Two Essays on Problems of Deregulated Electricity Markets

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1 Essay: Empirical Test of Models of Unilateral and Collusive Market Power in California's Electricity Market in 2000

Abstract

The data from California energy crisis of 2000 suggests that the largest departures of observed electricity prices from the estimates of the competitive price occur when demand approaches market capacity. This paper studies models of unilateral and collusive market power applicable to electricity markets. Both suggest a unique mechanism explaining the increase of the price-cost margin with demand. The empirical test of these models provides more evidence for unilateral market power than for behavior suggesting tacit collusion.

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1.1 Introduction

In the summer of 2000 wholesale electricity prices in California increased to about four times above their usual summer levels. That signaled the beginning of the notorious California energy crisis. Now, four years after that it is still not clear to what extent this price increase was explained by market power exercised by California independent generators and if the market power was in fact exercised, then what generator or generators contributed to that the most.

Electricity markets are characterized by a set of unique properties which make them extremely vulnerable with respect to market power exercise: electricity is prohibitively costly to storable in any significant amounts, short-run demand elasticity is close to zero as is the supply elasticity at output levels close to capacity. In addition, energy markets are usually cleared on an hourly basis and such high frequency of interactions facilitates tacit collusion among the players.

The California energy crisis drew additional attention to these features of electricity and the resulting risk of market power exercise in such energy markets. Further steps in the energy deregulation process are now taken with caution in the US and elsewhere in the world.

This paper studies the models of unilateral market power exercise applicable for electricity markets as well as a model of collusive behavior and tests which kind of market power was likely exercised in California in 2000.

A conventional Cournot model gives poor results when applied to a market with inelastic demand. Supply Function Equilibrium (SFE) is the general case of models of unilateral market power. Cournot may be viewed as a particular case of it. I suggest a model of withholding equilibrium, which is another particular case of SFE. This model is both more tractable than general SFE and can be applied to the cases with inelastic demand as opposed to the Cournot model.

The possibility of collusive behavior among generators is of great concern. According to the Folk Theorem, the generating firms can sustain any level of pricing between monopoly and competitive in an infinitely repeated game. Energy markets can be considered infinitely repeated games with a discount rate between days or hours very close to one. However, the degree of coordination required for successful collusion depends on how high the demand is relative to the total system capacity and the size of generators.

For instance, in a particular hour the supply margin, measured as the difference between the total system capacity and the inelastic demand, may be smaller than the individual capacity of some generator firms. In that case any such firm can profitably raise prices up to the price cap. Conventionally, each of such firms is called a pivotal supplier. Not much coordination is needed between the firms in this case since any one can exercise market power unilaterally, and I call such situation a pivotal monopoly.

However, the supply margin may be larger than the capacity of any individual generator but smaller than the cumulative capacity of two generators. In that case any two generators whose cumulative capacity is larger than the supply margin have to act in concert to raise market prices profitably. This situation may be called a pivotal duopoly and the implicit coordination between the generators in this case is harder to achieve than in the case of pivotal monopoly.

I provide a stylized model of the electricity market that links the number of firms in the pivotal oligopoly with the resulting degree of market power exercise. The model suggests that significant market power can be expected even with two and more firms in the pivotal oligopoly.

Both the model of unilateral withholding equilibrium and the pivotal oligopoly model of collusive behavior suggest that the price-cost markup should increase as the demand gets closer to market capacity. I use the level output data from the California energy market in 2000 to check empirically which of the two models is supported by the data. The data suggest that the market power is mostly exercised unilaterally during the hours when the system can be considered a pivotal monopoly. During hours when the number of firms in the pivotal oligopoly is two or more, unilateral withholding is often not profitable and collusive behavior is not very significant.

1.2 Unilateral models of market power: SFE, Cournot, and withholding equilibrium

Cournot Model

The conventional Cournot model of oligopoly assumes that firms choose output quantity as a strategic variable. That results in a perfectly inelastic supply of the oligopoly. Given that in electricity markets the short-run demand is also very close to being perfectly inelastic the predictions of the Cournot model in the energy markets are often misleading. Indeed, the most often used result of the Cournot model links the Herfindahl-Hirschman Index, calculated as the sum of squared market shares of the participating firms with the predicted price-cost markup:

$$\frac{p-c}{p} = \frac{\sum s_i^2}{\varepsilon},$$

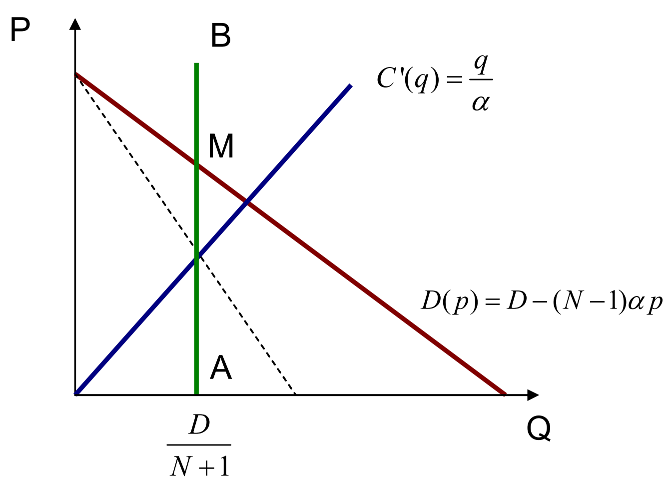
where ε is the demand elasticity. As the demand elasticity gets closer to zero, the Cournot model will predict a price markup close to infinity making the use of Herfindahl-Hirschman Index for measuring market power irrelevant (Borenstein and Bushnell (1998), Borenstein, et al. (1999)).

The mechanism of this prediction of the Cournot model can be better understood by studying the best response functions. Suppose demand is inelastic and equal to D, and N firms have cost functions $c_i(q_i) = \frac{q_i^2}{2\alpha}$ [1]. If $N-1$ firms behave competitively and supply at their marginal cost then the best response of the remaining firm is given by the quantity maximizing the profits given the

[1] I will illustrate the relationship between the Cournot, SFE, and withholding equilibrium on simple linear and symmetric examples going into more details

residual demand $D(p) = D - (N-1)\alpha p$, which is $q^{BR} = \frac{D}{N+1}$. The best response can be viewed as a perfectly inelastic supply function on the side of the remaining firm AMB (Figure 1-1). However, different parts of this supply function serve different purposes. Segment AM has no strategic effect; point M is actually maximizing the profits given the residual demand; finally, segment MB serves to decrease the elasticity of the residual demand faced by the rest of the firms. If all firms follow the same strategy, then all firms are facing inelastic residual demands because of the MB segments of the supplies of their rivals and the resulting price becomes infinity given perfectly inelastic total demand.

Figure 1-1. Cournot best response



Supply Function Equilibrium is a model of oligopoly that suggests elastic MB segments in the best response supply functions and therefore avoids the problem of the Cournot model dealing with inelastic demands.

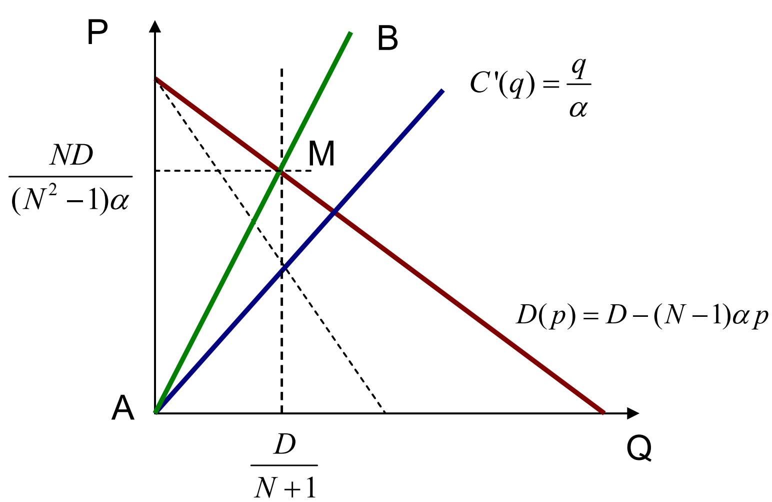
Supply Function Equilibrium

In energy markets firms submit bids for their generation that consist of both quantity and price. Essentially, firms choose the supply schedules as their strategic variable. That led Green and Newbery (1992) and Rudkevich, et al. (1998) to apply the Supply Function Equilibrium (SFE) model of oligopoly developed by Klemperer and Meyer (1989) to the electricity markets in the United Kingdom and Pennsylvania-Maryland-New Jersey Independent System Operator.

In the case of deterministic demand, the SFE provides a multiplicity of equilibria. This happens because only one point on the best response supply function is actually a price-quantity point maximizing profit with respect to residual demand. The rest of the points on the supply function are irrelevant for the immediate profit maximization but may serve to influence the residual demand faced by the other firms.

Supply Function Equilibrium works the best when demand is stochastic. In that case each point on the best response supply function is a price-quantity pair maximizing the profits under some demand realization. Similarly to the Cournot example above assume that N firms have the cost functions $c_i(q_i) = \frac{q_i^2}{2\alpha}$, and total demand is D, which is now stochastic on $[0,\infty)$ support. If N-1 firms behave competitively bidding their marginal costs then for the remaining firm the price-quantity pair maximizing the profit with respect to the residual demand function will be $\left[D\frac{N}{\left(N^2-1\right)\alpha}, D\frac{1}{N+1}\right]$. Connecting these points for every demand realization one gets the best response supply function $s^{BR}(p) = \frac{N-1}{N}\alpha p$ shown by the line AMB on Figure 1-2.

Figure 1-2. Supply Funciton Equilibrium best response



At any given demand realization D only point M on the best response supply function maximizes the profit with respect to the residual demand. As opposed to the Cournot model the segment MB is now elastic and its elasticity is explained by the fact that the firm realizes that there is a non-zero probability of the demand being different from D.

In this example with linear marginal costs and inelastic demand, the Supply Function Equilibrium can be written in closed form. Conjecture that for every firm the equilibrium supply function is $s_i^*(p) = \beta p$. The residual demand faced by a single firm will then be $D(p) = D - (N-1)\beta p$. Given the cost function $c_i(q_i) = \frac{q_i^2}{2\alpha}$ the profit maximizing price-quantity pair for a realization of D will be:

$$\left[D\frac{\alpha + (N-1)\beta}{(N-1)\beta(2\alpha + (N-1)\beta)}, D\frac{\alpha}{2\alpha + (N-1)\beta} \right],$$

which suggests a best response supply function:

$$s^{BR}(p) = \frac{\alpha\beta(N-1)}{\alpha + (N-1)\beta}\, p$$

Solving for β from

$$\beta = \frac{\alpha\beta(N-1)}{\alpha + (N-1)\beta},$$

the equilibrium supply functions become[2] $s_i^*(p) = \alpha\frac{N-2}{N-1}p$. The total industry demand is therefore

$$S^*(p) = \alpha N\frac{N-2}{N-1}p \tag{2.1}$$

[2] For more on linear Supply Function Equilibria see Rudkevich, Aleksandr, Max Duckworth, and Richard Rosen, 1998, Modeling Electricity Pricing in a Deregulated Generation Industry: The Potential for Oligopoly Pricing in a Poolco, *Energy Journal* 19, 19-48. and Baldick, Ross, Ryan Grant, and Edward Kahn, 2000, Linear Supply Function Equilibrium: Generalizations, Application, and Limitations, POWER.

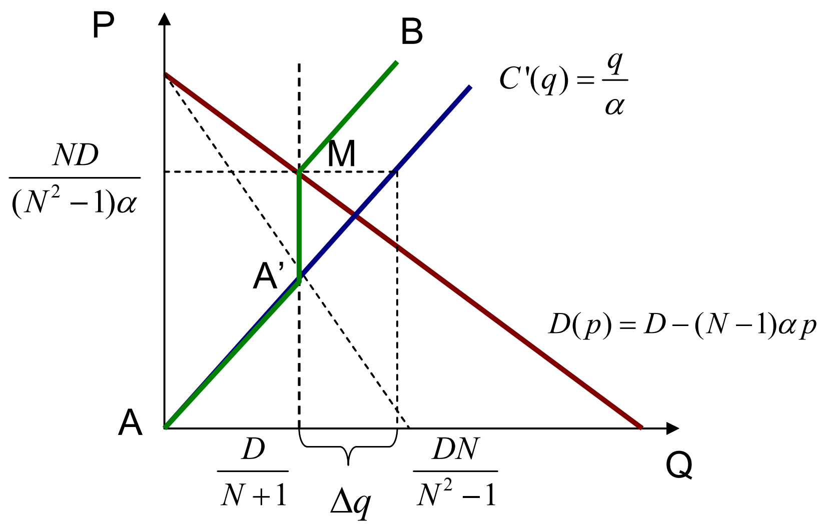
The computation of the Supply Function Equilibrium quickly becomes quite involved when the linearity assumption is dropped. In the general case it requires numerical solution of systems of differential equations. The main idea, that firms maximize their profits with respect to the expected residual demand they face, remains the same as in the Cournot model. However, the rationale for the firms to submit elastic segments of the supply functions above the profit maximizing price-quantity point comes at too high a computational cost.

Withholding Equilibrium

I suggest an alternative oligopoly model that both can be applied to the cases of inelastic demand as opposed to the Cournot model and is less computationally involved than the Supply Function Equilibrium since in my model the strategic variable is a number rather than a function. In fact, the withholding equilibrium can be thought of as a particular case of the Supply Function Equilibrium in which the supply functions can be obtained by cutting out a segment of the marginal cost function assuming that that part of the capacity is strategically withheld from the market.

Strategic withholding in the electricity market can be of two kinds: physical withholding in the cases when an outage in generating capacity is announced for strategic reasons and economic withholding when capacity is strategically overpriced without intention to sell the capacity at that price but merely to drive the market price up and enjoy extra profits on the remaining capacity. In the following models I make no explicit distinction between the two types of withholding.

Figure 1-3. Withholding Equilibrium best response



Consider an example similar to the ones presented above. N identical firms have quadratic cost functions $c_i(q_i) = \frac{q_i^2}{2\alpha}$, total deterministic demand D; $N-1$ firms competitively bid their marginal costs leaving the remaining firm with residual demand $D(p) = D - (N-1)\alpha p$. This time assume that the profit-maximizing supply function is obtained by withholding amount of capacity $\Delta q = \frac{D}{N^2-1}$ (Figure 1-3). The resulting supply function AA'MB goes through the point M that maximizes the profit with respect to residual demand. The segment MB that determines the residual demand faced by other firms is in fact the segment of the marginal cost curve shifted leftwards by the amount of the withheld capacity Δq.

The rationale for choosing such form of the best response supply functions is that it first of all maximizes the profits with respect to the residual demand. In the case of convex cost curves this form of the supply functions leaves other firms with more elastic residual demand than under Cournot and SFE.

Nash equilibrium in this game can be calculated as follows: Suppose there is an equilibrium withholding quantity Δq^* and $N-1$ firms has already withheld it. This total withholding can be viewed as a outward shift of the total inelastic demand to $D' = D + \Delta q^*(N-1)$. Therefore, now the remaining firm faces the residual demand $D'(p) = D' - (N-1)\alpha p$ and the best response withholding is

$$\Delta q^{BR} = \frac{D'}{N^2 - 1}$$

The Nash equilibrium is then found by solving for

$$\Delta q^* = \frac{D + \Delta q^*(N-1)}{N^2 - 1},$$

which gives

$$\Delta q^* = \frac{D}{N(N-1)},$$

and the total withholding

$$\Delta Q^* = N\Delta q^* = \frac{D}{N-1}$$

The total industry supply for each demand realization is then

$$S^*(p) = \alpha(N-1)p \tag{2.2}$$

This is a higher supply than in the case of Supply function Equilibrium (2.1) but lower than the competitive supply $S^C(p) = \alpha(N-1)p$.

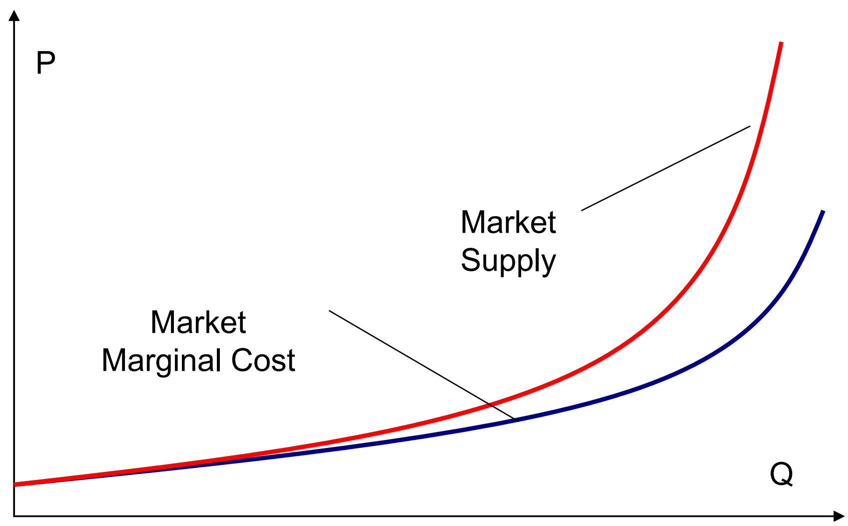
Withholding Equilibrium with nonlinear and asymmetric marginal costs

The example of linear symmetric marginal costs presented above is illustrative but not realistic. The energy markets are characterized by "hockey-stick" marginal costs that are relatively flat at low and medium output levels and very steep at high output levels. So, it may be important to check equilibrium withholding behavior at different levels of curvature of system marginal costs.

It turns out that for a class of industry marginal cost functions $mc(Q) = AQ^\varepsilon + B$, where A, B, and ε are constants and with the affine assumptions on the individual marginal costs functions the withholding equilibrium can be solved in the closed form. Parameter ε can be viewed as a measure of the relative curvature of the industry marginal cost curve measured as $\varepsilon - 1 = \dfrac{mc''(Q) \cdot Q}{mc'(Q)}$. It can be shown that in such a setting the total equilibrium withholding is proportional to the total demand and depends strongly on the relative curvature of the marginal cost curve. In addition, the total withholding depends on market concentration. See Appendix A for proofs.

The resulting industry supply is therefore overpriced the most at high values of output at which the curvature of marginal cost is high and approaches the competitive supply as the output decreases (Figure 1-4).

Figure 1-4. Industry supply function under withholding equilibrium



1.3 Collusive behavior in electricity markets: Pivotal suppliers model

The withholding equilibrium presented above and its more general case, Supply Function Equilibrium, are the cases of unilateral market power and are based on the individual incentive to profitably withhold capacity economically or physically given the residual demand. However, energy markets involve repeated interaction with a handful of players, and a high frequency of these interactions (daily and hourly) may significantly facilitate the tacit collusion Tirole (1988).

In an efficient tacit collision firms maximize joint profits. Although each firm has an incentive to deviate from the joint profit maximization to increase its instantaneous profit, such incentive is balanced with the threat of the price war that such deviation can initiate. If the loss in the expected present discounted value of the future profits as a result of the deviation outweighs the instantaneous profits of the deviation, the tacit collusion can be sustained. High frequency of interactions means a discount rate between the periods close to one and therefore increases the present value of the profit losses from deviation and stability of tacit collusion.

Energy markets are also characterized by the firms facing capacity constraints and high variability of demand, which is partly due to the predictable seasonal demand variations but to a large part is stochastic with the actual realization after the quantity and price bids are submitted[3].

Green and Porter (1984) study tacit collusion in which firms do not observe the rivals' actions observing rather the resulting market price or their own realized market share. In such a setup

[3] In fact, a lot of uncertainty in the supply-demand conditions like unplanned outages is only realized after the fact

upward demand shocks can switch the market into the collusive state and downward demand shocks into the price war state. Rotemberg and Saloner (1986) study collusive regimes under predictable future demand changes. They suggest that high current demand provides a larger incentive to deviate from collusive strategy, since the deviation profits are high and the future profit loss as a result of the price war is low because of the expected low future demand. As a result, collusion can be sustained at a higher price level during low and increasing demand. Brock and Scheinkman (1985) and Staiger and Wolak (1992) study the effects of capacity constraints faced by firms on the resulting tacit collusion. In general, capacity constraints change the results as they change both the deviation profits and loss of future profits as a result of the price war.

It is likely that if tacit collusion exists in energy markets, it is easier to sustain during periods of high demand rather than during periods of low demand. To see that, one needs to remember that generating firms in energy markets operate under tight capacity constraints and the lead time of new capacity entry exceeds by the factor of thousand the frequency of interaction.

Consider the situation in which hourly inelastic demand is so close to the market capacity that the supply margin defined as the difference between the market capacity and the demand is less than capacity of an individual generator. Even if everyone else behaves competitively, this generator faces inelastic residual demand and has an incentive to place a price bid on his capacity up to the price cap. Such bid would be accepted and would set the market price. This would be the case of unilateral market power described above.

Consider the situation in which all but two firms behave competitively and the supply margin is larger than the capacity of any of these two firms but smaller than their cumulative capacity. In this case two firms may engage in tacit collusion, both bidding their capacity up to the price cap. Given a discount rate very close to one between the interactions, such tacit collusion can be sustained easily. I will refer to this situation as to pivotal duopoly, implying that two firms have to act in concert to exercise market power in this case. Likewise, lower demand level corresponds to higher number of firms in pivotal oligopoly.

