Artificial Intelligence for Engineering Design, Analysis and Manufacturing (1999), *13*, 241–251. Printed in the USA.

Languages and semantics of grammatical discrete structures

KRISTINA SHEA[1] AND JONATHAN CAGAN[2]

[1]Engineering Design Centre, Department of Engineering, University of Cambridge, Trumpington St., Cambridge CB2 1PZ, United Kingdom

[2]Computational Design Lab, Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

(RECEIVED June 18, 1998; FINAL REVISION March 10, 1999; ACCEPTED April 1, 1999)

Abstract

Applying grammatical formalisms to engineering problems requires consideration of spatial, functional, and behavioral design attributes. This paper explores structural design languages and semantics for the generation of feasible and purposeful discrete structures. In an application of shape annealing, a combination of grammatical design generation and search, to the generation of discrete structures, rule syntax, and semantics are used to model desired relations between structural form and function as well as control design generation. Explicit domain knowledge is placed within the grammar through rule and syntax formulation, resulting in the generation of only forms that make functional sense and adhere to preferred visual styles. Design interpretation, or semantics, is then used to select forms that meet functional and visual goals. The distinction between syntax used in grammar rules to explicitly drive geometric design and semantics used in design interpretation to implicitly guide geometric form is shown. Overall, the designs presented show the validity of applying a grammatical formalism to an engineering design problem and illustrate a range of possibilities for modeling functional and visual design criteria.

Keywords: Engineering Grammars; Design Languages; Structural Semantics; Structural Layout

1. INTRODUCTION

Shape annealing was developed as a technique for the configuration of optimally directed designs of shape (Cagan & Mitchell, 1993). Using a combination of a shape grammar formalism and simulated annealing, a stochastic search mechanism, performance-driven designs are generated. In this application to structural layout, a shape grammar defines a language of discrete structures by specifying spatial design transformations that implicitly represent form–function relations for truss design. The function of structural forms generated from this grammar is then interpreted using a semantic model that includes behavioral interpretation as well as a combination of performance goals. Because a structure is a visual and physical object, performance goals consider spatial and functional goals, which can be generally categorized in terms of efficiency, economy, utility, and elegance (Billington, 1983). This paper presents an exploration of design languages for discrete structures and their interpretation through structural semantics for the generation of purposeful structures.

Production systems are mechanisms that specify a set of designs, called a design language, by the transformations required to generate that set (Stiny & Gips, 1980). Applied to spatial design, shape grammars define a set of allowable shape transformations that in turn can be used to generate a language of spatial designs (Stiny, 1980). Since their introduction, shape grammars have been used extensively in architecture to define languages of architectural form and style (Stiny & Mitchell, 1978; Flemming, 1986). An advantage of using grammars as design production systems is that the language of designs defined by a grammar contains both known designs, from which the grammar was derived, and new designs with similar style. For engineering design problems, our interest in specifying a set of designs is that this set defines a design space that can be searched to determine optimal and innovative designs.

Applying grammatical formalisms to engineering problems presents several interesting extensions because engi-

Reprint requests to: Kristina Shea, Engineering Design Centre, Department of Engineering, University of Cambridge, Trumpington St., Cambridge CB2 1PZ, United Kingdom. Phone: +44 1223 332600; Fax: +44 1223 332662; E-mail: Kristina.Shea@epfl.ch

neering designs have behavioral, functional, and spatial goals. Because the function of a design object cannot always be implied from its shape, one approach is to explicitly describe function in a grammar through the use of syntax, that is, labels and symbols. Mitchell (1991) presents functional grammars as shape grammars that are limited to the generation of realizable and functional designs that he illustrates with an example of a functional grammar to design primitive huts. Functional grammars have also been used for the conceptual design of framed structures (Fenves & Baker, 1987), discrete planar structures (Rinderle, 1991), and mechanical systems (Finger & Rinderle, 1989; Schmidt & Cagan, 1997). The approach taken in shape annealing is different from previous works cited because design function is implicit in the representation as well as interpreted with a semantic model. Separating functional attributes, other than those that make designs valid, from spatial form allows for a wider range of designs to be generated and multiple interpretations of the same design language depending on varying definitions of design purpose.

Generative grammars are an extension of the original grammatical formalism that not only define a design language but also use control mechanisms to computationally search this language for feasible and in some cases optimal solutions. In architecture, generative systems have been created that generate feasible spatial designs for building layouts (Flemming et al., 1988) and three-dimensional buildings (Hiesserman & Woodbury, 1994). Adding design goals used to direct grammatical generation has led to the generation of optimal mechanical systems (Schmidt & Cagan, 1998) and optimized process plans for machining designs (Brown & Cagan, 1997) defined by a language of machineable parts (Brown et al., 1995). Directed generation incorporates goals in the generation process and requires the use of semantic models or interpretation to provide an understanding of a design language. Semantic models will be discussed in Section 5.

In the domain of structural topology layout, early analytical work by Michell (1904) based on Maxwell's theorem (1864) provided means for optimal structural layout of single-purpose truss structures. Current representations for structural layout use either continuous or discrete representations. Continuous methods discretize a defined space of material and optimize the distribution, making them advantageous for the layout of monolithic parts; see Bendsoe (1995). Most discrete methods reduce topology layout to sizing a highly connected ground structure; reviews of current methods can be found in Kirsch (1989), Topping (1993), and Bendsoe (1995). With discrete methods, topology changes occur by member cross sections reducing to a specified minimum, implying that they can be removed from the structure after the optimization process. Disadvantages of discrete methods are a strong dependence of results to the initial ground structure (Bendsoe et al., 1994) and hierarchical consideration of shape and sizing variables. Applying a grammatical formalism to structural topology layout has the advantages of (1) introducing new members and joints throughout the design process so that generated designs are not biased by the initial structure, and (2) providing simultaneous optimization of topology, shape, and size variables. In a comparison to other discrete optimization methods, shape annealing was shown to provide improved results (Shea & Cagan, 1998).

