K. Shea

J. Cagan

Department of Mechanical Engineering

S. J. Fenves

Department of Civil and Environmental Engineering,

Carnegie Mellon University, Pittsburgh, PA

A Shape Annealing Approach to Optimal Truss Design With Dynamic Grouping of Members

A shape annealing approach to truss topology design is presented that considers the tradeoff between the mass of the structure and the grouping of members, where all members of a group are given the same size. The problem of optimal grouping involves finding a structural design with an optimal number of groups and the optimal sizes for each group. In this paper cross-sectional area is considered as the measure of group size. Designs incorporating multiple members with the same cross-sectional area are advantageous when considering the cost of purchasing and fabricating materials. The shape annealing method is used as an approach to solve this problem by incorporating a method for dynamic grouping of members based on cross-sectional area that creates a tradeoff between mass and the number of groups through a weighted objective function that includes a group penalty function. This method is demonstrated on transmission tower and general truss problems.

1 Introduction

Structural design can be described as an attempt to achieve a balance between three ideals: efficiency, economy, and elegance (Billington, 1983). Most structural optimization problems focus on the first ideal, that is to find a design with the minimum amount of material required for a specified performance. The problem of optimal grouping begins to incorporate the second ideal, economy, into the problem formulation by modeling the influence of the purchase and fabrication costs of materials and not just the mass. A group consists of members with the same cross-sectional area. Given a problem specification, the optimal number of groups and the cross-sectional area of each group are determined during the design process. By limiting the number of distinct member sizes through groups, a tradeoff is created between achieving the minimum mass of the structure and having fewer distinct member cross-sectional areas with a corresponding economy of scale.

This paper focuses on the truss topology optimization problem with special sizing characteristics through an exploration of the optimal grouping of members. Currently, topology layout methods generate topologies using either continuous or discrete cross-sectional area variables. Frequently, members are placed a priori into *predefined* groups, with the cross-sectional area of all members in a group linked to the same design variable. The problem of optimal grouping is different from the previous problems because as a topology is generated the members are grouped *dynamically,* where individual members within a group are assigned a common cross-sectional area but still maintain an independent cross-sectional area variable. For this paper the common cross-sectional area can be any positive value and will result in cost savings if members are to be fabricated. If materials are to be purchased additional cost benefits could be gained if the common cross-sectional area were assigned based on gauge sizes.

The optimal number of groups versus optimal mass is influenced by the designer. By setting the weight of the group penalty function in the objective function the designer specifies a tradeoff between performance, i.e., mass, and economies of scale, i.e., the number of groups. This problem is solved with the shape annealing method. Shape annealing generates optimally directed structural topologies, that is structures that are in the range of a global optimum but may not be the exact optimum, through a simulated annealing optimization on a shape grammar representation (Reddy and Cagan, 1995). The optimal grouping problem is solved by ordering the members into groups and assigning a common group size that is the cross-sectional area. After a review of related work the approach will be presented in detail and applied to two structural design problems.

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2 Related Work

Recent approaches to sizing in the structural layout optimization problem use both continuous and discrete sizing variables. The general structural layout problem has been investigated as early as Michell's (1904) analytical work. According to Kirsch (1989), structural topology optimization methods can be broken down into two categories: distributed parameter optimization and discrete topology parameter optimization. Distributed parameter optimization treats the problem as a continuum of material broken down into a grid of elements. Discrete parameter optimization uses either a ground structure on which the design is based or heuristics to introduce new members. Among the distributed parameter methods much work has been done with the homogenization method which discretizes a specified space into finite elements. The optimal density of each element is then determined from the stress limit resulting on the principal stresses (Bendsoe and Kikuchi, 1988; Bremicker et al., 1991; Diaz and Belding, 1993). Another similar approach uses a genetic algorithm to generate the topology, finding optimal stress contours for a given space of finite elements (Chapman et al., 1994). Discrete parameter methods generate optimal structures by starting with a highly connected ground structure and removing members based on stress limits (Hemp, 1973), or using a grid of allowable nodes to lay out members (Dorn, 1964; Pederson, 1992). Another method uses heuristics based on a grammatical approach that introduces new members (Spillers, 1985). A review of discrete parameter optimization problems can be found in Kirsch (1989).

