OPTIMAL PLUG-IN HYBRID ELECTRIC VEHICLE DESIGN AND ALLOCATION FOR DIVERSE CHARGING PATTERNS

Nikhil Kaushal  
Graduate Student  
Mechanical Engineering  
Carnegie Mellon University  
nkaushal@cmu.edu

Ching-Shin Norman Shiau  
PhD Student  
Mechanical Engineering  
Carnegie Mellon University  
cshiau@cmu.edu

Jeremy J. Michalek  
Assistant Professor  
Mechanical Engineering  
Engineering & Public Policy  
Carnegie Mellon University  
jmichalek@cmu.edu

ABSTRACT

Plug-in hybrid electric vehicle (PHEVs) technology has the potential to address economic, environmental, and national security concerns in the United States by reducing operating cost, greenhouse gas (GHG) emissions and petroleum consumption. However, the net implications of PHEVs depend critically on the distances they are driven between charges: Urban drivers with short commutes who can charge frequently may benefit economically from PHEVs while also reducing fuel consumption and GHG emissions, but drivers who cannot charge frequently are unlikely to make up the cost of large PHEV battery packs with future fuel cost savings.

We construct an optimization model to determine the optimal PHEV design and optimal allocation of PHEVs, hybrid-electric vehicles (HEVs) and conventional vehicles (CVs) to drivers in order to minimize net cost, fuel consumption, and GHG emissions. We use data from the 2001 National Household Transportation Survey to estimate the distribution of distance driven per day across vehicles. We find that (1) minimum fuel consumption is achieved by assigning large capacity PHEVs to all drivers; (2) minimum cost is achieved by assigning small capacity PHEVs to all drivers; and (3) minimum greenhouse gas emissions is achieved by assigning medium-capacity PHEVs to drivers who can charge frequently and large-capacity PHEVs to drivers who charge less frequently.

Keywords: Plug-in Hybrid Electric Vehicle; Design Optimization; Vehicle Design; Greenhouse Gases

1. INTRODUCTION

Plug-in hybrid electric vehicle (PHEV) technology has been considered a promising route to addressing U.S. dependency on foreign oil and global warming in the transportation sector. PHEVs use large battery packs to store energy from the electricity grid and propel the vehicle partly on electricity instead of gasoline. Under the average mix of electricity sources in the United States, which in 2006 included 49% coal, 20% nuclear, 20% natural gas, 7% hydroelectric, 3% renewables, 2% oil, and 1% other (EIA, 2008b), vehicles can be driven with lower operation cost and fewer greenhouse gas (GHG) emissions per mile when powered by electricity than by gasoline (Samaras and Meisterling, 2008). In the best case scenario, PHEV may be able to displace a large portion of the gasoline consumed by the transportation sector with electricity, since approximately 60% of U.S. passenger vehicles travel less than 30 miles per day (US DOT, 2003).

PHEVs have potential as a near-term strategy for addressing global warming and oil dependency, as compared to fuel cell vehicle and hydrogen-powered vehicles, because the technology exists today at not-unreasonable costs: Several automobile manufacturers have announced plans to produce PHEVs commercially, including General Motors’ Chevrolet Volt, which will carry enough battery modules to store 40 miles worth of electricity (Bunkley, 2008) and Toyota’s PHEV version of the Prius, which will carry enough batteries for closer to 7-10 miles of electric travel (Maynard, 2008). The structure of a PHEV is similar to that of a regular hybrid electrical vehicle (HEVs) except for larger battery capacity and plug-charging capability (Frank, 2007). HEVs comprise of an electric motor coupled with an internal combustion engine and are arranged either in series, parallel or in a split...
series/parallel configuration. In this study we use the split parallel/series powertrain configuration based on the popular 2004 Toyota Prius, and we add additional lithium-ion batteries to predict performance of PHEVs. A larger battery pack allows a PHEV to travel farther on electricity from the grid, but batteries are expensive and heavy. We account for the effects of adding additional batteries on vehicle weight, cost and efficiency in order to predict net implications for cost, petroleum consumption and greenhouse gas emissions.

In our model, a PHEV with a fully charged battery operates in charge-depleting mode (CD-mode), drawing energy from the battery to power the vehicle until the battery reaches its target state of charge (SOC). Once the battery is depleted to the target SOC, the PHEV switches to operate in charge-sustaining mode (CS-mode), using the gasoline engine to provide net propulsion energy and using the battery and motor for transient conditions, as a HEV does. PHEVs can be classified as range extended or blended based on their energy management strategy in CD-mode. A range extended PHEV operates as an electric vehicle in CD-mode utilizing only the electric energy stored in the battery without using engine power and fuel for propulsion. The distance the vehicle can travel on electricity alone is called the all-electric range (AER). In contrast to range extended PHEV, a blended strategy PHEV uses a mix of the electric motor and gasoline engine to power the vehicle in CD-mode. In CS-mode, both types of PHEVs operate similar to standard HEVs where the internal combustion engine is enabled to provide a portion of the propulsion power and also maintain the target SOC. For simplicity and clear comparison between HEV and PHEV, we restrict our attention to range extended PHEVs.

