Introduction to Biological Physics: Collection of physics equations for final exam

$k_BT \approx 1/40$ eV ≈ 4.1 pN·nm $= 4.1\cdot10^{-21}$ J; $k_B \approx 1.38\cdot10^{-23}$ J/K; $N_A = 6.022\cdot10^{23}$/mol; $g = 9.81$ m/s^2 ≈ 10 m/s^2.

Coulomb force: $\vec{F} = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{q_1q_2}{R^2}\hat{r}$, potential: $U = \frac{1}{4\pi\varepsilon\varepsilon_0}\frac{q_1q_2}{R}$; $e = 1.6\cdot10^{-19}$ C; $\varepsilon_o = 8.85\cdot10^{-12}$ As/Vm.

Bjerrum length: $l_B = \frac{e^2}{4\pi\varepsilon_0\varepsilon k_BT} \approx 7$ Å in water at $T = 300$ K. Born energy of an ion: $U = \frac{q^2}{8\pi\varepsilon_0\varepsilon r}$.

dipole field: $\vec{E} = \frac{3(\vec{\mu}\cdot\hat{r})\cdot\hat{r} - \vec{\mu}}{4\pi\varepsilon_0\varepsilon r^3}$; interaction energy (IE) for static dipoles: $U = -\frac{\vec{\mu}_1\vec{\mu}_2}{2\pi\varepsilon_0\varepsilon r^3}$

IE between charge and rotating dipole: $U = -\frac{1}{(4\pi\varepsilon_0\varepsilon)^2}\frac{q^2\mu^2}{6k_BT}\frac{1}{r^4}$; polarizability (Bohr atom): $\alpha = 4\pi\varepsilon_0 a_0^3$

IE btw. charge and polarizable molecule: $U = -\frac{\alpha q^2}{(4\pi\varepsilon_0\varepsilon)^2}\frac{1}{r^4}$; Keesom energy: $U = -\frac{1}{(4\pi\varepsilon_0\varepsilon)^2}\frac{\mu_1^2\mu_2^2}{3k_BT}\frac{1}{r^6}$;

Debye energy: $U = -\frac{1}{(4\pi\varepsilon_0\varepsilon)^2}\frac{\mu^2\alpha}{r^6}$; London energy: $U = -\frac{3}{2(4\pi\varepsilon_0\varepsilon)^2}\frac{\alpha_1\alpha_2}{n^4}\frac{1}{r^6}\frac{I_1I_2}{I_1+I_2}$;

Lennard-Jones: $U(r) = 4\varepsilon\left[(r/r_0)^{-12} - (r/r_0)^{-6}\right]$; Gauss distribution: $P(x) = 1/\left(\sqrt{2\pi}\sigma\right)\cdot \mathrm{e}^{-(x-x_0)^2/2\sigma^2}$

statistical mean: $\langle f(x)\rangle = \sum_i f(x_i)P(x_i) = \int f(x)P(x)\,\mathrm{d}x$; variance: $\mathrm{var}(x) = \left\langle (x-\langle x\rangle)^2\right\rangle = \langle x^2\rangle - \langle x\rangle^2$

ideal gas: $\langle E_{kin}\rangle = 3/2\,k_BT$; speed distribution: $P(v)\mathrm{d}v = \frac{4\pi v^2}{\left(\sqrt{2\pi\,k_BT/m}\right)^3}\mathrm{e}^{-\frac{mv^2}{2k_BT}}\mathrm{d}v$; $\langle v\rangle = \sqrt{8k_BT/\pi m}$

generally: $\langle E\rangle = 1/2\,k_BT$ per degree of freedom; statistical weight/Boltzmann distribution: $w_i \propto \mathrm{e}^{-E_i/k_BT}$

partition function: $Q = \sum_i \mathrm{e}^{-E_i/k_BT} = \sum_E g_E \mathrm{e}^{-E/k_BT}$