However, unlike the above example, the total number of firms is usually larger than the number of firms needed to form a pivotal oligopoly. Therefore, coordination becomes a problem. Suppose, there is a total of three firms of identical capacity and two are needed to form a pivotal oligopoly. Some coordination is required as to which two of the three firms will be bidding the high price on their capacity. The coordination is even harder to achieve given that players only observe market price as a correlated signal of their actions but not the actions themselves. I suggest a model that uses the notion of symmetric mixed strategy Nash equilibrium to quantify the coordination problem in tacit collusion in energy markets.

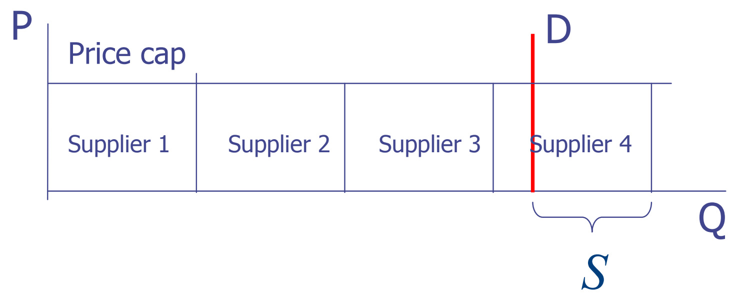
Model of pivotal oligopoly

I model the market price resulting from a real-time uniform-price electricity auction as symmetric, mixed strategy Nash equilibrium of a simultaneous move static game. The goal is to quantify the degree of coordination between tacitly colluding firms depending on how many firms have to collude to successfully exercise market power, in other words how many firms need to constitute the pivotal oligopoly.

There are N identical firms in the market, each with zero marginal cost of generation and one unit of capacity. The market price cap is set at $\overline{p} = 1$ without loss of generality. Demand $D \leq N$ is completely inelastic.

First, consider the case of a pivotal group monopoly. This implies that the difference between the system capacity N and the demand D is less than the unit capacity of an individual generator (Figure 1-5).

Figure 1-5. Pivotal monopoly



I model the strategies available to generators as choosing between two points: "Compete" and bid a price equal to marginal cost ($p = 0$) for all capacity or "Collude" and bid a price equal to the price cap ($p = 1$). The payoff of each firm depends on how many other firms "Collude". The payoff matrix of each firm is as shown in Table 1-1. The payoff depends on each firm's own strategy and on the number of other colluding firms.

Any firm gets zero profit if all firms (including itself) compete bidding zero. If one of the firms, say, Firm 1 bids zero and one or more other firms chooses to "Collude", Firm 1 gets one unit of profit. This happens because under the uniform-price auction, the firm or firms who bid at the unit price cap set the market-clearing price as long as the total capacity priced at the price cap exceeds the supply margin. The resulting market-clearing price then has to be paid for all purchased power, including that from firms bidding zero.

Table 1-1. Payoff matrix for the pivotal monopoly

		# of firms other than Firm 1 playing "Collude"			
		0	1	2	3
Firm 1	Compete	0	1	1	1
	Collude	1-s	1-s/2	1-s/3	1-s/4

If the Firm 1 colludes, it sets the market-clearing price at one, but only part of its capacity is purchased, since those bidding zero are rewarded. In fact, Firm 1 can only sell as much as $1 - s$ of the capacity and receive the unit price for it, where s is the supply margin (Table 1-1).

If two or more firms bid collude, then the market takes only a part of the capacity of these firms. I assume that the system operator running the market uses the following rationing rule when more than one firm colludes. The amount of power which is not purchased from each colluding generator is simply proportional to supply margin and inversely proportional to the total capacity priced at the price cap. That explains the payoffs of Firm 1 $1-s/2$, $1-s/3$ and $1-s/4$ when it colludes together with one, two and three other firms.

Mixed strategy Nash Equilibrium

In the symmetric Nash equilibrium each firm colludes with probability q. For q to be equilibrium, each firm should have the same expected payoff from playing "Collude" and "Compete". That is, since the probability that k out of N - 1 firms simultaneously bid "High" is

$$C_{N-1}^{k}(1-q)^{N-1-k}q^{k} \tag{3.1}$$

where C_N^k is the number of combinations of N taken k at a time calculated as

$$C_N^k = \frac{N!}{k!(N-k)!}$$

q should be the root of the following polynomial:

$$(1-s)(1-q)^{N-1} - \frac{s}{2}C_{N-1}^{1}(1-q)^{N-2}q - \frac{s}{3}C_{N-1}^{2}(1-q)^{N-3}q^{2} - \ldots - \frac{s}{N}q^{N-1} = 0 \tag{3.2}$$

The probability of having the market-clearing price at the price cap is then equal to the joint probability of having at least one player bidding "Collude":

$$P = 1-(1-q)^{N}\ , \tag{3.3}$$

where q is the solution to the equation (3.2).

Two or more firms in a pivotal oligopoly

When the demand is such that the supply margin exceeds the capacity of a single generator, a group of several generators have to act together to be pivotal. The probability of exercising market power and the expected market price when two or more firms have to jointly become pivotal can be also derived from the symmetric mixed Nash equilibrium. The case of two firms in the pivotal group, that is, when $s \in [1,2)$, is illustrated in Figure 1-6, and the resulting matrix of payoffs is shown in Table 1-2.

Figure 1-6. Two firms in pivotal oligopoly

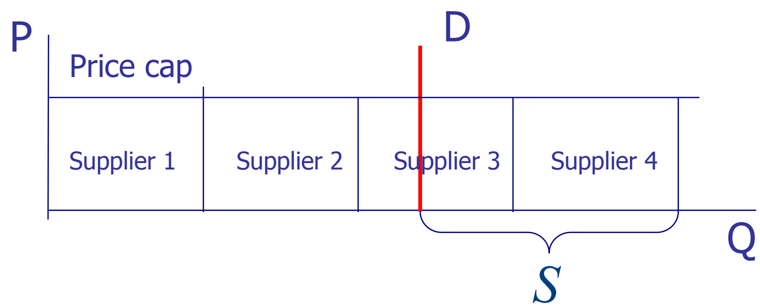


Table 1-2. Payoff matrix for the case of a pivotal duopoly

# of firms other than Firm 1 playing "Collude"

		0	1	2	3
Firm 1	Compete	0	0	1	1
	Collude	0	1-*s*/2	1-*s*/3	1-*s*/4

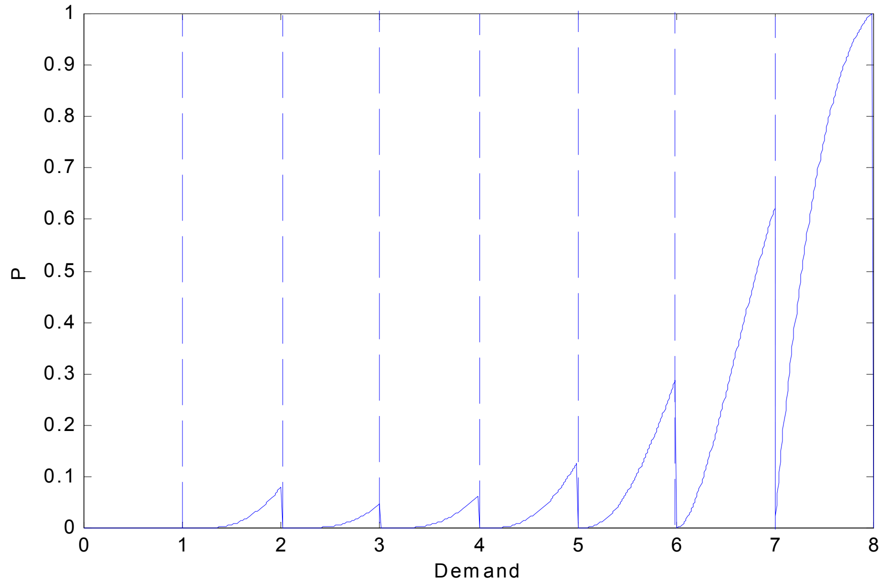
At least two firms now need to collude in order to get the market price equal to the price cap. In such case each of the colluding firms receives a profit of $1-\frac{s}{m}$, where m is the number of firms colluding firms, while each of the generator that plays "Compete" gets one unit of profit.

In general, if the minimum number of firms needed to form a pivotal oligopoly is g and the total number of firms is N, the equilibrium probability of each firm's playing "Compete" in the mixed strategy Nash equilibrium is given similarly to (3.2) by the polynomial:

$$\left(1-\frac{s}{g}\right)C_{N-1}^{g-1}(1-q)^{N-1}q^{g-1}-\frac{s}{g+1}C_{N-1}^{g-2}(1-q)^{N-1-g}q^{g}-\frac{s}{g+2}C_{N-1}^{g-3}(1-q)^{N-2-g}q^{g+1}-\ldots-\frac{s}{N}q^{N-1}=0 \quad (3.4)$$

where s is the supply margin.

Figure 1-7. Expected market prices as a function of market demand



The probability of having the market price equal to the price cap is then

$$P = C_n^g (1-q)^{N-g} q^g + C_N^{g+1}(1-q)^{N-g-1} q^{g+1} + ... + C_N^N q^N \tag{3.5}$$

The polynomial (3.4) has only one real root in the interval [0, 1], which is taken to be the mixed strategy Nash equilibrium. I then calculate the probability of market power exercise for any given demand level. The resulting "supply function" is plotted in Figure 1-7, which illustrates the case of eight firms, each of which has unit capacity.

When the demand is at the capacity margin ($s = 0$) the probability of having the market price at the cap is one. The probability of a high market-clearing price is decreasing in the supply margin, although not uniformly. The graph of the probability of high price has discontinuities every time when the number of firms in pivotal oligopoly changes. For instance, the number of firms in the pivotal oligopoly changes from 2 to 1 when demand changes from 6.99 to 7.01. When the demand is 7.01, to "Compete" becomes a dominant strategy. The "Collude" strategy does not give any extra profit over competing in any state. When the demand level is 6.99, "Collude" gives a higher profit than "Compete" when one other generator colludes. In other worlds, the probability of market power exercise at each g is maximized when $s = (g-1)_-$ and is zero when $s = g_+$.

Appendix B compares the effect of the total number of firms in the market and the number of firms in the pivotal oligopoly on the expected market price. The result is that the number of firms in pivotal oligopoly rather than the total number of firms determines the degree of tacit collusion stability and therefore the expected market price for demand levels exceeding one third of market capacity.

Blumsack, et al. (2002) provide an analysis of the prevailing number of firms in pivotal oligopolies in electricity markets of California Independent System Operator (CAISO), New York ISO (NYISO) and Pennsylvania-New Jersey-Maryland ISO (PJM). PJM and NYISO seem to have enough excess generating and transmission capacity so that the situations of pivotal monopoly and pivotal duopoly never occur. In California the number of firms in pivotal oligopoly was two or less about 12% of the time in 2000.

1.4 The California Energy Market in 1998-2000

Prior to April 1, 1998 electricity in California was generated and delivered to most customers through transmission and distribution networks by one of the three major investor owned utilities: Pacific Gas & Electric, Southern California Edison, and San Diego Gas & Electric. The three utilities owned generation, transmission and distribution assets and were subject to government regulation of the retail rates and investment decisions.

In April 1998 California opened a restructured electricity market. The three investor-owned utilities were directed to divest their natural gas generation assets to five private generator companies: AES, Duke, Dynegy, Reliant, and Mirant that were supposed to compete in selling power. The three investor-owned utilities still operated their distribution networks and served their customers by buying power in the markets from the private generators and selling it to customers under regulated rates. The operation of the transmission network was delegated to the California Independent System Operator (CAISO).

Table 1-3. CAISO capacity, 1999

July 1999 - online capacity (MW)

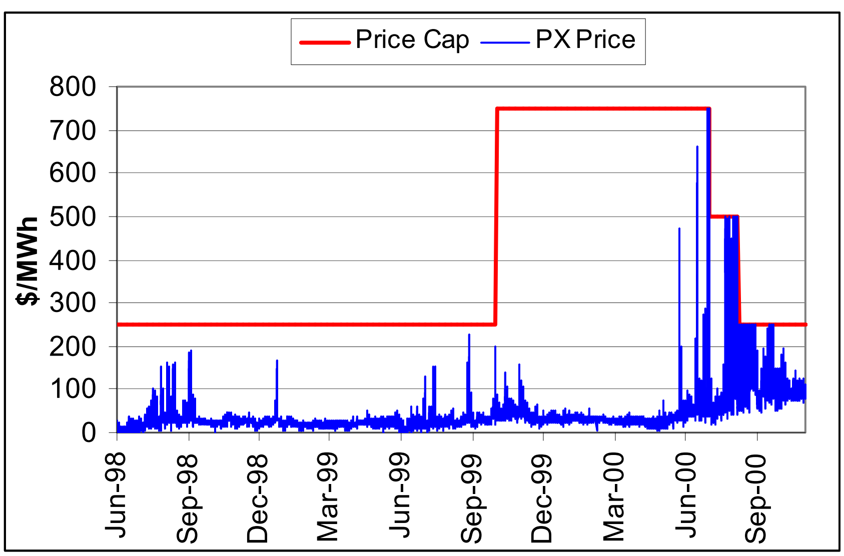
Firm	Fossil	Hvdro	Nuclear	Renewable	Total
AES	4,071	-	-	-	4,071
Duke	2,950	-	-	-	2,950
Dynegy (DST)	2,856	-	-	-	2,856
PG&E	580	3,878	2,160	793	7,411
Reliant	3,531	-	-	-	3,531
SCE	-	1,164	1,720	-	2,884
Mirant	3,424	-	-	-	3,424
Other	6,617	5,620	430	4,888	17,555
Total	24,029	10,662	4,310	5,681	44,682

As of 1999 the ownership of generating capacity under CAISO jurisdiction was distributed among private generators and investor-owned utilities as shown in Table 1-3.

The Power Exchange (PX) was instituted for day-ahead energy trading. One day before delivery generating firms submitted their supply schedules for every hour of the next day. Likewise, parties purchasing power, like the investor-owned utilities, submitted demand schedules for every hour of the next day. The PX computed the aggregate supply and demand schedules, found the equilibrium and computed the day-ahead hourly market clearing prices and quantities at which the trades were settled.

The schedules were then submitted to the CAISO, which checked whether the transmission grid capacity and reliability is violated as a result of the market-clearing schedule of the PX in which case managed the congestion. The CAISO also ran the real-time market to ensure that the actual realized load is matched with enough generation. The vast majority of power (80-90%) in 1998-2000 was sold through the PX. A price cap were established in the CAISO real-time market at $250/MWh originally, which was later lifted to $750/MWh and lowered again first to $500/MWh and then to $250/MWh as the prices soared in summer 2000. Because of arbitrage, the price cap in the ISO real-time market effectively capped the PX prices (Figure 1-8).

Figure 1-8. Historic PX price and CAISO price cap



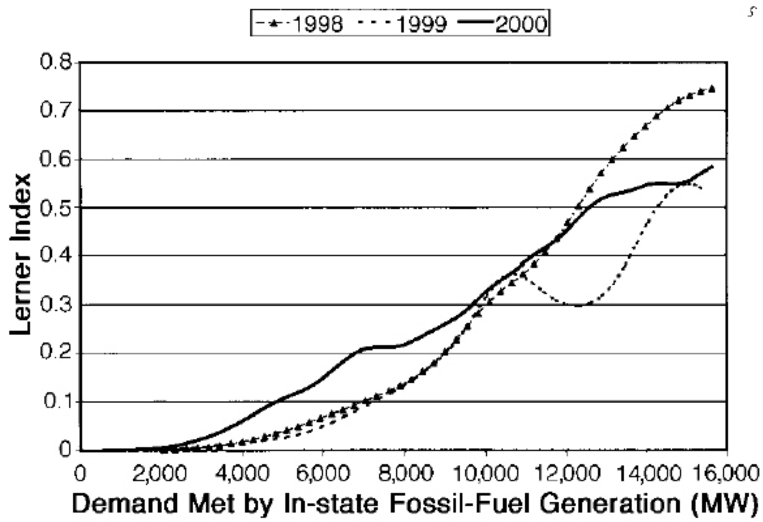
Several groups studied the sharp increase in California wholesale energy prices in summer 2000 (Joskow and Kahn (2002), Borenstein, et al. (2002), and Sheffrin (2001b)) and concluded that a large part of this price increase was due to the market power exercised by private generating firms.

These studies were based on the premise that the fossil fuel power plants operated by the five private generators constituted the high end of the marginal cost curve of CAISO generation mix and therefore set the market price most of the time. They further used maximum capacity, heat rates and emission rates of fossil fuel power plants together with the data on California prices of natural gas and NOx allowances to construct the CAISO marginal cost curve. The marginal cost is then intersected with the residual demand faced by the fossil fuel generators to establish the competitive benchmark price. The actual price was then compared with the competitive benchmark (Wolfram (1999)).

Joskow and Kahn (2002) found that the gap between the average competitive benchmark and the actual price significantly widened in summer 2000. They also analyzed the output of generators and pointed to a gap between the average output of some generators and the output that would be expected under competitive operations. They attributed this gap to economic or physical withholding with the purpose of market power exercise.

Figure 1-9 presents one of the findings of Borenstein, et al. (2002) showing the kernel regression of the Lerner Index as a function of the output of the private fossil fuel generation in 1998, 1999, and 2000. This suggests that the price tends to exceed the competitive benchmark when the market is tight and the demand approaches capacity. The high prices increase in 2000 can therefore be explained by unusually high demand faced by the fossil fuel plants. However, the question remains whether the excess of the price over the competitive benchmark is the result of market power exercise.

Figure 1-9. Kernel regression of Lerner Index, CAISO 1998-2000



Harvey and Hogan in their papers (Harvey and Hogan (2001) and Harvey and Hogan (2002)) assert that both the gap between the competitive and actual output of generators and the increase of price over the competitive benchmark as the output gets closer to capacity can be explained by reasons other than market power. Their main point is that, in addition to the fuel costs of energy, generators should include significant opportunity costs in their bid price. In particular, Harvey and Hogan provide the following possible scenarios of why a generator may bid above the marginal cost, and as a result generate less without the intent to exercise market power.

Simulations of the competitive benchmark price usually include the price of NOx tradable allowances. However, some generating plants face environmental constraints other than NOx that effectively cap their total annual output. When the plant's production reaches the cap, the generator has to include the opportunity cost in his bid since by generating now it foregoes the future profits. As a result, the generator bids above the marginal cost and generates less than it would if bid its marginal costs.

Likewise, extreme plant operations such as operating for an extended time or at or above capacity may increase the risk of an unplanned outage and the loss of profits. The expected costs of the increased outage risk can also be included in the bids and result in an output gap.

Finally, generators often face operational restrictions. For instance, some steam generating units may have slow ramp rates that keep them from getting from shut down to full within an hour. Starting a plant is a costly procedure itself. As a result, generators may include startup costs and the opportunity costs reflecting their ramp rates in their bids.

Harvey and Hogan's conclusion is that it is misleading to compare the simulated competitive benchmark with the realized price to prove anything about market power exercise unless all opportunity costs are taken care of.

In addition to the studies measuring market power on the market level Puller (2001) studies market power in California on the firm level. He empirically estimates the first order conditions of California private generators in a general form to see whether their behavior is competitive, represents a case of static (Cournot) market power, or perfect collusion. He finds that firms behave according to static Cournot; the estimated conduct parameters do not show perfect collusion, but instead suggest an imperfect collusion leading to Cournot profits just a little above.

1.5 Empirical test of the nature of market power exercise

I use firm level output data as well as PX price data to test the nature of market power exercise by the firms in California. I particularly focus on tracking changes in firms' behavior with the demand faced by private generating firms. I also check whether market power exercise had a unilateral character explained by the models like withholding equilibrium or Supply Function Equilibrium or a collusive character according to the pivotal suppliers model.

The analysis shows whether the decrease in each firm's output was significantly associated with a price increase after controlling for price determinants like input prices and the residual demand faced by the fossil fuel generators. I do not estimate the cost of the firms since I do not have enough data to accurately estimate the opportunity cost of generator units, which may constitute a significant part of the total cost according to Harvey and Hogan. As a result I cannot always tell whether a high price increases firm's profit. Instead I look at the potential revenue increase assuming that if a decrease in output is associated with an increase in revenue it would also increase the profits. As a result, in some cases I have instances when firms' output decreases are associated with price increases which do not increase revenues. I assume that little can be said in these cases.

To check collusive behavior, I test whether a simultaneous decrease in output by groups of generators is associated with a significant price increase. In particular, as the evidence of collusive behavior, I use situations when price increases, resulting from the cumulative output decrease by several firms significantly exceeds the sum of the price increases resulting from the individual actions of the same firms.

Data

For my analysis I use data collected by Borenstein, Bushnell and Wolak for their paper. The data includes hourly observations of system load in the CAISO area, the part of the load met by the fossil fuel generators, PX price, and price of NOx allowances. For the hourly energy output of each generator I use the public data available from Environmental Protection Agency's Continuous Emissions Monitoring system (CEMS). CEMS contains hourly output and emission form most of the California fossil fuel units except small gas turbines. I use the data on daily natural gas prices in California from San Diego Gas and Electric. I study the period between January 1, 2000 and October 31, 2000.