Shiau et al. (2009) demonstrated that charging frequency has critical implications on total cost, fuel consumption, and greenhouse gas emissions from PHEV operation: Vehicles that are charged frequently can drive most of their miles on electric power, even with a relatively small battery pack. In contrast, vehicles that are not charged frequently would require larger battery packs to cover longer distances with electric power. This study argues that properly-sized vehicles should be targeted to the right drivers based on charging frequency. In this paper, we identify the optimal solution of (1) PHEV designs and (2) allocation of PHEVs, HEVs, and conventional vehicles (CVs) to drivers in order to minimize total cost, fuel consumption, or greenhouse gas emissions.

2. METHOD

We describe our approach by first developing a general optimization formulation and then defining models for each element. The assumptions in our model are: (1) vehicle maintenance expenses over the vehicle lifetime are not included; (2) the cost of scaling the engine and motor is relatively small compared to the battery cost and hence neglected from the model; (3) PHEV drivers drive the same distance every day and charge their vehicle only once a day; and (4) a single battery lasts the life of the vehicle (150,000 miles), and no battery replacement is required.

2.1. Optimization Model

To optimize a single vehicle for minimum net cost, fuel consumption or greenhouse gas emissions over the population of drivers, we minimize the integral of the quantity per mile at each driving distance \( f(x,s) \) times the driving distance \( s \) over the distribution of driving distances \( f_s(s) \) in the population of drivers.

\[
\min_x \int_0^{s_{\text{max}}} s f(x,s) f_s(s) \, ds \\
\text{subject to } g(x) \leq 0
\]

where \( x \) is a vector of design variables that define the vehicle, \( s \) is the distance the vehicle is driven between charges, \( f(x,s) \) is the value of the objective (cost, fuel consumption, or GHG emissions) per mile for vehicle design \( x \) when driven \( s \) miles per day, \( f_s(s) \) is the probability density function for the number of miles driven per day, and \( g(x) \) is a vector of design constraints that must be satisfied.

To extend this model to the case where different drivers are assigned different vehicles based on the number of miles driven per day, we incorporate a variable \( s_i \) that defines the cutoff point such that drivers who travel less than \( s_i \) per day are assigned the vehicle defined by \( x_i \) and drivers who travel more than \( s_i \) per day are assigned the vehicle defined by \( x_{i+1} \). The formulation for design and ordered allocation of multiple vehicles \((n>1)\) is given by:

\[
\min_{x_i \forall i \in \{1,...,n\}, s_i \forall i \in \{1,...,n-1\}} \sum_{i=0}^{n-1} \left( \int_{s_i}^{s_{i+1}} s f(x_i,s) f_s(s) \, ds \right) \\
\text{subject to } g(x_i) \leq 0; \forall i \in \{1,...,n\} \\
0 \leq s_i \leq s_{\text{max}}; \forall i \in \{1,...,n-1\} \\
\text{where } s_0 = 0; s_n = s_{\text{max}}
\]

In general, each design vector \( x_i \) may include a discrete variable defining vehicle type – for example, PHEV, HEV or CV. However, to avoid the resulting nonconvex mixed-integer nonlinear programming (MINLP) problem in our case study, we test all relevant combinations of vehicle types for the case \( n=2 \) \( \{\text{PHEV, PHEV}\}, \{\text{PHEV, HEV}\}, \{\text{PHEV, CV}\}, \text{etc.} \), and we alter the design vector \( x_i \) accordingly in each case, as defined later.

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1. For batteries that are fully charged and discharged frequently, degradation may be significant, and inclusion of degradation effects warrants further investigation.

2. The structure of cost, fuel consumption and GHG emissions per mile over the vehicle life time is based on the models presented in Shiau et al. (2009).
In the remainder of this section we define the functions in the optimization formulation as follows: We first define the probability density function \( f_s(s) \) for miles traveled per day; we then define each of the objective functions \( f(x,s) \) in terms of vehicle electrical efficiency \( \eta_e \) and gasoline efficiency \( \eta_G \); we next define efficiency as a function of vehicle design variables \( x \) using vehicle simulation; and finally we define the constraints on acceleration performance \( g(x) \) that enable us to compare vehicles of comparable performance characteristics.

### 2.2. Distribution of Vehicle Miles Travelled per Day

We use data from the 2001 National Household Transportation Survey (NHTS) (US DOT, 2003) to estimate the distribution of distances driven per day over the population of drivers. The survey collected data by interviewing 69,817 households across several U.S. cities on the mode of transportation, duration, distance and purpose of the trips taken on a particular day. The distribution below represents the probability density function (PDF) for vehicle miles traveled on the day surveyed. We fit the driving data\(^3\) using the Weibull distribution\(^4\). The Weibull PDF is defined for \( s \geq 0 \) by:

\[
f_s(s) = \frac{\beta}{\lambda} \left( \frac{s}{\lambda} \right)^{\beta-1} \exp\left( -\left( \frac{s}{\lambda} \right)^{\beta} \right)
\]

The two parameters at optimal fit are \( \lambda = 35.28 \) and \( \beta = 1.08 \) with a 99% confidence level. Figure 1 shows the Weibull curve and the histogram of the surveyed daily vehicle driving miles.

