RESEARCH

Economic Analysis of Quality Innovation

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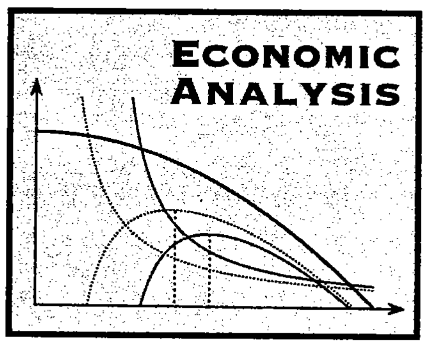
THE AUTHORS REVIEW DIFFERENT PERSPECTIVES ON INDUSTRIAL QUALITY AND ADOPT A FORMALISM IN WHICH SOCIAL AND CORPORATE OPTIMUM CAN BE COMPARED FROM AN ENGINEERING STANDPOINT. THE POTENTIAL BENEFITS TO A MANUFACTURER FROM IMPROVING THE QUALITY OF ITS PRODUCTS ARE STUDIED UNDER SEVERAL MARKET CONDITIONS. THE INCENTIVE TO IMPROVE QUALITY IS THE STRONGEST IN A COMPETITIVE ENVIRONMENT IN WHICH THE BENEFITS OF QUALITY INNOVATION ARE TWOFOLD: IT INCREASES CONSUMER DEMAND AND ALLOWS THE MANUFACTURER TO KEEP MORE SUBSTANTIAL PROFIT MARGINS.

More than ever before, industrial quality is one of the critical elements in the fierce competition among rival manufacturers. From an engineering standpoint, as well as from a management perspective, quality is a challenge that cannot be ignored: a lack of competitive success can often be attributed to quality problems. In such cases, the first logical approach is to question manufacturing and design practices. An abundant literature deals with systematic methods to improve quality at zero or low cost (e.g., [1–4]). However, this article addresses a different problem that may arise, but has received much less attention: Several alternate production modes can often be employed to manufacture the same product, resulting in costs and quality levels that are significantly different. From this perspective, the choice of a production mode, and thereby a quality level, consists of trading off cost for quality. Decisions to increase production expense, such as choosing a tighter manufacturing process or screening a production, are justified by higher expected profits. The need for economic



analysis has been emphasized by several authors (e.g., [3]), and the justification of engineering decisions by economic theory has been introduced in [5]. This article extends that economic approach and models the consequences of engineering decisions regarding quality in different economic contexts.

PRODUCT VALUE TO SOCIETY

Experience has shown that decisions about manufacturing quality cannot rely on simple accounting practices: the gains and losses when dealing with quality are not expressed directly in terms of dollars. Generally, the costs involved can be easily estimated, but the benefits are somewhat intangible (*e.g.*, consumer satisfaction). More often than not, a change in consumer satisfaction is only detected by a change in the number of sales during a given period—well after production decisions have been made. Consequently, the profitability of manufacturing decisions involving quality is at best uncertain, and there is a need to better assess the impact of these decisions. More precisely, it is necessary to ascertain beforehand—that is, before production begins—what quality level a product should have to be a potential commercial success.

As described below, economic theory provides a method for estimating the value that consumers attach to a product with given characteristics [6]. For example, in considering a market of N potential consumers, let us assume that the consumers are ranked according to their willingness to pay for a product. We denote by $P(n)$ the maximum price that the n^{th} consumer is willing to pay, as shown in Fig. 1. An aggregate measure of the value that consumers attach to the product is:

$$\int_0^N P(n)dn.$$

Suppose the actual price of the product is P_0. Only n_0 consumers will buy the product. The difference between what they are willing to pay and what they actually pay is the consumer surplus, S. This surplus measures the satisfaction provided by the product when sold for a price P_0:

$$S = \int_0^{n_0} P(n)dn - n_0P_0.$$

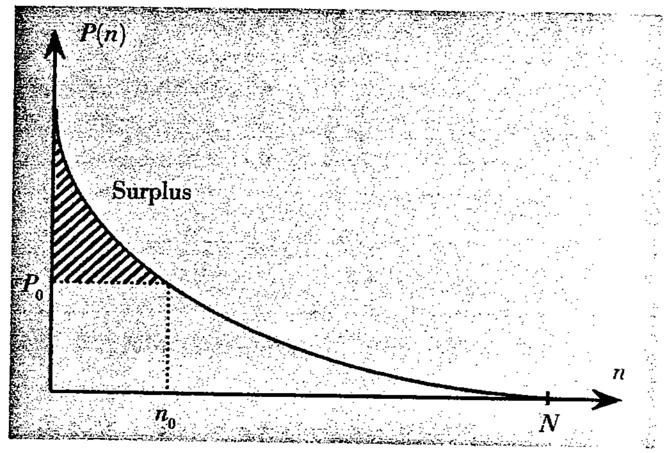


Fig. 1. Willingness to pay for a particular product.