This paper presents an exploration of structural design languages, defined by shape rules and their syntax, as well as structural semantics. First, the shape annealing method will be presented along with current capabilities to illustrate the role that syntax and semantics play in design generation. Next, the definition of valid topologies in a structural language will be investigated by exploring the effects of modifying the type of topologies found in a language on the designs generated. The second aspect of language definition, geometric syntax, will then be explored by constraining the language to contain only designs with desired geometric properties that model functional and visual design goals. This will be shown through examples of constraining joint angles and shape proportions. Interpretation of design languages will then be investigated by varying visual and behavioral semantic models. Finally, a discussion of problem modeling and future extensions will be presented. Because structural layout is a simplified engineering design configuration problem, this investigation contributes knowledge about the syntax and semantics needed for the grammatical generation of more functionally elaborate engineering designs.

2. SHAPE ANNEALING FOR THE DESIGN OF DISCRETE STRUCTURES

Shape annealing is a design technique that combines a generative shape grammar with simulated annealing (Kirkpatrick et al., 1983; Swartz & Sechen, 1990) to produce optimally directed designs (Cagan & Mitchell, 1993; Reddy & Cagan, 1995). A shape grammar is a means of representing the relation between form and function in structural design through the specification of design transformations that define a language of discrete structures (Stiny, 1980). The two tasks of the shape annealing method applied to structural design are (1) grammatical transformation of structural topology and geometry, and (2) interpretation of structures using design objective metrics and constraint functions, which include structural behavior (Fig. 1). Structural analysis (Gobat & Atkinson, 1994) is used as input to the performance evaluation because final structures must be functionally feasible. Given a structural design problem, knowledge can be divided into specifications, such as external loads or material properties; constraints, such as physical limitations on stress and buckling; and design objectives. Design objectives define the interpretation of the structural design language and can include metrics for structural efficiency, economy, utility, and elegance. An overview of the implemented specifications, constraints, and design objectives is shown in Table 1.

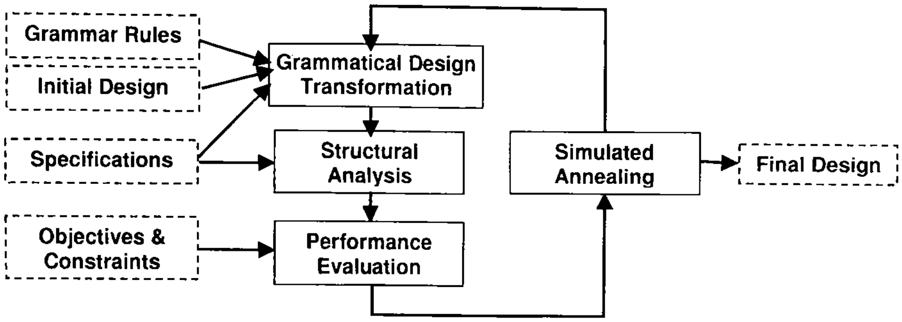


Fig. 1. Overview of Shape Annealing.

The grammar rules used for design transformation define a language of structures, which can be considered as the design space. Two shape grammars are presented: (1) a planar truss grammar shown in Figure 2, and (2) a single-layer space truss grammar shown in Figure 3. In both grammars the design transformation types are divided into geometry and topology. The allowable geometric transformations include a shape modification rule that changes the location of a single joint in a design and a size modification rule that changes the cross-sectional area of a single member. Multiple instantiations of the geometric rules exist with different ranges of geometric transformation. Together these grammar rules with the annealing algorithm perform shape and sizing optimization of a structure with a fixed topology. Topology exploration occurs by applying topology modification rules that transform triangles while implicitly adhering to the form–function stability relation for truss design. A different topology rule is applied depending on whether the division within the triangle is associated with a free or fixed point; see Figure 4 for illustrative rule applications. A free line, denoted by the label "*f*," indicates an exterior line in the design. The lines in the shape grammar represent truss elements. Note that the topology rules are created in pairs, except rule 5, so that any modification can be reversed. Rule 5 is its own reverse. The topology layout for a single-layer space structure using the grammar in Figure 3 is designed on an x–y plane and then projected in z onto a desired surface that is defined as a dependent function of the planar x and y dimensions.

The performance interpretation of structures determines the search direction in the annealing optimization method. A metric for design performance is formulated as the summation of design objectives and weighted constraint violations:

$$\textit{cost function} = \textit{objective cost} + \textit{constraint cost}, \tag{1}$$

$$\textit{objective cost} = \sum_{i=1}^{I} (\textit{objective weight}_i \cdot \textit{objective value}_i), \tag{2}$$

Table 1. *Modeling structural design knowledge in shape annealing*

Specifications (Syntax)	**Constraints (Semantics)**
material properties	stress
number of supports and locations	Euler buckling
symmetry	displacement
joint angles	geometric obstacles
Objectives (Semantics)	
efficiency	*aesthetics*
minimum mass	uniformity metric = σ(member lengths)[1]
economy	golden ratio metric =
minimum number of distinct cross-sections	$\sum_{numshapes} \left\lvert\phi - \frac{b}{a}\right\rvert + \left\lvert\phi - \frac{b}{c}\right\rvert + \left\lvert\phi - \frac{a}{b}\right\rvert$[2]
minimum number of distinct lengths	
utility	
maximum enclosure space	
minimum surface area	

[1]Sigma, σ, is defined as the standard deviation of all member lengths in a design.
[2]The golden ratio, ϕ, is defined as 1.618....

