Robust Design for Profit Maximization With Aversion to Downside Risk From Parametric Uncertainty in Consumer Choice Models

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In new product design, risk averse firms must consider downside risk in addition to expected profitability, since some designs are associated with greater market uncertainty than others. We propose an approach to robust optimal product design for profit maximization by introducing an α-profit metric to manage expected profitability vs. downside risk due to uncertainty in market share predictions. Our goal is to maximize profit at a firm-specified level of risk tolerance. Specifically, we find the design that maximizes the α-profit: the value that the firm has a $(1-\alpha)$ chance of exceeding, given the distribution of possible outcomes. The parameter $\alpha \in (0,1)$ is set by the firm to reflect sensitivity to downside risk (or upside gain), and parametric study of α reveals the sensitivity of optimal design choices to firm risk preference. We account here only for uncertainty of choice model parameter estimates due to finite data sampling when the choice model is assumed to be correctly specified (no misspecification error). We apply the delta method to estimate the mapping from uncertainty in discrete choice model parameters to uncertainty of profit outcomes and identify the estimated α-profit as a closed-form function of decision variables for the multinomial logit model. An example demonstrates implementation of the method to find the optimal design characteristics of a dial-readout scale using conjoint data. [DOI: 10.1115/1.4007533]

Keywords: design for market systems, delta method, logit, design optimization, robust design, design under uncertainty, discrete choice model

1 Introduction

Over the last three decades, a significant portion of the new product development (NPD) literature has been dedicated to the integration of engineering design and marketing processes for differentiated markets. Simple models to determine the most profitable characteristics of a single new product [1,2] have progressed to account for issues such as product-line design and preference heterogeneity [3–7], competitor reactions [8–10], cost structure [11,12], distribution channels [9,13–16], choice-set-dependent preferences [17], and coordination with constrained engineering design decisions [18–26].

As Hsu and Wilcox [27] argue, the trend toward estimating marketing models at lower levels of aggregation that are more structural[2] in consumer behavior representation (as opposed to high-level macro supply and demand equations) has led to models with many parameters and consequently greater uncertainty of those parameters. However, despite the advances in NPD methods, the research has not given much consideration to the intrinsic parameter uncertainty of the demand models. Demand uncertainty directly affects the risk of introducing a new product into the market, and firms evaluate potential projects not only in terms of expected return, but also in terms of risk.

The purpose of this work is threefold. First, we define a robust *a*-profit metric and propose a general framework to incorporate demand uncertainty arising from choice model parameter estimation into the design decision process such that it accounts for varying risk tolerance profiles. Second, we apply the delta method to approximate the *a*-profit function in closed-form for multinomial logit (MNL) choice models to be used efficiently in numerical optimization routines. Finally, we show how ignoring demand uncertainty can lead to suboptimal decisions for risk averse firms.

We do not intend to consider all the various sources of demand model uncertainty [28], and several questions will remain open. In particular, we assume the discrete choice model is correctly specified and ignore uncertainty due to model misspecification, and we assume that the model parameters do not change over time or from the context in which the data were collected to the context in which predictions will be made. Nevertheless, the proposed methodology can be useful, and it serves as a step in addressing design for profit maximization under demand model uncertainty.

This paper begins by discussing the relevant literature on product design, pricing under uncertainty and incorporation of firm risk tolerance in Sec. 2. Section 3 describes the proposed methodology for finding optimal designs for varying levels of tolerated product profit uncertainty and applies it to multinomial logit demand models. Section 4 presents an example application using the multinomial logit demand model to determine the optimal attributes of a dial-readout bathroom scale from the literature for different levels of risk aversion. Section 5 discusses conclusions, limitations, and future work.

2 Literature Review

Demand uncertainty is caused by several factors such as preference dynamics [29], demand model misspecification [30,31], choice context [32,33], response variability [34,35], and sampling errors associated with the estimation procedure [36]. As a result,

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[2] "Econometric models that are based explicitly on the consumer's maximization problem and whose parameters are parameters of the consumers' utility functions or of their constraints are referred to as structural models." [52]

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Table 1 Papers that consider choice model parameter uncertainty as a source of demand uncertainty

References	Treats demand uncertainty as	Design attributes	Design objective(s)
Hsu and Wilcox [27]	Probability distribution of market share obtained by simulation	NA	NA
Luo et al. [36]	Interval estimates of market shares obtained using 95% confidence levels for the utility function	Discrete	—Maximize nominal market share —Minimize performance variance —Maximize worst-case performance
Besharati et al. [21]	Interval estimates of market shares obtained using 95% confidence levels for the utility function	Discrete	—Maximize nominal share —Minimize share variance —Maximize nominal performance
This paper	Probability distribution of market share estimated by delta method	Continuous	—Maximize profit at specified downside risk tolerance level

several researchers have considered the impact of demand uncertainty on optimal pricing strategies [29,30,37,38]. However, in contrast to prices, design decisions are difficult to change post hoc, especially in durable-goods markets. Products with high start-up capital costs can have virtually unchangeable characteristics, and producers are incentivized to consider demand uncertainty during the initial stages of the design process (e.g., car manufacturers invest a significant portion of capital up front in production equipment, and changing a characteristic such as the footprint of a car leads to very high costs).

Hazelrigg [39] proposed applying von Neumann-Morgenstern [40] utility theory as the frame for selecting among competing design alternatives by taking the firm's profit (net present value) as the sole design objective and using a firm-level (single-attribute) utility function to manage risk of uncertain profit outcomes. This decision-based design (DBD) framework and its variants have been explored and implemented in subsequent literature (e.g., Ref. [41]), some of which has applied discrete choice models to predict demand as a function of product attributes (e.g., Refs. [19,42]).

The use of firm utility functions to describe risk preference has advantages. In particular, specifying a utility function over profit outcomes is a flexible approach, accounting for risk sensitivity over the entire distribution of outcomes. However, in practice a firm's utility function can be difficult to identify. This is in part because firm preferences do not necessarily satisfy utility axioms (e.g., ability to express consistent, transitive preferences over all possible outcomes); managers are not accustom to specifying utility functions or answering lottery questions consistently; and it is not straightforward to assess uncertainty caused by error and misspecification of the firm's utility function[3]. Indeed, firm utility functions in the DBD literature are typically fictitious or left unspecified (e.g., Refs. [19,43]).

We take an alternative approach, instead asking the decision-maker to specify a single parameter α to represent risk preference and then conducting parametric studies to help the decision-maker understand how the optimal design changes with different choices of α. The main restriction is that we assess the distribution of profit outcomes at a single critical point, rather than assessing the entire distribution. However, the advantages include (1) closed-form solutions that enable efficient optimization; (2) improved intuition and ease of managerial interpretation and specification; and (3) ease of parametric study to understand the sensitivity of design choices to risk preference.

We are interested in uncertain profit outcomes that result from uncertainty in product demand predictions. Several authors have addressed product demand uncertainty resulting from variation in engineering design model parameters (e.g., due to manufacturing variability or usage conditions) [19,21,36,39]. Two of these publications also account for uncertainty in the marketing model parameters: Luo et al. [36] and Besharati et al. [21], and both model this uncertainty using intervals.

In particular, Luo et al. [36] use the parameter covariance matrix of part-worth utility point estimates to obtain 95% confidence intervals around the point estimates from the design parameter best- and worst-case scenarios for a set of product alternatives under consideration. The greatest utility under the best-case scenario and lowest utility under the worst-case scenario within the confidence interval are compared to the similarly constructed estimates of utility for competitor products. The highest own-utility is compared to the sum of the lowest competitor-utilities and vice versa to construct interval estimates of market shares (these no longer represent statistical confidence intervals for market share). They then use pair-wise comparisons to eliminate dominated alternatives (defined as alternatives that have a best-case market share worse than an alternative's worst-case market share, perform worse on worst-case performance, and have higher performance variability). All nondominated designs are then considered for prototyping and further subjective evaluation.

Besharati et al. [21] use a framework similar to Luo et al. [36], but they change the optimization criteria arguing that looking for the best performance on the worst-case condition might be too conservative. Alternatively, they replace the design objectives of worst-case performance and performance variability with multi-objective optimization of nominal performance characteristics. The marketing model is also treated as a multi-objective optimization problem of maximizing nominal market share and minimizing the market share variance (penalizing both positive and negative variation) resulting from uncertainty in both engineering design parameters and part-worth utility estimates. Finally, they develop a ranking system for pair-wise comparison of designs on the design and marketing criteria.

