Introduction

Solid oxide fuel cells (SOFC) hold potential as technology for distributed power generation in view of their high efficiency, ultralow emissions, and modular nature [1]. The present challenge is to make the cost of SOFC competitive with that of other power generation technologies, without compromising on the performance. This calls for a multipronged approach by considering such things as new materials, new cell designs, and varying cell dimensions. Computational modeling of the fuel cell can assist in understanding several of these issues. A large number of papers are available in the literature modeling various aspects of SOFC power systems. These models can be broadly classified in two categories: systems models and mechanistic models [1]. Systems models are macroscopic and are based on a mass and energy balance of the SOFC power plant. They are concerned with the overall performance of the plant subject to thermodynamic and process variables such as pressure, temperature, and fuel and oxidant composition [1]. Mechanistic models, on the other hand, focus on detailed modeling of an individual component of the cell such as kinetics of the natural gas reformer, electrical performance of the cell, or heat and mass transport in the electrodes. Several researchers have developed systems models for stand-alone SOFC power systems and have done parametric analysis with respect to key process variables [2–7]. Others have considered hybrids of SOFC with gas turbines or microturbines [8–17]. The individual cell design and impact of dimensions on cell performance also have been subjects of investigation [18–23]. Finally, there are some finite-element/CFD models for heat and mass transport inside the electrodes and the internal re-forming process [24]. Some of these models are fairly detailed and computationally intensive. Most systems models have predicted an electrical efficiency of 45%–50% (based on lower heating value) for stand-alone SOFC power plants and 60%–70% for SOFC–gas turbine hybrids. These numbers closely match the performance characteristics quoted by the manufacturers and are clearly well above the efficiency of other currently available technologies for distributed power generation, such as reciprocating engines and microturbines. However, from the point of view of making the SOFC cost competitive, it is perhaps more useful to develop mechanistic performance models that can be implemented with relative ease and which predict the performance of key SOFC components whose cost is a major concern. At present, the SOFC material cost is still very high and, therefore, the cell dimensions are key design variables.

Therefore, in this paper, we have developed a computational model to predict the electrical performance (voltage and current characteristics) of a tubular SOFC as a function of the cell dimensions. Such a model, in conjunction with a cost model, would be a powerful tool to understand the interlinking of performance and cost.

Cell Performance Model

In this section, we explain in detail the model to predict the electrical performance of the tubular SOFC. Figure 1 shows the schematic of the tubular SOFC. Air flows in the inner hollow tube and partially re-formed natural gas (produced in the internal reformer) flows on the surface of the outer tube (Fig. 1). The cell is discretized into slices of thickness dx. At each location x, a local Nernst voltage is generated, the magnitude of which depends on the temperature and on the fuel and oxidant concentrations at that location. Thus, the Nernst voltage varies along the axial location x and for a given cell operating voltage (\(V_{op}\)), the current generated also varies with x. The total cell current is then obtained by integrating the current generated by each slice over the length of the cell. The following sections describe the procedure for calculating the cell current (\(I_0\)) at each such slice.

Figure 2 shows the cross section of one such slice of the tubular SOFC with a thickness of dx and located at a distance x. The cell is cathode-supported and consists of concentric layers of the cathode (doped LaMn\textsubscript{3}O\textsubscript{4}), electrolyte yttria-stabilized zirconia, and the anode (Ni/doped zirconia). The cell’s outer radius is R and the thicknesses of cathode, electrolyte, and anode are \(\delta_c\), \(\delta_e\), and \(\delta_a\), respectively. The anode surface extends to an angle of \(\theta_0\) deg.

For convenience, we define the following average radii of the three layers:

\[
R_1 = R - \frac{\delta_0}{2}, \quad \text{(cathode)}
\]

\[
R_2 = R - \delta_0, \quad \text{(electrolyte)}
\]

\[
R_3 = R - \frac{\delta_0}{2}, \quad \text{(anode)}
\]

The interconnect contact connects one cell to the next adjoining. The width of the contact is \(W_I\) and the height is \(H_I\).