thermodynamic energy/free energy: $\langle E\rangle = k_BT^2\frac{\partial}{\partial T}\ln Q$; $\langle F\rangle = -k_BT\ln Q$; entropy: $\frac{\partial F}{\partial T} = -S$

end-to-end distances – freely jointed chain: $R_e = \sqrt{\langle \vec{R}_N^2\rangle} = \sqrt{N}l$; freely rotating chain: $l_{eff} = l\cdot\sqrt{\frac{1+\cos\vartheta}{1-\cos\vartheta}}$

persistence length: $l_p = \lim_{N\to\infty}\left\langle \frac{\vec{r}_1}{l}\sum_n \vec{r}_n\right\rangle$; Kuhn length: $a = 2l_p$; diffusion of spherical particle: $D = \frac{k_BT}{6\pi\eta R}$

end-to-end distance distribution: $P_N(R)\mathrm{d}R = \frac{4\pi R^2}{\left(\sqrt{2\pi N}l\right)^3}\mathrm{e}^{-\frac{R^2}{2Nl^2}}\mathrm{d}R$; radius of gyration: $R_G^2 = \frac{1}{6}R^2_{\text{end-to-end}}$

statistical entropy: $S = k_B\ln W$; probability of knot formation: $P_N(0)\mathrm{d}V = 1/\left(\sqrt{2\pi N}l\right)^3$

free energy of deformed polymer coil: $F(x) = \frac{k_BT\cdot x^2}{2Nl^2}$; resulting reactive force: $f(x) = \frac{k_BT\cdot x}{Nl^2}$

entropy associated with statistical weight: $S = k_B\ln W$; binomial distribution: $P(N) = \frac{1}{2^N}\frac{N!}{(N/2)!\times(N/2)!}$

Stirling's formula: $\ln N! \approx N\ln N - N + 0.5\ln(2\pi N)$; displacement in n-dim. random walk: $\langle \Delta x^2\rangle = 2nDt$

Stokes formula: $\zeta = 6\pi\eta R$; Einstein relation: $D \cdot \zeta = k_B T$; Stokes-Einstein relation: $D = k_B T / (6\pi\eta R)$

continuity equation: $\frac{\partial c(\vec{r},t)}{\partial t} = -\nabla \vec{j}(\vec{r},t)$; Fick's laws: (1) $\vec{j}(\vec{r},t) = -D\nabla c(\vec{r},t)$; (2) $\frac{\partial c(\vec{r},t)}{\partial t} = D\Delta c(\vec{r},t)$

membrane permeation: $j_s = -P_s \Delta c = -D\frac{\Delta c}{L}$; ion mobility: $v_{el} = \frac{zeE}{\zeta}$ (ζ : friction coefficient)

Einstein: $D = \frac{k_B T}{\zeta}$; Nernst-Planck formula: $j = D\left(\frac{q}{k_B T} Ec - \frac{\mathrm{d}c}{\mathrm{d}x}\right)$; Nernst equation: $U = U_0 - \frac{k_B T}{ze}\ln(c/c_0)$

information content of sequence: $I = \frac{1}{\ln 2} N \ln M = \frac{1}{\ln 2}\ln\Omega = \frac{1}{\ln 2}\left(\ln N! - \sum_{j=1}^{M}\ln(N_j!)\right)$

Shannon's formula: $I/N = -\frac{1}{\ln 2}\sum_j p_j \ln p_j$

statistical definition of entropy: $S = k_B \ln\Omega$; Ideal Gas: $S = k_B \ln\left[(2mE)^{3N/2} \cdot V^N\right] + const$ (Sakur-Tetrode)