Hsu and Wilcox [27] use the estimation error associated with the parameter estimates to find the stochastic market share prediction in a multinomial logit framework. They use a simulation-based approach for approximating the distribution efficiently.

Table 1 compares the above papers that consider the uncertainty in demand model parameters as a source of demand uncertainty and positions our contribution against this prior work. We address variance of profit estimates but do not seek to minimize it as a means to improve robustness because profit uncertainty is harmful to a firm only in the negative tail—i.e., when product demand is less than expected – and we avoid penalizing uncertainty that could lead to higher than expected profits.

We apply an α-profit metric in conjunction with discrete choice models as a means to incorporate firm risk tolerance into the new product design optimization process. This allows us to develop a framework to find optimal product characteristics and price in a continuous domain, instead of requiring a discrete set of product alternatives; and in contrast to Luo et al. [36] and Besharati et al. [21], we can treat demand uncertainty as a continuous probability distribution instead of representing it as an interval. We use the delta method to derive a closed-form approximation for points on the market share distribution, since a simulation-based approach such as the one used

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[3] Though it is possible to conduct sensitivity analysis on the parameters defining the firm's utility function, misspecification of functional form remains, and interpretation of parametric sensitivity is generally cumbersome.

by Hsu and Wilcox [27], though efficient for estimating the stochastic distribution of a single design, would be computationally expensive and noisy when used as an intermediate function in a numerical optimization loop. Our framework focuses on demand models derived from random utility theory, particularly MNL models [44].

The α-profit methodology can be extended to multinomial probit (MNP) [45], mixed logit (MIXL) [46], and generalized logit (G-MNL) [47] models; however, any functional forms that require numerical simulation to compute may be computationally burdensome and introduce potential numerical issues when embedded within an optimization loop.

3 The Proposed Methodology

We want to find the characteristics of a new product in order to maximize a firm's profit; however, the uncertainty present in the demand model parameter estimates will result in uncertainty about predicted market share and resulting predicted profit, which we model as a distribution of potential profit outcomes for each design alternative. (A similar framework can also be used for maximizing alternative objective functions, such as market share.)

3.1 General Mathematical Formulation. Our goal is to find the design whose predicted profit distribution maximizes the α-profit: the value below which less than an α fraction of the cumulative profit distribution falls. The parameter α is set by the firm to reflect sensitivity to downside risk (or upside gain), and parametric study of α reveals the sensitivity of optimal design choices to firm risk preference. We define the α-profit $\pi_j^\alpha(\hat{\boldsymbol{\beta}}, \mathbf{X})$ as the value of the profit distribution at level $\alpha \in (0,1)$ for product $j \epsilon \{1, 2, \ldots J\}$ given the column vector of random variables $\hat{\boldsymbol{\beta}} \sim N(\bar{\boldsymbol{\beta}}, \boldsymbol{\Sigma})$ that define the choice model parameter estimates and the values of the n attributes (including price) for each of the J products available in the market $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_j, \ldots, \mathbf{x}_J] \in \mathbb{R}^{J \times n}$. Specifically, if $\hat{\pi}_j$ is a random variable with cumulative distribution function $F_{\hat{\pi}|\hat{\boldsymbol{\beta}},\mathbf{X}}(\pi)$ representing the distribution of profit outcomes conditional on $\hat{\boldsymbol{\beta}}$ and $\mathbf{X}$, then $\pi_j^\alpha(\hat{\boldsymbol{\beta}}, \mathbf{X})$ is the maximum value of π_j for which $F_{\hat{\pi}|\hat{\boldsymbol{\beta}},\mathbf{X}}(\pi) \le \alpha$, i.e., for which $\Pr(\hat{\pi}_j < \pi_j) \le \alpha$ (see Fig. 1). If $F_{\hat{\pi}|\hat{\boldsymbol{\beta}},\mathbf{X}}(\pi)$ is continuous and invertible, then $\pi_j^\alpha\left(\hat{\boldsymbol{\beta}}, \mathbf{X}\right) = F^{-1}_{\hat{\pi}|\hat{\boldsymbol{\beta}},\mathbf{X}}(\alpha)$.

Our objective is to find the product attributes and price that maximize the robust profit given the α level that reflects firm sensitivity to downside risk. That is, we seek the robust optimal new product characteristics $\mathbf{x}_j^{\alpha*}$ at level α, where $\mathbf{x}_j^{\alpha*} = \operatorname{argmax}_{\mathbf{x}_j}\left(\pi_j^\alpha(\hat{\boldsymbol{\beta}}, \mathbf{X})\right)$; i.e., $\mathbf{x}_j^{\alpha*}$ is the design that maximizes the value of profit that the model predicts a $(1-\alpha)$ chance of exceeding. For illustration, Fig. 2 shows the probability density function of profit for two alternative designs. Design 1 is preferred over design 2 when optimizing for the expected value of profit. However, design 1 has more downside risk, and a risk averse firm optimizing for the α-profit with small α would prefer design 2.

Defining for product j the random variable describing the distribution of market share outcomes, $\hat{s}_j$; market share at level α, s_j^α; price, p_j; variable cost, $c_j = f_{\mathrm{VC}}(\mathbf{x}_j)$; fixed cost, C_j, and total market size, m; we have $\hat{\pi}_j = m(p_j - c_j)\hat{s}_j - C_j$ and $\pi_j^\alpha = m(p_j - c_j)s_j^\alpha - C_j$. Assuming that there is no uncertainty on product price and costs and that $p_j > c_j$:

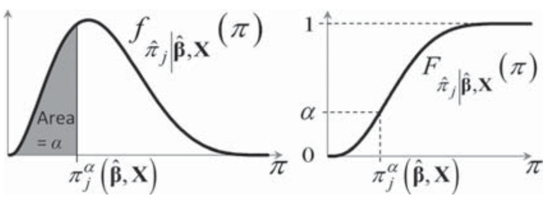

Fig. 1 α-profit shown for (*a*) probability density function of profit and (*b*) cumulative distribution function of profit

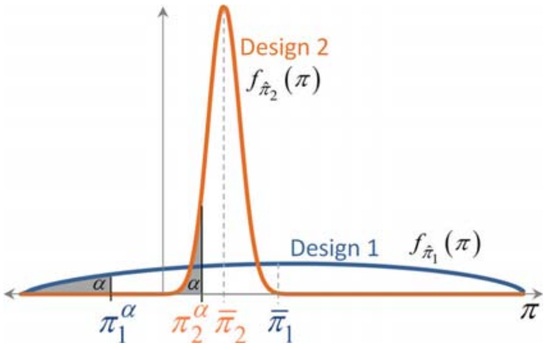

Fig. 2 Expected profit vs. downside risk: the expected profit for design 1 is higher than the expected profit for design 2 ($\pi_1 > \pi_2$); however, design 2 has a higher profit at the α-level than design 1 ($\pi_2^\alpha > \pi_1^\alpha$)

$$\Pr\left(\hat{\pi}_j < \pi_j^\alpha\right) = \Pr\left(m(p_j - c_j)\hat{s}_j < m(p_j - c_j)s_j^\alpha\right) = \Pr\left(\hat{s}_j < s_j^\alpha\right) \tag{1}$$

Therefore

$$\Pr\left(\hat{\pi}_j < \pi_j^\alpha\right) \le \alpha \Leftrightarrow \Pr\left(\hat{s}_j < s_j^\alpha\right) \le \alpha \tag{2}$$

In the Secs. 3.2–4.3, we will show how to find the robust optimal new product attributes, as defined in this section, for the MNL demand model, given uncertainty in the estimated parameters.

3.2 Application to Multinomial Logit Demand Model. We apply the proposed methodology to the MNL model [44] for several reasons: (1) it is among the most simple discrete choice model specifications, permitting closed-form choice probabilities and closed-form expressions for alpha profit in our applications; (2) it is the most widely used discrete choice model broadly and within the NPD literature specifically [14,48,49], due to its closed-form choice probabilities and interpretability [50]; and (3) several discrete choice models evolved from MNL, such as MIXL and G-MNL, and a better understanding of how uncertainty affects NPD under MNL models may be useful in understanding the effects of uncertainty under its variants. For the purposes of this paper, we assume that the model is correct and that the uncertainty arises from the parameter estimation and not model misspecification.

