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An Analysis of the 'Failed States Index' by Partial Order Methodology

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Abstract
Often objects are to be ranked. However, there is no measurable quantity available to express the ranking aim and to quantify it. The consequence is that indicators are selected, serving as proxies for the ranking aim. Although this set of indicators is of great importance for its own right, the most commonly used practice to obtain a ranking is an aggregation method. Any aggregation, however suffers from the effect of compensation, because the aggregation technique is in the broadest sense an averaging method. Here an alternative is suggested which avoids this averaging and which is derived from simple elements of the theory of partially ordered sets (posets). The central concept in partial order is the 'concept of comparison' and the most general outcome is a web of relations between objects according to their indicator values, respecting the ranking aim.

As an example the 'Failed State Index' (FSI), annually prepared by the Fund of Peace is selected. The FSI is based on twelve individual contextual different indicators, subsequently transformed into a single composite indicator, by simple addition of the single indicator values. Such an operation leaves space for compensation effects, where one or more indicators level out the effect of others. Hence, a comparison between the single states (in total 177) based on their mutual FSI ranking has its limitations as the comparisons are made based on the composite indicator. We show that brain drain is one of the indicators in the FSI-study that plays a crucial role in the ranking, whereby the ranking aim is the stabilization of nations.

Keywords
Partial order, averaged ranks, indicator analyses, multicriteria analyses, decision support tools.

Introduction

Often objects are to be ranked without an available measurable quantity, expressing the ranking aim. Typically a set of indicators is then selected, where the indicators are considered as proxies for the ranking aim (this set of indicators is often called an information basis, IB). Definition and quantifying these indicators are difficult and time expensive. Therefore the multi-system of indicators (MIS: multi-indicator system) is of high value for its own right. Nevertheless a ranking on the basis of a MIS cannot directly be performed. Therefore in many ranking methods an aggregation of these indicators is performed, for example by determining the weighted sum of indicator values of each object.

Obviously, provided a one-dimensional scale after any aggregation such as a simple addition of the single indicators, ranking is easy and straightforward. However, what are the consequences of this simplicity? In the best case some valuable information is lost. More unfortunate is that such a simple addition of the indicator values may lead to quite erroneous conclusions as high score(s) in certain indicator(s) may be leveled off by low scores in other indicator(s), without taking into account that these indicators point towards quite different topics albeit expressing the same ranking aim. In plain words such a simple addition is adding apples and oranges, the eventual result being bananas ranked according to their length. This is a general problem, which holds to a different extent for all multicriteria decision tools and is called the degree of compensation (Munda, 2008). Compensation effects appear to different degrees in all decision support systems where a set of indicators is mapped onto a single scale. Munda (2008) analyzes many of the often used multicriteria decision methods and he founds that the construction of composite indicators by weighted sums of individual ones has the highest degree of compensation.

In this paper a methodology is presented, which is based on simple elements of partial order theory. Partial order theory is considered as a discipline of Discrete Mathematics. The central idea is, to avoid any mapping of indicators on a single scale and extract as much information from the set of indicators respecting at the same time the ranking aim. Partial order theory also provides a technique to derive rankings (where ties are not excluded), which avoids the need of a weighting of the indicators of the information basis. We will outline basics of this theory in the methodology.

As an example we selected the Failed State Index from 2011 (FFP, 2013a), which is generated as a sum of twelve individual indicators, serving as proxy for the not immediately accessible ranking aim "Failed Nations" (or in a dual sense: "Stabilization of nations") (see below).

Methodology

Multi-Indicator Systems

In a multi-indicator system (MIS) the main part of information about the objects is the setting of the indicators and their quantification to obtain appropriate indicator values for the single objects under investigation. In many multicriteria decision systems this valuable and detailed information is mapped onto a single constructed indicator, whereby the information, originally included in the MIS is lost. However, in general metric information is kept.

Partial order theory applied on a MIS is an alternative way to analyze the MIS by keeping the information of the set of indicators, but by providing ordinal information instead of a metric one.

Partial Order

The analysis of partially ordered sets (posets) is a relatively new branch in Discrete Mathematics. The first steps were taken at the end of the 19th century, but only in the mid of the 20th it received a more widespread attention in mathematics. The contributions of Birkhoff (1984) and of Hasse (1967) may be considered as important mile stones. In physics and mathematical chemistry partial order plays some role (see e.g. Ruch, 1975). However broader applications arise when the concept of diversity (Patil and Taillie, 1982; Patil, 2002) and decision making in environmental sciences were put as the focus (Halfon and Reggiani, 1986).

The central concept in partial order is the 'concept of comparison'. Objects are mutually compared, and we assume transitivity, i.e., we assume that if nation x is better than nation y, and nation y is better than nation z then nation x is better than nation z. However, we stress that a comparison alone, i.e., without requiring transitivity is also possible and is applied in the theory of tournaments for example in sports, where team A may beat team B and team B beats team C. However, this does not imply that team A beats team C.

Basic Definition

It is convenient to consider a set of objects x,y,... as elements of X. We define:

$x < y$ if for all indicators q_i ($i=1,\ldots,m$) $q_i(x) \leq q_i(y)$,
with at least one indicator q_i^*, for which a strict inequality $q_i^*(x) < q_i^*(y)$ holds. (1)

If for all i $q_i(x) = q_i(y)$, we consider x as equivalent with y. The equality in all indicator values leads to equivalence classes, which obviously may contain more than one element (a nontrivial equivalence class). In those cases the analysis subsequently is performed with one representative element out of any equivalence class. The other elements of a nontrivial equivalence class are regarded, when contextually needed. The set of representative elements is called X_r.

It is convenient to introduce the set $(X_r,<)$ as $\{(x,y)$ where $(x,y) \in X_r^2$ and $x < y\}$.

A partial order defined in this way is called product order or component-wise order (the latter name is quite obvious, as each single indicator q_i is checked alone) and is in applied sciences also known as Hasse diagram technique (HDT) (see, e.g., Brüggemann et al., 2001; and Brüggemann and Voigt, 2008). By this procedure the logical concept behind each indicator is not mixed with that of another indicator (and therefore there is no compensation), although metric information is lost.

Often two objects do not obey the definition, given above. Consider two indicators and two objects x and y. Object x has the indicator values (2,3) and y (1,4). Then neither $x<y$ nor $y<x$ nor $x=y$. In this case the two objects are incomparable (notation: $x||y$). Incomparabilities indicate conflicts among the involved objects, as at least one indicator favors one object, and another indicator favors the other object with respect to the ranking aim. When a comparability between two objects should be indicated, without specifying the orientation by the <- or >-relation, the notation $a \perp b$ is used. Some additional concepts should be mentioned.

Data profile:
The ordered indicator values of any object play a decisive role in getting a partial order.
Maximal elements:
An object x for which no object y can be found such that $x<y$, is called an maximal element.
Minimal element:
An object x for which no object y can be found such that $y < x$, is called minimal element.
Isolated element:

An object which is at the same time a maximal and minimal element is called an isolated element, i.e. it is not comparable to any other element in the set studied.

Composite indicator (CI):

A weighted sum of indicator values. Most often the weights g_i are taken from an interval [0,1], obeying

the constraint that their sum has to be 1. In that case the indicators should be considered as [0,1]-normalized:

Let CI(x) be the composite indicator value of object x, then:

$$CI(x) = \sum_i g_i \cdot q_i(x) \qquad (2)$$

Hasse Diagram

Partially ordered sets can often be conveniently visualized by Hasse diagrams (Halfon and Reggiani, 1986) by applying the rule that in a geometric plane, a<b is drawn by locating object a below object b and connecting them. For more details, we refer to Davey and Priestley (1991) and Brüggemann and Voigt (2008). A detailed construction of Hasse diagrams in nine steps is in detail explained by Brüggemann and Voigt (1995).

In order to introduce the concept of Hasse diagrams it may be useful to give a little fictitious example: We select X = {a,b,c,d,e,f} and a multi-indicator system (MIS), consisting of two indicators q_1 and q_2. The data matrix is given in Table 1.

Table 1. Data matrix of the fictitious example.

	q_1	q_2
a	2	1
b	3	2
c	2	1
d	2	3
e	4	0
f	2	1

It is immediately noted (Table 1) that the objects a, c and f constitute a nontrivial equivalence class. Using the first element, a, as a representative element for the class, a reduced data matrix is obtained (Table 2):

Table 2. Reduced data matrix, only the representatives of equivalence classes are considered.

	q_1	q_2
a	2	1
b	3	2
d	2	3
e	4	0

Checking Table 2 for the component-wise order it is clear that $a<b$ and $a<d$. Element e, on the other hand, cannot be compared with neither a, b nor d. Thus, element e is in this connection somewhat special, as it has in q_1 the maximal, but in q_2 the minimal value within the data set.

Hence, in our example X= {a,b,c,d,e,f} and X_r is the set of representatives, X_r={a,b,d,e}, and the equivalence class is [a,c,f]. The partial order can be described by $(X_r,<) = \{(a,b),(a,d)\}$. The partially ordered set of our fictitious example is visualized by the Hasse diagram depicted in Figure 1. The location of the isolated element e is arbitrary. However, following drawing conventions the object e is located in the top level (see below).

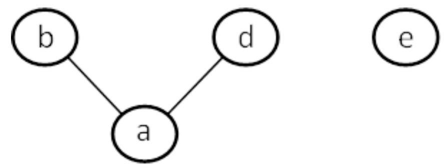

Figure 1. Hasse diagram of the fictitious example, based on the component-wise order (see text).

Figure 1 shows nicely that b and d both have larger values for their two indicators than the corresponding values for a. On the other hand, objects b and d cannot be compared; they are “incomparable”, because b has a higher value in q_1 than d, whereas d has a higher value in q_2 than b. As a common scale for both indicators does not prevail, it is appropriate to keep the information of the indicators well separated to indicate this “data conflict” (see above). The peculiar role of object e is evident, as it cannot be compared with any other element.

As seen, order relations can be visualized as graphs, e.g., by Hasse diagrams. Whereas the graphs as algebraic concept are uniquely defined, the drawing of graphs can be done in different manner. Consequently, a software code must contain rules to set up a graph drawing. These rules can be traced back to the early paper by Halfon and Reggiani (1986). Especially the possible crossing of lines or the location of vertices in levels can be confusing. The software PyHasse (see Appendix 3) is following the traditional rules. Because of the problems mentioned above, PyHasse provides an interface to the well-known program Graphviz (Gansner and North, 1999), which beside others is based on graph theoretical methods of drawing hierarchies (Warfield, 1973; 1974a; 1974b) and Sugiyama et al.,1981). In Appendix 1, an illustrative example is given demonstrating how Graphviz can be applied to reduce the number of crossing lines to a minimum as well as placing ‘children’ near ‘parents’. Finally a word concerning Hasse diagrams as oriented, acyclic, transitively reduced and therefore triangle free, graphs may be useful. Graphs are usually not considered as embedded in a coordinate system. Therefore, a question for a coordinate for any representation of a Hasse diagram is not directly applicable. When later horizontal and vertical analyses are discussed then this is related to the standard drawings of Hasse diagrams. A variant of partial order theory that aims at, sometimes approximately, constructing coordinates, is the well-known POSAC method (Partially Ordered Scalogram Analysis with Coordinates) (see e.g. Brüggemann and Patil, 2011). POSAC is not applied in the present study.