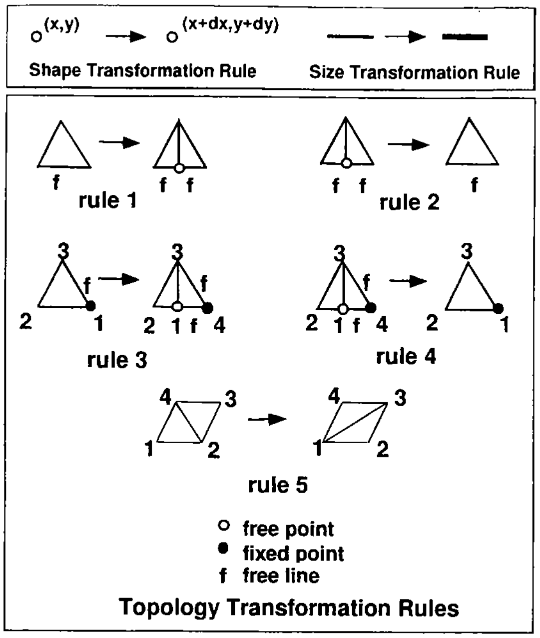


Fig. 2. Planar truss grammar (Shea et al., 1997).

$$\textit{constraint cost} = \sum_{j=1}^{m} (\textit{constraint weight}_j \cdot \textit{constraint violation}_j), \tag{3}$$

where:

l = number of objectives, and m = number of constraints.

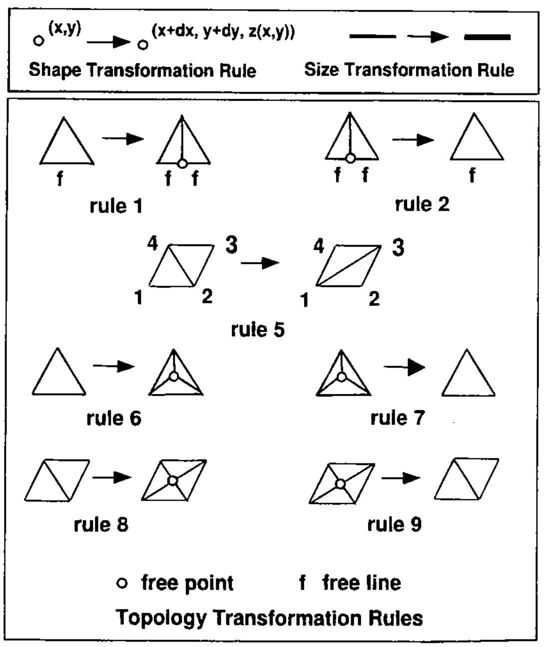


Fig. 3. Space truss grammar (Shea & Cagan, 1997).

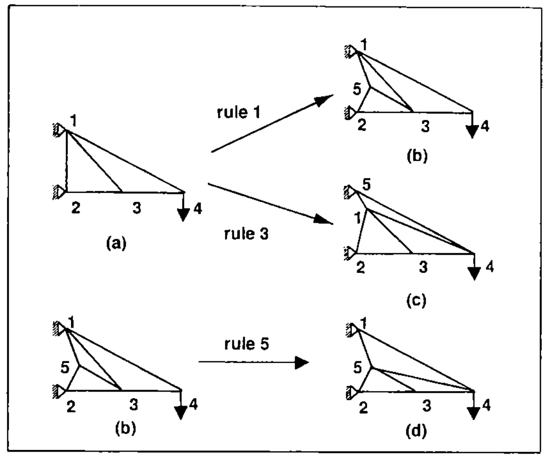


Fig. 4. Example of planar rule applications.

Weighting factors set by the designer are used to determine the relative tradeoff among design objectives. Utility schemes could also be used (Thurston, 1991). Details of these techniques and further implementation details of the algorithm are beyond the scope of this paper, but can be found in Shea et al. (1997) and Shea (1997).

3. LANGUAGES OF DISCRETE STRUCTURES: TOPOLOGY

The purpose of a shape grammar is to define a language of shapes that in this application are interpreted as truss structures. A language of truss structures is defined in terms of valid topology and valid geometry. Valid topologies are determined from the topology rules in the grammar, including their syntax, and the maximum number of members allowed during design generation. This section discusses changes in the topological properties of a design language and their effect on design generation. The goal in formulating a grammar is to fully define all designs of interest by modeling knowledge about how designs are generated from combinations of topology and geometry transformations. This suggests that a purpose of the shape grammar is to model designer intent. In directed design generation the implication is that if a desired topology cannot be configured from the grammar, then it is not necessarily the case that the optimization does not find that topology optimal, but rather that it does not exist within the language of designs being explored. For this reason, testing of the modeling capabilities of a grammar is important. Two means of testing are: (1) to generate by hand standard or known configurations using the shape grammar (Fig. 5), and (2) to investigate the effects of removing and adding rules in the shape grammar.

Considering the two rule pairs for planar design, rules 1 and 2, and rules 3 and 4 from Figure 2, truss designs can be configured with shape annealing using all rules, including rule 5, or using only one rule pair at a time. Least weight

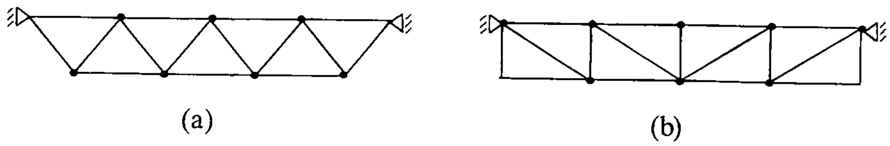


Fig. 5. Conventional planar trusses in the structural language defined by the shape grammar in Figure 2; (a) Warren Truss; (b) Pratt Truss.

designs were generated from different structural languages (Figs. 6–8) using solid steel truss members with material properties of: modulus of elasticity, E, 6.88 E^6 N/cm^2; allowable tensile and compressive stress, 14,880 N/cm^2; and mass density, ρ, .00785 kg/cm^3. Additionally, constraints on the maximum number of members, 50, and minimum angle between members, 1°, were used. Using all topology rules, 1 through 5, resulted in the design shown in Figure 6 with a mass of 2023 kg. Limiting a design to only be generated from the divide rule-pair, rules 1 and 2, resulted in the design shown in Figure 7 with a mass of 1802 kg. Using only the add rule-pair, rules 3 and 4, resulted in the design shown in Figure 8 with a mass of 3056 kg. All designs presented are the best designs generated from a total of six.

