Lab 2:

LENSES, LENS SYSTEMS, and SIMPLE OPTICAL INSTRUMENTS

(2 Lab Periods)

Objectives Measure focal lengths and magnifications of thin lenses and thin lens combinations. Construct a simple optical instrument from thin lenses and measure some of its properties. Investigate the aberration of a thick lens.

References Hecht, sections 5.2, 5.7, 6.3

(A) Basic Equations

1. Gaussian Lens Formula

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \tag{2.1}$$

where f is the focal length of a thin lens. $f > 0$ for a convex (converging) lens and $f < 0$ for a concave (diverging) lens. s_o and s_i are the distances between the central plane of the lens and an object and its image, respectively.

2. Newtonian Lens Formula

$$x_o x_i = f^2 \tag{2.2}$$

where x_o and x_i are the distances between an object and its image, respectively, to the appropriate focal points.

3. Transverse Magnification

$$m_T = -s_o/s_i = -x_i/f = -f/x_o \tag{2.3}$$

(B) Equipment

Optical bench, an illuminated target, several lenses, and an image board. There are only a few thick lenses in the laboratory, so they will have to be shared.

(C) LAB SAFETY: Do not look into the Laser beam. Eye injury and blindness may result.

(D) Pre-Lab Exercises

1. Consider a positive (converging) thin lens of focal length f.

Starting with the Gaussian formula for a thin lens, derive a general expression for the image distance s_i in terms of s_0 and f and use it to write a general expression for the object-image distance $L = s_0 + s_i$ in terms of s_0 and f.

Show that there is an extremum in L for variations in s_0. Is it a maximum or a minimum? Determine the value of L at this extremum, the value of s_0 at which it occurs and the corresponding value of s_i.

A pair of object and image points is sometimes called a pair of conjugate points; your results should show that the separation L between a pair of conjugate points conforms with $L \geq 4{\cdot}f$.

Show that if you have a positive lens of focal length f, and you place it between an object and an image board that are separated by a fixed distance $L > 4f$, there are two locations of the lens that will give a focused image. Determine the two values of s_0 that will give a focused image for a fixed object–image distance $L > 4{\cdot}f$.

Show that for $L > 4{\cdot}f$ these two possible locations of the lens are separated by a distance d given by the expression

$$d^2 = L^2 - 4{\cdot}Lf$$

What happens when $L = 4{\cdot}f$?

Solve for f in terms of L and d. Determine expressions for the transverse magnification for each location of the lens. How are these two expressions related?

This result can be used to measure the focal length of a lens; it is particularly useful because it does not require that any measurements be made to a lens surface.

2. Consider an object located at the front focal point of a thin positive lens, with a plane mirror located on the opposite side of the lens at an arbitrary distance. Draw enough rays to show that the final image has the following properties:

(a) it is formed in the plane of the object

(b) it is the same size as the object

(c) it is inverted.

Explain your placement of the rays!

Note: (b) and (c) together tell us that the transverse magnification is –1.

3. Consider two thin lenses, of focal lengths $f_1 > 0$ and $f_2 > 0$, respectively, separated by a distance $d >> (f_1 + f_2)$. Let s_{01} be the object distance (measured from the object to the first lens). Determine an expression for the final image distance s_{i2} (measured with respect to the second lens). Express your answer in terms of f_1, f_2, d, and s_{01}. Hint: the solution to this problem requires two successive applications of the Gaussian lens formula, in which the image formed by the first lens serves as the object for the second lens. The object distance for the second lens will depend on the image distance for the first lens and the separation between the two lenses. Be careful of the sign.

4. Consider again Problem 3, but for the case when $d < (f_1 + f_2)$. Draw a ray diagram clearly indicating the rays from the object through the first lens to an image. Draw another diagram indicating the rays from that first image, through the second lens, to another image. Identify all images and objects as real or virtual. Derive an expression for the final image distance s_{i2}; is this expression different or the same as in Problem 3?