Estimation of inverse supply of fossil fuel generators

First, I net out the effects of the fundamental determinants of the energy price, such as the price of inputs like natural gas and emission allowances and the demand met by the instate fossil fuel plants. I estimate the model:

$$lmPXP_t = \beta_0 + \beta_1 Total_t + \beta_2 Total_t^2 + \beta_3 Total_t^3 + \beta_4 Total_t^4 + \beta_5 \ln Pgas_t + \beta_6 \ln Pnox_t + \varepsilon_t \ ,$$

where *PXP* is the hourly Power Exchange price in \$/MWh and *Total* in MWh is the total output of California private fossil fuel generators. The estimated marginal cost curve of the fossil fuel capacity is highly convex and so is the actual inverse supply. To reflect that I approximate the logarithm of the price by a fourth order polynomial in total fossil fuel output. I also model constant elasticity in the price of natural gas *Pgas* and the price of NOx allowances *Pnox*. I use the generalized method of moments to estimate the inverse supply of gas fired capacity instrumented by the total load in the California ISO with heteroskedasticity and autocorrelation adjusted weighting matrix. The result of this estimation is presented in Table 1-4.

Table 1-4. Estimation of the inverse supply of the California fossil fuel capacity

Dependent Variable: LPXP

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.011208	0.178662	5.659887	0.0000
TOTAL	0.000948	0.000148	6.429851	0.0000
TOTAL^2	-1.85E-07	4.13E-08	-4.492384	0.0000
TOTAL^3	1.46E-11	4.24E-12	3.451478	0.0006
TOTAL^4	-3.56E-16	1.43E-16	-2.486009	0.0129
LPGAS	0.826341	0.100790	8.198655	0.0000
LPNOX	0.033452	0.028472	1.174909	0.2401

R-squared	0.847514	Mean dependent var	3.992519
Adjusted R-squared	0.847389	S.D. dependent var	0.779891
S.E. of regression	0.304668	Sum squared resid	678.8135
Durbin-Watson stat	0.196527	J-statistic	5.71E-26

The energy price is almost proportional to the gas price given the demand met by the fossil fuel plants, however is less than proportional to the price of NOx allowances since only a fraction of California power plants is subject to the NOx trading program.

Test of unilateral market power

To test the firms' behavior I regress the residuals from the inverse supply regression on the logarithm of the actual output of individual firms. To reflect the possible effect predicted by the model of pivotal oligopoly I divide the total range of the demand met by the five private generators into five equal regions and test separately the firms' behavior in each of the regions. In each region I effectively estimate the equation:

$$\ln PXP_t|_{Total,Pgas,Pnox} = \gamma_0^i + \gamma_1^i \ln Gen_t^i + \nu_t^i, i = 1,..,5 \,, \tag{5.1}$$

where Gen_t^i is the output of one of the generators: AES, Duke, DST, Reliant, and Mirant. The results of such regressions for the values of fossil fuel demand over 12,000MWh roughly corresponding to a pivotal monopoly are presented in Table 1-5. For three out of five generators decreases in output are significantly associated with price increases. This price increase also increases revenue since γ characterizes the inverse residual demand (Wolak (2003)) to the extent exploited by the firms is less than minus one (significantly for Reliant). AES and Duke seem to behave competitively, increasing the output in response to the price increases.

Table 1-5. Unilateral firm behavior: pivotal monopoly

Total > 12,000MW

Firm	γ	Std	t	P
AES	0.5562	0.3548	1.5677	0.1175
Duke	1.4001	0.6946	2.0157	0.0443
DST	**-1.2763****	0.2633	-4.8478	0.0000
Reliant	**-1.9466****	0.3297	-5.9046	0.0000
Mirant	**-1.3541****	0.3072	-4.4081	0.0000

Table 1-6 through Table 1-9 show similar estimates for the cases of two to five firms in the pivotal oligopoly. In the case of pivotal duopoly only withholding by DST and Reliant seem to increase the price significantly. In cases of three, and four firms in pivotal oligopoly only Reliant's decrease in output is significantly associated with price increases. Interestingly, when the demand faced by the five firms is the least, that is, in the case of five firms in pivotal oligopoly the output AES, Reliant and Mirant is significantly negatively correlated with the price. However, in none of the cases of more than one pivotal firms the absolute value of γ is significantly greater than 1, meaning that the price increase associated with the output decrease is not revenue increasing, however may be profit increasing depending on the costs

Table 1-6. Unilateral behavior: two firms in pivotal oligopoly

Total > 9,000MW and < 12,000MW

Firm	γ^i	Std	T	P
AES	-0.0583	0.1388	-0.4201	0.6745
Duke	0.0519	0.1271	0.4087	0.6828
DST	**-0.1578****	0.0674	-2.3411	0.0194
Reliant	**-0.2361****	0.0974	-2.4235	0.0155
Mirant	0.1124	0.0813	1.3835	0.1668

Table 1-7. Unilateral behavior: three firms in pivotal oligopoly

Total > 6,000MW and <9,000MW

Firm	γ^i	Std	T	P
AES	0.07928	0.01820	4.35546	0.00000
Duke	0.09682	0.04970	1.94811	0.05160
DST	-0.02295	0.02860	-0.80259	0.42230
Reliant	**-0.23577****	0.03722	-6.33507	0.00000
Mirant	0.23280	0.02594	8.97475	0.00000

Table 1-8. Unilateral behavior: four firms in pivotal oligopoly

Total > 3,000MW and <6,000MW

Firm	γ^i	Std	T	P
AES	-0.0122	0.0098	-1.2435	0.2138
Duke	0.1311	0.0140	9.3480	0.0000
DST	0.0099	0.0170	0.5841	0.5592
Reliant	-0.0731**	0.0179	-4.0795	0.0000
Mirant	0.0903	0.0166	5.4454	0.0000

Table 1-9. Unilateral behavior: five firms in pivotal oligopoly

Total < 3,000MW

Firm	γ^i	Std	T	P
AES	-0.0671**	0.0197	-3.4104	0.0007
Duke	0.0350	0.0145	2.4106	0.0160
DST	0.0547	0.0292	1.8716	0.0614
Reliant	-0.1196**	0.0288	-4.1499	0.0000
Mirant	-0.0972**	0.0254	-3.8287	0.0001

Test of collusive behavior

Testing the model of pivotal oligopoly requires looking at whether the simultaneous actions of generators resulted in significantly higher price increases than the total of the price increases they could individually achieve. Model of pivotal oligopoly suggests that unilateral withholding should not result in much price increase when supply margin is larger than the capacity of a single generator. However, joint withholding should. To test that, I run the regressions (5.2).

$$\begin{aligned}
\ln PXP_t|_{Total,Pgas,Pnox} &= \gamma_0^{ij} + \gamma_1^{ij}\ln\left(Gen_t^i + Gen_t^j\right) + \nu_t^{ij}, \quad i,j = 1,..,5 \\
\ln PXP_t|_{Total,Pgas,Pnox} &= \gamma_0^{ijk} + \gamma_1^{ijk}\ln\left(Gen_t^i + Gen_t^j + Gen_t^k\right) + \nu_t^{ijk}, \quad i,j,k = 1,..,5 \\
\ln PXP_t|_{Total,Pgas,Pnox} &= \gamma_0^{ijkn} + \gamma_1^{ijkn}\ln\left(Gen_t^i + Gen_t^j + Gen_t^k + Gen_t^n\right) + \nu_t^{ijkn}, \quad i,j,k,n = 1,..,5
\end{aligned} \tag{5.2}$$

I further test the hypotheses:

$$H_0 : \gamma_1^{ij} \geq \gamma_1^i + \gamma_1^j, i,j = 1,..,5; \quad H_A : \gamma_1^{ij} < \gamma_1^i + \gamma_1^j, i,j = 1,..,5$$
$$H_0 : \gamma_1^{ijk} \geq \gamma_1^i + \gamma_1^j + \gamma_1^k, i,j,k = 1,..,5; \quad H_A : \gamma_1^{ijk} < \gamma_1^i + \gamma_1^j + \gamma_1^k, i,j,k = 1,..,5$$
$$H_0 : \gamma_1^{ijkn} \geq \gamma_1^i + \gamma_1^j + \gamma_1^k + \gamma_1^n, i,j,k,n = 1,..,5; \quad H_A : \gamma_1^{ijkn} < \gamma_1^i + \gamma_1^j + \gamma_1^k + \gamma_1^n, i,j,k,n = 1,..,5 \tag{5.3}$$

Here H_0 hypotheses correspond to the case of no collusion (see Appendix C for proofs). Table 1-10, Table 1-11, and Table 1-12 provide the results of the regressions (5.2) for different levels of the demand faced by the five private firms corresponding to different numbers of firms in the pivotal oligopoly. The cumulative output of two, three, and four firms is often significantly negatively correlated with the price when demand faced by the fossil fuel generators exceeds 12,000MW, which corresponds to having a pivotal monopoly. In most of the cases the implied inverse demand elasticity is significantly over 1, meaning that price increases as a result of output decrease increased the revenue of the firm combinations.

The model of pivotal suppliers suggests that in the cases of a pivotal duopoly, that is, the total demand between 9,000MW and 12,000MW, we should see the potential to increase prices by joint withholding of two and more firms similar to the one single firms have during the hours with the demand over 12,000MW. However, just three out of ten combinations of two firms are significant: AES+DST, AES+Reliant and DST+Reliant; three out of ten combinations of three firms: AES+DST+Reliant, Duke+DST+Reliant, and DST+Reliant+Mirant; and two out of five combinations of four: AES+Duke+DST+Reliant and AES+DST+Reliant+Mirant are significant at 10% confidence level. None of the price increases resulting from the joint withholding increased the revenue of the group of firms (but may be profit increasing depending on the costs).

When demand corresponds to the cases of pivotal oligopoly of three and four firms the situation is similar. There is no significant price increase resulting from withholding by most of the combinations of three and four. In cases when such withholding is significantly price increasing the magnitude is not higher than in the cases of lower number of firms in pivotal oligopoly as the mode suggests.

Apparently, when demand is less than 3,000MW, that is, there are all five firms must be in the pivotal oligopoly, about half of the combinations of two and three firms and two out of five combinations of four firms are significantly price increasing. This may seem in line with the pivotal oligopoly model that suggests that when the number of firms in the pivotal oligopoly is the same as the total number of firms there is no coordination problem and the firms get to the joint profit maximization easier.

Table 1-13 through Table 1-17 show the results of the independent *t*-tests (5.3) that look at whether the price increase resulting from the joint withholding of a combination of firms was significantly larger than the sum of the results of the individual withholdings. Table 1-13 shows that when demand is over 12,000MW, corresponding to the case of pivotal monopoly, many combinations of two to four firms are significantly more profitable when they act together rather than when they act individually.

However, this result does not hold for lower demand levels corresponding to two and more pivotal firms. Some combinations of firms acting together achieve a price increase larger than the sum of the individual price increases but not statistically significant one.

1.6 Conclusion

This paper studies the models of market power that account for the unique features of the electricity markets and tests them on the firm level data from the California Independent System Operator for 2000.

I address the question of what causes the increase in the Lerner Index with demand shown by Borenstein, et al. (2002) (BBW) (Figure 1-9): unilateral market power (withholding equilibrium or SFE), collusive behavior explained by the model of pivotal oligopoly, or the fact that the opportunity costs are not accounted for in the BBW simulations (Harvey and Hogan (2001)).

Depending on the result this could mean different things in terms of policy implications:

If, as in Harvey and Hogan, the high Lerner Index at high demand levels is explained by the costs and operation constraints not accounted for in the cost simulations of BBW such as opportunity costs, startup costs, etc., then no extra mitigation of market power is needed. This would mean that the prices observed in California in 2000 simply indicate a severe capacity shortage and were effective signal for new capacity investments.

If the unilateral market power were the main reason for high price markup, the policy implications would be different. As suggested by the withholding equilibrium, the most market power would be exercised during the high demand hours when the residual demand faced by the firms is the less elastic. In particular, the hours in which a single firm is pivotal would be the most critical with respect to market power exercise as they correspond to the case of inelastic demand faced by pivotal monopolist. During hours of more than one firm in pivotal oligopoly, individual firms face elastic demand, and therefore, the risk of market power is significantly lower. Policymakers like the Federal Energy Regulatory Commission (FERC) and California Independent System Operator (CAISO) have recognized this fact. FERC suggests using a Supply Margin Assessment (SMA) test in making a decision to allow firms market based bidding. The SMA test essentially looks at whether the supplier is pivotal during the hours of peak demand. CAISO uses the Residual Supply Index that measures the extent of pivotality of an individual firm (Sheffrin (2001a)). CAISO uses RSI to monitor the market and proposes to use it as a trigger of market mitigation procedures. Both SMA and RSI look at the uncommitted capacity of firms, that is, the capacity that is not bound with a long term contract since the contracted capacity cannot earn market power premium.

A natural mitigation of unilateral market power would be ensuring that the supply margin does not often shrink to values smaller than the uncommitted capacity of individual generators. This may include increasing capacity margins, reducing individual uncommitted capacity by disinvestment or by creating favorable environment for long term contracting (Wolak (2000)). However, to some extent unilateral market power can also be exercised in long term contracts as well.

The model of pivotal oligopoly suggests that the firms participate in tacit collision. Therefore, if the high Lerner Index at high demand levels is explained by the pivotal oligopoly game then one of the policies to mitigate such market power would be to decrease the interaction frequency. This is hard to do completely because of the instantaneous nature of the electricity markets. However, long-term contracts can help here to decrease the inframarginal capacity. A pivotal supplier game in the capacity under long term contracts is less likely since the interaction frequency required for the pivotal suppliers game drops significantly in such contracts. A natural mitigation of the pivotal suppliers game would also include expanding the supply margin by building extra generation or transmission capacity to make the low number of firms in pivotal oligopoly happen less often (Blumsack, et al. (2002)).

The empirical analysis of this paper suggests that the most evidence of unilateral market power can be found during the hours of pivotal monopoly. When several firms simultaneously withhold capacity, they achieve even higher price increase than they could by acting independently. So, there is also an evidence of some collusive behavior during the hours of pivotal monopoly.

However, during the hours corresponding to more than one firm in pivotal oligopoly, the unilateral withholding is not profitable in general and the joint actions of several firms do not significantly increase the prices more than the independent actions could do.

These arguments suggest the firms' behavior in California is the best explained by the unilateral models of market power. Puller has arrived at similar results using a different method. He rejected the hypothesis of perfect collusion in CAISO market and failed to reject the unilateral market power, The results of the empirical analysis obtained for the hours with a single firm in the pivotal oligopoly allows making some conclusion on the behavior of individual firms. Table 1-5 suggests that DST, Reliant and Mirant were involved in profitable unilateral withholding. Table 1-13 also shows that the actions of Duke in combinations with these three firms were quite successful in raising the, price although unilaterally. However, alone Duke behaves competitively according to Table 1-5. Unilaterally AES is behaving competitively, however, some combinations including AES such as AES+Duke+Reliant and AES+Duke+Reliant+Mirant are more profitable than the result of the individual actions. However, this can be largely attributed to the participation of Duke in these combinations.

1.7 References

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Appendix A

Withholding equilibrium with non-linear asymmetric marginal costs

Closed form solution for the withholding equilibrium is general case cannot be obtained. I restrict the analysis to the affine marginal costs of individual firms. Assume the industry marginal cost being $C'(Q) = mc(Q)$. Assume also that each firm i takes the share β_i of the total marginal cost. That is, individual marginal cost of firm i is $c_i'(q_i) = mc\left(\frac{q_i}{\beta_i}\right)$. Total market competitive supply is given by the inverse marginal cost and is $q = S(p) = mc^{-1}(p)$ making the competitive supply of individual firms $q(p) = \beta_i S(p)$.

If all firms but j have withheld their equilibrium quantities Δq_i^*, then firm j faces the residual demand

$$D_j(p) = D + \sum_{i \neq j} \Delta q_i^* - \sum_{i \neq j} \beta_i S(p) = D_j' - (1 - \beta_j) S(p)\,,$$

where D is the market deterministic inelastic demand and $D_j' = D + \sum_{i \neq j} \Delta q_i^*$ is the effective market demand after the equilibrium withholding by firms other than firm j. Inverse residual demand is given by

$$p = mc\left(\frac{D_j' - q_j}{1 - \beta_j}\right)$$

The profit of firm j is

$$\pi_j(q_j) = mc\left(\frac{D_j' - q_j}{1 - \beta_j}\right) q_j - c_j(q_j)$$

And profit maximizing quantity is given by

$$mc\left(\frac{D_j' - q_j}{1 - \beta_j}\right) - \frac{1}{1 - \beta_j} mc'\left(\frac{D_j' - q_j}{1 - \beta_j}\right) - mc\left(\frac{q_j}{\beta_j}\right) = 0 \qquad \text{(A.4)}$$

The profit maximizing withholding can be calculated from the condition

$$mc\left(\frac{D_j' - q_j}{1-\beta_j}\right) = mc\left(\frac{q_j + \Delta q_j}{\beta_j}\right) \tag{A.5}$$

The left-side of the expression is the inverse residual demand and the right side is the firm j's supply function after the withholding. Equations (A.4) and (A.5) give the condition for the optimal withholding Δq_j:

$$\left(D_j' - \frac{1-\beta_j}{\beta_j}\Delta q_j\right)mc'\left(D_j' + \Delta q_j\right) - \frac{1-\beta_j}{\beta_j}\left(mc\left(D_j' + \Delta q_j\right) - mc\left(D_j' - \frac{1-\beta_j}{\beta_j}\Delta q_j\right)\right) = 0 \tag{A.6}$$

Formula (A.6) simplifies a lot with the substitution:

$$\begin{aligned} y_j &= D_j' + \Delta q_j \\ z_j &= \frac{\Delta q_j}{\beta_j} \end{aligned} \tag{A.7}$$

The result of this substitution is

$$mc'\left(y_j\right)\left(y_j - z_j\right) = \frac{1-\beta_j}{\beta_j}\left(mc\left(y_j\right) - mc\left(y_j - z_j\right)\right) \tag{A.8}$$

Consider the constant relative curvature form of the marginal cost function $mc(Q) = AQ^{\varepsilon} + B$. For this type of marginal cost function the equilibrium withholding is proportional to the demand. To see that conjecture $z_j = (1-\alpha_j)y_j$, where α_j is the generator specific constant. With this conjecture condition (A.8) boils down to the equation in terms of α_j

$$\varepsilon\alpha_j = \frac{1-\beta_j}{\beta_j}\left(1-\alpha_j^{\varepsilon}\right) \tag{A.9}$$

With determined from (A.9) the unilateral optimal withholding becomes proportional to the market demand:

$$\Delta q_j^{BR} = \frac{D_j'(1-\alpha_j)\beta_j}{1-(1-\alpha_j)\beta_j} \tag{A.10}$$

To find the vector of equilibrium withholdings the following system of linear equations should be solved:

$$\Delta q_j^* \frac{(1-\alpha_j)\beta_j\left(D + \sum_{i\neq j}\Delta q_j^*\right)}{1-(1-\alpha_j)\beta_j}, \forall j \tag{A.11}$$

or

$$\begin{pmatrix} \left(1-\frac{1}{\gamma_1}\right) & 1 & 1 & \dots & 1 \\ 1 & \left(1-\frac{1}{\gamma_2}\right) & 1 & \dots & 1 \\ & & \dots & & \\ 1 & 1 & 1 & \dots & \left(1-\frac{1}{\gamma_N}\right) \end{pmatrix} \begin{pmatrix} \Delta q_1^* \\ \Delta q_2^* \\ \vdots \\ \Delta q_N^* \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} D, \text{ where } \gamma_i = (1-\alpha_i)\beta_i \qquad \text{(A.12)}$$

The total equilibrium withholding can be found in the close form noting that from (A.12)

$$\gamma_j \left(\Delta Q^* + D \right) = \Delta q_j^*, \forall i$$

Summing this up over i gives:

$$\Delta Q^* = \frac{\sum \gamma_j}{1 - \sum \gamma_j} D = \frac{\sum (1-\alpha_j)\beta_j}{1 - \sum (1-\alpha_j)\beta_j} D \qquad \text{(A.13)}$$

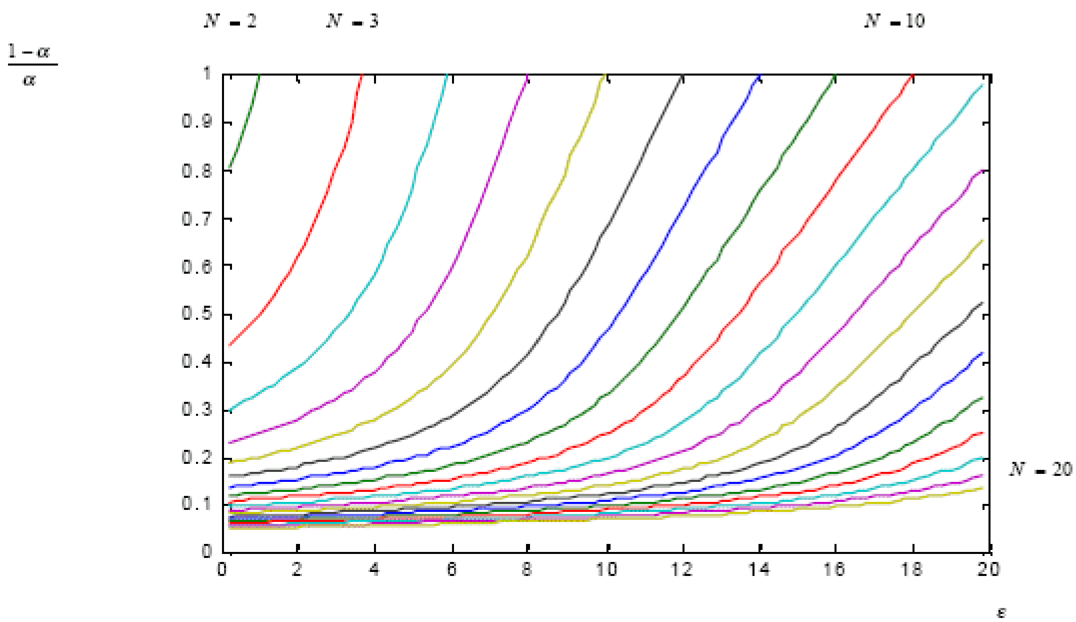
Effect of curvature

To check how the equilibrium withholding depends on the curvature parameter ε consider symmetric case, where $\beta_j = \frac{1}{N}, \forall i$. The total market withholding is then

$$\Delta Q^* = \frac{D(1-\alpha)}{\alpha}$$

Figure 1-10 shows that the multiple resulting from (A.9) increases with the curvature parameter and decreases with the number of firms N.