**Figure 1: Probability density function for vehicle miles traveled per day**

In order to estimate the charging frequency for PHEV owners, we further assume that (1) each driver travels the same distance each day and (2) each driver has the potential to charge a plug-in vehicle once per day. Under these assumptions the distribution of miles driven between charges for a PHEV equals the distribution of miles driven per day in the NHTS survey.

#### 2.3. Objective Functions

The objective functions of minimum cost, fuel consumption, and greenhouse gas emissions can be calculated as functions of vehicle design variables \( x \) and the distance traveled per day \( s \). The integration of each objective function is computed by numerical integration\(^5\).

To calculate each objective function, we first define the distance driven on electric power \( s_E \) and the distance driven on gasoline \( s_G \) as a function of the vehicle’s AER \( s_{AER} \) and the total distance driven per day \( s \).

\[
s_E = \begin{cases} s & \text{if } s \leq s_{AER} \\ s_{AER} & \text{if } s > s_{AER} \end{cases}
\]

\[
s_G = \begin{cases} 0 & \text{if } s \leq s_{AER} \\ s - s_{AER} & \text{if } s > s_{AER} \end{cases}
\]

We treat the HEV and CV as special cases with \( s_{AER} = 0 \), so that \( s_E = 0 \) and \( s_G = s \).

**Fuel Consumption:** The average fuel consumed per mile \( f_G(x,s) \) is calculated as:

\[
f_G(x,s) = \frac{1}{s} \frac{s_G}{\eta_G(x)}
\]

For a PHEV, below the AER the vehicle consumes no gasoline. However beyond the AER fuel is consumed at a greater rate for the heavier vehicle. For HEV and CV cases Eq. 5 reduces to \( 1/\eta_G \).

**Average Lifetime Cost:** The net present value of lifetime cost per mile includes base vehicle cost, battery purchase cost, net present value of operations cost and costs associated with imposing a potential tax on CO\(_2\) is calculated as:

\[
f_C(x,s) = \frac{1}{s_{LIFE}} \left( c_{VEH} + c_{BATT} \right) + \sum_{m=1}^N \frac{c_{OP} + \rho V_{OP}}{1+\rho} s_{ANUL}
\]

\[
+ \rho V_{VEH} + V_{BATT}
\]

where operating cost per mile \( c_{OP} \) is

\[
c_{OP} = \frac{1}{s} \left( \frac{s_E}{\eta_E(x)} + \frac{s_G}{\eta_G(x)} \right)
\]

---

\(^3\) We excluded drivers who traveled zero miles or more than 200 miles

\(^4\) Different asymmetric distributions, such as lognormal and Weibull have been tested for fitting the NHTS driving data. We found that the Weibull distribution provides the best fit over the data set.

\(^5\) We use the trapezoid method with a step size \( 10^{-4} \) to approximate the integral. Smaller step sizes were tested, and we found no significant effects on the solution.
We assume that the annual vehicle miles traveled $s_{ANUL} = 12,500$ (EPA, 2005) and the vehicle lifetime $N = 12$ year, thus resulting in a lifetime vehicle miles traveled $s_{LIFE} = 150,000$. To calculate the vehicle purchase cost represented by the first term in Eq. (6), we take the vehicle base cost, excluding any battery cost, as $c_{VEH} = \$17,600$, based on the Prius MSRP less its Nickel-metal hydride (NiMH) battery cost of $\$3,900$ (Naughton, 2008). Additional cost for Li-ion batteries is assumed to be an additional $c_{BAT} = \$1,000$ per kWh (Lemoine et al., 2008) times the battery size $k$. The second term in Eq. (6) represents the net present value of operating cost $c_{OP}$ (Eq. (7)) plus the carbon tax paid for operations over the lifetime of the vehicle. The carbon tax is estimated by taxation rate $\rho$ per kg of CO$_2$-eq and operating GHG emission per mile $v_{OP}$ (Eq. (8)) with conservatively assuming the customer would bear the full cost of tax imposed on the producers. The average operating cost per mile $c_{OP}$ (Eq. (7)) represents the average consumer expenses per mile associated with gasoline and electricity used to propel the vehicle. The base case parameters include discount rate $r = 5\%$, retail electricity price $c_{EL} = \$0.11$ per kWh, charging efficiency between outlet and battery $\eta_{C} = 88\%$ (EPRI, 2007) and gasoline price $c_{G} = \$3.00$ per gallon. The total operating cost to travel a particular distance is the sum of the cost of electricity needed to charge the battery and the cost of gasoline used. For PHEVs, we assume that for distances less than the AER the battery is charged as much as needed for the trip n and the battery is fully charged for distances greater than the AER. For HEVs and CVs, there is no electrical travel ($s_{E} = 0$), and therefore operating cost consists only of gasoline cost. The third term in Eq. (6) represents the carbon tax cost imposed on GHG emissions associated with vehicle and battery manufacture, $v_{VEH}$ and $v_{BAT}$, respectively.