Current enters the cell from the lower adjoining cell through the interconnect contact. If the depth of the cross section slice is \(\Delta x\), the total current entering the cell is \(2I_0\Delta x\), where \(I_0\) is the current entering the half-cell per unit depth (A/m). We assume that there...
is no current transport in the axial direction. Further, we assume that the current flows along the circumference in the cathode and anode, while it flows in the radial direction in the electrolyte layer. In the cathode and anode, the current transport occurs due to flow of electrons, while in the electrolyte it is ionic.

The current distributes itself along several possible paths, finally leaving the cell at the upper interconnect contact to the next adjoining cell. Consider one such current path (Fig. 2), in which the current travels along the circumference on the anode until the angular location \( \theta \). There is an Ohmic voltage drop across this path, which depends on the anode resistivity and the anode thickness. At this point, the current crosses radially into the electrolyte and transforms into an ionic current. An ideal Nernst voltage \( E_r \) is generated by the electrochemical reaction at the interface of the electrolyte and the anode. In the electrolyte there are resistive loss and activation and concentration polarization losses. The current then flows along the circumference of the cathode and there are further Ohmic voltage drops in the cathode. Finally, the current flows through gap and interconnect to the next cell. In each of the possible current paths, the total voltage difference is the same and equals the cell operating voltage \( V_{op} \):

\[
V_{op} = E_r - \eta_{act}(\theta) - \eta_{conc}(\theta) - \eta_{ohm}(\theta)
\]

(2)

Here, \( E_r \) is the Nernst voltage, \( \eta_{act} \) and \( \eta_{conc} \) are the activation and concentration polarizations, respectively, in the particular current path, and \( \eta_{ohm} \) is the total Ohmic voltage loss in the anode, cathode, electrolyte, gap, and interconnect:

\[
\eta_{ohm} = \eta_{anode} + \eta_{electrolyte} + \eta_{cathode} + \eta_{gap} + \eta_{interconnect}
\]

(3)

Figure 3 shows the differential current element located at an angle \( \theta \). In Fig. 3, \( I_a(\theta) \) and \( I_c(\theta) \) are the anode and cathode currents, respectively, per unit cell depth (A/m). Thus, the actual anode current is \( I_a \Delta \theta \), and the cathode current is \( I_c \Delta \theta \). The local current density at the angular location \( \theta \) is \( i(\theta) \) (A/m²) and hence the current flowing in the branch is \( i(\theta)R_e \Delta \theta dx \).

\( R_e \) and \( R_a \) are the differential anode and cathode resistances, respectively:

\[
R_e = \frac{\rho_e R_e \Delta \theta}{\delta_e \Delta x}
\]

(4)

\[
R_a = \frac{\rho_a R_a \Delta \theta}{\delta_a \Delta x}
\]

(5)

The sum of the anode and cathode currents is \( I_0 \), which is the total current entering the half-cell.

The fundamental equations governing the current distribution in each differential element of the cell are Kirchoff’s current law and Kirchoff’s voltage law. These equations apply to each differential element, as shown in Fig. 3.

\[
I_a(\theta + \Delta \theta) = I_a(\theta) - (i(\theta)R_e \Delta \theta)
\]

\[
I_c(\theta + \Delta \theta) = I_c(\theta) + (i(\theta)R_e \Delta \theta)
\]

(6)

\[
E_r(\theta) = E_r(\theta + \Delta \theta) - \eta_{act}(\theta) - i(\theta)\rho_e \delta_e - I_a(\theta)R_e \Delta x
\]

(7)

Concentration polarization \( \eta_{conc} \) is given by

\[
\eta_{conc}(\theta) = \frac{RT}{nFE} \ln \left( 1 - \frac{i(\theta)}{i_i} \right)
\]

(8)

Here, the limiting current is obtained from Eq. (10). The diffusivity of oxygen accounts for the porosity and tortuosity of cathode material [25].

Fig. 1 View of the SOFC showing the Nernst voltages (V) and current (A/m) at the inlet and exit and also a slice (dx) located at a distance x

Fig. 2 Cross section of a tubular SOFC showing the different layers

Fig. 3 Differential element located at angular location \( \theta \) showing flow of current between anode and cathode
\[ i = n F D_{\text{eff}} C_{a 2, n} \frac{\delta_c}{\delta} \]

\[ D_{\text{eff}} = D_0 \left( \frac{T}{273} \right)^{1.5} \left( \frac{\varepsilon}{\xi} \right) \]  

\( D_{\text{eff}} \) is the effective diffusivity of oxygen in stagnant air at cell temperature, \( \varepsilon \) is the porosity, and \( \xi \) is tortuosity of cathode material.