Comparing the two designs in Figures 7 and 8, we can recognize topological differences. While the layout in Figure 7 resembles a tied arch, the layout in Figure 8 is a combination of truss patterns and an arch. Comparing all three designs, the best design, Figure 7, was generated using only the divide rule. It is difficult to foresee the combined effects of different topology rule applications, especially when combined with simultaneous shape optimization. Separating the application of grammar rules provides a means of testing the individual effects of rules and their impact on solving design problems.

From this example, it can be seen that topology rules affect the definition of the structural language that is used for design generation. While the number of topologies that can be generated by the rules in Figure 2 is large, the grammar is not inclusive of all possible truss topologies. For instance, the topology shown in Figure 9 with two intersecting members without a joint cannot be generated from the current grammar. If this topology is not of interest, then the grammar fully models the designer's intention. However, in the case of a transmission tower design, this is a common topology and further rule formulation is under investigation to generate topologies specific to that domain.

4. LANGUAGES OF DISCRETE STRUCTURES: GEOMETRIC SYNTAX

The second aspect of defining a structural design language is the incorporation of valid geometric relations used for parametric control. Through the use of geometric rules and geometric syntax, a structural language is constrained to

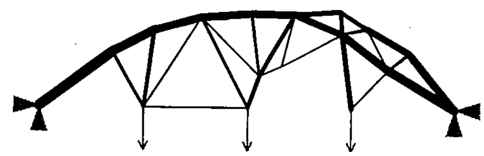


Fig. 6. Truss design generated using all rules; mass = 2023 kg.

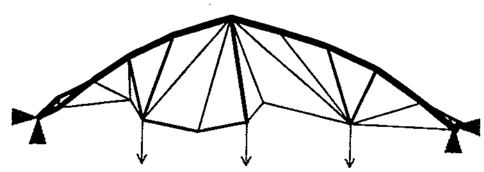


Fig. 7. Truss design generated using rules 1 and 2; mass = 1802 kg.

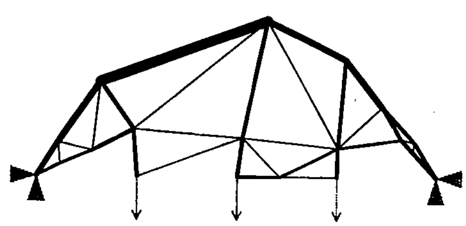


Fig. 8. Truss design generated using rules 3 and 4; mass = 3056 kg.

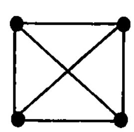


Fig. 9. Truss topology that cannot be generated from the rules in Figure 2.

reflect modeled parametric properties of structural topologies. The term "syntax" in generative grammars correlates to its use in natural language, where it determines valid sentence structure. Changing syntax in the grammar thus creates a new language of structures. In topology rules, syntax is used to model implicit function such that the shapes in the design language correspond to valid truss structures, but can also be used to model local and global parametric constraints that arise from functional and visual considerations. Examples of design properties modeled with geometric syntax are constraints on angles to generate realizable designs and proportions on shapes to generate designs with desired visual styles.

Incorporating geometric shape relations and their corresponding function in the design generation is investigated two ways: (1) as a fixed constraint on the design generation and (2) as a grammatical rule for shape improvement. Consider the design of a least weight planar tower. While it can be assumed that form follows function and that this will yield the lightest structure that meets the required function, maximally efficient designs could contain small angles between members that are difficult to construct. Modeling the consideration of joint construction in design generation, syntax is added to all grammar rules such that a minimum angle is maintained throughout the structure. The generated structures will now take on different forms because previous solutions that violate this constraint no longer exist in the design language; compare the design in Figure 10a, where no minimum angle is imposed, with that in Figure 10b, where a minimum angle of 30° is imposed. These designs were generated using tubular members with $d/t = 10$; modulus of elasticity, E, 2.067 E^7 N/cm^2; allowable tensile stress, 13,790 N/cm^2; allowable compressive stress, 10,340 N/cm^2; and mass density, ρ, .0083 kg/cm^3. A maximum of 50 members was placed on design generation. This example illustrates the sensitivity of the generated designs to the grammar syntax, and more generally the defined design language. The designs shown are the lightest designs from a total of ten designs generated.

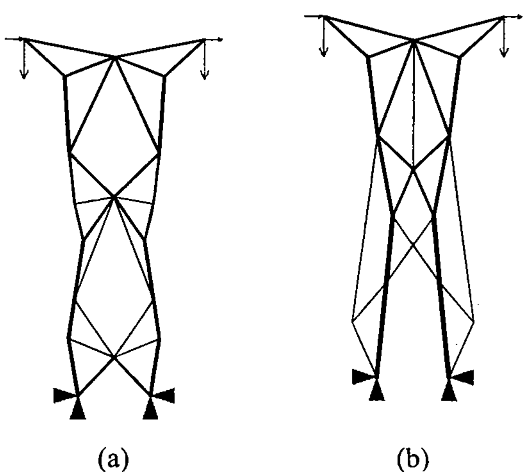


Fig. 10. Using grammar syntax to limit the angle between members; (a) no minimum angle; mass = 850 kg; (b) minimum angle = 30°; mass = 938 kg.

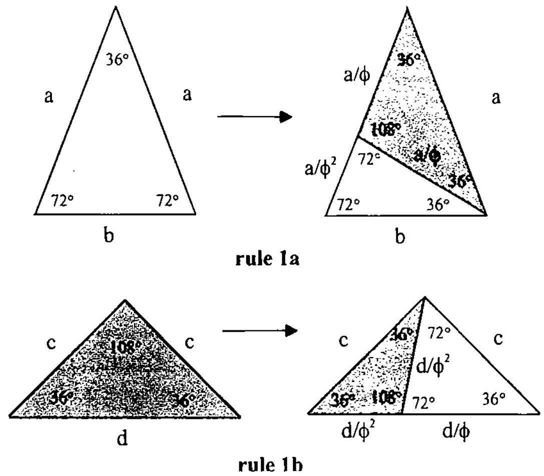


Fig. 11. Syntax for strict golden section rule where the golden ratio, ϕ, is defined as 1.618.