**Greenhouse Gas Emissions:** Total lifetime GHG emissions per mile include GHG emissions associated with production and use of the vehicle. The operating GHG emissions per mile $v_{OP}$ (Eq. (8)) represent the average GHG emissions per mile associated with gasoline and electricity used to propel the vehicle:

$$v_{OP} = \frac{1}{s} \left( \frac{s_{E}}{\eta_{E}(x)} + \frac{s_{G}}{\eta_{G}(x)} - v_{G} \right)$$

(8)

The average life cycle GHG emissions are expressed in kg CO2 equivalent (CO2-eq) is represented by Eq. (9).}

$$f_{V}(x,s) = \frac{V_{VEH} + V_{BAT}k}{s_{LIFE}} + v_{OP}$$

(9)

We assume the battery lasts the lifetime of the vehicle, and no battery replacement occurs.

The emissions calculation in this study assume $v_{VEH} = 8,500$ kg CO$_2$-eq per vehicle for vehicle manufacture (excluding emissions from battery production), $v_{BAT} = 120$ kg CO$_2$-eq per kWh of battery capacity produced, battery charging efficiency $\eta_{C} = 88\%$, $s_{LIFE} = 150,000$ miles, $v_{G} = 0.730$ kg of CO$_2$-eq emitted per kWh of electricity, and $v_{G} = 11.34$ kg of CO$_2$-eq per gallon (Samaras and Meisterling, 2008).  

### 2.4. Vehicle Performance Models

To calculate vehicle efficiency and acceleration characteristics, we perform vehicle performance simulations using the Powertrain System Analysis Toolkit (PSAT) vehicle physical simulator developed by Argonne National Laboratory (Argonne National Laboratory, 2008). The body, powertrain and vehicle parameters for all PHEV and HEV simulations are based on the 2004 Toyota Prius model. A kg of structural weight is added per kg increase in battery, motor and engine weight of the vehicles. The CV model is based on the Honda Civic model, and the parameters that define the frontal area, drag coefficient and base weight are adjusted to match the Prius for fair comparison. Detailed vehicle parameters are presented in Table A2 in Appendix.  

For the PHEV and HEV, we use the SAFT Li-ion battery model in the PSAT package, in which each module contains three cells and all cells and modules are connected in series. Each cell in the module weighs 0.378 kg and has a capacity of 9.6 Wh with a nominal output voltage of 3.6 volts. The weight of each 3-cell module is 1.42 kg after accounting for a packaging factor of 1.25. The battery size and capacity are scaled by specifying the number of modules that go into the battery. For the PHEV, we examine the range from 100 to 1000 cells.

The base engine is a 1.4 liter four cylinder engine with a 57 kW maximum power. The engine was scaled by changing the peak power, and the performance map and weight were linearly scaled simultaneously. We define the design range of maximum engine power values between 15 kW and 60 kW.

The base motor is a permanent magnet type with a maximum peak power of 52.35 kW and a weight of 40 kg including a 5 kg controller. Like the engine the motor was scaled by the peak power output of the motor, and the performance map and weight were scaled linearly. The peak motor power ranged from 52.35 kW to 120 kW. The performance maps and weight characteristics of larger motors and engines needed for different PHEV cases are predicted using a motor scaling parameter.

To simulate PHEV performance, we use the standard Urban Dynamometer Driving Schedule (UDDS) driving cycle...
(EPA, 1996) to calculate simulated electrical efficiency $\eta_E$ (miles/kWh) in CD-mode for PHEVs, and fuel efficiency $\eta_G$ (mpg) in CS-mode for PHEVs as well as for HEVs and CVs. We also perform a simulated performance test to calculate the time required to accelerate the vehicle from zero to sixty miles per hour $t_{0:60}$, and we define $g(x)$ as $t_{0:60} \leq 10$ (seconds) for all vehicles.

For the PHEVs, the design variables $x$ consist of $x_1 =$ battery scaling factor, $x_2 =$ motor scaling factor, and $x_3 =$ engine scaling factor. Because the fuel consumption, cost, and greenhouse gas emissions per mile associated with HEVs and CVs are independent of the number of miles driven per day, we focus on PHEV design and take the HEV and CV to have fixed designs. The HEV is identical to the Prius model except NiMH batteries are replaced by Lithium batteries of equivalent energy for comparison purposes. Our HEV has $x_1 = 75$ cells, $x_2 = 52.35$ kW, $x_3 = 57$ kW, $\eta_G = 51.8$ miles per gallon, and $t_{0:60} = 10.5$ seconds. Similarly, our CV has $x_1 = 113$ kW (Toyota Corolla engine with an engine scaling factor of 1.256), $\eta_G = 28.33$ miles per gallon, and $t_{0:60} = 10.3$ seconds.

