Millikan Lecture 1994: Understanding and teaching important scientific thought processes

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I. INTRODUCTION

Physics is an intellectually demanding discipline and many students have difficulties learning to deal with it. Further, our instruction is often far less effective than we realize. Indeed, recent investigations have revealed that many students, even when getting good grades, emerge from their basic physics courses with significant scientific misconceptions, with prescientific notions, with poor problem-solving skills, and with an inability to apply what they ostensibly learned.[1–4] In short, students' acquired physics knowledge is often largely nominal rather than functional.

This situation leads one to ask: Why is this so, and what might be done about it? More specifically, it has led me to address the following two basic questions: (a) Can one understand better the underlying thought processes required to deal with a science like physics? (b) How can such an understanding be used to design more effective instruction?

These are the questions which have been the focus of my work during the last several years and which I want to discuss in the following pages.

A. Formulation of the instructional problem

Instruction is a problem that requires one to transform a system S (called the student) from an initial state S_i to a desired final state S_f where S can do things which S could not do initially. This transformation process can schematically be expressed in the form

$$S_i \rightarrow S_f . \qquad (1)$$

Although this may seem like a cold-blooded physicist's way of formulating the instructional problem, it is certainly *not* dehumanizing. On the contrary! Rather than dealing primarily with physics subject matter or curriculum, it focuses central attention on the human student S trying to deal with physics.

More important, the formulation (1) of the instructional problem makes apparent that a systematic approach to instruction needs to address the following issues.

(1) Analysis of desired performance (S_f). (a) One needs to specify clearly the desired final student abilities and observable performance. (b) On a more theoretical level, one needs to understand the underlying cognitive mechanisms (knowledge and thought processes) required to achieve these abilities.

(2) Analysis of the initial student (S_i). (a) One needs to describe adequately the characteristics and performance of students coming to instruction. (b) On a more theoretical level, one needs to identify what they know and how they think.

(3) Useful comparisons. A good analysis of desired performance (i.e., of S_f) allows several useful comparisons: (a) A comparison with actual expert performers. (This can suggest improved models of good performance, can help reveal "tacit knowledge" of which experts are unaware, and may sometimes disclose that experts are far from perfect.) (b) A comparison with novice students. (This can reveal anticipated learning difficulties and identify more precisely what needs to be taught.) (c) A comparison with prevailing methods of instruction. (This can reveal the deficiencies of such instruction, e.g., important skills that are never explicitly taught.)

(4) Design of instruction (the transformation process $\rightarrow$). (a) One needs to design an effective learning process whereby the student can acquire the knowledge and thinking skills required to achieve the desired final performance. (b) Finally, one needs to implement this design in practical settings.

The preceding approach to instruction is centrally based on an adequate understanding of the thought processes leading to the desired performance. The basic premise is that one cannot teach physics effectively without an adequate understanding of the thought processes needed in this field (no more than one can teach someone how to play good chess without an adequate understanding of the thought processes needed to play that game).

B. Outline of important issues

Let me then follow the preceding instructional approach to identify some of the specific issues important to the teaching of physics.

Instructional goals. The choice of instructional goals is a matter of judgment and depends also on the particular student audience. However, my central goal has been to help students acquire a modest amount of basic knowledge which they can *flexibly use*. There are at least two reasons why such flexible usability seems centrally important. (a) The goal of science is not the accumulation of various facts, but the ability to use a small amount of basic knowledge to predict or explain many diverse phenomena. (b) Students will have to function in a complex and rapidly changing technological world where they will profit little from knowledge that is rotely memorized or poorly understood. Any acquired physics knowledge will be useful to them only if it allows them to cope flexibly with any future courses or tasks encountered by them.