Activation polarization is obtained from the Butler–Volmer equation:

\[ i = i_0 \left[ \exp \left( \frac{\beta Z F \eta_{\text{act}}}{R T} \right) - \exp \left( (1 - \beta) Z F \eta_{\text{act}} \right) \right] \]  

However, this is an implicit equation for the activation polarization and therefore, we used the following form for computational simplicity:

\[ \eta_{\text{act}} = \frac{R T}{\beta n F} \ln \left( \frac{i}{i_0} \right), \quad \text{if } i \ll i_0 \]  

\[ \eta_{\text{act}} = \frac{R T}{n F} \ln \left( \frac{i}{i_0} \right), \quad \text{if } i \gg i_0 \]

Solving Eqs. (7) and (8) for each differential element subject to the condition that the total voltage across the current path is \( V_{\text{op}} \) (Eq. (2)) gives the solution for the currents and also for the total current \( I_0 \). However, the solution is complicated because of the presence of the nonlinear polarization terms [Eqs. (9) and (12)].

In this paper, we present two approaches for solving the above equations and compare the results to determine the relative merits.

1) **Constant Polarizations:** We assumed that the cell is operating in the linear region of the \( V-I \) curve and hence the activation and concentration polarizations at a particular cross section are almost constant and do not depend on the angular location \( \theta \). In this case, we can obtain a closed form analytical solution that can be used to construct the \( V-I \) and power–I curves.

2) **Variable Polarizations:** This is a more rigorous approach and assumes the activation and concentration polarizations to be variable. In this case, it is difficult to construct an analytical solution and therefore we used finite difference methods to obtain the \( V-I \) and power–I curves.

**Analytical Solution: Constant Polarizations.** Nisancioglu [21] used this approach, but the cell geometry was approximated as a flat plate. This paper develops a model using the circular coordinate system and hence uses the exact lengths of the anode and cathode surfaces and of the electrolyte layer. We make a reasonable guess for the values of the activation and concentration polarizations and of the Nernst voltage at a particular cross section. These values are the same for each of the several current paths and are independent of \( \theta \). It follows from Eq. (3) that the total Ohmic losses (as given by Eq. (3)) are the same for each current path in the cell. The Ohmic voltage drop in the anode can be expressed as

\[ \eta_{\text{anode}} = \int_0^\theta \left[ I_0 - \int_0^\theta i R_3 d\theta \right] \frac{\rho_a R_3}{\delta_a} d\theta \]  

Similarly, the cathode Ohmic losses are

\[ \eta_{\text{cathode}} = \int_0^{\theta} \left[ \int_0^\theta i R_2 d\theta \right] \frac{\rho_c R_1}{\delta_c} d\theta \]

The current flow in the electrolyte is in radial direction and therefore, the Ohmic voltage drop in the electrolyte is

\[ \eta_{\text{electrolyte}} = i \rho_e \delta_e \]

Therefore, the total voltage drop caused by Ohmic resistances is

\[ \eta_{\text{ohm}} = \int_0^\theta \left[ I_0 - \int_0^\theta i R_2 d\theta \right] \frac{\rho_a R_3}{\delta_a} d\theta + \int_0^\theta \left[ \int_0^\theta i R_2 d\theta \right] \frac{\rho_c R_1}{\delta_c} d\theta + i \rho_e \delta_e + I_0 (R_s + R_l) \]

Here, \( R_s \) is the resistance of the gap between interconnect and the active cell area and \( R_l \) is the resistance of interconnect.

\[ R_s = \frac{\rho_s \delta_s}{W_s} \]  

\[ \delta_s = (\pi - \theta_0) R_2 \]