In order to avoid the computationally expensive process of executing a PSAT simulation for each function evaluation in the optimization algorithm, we created a simple meta-model using data points from the simulation to estimate $\eta_E$, $\eta_G$, and $t_{0:60}$ as a function of $x$ for the PHEV. We evaluated the three output values using PSAT over a grid of values for the inputs $x_1 = \{100, 400, 700, 1000\}$, $x_2 = \{52.35, 60, 90, 120\}$, $x_3 = \{15, 30, 45, 60\}$, and multivariate polynomial functions were fit to the data using least squares. The resulting functions are represented in Eq. (10) below.

$$
\eta_E(x) = a_{111}x_1^3 + a_{112}x_1^2 + a_{113}x_1 + a_{114}x_1^4 + a_{115}x_2^3 + a_{116}x_2^2 \\
+ a_{117}x_3 + a_{118}x_3 + a_{119}x_3 + a_{120}x_4^2 + a_{121}x_5^2 + a_{122}x_6^2 \\
\eta_G(x) = a_{211}x_1^3 + a_{212}x_1^2 + a_{213}x_1 + a_{214}x_1^4 + a_{215}x_2^3 + a_{216}x_2^2 \\
+ a_{217}x_3 + a_{218}x_3 + a_{219}x_3 + a_{220}x_4^2 + a_{221}x_5^2 + a_{222}x_6^2 \\
t_{0:60}(x) = a_{311}x_1^3 + a_{312}x_1^2 + a_{313}x_1 + a_{314}x_1^4 + a_{315}x_2^3 + a_{316}x_2^2 \\
+ a_{317}x_3 + a_{318}x_3 + a_{319}x_3 + a_{320}x_5^2 + a_{321}x_6^2 \\
$$

where $a_{ij}$ represent the polynomial metamodel coefficients; the values of which are presented in Table A1 in Appendix. The maximum metamodel error among the test points is 0.1 miles/kWh, 0.7 miles/gallon, and 0.5 seconds for electrical efficiency, gasoline efficiency, and acceleration time, respectively. A plot comparing the metamodel and simulation data points for vehicle CD-mode efficiency, CS-mode efficiency and 0-60 mph acceleration time as a function of battery, motor and engine size is presented in Figure A1 in Appendix.

9 Our simulation results are generally optimistic for all vehicles in that they do not account for factors such as vehicle wear, improper maintenance and tire pressure, aggressive driving cycles, use of significant accessories, or terrain and weather variation.

3. RESULTS AND DISCUSSION

We examine three cases for dual vehicle model ($n=2$). It is not necessary to study any single vehicle cases, since they emerge as degenerate results for a dual vehicle case. It is also not necessary to examine cases where the first vehicle is a HEV or CV because it is known that PHEVs show best performance on fuel consumption, cost, and GHG emissions for drivers who charge frequently. Thus we are left with three cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PHEV</td>
<td>PHEV</td>
</tr>
<tr>
<td>2</td>
<td>PHEV</td>
<td>HEV</td>
</tr>
<tr>
<td>3</td>
<td>PHEV</td>
<td>CV</td>
</tr>
</tbody>
</table>

We optimize each of these cases separately for each of the three objective functions using a randomized multistart loop to avoid local minima. Table 2 summarizes results from the optimization study for each of the three objective functions. Total fuel consumption can be minimized by allocating a PHEV with the largest possible battery capacity to drivers who charge every 113 miles or less and a PHEV with a slightly smaller motor and a larger engine to the rest of the drivers. The relative contribution of the second, slightly-lighter PHEV to fuel savings is minimal ($-10^3$), and this solution can be seen for practical purposes as a single PHEV solution with maximum battery capacity. This is not surprising, since a large-capacity PHEV can travel long distances without using gasoline. This solution is bounded above by the maximum battery capacity considered in the study – larger battery capacities will in general reduce fuel consumption. For minimum cost, the optimal solution is to allocate a small PHEV to drivers who can charge frequently and a similar sized PHEV to drivers unable to charge frequently. The relative contribution of the second PHEV to vehicle cost is minimal ($-10^3$), and the solution can be seen for all practical purposes as a single PHEV with minimal battery capacity. This solution is limited by the range of battery capacity studied: the optimal battery capacity is bounded below by 100 cells. Further study is needed to understand the extent to which the size of the battery can be reduced and its effects on battery life. Finally, the optimal solution for minimum GHG emissions is to allocate a medium-sized PHEV with an AER of 30 to drivers who can charge every 35 miles or less and allocate a large PHEV with an AER of 53 to drivers who charge less frequently. The impact of vehicle design on different objectives summarized in Table 2 indicates that the optimal vehicle designed for different objectives results in the corresponding minimum objective value.