Free Energy: $F = E - TS$; Free Enthalpy: $G = F + pV$; $T = \left(\frac{\mathrm{d}S}{\mathrm{d}E}\right)^{-1}$; $p = T\frac{\mathrm{d}S}{\mathrm{d}V}$; $f_a = -\frac{\mathrm{d}F_a}{\mathrm{d}L}$

level occupancy in Two-Level System ($1 \rightleftharpoons 2$; state 1 is low-energy level): $\frac{N_{1,eq}}{N_{2,eq}} = K = \frac{k_{2\to 1}}{k_{1\to 2}} = \mathrm{e}^{\Delta F/k_B T}$

probability ratio in TLS: $p_1/p_2 = \exp(\Delta E/(k_B T))$; probability to find system in state 1: $p_1 = \frac{1}{1+\mathrm{e}^{-\Delta E/k_B T}}$

probability ratio for two generalized states with Free Energies F_1 and F_2: $p_1/p_2 = \exp((F_2 - F_1)/(k_B T))$

Van't Hoff equation: $p_{eq} = c \cdot k_B T$; flux through semipermeable membrane: $j_v = -L_p(\Delta p - \Delta c \cdot k_B T)$

Poisson eqn.: $\Delta\Psi = -\rho_e(\vec{r})/(\varepsilon_0\varepsilon)$; Poisson-Boltzmann eqn.: $\Delta\Psi = -\frac{c_0 e}{\varepsilon_0\varepsilon}\left[\exp(-e\Psi/k_B T) - \exp(e\Psi/k_B T)\right]$

ionic strength of an electrolyte: $I = \sum_i c_0^{(i)} z_i^2$; Debye length: $\lambda_D = \left(\frac{\varepsilon_0\varepsilon k_B T}{e^2 \cdot 2c_0}\right)^{1/2} = (8\pi l_B \cdot I)^{-1/2}$

potential decays as $\Psi(x) = \frac{\sigma\lambda_D}{\varepsilon\varepsilon_0}\mathrm{e}^{-x/\lambda_D}$ with distance x; specific capacity of diffuse layer: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Psi_0} = \frac{\varepsilon_0\varepsilon}{\lambda_D}$

$\lambda_D \sim 3\text{Å}/\sqrt{c_0}$ for 1:1 electrolyte (c_0 in mol/L); Grahame eqn.: $\sigma = \left[8c_0\varepsilon_0\varepsilon k_B T\right]^{1/2} \cdot \sinh\left(\frac{e\Psi_0}{k_B T}\right)$

$S = -\left.\frac{\partial G}{\partial T}\right|_p$; $V = \left.\frac{\partial G}{\partial p}\right|_T$; $\mu_\alpha = -T\left.\frac{\partial S}{\partial N_\alpha}\right|_{E,N_{\beta\neq\alpha}}$; $\mu(p) = \mu(p_0) + \int_{p_0}^{p} V(p)\mathrm{d}p = G(p_0) + k_B T \ln p/p_0$ (Ideal Gas)

$$\mathrm{d}G = \left.\frac{\partial G}{\partial T}\right|_{p,N_\alpha} \mathrm{d}T + \left.\frac{\partial G}{\partial p}\right|_{T,N_\alpha} \mathrm{d}p + \sum_\alpha \left.\frac{\partial G}{\partial N_\alpha}\right|_{p,N_{\beta\neq\alpha}} \mathrm{d}N_\alpha$$; Arrhenius equation: $$\frac{\partial \ln K}{\partial (1/T)} = -\frac{\Delta H^0_{react}}{R}$$

$$G^{react}_{tot} = \sum_i n_i G^0_i - T \cdot R \sum_i n_i \ln\left(n_i \Big/ \sum_i n_i \right)$$

ligand binding: $K_d = \frac{[L]\cdot[R]}{[LR]}$; $p_b = \frac{[L]/K_d}{1+[L]/K_d} = \frac{(c/c_0)\mathrm{e}^{-\Delta E/k_B T}}{1+(c/c_0)\mathrm{e}^{-\Delta E/k_B T}}$

multiple ligand binding sites, n: $K^n_d = \frac{[L]^n\cdot[R]}{[LR]}$; $p_b = \frac{([L]/K_d)^n}{1+([L]/K_d)^n}$