Research has also been done on the discrete sizing problem which uses a fixed topology, in that no new members are introduced, to generate the optimal discrete sizes of members picked from an allowable set. The topology only varies by allowing a member to have zero area, implying that the member can be removed. A review of discrete sizing methods including branch and bound, approximations using branch and bound and ad-hoc methods can be found in Vanderplaats and Thanedar (1991). A different approach to the discrete sizing problem is a mixed integer formulation where the allowable set of sizes contains only integer sizes (Ghattas and Grossman, 1991). Another approach uses a genetic algorithm to perform discrete sizing from an allowable set of gauge sizes (Grierson and Pak, 1993). None of the above-mentioned approaches have been applied to the problem of generating optimally directed structural topologies while dynamically assigning members to groups based on cross-sectional area. The shape annealing method will be shown to be capable of solving this problem.

3 Shape Annealing

The solution to optimal grouping builds on the shape annealing method, a discrete, stochastic optimization method that uses a grammatical approach for structural topology modifications (Reddy and Cagan, 1995). The modified shape annealing method in this paper includes a formal grammar, an improved annealing schedule and constraint penalty functions as well as an additional method to dynamically group members. The shape annealing approach builds structures using a shape grammar (Stiny, 1980), optimizes the structures using the stochastic optimization method of simulated annealing (Kirkpatrick et al., 1983) and analyzes the structures using external finite element tools.[1] The iterative process involves the following algorithm: First, an initial structure is generated from a minimal connection of truss members between the applied loads and support points of the problem specification. The members are then grouped by cross-sectional area and their cross-sectional areas are reset to the calculated group size. Next, the structure is loaded and analyzed using the finite element method. The cost function is then evaluated from the structural behavior of the design and the geometric constraints of the problem. Next, a rule from the shape grammar is applied to the structure to create a new design which is analyzed and its cost is calculated. The costs of the new design and the previous design are used in the simulated annealing algorithm to determine whether to accept the new design or revert to the previous design. A better design is always accepted while a worse design may be accepted based on a probability function. A rule from the shape grammar is applied again to the structure to create a different design and the process continues iteratively. This process is completed when either the design has converged based on the freezing criteria or the annealing schedule has terminated. In practice, simulated annealing does not guarantee that a global optimum will be found, although results will show that optimally directed designs are generated. After the design process has been completed, members that have the minimum allowable cross-sectional area can be removed by the designer as long as the structure remains stable.

The current implementation uses a modified Lam Delosme annealing schedule (Swartz and Sechen, 1990). Freezing criteria are used to detect if the design has reached its minimum, allowing for the same annealing schedule to be used for problems of varying difficulty. The modified Lam Delosme schedule is based on following a specified accept rate function, where the accept rate is defined as the number of accepted moves out of total moves over a fixed statistical interval. The target accept rate function versus the actual accept rate for one annealing run is shown in Fig. 1. At each iteration the temperature is adjusted based on the comparison between the actual accept rate and the target accept rate. If the actual accept rate is higher than the target this implies that too many designs are being accepted and the temperature is decreased, thus "cooling" the process. In contrast, if the actual accept rate is lower than the target not enough designs are being accepted and the temperature is increased, thus "heating" the process. This annealing schedule is characterized by three regions of temperature: an initial quench, a simmer, and a freeze as shown in Fig. 2. During the initial quench large moves have a high probability of being accepted so that the design space is explored and the design moves sufficiently away from the initial structure so as not to bias the final topology. The simmer region is where most of the profitable design takes place since it is more controlled than the initial quench but not as restrictive as the freeze region.