In a multinomial logit model, given some competitive set of J products, the predicted market share s_j for product j can be computed as

$$s_j(\mathbf{v}) = \frac{e^{v_j}}{\sum_{k=1}^{J} e^{v_k}} \tag{3}$$

where $\mathbf{v} = (v_1, \ldots, v_J)$ is the vector of observable utility point estimates of the respective products and the no-choice option (outside good) is not included.

The utility function is often specified to be linear in parameters: $v_k = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k$, resulting in predicted market share $s_j(\mathbf{X})$ for product $j \in \{1, \ldots, N\}$:

$$s_j(\mathbf{X}) = \frac{e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_j}}{\sum_{k=1}^{J} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}} \tag{4}$$

Ignoring constant fixed costs without loss of generality, in a multinomial logit demand model the predicted profit π_j can be computed as

$$\pi_j(\mathbf{X}) = m(p_j - c_j)s_j(\mathbf{X}) = m(p_j - c_j)\frac{e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_j}}{\sum\limits_{k=1}^{J} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}} \tag{5}$$

The classical practice is to use maximum-likelihood methods to estimate the parameters $\boldsymbol{\beta}$ in multinomial logit models [27]. Train [50] notes that the estimates are easily obtained since the log-likelihood function is concave for linear utility specifications, and Wooldridge [51] proves that the maximum-likelihood estimator $\hat{\boldsymbol{\beta}}$ is asymptotically normally distributed with distribution $\hat{\boldsymbol{\beta}} \sim N(\bar{\boldsymbol{\beta}}, \boldsymbol{\Sigma})$, where $\bar{\boldsymbol{\beta}}$ is the vector of means and $\boldsymbol{\Sigma}$ is the covariance matrix, implying that $\hat{v}_j \sim N(\bar{\boldsymbol{\beta}}^{\mathrm{T}}\mathbf{x}_j, \mathbf{x}_j^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{x}_j)$.

The exact distribution of $\hat{s}_j$ is unknown, but the delta method enables analytic approximation of a transformed distribution using a linear approximation of the mapping function. This frees us from the computational burden of simulating a market share distribution for each choice of product attributes in the optimization loop, as would be required by the method in Hsu and Wilcox [27]. The delta method states that any function of a normally distributed random variable (in this case the estimated parameters) converges asymptotically to a normal distribution (see Ref. [51] for proof). The delta method relies on a Taylor series expansion of the mapping function g. If the function of the expected value of the parameters is $g(\bar{\boldsymbol{\beta}})$, then $g\left(\hat{\boldsymbol{\beta}}\right) \cong g(\bar{\boldsymbol{\beta}}) + \nabla g(\bar{\boldsymbol{\beta}})\left(\hat{\boldsymbol{\beta}} - \bar{\boldsymbol{\beta}}\right)$ where $\nabla g(\boldsymbol{\beta})$is a row vector. The mean and variance of $g(\hat{\boldsymbol{\beta}})$ can be calculated as

$$\mathrm{E}\left[g\left(\hat{\boldsymbol{\beta}}\right)\right] \cong \mathrm{E}\left[g(\bar{\boldsymbol{\beta}}) + \nabla g(\bar{\boldsymbol{\beta}})\left(\hat{\boldsymbol{\beta}} - \bar{\boldsymbol{\beta}}\right)\right] = g(\bar{\boldsymbol{\beta}}) \tag{6}$$

$$\begin{aligned}
\mathrm{Var}\left[g\left(\hat{\boldsymbol{\beta}}\right)\right] &\cong \mathrm{Var}\left[g(\bar{\boldsymbol{\beta}}) + \nabla g(\bar{\boldsymbol{\beta}})\left(\hat{\boldsymbol{\beta}} - \bar{\boldsymbol{\beta}}\right)\right] \\
&= \mathrm{Var}\left[g(\bar{\boldsymbol{\beta}}) + \nabla g(\bar{\boldsymbol{\beta}})\hat{\boldsymbol{\beta}} - \nabla g(\bar{\boldsymbol{\beta}})\bar{\boldsymbol{\beta}}\right] \\
&= \mathrm{Var}\left[\nabla g(\bar{\boldsymbol{\beta}})\hat{\boldsymbol{\beta}}\right] \\
&= \nabla g(\bar{\boldsymbol{\beta}})\mathrm{Var}\left[\hat{\boldsymbol{\beta}}\right]\nabla g(\bar{\boldsymbol{\beta}})^{\mathrm{T}} \\
&= \nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{\mathrm{T}}
\end{aligned} \tag{7}$$

As with any linear approximation to a nonlinear function, the approximation may lead to significant distortion of the function outside the neighborhood of $g(\bar{\boldsymbol{\beta}})$.

The quantity of interest $\hat{s}_j$ is itself a function of $\hat{\boldsymbol{\beta}}$, but $s_j \in (0, 1)$, which does not match the domain of the normal distribution. Instead, we select the intermediate function $g(\boldsymbol{\beta}) = \ln(1/s_j - 1) \in (-\infty, +\infty)$ so that it has the same domain as a normal distribution and so that in the case of a monopolistic single-product firm with an outside good, the approximation leads to the exact distribution of $g(\boldsymbol{\beta})$.

$$g(\boldsymbol{\beta}) = \ln\left(\frac{1}{s_j} - 1\right) = \ln\left(\frac{\sum\limits_{k=1}^{J} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}}{e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_j}} - 1\right) = \ln\left(\sum_{k \in J \setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k - \mathbf{x}_j)}\right) \tag{8}$$

By the delta method, we know that

$$g\left(\hat{\boldsymbol{\beta}}\right) \overset{a}{\sim} N\left(g(\bar{\boldsymbol{\beta}}), \nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right) \tag{9}$$

Since

$$\nabla g(\boldsymbol{\beta}) = \frac{\sum\limits_{k \in J \setminus j} (\mathbf{x}_k - \mathbf{x}_j)e^{\boldsymbol{\beta}^{T}\mathbf{x}_k}}{\sum\limits_{k \in J \setminus j} e^{\boldsymbol{\beta}^{T}\mathbf{x}_k}} \tag{10}$$

we can approximate the variance of g for any given $\mathbf{X}$. See Appendix A for details and Appendix B for formulation when an outside good is present. Because

$$\hat{s}_j < s_j^{\alpha} \quad \Leftrightarrow \quad \left(\frac{1}{\hat{s}_j} - 1\right) > \left(\frac{1}{s_j^{\alpha}} - 1\right) \quad \Leftrightarrow \quad g\left(\hat{\boldsymbol{\beta}}\right) > \ln\left(\frac{1}{s_j^{\alpha}} - 1\right) \tag{11}$$

we can calculate

$$\Pr\left(\hat{s}_j < s_j^{\alpha}\right) = \alpha \quad \Leftrightarrow \quad \Pr\left(g\left(\hat{\boldsymbol{\beta}}\right) > \ln\left(\frac{1}{s_j^{\alpha}} - 1\right)\right) = \alpha \tag{12}$$

Normalizing the right hand equation

$$\Rightarrow \Pr\left(\frac{g(\bar{\boldsymbol{\beta}}) - g\left(\hat{\boldsymbol{\beta}}\right)}{\left(\nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)^{\frac{1}{2}}} < \frac{g(\bar{\boldsymbol{\beta}}) - \ln\left(\frac{1}{s_j^{\alpha}} - 1\right)}{\left(\nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)^{\frac{1}{2}}}\right) = \alpha \tag{13}$$

Since

$$\left(\frac{g(\bar{\boldsymbol{\beta}}) - g\left(\hat{\boldsymbol{\beta}}\right)}{\left(\nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)^{\frac{1}{2}}}\right) \overset{a}{\sim} N(0, 1) \tag{14}$$

The probability expression is the cumulative distribution of a standard normal, thus

$$\Pr\left(\hat{s}_j < s_j^{\alpha}\right) = \alpha \quad \Leftrightarrow \quad \Phi\left(\frac{g(\bar{\boldsymbol{\beta}}) - \ln\left(\frac{1}{s_j^{\alpha}} - 1\right)}{\left(\nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)^{\frac{1}{2}}}\right) = \alpha \tag{15}$$

where Φ is the cumulative distribution function of the standard normal distribution. Solving for s_j^{α}

$$s_j^{\alpha} = \left(1 + \exp\left(g(\bar{\boldsymbol{\beta}}) - \Phi^{-1}(\alpha)\left(\nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)^{\frac{1}{2}}\right)\right)^{-1} \tag{16}$$

