Application of system identification and numerical optimization to a floor radiant heating control in a Solar House

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SUMMARY

A water-based floor radiant heating system (WFRH) is used in a Solar Decathlon House for temperature and thermal comfort control. To minimize the energy consumption, while maintaining the thermal comfort, computer simulations are utilized to evaluate different control strategies before actual implementation. However, there are two potential challenges which have not been addressed before: first, the availability and credibility of material thermal property data or sensor data could be problematic. Second, the optimum control algorithm comes mainly from a candidate pool based on and limited by operator’s experience. This paper presents the study of system identification and numerical optimization application in the WFRH. The combined approach is to improve the reliability of building simulation in a real system operation. The formulating process of system identification and optimization has been detailed in this paper. The integral method can be generalized to other building control systems.

INTRODUCTION

Water-based floor radiant heating systems (WFRH) have been increasingly used recently. The reason for the preference is the improved comfort of having a warm floor and a more uniform temperature distribution in the heated room (Olesen, 1983, 2002). Compared to a traditional convective heating system, a WFRH can reach the same level of operative temperature at a lower air temperature. Therefore, a higher relative humidity in winter and less energy loss due to air change may be achieved (Cho, 1997).

A Solar Decathlon House erected in 2005 is reused as a living laboratory for heating, ventilation and air-conditioning system (HVAC) control related studies. To minimize the energy consumption of the system, computational simulation is considered as a good tool evaluating different operation strategies. However, the multilayer envelopes, roof and floor, with interlaced architectural construction, make it difficult to obtain trustable building material properties for such an energy modeling. Meanwhile, there is no scientific reference but experience on how an operation should be performed to ensure low energy consumption and acceptable system performance when indoor and outdoor conditions vary from day to day. A combined two-step approach with system identification and numerical optimization is designed in this study to overcome the two obstacles.
There are several parameter identification methods in the literature and can be divided into two main categories: black-box and grey-box. Mechaqrance and Zouak (2002) developed a neural network auto regressive with exogenous input (NNARX) model to predict the indoor temperature of a residential building. The summed square error is close to 0.9. Recently, Jimenez, Madsen and Andersen (2008) presented the application of the IDENT Graphical User Interface of MATLAB to estimate thermal properties of building thermal components from outdoor dynamic testing, imposing appropriate physical constraints and assuming linear and time invariant parametric models. However, this black-box approach requires a long period of training to improve the performance accuracy. In addition, the black-box may have more complex model structure than lower-order models such as gray-box models, which make the model analysis difficult.

With these disadvantages of black-box model, there are some other studies on developing and validating grey-box model. Braun and Chaturvedi (2002) developed a thermal network model for transient building load prediction. This inverse grey-box model needs one week of data to train with rich zone temperature variations or two to three weeks of data to train with limited zone temperatures variations. Wang and Xu (2006) developed a simplified model of the building thermal load on heat transfer of building envelope and internal mass. The parameters of building thermal network models for building envelope are determined by frequency characteristic analysis; the parameters of thermal network models for lumped internal mass are identified with generic algorithm. McKinley and Aleyne (2008) presented an alternative approach using optimization search process (hill climbing algorithm) to identify building thermal model parameters and loads based on site measurement.

Thermal system control optimization is special due to its nonlinear and non-convex features. State variables, such as room air temperature, wall surface temperature, etc, are implicitly embedded in a dynamic system expressed as differential equations. The presence of state-dependent inequality constraints complicates the treatment since analytical methods require previous knowledge of the number and sequence of state-constrained arcs, which are normally unknown beforehand (Bryson and Ho, 1975). One approach, referred to as a sequential method. Control vector is parameterized as a trajectory leaving the state equations in the form of the original differential algebraic equation (DAE) system (Goh and Teo, 1988). The second approach is called simultaneous method, where both the control and state variables are discretised using polynomials (e.g., Lagrange polynomials) of which the coefficients become the decision variables in a much larger Nonlinear Programming problem (NLP). The resulting formulation is algebraic in nature and can be solved using known nonlinear programming methods (Cuthrell and Biegler, 1987).

To obtain an optimal solution of the formed NLP numerically, interior point and penalty function method can be used. In penalty function method, the original constrained problem is converted into an unconstrained by augmenting the objective function with a term that becomes positive and large when any inequality constraint is violated. The disadvantage is that this usually causes ill conditioning problems (Bazarraa, et al., 1993). Interior point method, or called barrier method, reaches an optimal solution by traversing the interior of the feasible region and does not possess the shortcoming of penalty function method (Forsgren, et al., 2002).