Figure 1-10. Effect of relative curvature on the total equilibrium withholding



Appendix B

The model of pivotal oligopoly presented above suggests that the probability of successful tacit collusion depends on both the total number of firms and the number of firms in the pivotal oligopoly. I examine the contribution of these two factors to see under what conditions the number of firms in the pivotal oligopoly is a more important indicator of the market power potential than the total number of firms. This would allow a link between the conventional measures of market concentration like HHI and the indices based on the supply margin, as well as determining the domains where one or the other is best applied.

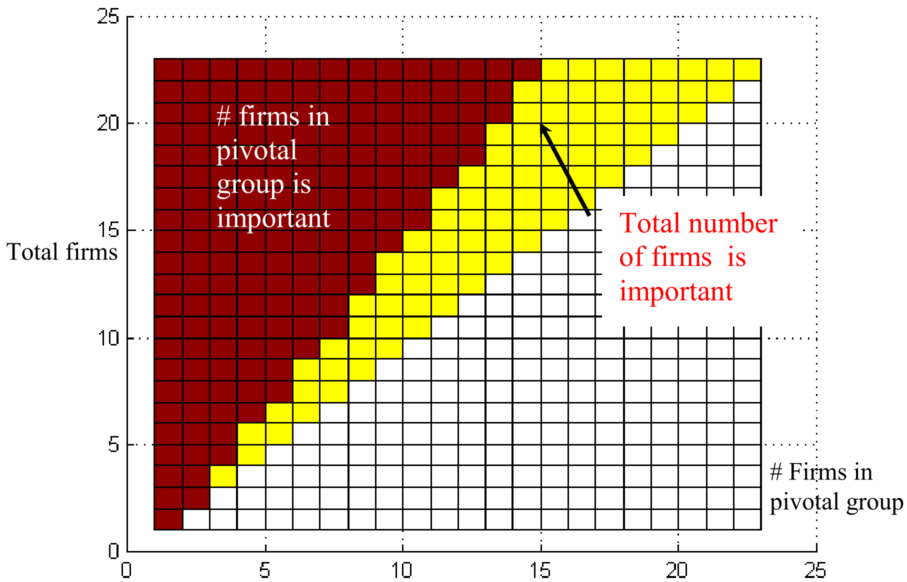
For that purpose I construct the matrix of maximum expected market prices for $s = g - 1, g = 1, 2, ..., N - 1$, assuming different market sizes in terms of total number of firms and demand levels corresponding to different sizes of the pivotal group:

$$P_{N,g}, g \leq N - 1$$

where N is the total number of firms and g is the number of firms in the pivotal group. I further take the partial differences $P_{N,g}^{N} = P_{N,g} - P_{N+1,g}$ and $P_{N,g}^{g} = P_{N,g} - P_{N,g+1}$, which characterize the marginal effects of the total number of firms and the number of firms in pivotal group on the probability of market exercise.

Figure 1-11 shows the total number of equally sized firms in the market and the number of firms in the pivotal oligopoly. When there are N firms in the market and g firms in the pivotal group, total demand is assumed to be $N - g + 1$ ($s = (g-1)_{-}$ to achieve the maximum expected market price for each pivotal oligopoly). Only the area of the graph where $g < N + 1$ is relevant. The dark-shaded area corresponds to the situations where the marginal effect of the number of firms in the pivotal group on the probability of a high market-clearing price outweighs the marginal effect of the total number of firms. That is, the dark-shaded area corresponds to situations in which a supply margin measure of market structure is a better predictor of the exercise of market power than are conventional measures of market structure. This happens at high demand levels, at which $g < 2N/3+1$. In the light-shaded area corresponding to the demand levels with $2N/3+1<g<N+1$, the marginal effect of the total number of firms on the expected market price dominates and the conventional HHI is a better predictor of market power exercise. However, at that demand level the probability of getting the price rising to the price cap is trivial according to the model.

Figure 1-11. Marginal effect of total number of firms and the number of firms in pivotal oligopoly



Appendix C

Hypotheses (5.3) use the fact that when several firms collude, they face the residual demand with higher absolute value of the inverse elasticity than the sum of inverse elasticities of residual demand faced by the firms participating in the coalition

Suppose the two firms have competitive supply functions $s_1(p)$ and $s_2(p)$ and they face a residual demand $D(p)$. At the competitive price p^* given by $D(p^*) = s_1(p^*) + s_2(p^*)$ the inverse elasticity faced by the coalition is

$$\gamma_{12} = \frac{1}{\varepsilon_{12}} = \frac{D(p^*)}{D'(p^*)p^*}$$

Firm 1 faces the residual demand $D_1(p) = D(p) - s_2(p)$ and its inverse elasticity of the residual demand is

$$\gamma_1 = \frac{D(p^*) - s_2(p^*)}{\left(D'(p^*) - s_2'(p^*)\right)p^*}$$

Similarly, the inverse elasticity of residual demand faced by firm 2 is

$$\gamma_2 = \frac{D(p^*) - s_1(p^*)}{\left(D'(p^*) - s_1'(p^*)\right)p^*}$$

If both firms have upward sloping supply functions then

$$\gamma_1 + \gamma_2 = \frac{D(p^*) - s_2(p^*)}{\left(D'(p^*) - s_2'(p^*)\right)p^*} + \frac{D(p^*) - s_1(p^*)}{\left(D'(p^*) - s_1'(p^*)\right)p^*} \geq \frac{D(p^*) - s_2(p^*) + D(p^*) - s_1(p^*)}{D'(p^*)p^*} = \frac{D(p^*)}{D'(p^*)p^*} = \gamma_{12}$$

Since $\gamma_1, \gamma_2,$ and $\gamma_{12} < 0$, this suggests that $|\gamma_{12}| \geq |\gamma_1| + |\gamma_2|$.

Appendix D: Collusion Regressions Tables

Table 1-10. Regression for collusive behavior: two firms

Total > 12,000MW

Firms	γ^i	Std	P
AES+Duke	1.4879	0.5935	0.0124
AES+DST	-0.2902	0.4490	0.5183
AES+Reliant	**-1.5166****	0.4874	0.0019
AES+Mirant	-0.5797	0.5403	0.2837
Duke+DST	**-1.4469****	0.6051	0.0171
Duke+Reliant	**-2.8321****	0.5741	0.0000
Duke+Mirant	**-1.6338****	0.6174	0.0083
DST+Reliant	**-2.5084****	0.3532	0.0000
DST+Mirant	**-2.1357****	0.3328	0.0000
Reliant+Mirant	**-3.2317****	0.3388	0.0000

Total > 6,000MW and <9,000MW

Firms	γ^i	**Std**	**P**
AES+Duke	0.2896	0.0542	0.0000
AES+DST	0.1056	0.0259	0.0000
AES+Reliant	0.0103	0.0303	0.7345
AES+Mirant	0.2319	0.0267	0.0000
Duke+DST	0.1096	0.0597	0.0667
Duke+Reliant	**-0.2285****	0.0611	0.0002
Duke+Mirant	0.5332	0.0525	0.0000
DST+Reliant	**-0.2736****	0.0454	0.0000
DST+Mirant	0.2088	0.0324	0.0000
Reliant+Mirant	0.0236	0.0555	0.6708

Total > 9,000MW and < 12,000MW

Firms	γ^i	**Std**	**P**
AES+Duke	0.0231	0.2515	0.9270
AES+DST	**-0.2661***	0.1469	0.0704
AES+Reliant	**-0.4606****	0.2149	0.0323
AES+Mirant	0.0890	0.1454	0.5406
Duke+DST	**-0.0647**	0.1759	0.7132
Duke+Reliant	**-0.1692**	0.1388	0.2230
Duke+Mirant	0.3171	0.1941	0.1026
DST+Reliant	**-0.3545****	0.1082	0.0011
DST+Mirant	0.0141	0.0908	0.8769
Reliant+Mirant	**-0.1545**	0.1408	0.2724

Total > 3,000MW and <6,000MW

Firms	γ^i	**Std**	**P**
AES+Duke	0.1604	0.0161	0.0000
AES+DST	**-0.0200**	0.0181	0.2682
AES+Reliant	**-0.0481****	0.0152	0.0016
AES+Mirant	0.0368	0.0159	0.0206
Duke+DST	0.1824	0.0185	0.0000
Duke+Reliant	0.1070	0.0139	0.0000
Duke+Mirant	0.2670	0.0232	0.0000
DST+Reliant	**-0.1135****	0.0251	0.0000
DST+Mirant	0.1118	0.0217	0.0000
Reliant+Mirant	**-0.0139**	0.0187	0.4577

Total < 3,000MW

Firms	γ^i	**Std**	**P**
AES+Duke	0.0178	0.0233	0.4444
AES+DST	**-0.1013****	0.0355	0.0044
AES+Reliant	**-0.1411****	0.0312	0.0000
AES+Mirant	**-0.1487****	0.0286	0.0000
Duke+DST	0.0580	0.0220	0.0084
Duke+Reliant	**-0.0197**	0.0285	0.4896
Duke+Mirant	0.0346	0.0276	0.2097
DST+Reliant	**-0.1428****	0.0442	0.0013
DST+Mirant	**-0.0533**	0.0360	0.1393
Reliant+Mirant	**-0.1794****	0.0371	0.0000

Table 1-11. Regression for collusive behavior: three firms

Total > 12,000MW

Firms	γ^i	Std	P
AES+Duke+DST	0.0811	0.6301	0.8976
AES+Duke+Reliant	**-1.3618****	0.6582	0.0390
AES+Duke+Mirant	**-0.2334**	0.7373	0.7516
AES+DST+Reliant	**-2.2584****	0.5060	0.0000
AES+DST+Mirant	**-1.4662****	0.5386	0.0067
AES+Reliant+Mirant	**-3.1088****	0.5093	0.0000
Duke+DST+Reliant	**-3.3192****	0.5432	0.0000
Duke+DST+Mirant	**-2.5734****	0.5827	0.0000
Duke+Reliant+Mirant	**-3.9150****	0.5197	0.0000
DST+Reliant+Mirant	**-3.4463****	0.3604	0.0000

Total > 6,000MW and <9,000MW

Firms	γ^i	Std	P
AES+Duke+DST	0.3191	0.0556	0.0000
AES+Duke+Reliant	0.0611	0.0608	0.3156
AES+Duke+Mirant	0.5841	0.0558	0.0000
AES+DST+Reliant	0.0037	0.0364	0.9193
AES+DST+Mirant	0.2360	0.0306	0.0000
AES+Reliant+Mirant	0.1515	0.0392	0.0001
Duke+DST+Reliant	**-0.2612****	0.0687	0.0001
Duke+DST+Mirant	0.4984	0.0624	0.0000
Duke+Reliant+Mirant	0.2023	0.1089	0.0635
DST+Reliant+Mirant	0.0058	0.0602	0.9238

Total > 9,000MW and < 12,000MW

Firms	γ^i	Std	P
AES+Duke+DST	**-0.1848**	0.2764	0.5040
AES+Duke+Reliant	**-0.3198**	0.2515	0.2037
AES+Duke+Mirant	0.2998	0.2874	0.2971
AES+DST+Reliant	**-0.6263****	0.1947	0.0013
AES+DST+Mirant	**-0.0662**	0.1316	0.6152
AES+Reliant+Mirant	**-0.3540**	0.2356	0.1331
Duke+DST+Reliant	**-0.2896***	0.1536	0.0597
Duke+DST+Mirant	0.1456	0.1914	0.4468
Duke+Reliant+Mirant	**-0.0645**	0.1964	0.7428
DST+Reliant+Mirant	**-0.2228***	0.1270	0.0797

Total > 3,000MW and <6,000MW

Firms	γ^i	Std	P
AES+Duke+DST	0.2162	0.0203	0.0000
AES+Duke+Reliant	0.0935	0.0170	0.0000
AES+Duke+Mirant	0.2453	0.0243	0.0000
AES+DST+Reliant	**-0.0721****	0.0202	0.0004
AES+DST+Mirant	0.0544	0.0211	0.0099
AES+Reliant+Mirant	**-0.0200**	0.0182	0.2731
Duke+DST+Reliant	0.1271	0.0181	0.0000
Duke+DST+Mirant	0.2979	0.0267	0.0000
Duke+Reliant+Mirant	0.2141	0.0228	0.0000
DST+Reliant+Mirant	**-0.0136**	0.0221	0.5375

Total < 3,000MW

Firms	γ^i	Std	P
AES+Duke+DST	0.0219	0.0301	0.4670
AES+Duke+Reliant	**-0.0432**	0.0332	0.1925
AES+Duke+Mirant	**-0.0102**	0.0339	0.7625
AES+DST+Reliant	**-0.1643****	0.0420	0.0001
AES+DST+Mirant	**-0.1468****	0.0389	0.0002
AES+Reliant+Mirant	**-0.1848****	0.0357	0.0000
Duke+DST+Reliant	**-0.0145**	0.0342	0.6715
Duke+DST+Mirant	0.0478	0.0327	0.1445
Duke+Reliant+Mirant	**-0.0458**	0.0362	0.2052
DST+Reliant+Mirant	**-0.1806****	0.0473	0.0001

Table 1-12. Regression for collusive behavior: four firms

Total > 12,000MW

Firms	γ^i	Std	T	P
AES+Duke+DST+Reliant	**-2.3455****	0.6752	-3.4736	0.0005
AES+Duke+DST+Mirant	**-1.3762***	0.7272	-1.8924	0.0589
AES+Duke+Reliant+Mirant	**-3.1201****	0.6827	-4.5701	0.0000
AES+DST+Reliant+Mirant	**-3.4329****	0.5067	-6.7753	0.0000
Duke+DST+Reliant+Mirant	**-4.0854****	0.5164	-7.9117	0.0000

Total > 9,000MW and < 12,000MW

Firms	γ^i	Std	T	P
AES+Duke+DST+Reliant	**-0.4962****	0.2523	-1.9671	0.0494
AES+Duke+DST+Mirant	0.0705	0.2580	0.2734	0.7846
AES+Duke+Reliant+Mirant	**-0.2135**	0.3097	-0.6895	0.4906
AES+DST+Reliant+Mirant	**-0.4443****	0.1850	-2.4016	0.0165
Duke+DST+Reliant+Mirant	**-0.1744**	0.1853	-0.9412	0.3468

Total > 6,000MW and <9,000MW

Firms	γ^i	Std	T	P
AES+Duke+DST+Reliant	0.0601	0.0655	0.9181	0.3587
AES+Duke+DST+Mirant	0.5627	0.0548	10.2649	0.0000
AES+Duke+Reliant+Mirant	0.3809	0.0714	5.3309	0.0000
AES+DST+Reliant+Mirant	0.1453	0.0435	3.3406	0.0009
Duke+DST+Reliant+Mirant	0.1701	0.1049	1.6216	0.1051

Total > 3,000MW and <6,000MW

Firms	γ^i	Std	T	P
AES+Duke+DST+Reliant	0.1135	0.0201	5.6466	0.0000
AES+Duke+DST+Mirant	0.2884	0.0282	10.2316	0.0000
AES+Duke+Reliant+Mirant	0.1616	0.0227	7.1203	0.0000
AES+DST+Reliant+Mirant	**-0.0222**	0.0213	-1.0449	0.2962
Duke+DST+Reliant+Mirant	0.2329	0.0255	9.1408	0.0000

Total < 3,000MW

Firms	γ^i	Std	T	P
AES+Duke+DST+Reliant	**-0.0444**	0.0387	-1.1460	0.2520
AES+Duke+DST+Mirant	**-0.0047**	0.0390	-0.1215	0.9033
AES+Duke+Reliant+Mirant	**-0.0733***	0.0393	-1.8641	0.0625
AES+DST+Reliant+Mirant	**-0.1935****	0.0443	-4.3665	0.0000
Duke+DST+Reliant+Mirant	**-0.0400**	0.0411	-0.9747	0.3299

Table 1-13. Collusion hypothesis test: pivotal monopoly

	γ^{ij}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$ [4]	t	P
AES+Duke	1.4879	1.9563	**-0.4683**	0.9801	-0.4779	0.3164
AES+DST	-0.2902	-0.7202	0.4299	0.6299	0.6826	0.7526
AES+Reliant	-1.5166	-1.3904	**-0.1262**	0.6871	-0.1836	0.4272
AES+Mirant	-0.5797	-0.7980	0.2182	0.7157	0.3049	0.6198
Duke+DST	-1.4469	0.1238	**-1.5707***	0.9581	-1.6395	0.0506
Duke+Reliant	-2.8321	-0.5465	**-2.2856****	0.9595	-2.3820	0.0086
Duke+Mirant	-1.6338	0.0460	**-1.6798****	0.9788	-1.7162	0.0431
DST+Reliant	-2.5084	-3.2229	0.7145	0.5503	1.2986	0.9030
DST+Mirant	-2.1357	-2.6305	0.4947	0.5239	0.9444	0.8275
Reliant+Mirant	-3.2317	-3.3007	0.0691	0.5638	0.1225	0.5488

	γ^{ijk}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	t	P
AES+Duke+DST	0.0811	0.6800	**-0.5988**	1.0366	-0.5777	0.2817
AES+Duke+Reliant	-1.3618	0.0097	**-1.3715***	1.0725	-1.2787	0.1005
AES+Duke+Mirant	-0.2334	0.6021	**-0.8356**	1.1163	-0.7485	0.2271
AES+DST+Reliant	-2.2584	-2.6668	0.4083	0.7482	0.5457	0.7074
AES+DST+Mirant	-1.4662	-2.0743	0.6081	0.7613	0.7988	0.7878
AES+Reliant+Mirant	-3.1088	-2.7446	**-0.3642**	0.7670	-0.4748	0.3175
Duke+DST+Reliant	-3.3192	-1.8228	**-1.4964***	0.9775	-1.5309	0.0629
Duke+DST+Mirant	-2.5734	-1.2303	**-1.3431***	0.9928	-1.3527	0.0881
Duke+Reliant+Mirant	-3.9150	-1.9006	**-2.0144****	0.9775	-2.0606	0.0197
DST+Reliant+Mirant	-3.4463	-4.5771	1.1307	0.6342	1.7829	0.9627

	γ^{ijkn}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	t	P
AES+Duke+DST+Reliant	-2.3455	-1.2666	**-1.0788**	1.1146	-0.9679	0.1665
AES+Duke+DST+Mirant	-1.3762	-0.6742	**-0.7020**	1.1405	-0.6155	0.2691
AES+Duke+Reliant+Mirant	-3.1201	-1.3445	**-1.7756***	1.1303	-1.5710	0.0581
AES+DST+Reliant+Mirant	-3.4329	-4.0209	0.5880	0.8093	0.7266	0.7663
Duke+DST+Reliant+Mirant	-4.0854	-3.1769	**-0.9084**	1.0107	-0.8988	0.1844

[4] In Table 1-13 through Table 1-17 standard deviation of $\Delta\gamma$ is calculated as $S_{\Delta Y}^2 = S_{Yi}^2 + S_{Yj}^2 + S_{Yij}^2$

Table 1-14. Collusion hypothesis test: pivotal duopoly

	γ^{ij}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke	0.0231	-0.0063	0.0294	0.3141	0.0936	0.5373
AES+DST	-0.2661	-0.2161	**-0.0500**	0.2131	-0.2346	0.4073
AES+Reliant	-0.4606	-0.2944	**-0.1662**	0.2738	-0.6071	0.2719
AES+Mirant	0.0890	0.0542	0.0348	0.2168	0.1607	0.5638
Duke+DST	-0.0647	-0.1059	0.0412	0.2273	0.1813	0.5719
Duke+Reliant	-0.1692	-0.1842	0.0149	0.2119	0.0705	0.5281
Duke+Mirant	0.3171	0.1644	0.1527	0.2459	0.6212	0.7328
DST+Reliant	-0.3545	-0.3940	0.0394	0.1605	0.2459	0.5971
DST+Mirant	0.0141	-0.0454	0.0594	0.1393	0.4268	0.6653
Reliant+Mirant	-0.1545	-0.1237	**-0.0309**	0.1895	-0.1629	0.4353