Basics of Partially Ordered Sets, Extended

Chains: In component-wise orders a linear ranking is not generally available, because some objects may be mutually incomparable. Nevertheless, it is of interest to extract as much as possible ranking information out of the partially ordered set. The theoretical concept behind this idea is the concept of chains. Chains are subsets of X_r where any object can be compared with any other object. For example in Figure 1 we find two chains with two elements, i.e., Chain 1: a<b and Chain 2: a<c, respectively. This ordering is valid for all indicators. Therefore, chains are those subsets of X_r, where all the indicator values are weakly monotonously increasing, starting from the bottom element and proceeding upwards.

Often additional information is useful, namely rendering a chain statistics, i.e., calculating the number and lengths of chains (comparable objects). Chains extracted from posets can be considered as ranking within subsets of X_r and are an analyzing tool of Hasse diagrams in a vertical dimension (following the drawing rules of Hasse diagrams).

Antichains: If indicator values are in conflict for any two objects then these objects are incomparable. With other words, an analysis of sets of objects where no object is comparable with another (called an "antichain") is as important as the chain analysis, because here conflicts can be identified.

We call this kind of analysis a "horizontal" analysis as the focus is not on ranking but on conflicts, which imply the extension of the Hasse diagram in a second dimension.

The vertical (ranking oriented) and horizontal (conflict oriented) analysis may be made more vivid by Figure 2.

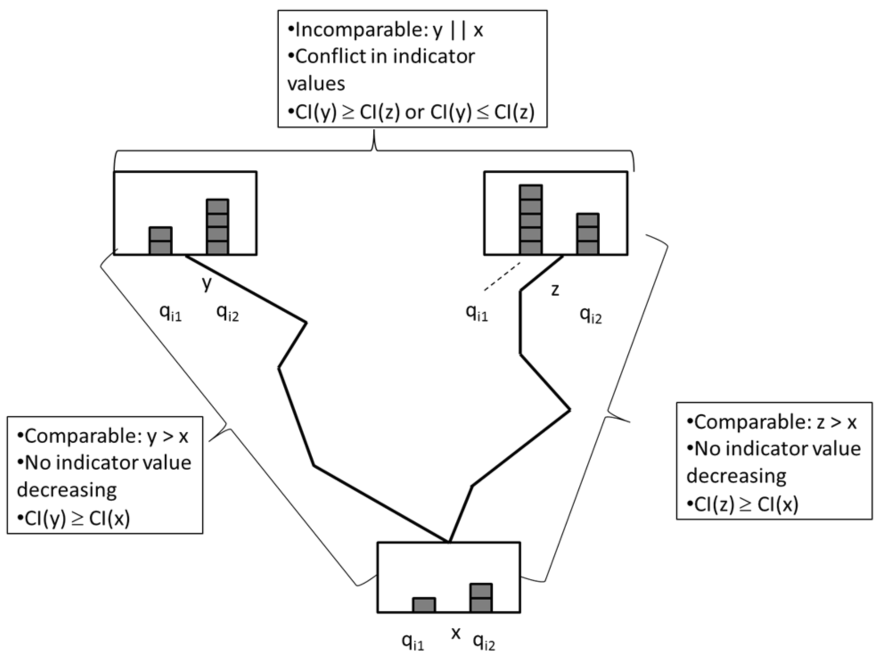

Figure 2. The two polygonal lines symbolize an upwards oriented path in a Hasse diagram according to x<y and x<z. Two individual indicators q_{i1}, q_{i2} out of IB symbolize the data profiles of the three objects. Vertical: Comparability, ranking of subsets (see text). Horizontal: Conflict identification.

Levels: In finite posets, chains of a maximum number of elements can be defined. This number defines the number of levels, NLev. As in detail explained in Brüggemann and Patil (2011) the assignment of objects to levels can be done by starting from the maximal elements. These elements get the number nLev = NLev. Eliminate the maximal elements from X_r and identify the new maximal elements. These new maximal elements get nLev= NLev-1.

This process is repeated, until X_r is exhausted and the result is an assignment, where each object is characterized by nLev = 1,...,NLev.

Following this procedure, the Hasse diagram as in Figure 1 is constructed. Therefore, having a clear Hasse diagram at hand, the levels are simply the elements that are positioned in the same vertical height of the drawing.

Order preserving maps: Incomparable elements may be given an order relation, obeying the transitivity axiom. For example, in Figure 1 object e is set > d; then the axiom of transitivity demands for setting object e > a. When such a procedure is followed, the already existing order relations are preserved and new order relations are constructed. When this procedure ends with a linear order (all elements of X_r are in a chain) then the original poset is order preserving mapped onto a linear order. Thus, it is "enriched." Linear orders deduced from a poset are called linear extensions. From a poset usually several linear extensions can be obtained. Each object has a certain height in any of these linear extensions and the average height of an object is obtained as the average over all single heights (Winkler, 1982) (see below). It is well known (Brüggemann and Patil, 2011) that an aggregation of indicators of IB by a weighted sum does not contradict the order relations deduced by applying definition (1).

The aggregation (2) is an enrichment of the poset, i.e., $x < y \Rightarrow CI(x) \leq CI(y)$; however, as the set of linear extensions of a poset can be very large (at maximum $|X_r|!$) the order deduced from CI is at best just one of the many linear extensions. Furthermore, it can be shown that a weighted sum of indicators of the information basis IB of the MIS cannot represent all linear extensions, even if all weights are varied freely taking into regard only the constraints $1 \geq g_i \geq 0$, $\Sigma g_i = 1$ and q_i, normalized in the range [0,1] (Brüggemann and Patil, 2011).

Navigation tools: The level structure, the number of successors of each element (downward comparable elements) and predecessors (upward comparable objects), as well as the identification of incomparabilities, are the main results in every posetic study. Hence, PyHasse (see Appendix 3) offers information on the following important features:

- Principal order filters (synonym is principal upsets, i.e., objects being upwards comparable to a given object studied)
- Principal order ideals (synonym is principal downsets, i.e., objects being downward comparable to a given object studied)
- Order interval graphs (i.e., objects order theoretically between two given objects studied)
- Level structure (NLev and the objects having the same value of nLev)
- Set of elements incomparable with a selected element

Weak order: A weak order is an ordered sequence of objects, where some objects are tied. Following the above example we could write a=c=f < b derived from one of the two chains.

Based on the numbers nLev a weak order can be found, encompassing all elements of X_r. Thus, the level construction is a simple way to obtain a weak order and to perform a ranking based on nLev. The advantage is the simplicity of this construction; the disadvantage is the appearance of many ties and the dependency on heuristic rules. Following Table 2 and Figure 1, a weak order related to X_r based on the levels can be derived: a < b=d=e.

Another more complicated way to get weak orders for all elements of X_r is a method where the number of ties is strongly reduced (in comparison with the weak order obtained from the level construction). The method is based on the calculation of averaged heights, hav, which can be obtained directly by a counting procedure (most often computationally intractable), from Monte Carlo simulation of a large number of linear extensions (Bubley and Dyer, 1999, Denouex et al., 2005, Lerche et al., 2003) or by the local partial order methods LPOM0 (Brüggemann et al., 2004) or LPOMext (Brüggemann and Carlsen, 2011) or following De Loof et al., 2011.

In our fictitious example (Table 2), such linear orders are: a<b<d<e, a<d<b<e, e<a<b<d, ... In total, eight linear orders exist for this example. In each of the linear extensions, the four objects have a "height" (rank). In the first linear extension, hav(a, le_1) = 1, where hav(a,le_1) is the height of the object a in linear extension le_1, hav(b, le1) = 2, hav(d, le_1) = 3 and hav(e, le_1)= 2. In the next linear extension another distribution of the heights is found, for example hav(a, le_2) = 1, hav(b, le_2) = 3, hav(d, le_2) = 2 and hav(e, le_2) = 4, etc. Averaging over all heights of a certain element leads to the so-called averaged rank of that object. As this is possible for each object, an ordering index is obtained by taking the averaged heights.

In Table 3 the average heights (interpreted as average ranks) can be seen. Hence, in Table 3, the cell, obtained from the second row (object b) and the fourth column (height = 3) has the value 3, meaning that in 3 of 8 linear extensions object b has height = 3. Consequently, the average height of b is (0*1+2*2+3*3+3*4)/8 = 25/8.

Table 3. Averaged ranks of the objects of the fictitious example.

	height=1	height=2	height=3	height=4	average height
a	6	2	0	0	10/8
b	0	2	3	3	25/8
d	0	2	3	3	25/8
e	2	2	2	2	20/8

Based on the data in Table 3, the following weak order, based on the average heights: a < e < b = d.

The tie between b and d is obvious, as both objects are order theoretically equivalent; there is no change in the poset if objects b and d are interchanged.

Sensitivity Analysis

Obviously for a multi-indicator system (MIS), information about the relative importance of the single indicators are of utmost interest as it provides crucial input, e.g., in connection with decisions about where a possible specific effort should be made in order to change an unwanted situation, as will be discussed in connection with our analyses of the FSI data.

The sensitivity expresses how important the single indicators are for the ranking, e.g., which indicator is more important for the structure of the partial order (i.e., the system of levels, chains and antichains). Further details, concerning the sensitivity analysis have been reported by Brüggemann et al., 2001.

The leading idea is to find distances among posets, where one poset is the original one with all indicators and the others are those posets where one indicator is left out. Obviously the indicator, whose elimination from the data matrix leads to the maximal distance to the original one, is most important for the structure of the MIS. This kind of sensitivity analysis may be called 'indicator-related sensitivity,' because the effect of one indicator is studied. Clearly also a change in the values of indicators have an influence on the structure of a poset. Hence, a recently published concept that unifies both concepts: the variance based sensitivity (Annoni et al., 2011).

Partial Order and Other Approaches

Overview

In more general terms we can define three groups of methods that can be thought of as useful in ranking studies:

1. Approaches using tools of network theory
2. Multivariate approaches (Principal Component analysis, factor analysis, Cluster analysis, etc.) and - what will be done in this paper)
3. Order theory (not necessarily HDT).

Order Theory

We start with order theory. In order elucidate other concepts we first critically illuminate some concepts in HDT.

- The graph, which may be derived from the order relations is not only representing a web or network of “good”/”bad” relations, according to the ranking aim, but is also related to the indicator values of individual objects. When two vertices (objects, states) are connected, we not only know their evaluation status, but also some information about the corresponding indicator values. Two vertices not connected means there is a conflict in the data profile (see Figure 2).
- In the case of only minor numerical differences in indicator values objects could advantageously be regarded as being equivalent. Procedures to handle numerical differences of indicator values in a systematic way are well known and are based on fuzzy theory (Van de Walle, 1995; De Baets and De Meyer, 2003; De Meyer et al., 2004; Brüggemann and Patil, 2011; Brüggemann et al., 2011; Wieland and Brüggemann, 2013).
- The relation between the network and data profile can be formalized, and a Galois connection can be established (Wolski, 2004), which may, e.g., lead to the well-known Formal Concept Analysis (Ganter and Wille, 1996). In this context Annoni and Brüggemann (2008) may be cited: “Formal Concept Analysis (FCA) is a tool to investigate relationships among attributes and objects in a symmetric way. It provides a parameter free exploration of data and a powerful graphical representation of data.” Formal Concept Analysis is left for future work, because there is still programming work to be done.