Abilities facilitating flexible usability. What kinds of thought processes are required to ensure that scientific knowledge can be flexibly used? My work suggests that the cognitive abilities summarized in Fig. 1 are of particular importance. These include the basic abilities required to interpret properly scientific concepts and principles, to describe knowledge effectively, and to organize it effectively. These are necessary prerequisites for more general problem-solving abilities, including the abilities to analyze problems, to construct their solutions, and to check these solutions.

Overview of this paper. In the following pages, I shall examine more closely each of the preceding abilities, pointing out why each of these is important and more complex than one might naively believe. In each case, consideration of the instructional problem $S_i \rightarrow S_f$ will lead me to do the following: (a) Indicate some common inadequacies of students' initial abilities. (b) Analyze the thought processes re-

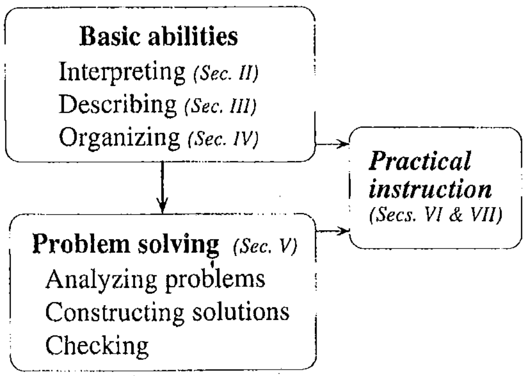

Fig. 1. Central cognitive issues important for scientific work.

quired to achieve the desired abilities to be finally acquired by students. (c) Examine the instructional implications for designing an effective learning process.

As outlined in Fig. 1, Sec. II will examine the interpretation of scientific concepts and principles, Sec. III will deal with effective methods of description needed for scientific work, and Sec. IV will describe useful forms of knowledge organization. Section V will then discuss problem solving (i.e., methods for analyzing problems, for constructing their solutions, and for checking these). The examination of the preceding issues prepares one to consider how all of them may be jointly incorporated in the design of practical instruction. Correspondingly, Sec. VI will describe work done to implement such practical instruction and Sec. VII will mention some of the difficulties faced by attempts at effective implementation. Finally, Sec. VIII will summarize the discussion with some brief concluding remarks.

II. INTERPRETATION OF SCIENTIFIC CONCEPTS OR PRINCIPLES

The basic building blocks of scientific knowledge are special concepts and principles. These are ordinarily quite abstract in order to provide the desired scientific generality (i.e., to ensure that very few such concepts or principles are sufficient to predict and explain many diverse phenomena).

Abstractness as such does not present undue difficulties to people. Many concepts commonly used in everyday life are also quite abstract (e.g., love, truth, beauty, justice, etc.). The difficulty is that one must be able to interpret a scientific concept unambiguously in any particular instance, a requirement *not* imposed on everyday concepts. For example, in daily life there may be many disagreements about whether something is a case of "true love" or whether a particular action is "just." But scientific work does not tolerate similar ambiguities about the proper identification of a scientific concept.

In the sense in which I use it, "interpreting a concept" means identifying or generating the concept in any particular instance. For example, suppose that somebody tells me that a triangle is a three-sided polygon. However, the person cannot recognize a triangle among some other geometric figures, nor construct a triangle with three sticks. Then I would say that the person has some nominal knowledge about a "triangle," but does *not* know how to interpret this concept.

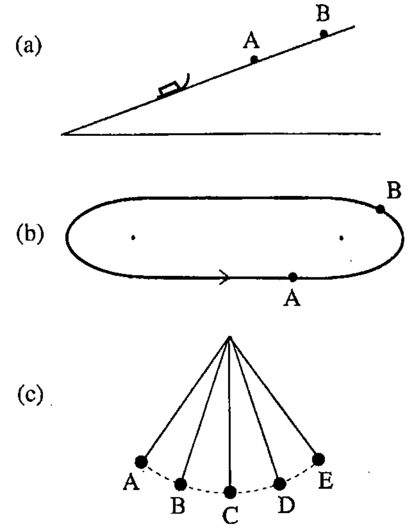

Fig. 2. Situations used for testing the interpretation of acceleration. (a) Sled sliding along a hill. (b) Car traveling around a horizontal racetrack. (c) Oscillating pendulum bob.