First and second derivatives of Eq. (16) results in

\[ I_0 - \int_0^\theta i R_2 d\theta \frac{\rho_a R_3}{\delta_a} - \int_0^\theta i R_2 d\theta \frac{\rho_c R_1}{\delta_c} + \rho_a \delta_a \frac{d}{d\theta} = 0 \]

\[ \frac{d^2 i}{d\theta^2} + b^2 i = 0 \]  

where

\[ b^2 = \left( \frac{1}{\rho_c \delta_c} \right) \left[ R_s R_2 \rho_a + R_s R_3 \rho_c \right] \]

This differential equation has a closed form solution for \( i \):

\[ i(\theta) = C_1 e^{b \theta} + C_2 e^{-b \theta} \]

The boundary conditions for solving the above system are:

1) **Cathode current at \( \theta = \theta_0 \) is \( I_0 \).**

\[ I_s(\theta = \theta_0) = I_0 \]

\[ I_0 = \left( \frac{R_s}{b} \right) \left[ C_1 (e^{b \theta_0} - 1) - C_2 (e^{-b \theta_0} - 1) \right] \]

2) **Total cell operating voltage is \( V_{\text{op}} \).** This, in conjunction with Eq. (1), concludes that the total Ohmic loss in each possible current path is a constant and is equal to the Nernst voltage reduced by the activation and concentration polarizations.

We then substitute Eqs. (17) and (18) in Eqs. (12) and (14), and after considerable simplification obtain

\[ C_1 = \frac{E_v - \eta_{\text{act}} - \eta_{\text{conc}} - V_{\text{op}}}{A_1 + A_2 Q} \]

\[ C_2 = Q C_1 \]
Therefore, at a certain cross section, we can obtain the total current \((2I_{0}\Delta x)\) for a given value of the cell operating voltage \((V_{op})\) and assumed values of Nernst voltage \((E_{n})\), activation polarization \((\eta_{act})\) and concentration polarizations \((\eta_{conc})\).

**Finite Difference Method.** The constant polarization solution as obtained above works well in the linear region of the \(V-I\) curve. However, this may not give a good estimate of the current densities in the nonlinear regions. Therefore, we considered a finite difference approach for modeling the nonlinear polarization losses in the electrolyte. This is based on the “distributed parameters” model developed by Sverdrup et al. [23].

We discretize the cell into \(n\) (~300) loops as shown in Fig. 2. Therefore, there is a system of \(3n+1\) simultaneous equations, out of which \(n+1\) equations are nonlinear. We solved the above system using an \(n\)-dimensional Newton-Raphson method.

**Solution Procedure.** The two procedures explained above obtain the current and hence power (per unit length) at a particular cross section, if the local Nernst voltage and the cell operating voltage are known. However, to know the total cell current, we need to integrate the current and power at each such slice over the length of the cell (Fig. 1). This can be accomplished precisely if the Nernst voltage and cell temperature are known as a function of \(x\). However, as a first approximation, we assume that the Nernst voltage varies linearly between the extreme values at \(x=0\) and \(x=L\). We assumed the Nernst voltage values at the cell inlet and exit to be 0.90 and 0.77 V, respectively.

In the case of analytical model, we need to assume the activation and concentration polarizations at each cross section. This can be accomplished by estimating the activation and concentration polarizations at a slice based on the current density obtained in the previous slice using Eqs. (9) and (12). Thus, we vary activation and concentration polarizations with the axial direction \((x)\), while keeping them constant at a particular cross section (and hence independent of \(\theta\)).

**Source of Uncertainties in Model.** There are two main sources of uncertainties because of which the predictions of the model may differ from the actual cell performance. We assumed the cell to be isothermal at 900°C, while in reality, there is a temperature profile. The partially re-formed fuel entering the anode at \(x=0\) is fairly cool since re-forming is endothermic. Therefore, the entry region of the cell is relatively cool and the hottest region occurs upstream. The cell temperature will have an effect on the Nernst voltage, kinetics of the electrode reactions, and resistivity of the three layers. If the temperature profile is available, it can be included in the above models to obtain more accurate results.

Secondly, the resistivity of the three layers is taken from the literature. However, with the exception of the electrolyte, the other two layers are doped cermet and hence their resistivity depends on the composition, which may vary.

We solved the model for a base case as given in Table 1. The cell outer diameter is 22 mm and the length is 1.5 m. Subsequently, the cell dimensions are varied from these base values to study the impact on cell performance.