The cost function consists of a multicriterion objective function and dynamically weighted constraint violation penalties. The objective function is the weighted summation of the mass and a group penalty function:

$$\text{cost} = \text{mass} + (\text{group\_weight}*\text{group\_penalty}) + \sum_i (\text{constraint\_weight}_i*\text{constraint\_violation}_i). \quad (1)$$

Constraints included in the following examples are stress, buckling and overlap of geometric obstacles. All constraints are soft constraints, implying that they can be violated during the design process. Each constraint violation has an associated weight, constraint_weight, which acts as a penalty function that combined with the optimization pushes the constraint violation to zero as the design progresses. The constraint weights are dynamic and are modified based on feedback from a predefined decreasing target violation (Ochotta, 1994). If the violation of a constraint is higher than the target violation the constraint weight is increased in order to increase the influence of this constraint violation in the cost function; if the constraint violation is equal to the target violation the constraint weight is not changed since it is working effectively; if the constraint violation is lower than the target violation the constraint weight is decreased thus decreasing the influence of the constraint violation in the cost function and allowing more designs with this violation to be accepted. Each constraint weight changes individually allowing the search to be driven by the largest term in the cost function: either the constraint with the greatest violation or the dominating design objective value. Simulated annealing attempts to minimize each term in the cost function causing constraint violations to be driven to zero by the end of the schedule. This implies that the only feasible design generated may be the final design.

The shape grammar shown in Fig. 3 is a formalization of the grammar presented in Reddy and Cagan (1995) that is used in the shape annealing method to introduce new members in the design. For the grammar presented the only shape represented is a triangle; thus, the designs can only consist of a composition of triangles. Additionally, the lines in the shape grammar represent truss members allowing only truss topologies to be generated from this grammar. The rules of the grammar described below are divided into shape and topology modification rules. The shape modification rules take a free point and move it some distance or change the diameter of a member cross-section based on the labels "+" or "−" that indicate the direction of the

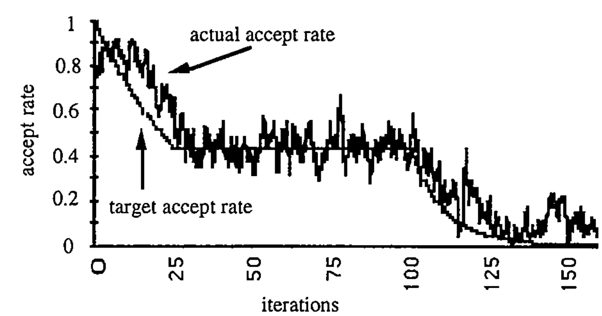

Fig. 1 Accept rate over annealing schedule

[1] Interfaces for MSC/NASTRAN™ and FElt (Gobat and Atkinson 1994) have been built. FElt has been used as the FEA tool for the examples in this paper.

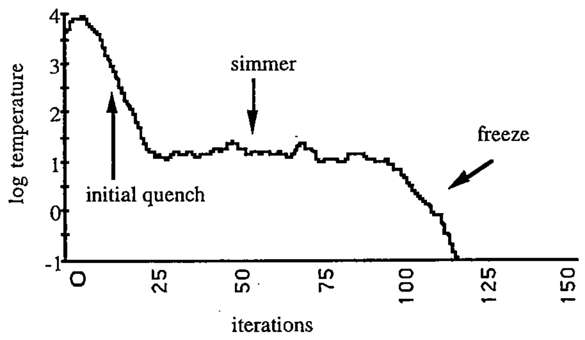

Fig. 2 Temperature over annealing schedule

change. If a member is under both the stress and buckling limits the label is set to "−" to decrease the cross-sectional area; otherwise the label is set to "+". These rules together with the shape annealing algorithm perform shape and sizing optimization of a structure with a fixed topology. The topology modification rules take a triangle within the design and transform it into a different configuration of triangles based on the general rule of truss design that is to add one node and two members to the current topology. There is a different rule applied depending on whether the division within the triangle is associated with a free or fixed point. A free line, denoted by the label "*f*", is an exterior line in the design. Note that the topology rules are created in pairs so that any modification can be reversed. In theory, through the sequential application of reverse rules any previous topology design state can be reached. The application of this grammar to the layout of a transmission tower and general planar trusses will be shown later. Additional details on the use of shape grammars can be found in Reddy and Cagan (1995) or Stiny (1980).