The ability to interpret a scientific concept is clearly an essential prerequisite for using the concept to make complex inferences or to do any scientific work with it. Hence one may ask the following question: How well can students interpret the physics concepts which they have ostensibly learned?

A. Observed interpretation deficiencies

To examine this question in some detail, let me focus specific attention on the concept "acceleration." This is a very basic concept, of fundamental importance in Newtonian mechanics and commonly taught at the beginning of any introductory physics course. The concept is specified by its familiar definition that "acceleration is the rate of change of velocity with time," a statement which can also be summarized by the equation

$$\mathbf{a}=\frac{d\mathbf{v}}{dt}. \tag{2}$$

Someone able to interpret the concept acceleration should be able to identify the acceleration of a particle in various specific cases, such as those illustrated in Fig. 2. For instance, Fig. 2(a) shows a sled which moves up along a hill, passes the point A with decreasing speed, comes momentarily to rest at the point B, and then slides down again. Figure 2(b) shows a car passing the points A and B while moving with constant speed around a horizontal racetrack. Figure 2(c) shows an oscillating pendulum bob which is momentarily at rest at the extreme point A of its circular arc, passes the point B with increasing speed, reaches its maximum speed at its lowest point C where the string is vertical, continues past the point D, and is again momentarily at rest at the point E.

In a study carried out by me and some co-workers, we presented 15 such specific situations to various persons and observed their responses in detail. The person was asked to

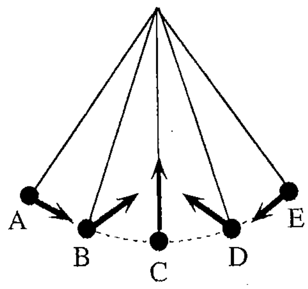

Fig. 3. Accelerations of a pendulum bob.

specify whether the acceleration is zero at the indicated points, or to specify its direction if it is nonzero.

The observed individuals were either students or professors at the University of California at Berkeley. The students, enrolled in an introductory college physics course for prospective scientists or engineers, had been working with acceleration for at least two months. The professors had all taught an introductory physics course in the recent past.

The main results of this study (discussed at length in a paper by Reif and Allen)[5] were the following: The students could answer correctly at most only 35% of such questions. The professors were very much better, but not perfect. (For example, one of them answered correctly only 10 of the 15 questions.)

Recent observations at the University of Washington[6] support these conclusions by more extensive data about various individuals presented with the pendulum problem of Fig. 2(c). Of 124 students who had studied acceleration in the introductory physics course, none could answer this problem correctly; of 22 graduate-student teaching assistants, only 15% could answer it correctly; and of 11 graduate students on their Ph.D. qualifying examination, only 20% could answer it correctly. (Even some experienced physicists have difficulty identifying the acceleration of the pendulum bob. The arrows in Fig. 3 indicate the directions of these accelerations.)

The preceding data indicate that the proper interpretation of a scientific concept is no easy task and that many students do not acquire the ability to interpret the scientific concepts supposedly learned by them.

What are some of the reasons for the observed interpretation deficiencies? One common reason is that students retrieve remembered or plausible knowledge fragments which are often incorrect and which are rarely checked against a definition of the concept. For example, many students deem it obvious that a particle's acceleration is zero whenever its velocity is zero. Or they simply retrieve the fact that the acceleration in circular motion is directed toward the center (without heeding the fact that this is only true if the speed is constant).

Even when students do invoke the definition of a concept, they are often unable to interpret it properly. For example, one student, when considering the acceleration of the pendulum at the extreme point A of its swing, said the following:

> "The velocity is zero, so the acceleration has to be zero. Because acceleration equals the change in velocity over the change in time···I mean, acceleration is the derivative of the velocity over time. And the derivative of velocity is zero."