More elaborate aesthetic syntax can be formulated by modeling proportional systems within the grammar. For example, designs can be generated using the golden ratio as a preferred set of proportions, thus defining a language of golden structures. Modifying rule 1 in the shape grammar from Figure 2 to reflect golden proportions results in the rules shown in Figure 11. Similar modifications could be made to reflect other proportional systems and aesthetic styles. Due to the nature of the golden ratio, using only these new rules and an initial shape consisting of only golden triangles, shape annealing explores designs consisting only of golden triangles. An example spatial layout is shown in Figure 12 using an initial layout that consists of two golden triangles. The center point is elevated such that the struc-

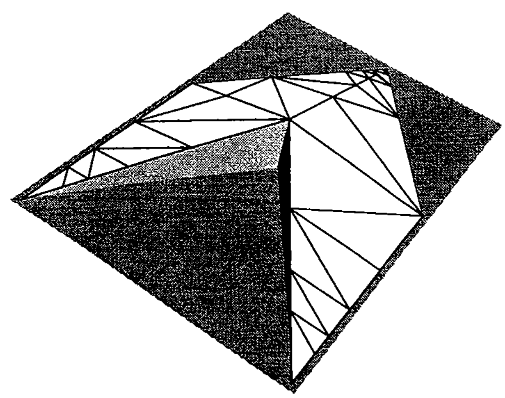


Fig. 12. Roof layout constrained to golden proportions.

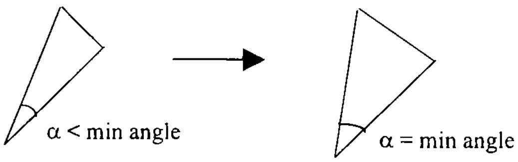


Fig. 13. Shape improvement rule.

ture could be used as a novel entranceway. The design objective is to minimize mass under loading from self-weight. A set of discrete sized tubes was used for selecting structural members having material properties of: modulus of elasticity, E, 2.067 E^7 N/cm^2; allowable tensile and compressive stress, 14,880 N/cm^2; and specific gravity, ρ, .00785 N/cm^3. A maximum of 150 members was allowed. Using shape annealing for pattern generation, spatial layouts that reflect desired styles as well as meet structural purpose can be generated.

Constraining angles and parametric proportions influence global spatial styles. At a local level, parametric grammar rules can be used to model form–function relations based on the behavior of a single structural shape. A simple rule is formulated for shape improvement that locates a shape in a design with an angle less than the desired minimum angle and then rotates one member in this angle such that the new angle is equal to the minimum desired angle (see Fig. 13). This minimum angle can be determined either analytically or based on heuristics such as those found in finite element modeling. One drawback to this rule is that it only results in local improvements because effects on connecting shapes are not considered. The optimization is then used to decide which transformations are globally beneficial.

Comparing to the previous transmission tower example (Fig. 10), a new design is generated using the shape improvement rule with a minimum desired angle of 30° and is shown in Figure 14. In this design shapes that do not meet the minimum angle result because there is no hard constraint on angles and application of the rule is not forced (note the shape that is attached to the load points). The design shown is the lightest design from a total of 10 designs generated.

We can see with this example that although it seems attractive to incorporate parametric form–function knowledge in the grammar, it does not necessarily yield the desired effects. Using functional feedback from the design evaluation concerning the behavioral performance of forms provided sufficient knowledge to the design process in generating good designs. However, between the two examples where angles were restricted, the best designs were found when constraining the design to shapes with angles of 30°. While there may be advantages to using a hard constraint on angles between members for joint construction purposes, in general, it seems a better approach to allow the optimization to determine the appropriate angles suitable to a design problem. Parametric rules are useful for modeling desired proportional styles, but are not as useful for implicitly modeling local function.

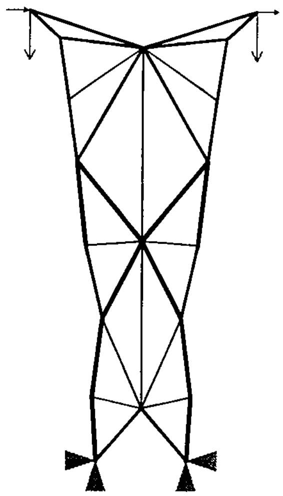


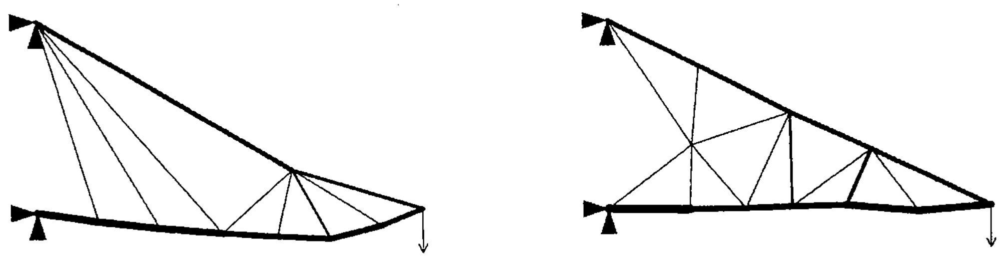
Fig. 14. Tower designs generated with the shape improvement rule; 30° desired minimum angle; mass = 954 kg.

5. SEMANTICS OF DISCRETE STRUCTURES

The language of truss structures defined by the grammar is immense due to the infinite number of geometric variations for any single topology. Thus, a means of design interpretation must be used to generate purposeful forms. Semantic models are used to provide an interpretation, or understanding, of a design language. As opposed to syntax, which defines new design languages, semantics interpret spatial forms within a language to attach meaning relative to performance goals. An engineering grammar requires this interpretation on functional and spatial levels. Defining a semantic model for design interpretation independent of the grammar allows for multiple interpretations of the same design language. The structuring and order of the semantic space is dependent on this model and changes with modifications to the weights among performance goals as well as with the addition and deletion of performance goals. In a previous paper, semantics were used to generate domes that meet desired functional aspects (Shea & Cagan, 1997). In the examples that follow, semantics are used to interpret the visual form as well as the physical behavior of structures.