The total value of the product to society, or welfare provided by the product, W, is obtained by adding the manufacturer's profit (sales revenue minus cost) to the consumer surplus:

$$W = \int_0^{n_0} P(n)dn - n_0P_0 + n_0\left[P_0 - C\right]$$
$$= \int_0^{n_0} P(n)dn - n_0C,$$

where C is the average long run manufacturing cost.

QUALITY LOSS

Taguchi relates quality to the overall well-being of society: "Quality, both tangible and intangible, may be defined in terms of the loss imparted to all people after a product leaves the hands of an enterprise [7] ." Taguchi defines quality loss as the expense resulting from 1) the variability of the function of the product, and 2) the possible harmful side effects of the product [3]. Examples of losses to be considered include the average maintenance cost or cost of repair. In addition to this concept, a more accurate measure of welfare should consider the whole service life of the product and should, therefore, include the expected quality loss, L, that each product will generate:

$$W = \int_0^{n_0} P(n)dn - n_0\left[C + L\right] \tag{1}$$

Taguchi uses the quality loss to determine the amount that a manufacturer should spend on quality control. A good illustration is the setting of manufacturer's tolerances, as opposed to consumer's tolerances [3]. A manufacturer should set an internal tolerance on his product in such a way that the product will be reworked when the cost of rework is lower than the loss society would incur if the product were not reworked. This principle corresponds to a maximization of $C + L$, which maximizes welfare as defined in Eq. 1. Since welfare maximization is not necessarily compatible with the objective of a profit maximizing firm, corporate decision makers may be reluctant to follow this method.

A different approach is generally used in economics to include quality considerations in the computation of welfare. Consumers are supposed to be well informed about the quality of the products they buy, and therefore quality is viewed as a characteristic influencing consumers' willingness to pay. The quality of a product is represented by the scalar, q (or vector when several characteristics are of importance). As described in [8], consumer surplus can be written as:

$$S = \int_0^{n_0} P(n, q)dn - n_0P_0.$$

This model assumes that the negative impact of the quality loss, as viewed by Taguchi, influences the consumer demand in which it becomes incorporated. This leads to the following expression for welfare:

$$W = \int_0^{n_0} P(n, q)dn - n_0C(q). \qquad (2)$$

Taguchi criticizes this approach on the grounds that it makes any engineering application impossible because of the wide range of values expressed by consumers [3]. However, this difficulty can be avoided by considering only a market segment for which the values expressed by consumers are more homogeneous, and can be more easily aggregated. Thus, we assume that for a particular market segment, the consumers' willingness to pay are all identical and can be expressed by a function $V(q)$. As used in [9, 10] or [5], $V(q)$ is a value function representing the highest price that consumers are willing to pay for a product of quality q. The determination of V is a difficult problem generally solved by selecting a priori a functional form for V. The appropriate function is determined by using econometric techniques [9, 11] or interpolation [12].

For this homogeneous market segment, Eq. 2 can be rewritten as

$$W = n_0V(q) - n_0C(q).$$

Again, welfare maximization is achieved by minimizing the sum of the manufacturing cost inside the plant and the loss outside the plant. However, the influence of q on corporate profits is still not clear.

AN OPTIMUM QUALITY LEVEL

A quality level that maximizies welfare and profit to the firm can be determined. The conditions for its existence are described below in this section. Without loss of generality, the quality level of a product can be described by a quality indicator q such that $0 \quad q \quad 1$. The highest possible quality is achieved when q is zero (i.e., zero defect, or no significant deviation from expected performance). A value of unity for q corresponds to the lowest quality level (i.e., 100 percent defective products). The cost, C, of manufacturing the product studied is a decreasing function of q, and denotes the lowest possible production cost associated with an output at quality level q. We assume $C(q)$ to be differentiable and convex, as in [13]. The convexity conveys the idea of diminishing returns to the increase of the manufacturing cost, with a hypothetical infinite cost for the perfect product.