Combining the shape grammar with the simulated annealing algorithm presented, the rules of the shape grammar are randomly chosen based on probabilities for selecting rules that change over the annealing schedule. The probability of selecting a topology modification rule is greater in the beginning of the annealing process, while the probability of picking a shape modification rule is greater at the end of the schedule. These probabilities lead to large moves through topology modification at the beginning of the annealing and small perturbations on the design through shape modification at the end. The probabilities for rule selection do not change during the simmer section of the annealing schedule, although the magnitude of the shape modification rules (i.e., change in node location or cross-sectional area) decreases.

4 Optimal Grouping

The incorporation of optimal grouping in the shape annealing method determines the optimal number of groups and the optimal cross-sectional area for each group. The weighted penalty function shown below has been added to the objective function to create a tradeoff between minimizing the number of groups and minimizing the mass in the structural design process:

$$\text{group\_objective} = \text{group\_weight} * \text{group\_penalty}; \quad (2)$$

$$\text{group\_penalty} = e^{(\text{iteration/max\_iterations})} * \text{number\_of\_groups}^2. \quad (3)$$

The constant, group_weight, allows the designer to define the relative importance between the competing objectives, number of groups and mass. The value of group_weight determines the relative cost of adding a new group to a structure. By increasing group_weight, the cost of adding a group increases and thus the tradeoff with mass, that is, the amount of mass that the structure must be reduced by in order for the design to have the same objective value, also increases. The group_penalty function was chosen to exhibit the following behavior: a strong