5.1. Visual semantics

In the previous examples, structural form resulted from functional goals. Using visual semantics geometric form becomes an explicit design goal and thus structural form reflects functional goals and modeled aesthetic metrics. To illustrate the impact of visual semantics, consider the design of a cantilever truss. While the designs can be evaluated for a



(a) (b)

Fig. 15. Using semantics to generate structural forms with different characteristics; (a) design for maximum efficiency; mass = 853 kg; (b) design for maximum efficiency and visual uniformity; mass = 1038 kg.

single goal of efficiency, such as the design shown in Figure 15a, evaluating performance using multiple goals of efficiency and visual uniformity results in a different form shown in Figure 15b. The members used in the designs have material properties of: modulus of elasticity, E, 6.88 E^6 N/cm^2; allowable tensile and compressive stress, 17,200 N/cm^2; and mass density, ρ, .0027 kg/cm3. A maximum of 25 members was allowed and the minimum angle between numbers was 1°. The two designs were generated from the same design language, where different forms resulted from differences in the semantic models.

An alternative to visual uniformity is interpretation of form based on similarity to known proportional systems. Using the golden proportion again, an aesthetic metric is formulated to generate designs that reflect golden proportion but do not strictly adhere to it, as was the case in the design presented previously (Fig. 12). Each shape in a design is interpreted based on its deviation from the golden ratio; the total aesthetic measure for a design is then the summation of all deviations. Given a triangle with sides a, b, and c, an aesthetic measure is calculated for each shape in a design as:

$$\text{aesthetic measure} = \left|\phi - \frac{b}{a}\right| + \left|\phi - \frac{b}{c}\right| + \left|\phi - \frac{a}{b}\right|. \qquad (4)$$

The aesthetic design goal is then to minimize the deviation from the golden ratio to generate aesthetically pleasing structures. Based on discussions of computational aesthetics found in Stiny and Gips (1978), this implies that if design A has a lower aesthetic measure than design B, it can be said that it is more aesthetically pleasing. The aesthetic measure is calculated using dimensions in three-space, but could also use different planar projections. While an explicit aesthetic measure of single shapes is calculated there is an implicit reflection of the relative proportions between adjacent shapes because a single line often lies in multiple shapes. The number of alternative geometric patterns that will meet the aesthetic design goal is large considering the allowance for multiple ratios of lengths to satisfy the golden ratio.

To illustrate the variation in visual effects that result from the different aesthetic metrics, two glass gallery roof designs are shown. The design problem is to cover two 14.5-m passages each with a glass-vaulted barrel roof consisting of a steel truss-work covered with half-inch plate glass. The loading consists of a surface load from the glass panels and snow as well as the self-weight of the steel members. The structural members have the material properties: modulus of elasticity, E, 2.067 E^7 N/cm^2; allowable tensile and compressive stress, 14,880 N/cm^2; and specific gravity, ρ, .00785 N/cm^3. A maximum of 150 members was used along with a minimum angle between members of 10°. Two designs are presented in Figures 16 and 17: The first, Figure 16, uses the aesthetic metric based on visual uniformity while the second, Figure 17, reflects golden proportions according to Eq. (4). The two designs, although similar in mass, have different visual impacts due to the difference in aesthetic interpretations.

5.2. Functional semantics

Specific to engineering grammars is the incorporation of physical behavior in the grammar syntax and design inter-

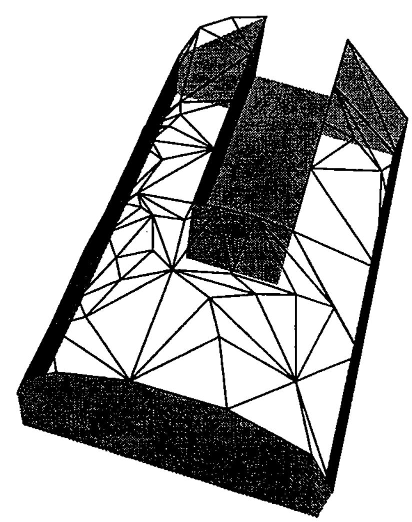


Fig. 16. Gallery design reflecting visual uniformity; mass = 24,017 kg.

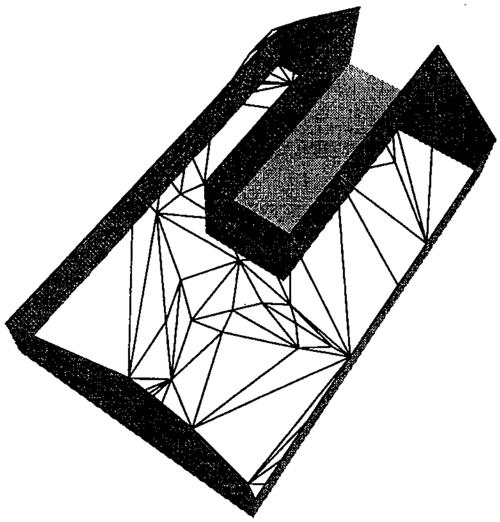


Fig. 17. Gallery design reflecting golden proportions; mass = 27,280 kg.

pretation. Until this point we have interpreted the forms within the design language as truss structures. Because the analysis is separate from design configuration, the same design language for truss structures can be interpreted as frame structures. We will now investigate the effects of changing the element type and analysis model on the resulting form of generated structures. The same grammar for planar trusses is used, but now the analysis will model the lines as beam elements that are capable of supporting bending and shear forces. The joints in the structure are now rigid, that is, in contrast to a pin-jointed structure, such that there is no rotation between elements.

The analysis model was created by dividing each member in a design into three beam elements. Solid steel rods were used with material properties of: modulus of elasticity, E, 6.88 E^6 N/cm^2; allowable tensile, compressive, and bending stress, 14,880 N/cm^2; allowable shear stress, 9,920 N/cm^2; and mass density, ρ, .00785 kg/cm^3. These values incorporate safety factors of 1.67 for tensile, compressive, and bending stresses as well as 2.5 for shear stress (Merritt, 1972). A maximum of 50 members was used along with a 1° minimum angle between members. A beam design is shown in Figure 18 and can be compared to previously shown truss solutions in Figures 6 through 8. Comparing the beam solution to the best truss design (Fig. 7), the beam solution is lighter and has a different topology and geometric form.