Equation (16) enables a modeler to compute the estimated market share at the α risk level as a closed-form deterministic function of the decision variables using only the mean $\bar{\boldsymbol{\beta}}$ and covariance matrix $\boldsymbol{\Sigma}$ defining the choice model parameter estimates. Both $\bar{\boldsymbol{\beta}}$ and $\boldsymbol{\Sigma}$ are available from standard estimation procedures. The α-profit can then be computed as $\pi_j^{\alpha} = m(p_j - c_j)s_j^{\alpha} - C_j$. In the special case of a monopolistic single-product firm, this framework leads to the exact distribution of $g(\boldsymbol{\beta})$, since

$$\begin{aligned}
g\left(\hat{\boldsymbol{\beta}}\right) &= \ln\left(e^{-\hat{\boldsymbol{\beta}}^{\mathrm{T}}\mathbf{x}_j}\right) \\
&= -\hat{\boldsymbol{\beta}}^{T}\mathbf{x}_j \sim N\left(-\bar{\boldsymbol{\beta}}^{T}\mathbf{x}_j, \nabla g(\bar{\boldsymbol{\beta}})\boldsymbol{\Sigma}\nabla g(\bar{\boldsymbol{\beta}})^{T}\right)
\end{aligned} \tag{17}$$

which is identical to the delta method approximation in Eq. (9). Figure 3 illustrates the mapping for a model with a single parameter showing the normal distribution of the estimated model coefficient $\hat{\boldsymbol{\beta}}$, the resulting distribution of $g(\hat{\boldsymbol{\beta}})$, its normally distributed approximation via the delta method, and the resulting distribution of $\hat{s}_j$ and its (non-normal) approximation via the delta method.

As a result of the delta method formulation, the distribution of $\hat{s}_j$ depends on the variance of $g(\hat{\boldsymbol{\beta}})$, which depends on ∇g.

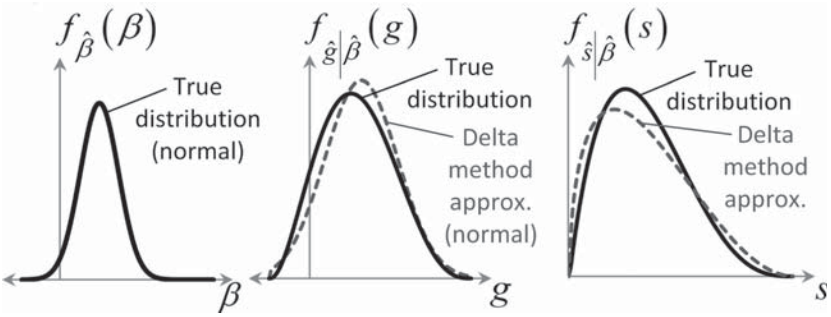

Fig. 3 Illustration of probability distribution functions and their approximations using the delta method

Because ∇g is proportional to $\sum_{k\in J\setminus j}(\mathbf{x}_\mathrm{k}-\mathbf{x}_\mathrm{j})^T e^{\boldsymbol{\beta}^\mathrm{T}\mathbf{x}_\mathrm{k}}$, the distribution of $\hat{s}_j$ is influenced by the distance of the new product's attributes from each of the attributes of existing products. All else being equal, greater differentiation implies higher uncertainty in market share predictions for the logit specification.

When an outside good is included in the problem formulation, the variance of $g(\boldsymbol{\beta})$ additionally depends on the differences between the firm's product attributes and the reference vector $\mathbf{0}$ (see Appendix C for details). This is problematic because the reference level can be arbitrarily chosen. As an example, for a set of products with a temperature attribute, the variance of $g(\boldsymbol{\beta})$ differs when temperature is measured in Kelvin with reference level 0 K versus measured in Celsius with reference level 273 K, even though a consumer would not perceive a difference in the product. The form of the logit model imposes a structure of increased uncertainty with distance from competitor attribute levels, and these levels are not meaningful with an outside good. For this reason, the method is recommended only for markets with no outside good, as in the example application.

4 Example Application

In this section, we examine the application of the proposed method to the optimal design of a dial-readout bathroom scale for a manufacturer using an engineering model and choice-based conjoint data from the literature [20]. It is assumed that the manufacturer is operating as a single-product firm and that it only competes with one other product. For consistency with the original notation of the example, we use $\mathbf{x}$ here to refer to the engineering design variables that define the product and $\mathbf{z}(\mathbf{x})$ to refer to product attributes observable by the customer.

4.1 Demand Side. Using the data from Michalek et al. [20], a conjoint survey was administered to 184 respondents. Each respondent was presented with 50 choice sets, consisting of three hypothetical analogue bathroom scales represented by the consumer attributes weight capacity, platform aspect ratio (ratio of length to width), platform area, gap between weight interval tick marks (readability), number size (weight reading), and price. In each choice set, the respondents selected which one of the three scales they would purchase or if they would not purchase any of the three (outside good option). The full data set consists of 184 respondents × 50 choice set responses per person for a total of 9200 choices—enough to identify logit model parameters with high certainty. We compare these highly certain results to a smaller data set of 250 choice observations, where the 50 choice set responses from five of the 184 respondents are randomly selected[4].

In the conjoint survey the six explanatory variables were presented to respondents at five discrete levels spanning the space of values under consideration (Table 2).

In order to select an appropriate utility function form specification, we begin by estimating a part-worth model. By visual inspection we choose an appropriate polynomial for each (linear in model parameters but not necessarily in attributes—see Appendix D for plots). Area and price are assumed linear while all other attributes are modeled as quadratic. If this model is misspecified, there will be additional uncertainty associated with model prediction, but we assume here correct specification and focus on uncertainty of parameter estimates due to missing data. Using the maximum-likelihood method for coefficient estimation, we obtain the results in Table 3. The attributes have been scaled for optimization stability and efficiency.

Table 2 Product characteristic and price levels

Desc.	Metric	Unit	Levels				
Capacity	Weight causing a 360 deg dial turn	Lbs	200	250	300	350	400
Aspect ratio	Platform length divided by width	—	6/8	7/8	8/8	8/7	8/6
Area	Platform length times width	in.^{2}	100	110	120	130	140
Gap	Distance between 1-lb tick marks	in.	2/32	3/32	4/32	5/32	6/32
Number size	Length of readout number	in.	0.75	1.00	1.25	1.50	1.75
Price	US dollars	$	10	15	20	25	30

Note: Source—Michalek et al. [20].

In the case of $n = 250$ choice observations, the constant, capacity, capacity^2, aspect ratio, aspect ratio^2, and price are statistically significant at the 0.01 level and number size is significant at the 0.05 level. The resulting utility curves for $E[\boldsymbol{\beta}]$ are plotted on top of the utility curves resulting from 500 draws of $\boldsymbol{\beta}$ from the multivariate normal distribution in Appendix D.

The information matrix obtained from the maximum-likelihood optimization problem is shown in Table 4. In the case of maximum-likelihood estimators, the information matrix is also

Table 3 Multinomial logit model coefficients

	$n = 250$			$n = 9200$		
Product attribute z	Coef.	Std. Error	t-stat	Coef.	Std. Error	t-stat
Constant	−21.49	4.24	−5.07	−12.95	0.59	−22.08
Capacity/100	5.16	1.51	3.42	2.94	0.21	13.92
(Capacity/100)^2	−0.76	0.24	−3.10	−0.45	0.03	−12.95
Aspect ratio	15.74	5.62	2.80	10.42	0.81	12.93
(Aspect ratio)^2	−8.22	2.72	−3.02	−5.44	0.39	−14.00
Area/100	0.50	0.67	0.74	0.05	0.10	0.50
Gap*10	2.46	1.50	1.64	2.43	0.22	10.87
(Gap*10)^2	−0.77	0.58	−1.31	−0.80	0.09	−9.14
Number size	5.42	2.49	2.18	4.84	0.36	13.62
Number size^2	−1.48	0.95	−1.57	−1.49	0.14	−10.82
Price/10	−0.71	0.14	−5.20	−0.79	0.02	−39.51

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[4] Results vary depending on which respondents are drawn.