5. Use your result for Problem 4 for the following: If the object is at infinity ($s_{01} = \infty$), the corresponding final image distance (measured from the second lens) is called the back focal length. If the final image is at infinity ($s_{i2} = \infty$), the corresponding object distance (measured from the first lens) is called the front focal length. Determine expressions for the front and back focal lengths, expressing your answers in terms of f_1, f_2, and d. Examine these expressions when $d = 0$, and show that for $d = 0$ the front and back focal lengths are identical. Show also that when $d = 0$ the two-lens combination can be treated as a single thin lens. Find its effective focal length in terms of f_1 and f_2.

(E) Procedures and Assignments

a. Measurements on a single thin lens

(1) Measure s_i for a positive lens using several various values of s_o and calculate the focal length f using Eq. (2.1). In each case measure m_T and compare the result with the value calculated from Eq. (2.3). For each s_o, you should make several measurements by repeatedly defocusing the image, refocusing, and then measuring the resulting s_i. For each s_o, determine the mean and uncertainty of your set of measurements of s_i. You might find it interesting to plot your resulting value of f as a function of s_o.

(2) For the same lens, measure the f by placing the lens between the source and a planar mirror. Adjust the lens position until the source is focused back on itself, and determine f as discussed in Pre-lab exercise 2. Again, you should repeat this measurement several times.

(3) Measure the focal length of this lens using the method of conjugate points, as discussed in Pre-lab exercise 1.

(4) Compare these three different methods for measuring the focal length. Estimate the uncertainty in your measurements and calculate the resulting uncertainty in each determination of f.

(5) For a second positive lens with a significantly different focal length, choose one of these methods and measure the f.

b. Thin-lens combinations

For a combination of two thin lenses, find the front and back focal lengths. Set up the two positive lenses whose focal lengths you have previously determined at some separation d between their central planes. Measure the image distance for a particular choice of object distances, and compare with the expressions derived in Pre-lab exercises 3 and 4. For d as close as possible to 0, measure f of the combination using any one of the methods from part A and compare your result again with the appropriate theoretical expression.

c. Focal length of negative lenses

Find a negative lens (what simple test shows that a lens is negative?). Set up an image board at the far end of your optical bench (away from the illumination source). *The image board stays fixed in this measurement*. Find a positive lens with a focal length f of about 10 cm. This focal length need not be measured precisely. Place an illuminated object at a distance $d > 4{\cdot}f$ from the image board. Place your positive lens between the object and the image board and obtain a focused image with a magnification > 1 just as you did in part **a. (3)** for the method of conjugate points. Adjust the position of the positive lens for best focus and lock it down; it ***must stay fixed*** throughout the rest of the procedure. Now place your negative lens between the positive lens and the object, but somewhat closer to the positive lens than to the object. The image will no longer be in focus on the image board. Measure the distance between the negative lens and the object and call this distance y_1. Now move the object until its image is again focused on the board. Measure the new distance between the negative lens and the object; call this distance y_2. The focal length f of the negative lens can be found from the expression

$$\frac{1}{f} = \frac{1}{y_2} - \frac{1}{y_1}$$

Derive this expression. Note that only two length measurements are necessary and that you do not need to know the exact focal length of the positive lens.

d. Simple optical instruments

Construct either an astronomical telescope or a Galilean telescope. Make a visual estimate of the magnification and compare this estimate with the value calculated from the focal lengths of the component lenses.

Using this telescope as a simple Laser beam expander, measure the beam expansion, and compare your result with the value calculated from the focal lengths of the component lenses. You will use this technique in a later laboratory.

e. Spherical aberration of a thick lens

Set up an illuminated target and an image board and find the best image you can.

Technical hints: Measure object and image distances with respect to the vertex of the convex surface. You may find it easiest to use a very distant source so that the image is formed at the back focal point.

Construct an annular mask with an aperture opening of about 3 cm diameter and use it to mask off all but the central part of the lens. Measure the object and image distances and find f. Construct a disk-shaped mask that blocks off the central part of the lens, leaving only a clear outer annular ring of width about 3 cm. Measure the object and image distances again and find f. Repeat these measurements with the lens reversed. For which orientation does the lens have the least spherical aberration?

See if you can observe the aberration known as *coma*; try it for both lens orientations. Comment on your observations. Do you observe any other aberrations in these measurements?