	γ^{ijk}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST	-0.1848	-0.1642	**-0.0206**	0.341106	-0.0603	0.4759
AES+Duke+Reliant	-0.3198	-0.2425	**-0.0773**	0.328852	-0.2352	0.4070
AES+Duke+Mirant	0.2998	0.1061	0.1937	0.353038	0.5488	0.7084
AES+DST+Reliant	-0.6263	-0.4522	**-0.1741**	0.266823	-0.6524	0.2571
AES+DST+Mirant	-0.0662	-0.1037	0.0375	0.218462	0.1717	0.5682
AES+Reliant+Mirant	-0.3540	-0.1820	**-0.1720**	0.301402	-0.5708	0.2841
Duke+DST+Reliant	-0.2896	-0.3420	0.0524	0.23195	0.2258	0.5893
Duke+DST+Mirant	0.1456	0.0066	0.1390	0.252838	0.5499	0.7088
Duke+Reliant+Mirant	-0.0645	-0.0717	0.0072	0.266141	0.0272	0.5109
DST+Reliant+Mirant	-0.2228	-0.2815	0.0587	0.191787	0.3063	0.6203

	γ^{ijkn}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST+Reliant	-0.4962	-0.4003	**-0.0959**	0.3363	-0.2853	0.3877
AES+Duke+DST+Mirant	0.0705	-0.0517	0.1223	0.3364	0.3635	0.6419
AES+Duke+Reliant+Mirant	-0.2135	-0.1300	**-0.0835**	0.3840	-0.2175	0.4139
AES+DST+Reliant+Mirant	-0.4443	-0.3398	**-0.1045**	0.2723	-0.3839	0.3505
Duke+DST+Reliant+Mirant	-0.1744	-0.2296	0.0552	0.2667	0.2069	0.5820

Table 1-15. Collusion hypothesis test: three firms in pivotal oligopoly

	γ^{ij}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke	0.2896	0.1761	0.1135	0.0757	1.4987	0.9330
AES+DST	0.1056	0.0563	0.0493	0.0427	1.1549	0.8759
AES+Reliant	0.0103	-0.1565	0.1668	0.0513	3.2493	0.9994
AES+Mirant	0.2319	0.3121	**-0.0802****	0.0414	-1.9345	0.0265
Duke+DST	0.1096	0.0739	0.0357	0.0828	0.4312	0.6669
Duke+Reliant	-0.2285	-0.1390	**-0.0896**	0.0871	-1.0281	0.1520
Duke+Mirant	0.5332	0.3296	0.2035	0.0768	2.6506	0.9960
DST+Reliant	-0.2736	-0.2587	**-0.0149**	0.0653	-0.2283	0.4097
DST+Mirant	0.2088	0.2099	**-0.0010**	0.0504	-0.0201	0.4920
Reliant+Mirant	0.0236	-0.0030	0.0266	0.0717	0.3706	0.6445

	γ^{ijk}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST	0.3191	0.1531	0.1660	0.0819	2.0265	0.9786
AES+Duke+Reliant	0.0611	-0.0597	0.1208	0.0888	1.3596	0.9130
AES+Duke+Mirant	0.5841	0.4089	0.1752	0.0811	2.1584	0.9845
AES+DST+Reliant	0.0037	-0.1794	0.1831	0.0621	2.9469	0.9984
AES+DST+Mirant	0.2360	0.2891	**-0.0531**	0.0525	-1.0116	0.1559
AES+Reliant+Mirant	0.1515	0.0763	0.0752	0.0626	1.1997	0.8849
Duke+DST+Reliant	-0.2612	-0.1619	**-0.0993**	0.0969	-1.0246	0.1528
Duke+DST+Mirant	0.4984	0.3067	0.1917	0.0886	2.1624	0.9847
Duke+Reliant+Mirant	0.2023	0.0939	0.1084	0.1280	0.8469	0.8015
DST+Reliant+Mirant	0.0058	-0.0259	0.0317	0.0805	0.3931	0.6529

	γ^{ijkn}	$\sum\gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST+Reliant	0.0601	-0.0826	0.1427	0.0964	1.4808	0.9307
AES+Duke+DST+Mirant	0.5627	0.3860	0.1767	0.0854	2.0687	0.9807
AES+Duke+Reliant+Mirant	0.3809	0.1731	0.2077	0.0998	2.0811	0.9813
AES+DST+Reliant+Mirant	0.1453	0.0534	0.0920	0.0714	1.2881	0.9012
Duke+DST+Reliant+Mirant	0.1701	0.0709	0.0992	0.1278	0.7757	0.7810

Table 1-16. Collusion hypothesis test: four firms in pivotal oligopoly

	γ^{ij}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke	0.1604	0.1188	0.0416	0.0235	1.7707	0.9617
AES+DST	-0.0200	-0.0023	**-0.0177**	0.0267	-0.6633	0.2536
AES+Reliant	-0.0481	-0.0854	0.0373	0.0255	1.4627	0.9282
AES+Mirant	0.0368	0.0781	**-0.0413****	0.0250	-1.6519	0.0493
Duke+DST	0.1824	0.1410	0.0414	0.0287	1.4422	0.9254
Duke+Reliant	0.1070	0.0579	0.0490	0.0267	1.8377	0.9669
Duke+Mirant	0.2670	0.2214	0.0456	0.0318	1.4340	0.9242
DST+Reliant	-0.1135	-0.0632	**-0.0503***	0.0352	-1.4291	0.0765
DST+Mirant	0.1118	0.1003	0.0115	0.0321	0.3579	0.6398
Reliant+Mirant	-0.0139	0.0172	**-0.0311**	0.0307	-1.0112	0.1560

	γ^{ijk}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST	0.2162	0.1287	0.0875	0.0315	2.7748	0.9972
AES+Duke+Reliant	0.0935	0.0457	0.0478	0.0300	1.5882	0.9439
AES+Duke+Mirant	0.2453	0.2092	0.0362	0.0340	1.0616	0.8558
AES+DST+Reliant	-0.0721	-0.0754	0.0033	0.0333	0.1001	0.5399
AES+DST+Mirant	0.0544	0.0880	**-0.0336**	0.0332	-1.0107	0.1561
AES+Reliant+Mirant	-0.0200	0.0050	**-0.0249**	0.0320	-0.7789	0.2180
Duke+DST+Reliant	0.1271	0.0679	0.0592	0.0336	1.7599	0.9608
Duke+DST+Mirant	0.2979	0.2313	0.0666	0.0383	1.7363	0.9587
Duke+Reliant+Mirant	0.2141	0.1483	0.0658	0.0362	1.8155	0.9653
DST+Reliant+Mirant	-0.0136	0.0271	**-0.0408**	0.0371	-1.1002	0.1356

	γ^{ijkn}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST+Reliant	0.1135	0.0556	0.0579	0.0362	1.6017	0.9454
AES+Duke+DST+Mirant	0.2884	0.2191	0.0693	0.0406	1.7061	0.9560
AES+Duke+Reliant+Mirant	0.1616	0.1360	0.0256	0.0375	0.6817	0.7523
AES+DST+Reliant+Mirant	-0.0222	0.0149	**-0.0371**	0.0379	-0.9803	0.1635
Duke+DST+Reliant+Mirant	0.2329	0.1582	0.0747	0.0416	1.7954	0.9637

Table 1-17. Collusion hypothesis test: five firms in pivotal oligopoly

	γ^{ij}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke	0.0178	-0.0320	0.0498	0.0338	1.4764	0.9301
AES+DST	-0.1013	-0.0124	**-0.0889**	0.0500	-1.7776	0.0377
AES+Reliant	-0.1411	-0.1867	0.0456	0.0468	0.9737	0.8349
AES+Mirant	-0.1487	-0.1643	0.0156	0.0430	0.3626	0.6415
Duke+DST	0.0580	0.0897	-0.0318	0.0393	-0.8078	0.2096
Duke+Reliant	-0.0197	-0.0845	0.0648	0.0431	1.5051	0.9339
Duke+Mirant	0.0346	-0.0622	0.0968	0.0402	2.4074	0.9920
DST+Reliant	-0.1428	-0.0649	**-0.0779**	0.0603	-1.2916	0.0983
DST+Mirant	-0.0533	-0.0425	**-0.0108**	0.0529	-0.2034	0.4194
Reliant+Mirant	-0.1794	-0.2168	0.0375	0.0534	0.7015	0.7585

	γ^{ijk}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST	0.0219	0.0227	**-0.0007**	0.0486	-0.0147	0.4941
AES+Duke+Reliant	-0.0432	-0.1516	0.1084	0.0503	2.1559	0.9845
AES+Duke+Mirant	-0.0102	-0.1293	0.1190	0.0489	2.4353	0.9926
AES+DST+Reliant	-0.1643	-0.1320	**-0.0323**	0.0619	-0.5217	0.3009
AES+DST+Mirant	-0.1468	-0.1096	**-0.0371**	0.0583	-0.6373	0.2620
AES+Reliant+Mirant	-0.1848	-0.2839	0.0991	0.0559	1.7700	0.9616
Duke+DST+Reliant	-0.0145	-0.0299	0.0154	0.0553	0.2773	0.6092
Duke+DST+Mirant	0.0478	-0.0075	0.0553	0.0527	1.0482	0.8527
Duke+Reliant+Mirant	-0.0458	-0.1818	0.1359	0.0547	2.4839	0.9935
DST+Reliant+Mirant	-0.1806	-0.1621	**-0.0184**	0.0676	-0.2726	0.3926

	γ^{ijkn}	$\sum \gamma^i$	$\Delta\gamma$	$S_{\Delta\gamma}$	**t**	**P**
AES+Duke+DST+Reliant	-0.0444	-0.0969	0.0526	0.0615	0.8553	0.8038
AES+Duke+DST+Mirant	-0.0047	-0.0746	0.0698	0.0601	1.1613	0.8772
AES+Duke+Reliant+Mirant	-0.0733	-0.2489	0.1755	0.0602	2.9170	0.9982
AES+DST+Reliant+Mirant	-0.1935	-0.2292	0.0357	0.0684	0.5221	0.6992
Duke+DST+Reliant+Mirant	-0.0400	-0.1271	0.0871	0.0650	1.3387	0.9097

2 Essay: Economics of hydro generating plants operating in markets for energy and ancillary services

Abstract

In order to preserve the stability of electricity supply, electric generators have to provide ancillary services in addition to energy production. Hydro generators are believed to be the most efficient source of ancillary services because of their good dynamic flexibility. This paper studies optimal operation decisions for river dams and pumped storage facilities operating in markets for energy and ancillary services as well as the change in the water shadow price in presence of ancillary services markets. The analysis is applied to valuation of the ancillary services provided by hydro resources in the Tennessee Valley Authority. A simulation of ancillary services markets shows that TVA's hydro resources providing ancillary services can allow for substantial savings in total costs of energy provision. Optimal hydro scheduling in markets for energy and ancillary services increases the value of TVA's hydro resources by 9% on average and up to 26% for particular units. As a result of hydro participation in ancillary services markets water shadow prices of river dams drop significantly allowing for tightening hydro constraints in favor of other water uses.

JEL Codes: D21, D24, D41, D58, Q25, Q41

Key words: hydro optimization ancillary service energy market simulation

2.1 Introduction

Electricity is different from other products in that its demand and supply must be matched every minute. Failure to do so may result in a costly system collapse. To ensure against such events, generating plants have to provide certain ancillary services in addition to electricity generation. As the industry deregulation advances, it is expected that the number of control areas administering simultaneous markets for energy and ancillary services will increase.

This paper studies the economics of providing ancillary services, such as regulation and frequency support, and operation reserves from hydroelectric resources. The definitions of these services according to Federal Energy Regulatory Commission (FERC) are the following (Hirst and Kirby (1996b), (1997b)):

Regulation and Frequency Support: The use of generation equipped with automatic-generation control (AGC) to maintain minute-to-minute generation/load balance within the control area to meet the control-performance standards of NERC (National Electric Reliability Council).

Operating reserve spinning: The provision of unloaded generating capacity that is synchronized to the grid that can respond immediately to correct for generation/load imbalances caused by generation and transmission outages and that is fully available within 10-15 minutes to meet NERC's disturbance-control standard.

Operation reserve supplemental: The provision of generating capacity and curtailable load used to correct for generation/load imbalances caused by generation and transmission outages and that is fully available within 10-15 minutes.

Provision of Ancillary Services (A/S) by power plants requires substantial dynamic flexibility. For instance, in order to provide spinning reserves or regulation a unit must be able to ramp up (or both ramp up and down in case of regulation) by the amount of the offered service within 5 to 15 minutes. Therefore, it is believed that hydro generators with their quick ramp rates have a significant advantage over fossil fuel plants and that provision of ancillary service by hydro power plants substantially increases their value.

This paper models the optimal operations of hydro generation plants in simultaneous markets for energy and ancillary services and their profits generated in these markets. The model can be useful for operators and owners of hydro facilities that operate in deregulated markets for energy and ancillary services in New York, New England, California and elsewhere. However, here the results are applied to valuation of hydro resources of Tennessee Valley Authority (TVA). Since TVA currently operates as a vertically integrated utility and does not have markets for ancillary services, a simulation of such markets is presented in this paper.

Studies of the ancillary services appeared in the literature on energy economics long before the deregulation of the electricity industry that started in the US in 1998 with creation of the electricity markets in California and Pennsylvania. A survey done in 1996 across 12 US investor-owned utilities estimated the cost of these services to be up to 10% of the total cost of energy generation and transmission (Hirst and Kirby (1996a)). It was realized that in order to avoid the problem of missing markets these services must be unbundled from the energy generation in the deregulated electricity markets (Hirst and Kirby (1996b)).

Currently, markets for ancillary services are administered in energy markets of California, New York, New England, Pennsylvania-New Jersey-Maryland and Texas (Cheung, et al. (2000), ISO-NE (2000), Kranz, et al. (2003), NYISO (1999), PJM (2000), (2002)). However, first attempts to run ancillary services markets were not always successful. Some markets provided wrong incentives and were susceptible to market power exercise (Brien (1999), Chao and Wilson (2002), Wilson (1998), and Wolak, et al. (2000)).

In this paper a market for ancillary services is simulated for a hydro-thermal generation mix. To run such simulation it is necessary to understand the nature of the costs of ancillary service provision from power plants as well as the economics of hydropower.

Studies of the costs of ancillary service provision from fossil fuel plants include Curtice (1997), El-Keib and Ma (1997), Hirst and Kirby (1997a), (1997b), and Hirst (2000). Hirst and Kirby (1997b) actually run a simulation of the market for energy and ancillary services for a fossil fuel mix and Hirst (2000) study the operation decisions and profits of a fossil fuel plant operating in markets for energy and ancillary services.

The difficulty of the market simulation for the hydro-thermal generation mix studied in this paper is that while the operation problem for fossil fuel plants is largely separable for each operation period (e.g. an hour), the hydro operation problem is dynamic. Hydro generators have a constraint on the amount of water available for generation over a time horizon. Therefore, current operation decisions affect the operation capabilities in future. One of the fundamental works on the economics of such problem has been done by Nobel Prize laureate Koopmans (1957). It has been recognized that reservoir dynamics can be relaxed by using an additional set of Lagrange multipliers that can be viewed as water shadow prices. In a simple setup, the water shadow price can be assumed to be constant over the cycle (Warford and Munasinghe (1982)). Horsley and Wrobel (1996), (1999) study a more realistic case where water shadow price changes over the cycle when the reservoir capacity constraints are reached. However, the water shadow price is still constant over the time frames over which the reservoir capacity is not binding. The authors provide derivations of the rental valuation of the fixed inputs such as turbine and reservoir capacity for both river dams and pumped storage units based on the water shadow prices.

In this paper the general idea of variable water shadow price of Horsley and Wrobel (1996), (1999) is adopted. The estimates of water shadow prices are calculated as constants over pre-specified time intervals during which reservoir capacity constraints are unlikely to be binding.

For the market simulation, the energy mix of the Tennessee River Authority is used. Although many of the river dams in the Tennessee River Authority are optimized simultaneously as they share a common watershed, this simulation assumes different water shadow prices for each hydro dam (El-Hawary and Christensen (1979)). The market simulation in this paper also assumes fixed coefficients of conversion of water into energy or fixed head for each dam. Cases of variable head are considered in Bauer, et al. (1984), Gferer (1984), Phu (1987).
The simulation of the ancillary services markets presented here also assumes perfect competition. The mechanisms of market power exercise by hydro plants are described in Bushnell (2003).

The remainder of the paper is structured as follows. Section 2 describes the nature of the cost of ancillary services provided by fossil fuel plants; Section 3 derives the water shadow prices for river dams and pumped storage facilities that provide both energy and ancillary services. Section 4 describes the linear programming model used for the simulation of hourly A/S markets, Section 5 presents the results, and Section 6 contains concluding remarks.

2.2 Cost of ancillary services provision from fossil fuel plants

In order to model the optimal operations of hydro plants in markets for energy and ancillary services, it is convenient to consider the cost of provision of ancillary services from fossil fuel plants and then to apply the findings to the hydro plants.

Regulation

Regulation is the use of online generating units that are equipped with automatic-generation control (AGC) that makes their output directly controllable by the system operator and that can change output quickly (MW/minute) to track the moment-to-moment fluctuations in customer loads and unintended fluctuations in generation. In so doing, regulation helps to maintain interconnection frequency, minimizing the differences between actual and scheduled power flows within the control area.

During the hours when a unit provides regulation, the unit must be online, synchronized to the grid and generating enough energy to be able to ramp generation up or down by the amount of scheduled regulation.

Regulation movements are usually symmetric in the long run, so on average, a unit providing regulation operates at the energy dispatch point. Thus, in order to be able to ramp up by the amount of regulation provided, the energy dispatch point must be lower than the High Operation Limit by the amount of provided regulation. To be able to ramp down, the dispatch point must be higher than the Low Operation Limit by the regulation capability. Finally, regulation capability is limited by the ramp rate of the unit. For instance, Tennessee Valley Authority requires the regulation capability to be available in 5 minutes, therefore 5 minutes times the MW/minute ramp rate sets the ramp limit to the regulation capability.

If y is energy dispatch, $y_{\min}$ and $y_{\max}$ are the Low and High Operation Limits, then the following must hold for regulation output r :

$$
\begin{aligned}
& y + r \le y_{\max} \\
& y - r \ge y_{\min} \ , \\
& r \le r_{\max}
\end{aligned}
$$

where $r_{\max}$ is the regulation limit set by ramp rates or other operational constraints.

Providing regulation incurs the following costs:

- Opportunity and Re-dispatch Cost
- Efficiency penalty and Energy Cost
- Wear and tear costs

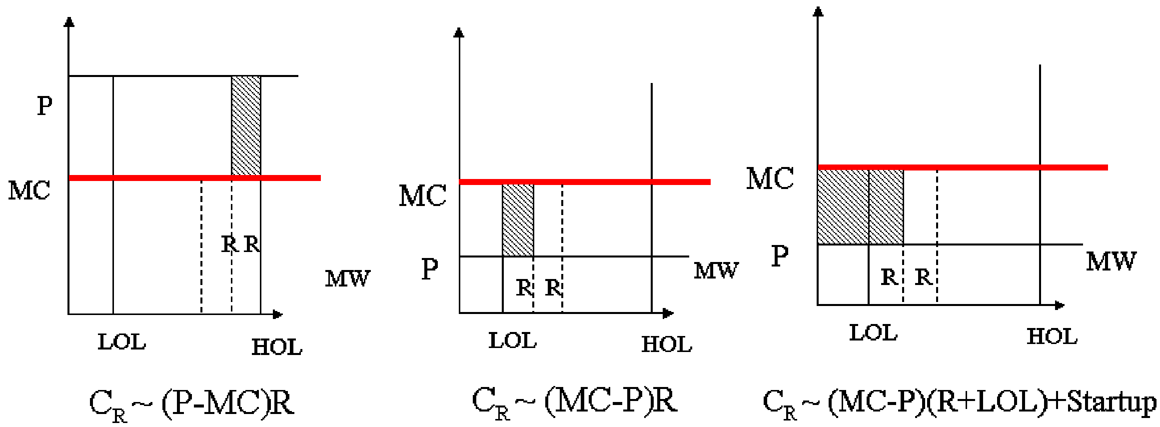
Opportunity and Re-dispatch Cost. One of the most significant costs to a generator providing regulation is opportunity and re-dispatch costs.

If the generator's marginal cost is lower than the market price, the generator would earn profits operating at full capacity. Therefore, reduction in the energy output necessary to provide regulation is associated with the opportunity cost of foregone profits (shaded area in Figure 2-1a). In that case the opportunity cost of regulation is roughly proportional to the difference between price and marginal cost of generation.

If generator's marginal cost is higher than energy market price, the generator may choose to still be online in order to avoid future startup costs. In that case the unit would be generating at the Low Operation Limit during the short-term energy price dips. To provide regulation, the generator would have to increase the energy output over the LOL by the amount of regulation provided (shaded area in Figure 2-1b). In that case the re-dispatch cost of regulation is proportional to the difference between marginal cost and price.