Networks

Network theory is widely used in socioeconomic studies, as networks formalize patterns of flows of different types—human movements, distribution of ideas, power/traffic networks—and (considered as ordinary graphs) allows to derive useful invariants such as diameter, eccentricity, centrality (see e.g. Naimzada et al., 2009, or De Nooy et al., 2009). The obvious commonality of networks with partial order is the appearance of directed edges. Incomparabilities and comparabilities give a “web of relations” by which objects are connected. Thus, in earlier days, Hasse diagrams were also called evaluation networks and partial order theory provides tools to derive data-driven orders from these “networks.” However, in partial order theory the concept of flow is not an obvious concept, therefore the conceptualization of a Hasse diagram as network of *flows* is of little importance (interestingly the well-known multicriteria method PROMETHEE talks of flows within the context of posets (Brans and Vincke, 1985.) The Hasse diagram itself cannot be used to derive graph theoretical invariants because the order relations are represented by the transitive lines. Nevertheless, the order theory defines invariants such as width and height (which may be compared with diameter). Components or leaves as typical graph-theoretical concepts can usefully be considered too. However, in general such invariants, which relate the structure of the graph (i.e., of the order relations) with the values of indicators or with indicators themselves, are of importance. The concept of separability (Brüggemann and Voigt, 2011) and of sensitivity (Brüggemann et al., 2001) may be mentioned in this context. As in network theory, partial order derives hierarchies; this facility in partial order methodology being one of the main objectives.

Beside the network of objects as vertices, partial order can generate other networks. One of the most important networks is derived from Formal Concept Analysis (Ganter and Wille, 1996; Wolski, 2004), which is an implication network: For a set of "if-then" statements, relating the indicators, a concept such as networks appears promising (see Brüggemann and Patil, 2011).

Multivariate Approaches

When data exploration is the focus without necessarily respecting the ranking purpose, the whole set of tools, provided by multivariate statistics should be considered to get maximum of information from the data matrix. Data matrices in general may hide unknown factors, which can be extracted by different methods, such as factor analysis or PCA. If then a single factor serves to derive a linear order then the task is done, without application of partial order techniques. However on the one side often the most important latent variable does not explain the variance in a satisfying manner and on the other side the extracted information has the same disadvantages as every scalar obtained by an aggregation process.

The same is true for another typical multivariate statistical approach, such as cluster analysis. Based on a distance concept, clusters are defined. Whether or not an order relation can be established is not the primary aim of these methods. A possibility is to reorder a posteriori the clusters as is shown for example in Mucha et al. (2005). However, distances or variances that are leading terms in multivariate statistics are rarely useful in order theory with its focus on an ordinal analysis. For example, when indicators of different scaling level are to be considered or when the role of error does not play a primary role then distances or variances are poorly defined. To our knowledge the well-known Gifi-system is based on multivariate concepts and allows considering simultaneously indicators of different scaling level. However, this and similar approaches are recently criticized (cf. Annoni, 2007).

Data

The organization Foreign Policy publishes annually the so-called 'Failed State Index' (FSI) (FSI, 2013a) that is an annual ranking of world's most vulnerable countries states prepared by the Fund For Peace (FFP, 2013b). The 2011 FSI that comprises 177 states is derived based on around 130,000 publicly available sources, the data originating from 2010. The data are collected into twelve so-called conflict assessment indicators or indicators of pressure (FFP, 2013c).

The group of indicators, q1 – q12, comprises **Social indicators** (1: Mounting Demographic Pressures, 2: Massive Movement of Refugees or Internally Displaced Persons, 3: Legacy of Vengeance-Seeking Group Grievance or Group Paranoia, 4: Chronic and Sustained Human Flight); **Economic indicators** (5: Uneven Economic Development Along Group Lines, 6: Sharp and/or Severe Economic Decline); and **Political/ Military Indicators** (7: Criminalization and/or Delegitimization of the State, 8: Progressive Deterioration of Public Services, 9: Suspension of the Rule of Law and Widespread Violation of Human

Rights, 10: Security Apparatus Operates as a "State within a State," 11: Rise of Factionalized Elites, 12: Intervention of Other States or External Political Actors) (FFP, 2013c).

All twelve indicators have values in the (continuous) range from 1-10, where 1 was assigned to the most stable and 10 to the least stable and thus the most at-risk of collapse and violence. Thus, the FSI may take on values from 12 to 120—the higher the value, the less stable the state. The location of the single states on a one-dimensional scale from 1-177 is then done by a simple addition of the single scores for a given country. Thus, for example, at the top of the list, ranked 1, we find Somalia, the individual indicators being 9.7, 10, 9.5, 8.2, 8.4, 9.3, 9.8, 9.4, 9.7, 10, 9.8, and 9.7 respectively; the total score being 113.5. Hence, a rather complex 12-dimensional indicator system is projected on a one-dimensional scale by simple addition of the single indicators resulting in a 'super' indicator representing the single countries.

The data matrix applied for the present study has been retrieved from the Foreign Policy's web site presenting the FSI 2011. The data matrix was downloaded as an Excel spread sheet (see Appendix 2) and subsequently saved as a tab-separated text file as input for the partial order studies, the latter being performed applying the PyHasse software (Voigt et al., 2010).

Results and Discussion

The FSI is, as mentioned above, based on a simple multi-indicator model where the single indicator values for the twelve included indicators are summed to a super-indicator, the index for the given state. The single indicators are assigned equal weight. Without doubt this is a useable and transparent method and it for sure gives some important indications of ranking of the single countries based on some kind of overall view. However, we claim that significant amount of valuable information is lost applying this simple aggregation approach. Taking into account the incredible amount of work being carried out behind the scenes to produce the data leading to the FSI, it deserves to be treated in a way that the data discloses as much information as possible of their encapsulated information. We will elucidate how this information, at least to some extent, may be revealed by applying partial order methodology.

The Hasse Diagram

In the case of the 'Failed States' we can obtain a Hasse diagram, thus visualizing the partial order applying the data matrix containing the values for the single states for the single indicators. Obviously, due to the relative high number of objects, as simple a picture as the one depicted in Figure 1 cannot be expected and it may virtually be impossible to get a detailed overview. To illustrate the complexity of the diagram, Figure 3 displays the Hasse diagram of an arbitrarily selected subgroup of 20 out of the 177 states.

Thus, looking at the results of the complete analysis, including all 177 states, we find, as expected many incomparable nations. Based on all twelve indicators, we find 6307 comparisons between states; whereas 9269 pairs of states prevail which cannot be compared, i.e., where conflicts between their indicator values appear.

The analysis further discloses that all 177 states appear as unique in this sense, i.e., we do not find any equalities (i.e. X_r= X). In the following we turn to other methods of characterization.

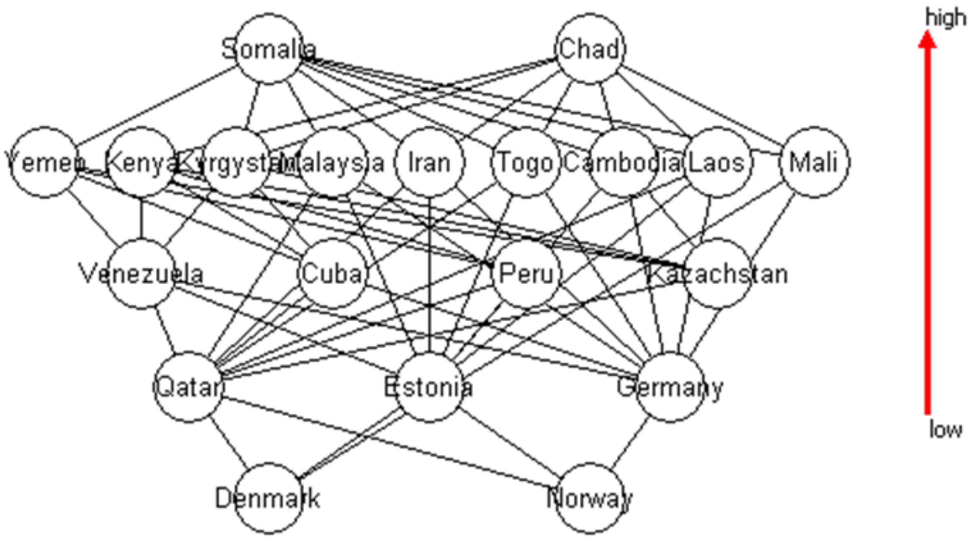

Figure 3. Hasse diagram based on the FSI data for 20 randomly selected states out of the 177, and the twelve indicators constituting the FSI.

It is noted that Figure 3 is organized in five levels. One may identify structures such as schematically shown in Figure 2 and deduce where a ranking of some objects is possible without the need of weighting (vertical analysis) and where conflicts appear (horizontal analysis). It should be noted that the analysis of the full set of 177 states leads to a diagram characterized by seven levels.

In Table 4, the results of the partial order analysis of the 177 states are summarized giving the level assignment (nLev) for the single states as well as the original FSI.

Table 4. Level structure (nLev) of the 177 states as disclosed by the partial order analysis.

Country	**FSI**	**Level (nLev)**	**Country**	**FSI**	**Level (nLev)**	**Country**	**FSI**	**Level (nLev)**
Somalia	113.5	7	Mauritania	88.0	5	Ghana	67.6	4
Chad	110.3	7	Egypt	86.8	5	Jamaica	67.1	4
Sudan	108.8	7	Laos	86.7	5	Seychelles	67.0	4
Congo (D. R.)	108.2	7	Georgia	86.4	5	Trinidad	63.7	4
Haiti	108.0	7	Syria	85.9	5	Antigua & Barbuda	59.9	4
Zimbabwe	107.9	7	Solomon Islands	85.9	5	Mongolia	59.6	4
Afghanistan	107.5	7	Bhutan	85.0	5	Panama	57.8	4
Central African Rep.	105.0	7	Philippines	85.0	5	Croatia	57.3	4
Iraq	104.9	7	Comoros	83.8	5	Malaysia	68.7	3
Guinea	102.5	7	Madagascar	83.2	5	Albania	66.2	3
Pakistan	102.3	7	Dijbouti	82.6	5	Romania	59.8	3
Nigeria	100.0	7	Ecuador	82.2	5	Kuwait	59.5	3

Niger	99.1	7
Myanmar	98.3	7
North Korea	95.6	7
Equatorial Guinea	88.0	7
Cote d'Ivoire	102.8	6
Yemen	100.4	6
Kenya	98.7	6
Burundi	98.7	6
Guinea Bissau	98.2	6
Ethiopia	98.2	6
Uganda	96.2	6
Timor-Leste	94.8	6
Cameroon	94.6	6
Bangladesh	94.3	6
Liberia	93.9	6
Nepal	93.7	6
Eritrea	93.5	6
Sri Lanka	93.1	6
Sierra Leone	92.1	6
Congo (Republic)	91.3	6
Malawi	91.1	6
Iran	90.1	6
Togo	89.3	6
Burkina Faso	88.5	6
Uzbekistan	88.3	6
Lebanon	87.7	6
Colombia	87.0	6
Angola	84.7	6
Israel/West Bank	84.5	6
Papua New Guinea	84.3	6
Zambia	83.9	6
Mozambique	83.5	6
Bolivia	82.9	6
Swaziland	82.6	6
Lesotho	80.4	6
China	80.1	6
Turkmenistan	79.7	6
Mali	79.3	6
Cape Verde	75.8	6
Guyana	72.5	6
Micronesia	71.9	6