Consider a situation where a product is manufactured at a quality level, q, and sold to consumers for a price, p. We restrict our analysis to a market segment where consumers have identical preferences and are indifferent between an increase in the quality level of the products and a price reduction. We do not consider the case where consumers require a precise quality level because of particular uses of the product. We also suppose that there is no income effect: no limit exists on the price that consumers are able to pay (as opposed to the price they are willing to pay). We model consumer behavior as a maximization of the surplus, $S(p, q) = V(q) - p$ [10], and manufacturer behavior as a maximization of the profit, $\pi(p, q) = p - C(q)$. Both objectives are functions of the independent variables p and q. However, the sum

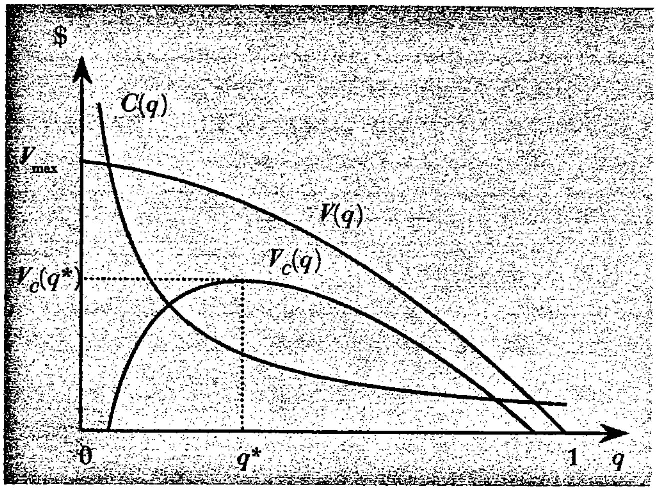


Fig. 2. Quality level maximizing the value created.

$S(p, q) + \pi(p, q)$ is only a function of q:

$$S(p,q) + \pi(p,q) = V(q) - C(q).$$

Therefore, the sum of the two satisfaction measurements depends exclusively on the quality level. $V(q) - C(q)$ can be viewed as the value created—denoted by $V_c(q)$—by the manufacture of a product. The benefits of this value creation are shared by manufacturers and consumers: q determines the amount to be shared, and p decides the allotment of this value.

Assuming that $V(q)$ is differentiable and strictly concave, $V_c(q)$ is a differentiable and strictly concave function over the compact set $[0, 1]$, therefore it has a unique maximum at q^*, as illustrated in Fig. 2.

Proposition: Let us denote by q^* the quality level that maximizes $S(p, q) + \pi(p, q)$. The q^* is the only Pareto efficient quality level, that is, given a price p and a quality level q, unless $q = q^*$, it is possible to find p^* such that, by manufacturing the product with a quality level q^* and selling it at the price p^*, either the consumer surplus or the manufacturer profit is increased, and neither of them is decreased.

Proof: In a situation where the quality level is q and the price is p, the consumer surplus is $V(q) - p$, and the manufacturer profit is $p - C(q)$. If the quality level is too high ($q < q^*$), then let us choose $p^* = V(q^*) - V(q) + p$. The consumer surplus is unchanged, and the manufacturer profit becomes $p^* - C(q^*)$ that is, $V(q^*) - V(q) + p - C(q^*)$. Because q^* is defined as the value that maximizes $S(p, q) + \pi(p, q)$, necessarily, $V(q^*) - C(q^*)$ $V(q) - C(q)$. Substituting $V(q^*) - C(q^*)$ by $V(q) - C(q)$ in the profit expression reveals that the profit is greater than $V(q) - C(q) - V(q) + p = p - C(q)$, which is the original profit. In other words, the manufacturers are better off and the consumers are not harmed. The case where the original quality level is lower than q^* can be handled in a similar way: one proves that by switching to the quality level q^*, the consumers are better off and the manufacturers are not harmed.

The concrete meaning of the existence of q^* is that there is a quality level that maximizes the potential commercial success of the product. The actual success also depends largely on marketing and advertising efforts. However, if a product is sold, the quality level q^* maximizes the consumer surplus and the manufacturer profit resulting from the sale. From an engineering perspective, q^* is the level at which the product should be designed and manufactured.