To further examine these solutions, we plot the derivative of the objective function with respect to driving distance per day in each case and compare the CV and HEV with the optimal PHEV design for that case. In each case the area under the curve is the objective function.
### Table 2: Optimization results for minimum fuel, cost, and GHG emissions objectives

<table>
<thead>
<tr>
<th>Optimization Objective</th>
<th>Minimum Fuel Consumption</th>
<th>Minimum Cost</th>
<th>Minimum GHG Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle Type</strong></td>
<td><strong>PHEV</strong></td>
<td><strong>PHEV</strong></td>
<td><strong>PHEV</strong></td>
</tr>
<tr>
<td>( s_1 ) Allocation to drivers (miles/day)</td>
<td>0-113.7</td>
<td>0-11.3</td>
<td>0-35.7</td>
</tr>
<tr>
<td>( s_1 ) Number of battery cells</td>
<td>1000*</td>
<td>100*</td>
<td>492</td>
</tr>
<tr>
<td>( s_2 ) Motor power (kW)</td>
<td>84.4</td>
<td>64.7</td>
<td>72.8</td>
</tr>
<tr>
<td>( s_3 ) Engine power (kW)</td>
<td>15*</td>
<td>15*</td>
<td>15*</td>
</tr>
<tr>
<td>( k ) Battery capacity (kWh)</td>
<td>21.6</td>
<td>2.2</td>
<td>10.6</td>
</tr>
<tr>
<td>( \eta_h ) EV efficiency (miles/KWh)</td>
<td>6.1</td>
<td>6.6</td>
<td>6.3</td>
</tr>
<tr>
<td>( \lambda_{\text{AER}} ) AER (miles)</td>
<td>59.1</td>
<td>6.4</td>
<td>30.0</td>
</tr>
<tr>
<td>( \lambda_{\text{EL}} ) HEV Efficiency (mpg)</td>
<td>58.0</td>
<td>61.5</td>
<td>60.2</td>
</tr>
<tr>
<td>( t_{0-60} ) HEV 0-60 mpg time (sec)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Fuel consumption (gallon per person day)</td>
<td>0.08</td>
<td>0.45</td>
<td>0.11</td>
</tr>
<tr>
<td>Cost (US$ per person day)</td>
<td>9.49</td>
<td>5.55</td>
<td>8.39</td>
</tr>
<tr>
<td>GHG emissions (kg CO(_2)-eq per person day)</td>
<td>7.43</td>
<td>7.86</td>
<td>7.30</td>
</tr>
<tr>
<td>Change with respect to CV only</td>
<td>-92%</td>
<td>-16%</td>
<td>-52%</td>
</tr>
</tbody>
</table>

* Variable limited by model boundary

### 3.1. Minimum fuel consumption

Figure 2 represents the derivative of net fuel consumed per person per day as a function of the distance travelled between charges, and Figure 3 represents the average fuel consumption per mile as a function of the distance travelled between charges. Fuel consumption is minimized by allocating a single large-sized PHEV over the entire range of drivers. The maximum capacity PHEV allows the largest number of drivers who travel distances less than the AER to avoid consuming any gasoline, and those drivers with longer distances still spend a significant portion of their trip miles using energy from the electricity grid instead of gasoline.

![Figure 2: Derivative of net fuel consumption per person per day by distance traveled between charges. The area under each curve is net fuel consumption per person per day](image)

![Figure 3: Average fuel consumption per mile as a function of the traveled driven between charges](image)

### 3.2. Minimum cost

The net cost of all drivers driving HEVs or the optimal PHEV is similar. In Figure 4, we observe that the net savings gained by allocating different vehicles to different drivers is a relatively small percent of the total cost. Figure 5 represents the average lifetime cost per mile as a function of the distance travelled between charges. The results show that the two PHEV curves coincide on indicating more or less identical designs. The PHEV is cheaper for all drivers, but the net savings gained by allocating different vehicles to different drivers is a relatively small percent of total cost.
3.3. Minimum GHG emissions

The net GHG emissions are smaller for PHEVs than HEVs and CVs, and there is an additional benefit to assigning medium-capacity PHEVs to drivers who charge frequently because reducing the number of unnecessary batteries in these vehicles reduces the emissions associated with battery production as well as the emissions associated with reduced vehicle efficiency caused by carrying heavy batteries. The largest gain is achieved by allocating PHEVs to drivers rather than HEVs or CVs, but a measurable additional gain is possible by allocating the right PHEV to the right driver. As indicated in Figure 6, the plot shows that the area under the optimized PHEV vehicle design curve is less than that of the HEV and conventional vehicle curve.

3.4. Sensitivity analyses

We performed several sensitivity analyses based on the parameters listed in Table 3 to verify the robustness of our optimal solutions. As shown in Fig. 8, the analysis results indicate that for most cases, including low-carbon electricity, low gasoline price, high/low discount rate and carbon tax, minimum cost and GHG solutions remain consistent, whereas a high gasoline price of $6.0 per gallon makes a PHEV12 more cost competitive than PHEV7 for drivers who charge every 24 miles or less. Another critical factor that affects our optimal PHEV allocations is battery cost. When the battery cost decreases from the base case $1000/kWh to $500/kWh, small capacity PHEVs (PHEV7 and PHEV9) are the most economic solutions. However, when lower cost battery technology ($250 per kWh) is available, medium capacity...
PHEV (PHEV16) become a better choice for drivers who charge once every 35 miles and small-sized PHEV is suitable for drivers who charge less frequently. Overall, the results of these sensitivity analyses demonstrate that small capacity PHEV is a robust choice while minimizing cost while medium to large capacity PHEVs are beneficial for reducing GHG emissions.

