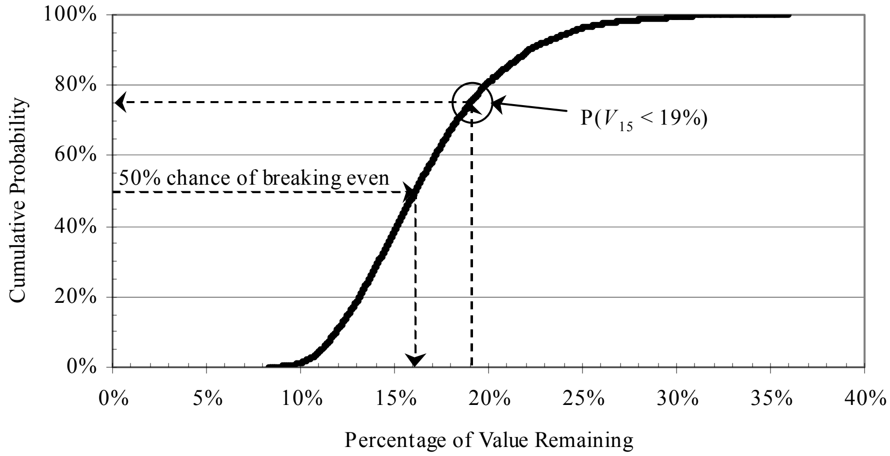

Figure 8: Prior and Posterior Parameter Values under Bayesian Updating

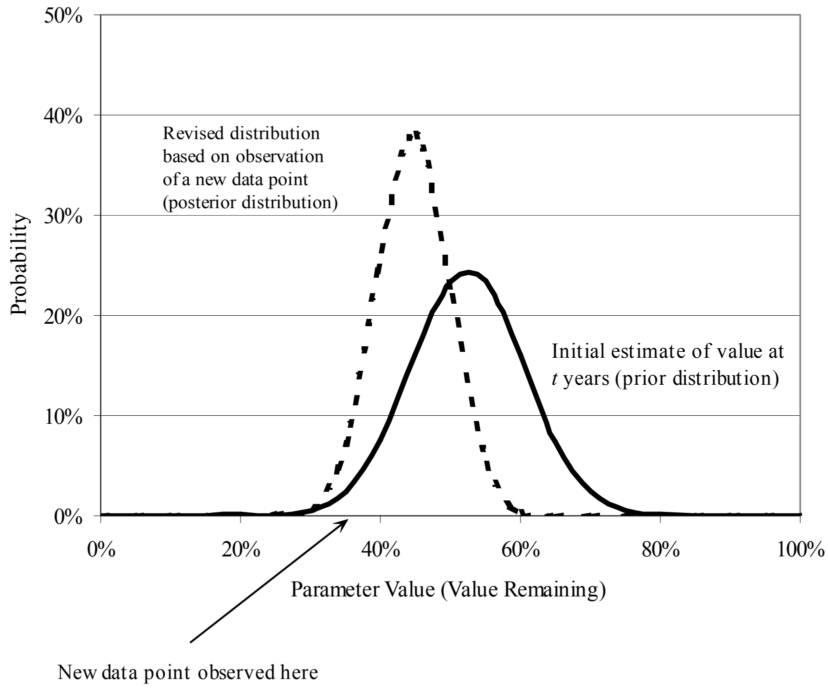

Table 1: Economic Adjustment Factors

Years	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Modal Adjustment	-0.02	-0.10	-0.10	-0.05	0.10	0.10	0.10	-0.20

Table 2: Correlation Matrix

	n	$\boldsymbol{\tau}$	$\boldsymbol{\kappa}$
n	1.0	0.7	0.4
$\boldsymbol{\tau}$	0.7	1.0	0.3
$\boldsymbol{\kappa}$	0.4	0.3	1.0

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[1] Thanks to Evan Dean for helpful suggestions.

[2] Naturally, because there is uncertainty in residual values, some assets are bound to be sold at a loss. However, if initial estimates were reasonably accurate, one might expect two conditions to hold. First, that actual residual value would be as likely to be above the estimated value as below (that estimates were not biased, as they appear when 75% of transactions end in losses). Second, that the relative magnitude of the losses would be small (but they are not – the average loss per vehicle number quoted by McReynolds [1999] is just under 10% of the average price of a new car, according to the National Automobile Dealers Association). Considering the margins on many cars, 10% is a significant number.