THE MONOPOLISTIC FIRM

Consider a market dominated by a single enterprise that manufactures and sells a single industrial product. Let us determine the profit maximizing values of the price, p, and the average quality level, q, of the product. If N is the consumer demand and C the average long-run cost of production of a single product (a function of q), the total profit π_T of the firm can be expressed as

$$\pi_T(p,q) = N[p - C(q)]$$

It is assumed that the firm can make accurate predictions about the consumer demand and can generate a supply to meet that demand. We assume the demand to be a strictly increasing function of the consumers surplus, S ($S = V(q) - p$):

$$N = F(S) = F[V(q) - p]$$

This model describes consumer behavior as a maximization of its surpluses. Under the assumption of differentiability for C, the profit maximizing values must satisfy the following optimality conditions:

$$\frac{\pi_T}{\partial p} = -F'(S)\left[p - C(q)\right] + F(S) = 0, \text{and}$$

$$\frac{\pi_T}{\partial p} = -V'(q)F'(S)\left[p - C(q)\right] - F(S)C' = 0.$$

Combining the two equations sets the quality level at the value of q satisfying

$$V'(q) - C'(q) = 0$$

which also corresponds to the maximization of the value created. Under the assumption that consumers maximize their surpluses, the profit maximizing quality level for a monopolist is also the Pareto quality level.

To determine the profit maximizing price, an additional assumption must be made on the demand function. Following [5], we set a limit on the number of products that can be sold [$\alpha\ V(0)$], and assume a linear demand function:

$$N(p,q) = \alpha\left[V(q) - p\right]$$

Under these conditions, as stated in [5], price should be set halfway between value and cost: manufacturer profits and consumer surplus are equal. The value created is equally shared between the manufacturer and the consumers.

It is questionable as to whether the production quality level can be adjusted as easily as its price. In most cases, major changes in the production processes are required, and retooling may be necessary. Therefore, the model provides an indication of the quality level toward which it is suitable to move. It can be noticed that no matter what the quality level is, the model recommends that the value created per product be shared equally by the manufacturer and the consumers. This is an engineering decision that will have long-term benefits to the manufacturer; short-term market influences are assumed irrelevant.

COMPETITIVE FIRMS

The Model

In the competitive case (here an oligopoly, in which a limited number of firms are in the market) a number of firms, n, compete in a given market to mass produce and sell a given product. This scenario can be modeled in a manner similar to the monopolistic case. We assume that the products sold by different firms are similar by their functions, but have different prices and different quality levels. To compare the results with the monopolistic case, we also model the demand function for firm i [$Ni(pi, qi)$], as a linear function of the consumer surplus S_i it offers:

$$N_i(p_i, q_i) = \alpha_i\left[V(q_i) - p_i\right] = \alpha_i S_i.$$

In the absence of competitors, firm i would sell a number of products equal to αS_i (demand in the monopolistic case). To accommodate the presence of competitors, the demand function can be modified by multiplying the potential number of products sold by the coefficient $1/n$, where n is the number of competitors:

$$N_i(p_i, q_i) = \frac{\alpha S_i}{n}.$$

However, this model is not really satisfactory because it only takes into account the number of competitors, and not their values relative to firm i. To this effect, we consider the coefficient

$$\frac{1}{\sum_{k=1}^{n} S_k} S_i$$

If firm i is just as good as its average competitor, the ratio is $1/n$; it becomes greater than $1/n$ if firm i compares favorably with its competitors, and smaller otherwise. Therefore, the demand for firm i,

$$N_i(p_i, q_i) = \frac{S_i \alpha S_i}{\sum_{k=1}^{n} S_k}$$

is the monopolistic demand αS_i, weighted by a coefficient reflecting the competitive position of firm i with respect to its competitors.