Azerbaijan	81.9	5
Indonesia	81.5	5
Tanzania	81.3	5
Moldova	81.2	5
Nicaragua	81.2	5
Fiji	81.1	5
Gambia	80.9	5
Bosnia & Herzegovina	81.0	5
Guatemala	80.0	5
Benin	80.0	5
India	79.3	5
Honduras	78.4	5
Thailand	78.3	5
Algeria	78.0	5
Russia	77.7	5
Dominican Republic	76.8	5
Senegal	76.8	5
Morocco	76.2	5
El Salvador	76.0	5
Maldives	75.7	5
Gabon	75.3	5
Saudi Arabia	75.2	5
Mexico	75.1	5
Jordan	74.6	5
Peru	73.6	5
Paraguay	72.4	5
Namibia	71.8	5
Cyprus	67.6	5
South Africa	67.6	5
Brunei	65.8	5
Brazil	65.1	5
Venezuela	78.1	4
Belarus	77.6	4
Cuba	76.6	4
Vietnam	76.0	4
Turkey	74.9	4
Sao Tome	74.4	4
Serbia	74.4	4
Armenia	72.3	4
Suriname	71.1	4
Macedonia	71.0	4

Bahrain	58.9	3
Bulgaria	59.0	3
Bahamas	56.5	3
Montenegro	56.3	3
Lativa	54.1	3
Barbados	52.8	3
United Arab Emirates	50.3	3
Estonia	49.4	3
Poland	46.9	3
Malta	45.5	3
Iceland	30.1	3
Costa Rica	50.6	2
Qatar	49.5	2
Oman	49.1	2
Hungary	48.7	2
Greece	47.4	2
Slovakia	47.0	2
Argentina	46.8	2
Italy	45.8	2
Lithuania	45.3	2
Mauritius	44.2	2
Spain	43.1	2
Czech Republic	42.3	2
Chile	40.8	2
Uruguay	40.4	2
South Korea	38.8	2
Slovenia	35.5	2
United States	34.8	2
United Kingdom	34.0	2
Belgium	34.0	2
France	34.0	2
Germany	33.9	2
Portugal	32.4	2
Netherlands	28.4	2
Singapore	35.1	1
Japan	31.0	1
Australia	28.1	1
Canada	27.7	1
Austria	27.3	1
Luxembourg	26.1	1
Ireland	25.3	1

Samoa	69.6	6	Kazakhstan	70.2	4	New Zealand	24.9	1
Grenada	66.4	6	Tunisia	70.1	4	Denmark	23.8	1
Kyrgyzstan	91.8	5	Ukraine	68.9	4	Switzerland	23.2	1
Rwanda	90.9	5	Libya	68.7	4	Sweden	22.8	1
Cambodia	88.6	5	Botswana	67.9	4	Norway	20.4	1
Tajikistan	88.3	5	Belize	67.7	4	Finland	19.7	1

Scrutinizing Table 4 it becomes clear that although we do see an overall tendency in the level structure that mimics that of the FSI ranking, pronounced differences are immediately noted. Thus, states that are ranked rather low at the FSI scale like Grenada (FSI=66.4) and Brazil (FSI=65.1) can be found at level 6 and 5, respectively. Further it is interesting to note that, e.g., the nine states that according to the FSI are the most 'failed, i.e., Somalia, Chad, Sudan, Congo (D. R.), Haiti, Zimbabwe, Afghanistan, Central African Rep. and Iraq, are not comparable (cf. Table 4) as they are all located at level 7 and constitute as such an antichain.

To disclose how a specific state is comparable to other states under investigation the order theoretical navigation tools, i.e., principal filters, ideals and intervals are applied. Thus, as an illustrative example Kazakhstan can be used. In Figure 4 the countries comparable to Kazakhstan both upwards (Figure 4A) and downwards (Figure 4B) are visualized.

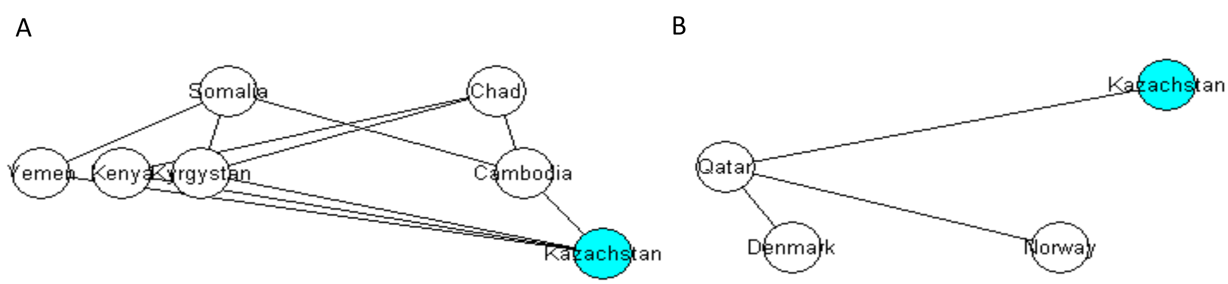

Figure 4. Order filter (A) and ideal (B) for Kazakhstan for the subset of 20 states (cf. Figure 3). The generating objects are marked blue.

Inspecting Table 4, it is clear that in the case of Kazakhstan, located at level 4, it is both upwards and downwards comparable to states located at several levels (cf. also Figure 4).

From the above Figures 3 and 4, it is further clear that a ranking based only on the level structure is very rough. Thus, this would give rise to only seven different ranks, i.e., a substantial number of states would assigned the same rank (cf. Table 4). Hence, we turn to the average rank to disclose a more detailed ranking.

Averaged Ranks

As a natural continuation of this discussion, it appears appropriate to compare the original FSI ranking based on the composite indicator (FSI) with the weak linear based on averaged ranks obtained directly from the data matrix (see Order Preserving Maps, above). We stress that partial order theory provides many techniques to obtain weak orders. First we applied the technique based on levels (see above). Now we apply deeper mathematical methods to find a weak order with a significantly higher degree of differentiation than that based on the levels. In Table 5, the calculated averaged ranks for all 177 states are summarized.

Table 5. Average ranks (Rkav) of the 177 states as calculated by the LPOMext method (Brüggemann and Carlsen, 2011).

Country	Rkav	Country	Rkav	Country	Rkav
Chad	1.52	Angola	51.71	Jamaica	113.45
Somalia	1.58	Solomon Islands	52.23	Paraguay	115.83
Congo (D. R.)	1.92	Lesotho	53.75	Albania	116.69
Zimbabwe	2.18	Bosnia & Herzegovina	56.58	Seychelles	118.21
Haiti	2.34	Dominican Republic	57.17	Gabon	119.78
Iraq	2.68	Philippines	57.58	Antigua & Barbuda	120.58
Sudan	2.92	Fiji	59.08	Brunei	126.60
Afghanistan	3.20	Ecuador	59.49	Romania	126.92
Guinea	4.19	Indonesia	60.61	Croatia	127.29
Pakistan	4.51	Dijbouti	61.51	Kuwait	134.36
Central African Rep.	5.31	Madagascar	61.61	Bulgaria	135.37
Cote d'Ivoire	5.50	Turkmenistan	64.01	Lativa	136.57
Nigeria	6.25	Swaziland	64.27	Panama	137.50
Kenya	7.00	Azerbaijan	65.82	Bahamas	138.50
Yemen	7.64	Comoros	65.87	Barbados	139.15
Niger	8.80	India	67.51	Montenegro	140.71
Myanmar	8.91	Venezuela	69.02	Estonia	143.22
Ethiopia	11.28	Tanzania	69.73	Bahrain	145.30
Bangladesh	11.78	Guatemala	72.62	Costa Rica	146.11
Guinea Bissau	13.84	Samoa	73.28	United Arab Emirates	149.51
Burundi	14.38	Cuba	74.38	Slovakia	151.28
Sierra Leone	15.78	Mexico	74.68	Malta	152.22
Cameroon	15.88	China	75.01	Mongolia	152.24
Uganda	17.21	Grenada	77.25	Poland	152.30
Nepal	17.70	Gambia	77.69	Qatar	152.36
North Korea	17.86	Morocco	77.78	Hungary	152.49
Liberia	20.03	Micronesia	77.98	Argentina	152.92
Eritrea	21.25	Honduras	78.27	Italy	153.58
Kyrgyzstan	22.49	Belarus	78.32	Spain	154.71
Sri Lanka	22.68	Macedonia	79.61	Lithuania	155.01
Malawi	24.59	Suriname	81.54	Greece	155.38
Cambodia	29.55	El Salvador	81.97	Chile	158.94
Iran	29.91	Namibia	82.03	Czech Republic	159.43
Papua New Guinea	31.10	Algeria	82.49	France	160.70
Congo (Republic)	31.32	Benin	82.61	Germany	161.27
Uzbekistan	31.36	Cyprus	82.84	United Kingdom	162.81
Rwanda	31.54	Jordan	85.44	Iceland	165.84
Timor-Leste	34.32	Sao Tome	85.52	Oman	166.03
Togo	34.92	Peru	87.13	South Korea	167.00
Burkina Faso	35.18	Senegal	87.83	Portugal	167.62
Tajikistan	37.08	Russia	88.35	United States	169.37
Mozambique	40.67	Turkey	88.68	Belgium	170.20
Lebanon	41.06	Vietnam	91.73	Mauritius	170.65

Egypt	41.90
Bolivia	42.38
Guyana	44.15
Zambia	44.68
Syria	45.59
Nicaragua	45.64
Bhutan	46.28
Equatorial Guinea	46.43
Laos	47.48
Moldova	47.60
Georgia	47.85
Mali	48.33
Israel/West Bank	48.83
Cape Verde	51.09
Mauritania	51.34
Colombia	51.68

Thailand	92.84
Saudi Arabia	93.09
Serbia	93.60
Armenia	94.18
Maldives	95.51
Belize	96.73
Ghana	98.86
Ukraine	99.20
Botswana	101.59
South Africa	106.00
Libya	106.15
Trinidad	107.52
Tunisia	107.95
Kazakhstan	108.72
Malaysia	111.22
Brazil	112.32

Netherlands	171.01
Uruguay	171.56
Singapore	171.73
Slovenia	172.68
Luxembourg	173.56
Austria	173.63
Japan	173.92
Ireland	173.99
Sweden	174.05
Canada	174.20
Australia	174.25
New Zealand	174.32
Denmark	174.57
Finland	174.81
Switzerland	175.33
Norway	175.79

The single states are ordered according to their average ranks and can be compared to the original FSI ranking (cf. Appendix 2). Thus, a plot of these two columns against each other immediately will visualize the possible correspondence between the FSI ranking and that based on averaged ranks (Figure 5).