**Results**

Figure 4 shows the \(V-I\) curves for the SOFC using the two approaches. The predictions of the two models are in good agreement in the linear regions of the curve. At higher current densities, there is a difference between the results, which is because the activation and concentration polarizations become significant. However, most commercial applications of the SOFC are likely to operate in the linear region and the relatively simple analytical model using constant polarizations gives results as good as the detailed and computationally intensive finite difference model.

The model predicts a power output of 123 W for an operating voltage of ~0.65 V and current density of 214 mA/cm². The Siemens Westinghouse tubular fuel cell is rated at 110 W with a current density of ~204 mA/cm² for an operating voltage of 0.65 V [1,27]. Therefore, the model predictions are reasonably good, and the difference with experimental results can be attributed to the isothermal assumption and errors in estimating resistivity of the layers.

Figure 5 shows the anode and cathode currents as a function of the angular location \((\theta)\) at a cross-sectional slice located at cell inlet \((x=0)\). As expected, the anode current continuously de-

\[
A_1 = \frac{A_2 \left( \rho_R R_S e^{-b_{h_0}} \right)}{b_2^e \delta_2} + \left( \rho_R R_S e^{-b_{h_2}} + \rho_R R_S e^{-b_{h_3}} \right) \frac{p_0 R_3}{b_2^e \delta_2} + \frac{p_0 R_3}{b_2^e \delta_2} + \left( \rho_R R_S e^{-b_{h_3}} \right) \frac{p_0 H_2}{W_1} \left( b_2^e \delta_2 \delta_3 \right)
\]

\[
Q = \frac{e^{b_{h_0} - 1 + L}}{e^{b_{h_2} - 1 + L}} \frac{L^2 \rho_S \delta_2 \delta_3}{\rho_R R_3}
\]

\[
I = \frac{A_1}{A_2}
\]

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Base case values of variables/parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cell length</strong></td>
<td>1.5 m [1,15]</td>
</tr>
<tr>
<td><strong>Cathode thickness</strong></td>
<td>1.8 mm [1,15]</td>
</tr>
<tr>
<td><strong>Anode thickness</strong></td>
<td>0.12 mm [1,15]</td>
</tr>
<tr>
<td><strong>Electrolyte thickness</strong></td>
<td>0.04 mm [1,15]</td>
</tr>
<tr>
<td><strong>Outer radius</strong></td>
<td>11 mm [1,15]</td>
</tr>
<tr>
<td><strong>Anode angle</strong></td>
<td>140 deg [1,15]</td>
</tr>
<tr>
<td><strong>Interconnector thickness</strong></td>
<td>5 mm [15]</td>
</tr>
<tr>
<td><strong>Interconnector height</strong></td>
<td>0.04 mm [15]</td>
</tr>
<tr>
<td><strong>Cell operating voltage</strong></td>
<td>0.65 V</td>
</tr>
<tr>
<td><strong>Anode resistivity</strong></td>
<td>(\exp(-6.03 - 1100/T)) (\Omega) cm [26]</td>
</tr>
<tr>
<td><strong>Cathode resistivity</strong></td>
<td>(\exp(-5.48 + 1210/T)) (\Omega) cm [26]</td>
</tr>
<tr>
<td><strong>Electrolyte resistivity</strong></td>
<td>(\exp(-6.01 + 1050/T)) (\Omega) cm [26]</td>
</tr>
<tr>
<td><strong>Interconnector resistivity</strong></td>
<td>(\exp(-4.51 + 4770/T)) (\Omega) cm [26]</td>
</tr>
<tr>
<td><strong>Porosity of cathode</strong></td>
<td>30% [19]</td>
</tr>
<tr>
<td><strong>Tortuosity of cathode</strong></td>
<td>6 [19]</td>
</tr>
<tr>
<td><strong>Cell temperature (C)</strong></td>
<td>900°C</td>
</tr>
</tbody>
</table>
creases and reaches zero at the point where the anode surface ends (\(\theta = 140\) deg). The cathode current keeps increasing and equals the total current at \(\theta = 140\) deg. The shapes of these two curves will depend on the relative resistivity and thickness of the two electrode layers.