This setting is consistent with the model of the monopolistic firm because the maximum number of products that can be sold (obtained by extrapolating the demand curve at maximum quality and zero price) is

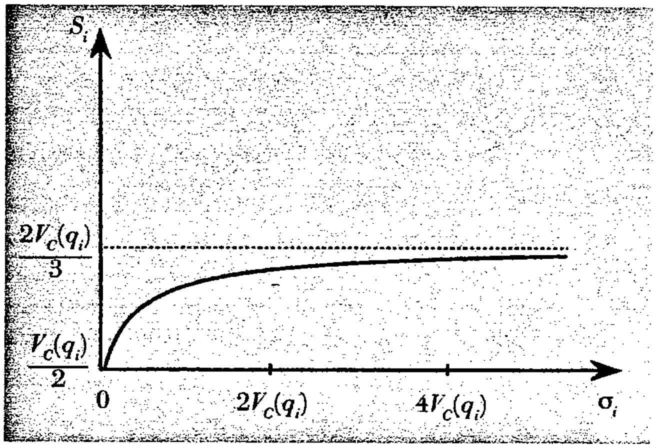


Fig. 3. Firm i *profit maximizing surplus as a reaction to the surplus offered by its competitors.*

again $\alpha V(0)$:

$$\sum_{i=1}^{n} N_i(0,0) = \alpha V(0).$$

Furthermore, over the domain of interest of the model (i.e., the surpluses offered by the competitors having the same order of magnitude), the main effect is a linear function of the consumer surplus offered—with perturbations due to the existence of competitors (see Appendix 1). The model assumes that an increase of the average consumer surplus results in an expansion of global market demand. An alternative model is described in [12]; this is examined more closely in the *Discussion* below.

This model is not meant to be a precise description of economic reality. Its purpose is the understanding of the economic consequences of quality variations. In particular, consumers' demand for differentiated products can be modeled much more accurately by using statistical methods [14].

Profit Maximizing Decisions

We will not consider the case where a collusion between firms artificially maintains high prices, because this case is not very different from a monopoly situation. Instead we will study the case where each firm defines its optimum quality level and price, taking its competitors' quality levels and prices as given and fixed. This setting is similar to that of the Cournot-Nash model [14], where the variables are prices and outputs. Profit is maximized when (see Appendix 2)

$$C'(q_i) - V'(q_i) = 0$$

The optimum value of q_i is once again the Pareto optimal solution. The profit maximizing surplus for firm i is now expressed by:

$$S_i = V(q_i) - p_i = \frac{\left[V_C(q_i) - 3\sigma_i + \sqrt{\left[V_C(q_i) - 3\sigma_i\right]^2 + 16V_C(q_i)\sigma_i}\right]}{4}, \quad (3)$$

where σ_i is the sum of the surplus offered by the other firms (again, see Appendix 2). It can be seen that when σ_i is zero, the profit maximizing price is equal to that of the monopolistic case. Furthermore, S_i is an increasing function of the sum of the surplus offered by the competitors. Figure 3 represents the variations of S_i as a function of σ_i. In a competitive situation, a firm offers a larger surplus than in the monopolistic case. Consumers can receive as much as two-thirds of the value created in the case of strong competition (versus only one-half in the monopolistic case). The well-known benefits for consumers of economic competition appear in this model, as in previous models, based on price and output.

This result is consistent with the studies carried out to compare consumer surplus and manufacturer profits in a competitive industry. For instance, in the semiconductor industry, Wilson et al. report that for Matallic Oxide Semiconductor (MOS) dynamic RAM, based on conservative linear demand functions, the manufacturer profits were significantly lower than the consumer surplus [16].

INCENTIVE FOR QUALITY INNOVATION

A manufacturer can take various actions to improve competitiveness. This section examines these courses of action and the consequences of quality innovation in a competitive environment.

Under the assumptions described in previous sections, improvement of manufacturing competitiveness is achieved by increasing $V(q) - C(q)$. Since consumer preferences (represented by $V(q)$) are more or less constant, competitiveness depends on reducing costs. Negotiating price reductions from suppliers or reducing overhead are classical steps taken to restore competitiveness. More interesting is the potential effect of manufacturing process improvement. By improving a process or introducing a new manufacturing method, a cost reduction may occur over some quality range, or a quality level that was not feasible before may become available. In particular, improving process yields may have a significant effect: besides avoiding production disruption, increased yields reduce product unit cost.

We denote by Y the probability for a product to be defective and by $C_0(q)$ the production cost of the product considered, whether or not it is defective. Assuming, to simplify the reasoning, that a defective product is scrapped, the resulting unit product cost is:

$$C(q) = \frac{C_0(q)}{Y}.$$

When Y increases, the unit cost decreases, and the magnitude of the cost reduction is higher for expensive products, in this case for high quality products. Such a change in the cost curve can lead to a shift of the optimum quality level q^* as shown in Fig. 4. Depending on the magnitude of the required investment to update the manufacturing equipment, it may be profitable to increase the quality of the products.