Immediately it is noted that the two rankings to some extent correspond to each other. However, it is also clear that significant differences must be noted. Thus, we note that the curve has a minor but significant S-shape. This finding demonstrates that partial order infers a new quality, which is not grasped by the simple aggregation as done with the FSI. It seems to be rather clear that extreme nations (most stable, most unstable) will be the same by any decision support method; hence, at the very end of the sigmoid shaped curve the deviations are small. In the range of FSI around 100 there are deviations, indicating that the additional knowledge (due to the weights given) contradicts the purely data-driven ranking. Such conflicts can only happen when nations are incomparable. Aggregating the indicators is thus eliminating the conflict in indicator values, which can lead to discrepancies in both directions giving FSI a higher rank then due to averaged ranks by partial order methodology or vice versa.

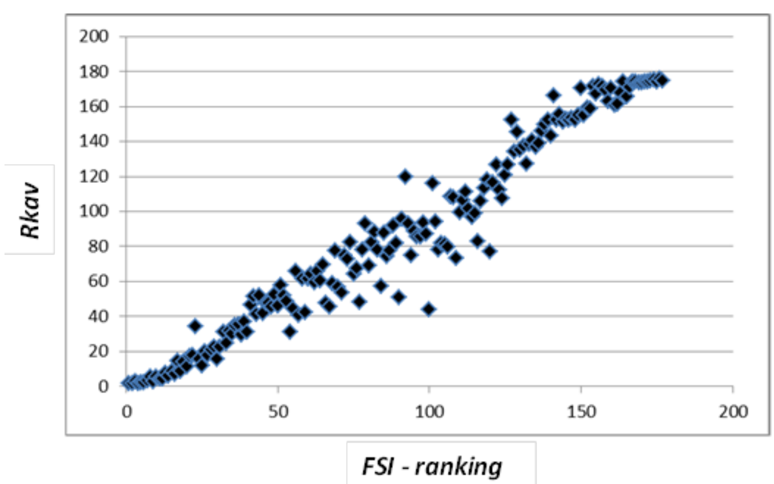

Figure 5. Original FSI ranking as a function of averaged ranks.

Chain and Antichain Analyses

Chain and antichain analysis are intended to extract ranking information (vertical analysis) and conflicts (horizontal analysis).

Chain Analysis

The chain analysis focuses on finding chains between two nations. Clearly a good starting point is to select nations from the bottom and top level. Here the states of The Netherlands and Somalia may serve as an illustrative example. The Netherlands is located in the bottom level, whereas Somalia is an element found at the top level. The chain analysis yields 27 chains of length ≥ 6, for illustration an arbitrary selection including seven chains is given in Table 6.

Table 6: Seven arbitrarily chosen chains between The Netherlands and Somalia.

Chain number	Count of states	
389	6	The Netherlands, Montenegro, Serbia, Kyrgystan, Cote d'Ivoire, Somalia
135	6	The Netherlands, Albania, Cuba, Kyrgystan, Cote d'Ivoire, Somalia
392	6	The Netherlands, Montenegro, Serbia, Georgia, Cote d'Ivoire, Somalia
194	6	The Netherlands, Romania, Venezuela, Kyrgystan, Cote d'Ivoire, Somalia
544	6	The Netherlands, Estonia, Venezuela, Kyrgystan, Cote d'Ivoire, Somalia
391	6	The Netherlands, Montenegro, Serbia, Egypt, Cote d'Ivoire, Somalia
568	6	The Netherlands, Estonia, Vietnam, Tajikistan, Cote d'Ivoire, Somalia

It is found that the 9th indicator (Suspension of the Rule of Law and Widespread Violation of Human Rights) increases along the chains from its lowest to its highest value. Analogously it is found that almost the full range of values starting from The Netherlands and ending in Somalia prevails for indicator 7 (Criminalization and/or Deligitimization of the State).

Antichain Analysis

For the antichain analysis the present study is illustrated by the top level with 16 nations. However, the principles illustrated here obviously are generally applicable. Clearly the selection of objects from only one level to build an antichain does not imply that these objects are comparable to all others in other levels.

As already mentioned, within an antichain every object is incomparable with every other. Hence, it is of interest to explore how many indicator conflicts are causing the incomparability between any two states of level 7. The number of indicator pairs causing incomparability between two objects $x \parallel y$ is called "severity of incomparability." In Table 7 an overview is given how severely (i.e. by how many indicator pairs) a pair of states is incomparable. The range of values is normalized to 1, i.e. 1 means with respect to all possible indicator pairs the object pair is incomparable. The lowest value 0 is not possible because then for the corresponding nations x, y, $x \perp y$ should be valid. Indeed the lowest value is 0.15.

Table 7. Distribution of severity of incomparable states.

	[0, 0.25]	[0.25, 0.5]	[0.5, 0.75]	[0.75, 1.0]
Relative number of pairwise indicator conflicts	1	19	39	61

From Table 7 we can deduct that from the 120 possible pairs of sixteen states taken from level 7 (= 16*15/2), only one pair has a striking low number of conflicts, whereas for the overwhelming numbers of pairs of states (61) either all possible indicator pairs (value = 1) or the majority (values >0.75) contribute to a conflict. Obviously we are here facing a high number of conflicts and thus a rich field of possible compensation effects, when the simple addition to the composite FS-indicator is done.

Clearly it is of interest to analyze which indicators are mainly contributing to the incomparabilities found in the antichain under investigation, e.g., level 7. It turns out that q5 (Uneven Economic Development along group lines) and q6 (Sharp and/or Severe Economic Decline), i.e., the economic status of the nations, are mostly responsible for the conflicts in the top level.

The incomparabilities between two states, not necessary located at the same level in the Hasse diagram, are easily visualized in simple bar diagrams. As an illustrative example the two neighboring states, Kyrgystan and Uzbekistan found at level 6 and level 5, respectively in the Hasse diagram (Table 4). However, despite the close geographical location of the two states they are not comparable when taking all indicators into account. In Figure 6, the twelve indicator values for the two states are summarized.

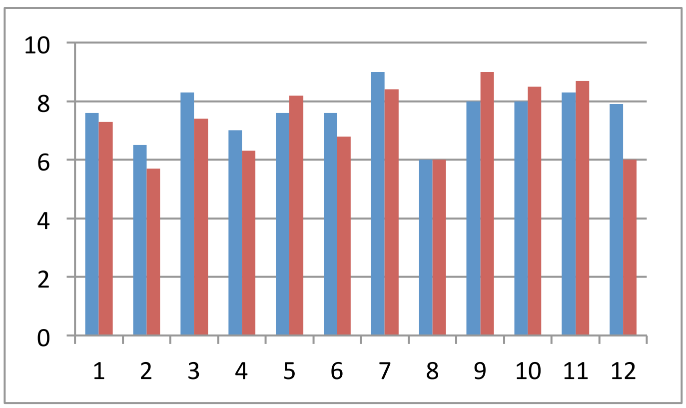

Figure 6. Indicator values for Kyrgystan (blue) and Uzbekistan (red).

From Figure 6 it is immediately clear that these two states as such are incomparable taking all indicators into account. Thus, we see that Kyrgystan is worse than Uzbekistan with respect to indicator 1, 2, 3, 4, 6, 7 and 12, whereas the reverse is true for indicator 5, 9, 10, and 11, i.e., Uzbekistan is worse than Kyrgystan. Indicator 8 has the same value for both states.

Sensitivity

A major count of controversy in the FSI ranking seems to be the fact that all indicators are given an equal weight. However, if different weights should be given to the single indicators it would often be a result of either rather subjective discussions or the outcome of hard, objective scientific work and clearly not arbitrary as correctly pointed out by Klöpffer (1998). An attempt to resolve this problem is to derive weights directly from the data matrix (see e.g. Sailaukhanuly et al., 2012) or approaches to make use of the correlations among indicators. Nevertheless, looking at the single indicators (FFP, 2013c) it is hard to believe that they actually should be equally important for rating the states according to their stability or most at-risk of collapse and violence, respectively.

Partial order methodology offers a unique possibility to disclose the relative importance of the single indicators both on a global and a local scale. In Figure 7, a graphical representation of the relative importance of the twelve indicators being the basis for the FSI is shown, the sensitivity values (i.e. distances) being the ordinate.

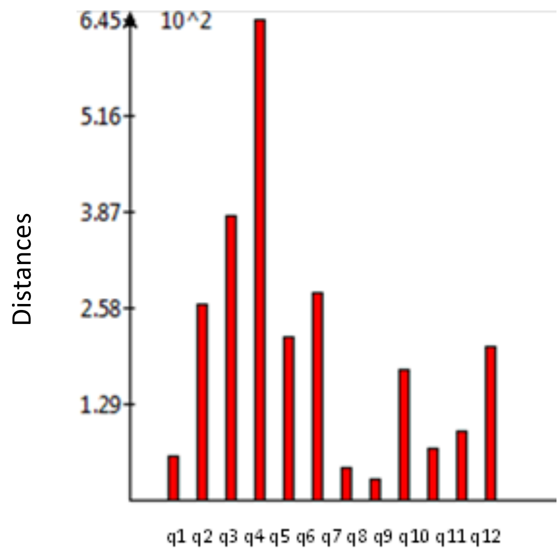

Figure 7. Relative importance of the twelve FS indicators as disclosed by partial order methodology.

It is immediately noted that on the global scale the absolute top rating among the indicators are indicator 4, which describes the 'Chronic and Sustained Human Flight,' i.e., brain drain, followed by indicator 3 ('Legacy of Vengeance-Seeking Group Grievance or Group Paranoia') and indicator 6 ('Sharp and/or Severe Economic Decline').

Maybe not surprising, but not directly intelligible from the original FSI, it is unequivocally disclosed that the brain drain apparently plays a crucial role in the stabilization of states. Thus, initiating programs that will secure not only the generation of human capital but of equally high priority that the human capital generated in the single countries is retained there for the benefit of the country appear as an area of high priority.

Focusing on the local scale, virtually the same picture develops. For the most vulnerable states the picture is as clear as the above, whereas for the most stable states the single indicators appear not surprisingly to be virtually of equal importance although the same overall trend is noted.

Based on above finding we may state a word-model. Hence, if a given state is very good in one of its twelve indicators (low score), this state is typically good in all indicators, whereas states being very bad in one of the indicators (high score) not necessarily must be bad in other indicators, perhaps in a reduced manner. Such a model fits very well with the finding of the local analysis: for states near the top, the pattern of sensitivity follows that found for all states, whereas for nations at the bottom of the Hasse diagram, the variations are so severely quenched that the clear sequence of importance (cf. Figure 8) is almost no more recognizable.

Partial Order vs. Principal Component Analysis

As an alternative principal component analysis may be used for aggregation of the indicators. The first two components explain 78.7% and 6.9%, respectively, of the total variance (R Core Team, 2012). Figure 8 displays the biplot of the first two components. Obviously, no specific groupings of the 177 states can be recognized.