Figure 6 shows the current density \(i(\theta)\) at two cross sections: the inlet \((x=0)\) and exit \((x=L)\) using the two approaches. This result shows the limitation of the analytical model using constant polarizations. The horizontal line is the limiting current density. Both the methods show a dish-shaped curve. The current density is highest at the interconnect point where the current enters the cell. As the current begins to flow from the anode to the cathode through the electrolyte, the current density keeps falling and reaches a minimum. However, subsequently, the current density begins to increase. This is because when the anode current becomes very low, the current density in the subsequent branches increases so as to satisfy the Kirchoff's voltage law in that differential element. The current density plot using the finite difference approach stays within the limiting current density, as expected. However, in the constant polarization case, the current density goes up significantly above the limiting current density, which obviously does not make physical sense. This is because we had assumed the activation and concentration polarizations as constant values and independent of the current density. Therefore, current density predictions using the analytical model are inaccurate. The current density on the cell surface has an impact on the local heat generation and thermal stresses. Therefore, if the analytical model is used, then it will give erroneous results, and this clearly is a limitation of this method. However, if the purpose of the model is to obtain the electrical performance in the normal linear range of operation, then the analytical model gives good results and is computationally much less intensive.

Figure 6 also shows the effect of the local Nernst voltage, which is 0.9 V at cell inlet \((x=0)\) and 0.77 V at cell exit \((x=L)\). At the cell inlet, the current densities are high and the local current density reaches a peak value of \(\sim 500\) mA/cm\(^2\) at \(\theta=0\). However, at the cell exit, the average current densities are lower and the maximum local current density is \(\sim 200\) mA/cm\(^2\), again at \(\theta=0\).

Figure 7 shows the current density graph for three operating voltages of 0.55, 0.6, and 0.65 V. As the operating voltage decreases (0.55 V), more current is drawn from the cell and the average current density at a particular cross section is higher. Consequently, the local current density at \(\theta=0\) approaches the limiting current (Fig. 7). Similarly when the cell operating voltage is high (0.65 V), the average current density is low.

We next attempt to study the relative importance of the various polarizations (Fig. 8). For this, we consider the cell inlet cross section \((x=0)\) and the follow the trajectories of seven current paths through the cell. Thus, for instance, a current path travels along the anode, crosses over to the cathode through the electrolyte at the angular location (\(\theta\)) of 25 deg, continues to travel through the cathode to the gap, and finally to the interconnect. Another current path crosses the electrolyte immediately at the lower interconnect (\(\theta=0\)) and this travels entirely through the cathode. The other extreme is a current path that travels entirely over the anode and crosses through the electrolyte at the point where the anode ends (\(\theta=140\) deg).

In each of these paths, the total voltage drop is 0.30 V as expected since the local Nernst voltage is 0.9 V, while cell operating voltage is 0.6 V. Ohmic losses in the three layers (anode, cathode, and electrolyte) account for most of the polarizations, 0.15 to 0.2 V. Activation polarizations appears to be the next main component, accounting for 0.08 to 0.10 V. The current density initially decreases with \(\theta\) and therefore activation and concentration polarizations also decrease. The voltage loss in interconnect and gap is about 0.05 V.

We now use the above models to study the impact of cell dimensions on performance (Figs. 9 and 10). Figure 9 compares the performance of two cells with cathode thicknesses of 1.8 and 1.6 mm. A thicker cathode implies lower resistance to current flow since the cathode current is circumferential. However, the limiting current for a thicker cathode will be lower (Eq. (15)) and therefore the concentration polarization losses will be higher for a thick
cathode layer. Figure 9 suggests that a cell with the cathode thickness of 1.8 mm gives more current and hence more power for the same operating voltage. A thick cathode may also be preferable from the material strength point of view, however, it implies a greater cost, since doped LaMnO$_3$ is an expensive material.

Cell diameter can have two contrasting effects on the performance. A larger diameter implies that the distance that the current has to travel in the cathode and anode will be larger and therefore Ohmic losses will be higher. However, a larger cell diameter also implies a larger cell surface area and hence the average current density will be lower. Therefore, the activation and concentration polarizations will be lower. Therefore, there is likely to be an optimum cell diameter where these two contrasting effects balance each other. Figure 10 shows the impact of cell diameter on power and power density. Here, the outer diameter is varied between 16 and 28 mm, keeping the other variables constant. The electrolyte and anode are already very thin and do not have much scope for any reduction in thickness. The cathode thickness is linked to the tube strength and therefore, has been kept a constant.