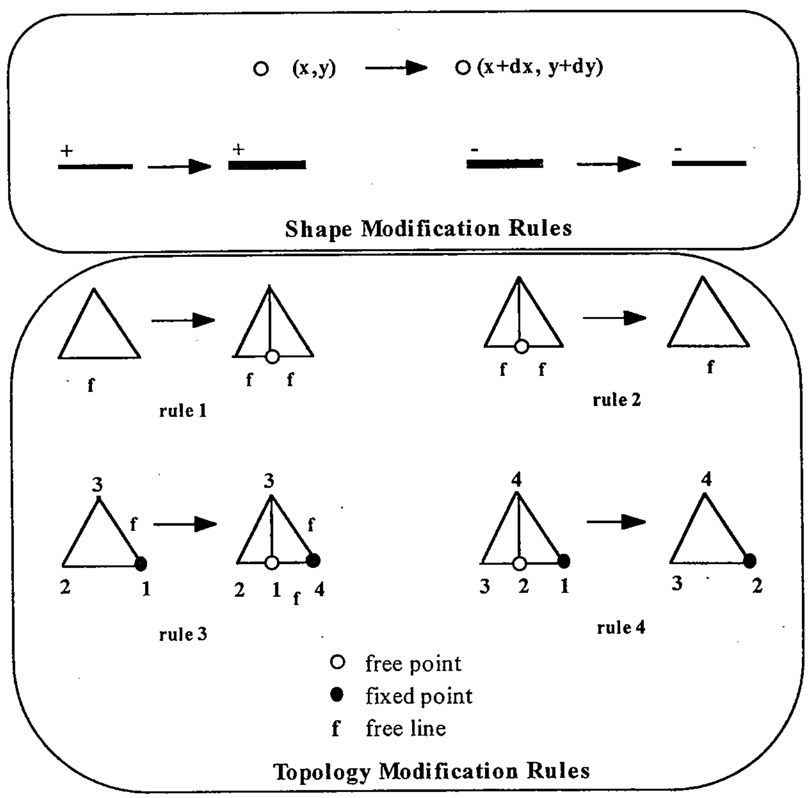

Fig. 3 Shape Grammar

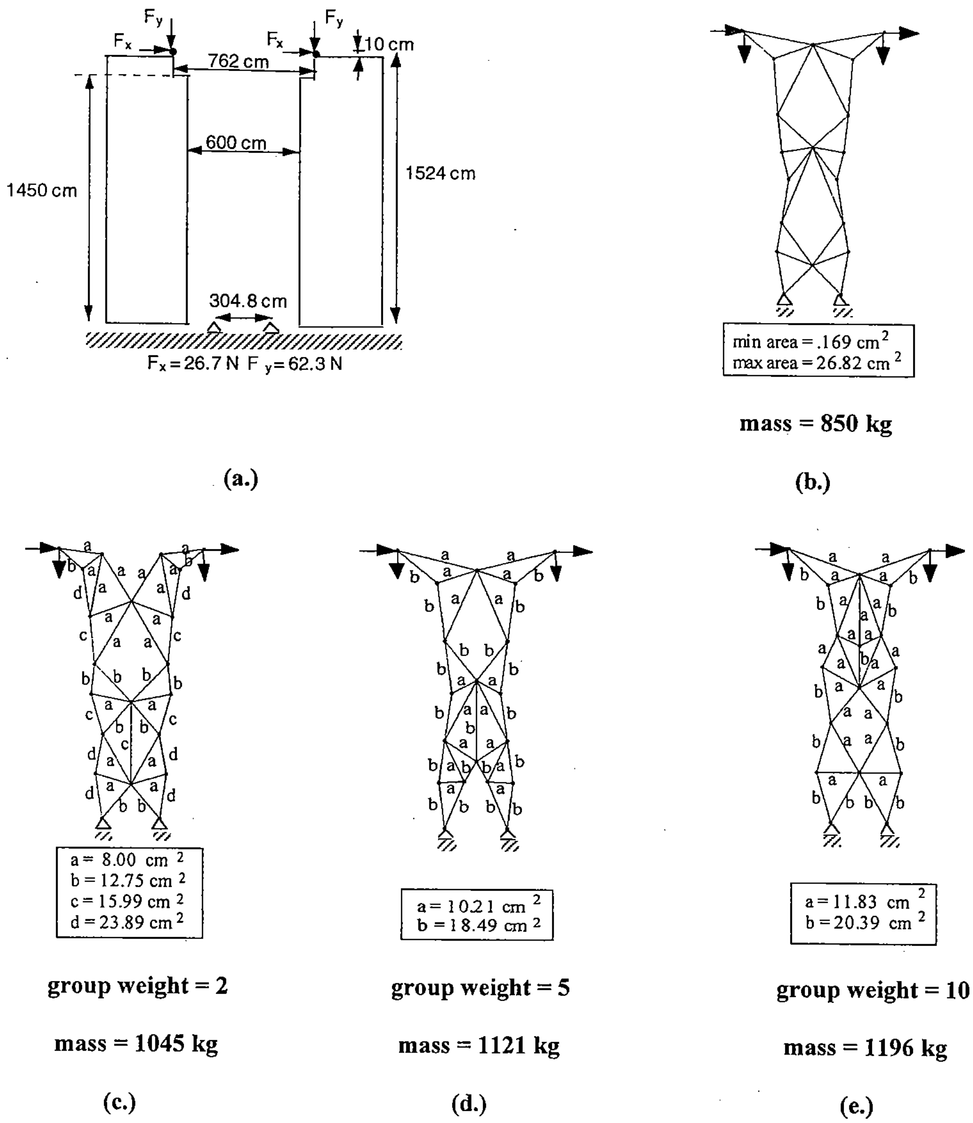

Fig. 4 Transmission Tower

effect with the number of groups and an increase in the relative importance of the number of groups in the objective function as the design progresses.

To examine the tradeoff between minimum mass and the number of groups, consider a fixed determinate structural topology under a single loading condition. If each member in the fixed layout is allowed to have a distinct cross-sectional area the optimal mass will be given by requiring maximum efficiency of all members through setting each member at its stress or buckling limit. Imposing a restriction on cross-sectional area decreases the efficiency of the structure by preventing some members from being at this limit. By varying the structural topology and shape, the structure can compensate to an extent for this restriction on cross-sectional area through a search for optimal groupings of members. Thus with group restrictions the design will not be optimal with respect to mass alone but rather with respect to the relative importance of mass and number of groups reflected in the weighted objective function. The tradeoff between mass efficiency and member grouping is less clear for indeterminate structures or structures with multiple loading conditions and will be explored in the examples.