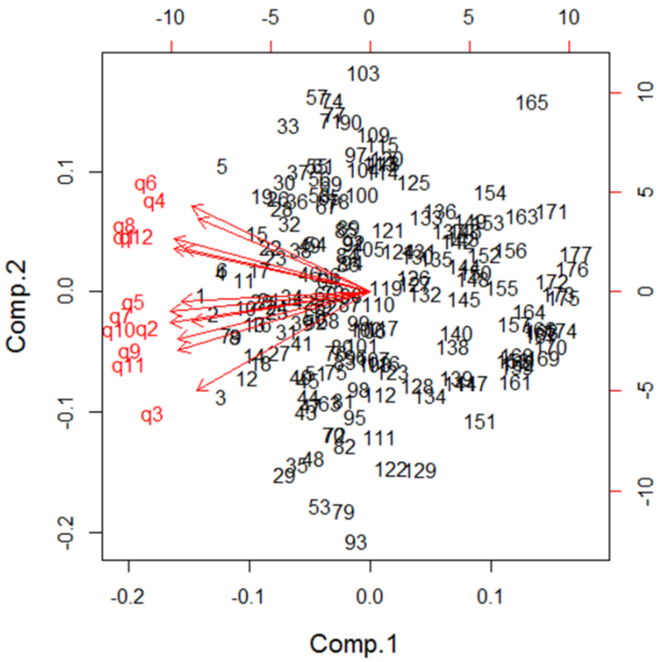

Figure 8. Principal Component Analysis. Biplot showing the first two components explaining 78.7% and 6.9%, respectively. The PCA has been carried out applying the freely available software R (R Core Team, 2012).

Above we have argued that important information is lost through the aggregation process of indicators done in the FSI. A similar effect can be noticed in the case of the PCA analysis. Thus, a correlation between the first components and the original FSI leads to a virtually perfect straight line (Figure 9), the correlation coefficient being -0.9999. Therefore, at least in the present case, PCA does not constitute as an attractive alternative to weak ordering of the 177 states.

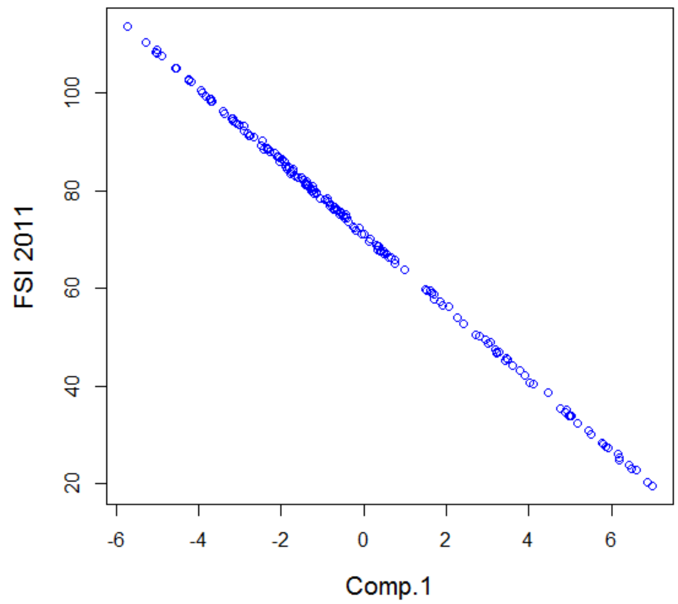

Figure 9. Scatterplot displaying the correlation between the Failed State Index and the first principal component. The plot has been produced applying the freely available software R (R Core Team, 2012).

Conclusions and Outlook

Partial order theory is an analytical concept that has been the leading concept in this paper. There is a composite indicator, the FSI, and in contrast to many other methods from the field of decision support, the technique to aggregate indicators by a weighted sum is transparent and finds a broad acceptance. However, we can move a step further by taking the concept of incomparability as the main starting point. Hence, indicators, as used for the construction of the FSI bear different information and partial order theory helps to extract information before the mixing with of indicators (due to the aggregation by summing them) is performed.

Each incomparable pair of states is sensitive to compensation effects. The question is only how far this compensation is relevant. Incomparabilities and comparabilities give a "web of relations" by which states are connected. Thus, in the present paper we disclosed the mutual comparabilities between the states (chains) as well as the incompabilities (antichains), respectively; the latter being explored by a pairwise analysis of the single states. Any chain means, without assuming additional information, that a partial ranking prevails. Any antichain means compensation effects can appear and the more severe the compensation the more pronounced the incomparability. Thus, chain and antichain analysis are the logical consequence of the study of the web of relations. If this, however, is accepted, then the natural question is to what extent the different indicators are influencing the structure of this web. Contextually, having a global view we found that 'brain drain,' 'Group Paranoia' and 'Severe Economic Decline' are of most important of the twelve indicators. We further disclosed that this was virtually true for all the states covered by the FSI.

Partial order methodology offers to construct a weak (linear) order based on averaged ranks of the single states. It has been visualized that although at a first glance the FSI ranking and a ranking based on averaged ranks look similar, significant differences prevail as a result of the presence of a high number of incomparabilities. Hence, once the concept of incomparability is accepted as some kind of valuable information, many aspects of partial order theory may be brought into play.

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Appendix 1

Using Graphviz

The built-in graphic facility in PyHasse may lead to Hasse diagrams with confusing crossing lines (cf. Fig 3).

Mathematically, the flexibility of drawing positions is governed by the Jordan Dedekind Chain Condition, JDCC (Birkhoff, 1984). If a (sub)poset is graded then there is no flexibility; a (sub)poset is graded if it fulfills the JDCC, i.e., all maximal chains between the same endpoints have the same finite length.

The software package PyHasse (see Appendix 3) offers four possibilities:

1) A simple (not to say primitive) graphical editor,
2) A "perturbed" HD, so that artificial overlapping lines are avoided,
3) The cover-matrix as a control facility, and
4) Interface to Graphviz.

Applying 4), i.e., Graphviz, crossing lines are automatically avoided to the best possible extent (Figure A1).

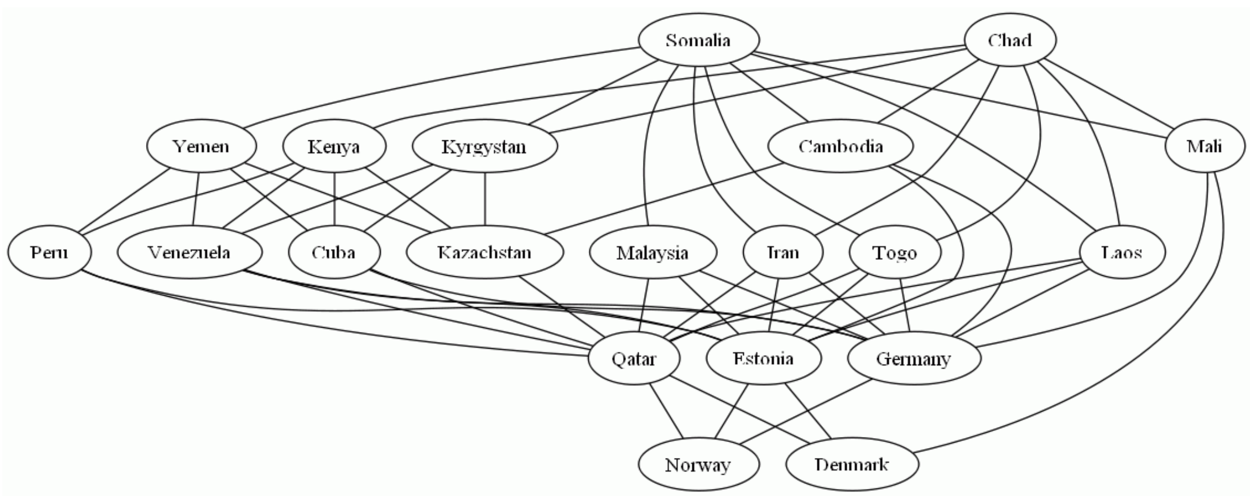

Figure A1. Same Hasse diagram as in Figure 3, however applying Graphviz.

It is immediately noted that the number of crossing lines has been minimized to three, and 'children' are near 'parents.'

Applying Graphviz, the resulting Hasse diagram still possesses a level structure. However, this level structure no longer follows any rule according to the evaluation of indicator values. Furthermore, the diagram now contains bended lines.

Despite the fact that the level structure no longer corresponds to the original partial order analysis (cf. Figure 2), the number of levels in the Graphviz-graphic (Figure 3) is identical to what is found in the PyHasse-generated diagram, i.e., five (Figure A1). This must be the case, because the maximum of all maximal chains have five elements.

Appendix 2

Original FSI data FSI, 2011; reproduced with permission from The Fund for Peace.

ID	Country	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12	FSI
1	Somalia	9.7	10	9.5	8.2	8.4	9.3	9.8	9.4	9.7	10	9.8	9.7	113.5
2	Chad	9.2	9.5	9.4	8	8.9	8.5	9.8	9.6	9.3	9.2	9.8	9.1	110.3
3	Sudan	8.5	9.6	9.9	8.2	9.1	6.4	9.4	9	9.7	9.6	9.9	9.5	108.8
4	Congo (D. R.)	9.7	9.6	8.3	7.7	9.2	8.7	9	8.9	9.2	9.6	8.8	9.5	108.2
5	Haiti	10	9.2	7.3	8.9	8.8	9.2	9.4	10	8	8.4	8.8	10	108
6	Zimbabwe	9.3	8.2	9	9.3	9.2	9	9.3	9	9.2	9	9.6	7.8	107.9
7	Afghanistan	9.1	9.3	9.3	7.2	8.4	8	9.7	8.5	8.8	9.8	9.4	10	107.5
8	Central African Rep.	8.9	9.6	8.6	5.8	8.9	8.1	9.1	9	8.6	9.7	9.1	9.6	105
9	Iraq	8.3	9	9	8.9	9	7	8.7	8	8.6	9.5	9.6	9.3	104.9
10	Cote d'Ivoire	8.1	8.5	8.7	7.9	8	7.7	9.5	8.4	8.6	8.6	9.1	9.7	102.8
11	Guinea	8.2	7.7	7.9	8.3	8.4	8.6	9.4	8.7	9.2	9.3	9.2	7.6	102.5
12	Pakistan	8.8	9.2	9.3	7.5	8.5	6.6	8.6	7.3	8.7	9.4	9.1	9.3	102.3
13	Yemen	8.7	8.4	8.6	6.9	8.3	7.7	8.6	8.7	7.7	9.3	9.3	8.2	100.4
14	Nigeria	8.3	6	9.6	7.7	9	7.3	9	9	8.6	9.1	9.5	6.9	100
15	Niger	9.8	6.6	7.8	6.2	7.9	8.9	8.9	9.5	8.2	8	8.6	8.7	99.1
16	Kenya	8.8	8.5	8.7	7.6	8.5	7	8.9	7.8	7.7	7.9	8.8	8.5	98.7
17	Burundi	9.1	8.7	8.2	6.2	8.1	8.5	8.2	8.8	8	7.7	8.2	9	98.7
18	Myanmar	8.2	8	8.7	6	9	7.9	9.7	8.3	9	8.5	8.3	6.7	98.3
19	Guinea Bissau	8.7	7.2	5.4	7.4	8.1	8.7	9.2	8.4	7.8	9.3	9.2	8.8	98.2
20	Ethiopia	9.1	8.2	8.4	7.2	8.2	7.7	7.5	8.4	8.5	7.9	9	8.1	98.2
21	Uganda	8.8	8	8	6.6	8.4	7.5	7.7	8.3	7.5	8.6	8.6	8.2	96.2
22	North Korea	8.2	5.3	6.9	4.7	8.5	9.2	9.9	9.3	9.5	8.1	7.4	8.6	95.6
23	Timor-Leste	8.5	8	7.1	5.8	7.3	7.9	8.8	8.7	6.8	8.3	8.3	9.3	94.8
24	Cameroon	8	7.3	7.8	7.8	8.4	7	8.8	8.3	8.1	7.8	8.5	6.8	94.6
25	Bangladesh	8.3	6.5	9.2	8.1	8.4	7.7	8	8	7.1	7.9	8.9	6.2	94.3
26	Liberia	8.3	8.6	6.8	7	8	8.4	7	8.8	6.3	7.3	8.1	9.3	93.9
27	Nepal	7.8	7.4	9	5.9	8.7	7.9	7.9	7.7	8.5	7.8	8	7.1	93.7
28	Eritrea	8.3	6.8	6.1	7.4	6.5	8.3	8.5	8.4	8.9	7.7	8.1	8.5	93.5
29	Sri Lanka	7	8.6	9.4	6.9	8.4	5.3	8.5	6.1	8.6	8	9.5	6.8	93.1
30	Sierra Leone	8.9	7.5	6.5	8	8.5	8	7.7	8.8	6.7	6	7.9	7.6	92.1
31	Kyrgyzstan	7.6	6.5	8.3	7	7.6	7.6	9	6	8	8	8.3	7.9	91.8
32	Congo (Republic)	8.5	7.7	6	6.7	8.2	7.3	8.9	8.3	7.5	7.3	6.7	8.2	91.3
33	Malawi	9.1	6.5	6	8.1	8	8.8	7.9	8.2	7	5.2	7.6	8.7	91.1
34	Rwanda	8.9	7.3	8.2	6.8	7.4	7	7.1	7.8	8.2	5.8	8.4	8	90.9
35	Iran	6.1	7.9	8.5	6.7	7	5.4	9.1	5.6	9	8.6	9.2	7	90.1
36	Togo	8.1	6.5	5.4	7	7.9	8	8	8.5	7.7	7.3	7.8	7.1	89.3
37	Burkina Faso	8.9	6.2	5.5	6.3	8.5	8	7.7	8.7	6.4	7	7.3	8	88.5
38	Cambodia	7.7	5.6	7.2	7.6	6.8	7.2	8.5	8.4	8	6.2	8	7.4	88.6
39	Tajikistan	7.7	5.9	7.2	6	6.8	7.4	8.9	6.9	8.5	7.4	8.6	7	88.3