Figure 10 shows that the power per cell reaches a peak value for a cell diameter of about 21 mm. At lower diameters, the average current density is high and consequently the activation and concentration polarization losses are high, even though the cathode and anode Ohmic losses are lower. Similarly, at higher cell diameters, the average current densities are lower and therefore, the activation and concentration polarizations are lower, but the cathode and anode Ohmic losses are higher. However, the power density (mW/cm$^2$) continuously decreases with the diameter.

Conclusions

We developed two solution procedures for the electrical performance of a SOFC as a function of the cell dimensions: an analytical model and a finite difference model. The former assumes that the nonlinear activation and concentration polarizations are constant at a cross section of the cell and hence are independent of the angle ($\theta$). In this case, it is possible to obtain a closed form analytical solution for the current density distribution and hence obtain the $V-I$ characteristics. The finite difference calculates the polarization terms as a function of the local current density and is computationally intensive. The results show that the $V-I$ characteristics predicted by the two methods are in good agreement in the linear region of the $V-I$ curve and also match with experimentally quoted results. The two models were compared to study the impact of cell dimensions (cathode thickness, diameter) on the electrical performance, and the results matched very well. The analytical model appears to be good for predicting the electrical performance of the cell and is computationally straightforward to implement. The main utility of this model therefore is to study the impact of cell dimensions on performance. Another potential benefit of this model is to combine with a cost model to carry out cost-performance analysis and hence identify variables whose cost is a prime concern.
Acknowledgments
We gratefully acknowledge the discussions with Dr. Gianfranco DeGuiseppe, Siemens Westinghouse Corporation, Pittsburgh. This work was funded by the US Department of Energy, Grant No. DE-FC26-00NT40935.

Nomenclature
\( C_{\text{CO}_2} \) = oxygen concentration in air
\( D_{\text{eff}} \) = effective diffusivity of oxygen in air
\( D_0 \) = diffusivity of oxygen in air at \( 0 \) °C
\( E_r \) = reversible Nernst voltage (V)
\( I_a \) = anode current per unit depth (A/m)
\( I_c \) = cathode current per unit depth (A/m)
\( I_0 \) = exchange current (A/m²)
\( I_{\text{act}} \) = total current per unit depth at a cross section (A/m)
\( R \) = internal radius of the cell
\( R_1 \) = radius of cathode layer
\( R_2 \) = radius of electrolyte layer
\( R_3 \) = radius of anode layer
\( R_a \) = anode resistance of a differential element per unit depth of cell
\( R_c \) = cathode resistance of a differential element per unit depth of cell
\( R_{\text{el}} \) = electrolyte resistance of a differential element per unit depth of cell
\( R_p \) = activation and polarization losses of a differential element per unit depth of cell
\( T \) = cell operating temperature
\( V_{\text{op}} \) = cell operating voltage (V)
\( i \) = current density (A/m²)
\( i_l \) = limiting current (A/m²)
\( \delta_a \) = anode thickness (microns)
\( \delta_c \) = cathode thickness (microns)
\( \delta_{\text{el}} \) = electrolyte thickness (microns)
\( \varepsilon \) = cathode porosity
\( \eta_{\text{act}} \) = activation polarization (V)
\( \eta_{\text{ohm}} \) = ohmic polarization in anode (V)
\( \eta_{\text{cat}} \) = ohmic polarization in cathode (V)
\( \eta_{\text{conc}} \) = concentration polarization (V)
\( \eta_{\text{el}} \) = polarization in electrolyte (V)
\( \eta_{\text{ohm}} \) = total ohmic voltage losses (V)
\( \theta \) = angular location (radians)
\( \theta_0 \) = angular extent of anode layer (radians)
\( \xi \) = cathode tortuosity
\( \rho_a \) = anode resistivity (Ω m)
\( \rho_c \) = cathode resistivity (Ω m)
\( \rho_{\text{el}} \) = electrolyte resistivity (Ω m)

References