5 Dynamic Grouping

To determine the groups of members within the shape annealing method, for each new structure the members are regrouped based on cross-sectional area. This process takes place at each iteration after a new design has been generated but before the analysis is performed so that the analysis reflects the cross-sectional areas imposed by the grouping. To group members, they are first sorted into increasing order by cross-sectional area. The first group is set to the smallest member cross-sectional area. The range of cross-sectional areas for this group has as its lower limit the first cross-sectional area and as the upper limit the first cross-sectional area plus a defined tolerance. Members are added to this group as long as they fall in this range. Once a member is found that is larger than the upper limit of the current group range a new group is created with the new cross-sectional area as the lower limit and the new cross-sectional area plus the tolerance as the upper limit. This process continues until all members are assigned to a group. The cross-sectional area variable for each member in a group is then reset to the calculated group size which is the average cross-sectional area of all members in the group. If gauge sizes were considered the group size would be assigned to the closest gauge size to the calculated average cross-sectional area. Members within a group still maintain independent cross-sectional area variables after the members are grouped. Members at or near the minimum allowable cross-sectional area are not considered in the grouping since they may be removed from the structure as long as the structure remains stable. If a member of minimum area

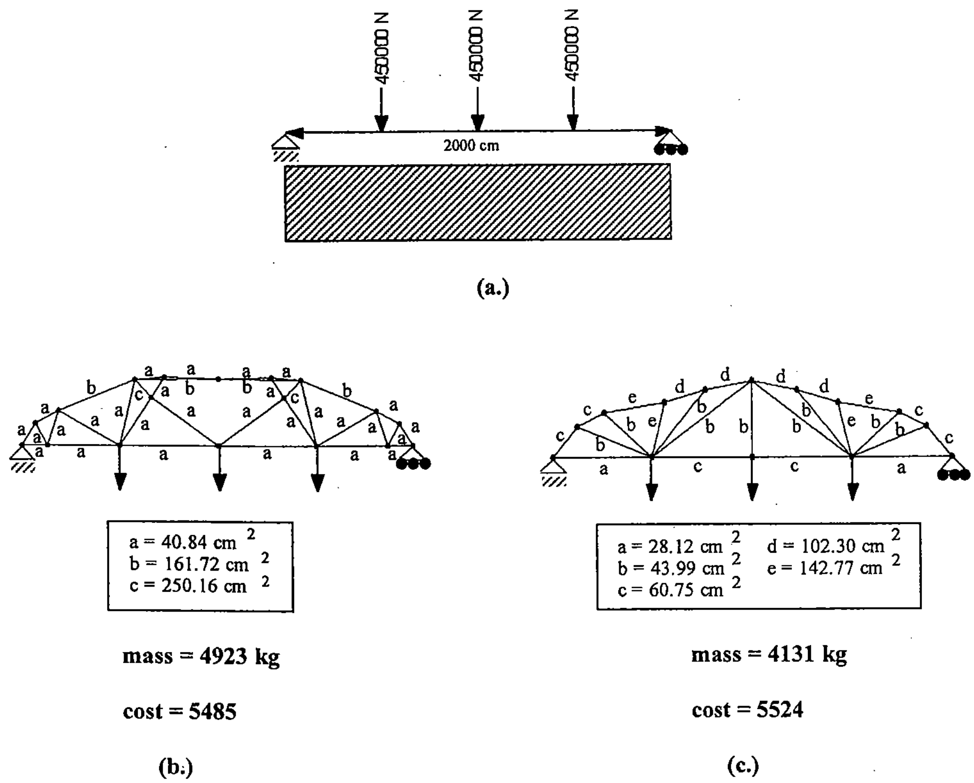

Fig. 5 Truss Problem

is not removed from the structure for stability or aesthetic reasons the cross-sectional area is reset to the smallest group cross-sectional area. Another alternative would be to allow an extra group of the minimum cross-sectional area.