In this study, the subspace trust region solver based on the interior-reflective Newton method (Branch et al., 1999) is chosen for system identification. Control variable parameterization technology combining interior point method is applied to the control optimization. The paper
is structured as followings: the solar house WFRH is briefly described with governing differential and algebra equations. System identification for the building thermal resistances, capacitances and coefficients of infiltration and solar radiation are conducted secondly. Night time system control optimization is formulated and solved with the identified parameters. Results of the combined approaches for this thermal system are presented followed by a study discussion.

**METHODS**

**Numerical Modeling of the House**

The system layout is illustrated in Figure 1.

![Diagram of the solar house radiant heating system](image)

**Room Air model**

Based on energy balance law, the sum of all energies entering and leaving the conference room is equal to the rate of change of stored energy. Following equation provides the general description for the system:

\[
Q_{in} + Q_{gm} - Q_{out} = \frac{dQ_{rm}}{dt} \tag{1}
\]

\[
\frac{dQ_{rm}}{dt} = C_{r,a} \cdot V_{r,a} \cdot \rho_{r,a} \cdot \frac{dT_{r,a}}{dt} \tag{2}
\]

**Building Envelope Model**

![Diagram of wall heat transfer model](image)

Figure 2: Wall Heat Transfer Model
A two-layer lumped capacitance approach is applied to model the heat transfer through the construction element for each wall and roof. The equivalent thermal network model is given in Figure 2.

If we define the relationship of $Q_{\text{sol, out}}$, $Q_{\text{int, rad}}$, solar radiation and internal heat gains as below, one of the purpose of system identification is to find out $\alpha_{ab, w}$, $\beta_{ab, w}$ and $\gamma_{ab, w}$

$$Q_{\text{sol, out}} = \alpha_{ab, w} Q_{\text{sol}}$$

$$Q_{\text{int, rad}} = \beta_{ab, w} Q_{\text{sol}} + \gamma_{ab, w} Q_{\text{gn}}$$

Then, the wall model can be written as:

$$C_{\text{out}} \frac{dT_{\text{sur, out}}}{dt} = \alpha_{ab, w} Q_{\text{sol}} + \frac{T_{\text{out}} - T_{\text{sur, out, i}}}{R_{\text{outer}}} - \frac{T_{\text{sur, out, i}} - T_{\text{sur, in, i}}}{R_{\text{wall, i}}}$$

$$C_{\text{in}} \frac{dT_{\text{sur, in}}}{dt} = \beta_{ab, w} Q_{\text{sol}} + \frac{T_{\text{sur, out, i}} - T_{\text{sur, in, i}}}{R_{\text{outer}}} - \frac{T_{\text{sur, in, i}} - T_{\text{r, a}}}{R_{\text{int, i}}}$$

The windows are simply modeled as a resistance. Finally, the heat lost through the building enclosure can be obtained:

$$Q_{\text{out}} = \sum_{i=1}^{n} \frac{T_{\text{sur, in, i}} - T_{\text{r, a}}}{R_{\text{int, i}}} + \frac{T_{\text{out}} - T_{\text{r, a}}}{R_{\text{win}}}$$

**Floor**

Heat is transferred from the hot water inside the embedded pipe to the concrete floor through the pipe surfaces. The thermal dynamic balance can be expressed as a set of equations:

$$M_{f} \cdot C_{f} \frac{dT_{f}}{dt} = Q_{\text{ht}} - \dot{Q}_{\text{in}}$$

Supportive functions are listed below as part of the thermal system:

$$\dot{Q}_{\text{in}} = U_{f} \cdot A_{f} \cdot (T_{f} - T_{r, a})$$

$$T_{w, in} = T_{f} - (T_{f} - T_{w, out}) \exp \left( -\frac{\alpha_{f} \nu D}{m_{w} C_{w}} L \right)$$

**Instant heater**

A compact instant water heater is installed which is rated at 5kW with around 90% efficient. When no additional heating required, the water simply passes through the heater passively. It is modeled as a pure heat source.

$$Q_{\text{ht}} = C_{w} \cdot m_{w} \cdot (T_{w, in} - T_{w, out})$$

**Pump**

In order to circulate the hot water in the radiant floor, a variable speed pump is equipped. Since there is no extra valve to throttle the water flow rate, the frictional and local resistance coefficient is a constant. The pump energy consumption is cubic to the rated power consumption:

$$Q_{\text{pump}} = k \cdot \left( \frac{m_{w}}{m_{w, \text{max}}} \right)^{3} \cdot 1$$