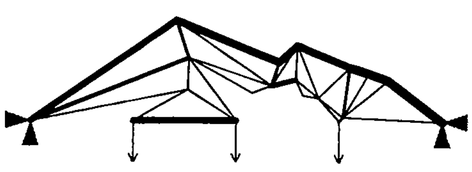


Fig. 18. Beam design; mass = 1783 kg.

6. DISCUSSION

The examples in this paper illustrate the use of syntax and semantics in the generation of purposeful structures. Syntax was used to model local parametric properties based on functional and visual considerations, whereas semantics were used to model global design goals. Relating the two to optimization, the difference between syntax and semantics is that syntax acts as a hard constraint, for example, a global minimum angle, while semantics act as soft constraints and design goals. Also, syntax changes the language of designs, or design space, that is searched whereas semantics change the search goals. A question in modeling a design problem is what to model with rule syntax and what to model as semantics. Generally, this can be decided using the same questions for determining hard and soft constraints: Which criteria must be satisfied (syntax) and which criteria can be traded off (semantics)? Comparing the spatial styles of the design in Figure 12 that constrained the layout to golden proportions and the design in Figure 17 that reflects golden proportions illustrates this difference.

While parametric proportions lead to designs that reflect modeled aesthetic styles, proportions were only considered on a local level. Parametric syntax used at a global level would create a dynamic grid based on relative proportions upon which members are laid out. One method for applying proportional syntax would be to assign proportion parameters to shapes as described by Stiny (1980) in his discussion of parametric grammars. This would enable structural design to occur on a topological level and then on a spatial level where different forms of a given topology meet required and desired proportions. Now, rather than allowing transformations of single points to govern the overall shape of a topology, relative proportions of shapes would govern point locations. While this would restrict the innovation of designs, the patterns generated could be intriguing and the functional implications of imposing one pattern over another could be investigated.

It was shown that the language of truss forms can be interpreted with beam elements, thus producing different design behaviors and styles. Rather than setting the structural element used for behavioral interpretation before design generation it would be more intriguing for shape annealing to determine the structural element that is advantageous for the problem specification. Extending the current grammar to the design of frames changes the underlying behavioral principles of the grammar that define the form–function relations and would create a new language of discrete structures. For the configuration of frames, there is greater topological latitude due to inherent stability from the joint construction; proposed examples are shown in Figure 19. This is an area of future investigation that could lead to further understanding of not only appropriate structural form

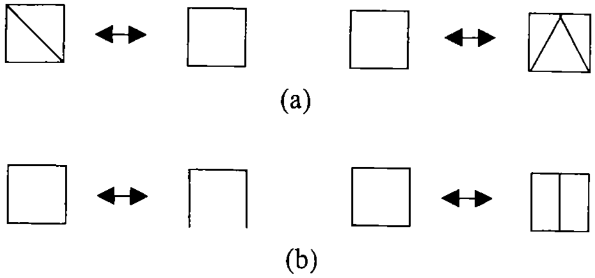


Fig. 19. Proposed frame grammar; (a) frame–truss transition; (b) frame rules.

for a given physical behavior, but functional principle as well.

Shape annealing has been used for the layout of structures ranging from 25 to 150 maximum members averaging 45 min to 5 h, respectively, on a DEC Alpha. Current investigation includes application to the redesign of a full-scale transmission tower consisting of 322 initial members and allowing a maximum of 600 members to be generated. This takes, on average, 4.5 h using an SGI Octane. The size of structures that can be generated using this method is mainly limited by time constraints. Considering the range of syntactic and semantic knowledge that can be incorporated, though, the uniqueness and practicality of structures generated in most cases will outweigh the cost of execution time.

The results of this work can be generalized for use in future grammatical applications to engineering design problems. Because the grammatical representation is based on the idea of a network of members with force flowing through them (Cagan & Mitchell, 1994), an extension to a general grammatical network approach for the configuration of mechanical systems can be made. Including mechanical design criteria in terms of syntax and semantics would create a method capable of the form and function configuration of optimal (high-quality, low-cost, reliable, and novel) mechanical systems. A grammatical network approach to functional and spatial synthesis of mechanical systems would provide a computational tool capable of generating languages of optimal mechanical systems and could lead to the discovery of new relations between efficient form and function in mechanical design.

7. CONCLUSION

Using shape annealing to explore structural design problems requires the modeling of domain knowledge in terms of grammar syntax and semantics. The separation of syntax and semantics has led to performance-driven design generation from a constrained structural language, where performance includes spatial and functional goals. For each structural application, syntax can be added to the grammar to limit the design language to a set of appropriate and desired design styles while semantic models enable this language to be interpreted from multiple viewpoints. Differentiation between syntax and semantics was essential for understanding how design models (grammar and semantics) affect design generation. Overall, this exploration illustrates the validity in applying grammatical formalisms to engineering design problems and shows a range of possibilities for modeling functional and visual design criteria.

ACKNOWLEDGMENTS

The authors acknowledge the National Science Foundation under grants EID-9256665, DDM-9300196, and DDM-9258090 and the Engineering Design Research Center, an NSF research center at Carnegie Mellon University for providing funding for this work.

REFERENCES

Bendsoe, M.P. (1995). *Optimization of structural topology, shape, and material.* Springer-Verlag, Berlin.

Bendsoe, M.P., Ben-Tal, A., & Zowe, J. (1994). Optimization methods for truss geometry and topology design. *Structural Optimization 7*, 141–159.

Billington, D.P. (1983). A new tradition: Art in engineering. In *The Tower and the Bridge: The New Art of Structural Engineering*, pp. 3–24. Basic Books, Inc., New York.