When demand is sensitive to price variations (denoted by a large value of α), cost and, therefore, price reduction is the logical main focus of the firm because of the large sales increase resulting from a price reduction. For a demand function relatively insensitive to price variations (a small value of α), quality innovation may be more appealing than efforts to reduce costs.

Let us assume that, in the context of the previous model described above in *Competitive Firms*, one of the competitors has been able to reduce its cost function, and takes advantage of the reduction to increase its quality level. Because the expressions involved are complex, we will not give exact analytical solutions, but rather, will focus on how the situation is changed compared to the case in which all the competitors create the same value. Also, we only consider the system in equilibrium, the point where no firm has any incentive to change its prices. Equation 3 shows that S_i is an increasing function of $V_c(q_i)$ as well as of σ_i: consumers always benefit from the quality improvement

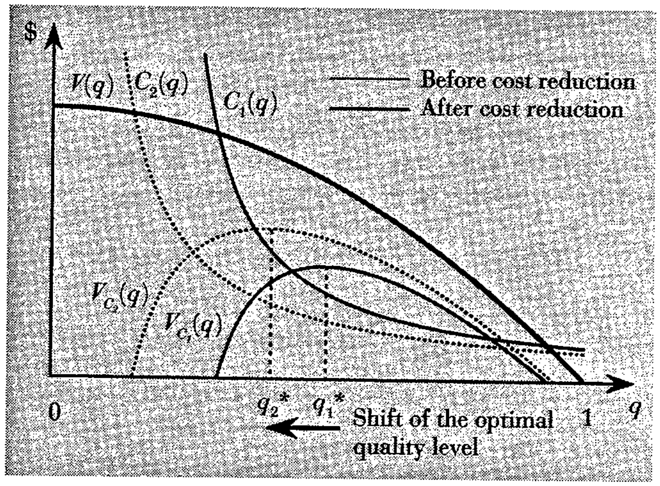


Fig. 4. A cost reduction can induce an improvement in the optimum quality level.

performed by one of the competitors. However, the expansion of market demand is mainly to the benefit of the firm that has improved the quality of its products. Moreover, it can be shown from Eq. 3 that the ratio of the surplus offered over value created decreases for the firm with the highest value created (see Appendix 3):

$$\frac{S_i}{V_c(q_i)} \text{ is a decreasing function of } \frac{V_c(q_i)}{\sigma_i}.$$

This indicates that the business conditions for the more advanced firm are more favorable because it can reduce the proportion of value created it passes on to the consumers. In fact, the more advanced the firm is, the closer its pricing policy gets to that of a monopolist.

As found in the previous sections, producing at a quality level other than q^* is a waste of resources. It hurts firm profitability in any kind of market, but even more so in a competitive one, in which there is a strong incentive to improve processes, thereby creating more value to escape from competition based on prices.

DISCUSSION

A good illustration of the model developed above is the manufacture of computer chips, where the impact of process improvement on product quality level is particularly clear. The quality of a chip can be assessed in many ways; one important measure is the number of functions available on the chip [16]. For a given state of technology, there is a minimum lithographic feature size, and therefore a maximum number, ρ, of components that can be implemented on the chip per unit area. Therefore, the quality of the chip is roughly inversely proportional to its area, A. The manufacture proceeds as follows: the chip is fabricated as many times as possible on a silicon wafer of area S. Due to several factors, including the amount of dust in the room and the accuracy when positioning the wafer, a certain number of defects occur on the wafer with density D. Under a few simplifying assumptions, the yield of the process is [17]

$$Y = e^{-DA}.$$

Since the products are inspected before shipment to the customers, the yield directly impacts the cost per chip within specifications: if C_0 is the processing cost per wafer, the cost per chip within specifications, C, becomes:

$$C = \frac{C_0 A}{SY} = \frac{C_0 A}{S} e^{DA}.$$

If n is the number of functions ($n = A\rho$), the cost per chip within specifications, as a function of n, is

$$C = \frac{C_0 n}{S\rho} e^{(nD/\rho)}.$$

This expression shows that if ρ (density of functions on the chip) is increased or D (density of defects on the wafer) is decreased due to process or design improvement, the impact of the cost reduction is higher for large values of n and therefore for high quality levels. This illustrates the relationship between process improvement, cost reduction, and quality increase. The global market expansion that is still occurring in the computer chip industry is probably not explained completely by quality improvements because product diffusion also plays an active part [18]. However, the frenzy of innovation that has taken place is a striking example of the double effect of quality innovation: 1) an increase in the overall market demand; and 2) improvement in the competitive position of a firm.