If the generator maximizes profits by shutting down when price is lower than marginal cost, then the re-dispatch cost of regulation may also include the startup cost and the cost of generation up to the Lower Operation Limit (Figure 2-1c).

Figure 2-1. Marginal cost curves and opportunity and re-dispatch costs of regulation



Efficiency Penalty. In order to be able to ramp up quickly in response to the AGC signal, a generator providing regulation may have to operate at reduced efficiency. This "efficiency penalty" is especially pronounced for steam units. The necessity of quick ramping times requires the valves to be half-open, rather than wide-open, that is, to be on "throttle reserve" (Curtice (1997)). Operation in throttle reserve mode reduces the efficiency of a unit and imposes an additional cost of providing regulation

Energy Cost. Regulation may require a generator to perform fast ramp-ups and ramp-downs within the regulation capability. Thus, units offering regulation may incur energy costs associated with both maintaining the output at the level required by the AGC signal and turbine acceleration and deceleration.

Wear and tear costs. Frequent output adjustments may incur additional wear and tear costs associated with providing regulation.

This paper focuses on the opportunity and re-dispatch costs of providing regulation. No data are available on the Efficiency Penalty, Energy Cost, and Wear and Tear Costs associated with regulation provision. These components of the cost will be considered negligible compared to the Opportunity and Re-dispatch costs.

Spinning reserves

Spinning reserves are a component of operation reserves, needed to replace power losses due to contingencies, such as loss of generation resources or transmission lines. Spinning reserves are provided by generating units under the following conditions: the units are online and synchronized to the grid; they are able to start increasing output immediately in response to changes in interconnection frequency; the units can be fully available within 10 or 15 minutes to correct for generation and load imbalances caused by generation or transmission outages. In principle, loads that are frequency-responsive or that are under control of the system operator could also help to provide this service. TVA considers 15-minute response appropriate for spinning reserves. Therefore, for spinning reserves output s the following must hold:

$$y + s \leq y_{\max}$$
$$s \leq s_{\max}$$
$$s = 0, \text{ if } y < y_{\min}$$

Similarly to regulation, the costs of providing spinning reserves include:

- Re-dispatch and Opportunity Costs,
- Efficiency Penalty,
- Energy costs

Re-dispatch and Opportunity costs faced by a unit providing spinning reserves are similar to those faced by units providing regulation. A unit may be competitive and profitably operate at full capacity. In this case, providing spinning reserves includes the opportunity cost equal to the lost profits from energy sales from the capacity designated for spinning reserves.

Otherwise, a unit may be less than competitive and stay online partially loaded. No re-dispatch is caused if such unit is willing to provide spinning reserves; therefore, no re-dispatch or opportunity cost is incurred.

Finally, a unit may be absolutely non-competitive and shut down. Providing spinning reserves from such a unit would incur the uplift cost of providing a minimum amount of generation, the cost of procuring generation at the Low Operation Limit plus the revenues from its sales. The cost of offering spinning reserves in this case may include the start-up costs necessary for attaining the minimum level of generation.

Efficiency Penalty. Similarly to regulation, the necessity for quick ramping times may require steam units to be operated in throttle mode, thus incurring an efficiency penalty.

Energy Costs. In the case of a major outage and activation of spinning reserves units providing spinning reserves may incur fuel costs. However, spinning events are not very frequent; major outages of 1,000 MW or more occur only a couple of times per month in the Eastern Interconnection (Hirst and Kirby (1997a), (1998)), so the expected energy costs of proving spinning reserves should not be very high.

Again, based on the available data, only opportunity and re-dispatch costs of spinning reserve provision will factor into the spinning reserve bid.

Supplemental or non-spinning reserve

Supplemental reserve is another component of operation reserves, which is usually required in addition to the spinning reserves. As opposed to the spinning reserve, units providing supplemental reserves do not need to be synchronized to the grid since they respond after the spinning reserve units pick up the outage. Hydro plants do not have a low operation limit and there is not much cost difference for them between providing spinning and non-spinning reserves. Therefore, the remainder of the paper does not deal with supplemental reserves.

2.3 Estimation of water shadow price for hydro plants

The previous section shows that, for fossil fuel plants, the opportunity cost of ancillary services provision depends on the energy market price and the cost of energy production. Compared to the fossil fuel plants, the direct cost of energy generated by hydro units is virtually zero. However, because of the scarcity of water supplies, the water shadow price can be viewed as a fuel cost. The idea behind the water shadow price is that generating one megawatt-hour now means not being able to generate one megawatt-hour at some point in the future. Therefore, for a hydro unit the foregone earnings from future generation constitute the opportunity cost of current energy generation. Once the opportunity cost of energy generation or water shadow price is known, the decision to provide generation or any of the ancillary services from a hydro plant is made the same way as for a combustion turbine with no startup time and costs (Horsley and Wrobel (1996), (1999)).

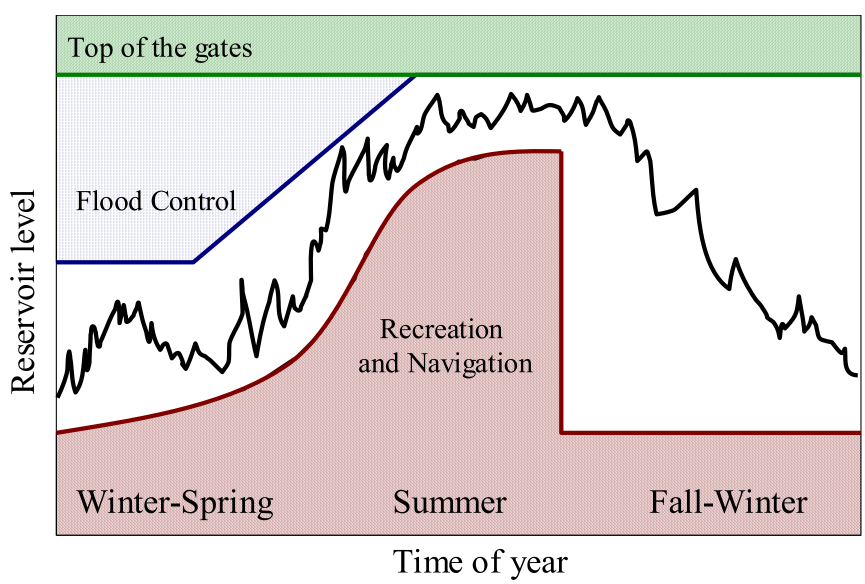
As opposed to the cost of fossil-fuel plants that is largely dependent on the fuel costs and is relatively stable over time, the shadow price of water in hydro plants varies a lot. It depends on the distribution of future prices of energy, as well as on the parameters of the hydro project, such as storage capability, generating and pumping (in case of pumped storage) capability and efficiency, expected natural water inflow and constraints on water use for navigation, flood control, recreation and other purposes.

In addition, as it will be shown in this section, the water shadow price depends on the ability of the hydro unit to participate in ancillary services markets and the distribution of the future prices in these markets.

River dams

River dams are characterized by a set of constraints and parameters. Maximum and minimum flow constraints determine the range of water flow that can be passed through the turbine in each hour. The level of the reservoir behind the dam also has to be within a certain range that can vary throughout the year. The upper bound of this range is determined by the top of the gates and the flood control constraints, which in the case of Tennessee River Authority are enforced during winter and spring ensuring that there is enough empty reservoir capacity to hold the floodwaters. The lower bound is determined by the navigation and recreation constraints that ensure that the reservoir level is high enough for navigation and recreation needs, the latter being particularly strict during summer. A typical "guide curve" for the reservoir level in Tennessee River Authority is shown on Figure 2-2.

Figure 2-2. TVA's typical guide curve



The supply of water in the dam is determined by the natural water inflow in the reservoir as well as the operation of the upstream dams. Some economic analysis of the cascaded hydro systems can be found in El-Hawary and Christensen (1979).

The efficiency of converting water into energy varies with reservoir level or head elevation since energy is proportional to the height of the water drop. In addition, the conversion efficiency depends on the turbine load. However, in the analysis below the conversion efficiency will be assumed to be constant.

Consider the optimization problem of a river dam. The outflow from the reservoir is

$$f(t) = y(t) - e(t) ,$$

where *y(t)* is the hydro plant's output and *e(t)* is reservoir water inflow. It is assumed that the hydro plant's maximum output capacity $y_{\max}$ is larger than $\sup|e(t)|$. This condition ensures that no spillage is ever necessary. With an initial reservoir level s_0 the reservoir level at t is:

$$s(t) = s_0 - \int_0^{\tau} f(\tau) d\tau$$

The hydro profit maximization problem given the deterministic continuous energy prices $p(t)$ over the time interval $[0,T]$ is therefore:

$$\begin{aligned} &\max_{y(t)} \int_0^T y(t)p(t)dt \\ &s.t. \\ &0 \le y(t) \le y_{\max} \qquad \alpha(t),\beta(t) \qquad , \\ &s_{\min}(t) \le s(t) \le s_{\max}(t) \qquad \gamma(t),\delta(t) \\ &s(T) = s_T, s_T \in [s_{\min}, s_{\max}] \quad \lambda \end{aligned} \tag{3.1}$$

where $s_{\min}$ and $s_{\max}$ are the reservoir lower and upper constraints, which may vary over time as in the case of the guide curves in TVA.

Horsley and Wrobel (1999) show that the solution to this problem is given by:

$$y(t) = \begin{cases} y_{\max}, & p(t) > \psi(t) \\ [0, y_{\max}], & p(t) = \psi(t) \\ 0, & p(t) < \psi(t) \end{cases} , \tag{3.2}$$

where $\psi(t)$ is the water shadow price given by

$$\psi(t) = \lambda - \gamma(t) + \delta(t) .$$

That is, “the hydro plant operates just like a thermal plant with a time-varying “fuel” price $\psi(t)$”.

It is easy to see, that if on the interval $[t_1, t_2]$ reservoir constraints are not binding, then shadow price is constant on that interval and equal to $\psi_{t_1,t_2} = \lambda$. On the other hand, Horsley and Wrobel (1999) show that $\psi(t) = p(t)$ if reservoir constraints are binding at time *t*.

Over each of the intervals $[t_1, t_2]$ where reservoir constraints are not binding the optimization problem (3.1) simplifies to:

$$\begin{aligned}
&\max_{y(t)} \int_{t_1}^{t_2} y(t)p(t)dt \\
&s.t. \\
&0 \le y(t) \le y_{\max} \qquad \alpha(t), \beta(t) \\
&\int_{t_1}^{t_2} y(t)dt = s_{t_1} - s_{t_2} + \int_{t_1}^{t_2} e(t)dt = S_{[t_1,t_2]} \quad \lambda
\end{aligned}, \tag{3.3}$$

where $S_{[t_1,t_2]}$ is the total amount of water available for the interval $[t_1, t_2]$. On such intervals the water price is constant $\psi(t) = \psi_{t_1,t_2} = \lambda$.

To solve (3.3) it is convenient to introduce the notation of a *price distribution function* over the interval $[t_1, t_2]$:

Definition:

A distribution function $F_p(a)$ of price $p(t)$ over the time interval $[t_1, t_2]$ is a measure of the subset of $[t_1, t_2]$ on which $p(t) < a$ relative to the total length of the interval:

$$F_p(a) = \frac{meas\{t : p(t) < a, t \in [t_1, t_2]\}}{t_2 - t_1}$$

A price distribution function is also sometimes called *price duration curve.*

A price distribution function defined this way has properties similar to the cumulative probability distribution function:

$$\begin{aligned}
&F_p(-\infty) = 0 \\
&F_p(\infty) = 1 \\
&F_p{}' \ge 0
\end{aligned}$$

Proposition 1

The solution to (3.3) is given by (3.2), where $\psi(t) = \psi^*_{t_1,t_2} = const$ is obtained from

$$y_{\max}\left(1 - F_p(\psi^*_{t_1,t_2})\right) = \frac{S_{t_1,t_2}}{(t_2 - t_1)}. \tag{3.4}$$

The solution of (3.4) exists and is unique if $F_p{}'(\cdot) > 0$.

Proof:

Suppose the solution to (3.3) is given by $\tilde{\psi} < \psi^*_{t_1,t_2}$. Then from (3.2) the amount of water used will be

$$\tilde{S} = \left(1 - F_p(\tilde{\psi})\right)(t_2 - t_1)y_{\max}.$$

Since F_p is nondecreasing, it follows that $\tilde{S} > S_{t_1,t_2}$, which violates the constraint of (3.3). Similarly, if $\tilde{\psi} > \psi^*_{t_1,t_2}$, then $\tilde{S} < S_{t_1,t_2}$, meaning that the objective in (3.3) is not maximized.

Since $\sup|e(t)| < y_{\max}$, it follows that $S_{t_1,t_2} < y_{\max}(t_2 - t_1)$. Therefore, at $\tilde{\psi} = 0$ the sign of (3.4) is positive and at $\tilde{\psi} = \sup\limits_{[t_1,t_2]}(p(t))$ is negative. Together with continuity of F_p this proves existence of the solution. The uniqueness follows from the fact that $F_p{}' > 0$.

The intuition behind the solution for the water shadow price in (3.4) is simple. The right hand side of the equation gives the proportion of the time that the hydro unit can generate at full capacity given the amount of water available. For optimality, generation must be scheduled during the time with highest energy price. That is ensured by the left hand side of the equation.

The length of the time interval $[t_1, t_2]$ on which the reservoir constraints are not binding is of the order of $\frac{s_{\max} - s_{\min}}{y_{\max}}$, that is, the time needed to drive the reservoir from the upper constraint to the lower while operating at full turbine capacity. For TVA's river dams this ratio is about one month, so the water shadow prices will be assumed to be constant over each calendar month.

Pumped Storage

Horsley and Wrobel (1996) study a similar problem applied to a pumped storage facility. A pumped storage facility is different from the river dams by the mechanism used to fill the reservoir. As opposed to the natural water inflow in river dams, pumped storage units have to reverse the turbines to pump water back into the storage. Not all of the energy expended to fill the reservoir can be recouped as the reservoir is emptied. Therefore, such reversion results in losses and the round-trip efficiency η [5] of pumped storage is less than one. As opposed to the river dams the reservoir capacity of pumped storage is constant and determined by its physical characteristics.

The profit maximization problem for a pumped storage unit would be:

$$\begin{aligned} &\max_{y} \int_{0}^{T} y(t)p(t)dt \\ &s.t. \\ &y \in [-u_{\max}, y_{\max}] \quad \alpha(t), \beta(t), \\ &0 \le s(t) \le s_{\max} \quad \gamma(t), \delta(t) \\ &s(T) = s_T, s_T \in [0, s_{\max}] \quad \lambda \end{aligned} \tag{3.5}$$

where y is the output at time t that can be positive up to $y_{\max}$ or negative down to $-u_{\max}$ depending on whether the pumped storage is in generating or in pumping mode. The reservoir level $s(t)$ is given by:

$$s(t) = s_0 - \int_{0}^{t} \left(y^{+}(\tau) - \eta y^{-}(\tau) \right) d\tau \tag{3.6}$$

According to Horsley and Wrobel (1996), the solution to this problem is:

$$y(t) = \begin{cases} y_{\max}, p(t) > \psi(t) \\ 0, \eta\psi(t) \le p(t) \le \psi(t), \\ -u_{\max}, p(t) < \eta\psi(t) \end{cases} \tag{3.7}$$

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[5] Round-trip efficiency η is the ratio of the transformation rates of water medium into energy during pumping and generating modes.

where $\psi(t) = \lambda - \gamma(t) + \delta(t)$ is water shadow price, that is shown to be unique and continuous if $p(t)$ is continuous.

Again, if the storage capacity constraints are not binding on $[t_1, t_2]$ then (3.5) simplifies to

$$\begin{aligned}
&\max_{y} \int_{t_1}^{t_2} y(t)p(t)dt \\
&s.t. \\
&y \in [-u_{\max}, y_{\max}] \quad \alpha(t), \beta(t) \\
&\int_{t_1}^{t_2} y^{+}(t)dt - \eta \int_{t_1}^{t_2} y^{-}(t)dt = s_{t_1} - s_{t_2} \quad \lambda
\end{aligned} , \tag{3.8}$$

and the water shadow price is constant over the interval $[t_1, t_2]$ and equal to $\psi(t) = \psi_{t_1,t_2} = \lambda$.

Proposition 2

The solution to (3.8) is given by (3.7), where $\psi(t) = \psi^{*}_{t_1,t_2} = const$ is obtained from:

$$\left(1 - F_p(\psi^{*}_{t_1,t_2})\right) y_{\max} - F_p(\eta \psi^{*}_{t_1,t_2}) \eta u_{\max} = \frac{s_{t_1} - s_{t_2}}{t_2 - t_1}. \tag{3.9}$$

The solution of (3.9) exists if $\max(y_{\max}, \eta u_{\max}) > \dfrac{s_{\max}}{t_2 - t_1}$ and is unique if $F_p{}'(\cdot) > 0$.

Proof:

Similarly to the proof of the Proposition 1 (3.9) solves (3.8). The condition $\max(y_{\max}, \eta u_{\max}) > \dfrac{s_{\max}}{t_2 - t_1}$ ensures that (3.9) is positive for $\tilde{\psi} = 0$ and negative for $\tilde{\psi} = \dfrac{\sup\limits_{[t_1,t_2]}(p(t))}{\eta}$, which together with continuity of $F_p(\cdot)$ ensures existence. Uniqueness follows from $F_p{}'(\cdot) > 0$.

Similarly to river dams, the length of the interval $[t_1, t_2]$ on which the reservoir constraints are not binding exceeds $\dfrac{s_{\max}}{\max(y_{\max}, \eta u_{\max})}$ or the time needed to empty or fill the reservoir operating at maximum generating or pumping capacity. For TVA's pumped storage facility Raccoon Mountain this value is on the order of one to three days. In the rest of the paper it is assumed that the water shadow price of the Raccoon Mountain is constant on each three-day interval of the year.

Ancillary services provision by river dams

If a river dam is capable of providing ancillary services such as regulation up to $r_{\max}$ and spinning reserves up to $s_{\max}$, and these services were priced at $p_r(t)$ and $p_s(t)$, its profit maximization problem over the period $[t_1, t_2]$ on which the reservoir capacity constraints are not binding modifies to:

$$\begin{aligned}
&\max_{y,r,s} \int_{t_1}^{t_2} \left(y(t)p(t) + r(t)p_r(t) + s(t)p_s(t) \right) dt \\
&s.t. \\
&0 \le y(t) \le y_{\max} \\
&0 \le r(t) \le r_{\max} \\
&0 \le s(t) \le s_{\max} \\
&y(t) + r(t) + s(t) \le y_{\max} \\
&r(t) + s(t) \le s_{\max} \\
&y(t) \ge r(t) \\
&\int_{t_1}^{t_2} y(t)dt = s_{t_1} - s_{t_2} + \int_{t_1}^{t_2} e(t)dt = S_{[t_1,t_2]} \quad \lambda
\end{aligned} \tag{3.10}$$

Here additional constraints require the sum of the energy and ancillary services output not to exceed the total generator capacity; the sum of ancillary services output not to exceed the spinning reserve capacity[6]; and the energy output to exceed the regulation output to provide the room for regulation downwards movements. The water shadow price is still constant $\psi_{t_1,t_2} = \lambda$ over the interval $[t_1, t_2]$, however, in general, is different from that in (3.3).

From now on it will be assumed that $s_{\max} = y_{\max}$ and $r_{\max} \le 0.5 y_{\max}$. The former is a reasonable assumption to make for hydro plants, whose dynamic characteristics allow them to

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[6] Spinning reserve capacity is limited by the 10-15 minutes ramp rate or the generator capacity, whatever is less. For hydro units it is likely that $s_{\max} = y_{\max}$

ramp up to the full capacity within 10-15 minutes; the latter comes from the fact that no unit can provide more regulation than a half of the total output capacity since both regulation upwards and downwards by the amount of regulation capacity is required.

It is also reasonable to assume that $p_r \geq p_s \geq 0$. The evidence of that is observed in the areas that currently administer markets for spinning reserves and regulation.