Table 3: Parameter levels for sensitivity analysis

<table>
<thead>
<tr>
<th>Sensitivity analysis parameters</th>
<th>Unit</th>
<th>Low level</th>
<th>Base level</th>
<th>High level</th>
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</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>%</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Gas price</td>
<td>$/gal</td>
<td>1.5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Total battery capacity cost</td>
<td>$/kWh</td>
<td>(250,500)</td>
<td>1000</td>
<td>-</td>
</tr>
<tr>
<td>CO₂ lifecycle emissions in kg/kWh</td>
<td>electricity</td>
<td>0.218</td>
<td>0.73</td>
<td>-</td>
</tr>
<tr>
<td>Carbon tax</td>
<td>$/ton</td>
<td>-</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

We construct an optimization model to determine the optimal PHEV design and optimal allocation of PHEVs, HEVs and CVs to drivers in order to minimize net cost, fuel consumption, and GHG emissions. We find that (1) minimum fuel consumption is achieved by assigning large capacity PHEVs to all drivers; (2) minimum cost is achieved by assigning small capacity PHEVs to all drivers; and (3) minimum greenhouse gas emissions is achieved by assigning medium-capacity PHEVs to drivers who can charge frequently and large-capacity PHEVs to drivers who charge less frequently. The optimal design and allocation could reduce fuel consumption by 92%, reduce cost by 16%, or reduce GHG emissions by 52% in the population represented by the NHTS survey data compared to exclusive use of the conventional vehicle.

It is important to note that these improvements cannot be made simultaneously because of tradeoffs among the objectives. Further analysis of multiobjective optimization or economic valuation of GHG emissions and fuel consumption externalities is warranted to better understand these tradeoffs. Optimal allocation may play a stronger role in resolving these tradeoffs than it does in any of the individual cases.

It is interesting to note that large-capacity battery packs that store 35 miles or more worth of energy for electric travel are not optimal for cost or greenhouse gas reduction, whereas PHEVs with small to medium sized battery packs play an important role in achieving both of these objectives. Technology for low cost, high specific energy and large SOC swing battery improvements will significantly improve the performance of PHEVs (Shiau et al., 2009).

In future work, additional sensitivity analysis will be performed to examine the impact of battery degradation over the life of the vehicle and the influence of new battery technologies with high specific energy batteries in order to further assess robustness of our study results.
ACKNOWLEDGMENTS
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NOMENCLATURE

\[ a \] = Polynomial coefficient in metamodel
\[ c_{\text{BAT}} \] = Battery cost ($/kWh)
\[ c_{\text{E}} \] = Retail electricity cost ($/kWh)
\[ c_{\text{G}} \] = Retail gasoline cost ($/gallon)
\[ c_{\text{OP}} \] = Average operating cost per mile ($/mile)
\[ c_{\text{VEH}} \] = Vehicle cost ($)\[ f(x,v) \] = Objective function per mile
\[ f_c(x,v) \] = Average lifetime cost per mile ($/miles)
\[ f_f(x,v) \] = Fuel consumption per mile (gallons/mile)
\[ f_p(x,v) \] = Probability density function (miles/day)
\[ f_t(x,v) \] = Total GHG emissions per mile (kg CO\textsubscript{2}-eq/mile)
\[ n \] = Number of vehicles
\[ \rho \] = Carbon tax rate (kg CO\textsubscript{2}-eq/ton)
\[ r \] = Discount rate (%)
\[ s \] = Distance traveled (miles)
\[ s_{\text{AER}} \] = AER distance (miles)
\[ s_{\text{ANUL}} \] = Annual vehicle distance traveled (miles)
\[ s_{\text{B}} \] = CD-mode distance (miles)
\[ s_{\text{E}} \] = CS-mode distance (miles)
\[ s_{\text{G}} \] = Cut-off distance between vehicle types (miles)
\[ s_{\text{LIFE}} \] = Vehicle life (miles)
\[ s_{\text{MAX}} \] = Maximum driving distance (miles)
\[ t_{0-60} \] = 0-60 mph acceleration time (sec)
\[ x_{\text{B}} \] = Number of battery cells
\[ x_{\text{E}} \] = Motor power variable (kW)
\[ x_{\text{G}} \] = Engine power variable (kW)
\[ \beta \] = Weibull distribution control parameter 2
\[ \eta_{\text{E}} \] = CD-mode efficiency (miles/kWh)
\[ \eta_{\text{G}} \] = CS-mode efficiency (miles/gallon)
\[ \kappa \] = Battery capacity (kWh)
\[ \lambda \] = Weibull distribution control parameter 1
\[ v_{\text{B}} \] = Life cycle GHG emissions in battery manufacturing (kg CO\textsubscript{2}-eq/kWh)
\[ v_{\text{E}} \] = CD-mode GHG emissions (kg CO\textsubscript{2}-eq/kWh)
\[ v_{\text{G}} \] = CS-mode GHG emissions (kg CO\textsubscript{2}-eq/gallon)
\[ v_{\text{OP}} \] = Operating GHG emissions (kg CO\textsubscript{2}-eq/gallon)
\[ v_{\text{VEH}} \] = Life cycle GHG emissions in vehicle manufacturing (excluding battery) (kg CO\textsubscript{2}-eq/vehicle)

REFERENCES
Argonne National Laboratory (2008) "Powertrain Systems Analysis Toolkit (PSAT)."

