

algorithm. Feasible truss topologies are generated through a shape grammar in an optimally directed manner using simulated annealing for minimum weight, subject to stress and Euler buckling constraints.

1 Introduction

The advent of computer methods of analysis and design have propelled intensive research in the field of structural optimization over the last three decades. The basic structural optimization problem is to minimize an objective function, such as the weight or cost of the structure, subject to a set of structural constraints: stress constraints, buckling constraints, and geometric constraints such as location of loads, support points, and obstacles. Shape optimization of such a problem is rather well understood, although not necessarily easy, for a given topology; generation and optimization of the topology itself is a difficult problem. This paper proposes a systematic way of generating truss topologies and integrating this process with shape optimization.

Previous efforts in topology optimization go back to the early 1900s (Michell, 1904). Recent efforts have taken three approaches: selection from predefined grid or node layouts (e.g., Hemp, 1973; Dorn et al., 1964; Pedersen, 1992; see Kirsch, 1989), heuristic approaches (Spillers, 1985; Shah, 1988; Rogers et al., 1988; Lakmazaheri and Rasdorf, 1990) and material distribution approaches (homogenization: Bendsøe and Kikuchi, 1988; Diaz and Belding, 1993; Rodrigues and Fernandes, 1993; genetic algorithms: Chapman et al., 1993; combinatorial particle/element approach: Anagnostou et al., 1992). In these latter approaches the structure emerges from within the predefined acceptable geometric bounds; however the resulting material must be further interpreted to determine the useful structure generated (e.g., Papalambros and Chirehdast, 1990).

In this paper we propose an algorithm for truss structural topology optimization based on the shape annealing algorithm introduced by Cagan and Mitchell (1993). The shape annealing algorithm provides a systematic way of generating truss topologies, integrated with shape optimization. This method combines the advantages of optimization-based techniques and production system-based techniques to generate optimally directed topological configurations.

2 Shape Annealing for Truss Generation

In shape annealing, knowledge of components and connectivity between components is modeled through shapes and their grammars (Stiny, 1980). A *shape* is a limited arrangement of straight lines which form an entity upon which Boolean operations can be applied; distinguishing information about the shape is defined through *labels*. A set of grammatical rules called *shape rules* are defined on the set of shapes to map one shape or sub-shape onto another shape or sub-shape. Optimally directed search of the resulting design space is accomplished through the stochastic optimization technique of simulated annealing (Kirkpatrick et al., 1983). Given a current design state, an eligible rule selected at random from the shape grammar is applied to the design. If the new design produced does not violate any constraints, it is evaluated and compared to the old design; acceptance is determined by the simulated annealing algorithm (it is accepted as the new current design if better, and accepted with a probability, which decreases over time, if worse). If the new design state violates any constraint then the old state is maintained as the current one. To allow for backtracking out of local minima, any discrete rule is defined with a counterpart rule that removes the effects of the original rule when applied successively. This paper demonstrates how shape

Optimally Directed Truss Topology Generation Using Shape Annealing

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This paper presents a technique for the generation and optimization of truss structure topologies based on the shape annealing

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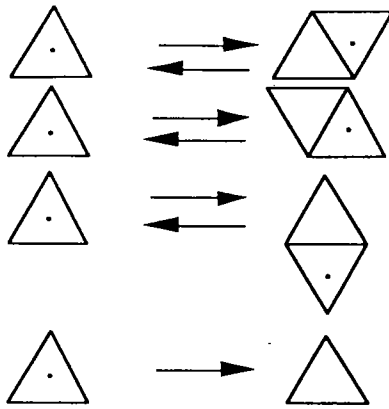


Fig. 1 Shape grammar for truss topology generation

annealing designs two-dimensional truss structures using a triangular shape grammar.