Proposition 3

Given the water shadow price ψ_{t_1,t_2} the optimal decision rule in (3.10) is:

$$(y(t), r(t), s(t)) = \begin{cases} (y_{\max}, 0, 0), & p(t) - \psi_{t_1,t_2} \geq p_r(t) \\ (y_{\max} - r_{\max}, r_{\max}, 0), & p_s(t) \leq p(t) - \psi_{t_1,t_2} < p_r(t) \\ (r_{\max}, r_{\max}, y_{\max} - 2r_{\max}), & 2p_s(t) - p_r(t) \leq p(t) - \psi_{t_1,t_2} < p_s(t) \\ (0, 0, y_{\max}), & p(t) - \psi_{t_1,t_2} < 2p_s(t) - p_r(t) \end{cases} \quad (3.11)$$

Proof:

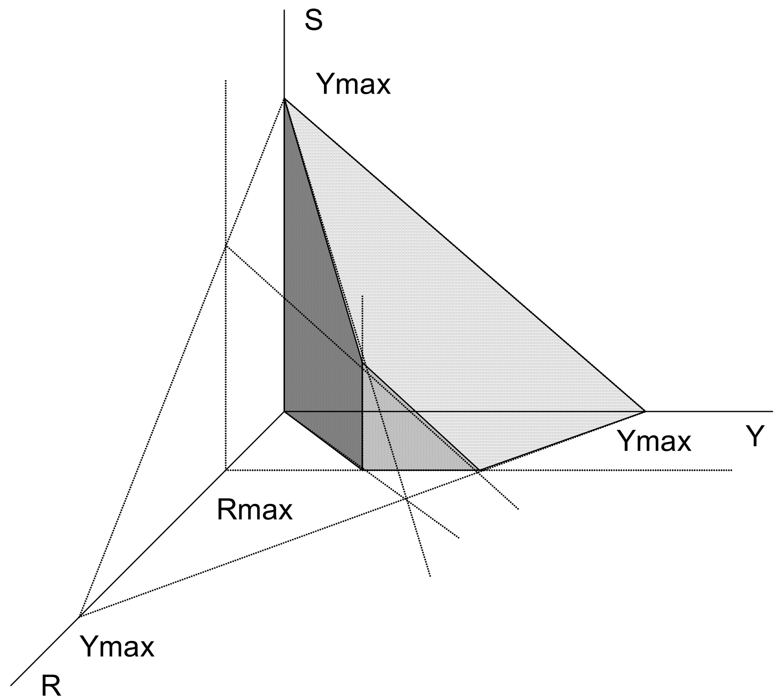
For reasons similar to those presented in Horsley and Wrobel (1999) it follows that problem (3.10) is equivalent to the following linear programming problem solved at each $t \in [t_1, t_2]$:

$$\begin{aligned} &\max_{y,r,s} y\left(p - \psi_{t_1,t_2}\right) + rp_r + sp_s \\ &s.t. \\ &0 \leq y \leq y_{\max} \\ &0 \leq r \leq r_{\max} \\ &0 \leq s \leq s_{\max} \\ &y + r + s \leq y_{\max} \\ &r + s \leq s_{\max} \\ &y \geq r \end{aligned} \quad , \quad (3.12)$$

that holds for every *t*. A solution to the linear programming problem (3.12) must lie on one of the corners of the polyhedron shown in Figure 2-3. This area is determined by the operation constraints in (3.12). The corners of this polyhedron are:

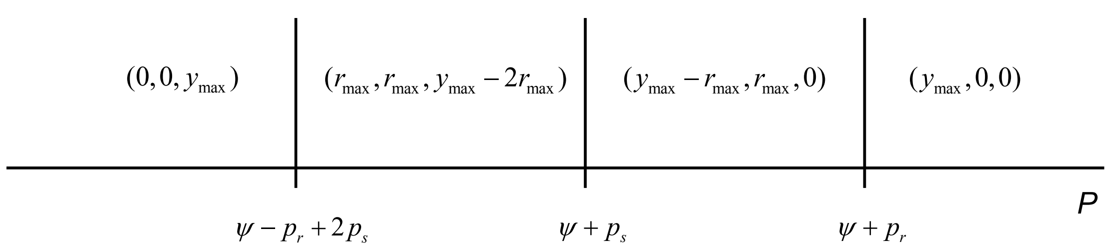
$$(0,0,0), (r_{\max}, r_{\max}, 0), (y_{\max} - r_{\max}, r_{\max}, 0), (y_{\max}, 0, 0), (0, 0, y_{\max}), (r_{\max}, r_{\max}, y_{\max} - 2r_{\max})$$

Figure 2-3 Simultaneous feasibility of energy and ancillary services output from a river dam



If the price of the spinning reserve is nonnegative, then the corners $(0,0,0)$ and $(r_{\max}, r_{\max}, 0)$ are dominated by $(0,0,y_{\max})$ and $(r_{\max}, r_{\max}, y_{\max} - 2r_{\max})$ leaving only four candidate solutions listed in (3.11). Having in mind that $p_r \geq p_s$, it is easy to check that each of the candidate solutions dominates the others on the intervals suggested in (3.11) illustrated on Figure 2-4.

Figure 2-4. Operation intervals for river dams



Proposition 4

If $s_{\max} = y_{\max}$ and $p_r \geq p_s \geq 0$, then the solution to (3.10) is given by (3.11), where $\psi(t) = \psi^*_{t_1,t_2} = const$ is obtained from:

$$G_{rd}\left(\psi^*\right) = \frac{S_{t_1,t_2}}{(t_2 - t_1)}, \tag{3.13}$$

where

$$G_{rd}\left(\psi^*\right) = y_{\max}\left(1 - F_{p-p_r}(\psi^*)\right) + (y_{\max} - r_{\max})\left(F_{p-p_r}(\psi^*) - F_{p-p_s}(\psi^*)\right) + r_{\max}\left(F_{p-p_s}(\psi^*) - F_{p-2p_s+p_r}(\psi^*)\right) \tag{3.14}$$

(the $_{t_1,t_2}$ subscript at ψ^* is dropped in (3.13) and (3.14)), and (3.13) has a unique solution.

Proof:

Similarly to Proposition 1 the form of (3.13) and (3.14) is dictated by the water balance constraint in (3.10) and the operation intervals in (3.11). The existence is shown as in Proposition 1. For uniqueness consider $G'_{rd}(\psi)$:

$$G'_{rd}(\psi) = -f_{p-p_s}(\psi)(y_{\max} - 2r_{\max}) - r_{\max}\left(f_{p-p_r}(\psi) + f_{p-2p_s+p_r}(\psi)\right) \tag{3.15}$$

Therefore, $G_{rd}(\psi)$ is non-increasing since $y_{\max} > 2r_{\max}$ from the property of regulation. If $p - p_s, p - p_r$, and $p - 2p_s + p_r$ do not have a plateaus, that is, $F'_{\bullet}(\cdot) > 0$ then $G_{rd}(\psi)$ is strictly decreasing and (3.13) has a unique solution.

Ancillary services provision by pumped storage

A similar solution for water shadow price for a pumped storage that is capable of ancillary service provision is slightly more involved.

According to Horsley and Wrobel (1996) and similarly to the proof of Proposition 3 the problem of the pumped storage with round-trip efficiency η for given water shadow price ψ is:

$$\max_{y,r,s} y^+(p-\psi) - y^-(p-\eta\psi) + rp_r + sp_s$$
$$\text{s.t.}$$
$$-u_{\max} \le y \le y_{\max}$$
$$0 \le r \le r_{\max}$$
$$0 \le s \le s_{\max} \tag{3.16}$$
$$y + r + s \le y_{\max}$$
$$r + s \le s_{\max}$$
$$y \ge r$$
$$r = 0 \text{ if } y < 0$$

The last constraint requires that no regulation can be provided when pumped storage is in the pumping mode. This is a technical constraint that is present in the Raccoon Mountain pumped storage facility studied further in this paper and perhaps is also present in other pumped storage facilities.

The operation constraints in (3.16) are represented by a polyhedron, which is non-convex because of the last constraint. However, solutions must still lie on the corners of the polyhedron. Assuming $p_s \ge 0$ and $s_{\max} = y_{\max}$, the set of candidate solutions becomes:

$$(y_{\max},0,0),(y_{\max}-r_{\max},r_{\max},0),(r_{\max},r_{\max},y_{\max}-2r_{\max}),(0,0,y_{\max}),(-u_{\max},0,y_{\max}+u_{\max})$$

Proposition 5

The solution to the operation problem of a pumped storage facility providing ancillary services (3.16) is given by:

$$(y,r,s) = \begin{cases} (y_{\max},0,0), & p > \psi + p_r \\ (y_{\max}-r_{\max},r_{\max},0), & \psi + p_s < p \le \psi + p_r \\ (r_{\max},r_{\max},y_{\max}-2r_{\max}), & \max(\psi - p_r + 2p_s, \tilde{p}) < p \le \psi + p_s \\ (0,0,y_{\max}), & \eta\psi + p_s < p \le \psi - p_r + 2p_s \\ (-u_{\max},0,y_{\max}+u_{\max}), & p \le \min(\eta\psi + p_s, \tilde{p}) \end{cases}, \tag{3.17}$$

where

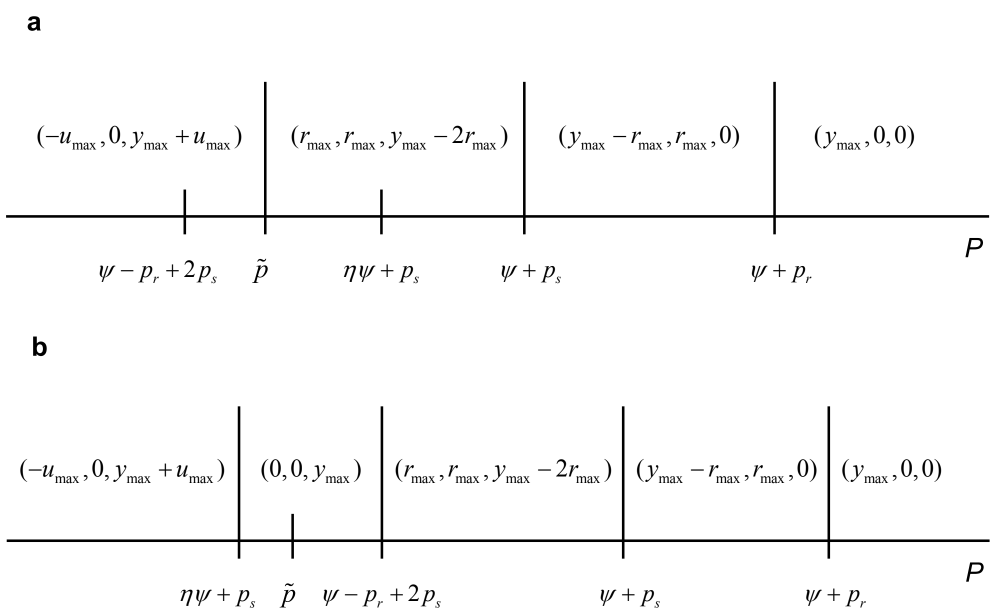
$$\tilde{p} = \frac{r_{\max}(\psi - p_r + 2p_s) + u_{\max}(\eta\psi + p_s)}{r_{\max} + u_{\max}}. \tag{3.18}$$

Proof:

See Appendix.

Optimal operation decisions for pumped storage for cases $\psi - p_r + 2p_s > \eta\psi + p_s$ and $\psi - p_r + 2p_s < \eta\psi + p_s$ are shown on Figure 2-5.

Figure 2-5 Operation intervals for pumped storage with AS



Proposition 6

The solution to the problem of a pumped storage facility operating in energy and ancillary services markets is given by (3.17), where ψ is found from

$$G_{ps}(\psi) = \frac{s_{t_1} - s_{t_2}}{t_2 - t_1}, \tag{3.19}$$

where

$$\begin{aligned} G_{ps}(\psi) = & \\ & = y_{\max}\left(1 - F_{p-p_r}(\psi)\right) + \\ & +(y_{\max} - r_{\max})\left(F_{p-p_r}(\psi) - F_{p-p_s}(\psi)\right) + , \\ & +r_{\max}\left(F_{p-p_s}(\psi) - F_{\min(p+p_r-2p_s,\breve{p})}(\psi)\right) - \\ & -\eta u_{\max} F_{\max((p-p_s)/\eta,\breve{p})}(\psi) \end{aligned} \tag{3.20}$$

where

$$\breve{p} = \frac{r_{\max}(p + p_r - 2p_s) + u_{\max}(p - p_s)}{r_{\max} + \eta u_{\max}}, \tag{3.21}$$

and the solution to (3.19) - (3.21) is unique.

Proof:

Same as in Proposition 4.

Change of water shadow price in presence of ancillary services markets

Unlike coal and gas generators whose cost is determined by the cost of fuel andis largely constant, for hydro units the shadow price of water depends on their operation capabilities. Formulas (3.4) and (3.13), (3.9) and (3.19) suggest that the water shadow price changes if a hydro unit is participating in the market for ancillary services.

To check the direction of the change in the water value with participation of hydro units in ancillary services markets examine

$$\frac{d\psi}{dp_s} \text{ and } \frac{d\psi}{dp_r},$$

assuming that $p_r, p_s = const$ and $p_r > p_s$.

Proposition 7

The shadow price of water goes down for both river dams and pumped storage with the increase of the price of spinning reserves.

Proof:

Consider $G(\psi)$ from (3.14) (subscript $_{ps}$ is omitted in this proof). With $p_r, p_s = const$, it transforms to

$$\begin{aligned} G(\psi, p_r, p_s) = & \\ & = y_{\max}\left(1 - F_p(\psi + p_r)\right) + \\ & +(y_{\max} - r_{\max})\left(F_p(\psi + p_r) - F_p(\psi + p_s)\right) + \\ & +r_{\max}\left(F_p(\psi + p_s) - F_p(\psi + 2p_s - p_r)\right) \end{aligned}$$

The change in ψ in response to the change in p_s is

$$\frac{d\psi}{dp_s} = -\frac{G_{p_s}}{G_\psi}$$

From the proof of Proposition 4 it is known that $G_\psi < 0$, therefore, consider G_{p_s}:

$$G_{p_s} = -(y_{\max} - 2r_{\max})f(\psi + p_s) - 2r_{\max}f(\psi + 2p_s - p_r)$$

But since $y_{\max} \geq 2r_{\max}$ by definition of regulation capability, $G_{p_s} < 0$ and $\frac{d\psi}{dp_s} < 0$.

The same logic applies to pumped storage for both cases: $\eta\psi + p_s > \psi - p_r + 2p_s$ and reverse.

The intuition behind this result is that the ability to profitably provide spinning reserves allows hydro units to save water that otherwise could have been used for generation. And additional supply of the saved water drives the water price down.

Proposition 8

The change of the water shadow price in the presence of regulation markets is undetermined for both hydro dams and pumped storage. If the energy price p has a unimodal distribution $f_p(\cdot)$ with the mode p_M then for river dams it can be stated that $\frac{d\psi}{dp_r} < 0$ if $\psi < p_M - p_r$ and $\frac{d\psi}{dp_r} > 0$ if $\psi > p_M - 2p_s + p_r$. A similar result applies to pumped storage units.

Proof:

Consider $G(\psi, p_r, p_s)$ as in Proposition 7.

$$G_{p_r} = -r_{\max}\left(f(\psi + p_r) - f(\psi + 2p_s - p_r)\right)$$

The sign of the expression $f(\psi + p_r) - f(\psi + 2p_s - p_r)$ is not determined. However, since $p_r \geq 2p_s - p_r$, the expression is positive and $\frac{d\psi}{dp_r} < 0$ if $f(\cdot)$ is unimodal with the mode p_M and $\psi < p_M - p_r$ since in this case both $\psi + p_r$ and $\psi + 2p_s - p_r$ are on the increasing segment of $f(\cdot)$. Likewise, $\frac{d\psi}{dp_r} > 0$ if $\psi > p_M - 2p_s + p_r$.

The intuition behind this result is the following. Consider the case when a hydro unit provides regulation in a given hour but in the absence of the regulation market it would choose not to generate in this hour. Therefore, the presence of a regulation market requires the unit to use water that otherwise would not be used. That increases the water value for the subsequent hours. The other situation is that in a given hour the hydro unit provides regulation that offsets the energy production in the absence of regulation markets. The hydro unit is therefore saving water that otherwise would have been used for energy decreasing the water value for future use.

The total effect of regulation markets on water shadow price is determined by the ratio of hours of both types. Units with low water availability and therefore high water shadow price have cases of the first type more prevalent, therefore the water price tends to increase in presence of regulation markets. Likewise, units with high water availability and low water prices enjoy more hours of the second type over the cycle and their water price decreases in presence of regulation markets.

2.4 Ancillary services market simulation

The purpose of this study is to evaluate the ancillary services that are or can be provided by hydro units in the Tennessee Valley Authority (TVA). However, since TVA currently operates as a vertically integrated utility, it does not have a market for ancillary services. Such a market is therefore simulated here using the analysis above to calculate the water shadow prices for the largest TVA's hydro units.

The market for energy and ancillary services is assumed to be competitive and cleared simultaneously. To simulate such markets an optimization program that minimizes the total cost of energy given the requirements for the ancillary services is run. Lagrange multipliers at the market clearing conditions for regulation and spinning reserves resulting from such optimization represent the correspondent market prices in the case of a perfectly competitive market.

The following linear programming problem is run at each hour t of the year (time index is omitted):

$$
\begin{aligned}
&\max_{y_i, r_i, s_i} \sum_{i=1}^{N} (p - mc_i) y_i \\
&s.t. \\
&1. \quad y_i, r_i, s_i \geq 0 \\
&2. \quad y_i + r_i + s_i \leq y_{\max}^i \\
&3. \quad y_i - r_i \geq y_{\min}^i \\
&4. \quad r_i \leq r_{\max}^i \\
&5. \quad r_i + s_i \leq s_{\max}^i \\
&6. \quad \sum_{i=1}^{N} r_i \geq R \qquad p_r \\
&7. \quad \sum_{i=1}^{N} s_i \geq S \qquad p_s
\end{aligned}
\tag{4.1}
$$

In problem (4.1) p is the energy price at hour t; mc_i is the marginal cost of energy production from i-th unit. For hydro units this is time-variant water shadow price; y_i, r_i, and s_i are the quantities of energy, regulation and spinning reserves produced by i-th unit. Constraint 1 requires the control variables to be nonnegative; constraint 2 requires that the total of the energy, regulation and spinning reserves provided by a unit not exceed the maximum energy output from that unit. Constraint 3 requires that the amount of energy exceed the amount of regulation by the amount of low operation limit. Constraint 4 states that the regulation output does not exceed the regulation capability and constraint 5 states that the total of the ancillary services output does not exceed the maximum spinning reserve capacity. Finally, constraints 6 and 7 are the market clearing conditions for regulation and spinning reserves.

The general constraints in (4.1) have to be modified for different resources. For instance, coal plants in TVA are relatively cheap in their energy cost but are expensive to start up. Therefore, it is assumed here that the coal plants are always online to avoid the startup costs even if energy price is below the energy cost. Therefore, for coal plants constraint 1 is modified to:

$$y_i \geq y_{\min}^i$$

Gas turbines on the other hand are cheap to startup but expensive to run; in addition, their low operation limit is above zero. Therefore the constraints for gas turbines must be modified to allow them to be offline with $y_i = 0$, perhaps providing the spinning reserves up to $y_{\max}^i$. Together with constraints 1 to 5 this will result in a nonconvex polyhedron for each gas turbine. However, since the solutions will still be on the corners of this polyhedron cutting planes may be found to "convexify" the original set of constraints. Thus, for gas turbines constraint 3 must be substituted by

$$\begin{aligned} &\frac{r_{\max}^i}{y_{\min}^i + r_{\max}^i} y_i - r_i \geq 0 \\ &y_i - \frac{y_{\max}^i + y_{\min}^i}{y_{\max}^i - y_{\min}^i} r_i - \frac{y_{\min}^i}{y_{\max}^i - y_{\min}^i} s_i \geq 0 \end{aligned} . \tag{4.2}$$

Gas units have good dynamic characteristics; therefore the corresponding constraint 5 is redundant. Constraint 4 however, is not redundant because not all gas turbines have regulation capability.

Because for river dams $y_{\min}^i = 0$, the general constraints in (4.1) hold without modification.

The most important changes are needed for the pumped storage case. Because TVA's pumped storage facility Raccoon Mountain cannot provide regulation during the pumping mode the constraint for the simultaneous feasibility of energy and ancillary service output (3.16) is essentially a mixed linear integer constraint. To solve such a problem in addition to (4.1) another program is solved:

$$\begin{aligned} &\max_{y_i, r_i, s_i} \sum_{i=1}^{N-1} (p - mc_i) y_i + (p - mc_N) y_N \\ &s.t. \\ &y_N \geq -u_{\max}; r_N, s_N \geq 0 \\ &y_N + r_N + s_N \leq y_{\max}^N \\ &r_N \leq 0 \end{aligned} , \tag{4.3}$$

where N is the index for the Raccoon Mountain Pumped Storage facility, while the constraints for the rest *N-1* units are the same as before. Solving this problem along with (4.1) essentially solves the mixed linear integer programming problem of the ancillary services market simulation by the branch and bound method.

Since the water shadow prices depend on the prices of ancillary services ((3.13), (3.19)), which in turn result from the market simulation (4.1) an iterative process is run in water shadow prices. In the initial stage the ancillary services capacity of all hydro units is assumed to be zero and the market simulation is run to obtain values of the ancillary services competitive prices that would occur if hydro units were not participating in these markets. These prices are used to obtain values of the water shadow prices using (3.13) and (3.19) which are further used to come up with the next iteration of ancillary services prices this time assuming hydro participation in all markets. The process continues until convergence in water shadow prices.

Since (3.13) and (3.19) assume continuous price distributions linear interpolation is applied to the sample distribution of the hourly prices used in the simulation (Judd (1998)). The total quantity of available water in (3.13) was calculated using the actual data of water usage by TVA's river dams over monthly cycles. In (3.19) the reservoir level of the pumped storage was assumed to be the same in the beginning and in the end of the three-day cycles.