**Parameter Identification**

The simulated indoor and wall surface temperature changes from the above equations are used to compare with the measured temperature. The optimized parameters are the thermal
resistances, capacitances and coefficients of infiltration and solar radiation. The objective function $f$ is defined as:

$$f = \min_x \|f(x)\|_2 = \min_x \left( \frac{f_1^2 + f_2^2 + \ldots + f_n^2}{n} \right)$$  \hspace{1cm} (13)

S.T. $X_L < X < X_h$

Where $f_n(x) = T_{\text{predict}}^n - T_{\text{measured}}^n$

Since most of the parameters are physical parameters which should be bounded in certain ranges, the lower and upper bound should have scale factors based on their initial engineering guess values. Table below shows the scale factor for different type of physical parameters (Braun, 2002).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{out}$</td>
<td>$0.5 C_{\text{initial}}$</td>
<td>$2 C_{\text{initial}}$</td>
</tr>
<tr>
<td>$C_{in}$</td>
<td>$0.1 C_{\text{initial}}$</td>
<td>$0.5 C_{\text{initial}}$</td>
</tr>
<tr>
<td>$R$</td>
<td>$0.5 R_{\text{initial}}$</td>
<td>$1.5 R_{\text{initial}}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.01$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.01$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0.01$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$0.01$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

### Numerical Optimization

The performance index is a scalar defined by the mechanical power consumption from the pump and the thermal power consumption from the instant heater:

$$J = \sum (q_{\text{pump}} + q_{ht})$$  \hspace{1cm} (14)

The control target is to minimize the overall energy cost by varying either the supply water temperature or the supply water flow rate.

$$\min_{U(t)} J$$  \hspace{1cm} (15)

$U(t)$ is the control variable vector, including the supply water temperature and the hot water flow rate:

$$U = (m_w)'$$  \hspace{1cm} (16)

$X(t)$ is the state variable vector, with floor surface temperature, the room air temperature, the inside wall surface temperature and the outside wall surface temperature:

$$X = (T_f \ T_{ra} \ T_s \ T_s)'$$  \hspace{1cm} (17)

The system algebra variables and the state variables are rearranged to form the dynamic constraint set. The format can be put as:

$$\dot{x} = f(x(t), u(t))$$  \hspace{1cm} (18)

Additional inequalities are also applied to the system:

$$18 \leq T_{ra} \leq 20$$  \hspace{1cm} (19)

$$T_f \leq 40$$  \hspace{1cm} (20)
To solve this optimization problem, control vector are parameterized. The control vector consists in an approximation of the control trajectory by a function of only a few parameters and leaving the state equations in the form of the original differential equation system. Once the NLP problem is obtained, the interior point method is applied to find the optimal solution.

Given the initial state \( x_0 \) and the input sequence \( \{u_0, u_1, ..., u_N-1\} \), it is possible to use the dynamic model to obtain the state sequence. With the input trajectory \( \{u_k\} \) and the state trajectory \( \{x_k\} \), the performance index and also the inequality and equality constraints can be evaluated.

RESULTS

Data Collection

The data is continuously collected every one minute or one and a half minutes (depending on the network legacy) since April 28, 2009. In this paper, one week continuous heating period was selected to do analysis.

System Identification

One of the important criteria to check the correctness of identified thermal properties is to compare the predicted temperature and measured temperature profiles. The measured inputs are: \( Q_{soh}, Q_{gph}, T_{out}, Q_{chr} \). The identified parameters are listed in Table 1. The results of system identification are divided into two parts: room air changes and floor surface temperature changes. Figure 3 and 4 shows the results from five days testing periods. Based on the identified parameters, the predicted temperature for both indoor and floor slab surface are well tracking the measured temperature.
Optimal Control Overnight

The system has a fixed 122°F (50°C) supply water temperature. During the day time, a PI controller modulates the supply water flow rate to satisfy the room air temperature set point. When it comes to the night operation period, there is no strict room air temperature requirement. However, the operation should not compromise the morning time set point when people come in the space at 8:00 am. Due to the big thermal mass possessed by the floor, it is hard to arbitrarily pick a fixed set point for the whole night, such as a setback. Numerical optimization can be utilized to run the system instead of a fix point PI control. The system has the following operation profiles for a default nighttime setback control, optional three-hour warm up control and the optimization control. All of the potential operations need to ensure the room air temperature above 65°F for the morning starting point.
Night setback and three-hour warm up have typical PI control profile and very different from the optimization control. It is found that a default night setback has the highest energy consumption around 13 kWh. It is about 200% more than the other two. Optimization operation shows great energy conservation capacity and consumes 17% less than a morning warm up operation.