Table 4 Coefficient variance-covariance matrix for $n = 250$ estimation data points

	β_const	β_cap.	β_cap^2	β_asp	β_asp^2	β_area	β_gap	β_gap^2	β_num	β_num^2	β_price
β_const	−17.97	3.89	−0.63	15.56	7.45	−0.68	−0.98	0.36	−4.48	1.66	−0.02
β_cap.	−3.89	2.27	−0.37	0.30	−0.13	0.07	0.01	0.00	0.45	−0.17	0.00
β_cap^2	0.63	−0.37	0.06	−0.06	0.03	−0.01	0.00	0.00	−0.07	0.03	0.00
β_asp	−15.56	0.30	−0.06	31.57	−15.23	−0.02	−0.84	0.31	−0.15	0.08	0.00
β_asp^2	7.45	−0.13	0.03	−15.23	7.42	0.02	0.42	−0.16	0.00	−0.02	0.00
β_area	−0.68	0.07	−0.01	−0.02	0.02	0.45	0.02	−0.01	0.07	−0.03	0.00
β_gap	−0.98	0.01	0.00	−0.84	0.42	0.02	2.24	−0.86	0.00	0.01	0.00
β_gap^2	0.36	0.00	0.00	0.31	−0.16	−0.01	−0.86	0.34	0.02	−0.01	0.00
β_num	−4.48	0.45	−0.07	−0.15	0.00	0.07	0.00	0.02	6.19	−2.34	−0.02
β_num^2	1.66	−0.17	0.03	0.08	−0.02	−0.03	0.01	−0.01	−2.34	0.90	0.01
β_price	−0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.02	0.01	0.02

Table 5 Engineering and marketing design variables

	Variable and description	Lower bound	Upper bound
Marketing variables			
z_1	Weight capacity	200 lbs	400 lbs.
z_2	Aspect ratio	0.75	1.33
z_3	Platform area	100 in.^{2}	140 in.^{2}
z_4	Tick mark gap	1/16 in.	3/16 in.
z_5	Number size	0.75 in.	1.75 in.
p	Price	\$5.00	\$35.00
Engineering variables			
x_1	Length from base to force on long lever	0.125 in.	21 in.
x_2	Length from force to spring on long lever	0.125 in.	21 in.
x_3	Length from base to force on short lever	0.125 in.	24 in.
x_4	Length from force to join on short lever	0.125 in.	18.175 in.
x_5	Length from force to joint on long lever	0.125 in.	18.175 in.
x_6	Spring constant	1.00 lb./in.	200 lbs./in.
x_7	Distance from base edge to spring	0.50 in	12 in.
x_8	Length of rack	1.00 in.	16.2 in.
x_9	Pitch diameter of pinion	0.25 in.	24 in.
x_{10}	Length of pivot's horizontal arm	0.50 in.	1.9 in.
x_{11}	Length of pivot's vertical arm	0.50 in.	1.9 in.
x_{12}	Dial diameter	1.00 in.	25 in.
x_{13}	Cover length	5.55 in.	19 in.
x_{14}	Cover width	7.4 in.	25 in.

Note: Adapted from Michalek et al. [20].

Table 6 Product attributes as a function of engineering design variables

Product attribute z	f(x)
z_1 (capacity)	$\frac{4\pi x_6 x_9 x_{10}(x_1+x_2)(x_3+x_4)}{x_{11}(x_1(x_3+x_4)+x_3(x_1+x_5))}$
z_2 (aspect ratio)	x_{13}/x_{14}
z_3 (platform area)	$x_{13}*x_{14}$
z_4 (tick mark gap)	$\pi*(x_{12}/z_1)$
z_5 (number size)	$\frac{\left(2\tan\frac{\pi y_{11}}{z_1}\right)\left(\frac{x_{12}}{2}-y_{10}\right)}{1+\frac{2}{y_{12}}\tan\frac{\pi y_{11}}{z_1}}$

Note: Adapted from Michalek et al. [20].

the variance-covariance matrix $\boldsymbol{\Sigma}$ of the estimators $\hat{\boldsymbol{\beta}}$ (see Ref. [51] for proof).

4.2 Supply Side. Following Michalek et al. [20], a scale is represented by 14 engineering design decision variables **x**, which map to the product attributes **z** observed by the consumer. Price is included directly as a consumer attribute. Tables 5 and 6 list these variables and the **x** to **z** mapping functions. A vector of engineering design parameters **y**, which is used in the mapping and optimization constraint functions, is included in Table 7.

The marginal cost to the manufacturer per scale is taken as \$3. Manufacturing equipment and economies of scale are omitted for simplicity, as cost modeling is not the focus of this paper (see Michalek et al. [20] for a discussion of cost considerations in this problem).

4.3 Optimization Results. The new scale is optimized according to the following formulation:

maximize $\pi_j^\alpha = m(p_j - c_j)s_j^\alpha - C_j$
with respect to design decision variables $\mathbf{x}_j$
subject to $\mathbf{g}(\mathbf{x}_j)$:
$g_1 - g_{20}$: Simple bounds given in Table 5
$g_{21}: x_{12} \le x_{14} - 2y_1$
$g_{22}: x_{12} \le x_{13} - 2y_1 - x_7 - y_9$

Table 7 Optimization problem parameters

Name	Description	Value	Units
y_1	Gap between base and cover	0.30	in.
$\mathbf{y_2}$	Minimum distance between spring and base	0.50	in.
$\mathbf{y_3}$	Internal thickness of scale	1.90	in.
$\mathbf{y_4}$	Minimum pinion pitch diameter	0.25	in.
$\mathbf{y_5}$	Length of window	3.00	in.
$\mathbf{y_6}$	Width of window	2.00	in.
$\mathbf{y_7}$	Distance between top of cover and window	1.13	in.
$\mathbf{y_8}$	Number of lb measures per tick mark	1.00	lb
$\mathbf{y_9}$	Horizontal distance between spring and pivot	1.10	in.
$\mathbf{y_{10}}$	Length of tick mark + cap to number	0.31	in.
$\mathbf{y_{11}}$	Number of lbs that number length spans	16.00	lb
$\mathbf{y_{12}}$	Aspect ratio of number (length/width)	1.29	—
$\mathbf{y_{13}}$	Minimum allowable distance of lever at base to centerline	4.00	in.

Note: Adapted from Michalek et al. [20]

Table 8 Optimal product characteristics for several α-levels ($n = 250$)

α	z_1 Cap (lbs)	z_2 Asp. ratio	z_3 Area (in.^{2})	z_4 Gap (in.)	z_5 Num. size (in.)	p Price ($)	Market share (%) s^{α}[a]	$E[\hat{s}]$[b]	Normalized profit ($) π^{α}[a]	π^{α}[a] for argmax ($E[\hat{\pi}]$)	$E[\hat{\pi}]$[b]
10%	272	1.05	140	0.115	1.35	20.89	29.4	46.7	5.26	4.79	8.36
20%	277	1.06	140	0.113	1.34	22.32	32.5	44.6	6.28	5.96	8.61
30%	280	1.06	140	0.112	1.32	23.49	34.8	42.8	7.12	6.92	8.76
40%	282	1.06	140	0.111	1.32	24.61	36.6	41.0	7.92	7.81	8.87
50%	284	1.06	140	0.110	1.31	25.77	38.4	39.2	8.73	8.69	8.93
60%	286	1.06	140	0.110	1.31	27.04	40.0	37.3	9.62	9.62	8.96
70%	288	1.06	140	0.109	1.30	28.55	41.7	35.0	10.67	10.65	8.95
80%	289	1.07	140	0.109	1.30	30.55	43.6	32.2	12.02	11.88	8.86
90%	292	1.07	140	0.108	1.29	33.81	46.0	27.8	14.18	13.59	8.56
Max $E[\hat{\pi}]$[b]	285	1.06	140	0.110	1.31	27.47	—	36.7	—	—	8.97

[a] Calculated using the delta method approximation.

