Homework 9 (due November 12, 2018)

(1) Ion diffusion across membranes (15 pts)

A lipid membrane separates two aqueous compartments with different ion concentrations. Assume that the membrane is permeable to a single ion species with valence $z$. When a constant electric field $E$ is applied between the two surfaces of the membrane, the net ion flux $j$ is the sum of the diffusive flux $j_{diff}$ of ions along the concentration gradient with diffusion constant $D$ and the electrophoretic flux $j_{el}$ of ions drifting in the electric field $E$ with a velocity, $v_d = zE/\zeta$.

a) Using Fick’s law, $j_{diff} = -D(dc/dx)$, and working out $j_{el}$ from the definition of the flux and Einstein’s formula, $D = k_BT/\zeta$, derive the net ion flux, i.e., the Nernst-Planck formula.

b) Under equilibrium (zero net ion flux across the membrane), the diffusive flux balances the electrophoretic flux. The voltage required to maintain a difference between $c_{in}$ and $c_{out}$ in equilibrium is

$$U = -\frac{k_BT}{Ze}\ln\left(\frac{c_{in}}{c_{out}}\right).$$

Show that the ion concentration ratio across the membrane follows a Boltzmann distribution in equilibrium and discuss the physical meaning of the exponent.

(2) Relaxation time and rate constants (15 pts)

In an isomerization, $A \rightleftharpoons B$, the conformers have an energy difference, $\Delta E = E_A - E_B$.

The forward ($A \rightarrow B$) and backward ($A \leftarrow B$) rates are $k_+$ and $k_-$.

(a) Derive the time dependence, $c_B(t)$, in terms of $k_+$ and $k_-$ by which the concentration of B relaxes to its equilibrium value $c_{B,eq}$ from an initial value $c_{B,0}$.

(b) Calculate $k_+$ and $k_-$ at room temperature for a reaction with $\Delta E = 2.2$ kJ/mol and a relaxation time, $\tau = 30$ s.