Polymer conformations

(a) The Kuhn length \( b \) of a polymer may not be confused with the physical bond length between monomers – most notably because the chemical bond angles are much more restricted than the idealized connections between the Kuhn segments in the Freely Jointed Chain. Derive the Kuhn length of a Freely Rotating Chain used in lecture,

\[
b = l \sqrt{\frac{1 + \cos \vartheta}{1 - \cos \vartheta}}
\]

where \( l \) is the physical bond length, \( \vartheta \) is the polar angle from the forward directions and the azimuthal angles on the cone with opening angle \( \vartheta \) are statistically independent (see sketch). For your derivation, observe that the projection of the direction of the \((n + 1)\)st segment on the direction of the \(n\)th segment is constant and use this property to construct a series.

(b) Derive the radius of gyration \( R_G \), defined as the mean-square distance of the hinge points from the chain’s center-of-mass, \( \bar{R}_{CM} \)

\[
R_G^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \bar{r}_i - \bar{R}_{CM} \right)^2
\]

of a polymer coil as a function of the Kuhn length \( b \) and show that \( R_G^2 = \frac{1}{6} \cdot R_c^2 \). Start by rewriting \( R_G^2 \) in Eqn. (2) as a function of \( \left( \bar{r}_i - \bar{r}_j \right)^2 \), then plug in the definition of \( R_c^2 \).