[b] Calculated using Monte Carlo simulation.

$$g_{23} : (x_4 + x_5) \leq x_{13} - 2y_1$$
$$g_{24} : x_5 \leq x_2$$
$$g_{25} : x_7 + y_9 + x_{11} + x_8 \leq x_{13} - 2y_1$$
$$g_{26} : x_8 \geq (x_{13} - 2y_1) - \left(\tfrac{x_{12}}{2} + y_7\right) - x_7 - y_9 - x_{10}$$
$$g_{27} : (x_1 + x_2)^2 \leq (x_{13} - 2y_1 - x_7)^2 + \left(\tfrac{x_{14} - 2y_1}{2}\right)^2$$
$$g_{28} : (x_1 + x_2)^2 \geq (x_{13} - 2y_1 - x_7)^2 + y_{13}^2$$

where

$$s_j^{\alpha} = \left(1 + \exp\left(g(\bar{\boldsymbol{\beta}}) - \Phi^{-1}(\alpha)\left(\nabla g(\bar{\boldsymbol{\beta}})^T \boldsymbol{\Sigma} \nabla g(\bar{\boldsymbol{\beta}})\right)^{\frac{1}{2}}\right)\right)^{-1}$$

$$g(\boldsymbol{\beta}) = \ln\left(\sum_{k \in J \backslash j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k - \mathbf{x}_j)}\right)$$

$$c_j = 3 \quad \mathbf{z}(\mathbf{x}_k) = [200 \quad 0.75 \quad 140 \quad 0.1875 \quad 1.75 \quad 5] \qquad (18)$$

MATLAB's *fmincon* function was used to solve the problem, and the results are summarized in Table 8. In this example, we define the competitor product as comprised of a set of selected attributes from their respective feasible intervals. Consumers choose to buy

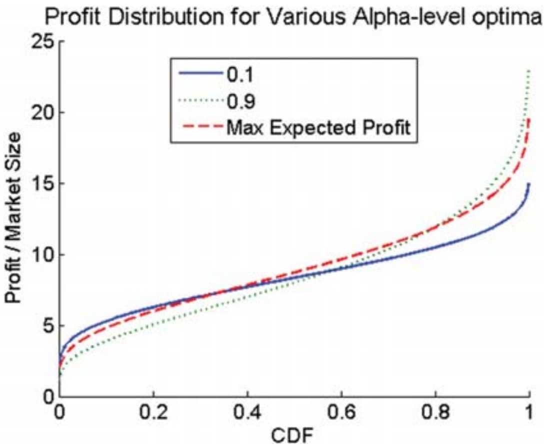

Fig. 4 CDF of profit distribution illustrating that different designs are preferred for $\alpha = 0.10$ versus $\alpha = 0.90$ and maximum expected profit for coefficients estimated using $n = 250$ data points

Table 9 Engineering characteristics for the $\alpha = 10\%$ and $\alpha = 90\%$ optimal products

	Variable and description	$\alpha = 0.1$	$\alpha = 0.9$	Exp. value	Lower bound	Upper bound
Marketing variables						
z_1	Weight capacity	272	292	285	200 lbs	400 lbs.
z_2	Aspect ratio	1.05	1.07	1.06	0.75	1.33
z_3	Platform area	140	140	140	100 in^2	140 in^2
z_4	Tick mark gap	0.12	0.11	0.11	0.0625 in.	0.1875 in.
z_5	Number size	1.35	1.29	1.31	0.75 in.	1.75 in.
p	Price	20.89	33.81	27.47	$5.00	$35.00
Engineering variables[a]						
x_1	Length from base to force on long lever	6.19	5.60	5.82	0.125 in.	21 in.
x_2	Length from force to spring on long lever	5.87	6.53	6.28	0.125 in.	21 in.
x_3	Length from base to force on short lever	13.84	6.73	13.81	0.125 in.	24 in.
x_4	Length from force to join on short lever	3.70	4.25	3.61	0.125 in.	18.175 in.
x_5	Length from force to joint on long lever	2.40	2.17	2.45	0.125 in.	18.175 in.
x_6	Spring constant	48.66	11.74	20.27	1 lb/in.	200 lbs./in.
x_7	Distance from base edge to spring	0.50	0.50	0.50	0.50 in	12 in.
x_8	Length of rack	6.11	6.16	6.19	1.00 in.	16 in.
x_9	Pitch diameter of pinion	0.69	1.50	1.91	0.25 in.	24 in.
x_{10}	Length of pivot's horizontal arm	0.91	1.24	0.83	0.50 in.	1.9 in.
x_{11}	Length of pivot's vertical arm	1.32	1.10	1.38	0.50 in.	1.9 in.
x_{12}	Dial diameter	9.95	10.03	10.00	1.00 in.	25 in.
x_{13}	Cover length	12.15	12.23	12.20	5.55 in.	19 in.
x_{14}	Cover width	11.53	11.45	11.48	7.4 in.	25 in.

[a] Optimal engineering variables are nonunique. See Ref. [20] for discussion. Adapted from Michalek et al. [20]

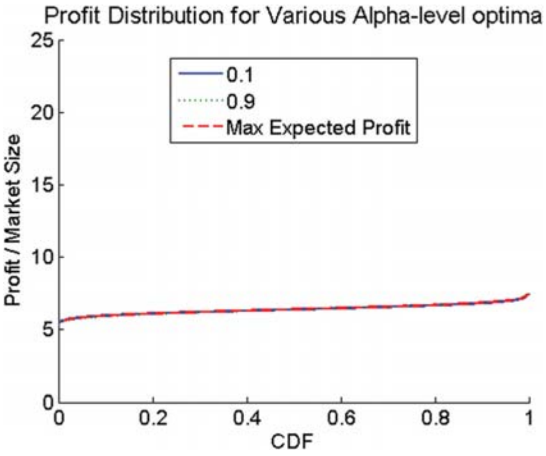

Fig. 5 CDF of profit distributions illustrating that $\alpha = 0.10$ and $\alpha = 0.90$ designs converge to the expected value design as data increase using $n = 9200$ data points

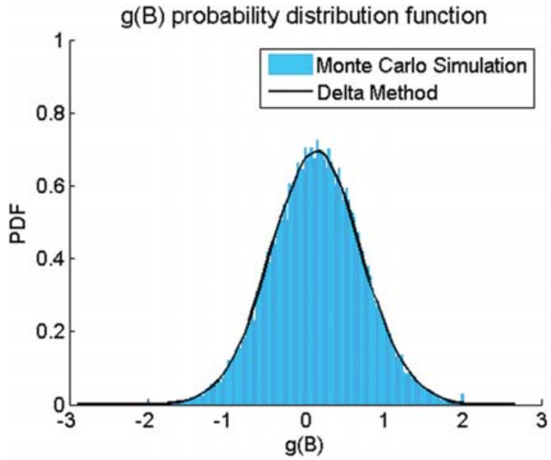

Fig. 6 Comparison of simulated and approximated g function

the new scale or the existing product on the market. A single-competitor market could represent the choice a consumer faces at a store that stocks only two options due to shelf-space limitations, and the two-option scenario mitigates issues with independence from irrelevant alternatives (IIA) restrictions to substitution patterns [50].

Because optimization results are independent of the constants for fixed cost C and market size m we report the normalized profit, defined as $s_j(p_j - c_j)$, which represents profit for a market size of one and fixed cost of zero. Profit for other values of these constants can be computed from the normalized profit post hoc. It should be noted that determining the correct market size is nontrivial and a subject of study in the marketing discipline.

Table 8 reveals that as the firm moves away from a risk neutral position (max $E[\hat{\pi}]$), either becoming more risk averse (lower α-values) or more risk seeking (high α-values), the expected value of profit decreases. The firm is sacrificing the expected value of profit in order to improve profit at the α-level (e.g., to reduce downside risk for small α). The table also shows the results of maximizing the expected value of profit (labeled max $E[\hat{\pi}]$) using a numerical simulation of the profit distribution with 50,000 draws from the beta distribution (predrawn to improve efficiency and stability). Note that the optimal solution using the expected value of the coefficients $\boldsymbol{\beta}$ (equivalent to setting $\alpha = 50\%$ in Table 8; see Eq. (16)) has lower expected profit than the solution that maximizes expected profit. Note also that the product optimized for any given π^{α} has greater profit at that α level than the product optimized for $E[\hat{\pi}]$ (labeled argmax($E[\hat{\pi}]$)).

The advantage of the proposed delta method approach over a simulation-based approach is illustrated in the example. While it takes approximately 1 min to find the optimal solution for a given α-level (using multistart with 20 random starting points); it takes approximately 30 min to find the solution that maximizes the expected profit using a function that calculates the average profit based on 50,000 values predrawn from the distribution of the coefficients using the same 20 starting points.

Figure 4 shows the cumulative profit distribution for $x_j^{\alpha *}$ at $\alpha = 10\%$ and 90% and the optimal solution maximizing the expected profit. The optimal design for $\alpha = 10\%$ has lower profit at $\alpha = 90\%$ and vice versa. Thus, the optimal design depends on a firm's sensitivity to risk. A risk averse firm would prefer the design resulting in the solid-line $\alpha = 10\%$ curve because thernpe is less loss associated with downside risk (where loss and upside are measured relative to expected value and not in absolute dollars). A risk seeking firm would prefer the design resulting in the dotted-line $\alpha = 90\%$ curve because it has the greatest upside potential (fatter tails). The dashed "Max Expected Profit" curve is the preferred design of a risk neutral firm. Optimal design attributes are listed in Table 9.