The tolerance used in grouping the members defines the range of cross-sectional areas within a group that will be forced to a common group size or cross-sectional area. The tolerance is determined from the user defined variables, the start tolerance and the end tolerance, where the start tolerance is larger than the end tolerance. The tolerance is then calculated over the annealing schedule and decreases linearly from the start tolerance to the end tolerance allowing looser groupings of members at the beginning of the annealing and tighter groupings towards the end. The tolerance is always smaller than the maximum change in cross-sectional area so that upon application of the rule to change the cross-sectional area of a member the member may move between groups.

6 Results

This method for generating topologies with optimal grouping will be demonstrated on two problem specifications, a symmetric transmission tower with asymmetric loading and a symmetric truss. Both examples have constraints on stress and Euler buckling. Since a perfectly symmetric design is not expected to be generated from this method both examples impose symmetry by representing only one-half of the design that is reflected to yield a full symmetric design. For analysis purposes symmetric boundary conditions are imposed if the loading is symmetric or the design is reflected to create a full analysis model if the loading is asymmetric. The maximum tolerance used in grouping the members is 3 cm^2 at the beginning of the annealing reducing linearly to a tolerance of .6 cm^2 at the end. The minimum area allowed is .06 cm^2. A maximum of fifty members are allowed to be generated during the design process. All topologies generated must maintain a connection between the fixed points and the applied loads of the problem specification. Thus, possible generated topologies range from a minimum topology connecting the fixed points to the loads to a maximum topology that also connects the fixed points and loads but consists of fifty members. Each design is generated in a maximum of 34,000 iterations, taking approximately eighty minutes on a DEC alpha, while some designs may converge earlier. The final cost of the structure is calculated from the cost function in equation (1), using the group_weight and the group_penalty in Eqs. (2) and (3).

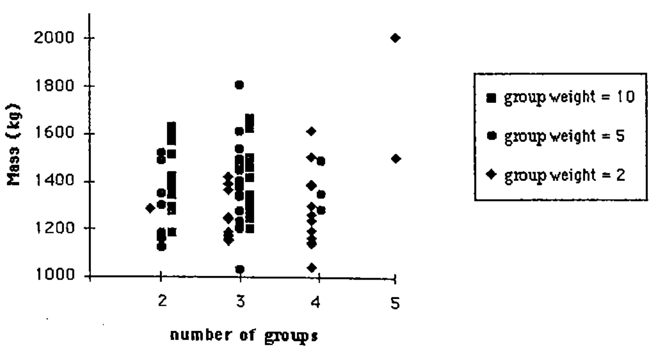

Fig. 6 Comparison of Group Weights

6.1 Transmission Tower. The transmission tower problem specifications shown in Fig. 4(a) are based on the planar tower problem in Vanderplaats and Moses (1972). The material used has a modulus of elasticity, E, of 20.67×10^6 N/cm^2, allowable tensile stress of 13790 N/cm^2 and compressive stress of 10340 N/cm^2 and mass density, ρ, of .0083 kg/cm^3. The structural members are tubes with continuous cross-sectional areas where $D/t = 10$ and the buckling factor is $10.1\pi EA/8L^2$. There are three geometric obstacles: two placed to the sides of the supports and one placed below the supports representing ground level. The side obstacles were chosen to allow access to the power lines that are represented by the applied load F_y. The second applied load, F_x, represents the wind load that cre-

Table 1 Convergence statistics for all tower results

	Group Weight		
	2	5	1 0
mean mass (σ)	1359 (210)	1418 (226)	1489 (203)
mean cost (σ)	1473 (251)	1720 (908)	3058 (5804)
mean number of groups (σ)	3.73 (.70)	2.90 (.61)	2.53 (.51)

ates an asymmetric loading. Since the structure is required to be symmetric the wind load can be supported from either direction.