[3] The long time between the initiation of a lease and the realization of the residual value raises important questions about agency problems and organizational strategy. Estimating higher residual values makes the cost of a lease appear lower, attracting clients and benefiting current managers, while simultaneously creating friction between sales and management personnel. If the realization of the actual residual value does not appear for ten or twenty years into the future, there may be little ability to tie control and compensation of current management to future profits or losses. This fosters an environment in which managers can set residual values for their own convenience to boost profits today, with the realization that they will have moved on by the time consequences arrive.

[4] Interestingly, this remains true in both directions. If a static forecast is *insufficiently* conservative, actual losses may be incurred. If, however, the forecast is *overly*

conservative, funds set aside for reserve against possible losses represent under-investment and an inefficient deployment of capital.

[5] For example, the comparable sales method is typically used to value residential real estate because most residential real estate does not have an "income" that is easily capitalized. Although in theory one could calculate an imputed rent or "rental equivalent income," the fact that the market for residential real estate is large and highly liquid renders such effort unnecessary. We should stress, however, that this is *not* to say that the income capitalization approach does not work for residential real estate (or, indeed, any other asset). Rather, it is that for many assets, simpler "proxy" valuation methods are available as a consequence of reasonably efficient markets.

[6] The data for the empirical curve is from automobile depreciation data taken from the NADA Used Car Appraisal Guides.

[7] Chilton noticed an empirical relationship present in an analysis of cost curves. Specifically, that on a log scale the cost curves for some process plants had an average slope of 0.6. Empirically derived heuristics such as this can often have enormous simplifying value, but to obtain that value, an understanding of the risks involved must be present. For example, the slopes in Chilton's sample ranged from 0.33 to 1.02, "but the bulk of them were closer to 0.6 and their overall average was close to 0.6" [American Association of Cost Engineers, 1978]. Without more precise information about the distribution of slopes, however, the mere statement of an average provides little information. Looking only at the information given, the possibility for errors of 50% or more exists, but there is no way of knowing the likelihood of any such errors from Chilton's information.

[8] Any residual left after period $\tilde{n}$ is customarily charged to period $\tilde{n}$.

[9] The triangular distribution takes parameters (minimum, mode, maximum).

[10] In fact, although we do not develop such an idea here, it may be advantageous to model the adjustment factors by including m multiple sources. Specifically, that $\boldsymbol{\kappa}$ may be a matrix:

$$\boldsymbol{\kappa}_{|m|} = \begin{pmatrix} \kappa_1^1 & \cdots & \kappa_{\tilde{n}}^1 \\ \vdots & & \vdots \\ \kappa_1^m & \cdots & \kappa_{\tilde{n}}^m \end{pmatrix}$$

In this case, there may exist correlations between the "sub-factors" as well as between the sub-factors and the technological factors and useful life variables. Obviously, this significantly complicates the implementation of the model, but does not fundamentally alter the implications of this framework.

[11] Clearly, this method takes some liberties with the formal statistical notion of correlation – particularity with regard to multiple correlation and the interpretation of a multiple R^2 statistic. A further complication in using this method arises from the fact that correlations determined individually via this method may not produce internally consistent or positive definite matrices. This complication, however, is dealt with relatively easily; we find the "matrix perturbation" method [Kim, Malz, and Mina, 1999] that makes use of the Rayleigh-Ritz theorem to be an attractive solution to this problem.

[12] The specific form of D depends on the type of depreciation method used. If the method is nonlinear (*e.g.*, sum-of-the-year's digits), then some transformation should be incorporated into D in order not to bias the regression results (by attaching a linear coefficient to a nonlinear variable).

[13] Clearly, there are several possible problems in doing this. Specifically, that the magnitude of the error may not persist. If the initial decline, for example, simply occurred faster than estimated, the remaining periods may experience slower than expected declines in value to compensate. Obviously managerial discretion and expert judgment are important in such scenarios.

[14] This graph should be interpreted strictly as a general example. In practice, the specifics of the change would be determined by the "degree of surprise" resulting from the location of the new data point in relation to the prior distribution. The basic point is: the larger the error, the greater the change.