When more data are used to estimate the beta coefficients and the uncertainty decreases, the optimal product solutions for various levels of α converge to the same design. This is because the

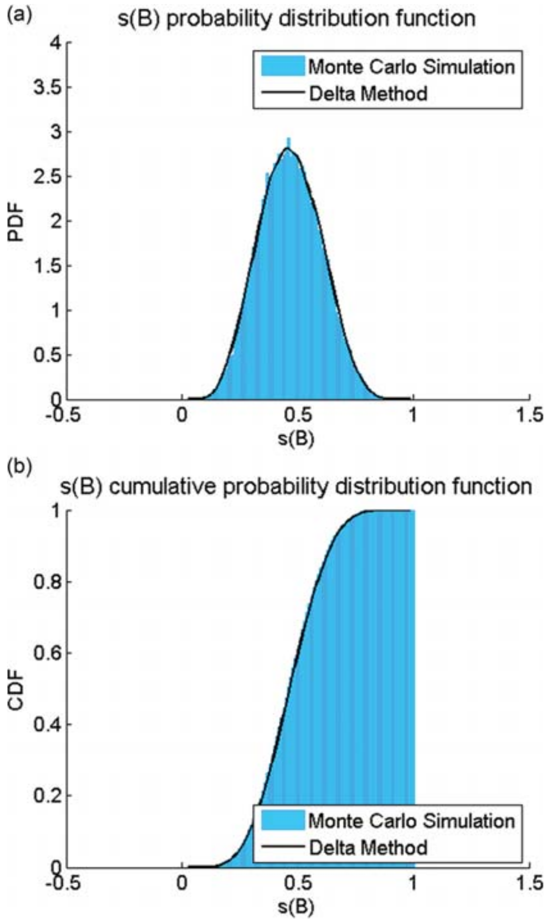

Fig. 7 (*a*) and (*b*) Comparison of simulated and approximated market share

distributions of profit become tighter about the mean and demand uncertainty reduces. In Fig. 5, the $\alpha = 10\%$, 90%, and max expected optimal product profit distributions lie on top of one another. This highlights the fact that our approach deals only with uncertainty of model parameter estimates. Any remaining uncertainty in model misspecification, context variation, respondent representativeness, or other sources of demand uncertainty are not captured here, and additional choice data are sufficient to reduce parametric uncertainty to near zero.

Table 10 Accuracy of the delta method at different alpha levels in the case study

	Market share at α level (%)		
α (%)	Estimated by delta method	Simulated (50,000 draws)	Error
10	29.4	29.4	0.03
20	32.5	32.4	0.08
30	34.8	34.7	0.03
40	36.6	36.6	0.01
50	38.4	38.3	0.03
60	40.0	39.9	0.08
70	41.7	41.7	0.07
80	43.6	43.6	0.05
90	46.0	46.0	0.02

4.4 Assessing the Delta Method Approximation in the Case Study. In order to check the quality of the delta method approximation, we compare the distribution obtained for the optimal design found at $\alpha = 10\%$ using a Monte Carlo simulation vs. the delta method.

First, we take 50,000 draws of the coefficients using the covariance matrix obtained in the logit estimation. We use these simulated draws to find a simulated distribution of the g function (Eq. (9)) and compare it with the delta method approximation. Using the g function distribution, we can also compute the market share distribution since $\hat{s}_j = (1 + e^{g(\hat{\boldsymbol{\beta}},\mathbf{X})})^{-1}$.

Figures 6 and 7 show, respectively, the comparisons of the simulated g function and market share distributions with those obtained by the delta method. The delta method approximation yields high accuracy in this example.

Table 10 shows the simulated market share at the α-levels estimated in the case study using the delta method. The delta method error is small (less than 0.1% for all alpha levels).

5 Conclusions

Uncertainty in consumer choice model predictions implies uncertainty about the profit a given product design would generate. We propose a method for incorporating discrete choice model parameter uncertainty in the design decision problem and for determining the optimal design of a product given a specified level of risk tolerance. In the proposed method, the modeler specifies the level of sensitivity to downside risk by setting $\alpha \epsilon (0, 1)$. Specifically, π_j^α is defined as the value below which α fraction of the profit distribution $\hat{\pi}_j$ lies, and the design is optimized to maximize π_j^α, rather than the expected value of profit. We apply the delta method to derive an estimated closed-form function for π_j^α in the case of the multinomial logit model. The closed-form function enables the optimization problem to be computationally efficient, and it is preferable over methods requiring a simulation-based approach when applicable.

We demonstrate the method in a simple scale design example, where the delta method is shown to yield a close approximation to the true distribution. We find that the optimal solution varies with α, and the optimal solution designed for one α-level may be significantly less profitable at another α-level. Thus, optimal design choices depend on risk preference. In the example, the delta method allows the optimization problem to be solved an order of magnitude faster than using simulation.

6 Limitations and Future Work

The proposed methodology addresses only the uncertainty of model parameter estimates caused by missing data; therefore, it is useful primarily in situations with limited data where model specification can be assumed to be correct, such as some conjoint experiments. Further, the relationship between uncertainty due to missing data and the resulting implications for downside risk of design alternatives can be sensitive to model specification assumptions, such as utility function form and error term specification. For example, the multinomial logit specification exhibits the independence of irrelevant alternatives property, which restricts substitution patterns [50].

The derived approximation for the multinomial logit model can be applied assuming any utility function linear in coefficients (e.g., $U_j = \beta_1 X_j + \beta_2 Y_j^2 + \beta_3 X_j Y_j + \beta_4 \log X_j$); therefore, it applies to a wide range of utility function specifications. While the α profit approach can be applied to alternative demand model specifications (e.g., probit, mixed logit), the closed-form approximation of the delta method applies only to the logit model. Future work may expand the method to be used with other choice models (e.g., mixed logit) and address other sources of uncertainty, such as model misspecification.

The delta method approximation was reasonably accurate for the presented case study; however, accuracy will vary with problem details, so similar validation simulations are needed to assess the accuracy of the approximation when applying the method to different data or functional forms.

Nomenclature

c = variable cost
C = fixed cost
F_π = cumulative distribution function of profit estimate
g = mapping function for delta method
j = product index
J = number of products
m = market size
n = number of attributes per product
p_j = price of product j
s_j = point estimate market share for product j
$\hat{s}_j$ = random variable market share estimate for product j
s_j^α = market share of product j at risk level α
$\mathbf{v}$ = vector of point estimates of utility for all products
ν_j = point estimate observable utility for product j
$\hat{v}_j$ = random variable observable utility estimate for product j
$\mathbf{x}_j$ = column vector of attributes for product j
$\mathbf{x}_j^{\alpha*}$ = optimal product attributes at level α
$\mathbf{X}$ = matrix of attributes for all products
α = profit risk tolerance parameter
$\boldsymbol{\beta}$ = column vector of choice model parameter point estimates
$\hat{\boldsymbol{\beta}}$ = random column vector of choice model parameter estimates
$\bar{\boldsymbol{\beta}}$ = mean of $\hat{\boldsymbol{\beta}}$ distribution
$\boldsymbol{\Sigma}$ = covariance matrix of $\hat{\boldsymbol{\beta}}$ distribution
π_j = point estimate of profit for product j
$\hat{\pi}_j$ = random variable profit estimate for product j
π_j^α = profit of product j at a level α
Φ = standard normal cumulative distribution function

Appendix A: Derivation of ∇*g* for a Multinomial Logit Model With No Outside Good

$$\text{Since } g(\boldsymbol{\beta}) = \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}\right)$$

$$\begin{aligned}
\Rightarrow \nabla_{\boldsymbol{\beta}} g &= \nabla_{\boldsymbol{\beta}} \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}} \nabla_{\boldsymbol{\beta}}\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}} \left(\sum_{k\in J\setminus j} \nabla_{\boldsymbol{\beta}}\left(e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}\right)\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}} \left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} \nabla_{\boldsymbol{\beta}}\left(\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)\right)\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}} \left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} (\mathbf{x}_k-\mathbf{x}_j)\right)\\
&= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}}\\
&= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}}
\end{aligned}$$

