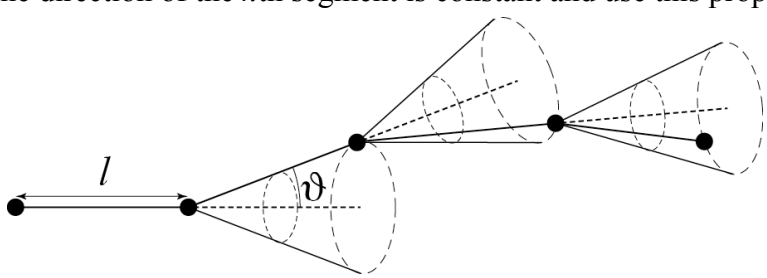
33-767, SS 2019, HW 3 (Feb-20, 2019)
due: Feb-27, 2019 (at start of class)

Polymer conformations **(30 pts)**

(a) The Kuhn length b of a polymer may not be confused with the physical bond length between monomers – most notably because the chemical bond angles are much more restricted than the idealized connections between the Kuhn segments in the *Freely Jointed Chain*. Derive the Kuhn length of a *Freely Rotating Chain* used in lecture,

$$b = l\sqrt{\frac{1+\cos\vartheta}{1-\cos\vartheta}} \tag{1}$$

where l is the physical bond length, ϑ is the polar angle from the forward directions and the azimuthal angles on the cone with opening angle ϑ are statistically independent (see sketch). For your derivation, observe that the projection of the direction of the (n + 1)st segment on the direction of the nth segment is constant and use this property to construct a series.



(b) Derive the radius of gyration R_G, defined as the mean-square distance of the hinge points from the chain's center-of-mass, $\vec{R}_{CM}$

$$R_G^2 = \frac{1}{N}\sum_{i=1}^{N}\left\langle(\vec{r}_i - \vec{R}_{CM})^2\right\rangle \tag{2}$$

of a polymer coil as a function of the Kuhn length b and show that $R_G^2 = 1/6 \cdot R_e^2$. Start by rewriting R_G^2 in Eqn. (2) as a function of $\left\langle(\vec{r}_i - \vec{r}_j)^2\right\rangle$, then plug in the definition of R_e^2.