33-353 — Intermediate Optics Thursday, Sept-23, 2010

HW solution, week 5 due: Wednesday, Sept-29, 2010 – before class

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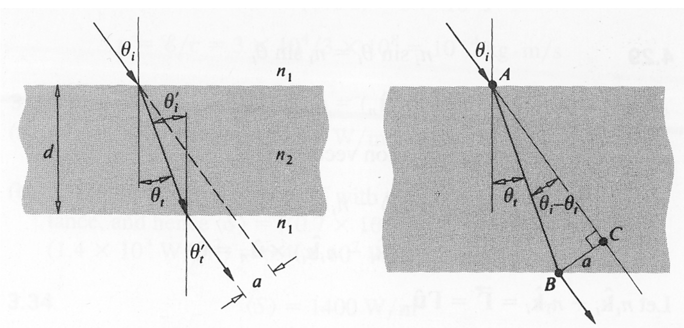
1. Planar glass plate **(4 pts)**

Using Snell's law, show analytically that a light beam impinging on a transparent planar glass plate exits that plate parallel to the incident direction. What is the parallel displacement of the beam behind the glass plate as a function of glass thickness d, glass index n and the angles θ_i, θ_t?

Discuss how the light path taken through the glass plate for an incident angle, $\theta_i > 0$, (which is longer than it would have been in air) complies with Fermat's principle.

Relating to the following geometry,



[A,B]: distance between A and B

we have

$$n_1 \sin\theta_i = n_2 \sin\theta_t \quad — \quad \theta_t = \theta_i'$$

$$n_2 \sin\theta_i' = n_1 \sin\theta_t', \text{ so } n_1 \sin\theta_i = n_1 \sin\theta_t' \text{ and } \theta_t = \theta_i'$$

$$\cos\theta_t = d/[A,B]; \; \sin(\theta_i - \theta_t) = a/[A,B] = a/d \cdot \cos\theta_t$$

$$a = d \cdot \sin(\theta_i - \theta_t)/\cos\theta_t$$

The ray crosses the glass plate at a sharper angle than it would in air, increasing the path lengths in air more than reducing the path length within the glass. Because the geometric path s_{gl} in glass is multiplied by $n_2 > n_1$, this actually *reduces* the optical path length and the longer path complies with Fermat's principle.

2. Dispersion in glass plate **(3 pts)**

Hecht, problem 4.20:

A narrow white beam strikes a slab of glass (d = 10 cm) at an angle $\theta_i = 60°$. The indices of refraction of the glass are $n_{\text{red}} = 1.505$ and $n_{\text{vio}} = 1.545$ for red and violet, respectively. Determine the diameter of the beam emerging out of the far glass interface.

4.20 (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so $\sin(60.0°) = n_t \sin G_\epsilon$. Diameter of emerging beam (D) is related to the difference in horizontal displacement of red and violet light (h) by $D\cos(60.0°) = h$ (See Problem 4.19). Red: $\sin\theta_{\text{red}} = \sin(60.0°)/n_{\text{red}} = (\sqrt{3}/2)/(1.505), \theta_{\text{red}} = 35.1°$; $\tan\theta_{\text{red}} = h_{\text{red}}/10.0$ cm so $h_{\text{red}} = (10.0 \text{ cm})\tan(35.1°) = 7.04$ cm. Violet: $\sin\theta_{\text{violet}} = \sin(60.0°)/n_{\text{violet}} = (\sqrt{3}/2)/(1.545)$; $\theta_{\text{violet}} = 34.1°$; $h_{\text{violet}} = (10.0 \text{ cm})\tan(34.1°) = 6.77$ cm. $D = h/\cos(60.0°) = (h_{\text{red}} - h_{\text{violet}})/\cos(60.0°) = (7.04 - 6.77)/(0.5) = 0.54$ cm.

3. Critical angle **(3 pts)**

Hecht, problem 4.58:

A glass block ($n_{\text{gl}} = 1.55$) is covered with a water layer ($n_{\text{gl}} = 1.33$). What is the critical angle at the glass/water interface?

$\sin\theta_c = n_t/n_i$; $\theta_c = 59°$.

4. Transmission and reflection amplitudes **(3 pts)**

From reviewing the electrostatic boundary condition at an interface that separates media of n_i and n_t ($> n_i$), show that the transmission and reflection amplitudes for a light beam with polarization of $\vec{E}$ out of the plane of incidence are related as $t_\perp - 1 = r_\perp$.

The tangential field across the interface is preserved to comply with $\oint_C \vec{E} d\vec{s} = 0$.

$\rightarrow E_{0,i}^{\perp} + E_{0,r}^{\perp} = E_{0,t}^{\perp}$, or $(E_{0,t}^{\perp}/E_{0,i}^{\perp}) - (E_{0,r}^{\perp}/E_{0,i}^{\perp}) = t_\perp - r_\perp = 1$.

5. Transmission and reflection amplitudes II (3 pts)

Show analytically by starting from the Fresnel equations for $t_\perp$ and $r_\perp$ that $t_\perp - 1 = r_\perp$ is true for all θ.

From

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \text{ and } r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

it follows that

$$t_\perp - 1 = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} - \frac{n_i \cos\theta_i + n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = r_\perp$$

6. Polarization angle (4 pts)

Show that the polarization angles for internal and external reflection at a given interface complement as $\theta_p + \theta_p' = 90°$.

$\theta_i + \theta_t = 90°$ for $\theta_i = \theta_p \rightarrow \sin\theta_t = \cos\theta_p$,

as θ_t and θ_p are complementing angles in a rectangular triangle. Therefore,

$n_i \sin\theta_p = n_t \sin\theta_t = n_t \cos\theta_p$ and $\tan\theta_p = n_t/n_i$.

Let this ratio be a at an externally reflecting interface. At an internally reflecting interface, $n_t/n_i = 1/a$, and correspondingly,

$\tan\theta_p = a$ while $\tan\theta_p' = 1/a$

Therefore,

$\tan\theta_p = 1/\tan\theta_p'$ and $\frac{\sin\theta_p}{\cos\theta_p} = \frac{\cos\theta_p'}{\sin\theta_p'}$

or

$\sin\theta_p \cdot \sin\theta_p' - \cos\theta_p \cdot \cos\theta_p' = \cos(\theta_p + \theta_p') = 0 \rightarrow \theta_p + \theta_p' = 90°$.