The shape grammar used in this paper, illustrated in Fig. 1, consists of combinations of triangular shapes to form new shapes. The label "•" on a triangle indicates that mating can take place from that triangle. Moving left to right on a rule is the application of the rule; a new triangle has been added onto the labeled existing triangle creating a new shape composed of two triangles. The label "•" has moved as well to indicate that further mating can take place from the new triangle. To encourage backtracking, each rule in the shape grammar has a counterpart rule within the shape grammar (moving right to left) except for the last *termination* rule where the label is removed. Each line of the triangle represents a bar (truss element) with all of the properties of truss bars. Alternative grammars are possible; the final characteristics of the truss structures are limited by the language produced by the shape grammar.

The current shape annealing method divides the problem space for truss generation and optimization into two spaces: the *shape* space and the *artifact* space. The shape space consists of a design generated by the shape grammar, while the artifact space consists of actual structures (trusses). The shape to artifact space has a one-to-one correspondence; every design in the shape space can be transformed into a corresponding truss. Truss topologies are generated and modified in the shape space through application of the shape rules, e.g., Fig. 2(a). These topologies are then transformed into actual truss structures (in the artifact space) through a process called *stretching*, Fig. 2(b), where the nearest node to each of the load and support points in the shape space configuration of triangles is stretched to move the node to that respective load and support point; the truss is then shape optimized, Fig. 2(c), and evaluated. The new truss structure is compared to the previous structure generated, and it is accepted or rejected based on the simulated annealing algorithm. The process of generating a new topology in the shape space, transforming it into the artifact shape, shape optimizing the truss in the artifact space, and comparing the solution with the previously existing design is done iteratively until the algorithm converges to the best design, or until the annealing temperature reaches a specified limit. A possible next shape generated in the shape and artifact space is shown in Figs. 2(d) and 2(e).

The shape optimization routine receives the topologies of the truss structures and the forces applied on them. The objective function used in this paper is minimization of the weight of the truss structure, but other objective functions could also be used. The variables are the cross-sectional areas of the bars and the locations of the nodes in the truss.

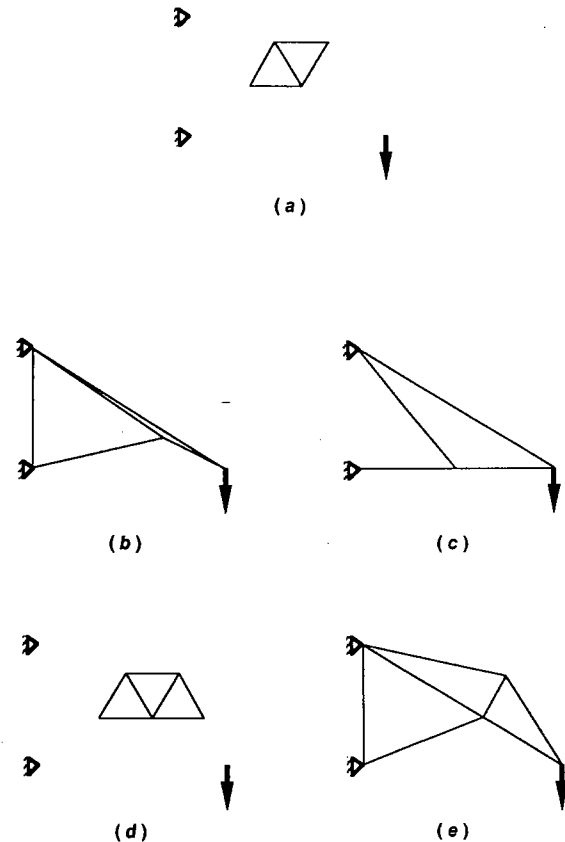


Fig. 2 Progression of steps within truss generation algorithm: (a) design in shape space, (b) stretched design (artifact space), (c) shape optimized design, (d) new shape space design, (e) stretched design (artifact space)

The constraints are stress and Euler buckling constraints (analyzed through a finite element analysis). The modified method of feasible directions is currently employed for optimization of the truss structure. After optimization of the truss topology, if the cross-sectional area of a bar gets below a limit then the bar is removed from the artifact space. The removal of bars in this manner results in the emergence of quadrilaterals and other polygons, which are not part of the shape grammar used, in the optimized structure.