The structure in Fig. 4(b) shows the best design generated using shape annealing without a restriction on cross-sectional area. This design, with a mass of 850 kg, is comparable to the best designs found by Vanderplaats and Moses (1972) using shape optimization with masses ranging from 844 kg to 866 kg. Figures 4(c) through 4(e) show the best design determined by the cost function from 30 designs generated for each group weight. Again, the group weight determines the relative importance of minimizing the group objective in the objective function. All of the designs with the group restriction result in an increase of mass over design 4(b), as expected. The group sizes determined by the algorithm for designs 4(d) and 4(e) with only two groups are in the middle of the range of the cross-sectional areas of design 4(b) where there is no group restriction. When the number of groups increases to four in design 4(c), the group sizes move towards the extremes of the range of cross-sectional areas in design 4(b). When the group weight is increased from two to five, Figs. 4(c) and 4(d) respectively, the algorithm finds a solution with an increase in mass but a decrease in the number of groups from four to two, resulting from the increase in importance of minimizing the number of groups. Comparing designs 4(d) and 4(e), increasing the group weight from five to ten results in the same number of groups and an increase in mass. This increase in mass could be due to the algorithm focusing more on minimizing the number of groups rather than mass causing certain moves that lead to design 4(d) to be rejected when generating design 4(e). Again, the shape annealing method does not guarantee that a global optimum will be found but rather an optimally directed design.

6.2 Truss Problem. This example shows the design of a standard truss structure, like that for a bridge, where symmetry is imposed in the structural layout and the loading. The material used is steel with a modulus of elasticity, E, of 2.0×10^7 N/cm^2, allowable tensile and compressive stress of 25000 N/cm^2 and mass density, ρ, of .00785 kg/cm^3. The structural members are solid bars with a buckling factor of $\pi EA/4L^2$. There is one geometric obstacle placed below the supports to represent ground level. The group weight is set to 20 for the designs shown in Fig. 5. To consider the tradeoff between mass and the number of groups for a particular objective function two designs with similar cost values but different topologies are shown. The design in Fig. 5(b) has a higher mass than that in 5(c) but two fewer groups, trading off economy of scale with mass. For results of a similar problem where symmetry is not imposed see (Shea et al., 1995).

6.3 Discussion. The results show that adding groups to the objective function in the structural layout optimization problem affects the mass of the structures generated. An investigation into the increase in mass associated with an increase in the importance of groups, through increasing the group weight in the objective function, is now shown. This analysis was performed for the transmission tower example specifications of Section 6.1. Group weights of two, five and ten were considered with 30 designs generated for each group weight. A comparison of mass and number of groups for all designs generated in this study is shown in Fig. 6. When the group weight is set to two, the majority of designs found have three and four groups, while when the group weight is increased to five, the majority of designs have two and three groups. When the group weight is increased to ten, only designs having two and three groups are found.

As the group weight increases the mean number of groups decreases and the mean mass increases. This is shown in Table 1, which includes statistics for all thirty designs for each group weight. Additionally, since the group sizes are not determined a priori, the range of group sizes increases as the number of groups increases, as shown in Fig. 4. Because simulated annealing can get trapped in local optima it is standard practice to take the best design out of every three designs generated. Using this heuristic, a second set of statistics is shown in Table 2 calculated from a total of 10 designs for each group weight. For the tower problem, the algorithm tends to get trapped in a local optimum on average one in ten times; thus, the convergence statistics are significantly improved by using the one in three heuristic.

8 Conclusions

A solution to a multicriterion structural optimization problem that incorporates efficiency and economy was shown. A tradeoff

Table 2 Convergence statistics for the best 1:3 of all tower results

	Group Weight		
	2	5	1 0
mean mass (σ)	1221 (111)	1262 (139)	1333 (118)
mean cost (σ)	1314 (105)	1394 (154)	1541 (112)
mean number of groups (σ)	3.40 (.70)	2.70 (.67)	2.40 (.52)

between the number of member groups, which reduces material purchase and fabrication costs, and the minimum mass of the structure was created by adding a weighted group penalty function to the objective function. The shape annealing method was modified to include a method for dynamic grouping of members based on cross-sectional area. The results show that the method succeeds in generating trusses with optimally directed topologies as well as number of groups and cross-sectional areas of each group with respect to the defined objective function.

Acknowledgments

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