Linear and Logarithmic Interpolation

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One is occasionally confronted with the task of extracting quantitative information out of graphs. More specifically, one has found a point in a graph one is interested in, and now wants to know which precise value on the horizontal and/or vertical axis it corresponds to. These notes explain how this is done for the two cases of linear and logarithmic scale.

Linear scale

The task of interpolating between tic-marks on the scale of a graph is quite straightforward if the axis in question has a linear scale, because then one just has to do a linear interpolation. Have a look at Fig. 1. Between two tic-marks x_1 and x_2 we want to know the precise x-value corresponding to the marked cross. We can measure the length of the intervals a and b (simply by using a ruler). If the horizontal axis is linear, we evidently must have

$$\frac{x_2 - x}{x - x_1} = \frac{b}{a} = \frac{1}{f} - 1 , \qquad (1)$$

where we also introduced the fractional division

$$f := \frac{a}{a+b} . \tag{2}$$

From this we get the simple linear interpolation formula

$$x = fx_2 + (1 - f)x_1$$
 (lin). (3)

Logarithmic scale

The situation is a little less straightforward if the axis is not on a linear scale but rather on a logarithmic scale. But in fact, the problem can be reduced to the previous one. A logarithmic scale simply means that values are not plotted at their "appropriate" location, but at a location proportional to the logarithm of their value.

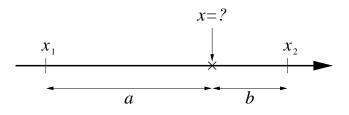


FIG. 1: Typical graphical read-off problem: What is the value of the abscissa at the point of the cross?

This, however, means that the logarithm of these values is on the appropriate linear scale! Hence, if the axis in Fig. 1 were in fact logarithmic, Eqn. 1 would have to be replaced by

$$\frac{\log x_2 - \log x}{\log x - \log x_1} = \frac{1}{f} - 1.$$
 (4)

Solving this for x, we find the logarithmic interpolation formula

$$x = x_2^f x_1^{1-f} \qquad (\log) . \tag{5}$$

If for instance $f = \frac{1}{2}$, *i.e.*, the cross is exactly between the two tic-marks, linear interpolation would simply yield $x = \frac{1}{2}(x_1 + x_2)$, the expected arithmetic average. In the logarithmic case we'd find $x = \sqrt{x_1x_2}$, the geometric average. This makes sense, because on a logarithmic scale the half-point between x_1 and x_2 has the property that if we get it by *multiplying* x_1 by some factor y, multiplying once more by the same factor gives us x_2 . Since in this case we have $y = x/x_1 = \sqrt{x_1x_2}/x_1 = \sqrt{x_2/x_1}$, we have $x_1y^2 = x_1(x_2/x_1) = x_2$ — as expected.