For more mature products, such as in the automotive industry, the dominant effect of a quality innovation is more a competitive position improvement than an overall market demand expansion. In that case, by improving quality, sales are gained from those lost by the competitors and firms compete essentially for market share, as described in [12]. Note that when there are no quality differences in the competing products, our model is the same as the one found in [12] (the slopes of the demand functions are the same for all the competitors).

CONCLUSION

We have used a classic economic framework to assess the value society attaches to the quality of a product based on engineering considerations. This approach allows us to examine the concept of quality loss from a societal and corporate perspective. Under a set of assumptions about consumer behavior, we define an optimum quality level. The existence of an optimum explains the propensity of manufacturing companies to produce at comparable quality levels, within a given market segment. This clustering results in competition through pricing, from which consumers benefit. Under these business conditions, we show that most of the value created is transferred to consumers through low prices. By using quality innovation manufacturers have an opportunity to escape from this situation. By creating more value, a firm increases the demand for its product and is able to maintain a pricing policy closer to that of a monopolist. However, this dominant position is probably only temporary because other firms are likely to follow the firm's lead and implement their own quality-based strategies.

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Appendix 1

The function used to model consumer demand for firm i is

$$N_i = \alpha \frac{S_i^2}{\sum_{k=1}^{n} S_k}$$

In this expression, the consumer surpluses offered by the various firms have the same order of magnitude, therefore $(S_k - S_i)/S_i$ is small compared to 1. N_i can be rewritten as:

$$\alpha \frac{S_i^2}{nS_i + \sum_{k=1,k\neq i}^{n} S_k - S_i}, \text{ or } \frac{\alpha S_i}{n} \frac{1}{1 + \frac{1}{n} \sum_{k=1,k\neq i}^{n} \frac{(S_k - S_i)}{S_i}}$$

Since for small x,

$$\frac{1}{1+x} \cong 1 - x,$$

over the whole domain where the surpluses have the same order of magnitude, N_i can be approximated by

$$N_i = \frac{\alpha S_i}{n} - \frac{\alpha}{n^2} \sum_{k=1,k\neq i}^{n} (S_k - S_i).$$

Appendix 2

Setting the partial derivatives of π_{Ti} to zero yields the following equations:

$$\left(S_i - 2\sum_{k=1}^{n} S_k\right)\left[p_i - C(q_i)\right] = -S_i \sum_{k=1}^{n} S_k, \quad (4)$$

and

$$\left(S_i - 2\sum_{k=1}^{n} S_k\right)\left[p_i - C(q_i)\right] = \frac{-S_i C'(q_i)}{V'(q_i)} \sum_{k=1}^{n} S_k. \quad (5)$$

The left hand side of Eq. 4 must equal the left hand side of Eq. 5, therefore

$$C'(q_i) - V'(q_i) = 0.$$

From Eq. 3, we can extract the expression for the surplus:

$$S_i = V(q_i) - p_i$$
$$= \frac{\left[V_C(q_i) - 3\sigma_i + \sqrt{\left[V_C(q_i) - 3\sigma_i\right]^2 + 16V_C(q_i)\sigma_i}\right]}{4}.$$

Appendix 3

From Eq. 3, we get

$$\frac{S_i}{V_C(q_i)} = \frac{\left[1 - 3\frac{\sigma_i}{V_C(q_i)} + \sqrt{\left[1 - 3\frac{\sigma_i}{V_C(q_i)}\right]^2 + 16\frac{\sigma_i}{V_C(q_i)}}\right]}{4}.$$

Since the function f such that

$$f(x) = \frac{1}{4}\left[1 - \frac{3}{x} + \sqrt{\left[1 - \frac{3}{x}\right]^2 + \frac{16}{x}}\right]$$

is a decreasing function of x, $\frac{S_i}{V_C(q_i)}$, is a decreasing function of $\frac{V_C(q_i)}{\sigma_i}$.