**DISCUSSION**

An integrated scientific method has been implemented in a Solar House WFRH system. The goal is to improve the accuracy of simulation as a tool in real system operation and identify an optimal operation strategy for lower energy consumption.

Multi sensors are installed in the system to obtain data for system parameter identification. The identified parameters are reused in the simulation to predict the system response and good tracking is observed. With the identified parameters, control variable parameterization plus interior point method is utilized to solve the NLP. The energy consumption under the new control strategy has much lower energy consumption than the original operation. The integrated approach can be generalized to similar HVAC applications.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{pa}$</td>
<td>specific heat of the air, $J/kg \cdot K$</td>
</tr>
<tr>
<td>$c_{f}$</td>
<td>specific heat of the floor, $J/kg \cdot K$</td>
</tr>
<tr>
<td>$c_{w}$</td>
<td>specific heat of water, $J/kg \cdot K$</td>
</tr>
<tr>
<td>$c_{in}$</td>
<td>inside wall heat capacitance, $J/m^2 \cdot K$</td>
</tr>
<tr>
<td>$c_{out}$</td>
<td>outside wall heat capacitance, $J/m^2 \cdot K$</td>
</tr>
<tr>
<td>$V_{pa}$</td>
<td>air volume, $m^3$</td>
</tr>
<tr>
<td>$\rho_{pa}$</td>
<td>air density, $kg/m^3$</td>
</tr>
<tr>
<td>$R_{in}$</td>
<td>inside wall surface thermal resistance, $m^2 \cdot K/W$</td>
</tr>
<tr>
<td>$R_{out}$</td>
<td>outside wall surface thermal resistance, $m^2 \cdot K/W$</td>
</tr>
<tr>
<td>$U_f$</td>
<td>floor heat transfer coefficient, $W/m \cdot K$</td>
</tr>
<tr>
<td>$A_f$</td>
<td>floor surface area, $m^2$</td>
</tr>
<tr>
<td>$m_{w,max}$</td>
<td>maximum water flow rate, $kg/s$</td>
</tr>
<tr>
<td>$h_{fl}$</td>
<td>heat transfer coefficient, water to pipe, $W/m^2 \cdot K$</td>
</tr>
<tr>
<td>$D$</td>
<td>water pipe diameter, $m$</td>
</tr>
<tr>
<td>$L$</td>
<td>water pipe length, $m$</td>
</tr>
<tr>
<td>$\phi_{in}$</td>
<td>the internal heat gain, $\text{kW}$</td>
</tr>
<tr>
<td>$\Phi_{out}$</td>
<td>the heat loss through the wall, $\text{kW}$</td>
</tr>
<tr>
<td>$Q_{fl}$</td>
<td>heat gain from the radiant floor</td>
</tr>
</tbody>
</table>
\( A_s \): wall surface area, \( m^2 \)  
\( M_f \): floor mass, kg  
\( T_{ra} \): room air temperature  
\( T_{o,e} \): outside air temperature  
\( T_{out,i} \): \( i \)th wall outside surface temperature  
\( T_{in,i} \): \( i \)th wall inside surface temperature  
\( \gamma_{ab,w} \): coefficient of absorbed internal heat gains from occupancy and equipments by inside surface of the wall  
\( \alpha_{ab,w} \): coefficient of absorbed solar radiation on the external surface of an external wall  
\( \beta_{ab,w} \): coefficient of absorbed transmitted solar radiation on the inside surface of an external wall  
\( Q_{pump} \): pump energy input, kW  
\( Q_h \): heater energy input, kW  
\( Q_{sol} \): solar radiation heat gain, kW  
\( Q_{rm} \): room energy change rate, kW  
n: number of measured data points.  
\( T_{predict} \): predicted temperature  
\( T_{measured} \): measured temperature.  
\( X \): vector of unknown parameters.  
\( X_L \) and \( X_U \): the lower and upper bound of unknown parameters, respectively

**ACKNOWLEDGEMENT**

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**REFERENCES**