Throughout the simulation, it is assumed that the energy price is given and does not depend on ancillary services markets. This assumption is reasonable keeping in mind the low quantity of ancillary services demanded compared to the total energy demand. E.g. in TVA the demand for regulation is set at 1% of the area peak energy demand which roughly corresponds to 300MWh. The total demand for spinning and non-spinning reserves amounts to 1,400MWh which is about 5% of the area peak energy demand. However, the assumption may result in more inaccuracy during high demand periods in which energy supply is inelastic.

2.5 Results

The data used for the simulation of the energy and ancillary services markets in Tennessee Valley Authority included the parameters of the fossil fuel plants such as low operation limits, maximum generating capacity, ramp rates, marginal cost, and whether the unit is equipped with automated generation control system (AGC) allowing it to provide regulation; parameters of hydro plants such as maximum energy output capacity, regulation capability, and maximum pumping capacity and roundtrip efficiency of the pumped storage facility Raccoon Mountain. Data on the actual hourly water usage by TVA's river dams in 2001 was used to calculate the total water availability in river dams over the cycles over which water shadow prices were assumed constant. Finally, TVA's hourly system marginal cost of energy (system lambdas) in 2001 were used to proxy the energy prices in the market simulation.

All of these data were obtained from TVA under a non-disclosure agreement. However, most of the components of this data can be obtained from public sources. E.g. a public database E-Grid provided by Environmental Protection Agency contains the data on capacity of all U.S. generating units (http://www.epa.gov/cleanenergy/egrid/index.htm). Combining the data on the heat rates of the fossil fuel units also available from E-Grid with the input prices from Energy Information Agency (www.eia.doe.gov), one can obtain estimates of marginal costs of the plants. Parameters of Raccoon Mountain can be obtained from TVA's website http://www.tva.gov/sites/raccoonmt.htm, and the data on hourly water usage of selected TVA's river dams is available at (http://home.hiwaay.net/~ksgrisse/lakeinfo.html). In order to preserve confidentiality a linear transformation was performed on the rest of the TVA's data.

Table 2-2 shows the total energy, regulation and spinning reserves capability of 59 coal plants, 72 gas plants, and 10 largest TVA's hydro plants along with the average energy cost of coal and gas plants and average water shadow price of hydro plants when hydro does not participate in ancillary services markets and when they participate. Only three hydro units have regulation capability. Nevertheless in the market simulation 1MW of regulation capability is assigned to each of hydro plants to measure their potential profits from regulation provision (Table 2-3). TVA's nuclear plants do not participate in the ancillary services market simulation because of their poor dynamic characteristics.

Total Cost of Ancillary Services

The market simulation allows calculating the total cost of ancillary services when hydro resources do not participate in the ancillary services markets and compare it to the case when hydro resources do participate in these markets. Table 2-1 shows these scenarios. Because we assume that energy prices do not change, the total cost of energy generation calculated as the total fuel cost of fossil fuel plants is the same in each scenario and is equal to $1,723 million per year. The cost of ancillary services, calculated as the total opportunity and re-dispatch cost of fossil fuel plants over the year is $145 million if all ancillary services are provided by fossil fuel plants and $24 million if hydro resources participate in ancillary services provision. Thus, hydro resources save 83% of the ancillary services cost or 7% of the total cost of energy provision in TVA.

Table 2-1. Energy and AS costs with and without hydro participation

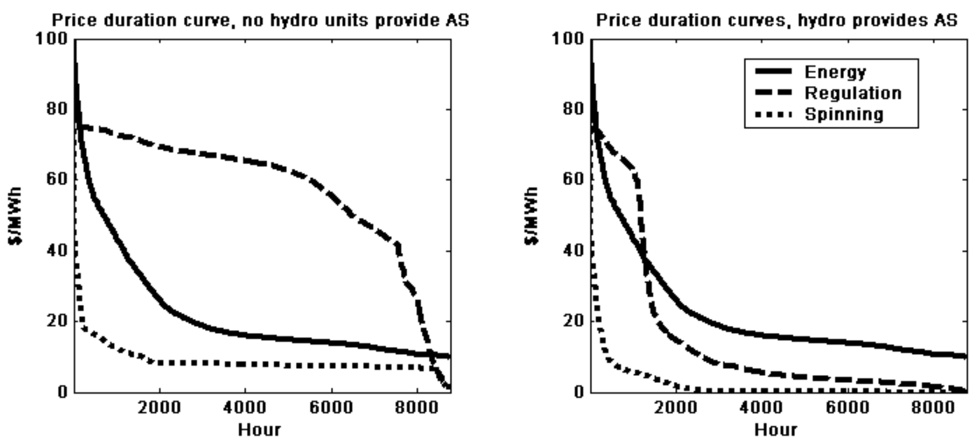
	No AS from hydro	Hydro provides AS
Total energy cost, million $	1,723	1,723
Total ancillary services cost, million, $	145	24

Prices of Ancillary Services

Figure 2-6 shows price duration curves for energy and ancillary services for scenarios in which hydro does not provide ancillary services and when hydro units provide ancillary services in TVA. Prices of regulation and spinning reserves are reduced dramatically due to hydro participation in ancillary services provision.

Figure 2-7 shows average hourly prices of energy, regulation and operating reserves for each hour of the day for the same two scenarios. On average the price of regulation is about $40 higher and the price of spinning reserve $8 higher if hydro resources do not participate in ancillary services provision. The regulation prices are low during the peak and high during the off-peak. This happens because gas turbines that have high energy cost have lots of regulation capacity that is cheap to provide regulation during the peak.

Figure 2-6. Energy and AS price duration curves with and without hydro provision of AS



In contrast, prices of operation reserves are high during the peak hours and are nearly zero on average during the off-peak. Hydro resources provide great amounts of cheap reserves during the off-peak hours when they are not generating. When Raccoon Mountain is in the pumping mode during the off-peak, it can provide spinning reserves up to its pumping capacity (1,530MW) plus generating capacity (1,530 MW) at no cost. Similarly, river dams that are shut down during the off-peak hours can provide spinning reserves at no cost.

Ancillary Services output by source

Table 2-2 shows the average hourly production of energy and ancillary services and capacity factors defined as the ration of average hourly output to the maximum hourly output. Hydro resources provide on average about two thirds of the regulation requirement followed by the coal and gas that provide 22% and 11% of regulation respectively. Hydro resources also provide the majority of spinning reserves.

The fact that hydropower is a relatively cheap source of ancillary services is confirmed by the highest utilization rates of hydro regulation and spinning capacity compared to coal and gas.

Although spinning reserves make up to about half of the hydro output, this market yields only 2% of the total hydro profits. The total regulation production amounts to 8% and gives 5% of total hydro profits. Ancillary services markets increase the value of hydro resources by 7%, the value of gas turbines by 133% and the value of coal plants by 1%.

Figure 2-7. Prices of energy and ancillary services depending on hydro participation

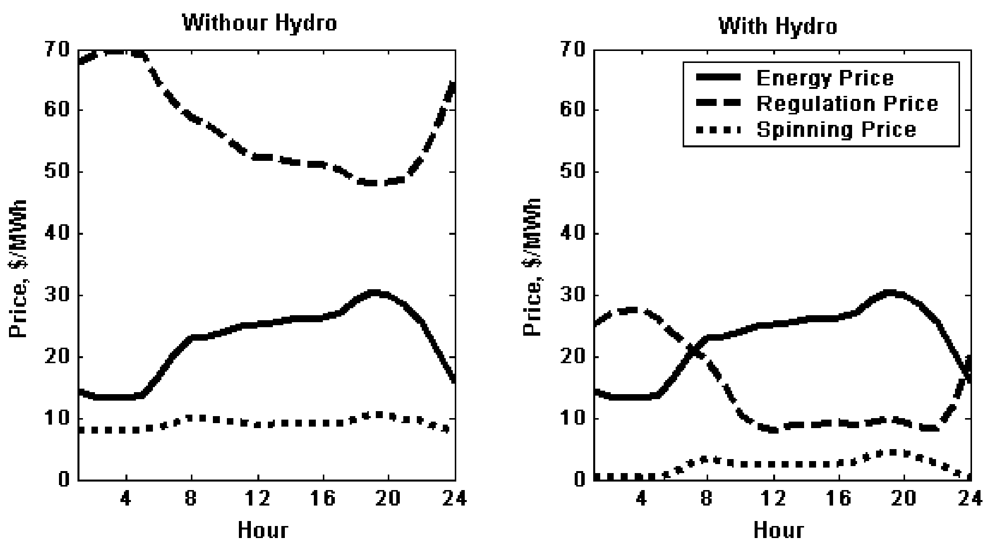


Table 2-3 shows the results of market simulation for 10 TVA's largest hydro generators along with one coal plant and one gas turbine for comparison. Two hydro dams, Wilson and Fontana, and the pumped storage facility at Raccoon Mountain have regulation capability. The Raccoon Mountain Pumped Storage facility alone has enough capacity to provide two thirds of the regulation demand and all spinning reserves requirements. Because Fontana does not have much water available per month relative to its generation capacity, it does not make much profit in energy market but compensates for that by profits made in regulation and spinning reserves markets. Wilson with higher water availability makes more profits from energy.

Value of additional regulation capability

The market simulation model presented here can be used for making budget decisions concerning capital improvements, such as adding regulation capability to more hydro units or fossil fuel units. Running the market simulation in which the 10 largest TVA's hydro facilities participate, one can calculate the marginal annual profit of these units from additional regulation capacity. Marginal profit from additional regulation capacity is also calculated for a coal plant with energy cost of $16/MWh, and a gas turbine with energy cost of $49/MWh.

The market simulation suggests that, for most of the selected units, adding a megawatt of regulation capacity would bring about $100,000 in annual profits, if the competitive markets for ancillary services existed. Although Raccoon Mountain is making a substantial share of its profits in regulation sales, it does not get much from its regulation capacity compared to the other hydro units. The reason is that Raccoon Mountain has to pump during the off-peak hours foregoing high regulation prices.

Apparently, adding a MW of regulation capacity to a coal plant pays about the same as for hydro units. A coal plant is provides regulation during off-peak hours when the regulation price is high. Finally, regulation capability on combustion turbines pays the least. Such unit would only provide regulation when the energy price is close to its cost ($49/MWh). This occasionally happens during the peak hours when regulation prices are low.

Change in water shadow price

Participation in markets for regulation and spinning reserves changes the water shadow price in hydro units. Table 2-3 shows the water shadow prices that would prevail without hydro participation in ancillary service provision and how they change as a result of such participation. Following Proposition 7, water shadow prices go down with units' participation in spinning reserves markets. For Wilson, Fontana, and Raccoon Mountain, this decrease is partially offset by the participation in regulation market. Water shadow price decreases the most for units with low water availability for which this price would be high in the absence of the ancillary services markets. This happens because such units operate during the peak hours when spinning prices are high. High spinning prices cause more reduction in value of water.

2.6 Conclusion

This paper studies the economics of hydro generating units operating in markets for energy and ancillary services. The paper provides the operation rules for river dams and pumped storage facilities that depend on prices of energy and ancillary services as well as water shadow prices. The paper also provides formulas for calculating the water shadow price in river dams and pumped storage facilities operating in these markets.

The analysis presented here provides an insight on why hydro resources are and should be used intensively for ancillary services provision. The main cost of ancillary services is associated with the opportunity and re-dispatch cost that is proportional to the absolute difference between the energy price and generation cost of a unit. However, for hydro units the water shadow price is determined by the prices of energy and ancillary services in next periods. That is, water shadow price, which can be considered as the generation cost of hydro units, follows the prices of energy and ancillary services, making the cost of ancillary services production small compared to that of fossil fuel plants on average.

The algorithm for water shadow price calculation was applied to the simulation of ancillary services markets in Tennessee Valley Authority, a vertically integrated electric utility with substantial amount of hydro generating capacity.

The simulation suggests that hydro resources should produce a substantial amount of TVA's requirements for ancillary services such as regulation and spinning reserves. The hydro resources can reduce the cost of ancillary services by $121 million per year.

Provision of ancillary services increases the total value of TVA's hydro resources by 7% with the value of some units (Fontana) going up by 26%.

The simulation also shows that the river dams and coal plants are better for investment in ancillary services capacity than pumped storage and gas turbines (per unit of regulation capacity). This analysis also suggest that adding regulation capability during the pumping mode to Raccoon Mountain Pumped Storage would also be quite valuable

Finally, the simulation suggests that the water shadow price generally goes down for hydro units in the presence of ancillary services markets. This means that the reservoir constraints that determine the amount water available for energy generation can be tightened in favor of other uses of water.

It is recommended that TVA routinely run a market simulation similar to the one presented in this report in order to obtain internal pricing for ancillary services and use these prices to optimize ancillary service procurement. This will also help both to estimate the value of TVA's generating resources associated with ancillary service provision and ensure a smooth transition to market-base provision of ancillary services.

The fact that the prices of ancillary services go down dramatically because of the presence of hydro resources in these markets suggests that TVA may profitably sell the ancillary services to the neighboring utilities that do not have much hydro capacity.

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Table 2-2. Simulation results by source type

	Coal Plants	Gas Plants	Hydro
# Units	59	72	10
Total Generation Capacity, MW	14,931	4,660	3,916
Total Regulation Capacity, MW	109.5	344	318
Total Spinning Capacity, MW	1,268.5	2,360	5,446
Average Generating Cost, $/MWh	14	52	27
Average Generating Cost, No AS Mkts	14	52	25
Average Energy Generation, MWh/h	13,024	330	1,205
Average Regulation Production, MWh/h	66	34	200
Average Spin Production, MWh/h	179	190	1,235
Energy Capacity Factor	0.87	0.07	0.31
Regulation Capacity Factor	0.60	0.10	0.63
Spinning Capacity Factor	0.14	0.08	0.23
Energy Profit, million $	1,123	21	376
Regulation Profit, million $	11	9	20
Spinning Profit, million $	1	19	8
Energy Profit per Capacity, $/MW	75,182	4,584	95,959
Regulation Profit per Capacity, $/MW	102,727	24,987	62,542
Spinning Profit per Capacity, $/MW	813	7,879	1,388

Table 2-3. Market simulation results for 10 hydro, one coal, and one gas generators

	Wilson	Wheeler	Pickwick	Fontana	Kentucky	Watts Bar	Hiwassee	Chickamauga	Douglas	Raccoon Mountain	Coal 16	Gas 49
Max Energy Output, MW	667.28	404.10	240.24	238.50	197.56	167.40	165.65	159.60	145.80	1530.00	167.00	63.00
Max Reg Output, MW	86.00	0.00	0.00	33.00	0.00	0.00	0.00	0.00	0.00	200.00	7.50	22.50
Hourly Energy Output, MW	271.45	139.32	117.28	69.96	123.35	64.86	42.02	73.07	43.41	-112.48	124.72	6.53
Hourly Regulation Output, MW	48.98	0.56	0.56	17.26	0.57	0.61	0.48	0.55	0.54	130.12	4.47	2.13
Hourly Spinning Output, MW	150.18	102.88	35.33	65.57	20.02	32.47	48.00	25.46	36.62	718.46	4.31	1.25
Energy profits/capacity, thousand $	108.86	102.64	130.28	69.67	153.64	108.51	83.06	124.26	88.86	77.57	60.20	2.57
Regulation profits/capacity, thousand $	102.43	105.09	100.20	101.10	99.92	102.53	103.32	100.50	103.39	35.14	105.12	23.88
Spinning profits/capacity, thousand $	0.83	1.29	0.02	8.67	0.00	0.61	2.73	0.03	1.67	2.36	1.09	1.12
Mean water price, no AS markets	26.07	28.12	21.68	37.53	15.97	28.48	34.21	24.03	34.08	24.71	16.21	49.00
Mean water price \w AS markets	26.17	26.23	21.48	29.93	16.01	27.22	30.43	23.83	29.72	23.54	16.21	49.00

Appendix

The following problem is solved:

$$\max_{y,r,s}(p-\psi)y^{+}-(p-\eta\psi)y^{-}+p_r r+p_s s$$

$$(y,r,s)=\begin{cases}1. & (y_{\max},0,0)\\ 2. & (y_{\max}-r_{\max},r_{\max},0)\\ 3. & (r_{\max},r_{\max},y_{\max}-2r_{\max})\\ 4. & (0,0,y_{\max})\\ 5. & (-u_{\max,},0,y_{\max}+u_{\max,})\end{cases}$$

Intervals of $p, p_r,$ and p_s are looked for on which each of the candidates 1. to 5. solve the above problem given $p_r > p_s$. Subscripts $_{\max}$ are omitted throughout the rest of the appendix.

1.

a.

$$\begin{aligned}&y(p-\psi)>(y-r)(p-\psi)+rp_r\\ &\Rightarrow \boxed{p>\psi+p_r}\end{aligned}$$

b.

$$y(p-\psi)=r(p-\psi)+r(p-\psi)+(y-2r)(p-\psi)>r(p-\psi)+rp_r+(y-2r)p_s$$

c.

$$y(p-\psi)>yp_s \text{ follows from } (p-\psi)>p_s$$

d.

$$y(p-\psi)>-u(p-\eta\psi)+p_s(y+u)$$

follows from

$$p-\eta\psi>p-\psi>p_r>p_s$$

2.

a.

$$
\begin{aligned}
&(y-r)(p-\psi)+rp_r > y(p-\psi)\\
&\Rightarrow \boxed{p<\psi+p_r}
\end{aligned}
$$

b.

$$
\begin{aligned}
&(y-r)(p-\psi)+rp_r > r(p-\psi)+rp_r+(y-2r)p_s\\
&\Rightarrow (y-2r)(p-\psi) > (y-2r)p_s \Rightarrow \boxed{p>\psi+p_s}
\end{aligned}
$$

c.

$$(y-r)(p-\psi)+rp_r > yp_s$$

follows from

$$y(\underbrace{p-\psi-p_s}_{>0})+r(\underbrace{p_r-p+\psi}_{>0})>0$$

d.

$$(y-r)(p-\psi)+rp_r > -u(p-\eta\psi)+(y+u)p_s$$

follows from

$$y(\underbrace{p-\psi-p_s}_{>0})+u(\underbrace{p-\eta\psi-p_s}_{>0})+r(\underbrace{p_r-p+\psi}_{>0})>0$$

3.

a.

$$
\begin{aligned}
&r(p-\psi)+rp_r+(y-2r)p_s > (y-r)(p-\psi)+rp_r\\
&\Rightarrow (y-2r)p_s > (y-2r)(p-\psi)\\
&\Rightarrow \boxed{p<\psi+p_s}
\end{aligned}
$$

b.

$$
\begin{aligned}
&r(p-\psi)+rp_r+(y-2r)p_s > yp_s\\
&\Rightarrow \boxed{p>\psi-p_r+2p_s}
\end{aligned}
$$

c.

$$r(p-\psi)+rp_r+(y-2r)p_s > y(p-mc)$$

follows from

$$r(\underbrace{p-\psi+p_r-2p_s}_{>0})+y(\underbrace{p_s-p+\psi}_{>0})>0$$

d.

$$
\begin{aligned}
& r(p-\psi)+rp_r+(y-2r)p_s > -u(p-\eta\psi)+(u+y)p_s \\
& \Rightarrow r(p-mc+p_r-2p_s) > u(p_s-p+\eta\psi) \\
& \Rightarrow \boxed{p > \frac{r(\psi-p_r+2p_s)+u(\eta\psi+p_s)}{r+u}}
\end{aligned}
$$

4.

a.

$$
\begin{aligned}
& yp_s > r(p-\psi)+rp_r+(y-2r)p_s \\
& \Rightarrow \boxed{p < \psi-p_r+2p_s}
\end{aligned}
$$

b.

$$
\begin{aligned}
& yp_s > -u(p-\psi)+(y+u)p_s \\
& \Rightarrow \boxed{p > \eta\psi+p_s}
\end{aligned}
$$

c.

$$
yp_s > y(p-\psi)
$$

follows from

$$
p-\psi < 2p_s-p_r < p_s
$$

d.

$$
yp_s > (y-r)(p-\psi)+rp_s
$$

follows from

$$
p_s > p-\psi
$$

5.

a.

$$
\begin{aligned}
& -u(p-\eta\psi)+(y+u)p_s > yp_s \\
& \Rightarrow \boxed{p < \eta\psi+p_s}
\end{aligned}
$$

b.

$$
\begin{aligned}
& -u(p-\eta\psi)+(y+u)p_s > r(p-\psi)+rp_r+(y-2r)p_s \\
& \Rightarrow u(p_s-p+\eta\psi) > r(p-\psi+p_\partial-2p_s) \\
& \Rightarrow \boxed{p < \frac{r(\psi-p_r+2p_s)+u(\eta\psi+p_s)}{r+u}}
\end{aligned}
$$

c.

$$-u(p-\eta\psi)+(y+u)p_s > (y-r)(p-\psi)+rp_r$$

follows from

$$\begin{aligned}
&u(p_s - p + \eta\psi) + y(p_s - p + \psi) + r(p - \psi - p_r) > \\
&> r(p - \psi + p_r - 2p_s + p - \psi - p_r) + y(p_s - p + \psi) = \\
&= (\underbrace{y - 2r}_{>0})(\underbrace{p_s - p + \psi}_{>0}) > 0
\end{aligned}$$

d.

$$-u(p-\eta\psi)+(y+u)p_s > y(p-\psi)$$

follows from

$$(y+u)p_s > (y+u)(p-\eta\psi) > y(p-\psi)+u(p-\eta\psi)$$