APPENDIX

Table A1: Polynomial meta-model coefficients

<table>
<thead>
<tr>
<th>( a ) coefficients</th>
<th>( b ) coefficients</th>
<th>( m ) coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{111} ) = -1.547</td>
<td>( a_{211} ) = 1.422</td>
<td>( a_{311} ) = -26.094</td>
</tr>
<tr>
<td>( a_{112} ) = 1.270</td>
<td>( a_{212} ) = 10.746</td>
<td>( a_{312} ) = 10.252</td>
</tr>
<tr>
<td>( a_{113} ) = 1.331</td>
<td>( a_{213} ) = 43.831</td>
<td>( a_{313} ) = 73.036</td>
</tr>
<tr>
<td>( a_{114} ) = 1.311</td>
<td>( a_{214} ) = 3.410</td>
<td>( a_{314} ) = 14.760</td>
</tr>
<tr>
<td>( a_{115} ) = 3.076</td>
<td>( a_{215} ) = 25.421</td>
<td>( a_{315} ) = 4.525</td>
</tr>
<tr>
<td>( a_{116} ) = 2.743</td>
<td>( a_{216} ) = 99.723</td>
<td>( a_{316} ) = 72.895</td>
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<tr>
<td>( a_{117} ) = 1.908</td>
<td>( a_{217} ) = 6.292</td>
<td>( a_{317} ) = 4.732</td>
</tr>
<tr>
<td>( a_{118} ) = 2.029</td>
<td>( a_{218} ) = 13.827</td>
<td>( a_{318} ) = 3.017</td>
</tr>
<tr>
<td>( a_{119} ) = 0.861</td>
<td>( a_{219} ) = 49.620</td>
<td>( a_{319} ) = 0.526</td>
</tr>
<tr>
<td>( a_{120} ) = 0.676</td>
<td>( a_{220} ) = 1.970</td>
<td>( a_{320} ) = 0.235</td>
</tr>
<tr>
<td>( a_{121} ) = -0.579</td>
<td>( a_{221} ) = 2.779</td>
<td>( a_{321} ) = 31.464</td>
</tr>
<tr>
<td>( a_{122} ) = 6.272</td>
<td>( a_{222} ) = 1.210</td>
<td>( a_{322} ) = 52.724</td>
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</tbody>
</table>


Table A2: Vehicle configurations in simulation

<table>
<thead>
<tr>
<th>Module</th>
<th>Description</th>
<th>CV</th>
<th>HEV</th>
<th>PHEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle</td>
<td>mass (kg)</td>
<td>960</td>
<td>960</td>
<td>960</td>
</tr>
<tr>
<td>Body &amp; Chassis</td>
<td>drag coefficient</td>
<td>0.26</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>frontal area (m²)</td>
<td>2.25</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>tire specification</td>
<td>P175/65 R14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR weight ratio</td>
<td></td>
<td>0.6/0.4</td>
<td>0.6/0.4</td>
<td>0.6/0.4</td>
</tr>
<tr>
<td>Battery</td>
<td>No. of cells</td>
<td>-</td>
<td>75</td>
<td>100-1000</td>
</tr>
<tr>
<td></td>
<td>mass (kg)</td>
<td>-</td>
<td>13</td>
<td>30-419</td>
</tr>
<tr>
<td>Motor</td>
<td>size (kW)</td>
<td>-</td>
<td>52.35</td>
<td>52.35-120</td>
</tr>
<tr>
<td></td>
<td>mass (kg)</td>
<td>-</td>
<td>65</td>
<td>40-143</td>
</tr>
<tr>
<td>Engine</td>
<td>size (kW)</td>
<td>113</td>
<td>57</td>
<td>15-60</td>
</tr>
<tr>
<td></td>
<td>mass (kg)</td>
<td>251</td>
<td>114</td>
<td>30-120</td>
</tr>
<tr>
<td>Misc.</td>
<td>mass (kg)</td>
<td>285</td>
<td>328</td>
<td>328</td>
</tr>
<tr>
<td></td>
<td>Net Weight (kg)</td>
<td>1496</td>
<td>1499</td>
<td>1430-2012</td>
</tr>
</tbody>
</table>

Figure A1: Metamodel and simulation data points for vehicle CD-mode efficiency, CS-mode efficiency, and 0-60 mph acceleration time as a function of battery size, motor size, and engine size.