Details of the general shape annealing algorithm can be found in Cagan and Mitchell (1993) and details of this application to truss generation can be found in Reddy and Cagan (1993).

4 Example: 10-Bar Truss

We apply shape annealing to the design of a truss for minimum weight under two loads and attached to two supports. The example demonstrates the algorithm's ability to generate alternative superior concepts to those previously considered. In Reddy and Cagan (1993) we compare the shape annealing topology to that of the homogenization method for a similar problem.

The specifications and the topology for the standard 10-bar truss problem are shown in Fig. 3(a). The stress limit is 172 MPa and the material density is 27 kN/m³. The 10-bar truss configuration after shape optimization without buckling constraints is shown in Fig. 3(b). The optimally directed structure obtained by the shape annealing algorithm, if only stress constraints are used, is shown in Fig. 3(c). Comparison of the designs shows an improvement over the 10-bar topology, i.e., the 16-bar truss generated by the shape annealing method

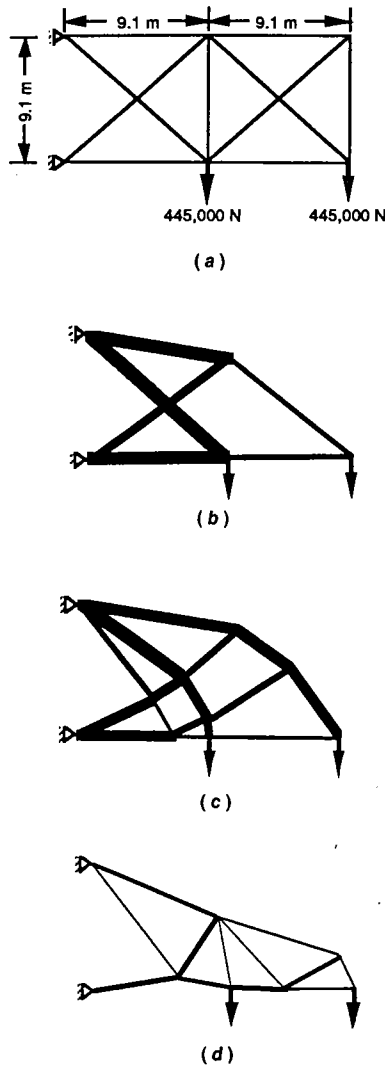


Fig. 3 (a) Problem specifications and 10-bar topology; (b) shape optimized 10-bar truss; (c) shape annealing topology; (d) shape annealing topology with Euler buckling constraints

has a weight of only 6573 N while the 10-bar truss topology gives a weight of 6880 N from shape optimization. If Euler buckling constraints are also considered, the structure generated by the shape annealing algorithm has a weight of 17315 N, Fig. 3(d). This compares well with the weight for the design found by shape optimization, including buckling constraints, of the original 10-bar truss topology (23963 N, figure not shown); and the design produced by shape optimization, including buckling constraints, of the truss topology in Fig. 3(c) (20515 N, figure not shown). In the figures, the thickness of the lines represents the relative cross-sectional areas of the corresponding bars.

5 Conclusions

Historically a bottleneck of truss structural design has been the problem of choosing a topology for the structure prior to optimization. In this paper, an evolutionary design process, shape annealing, is proposed to generate and optimize truss topologies. The shape annealing algorithm builds up the design from the initial state through addition and removal of triangular elements in accordance with the shape grammar, annealing criteria, and design constraints. Currently only stress and Euler buckling constraints are used but

it is also possible to incorporate other applicable constraints and knowledge.

The current algorithm is not able to avoid interfering objects between the loads and the support points; the resulting nonconvex evaluation space leads to a local optimum. A simulated annealing approach allocating penalties for constraint violations is being pursued to solve this problem, and to improve the efficiency of the overall algorithm by approximating the shape optimization process.

Presently the algorithm has been investigated for only two-dimensional truss structures. One immediate direction for future work is to extend the algorithm to three-dimensional truss structures. Other structural elements, such as frame, plate and shell elements, could conceivably be incorporated into the shape grammar, generating structures made up of various structural elements.

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