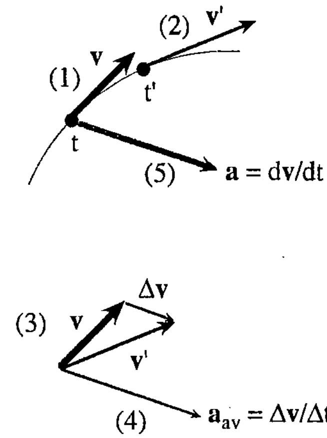

Fig. 4. Defining method for acceleration.

Thus the student invoked explicitly the definition of acceleration, even phrased in the formal mathematical language of a derivative, yet misinterpreted it to reach the wrong conclusion.

B. Cognitive analysis

The preceding detailed studies dealt with acceleration, but yield results consistent with students' observed misinterpretations of many other physics concepts.[1–4] To understand better the difficulties of concept interpretation and the reasons for misinterpretations, let us examine more closely the thought processes required for the proficient interpretation of scientific concepts.

Reliably accurate interpretation. The unambiguous specification of a scientific concept requires that the concept be explicitly specified with sufficient precision so that it can be properly interpreted in any specific instance. This requires an interpretation *method* (i.e., *procedural* knowledge) which specifies what one must actually *do* to identify or construct the concept in any particular instance. (The unambiguous specification of a scientific *principle* requires similarly an operational interpretation method.) The deliberate application of this method can then ensure the reliably accurate interpretation of the concept.

For example, the acceleration of a particle is specified by the defining statement (2). But this definition of the concept is inadequate without the specification of a corresponding interpretation method which involves the following five main steps (illustrated in Fig. 4).

(1) *Original velocity* $\mathbf{v}$. Identify the velocity of the particle at the time t of interest.
(2) *New velocity* $\mathbf{v}'$. Identify the velocity of the particle at a slightly later time t'.
(3) *Change of velocity* $\Delta\mathbf{v}$. Find the velocity change $\Delta\mathbf{v}=\mathbf{v}'-\mathbf{v}$ of the particle during the small time interval $\Delta t = t' - t$.
(4) *Average acceleration* $\mathbf{a}_{av}$. Find the ratio $\Delta\mathbf{v}/\Delta t$, the "average acceleration" of the particle during the time Δt.
(5) *Acceleration* $\mathbf{a}$. Determine the limiting value approached by the average acceleration if the time t' is chosen very close to t (so that Δt becomes infinitesimally small and

can be denoted by dt). The resultant ratio $d\mathbf{v}/dt$ is then called "the acceleration of the particle at the time t."

Application of this method is sufficient to determine the acceleration in all the situations (like those in Fig. 2) presented to the Berkeley students. Note how complex this method really is (especially the third step involving a vectorial subtraction of velocities and the fifth step involving a limiting process)! Yet all this complexity is hidden by the seemingly simple Eq. (2) or by the equivalent statement that "acceleration is the rate of change of velocity with time." No wonder that students have so much difficulty interpreting what this statement really means!

Efficient interpretation. The preceding interpretation knowledge is "formal," i.e., general, precise, and explicitly specified by a method for interpreting the concept in any particular instance. Deliberate application of this formal knowledge can ensure reliable accuracy, but it can be quite laborious. Thus one would also like to be *efficient*, i.e., able to interpret a concept rapidly and with little mental effort.

Cognitive efficiency is not just a luxury for people who prefer to be lazy and save time. It can also be essential for *effective* performance. Indeed, if we always had to spend much time and effort interpreting every concept, we would not have enough mental capacity left to deal with the more complex aspects of tasks in which these concepts are used. (Similarly, suppose that we had never learned to decode individual words and phrases more efficiently than six-year olds. How then would we have enough mental capacity left to read and understand an article in the *Physical Review?*)

Efficient concept interpretation can be achieved by compiling a repertoire of knowledge about special cases of the concept. An encountered situation which matches such a special case can then be recognized almost automatically. Hence such "compiled knowledge" can often be used to interpret the concept intuitively without the need for deliberate processing.