Brown, K.N. & Cagan, J. (1997). Optimized process planning by generative simulated annealing. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 11*, 219–235.

Brown, K.N., McMahon, C.A., & Sims Williams, J.H. (1995). Features, aka the semantics of a formal language of manufacturing. *Research in Engineering Design 7*, 151–172.

Cagan, J. & Mitchell, W.J. (1993). Optimally directed shape generation by shape annealing. *Environment and Planning B 20*, 5–12.

Cagan, J. & Mitchell, W.J. (1994). A grammatical approach to network flow synthesis. *IFIP Transaction 3 B-18*, 173–189.

Fenves, S. & Baker, N. (1987). Spatial and functional representation language for structural design. In *Expert Systems in Computer-Aided Design*, (Gero, J.S., Ed.) IFIP 5.2, pp. 511–529. Elsevier Science, North-Holland.

Finger, S. & Rinderle, J.R. (1989). A transformational approach to mechanical design using a bond graph grammar. *Proc. First ASME Design Theory and Methodol. Conf.*

Flemming, U. (1986). More than the sum of parts: The grammar of Queen Anne houses. *Environment and Planning B 14*, 323–350.

Flemming, U., Coyne, R., Galvin, T., & Rychener, M. (1988). A generative expert system for the design of building layouts—Version 2. In *Artificial Intelligence in Engineering: Design*, (Gero, J.S., Ed.), pp. 445–464. Elsevier, New York.

Gobat, J.I. & Atkinson, D. (1994). *FElt: User's guide and reference manual.* Report No. CS94-376. University of California, San Diego.

Hiesserman, J. & Woodbury, R. (1994). Geometric design with boundary solid grammars. In *Formal Design Methods for CAD (B-18)*, (Gero, J.S. and Tyugu, E., Eds.), pp. 85–105. Elsevier Science B. V., North-Holland.

Kirkpatrick, S., Gelatt, Jr., C.D., & Vecchi, M.P. (1983). Optimization by simulated annealing. *Science 220*, 671–679.

Kirsch, U. (1989). Optimal topologies of structures. *Applied Mechanics Review 42*, 223–238.

Maxwell, J.C. (1864). *Philosophical Magazine 27*, 294.

Merritt, F.S., ed. (1972). *Structural steel designer's handbook.* McGraw-Hill, New York.

Michell, A.G.M. (1904). The limits of economy of materials in frame structures. *Philosophical Magazine S.6, 8*, 589–597.

Mitchell, W.J. (1991). Functional grammars: An introduction. In *Computer Aided Design in Architecture '91: Reality and Virtual Reality*, (Goldman, G. and Zdepski, M., Eds.), pp. 167–176. Los Angeles, CA.

Reddy, G. & Cagan, J. (1995). An improved shape annealing algorithm for truss topology generation. *ASME Journal of Mechanical Design 117*, 315–321.

Rinderle, J. (1991). Grammatical approaches to engineering design, part II: Melding configuration and parametric design using attribute grammars. *Research in Engineering Design 2*, 137–146.

Schmidt, L.C. & Cagan, J. (1997). GGREADA: A graph-grammar-based machine design algorithm. *Research in Engineering Design 9*, 195–213.

Schmidt, L.C. & Cagan, J. (1998). Optimal configuration design: An integrated approach using grammars. *ASME Journal of Mechanical Design 120*, 2–9.

Shea, K. (1997). Essays of discrete structures: Purposeful design of grammatical structures by directed stochastic search. PhD Dissertation. Carnegie Mellon University, Pittsburgh, PA.

Shea, K. & Cagan, J. (1997). Innovative dome design: Applying geodesic patterns with shape annealing. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing 11*, 379–394.

Shea, K. & Cagan, J. (1998). Topology design of truss structures by shape annealing. *Proceedings of DETC98: 1998 ASME Design Engineering Technical Conf.*

Shea, K., Cagan, J. & Fenves, S.J. (1997). A shape annealing approach to optimal truss design with dynamic grouping of members. *ASME Journal of Mechanical Design 119*, 388–394.

Stiny, G. (1980). Introduction to shape and shape grammars. *Environment and Planning B 7*, 343–351.

Stiny, G. & Gips, J. (1978). *Algorithmic aesthetics*. University of California Press, Berkeley.

Stiny, G. & Gips, J. (1980). Production systems and grammars: A uniform characterization. *Environment and Planning B 7*, 5–18.

Stiny, G. & Mitchell, W.J. (1978). The palladian grammar. *Environment and Planning B 5*, 5–18.

Swartz W., & Sechen, C. (1990). New algorithms for the placement and routing of macro cells. *Proc. IEEE Conf. Computer-Aided Design*, 336–339.

Thurston, D.L. (1991). Multiattribute utility analysis in design management. *IEEE Transactions on Engineering Management 37*, 296–301.

Topping, B.H.V. (1993). Topology design of discrete structures. *Topology Design of Structures*, pp. 517–534. Academic Publishers, Dordrecht.

Kristina Shea is a lecturer in the Engineering Department at Cambridge University, UK. She received her B.S. (1993), M.S. (1995), and Ph.D. (1997) all in Mechanical Engineering from Carnegie Mellon University. Her principal research interest is computational design methods and tools in engineering; special interests include grammatical design, design optimization, artificial intelligence in design, and design theory and methodology.

Jonathan Cagan is a Professor of Mechanical Engineering at Carnegie Mellon University, with appointments in Biomedical Engineering and Computer Science. His research, teaching, and consulting are in the area of design theory, methodology, automation, and practice. He received his B.S. in 1983 and M.S. in 1985 from the University of Rochester, and his Ph.D. in 1990 from the University of California at Berkeley, all in Mechanical Engineering. Dr. Cagan is the recipient of the National Science Foundation's NYI Award and the Society of Automotive Engineer's Ralph R. Teetor Award for Education. He is a member of the Phi Beta Kappa, Tau Beta Pi, and Sigma Xi National Honor Societies, and the ASME, AAAI, SAE, and ASEE Professional Societies. Dr. Cagan is a registered Professional Engineer.