Appendix B: Derivation of *g*(***β***) and ∇*g* for a Multinomial Logit Model With Utility of the Outside Good Normalized to 0

$$s_j = \frac{e^{\boldsymbol{\beta}^{T}\mathbf{x}_j}}{\sum\limits_{k\in J} e^{\boldsymbol{\beta}^{T}\mathbf{x}_k} + 1}$$

$$\begin{aligned}
&\Rightarrow g(\boldsymbol{\beta}) = \ln\left(\frac{1}{s_j} - 1\right) = \ln\left(\frac{\sum\limits_{k\in J} e^{\boldsymbol{\beta}^{T}\mathbf{x}_k} + 1}{e^{\boldsymbol{\beta}^{T}\mathbf{x}_j}} - 1\right)\\
&= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}\right)\\
&\Rightarrow \nabla_{\boldsymbol{\beta}} g = \nabla_{\boldsymbol{\beta}} \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_j}} \nabla_{\boldsymbol{\beta}}\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}\right)\\
&= \frac{1}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}} \left(\sum_{k\in J\setminus j} \nabla_{\boldsymbol{\beta}}\left(e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)}\right) + \nabla_{\boldsymbol{\beta}}\left(e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}\right)\right)\\
&= \frac{\left(\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} \nabla_{\boldsymbol{\beta}}\left(\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)\right) + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j} \nabla_{\boldsymbol{\beta}}\left(-\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_j\right)\right)}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}}\\
&= \frac{\left(\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} (\mathbf{x}_k-\mathbf{x}_j) + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}(-\mathbf{x}_j)\right)}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j}}\\
&= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} - \mathbf{x}_j}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} + 1}
\end{aligned}$$

Appendix C: Demonstration that the Variance of g(*β*) is Affected by the Distance of the Products' Attributes From the Reference Level Only When an Outside Good Option is Present

Let $\mathbf{x}_0$ be an arbitrary constant vector, $\mathbf{y}_j = \mathbf{x}_j + \mathbf{x}_0$, and $\mathbf{y}_k = \mathbf{x}_k + \mathbf{x}_0$

C.1 No Outside Good

$$\begin{aligned}
g(\boldsymbol{\beta}) &= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}(\mathbf{y}_k-\mathbf{y}_j)}\right)\\
&= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}((\mathbf{x}_k+\mathbf{x}_0)-(\mathbf{x}_j+\mathbf{x}_0))}\right)\\
&= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}(\mathbf{x}_k-\mathbf{x}_j)}\right)\\
\nabla_{\boldsymbol{\beta}} g &= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{y}_k-\mathbf{y}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{y}_k}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{y}_k}}\\
&= \frac{\sum\limits_{k\in J\setminus j} \left((\mathbf{x}_k+\mathbf{x}_0)-(\mathbf{x}_j+\mathbf{x}_0)\right) e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k+\mathbf{x}_0)}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k+\mathbf{x}_0)}}\\
&= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_0}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_0}} = \frac{\sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k}}
\end{aligned}$$

C.2 Normalized Outside Good

$$\begin{aligned}
g(\boldsymbol{\beta}) &= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}(\mathbf{y}_k-\mathbf{y}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{y}_j}\right)\\
&= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}((\mathbf{x}_k+\mathbf{x}_0)-(\mathbf{x}_j+\mathbf{x}_0))} + e^{-\boldsymbol{\beta}^{T}(\mathbf{x}_j+\mathbf{x}_0)}\right)\\
&= \ln\left(\sum_{k\in J\setminus j} e^{\boldsymbol{\beta}^{T}(\mathbf{x}_k-\mathbf{x}_j)} + e^{-\boldsymbol{\beta}^{T}\mathbf{x}_j} e^{-\boldsymbol{\beta}^{T}\mathbf{x}_0}\right)\\
\nabla_{\boldsymbol{\beta}} g &= \frac{\sum\limits_{k\in J\setminus j} (\mathbf{y}_k-\mathbf{y}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{y}_k} - \mathbf{y}_j}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{y}_k} + 1}\\
&= \frac{\sum\limits_{k\in J\setminus j} \left((\mathbf{x}_k+\mathbf{x}_0)-(\mathbf{x}_j+\mathbf{x}_0)\right) e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k+\mathbf{x}_0)} - (\mathbf{x}_j+\mathbf{x}_0)}{\sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}(\mathbf{x}_k+\mathbf{x}_0)} + 1}\\
&= \frac{e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_0} \sum\limits_{k\in J\setminus j} (\mathbf{x}_k-\mathbf{x}_j) e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} - \mathbf{x}_j - \mathbf{x}_0}{e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_0} \sum\limits_{k\in J\setminus j} e^{\boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_k} + 1}
\end{aligned}$$

Appendix D: Part-Worth and Polynomial Utility Functions From the Conjoint Survey for n = 250 Respondents

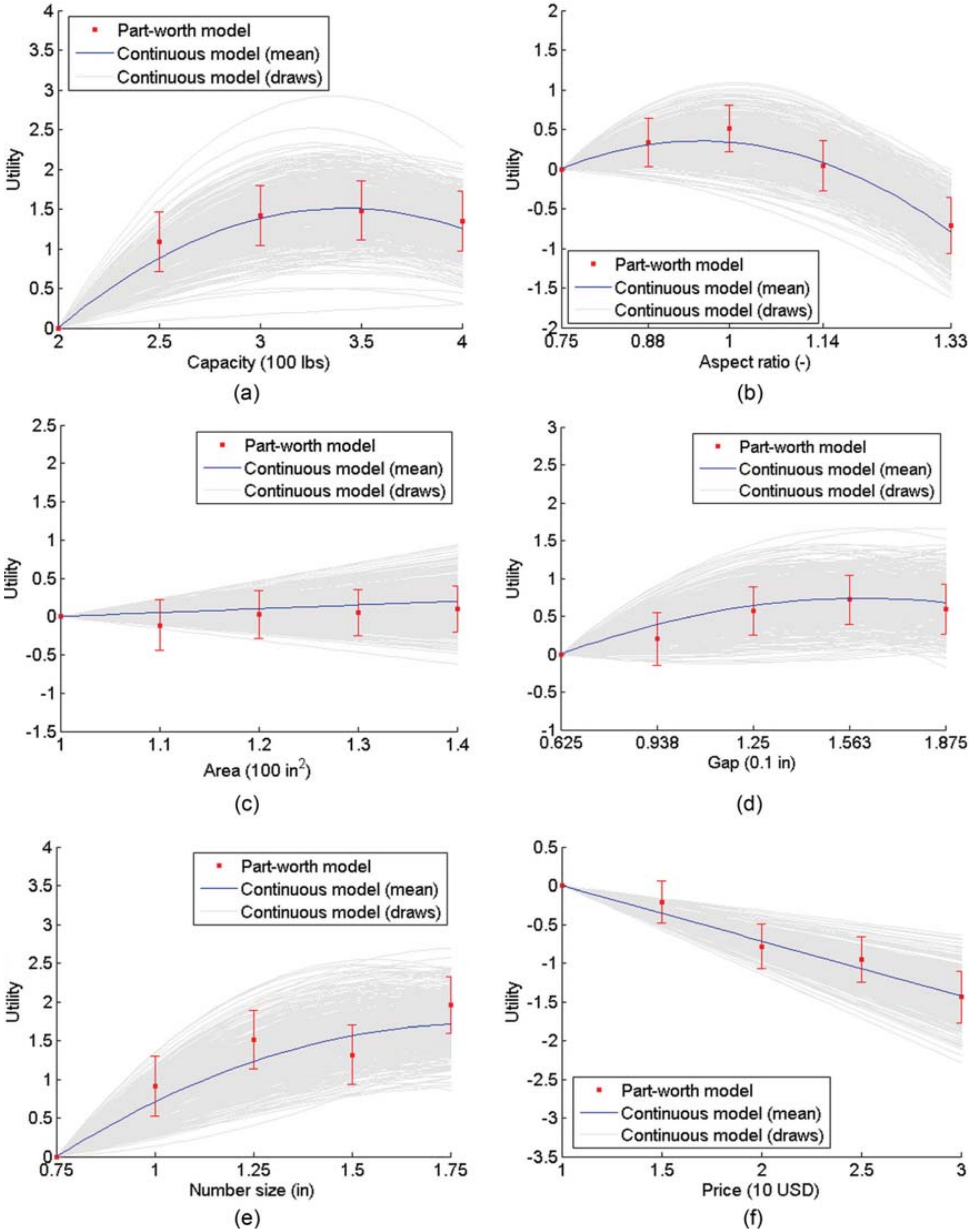

Fig. 8 (*a*)–(*f*) Utility levels

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