40	Uzbekistan	7.3	5.7	7.4	6.3	8.2	6.8	8.4	6	9	8.5	8.7	6	88.3
41	Equatorial Guinea	8.5	2.7	6.6	7.2	9.1	4.5	9.6	8.1	9.4	8.1	8.2	6	88
42	Mauritania	8.2	6.8	7.8	5.5	6.5	7.3	7.3	7.9	7	7.9	7.9	7.9	88
43	Lebanon	6.5	8.5	8.7	6.6	6.8	5.7	7	5.8	6.6	8.7	8.8	8	87.7
44	Colombia	6.7	8.7	7.5	7.9	8.6	4.1	7.5	5.6	7.2	7.5	8	7.7	87
45	Egypt	7.1	6.4	8.3	5.7	7.4	6.5	8.6	5.9	8.3	6.8	8	7.8	86.8
46	Laos	7.6	5.8	6.5	6.8	5.7	7.2	8	7.7	8.5	7.1	8.6	7.2	86.7
47	Georgia	5.8	7.5	8	5.5	6.9	6	8.4	6	6.9	7.9	9	8.5	86.4
48	Syria	5.6	8.5	8.7	6.3	7.4	5.8	8.3	5.8	8.6	7.5	7.9	5.5	85.9
49	Solomon Islands	7.9	4.5	6.8	5.1	8	7.6	7.9	8.1	6.5	6.7	8	8.8	85.9
50	Bhutan	6.6	6.9	7.8	6.8	8.2	6.9	6.6	6.9	7.6	6.2	7.5	7	85
51	Philippines	7.3	6.5	7.2	6.7	7.1	5.6	8.3	6.1	7.3	8.3	8.5	6.1	85
52	Angola	8.6	6.6	6.2	5.9	8.8	4.5	8.5	8.2	7.5	6.2	7	6.7	84.7
53	Israel/West Bank	6.8	7.6	9.6	3.8	7.8	4.3	7.3	6.5	7.9	7	8.1	7.8	84.5
54	Papua New Guinea	7.4	4.5	6.9	7.4	9.1	6.4	7.5	8.7	6.3	6.6	7.1	6.4	84.3
55	Zambia	8.9	7.6	5.7	6.8	7.3	7.7	7.6	7.8	6.1	5.3	5.8	7.3	83.9
56	Comoros	7.5	4	5.3	6.6	5.8	7.6	8	8.2	6.6	7.5	8	8.7	83.8
57	Mozambique	9	4	4.6	7.7	7.4	8.2	7.6	8.6	7	7.1	5.6	6.7	83.5
58	Madagascar	8.3	4.6	5.2	4.9	7.8	7.6	7.1	8.6	6	6.8	8	8.3	83.2
59	Bolivia	7.2	4.6	7.7	6.4	8.9	6.5	6.8	7.1	6.3	6.5	8	6.9	82.9
60	Dijbouti	7.8	7.2	6.2	5.2	6.8	6	7.2	7.2	7	6.2	7.5	8.3	82.6
61	Swaziland	9.2	4.6	3.9	5.9	6.5	7.8	8.5	7.5	8.2	6.6	7	6.9	82.6
62	Ecuador	5.9	6.4	6.9	7.1	7.7	6.3	7.5	7.2	5.7	7	8.2	6.3	82.2
63	Azerbaijan	5.8	7.9	7.5	5.4	6.9	5.5	7.7	5.7	7.2	7	7.8	7.5	81.9
64	Indonesia	7.4	6.6	6.6	6.9	7.5	6.4	6.7	6.5	6.3	7.1	7	6.5	81.5
65	Tanzania	8.1	7.4	6.1	5.8	6.3	7.4	6.5	8.6	6.2	5.5	6	7.4	81.3
66	Moldova	6.1	4.4	6.6	7.5	6.5	6.7	7.6	6.3	6.5	7.8	8	7.2	81.2
67	Nicaragua	6.9	4.9	6	7.2	8.2	7.3	7.3	7.3	6	6.2	6.8	7.1	81.2
68	Fiji	5.9	3.9	7.6	6.9	7.7	7	8.6	5.5	6.5	7	7.9	6.6	81.1
69	Gambia	7.9	6.4	4	6.5	6.6	7.1	7.5	7	7.5	6.1	6.8	7.5	80.9
70	Bosnia & Herzegovina	5	6.8	8.4	5.9	6.8	5.2	7.6	5	6.1	7	9.2	8	81
71	Lesotho	9	4.6	5	6.8	6.1	8.1	6.9	8.2	6	5.5	7	7.2	80.4
72	China	8.2	6.2	7.9	5.6	8.6	4.4	7.9	6.6	8.8	5.7	6.9	3.3	80.1
73	Guatemala	7.3	5.6	6.9	6.5	7.7	6.5	6.8	6.9	6.9	7.6	6	5.3	80
74	Benin	8.1	7.1	3.9	6.6	7.2	7.9	6.7	8.5	5.7	6	5	7.3	80
75	Turkmenistan	6.5	4.2	6.6	5.1	7.1	6	8.4	6.7	8.7	7.5	7.7	5.2	79.7
76	India	8	5	8.2	6.2	8.5	5.4	5.8	7.2	5.9	7.8	6.8	4.5	79.3
77	Mali	8.8	5.3	6	7.3	6.7	7.8	5.5	8.2	4.9	7.1	4.5	7.2	79.3
78	Honduras	7.6	3.9	5.3	6.6	8.1	7	7.3	6.6	6.3	6.5	6.3	6.9	78.4
79	Thailand	6.4	6.6	8	4.4	7.2	4	8.4	5	7.3	7.6	8.5	4.9	78.3
80	Venezuela	6	4.8	7	6.4	7.3	6.1	7.5	5.8	7.4	7	7.3	5.5	78.1
81	Algeria	6.4	6.1	7.8	5.7	6.8	5.2	7.1	6.1	7.5	7.2	6.8	5.3	78
82	Russia	6.3	5.1	7.6	5.7	7.6	4.6	7.8	5.3	8.1	7.2	7.8	4.6	77.7
83	Belarus	6.3	3.6	6.8	4.5	6.3	6.2	8.8	5.8	8	6.3	8	7	77.6