For example, most physicists have compiled knowledge about the acceleration of a particle in some special cases, such as that of circular motion with constant speed. When encountering a particle moving in this way, they then immediately recognize this situation and conclude that the particle's acceleration is directed toward the circle's center. All this is quickly done *without* any need to invoke the definition of acceleration or to engage in reasoning based on it.

Complementary use of formal and compiled knowledge. To interpret a concept both accurately and efficiently, general formal knowledge and case-specific compiled knowledge are used in complementary fashion. If one encounters a familiar situation which matches a particular case in one's compiled knowledge, then this compiled knowledge can be immediately applied. But if one encounters an unfamiliar situation, or runs into puzzling difficulties, or needs to make general arguments, then it is best to invoke one's formal knowledge and to reason from it.[7]

C. Instructional implications

Instruction must ensure that students can adequately interpret any concept or principle *before* they are asked to use it to perform more demanding problem-solving tasks. Otherwise, students are forced to deal simultaneously with the difficulties of concept interpretation as well as with other complexities, a situation which can transcend their learning capabilities and lead to frequent mistakes.

Explicit teaching of interpretation methods. The preceding analysis of the interpretation process suggests the following instructional strategy for teaching a scientific concept (or principle). (a) After motivating and introducing the concept, specify it explicitly together with the associated method required for its interpretation. (b) Let students themselves apply this method consistently in various special cases, including cases which are error prone. (Such error-prone cases include those which require fine discriminations, or which invite confusions with prior notions familiar from everyday life or earlier schooling.) (c) Ask students to summarize the results of their concept interpretations in these special cases so that they acquire a useful repertoire of compiled knowledge about the concept.

There is evidence that the preceding instructional method can be quite effective. For example, by applying this method in an experimental situation, we succeeded in substantially improving students' ability to interpret properly the concept acceleration (from about 40% correct interpretations before instruction to over 90% afterwards).[8,9] The method has also proved quite effective in actual classroom situations.

D. Assuring reliable compiled knowledge

As already mentioned, it is very useful to have compiled scientific knowledge about various specific situations so that these can be quickly recognized. In this way one can develop good scientific intuitions, and does not always need to engage in laborious reasoning from basic definitions or principles.

Need for quality assurance. Such compiled case-specific knowledge can, however, be unreliable unless it satisfies the following conditions: (a) It must be consistent with formal scientific knowledge. (b) It must be carefully discriminated from other kinds of intuitive knowledge used in everyday life or other contexts. (For example, the concept acceleration in physics has properties quite different from those associated with the word acceleration used in conversations with a taxicab driver.)

Most important, it is essential that one be able and willing to check whether intuitively applied compiled knowledge has, in fact, been correctly applied (as judged by consistency with explicitly specified definitions or principles). Otherwise, it is all too easy to ignore fine discriminations or validity conditions restricting the applicability of case-specific knowledge.

Instruction needs to ensure that the preceding conditions are achieved. This is no easy task, especially since students come to science from everyday life where intuitively used knowledge is not subjected to equally stringent requirements.

Fallibility of recognition processes. The ability to recognize a familiar or analogous situation can make concept interpretation fast and effortless. However, recognition processes can be error prone since they do not involve an explicit specification of which particular features should be heeded and which ones can be ignored. This is why recognition processes used in science need to be checked against more reliable interpretation methods. (Hence it can also be dangerous to introduce physics concepts by mere examples or analogies, without more explicitly specified definitions.) The following are some examples.

Figure 5(a) shows three vectors, of equal magnitudes, whose sum is zero. When students are shown the equilateral triangle in the first diagram and are asked about the angle between the vectors **A** and **B**, many say that this angle is 60°.