84	Dominican Republic	6.5	5.5	6.1	7.9	7.5	5.6	5.8	6.8	6.3	5.8	6.8	6.2	76.8
85	Senegal	7.6	6.4	6.3	6	7.2	6.5	5.9	7.8	6.2	6.3	4.5	6.1	76.8
86	Cuba	6.3	5.4	5.1	6.9	6.3	6	6.6	5.3	7.4	6.9	6.9	7.5	76.6
87	Morocco	6.4	6.5	6.4	6.4	7.5	6	6.9	6.6	6.4	5.9	6.3	4.9	76.2
88	Vietnam	6.7	5	5.7	5.7	6.2	6.1	7.5	6.4	7.7	6	6.9	6.1	76
89	El Salvador	7.6	5.3	5.8	7.1	7.6	6.3	6.5	6.9	6.7	7	4.3	4.9	76
90	Cape Verde	7.3	4.3	4.2	8.3	6.3	6.3	6.9	6.9	5.7	5.7	5.7	8.2	75.8
91	Maldives	6	5.9	4.9	6.8	5	6.7	7.4	6.9	7	5.7	7.6	5.8	75.7
92	Gabon	6.8	6.2	3.3	6.1	7.9	5.5	7.5	6.7	6.7	5.7	7.1	5.8	75.3
93	Saudi Arabia	6	5.8	7.5	3.2	7	3.4	7.9	4.2	8.9	7.5	7.9	5.9	75.2
94	Mexico	6.5	4.2	6.1	6.5	7.7	6	6.6	5.8	5.9	7.9	5.2	6.7	75.1
95	Turkey	5.9	6	8.3	4.5	7.4	5.5	5.9	5.7	5.2	7.4	7.5	5.6	74.9
96	Jordan	6.4	7.6	6.7	4.7	6.9	5.8	5.7	4.9	6.8	6	6.3	6.8	74.6
97	Sao Tome	7.1	4.3	4.8	7.3	6.2	6.9	6.9	7	4.9	5.8	6.3	6.9	74.4
98	Serbia	5.3	6.4	7.5	5	6.5	5.7	6.5	4.9	5.3	6.5	8	6.8	74.4
99	Peru	6.1	4.1	6.8	6.7	8	5.1	6.6	6.1	5.2	7.2	6.6	5.1	73.6
100	Guyana	6.4	3.6	5.9	8.4	7.4	6.4	6.5	5.5	5	6.3	5.1	6	72.5
101	Paraguay	5.9	1.9	6.5	5.5	8.3	5.9	7.9	5.5	6.4	6.4	7.7	4.5	72.4
102	Armenia	5.5	6.6	6	6.6	6.2	5.3	6.6	5	6.5	5.2	7	5.8	72.3
103	Micronesia	7.1	3.5	4.2	8	7.2	6.7	6.3	6.9	2.5	5.4	5.6	8.5	71.9
104	Namibia	7.2	5.6	5.3	7.1	8.5	6.3	4.4	6.7	5.5	5.5	3.5	6.2	71.8
105	Suriname	6	3.5	6.1	7	7.5	6.1	6.1	4.9	5.6	5.8	5.8	6.7	71.1
106	Macedonia	4.5	4.6	7.4	6.7	6.8	6.2	6.7	4.2	5	6	6.7	6.2	71
107	Kazakhstan	5.5	3.8	6	3.8	5.9	6.2	7.2	5.1	6.9	6.2	7.7	5.9	70.2
108	Tunisia	5.5	3.4	5.6	5.2	6.6	5	7.2	5.3	7.7	7	6.8	4.8	70.1
109	Samoa	7	2.7	4.8	8.3	6.6	5.9	6.2	4.7	4.2	5.5	5.1	8.6	69.6
110	Ukraine	5.3	3.1	6.5	6.3	5.9	6	7.4	4.1	5.5	4	8	6.8	68.9
111	Libya	5.5	4.6	6	3.9	6.9	4.6	7.3	4.3	8.3	5.9	7	4.4	68.7
112	Malaysia	6	4.8	6.7	4.2	6.7	4.9	6	5.1	6.9	6	6.4	5	68.7
113	Botswana	8.9	6.4	4.5	5.6	7.4	6.3	5	6	5	4.1	3.3	5.4	67.9
114	Belize	6.7	5.4	4.4	7	6.8	5.7	6	5.8	3.8	5.5	4.3	6.3	67.7
115	Ghana	6.8	5.5	5.5	7.6	6.3	6.1	4.8	7.7	4.5	3	4.2	5.6	67.6
116	Cyprus	4.4	4.4	7.6	5.3	7.3	5	5	3.3	3.3	5.3	7.9	8.8	67.6
117	South Africa	8.4	6.7	5.9	4.1	8.2	5.3	5.5	5.5	4.6	4.5	5.9	3	67.6
118	Jamaica	6.2	3.4	4.3	6.7	6.2	6.3	6.5	5.9	5.3	6.3	3.7	6.3	67.1
119	Seychelles	5.8	3.9	4.8	4.9	6.6	5.4	6.8	4.1	5.8	6.1	5.7	7.1	67
120	Grenada	5.8	3.2	3.9	8	6.5	5.7	6.2	4.2	4.3	5.3	5.6	7.7	66.4
121	Albania	5.5	3.1	5.1	6.8	5.4	5.9	6.4	5	5	5.4	6.3	6.3	66.2
122	Brunei	5.1	3.9	6.2	4.1	7.8	3.4	7.7	3.2	6.7	5.6	7.4	4.7	65.8
123	Brazil	6.1	3.5	6.5	4.5	8.5	3.9	5.9	5.8	5.1	6.5	4.9	3.9	65.1
124	Trinidad	5.3	3.2	4.7	7.7	6.9	4.5	5.5	4.9	5.1	5.5	5.6	4.8	63.7
125	Antigua & Barbuda	5.2	3	4.1	7.6	5.9	5.1	5.8	4.3	4.5	4.9	3.7	5.8	59.9
126	Romania	5.1	3.2	6	5	5.8	5.8	5.9	4.5	4	4.1	5.2	5.2	59.8
127	Mongolia	5.5	1.6	4	1.9	6.2	5.3	5.9	5.6	6	5	5.5	7.1	59.6

128	Kuwait	5.1	3.8	4.9	4.3	5.9	4	5.7	2.9	6.2	4.5	7.2	5	59.5
129	Bahrain	4.5	2.9	6.8	3.1	6	3.4	6.9	2.7	5.9	4.8	6.6	5.3	58.9
130	Bulgaria	4.1	3.6	4.3	5.5	5.7	5.3	5.9	4.6	4.3	4.9	5.3	5.5	59
131	Panama	6	3.9	4.6	4.9	7.4	4.9	4.6	5.2	4.5	5.7	2.5	3.6	57.8
132	Croatia	4.3	5.5	5.5	4.9	5	5.9	4.4	3.4	4.3	4.4	4.7	5	57.3
133	Bahamas	5.8	2.8	4.4	6.2	6.2	4.8	5.2	4.2	3.2	4.3	4.5	4.9	56.5
134	Montenegro	4.5	4.5	6.4	2.4	4.1	5.2	4.3	3.6	5	4.8	6.2	5.3	56.3
135	Lativa	4.2	3.9	4.9	4.8	5.7	5.8	5.3	3.9	3.6	3.3	4.3	4.4	54.1
136	Barbados	4.3	2.9	4.4	6.8	6.3	5	3.9	2.9	2.5	4.2	4.2	5.4	52.8
137	Costa Rica	5.1	4.3	4.1	4.1	6.5	4.9	3.5	4.2	3	2.5	3.5	4.9	50.6
138	United Arab Emirates	4.1	2.8	4.6	3	5.4	4.2	6.5	3.3	5.7	3	3.6	4.1	50.3
139	Qatar	4.2	2.7	4.9	3.1	5	3.7	6	2.3	5	3	5	4.6	49.5
140	Estonia	4.1	3.9	5.4	4.5	4.9	4.3	4.1	2.9	3	2.9	5.5	3.9	49.4
141	Oman	5.1	1.5	3	1.5	3	3.8	5.9	4.4	6.9	5.3	6.3	2.4	49.1
142	Hungary	3.1	3.1	3.5	4.5	5.5	5.4	5.4	3.7	3	2.5	4.7	4.3	48.7
143	Greece	4.1	2.6	4.5	4.4	4.3	5.1	4.9	3.8	3.1	3.8	2.5	4.3	47.4
144	Slovakia	3.8	2.3	5	5.1	5.2	4.6	3.9	3.6	3.6	2.3	3.7	3.9	47
145	Argentina	4.4	2.6	4.9	3.5	6	4.4	4	3.5	4	2.7	3	3.8	46.8
146	Poland	4.3	3.5	3.5	5.6	4.7	4.3	4.2	3.3	3.5	2.5	3.6	3.9	46.9
147	Italy	3.6	3.5	5.3	3.2	4.1	4.2	4.7	2.8	3.1	4.9	4.4	2	45.8
148	Malta	3.4	5.4	4	4.4	4.1	4.1	3.7	2.9	3.4	3.7	2	4.4	45.5
149	Lithuania	4.1	3.2	3.7	4.6	5.7	5.3	3.6	2.9	3.1	2.5	2.8	3.8	45.3
150	Mauritius	3.3	1.6	3.5	3	5.4	4.5	4.7	3.9	3.5	3.6	3.2	4	44.2
151	Spain	3.3	2.9	6	1.9	4.7	4.5	2.1	2.4	2.6	4.9	5.6	2.2	43.1
152	Czech Republic	3	2.8	3.8	4	3.8	4.6	3.7	3.9	3	2.1	3.8	3.8	42.3
153	Chile	5	3	3.5	2.8	5	4.6	2.1	4.3	3.3	2.5	1.4	3.3	40.8
154	Uruguay	3.9	1.7	2.4	5.3	4.7	3.8	2.5	3.3	2.5	3.7	2.7	3.9	40.4
155	South Korea	3.3	3	3.7	4.5	2.3	2.2	3.7	2.2	2.6	1.7	3.6	6	38.8
156	Slovenia	3.1	1.7	3.1	3.6	4.7	3.7	3	2.8	2.8	3	1.1	2.9	35.5
157	Singapore	2.5	0.9	3	2.8	3.4	3.6	3.9	2	4.7	1.5	4	2.8	35.1
158	United States	3.4	2.9	3.6	1.1	5.4	3.7	2.2	2.7	3.3	1.6	3.6	1.3	34.8
159	United Kingdom	2.9	3.3	4.4	2.1	4.2	3.3	1.4	2.2	2	2.7	3.6	1.9	34
160	Belgium	2.5	2.1	4.4	1.6	4.4	3.6	2.7	2.5	1.6	2	4	2.6	34
161	France	3.3	2.8	5.9	1.8	4.9	3.5	1.6	1.9	2.5	1.9	1.9	2	34
162	Germany	2.9	4.2	4.7	2.6	4.4	2.9	1.9	2	2	2.2	2.1	2	33.9
163	Portugal	3.3	2	2.5	2.5	3.6	4.8	1.6	3.3	3.3	1.6	1.4	2.5	32.4
164	Japan	3.6	1.1	3.9	1.8	2.3	3.5	2	1.7	3	2	2.6	3.5	31
165	Iceland	1.6	1.5	1	3.3	2.2	6.2	2	1.9	1.6	1	1.8	6	30.1
166	Netherlands	3	3	4.4	2.2	2.9	3.2	1.1	1.7	1	1.4	2.4	2.1	28.4
167	Australia	3.3	2.8	3.6	1.6	3.9	2.9	1.6	1.8	1.9	1.7	1.6	1.4	28.1
168	Canada	2.9	2.5	3.3	2.4	4.1	2.4	1.2	1.9	1.6	1.5	2.5	1.4	27.7
169	Austria	2.6	2.6	3.8	1.6	4.4	2.3	1.2	1.6	1.5	1.1	2.4	2.2	27.3
170	Luxembourg	1.7	2.1	2.8	1.5	2	2.3	2.5	1.9	1	2.3	3.4	2.6	26.1
171	Ireland	2.3	2	1.3	2.4	2.6	3.9	2	2.2	1.2	1.6	1.4	2.4	25.3

172	New Zealand	2	1.7	3.5	2.4	4	3.8	1.1	1.9	1.2	1.1	1.1	1.1	24.9
173	Denmark	2.9	2.1	3.3	2.1	1.7	2.5	1.2	1.6	1.3	1.5	1	2.6	23.8
174	Switzerland	2.1	1.9	3.5	2.1	2.8	2.4	1	1.6	2	1.4	1	1.4	23.2
175	Sweden	2.8	2.9	1.3	2	2.2	1.9	0.9	1.5	1.6	2.3	1.8	1.6	22.8
176	Norway	2	2	1.3	1.5	2.1	2.9	1	1.4	1.9	1.2	1.2	1.9	20.4
177	Finland	2	2.1	1.7	2.5	1.3	2.8	1	1.5	1.1	1	1.2	1.5	19.7

Appendix 3

Partial Order Software

The calculations are performed using the recently developed software PyHasse. The prefix “Py” indicates that Python is the programming language. Python is an interpreter language and can freely be obtained from the Internet. PyHasse includes mainly the HDT, but also simplified versions of multicriteria decision methods, such as PROMETHEE. PyHasse is based on four libraries, two of which were written by the second author; the other two are statistical libraries, which were freely available in the Internet. There are now more than 80 modules (programs) intended to support specific tasks in the ordinal analysis of data matrices, for example sensitivity analysis, chain and antichain analysis, techniques to obtain weak orders and many others. Any module has a graphical user interface that has so far as possible the same layout. Each module is supported by “about” and “help” functions.

PyHasse is freely available from the second author on request. First attempts are done to represent PyHasse in the Internet (see http://www.pyhasse.org). For further details on the PyHasse software, see for example Voigt et al., 2010.