1. **Motivation.** How do humans walk and run stably without falling down? How do humans walk without deviating too much from a roughly periodic steady motion, say, when walking on a treadmill? There are many hundred articles with mechanics-based mathematical models of bipedal walking and running, that demonstrate stable periodic motion, either due to passive dynamics or due to active control. Such theoretical work provide insight into how one might stabilize locomotion. However, it is not yet settled which among this zoo of stabilizing control schemes yield dynamics most similar to that of humans during locomotion.

Our goal is characterize, using appropriate human walking experiments, the dynamics near steady state walking motions. We seek a model of the dynamics that will be able to predict the transients back to steady state, not just reproduce the steady state motion. Aside being of general scientific interest, a characterization of walking dynamics might enable us to design prosthetic devices with human-like dynamics, or exoskeletons that do not fight these natural dynamics.

2. **State of the art.** One natural approach to this system identification problem is to apply external perturbations to the walking human, and then fit dynamical or control models to the transients back to steady state. There is a considerable literature on perturbation experiments (e.g., [1]). While most such experiments have not been mined for detailed dynamics, some have been examined for some control aspects. For instance, At Hof et al (e.g., [2]) show how sideways perturbations are controlled by sideways step width modulations, as has also been argued by Kuo and others.

As an alternative to perturbation experiments, Hurmuzlu [3], Dingwell [4], Revzen [5], and others have pioneered the use of steady state locomotion data to infer the dynamics. While this seems oxymoronic, the idea is that steady walking is not quite exactly periodic, and every step is slightly different from every other step. If this non-periodicity is attributable to process noise (whether of motor or sensory or external origins) — this process noise provides the ‘perturbations’ that let us explore the dynamics in the neighborhood of the periodic motion.

3. **Our approach.**

**Experiments.** We used treadmill motion capture of human walking at three speeds, for five subjects (so far), with at least three markers on each of seven body segments (two feet, two shanks, two thighs, and upper body). Subjects walked for a few minutes at each speed, corresponding to a few hundred steps. Joint centers and 2D joint angles were estimated, and we noted that step-to-step variability is systematically greater than the obvious sources of measurement or model errors, suggesting that there may be dynamical information in this step-to-step variability.

**Dynamics representation using Factorized Poincare maps.** Because linear time-periodic ODEs are susceptible to drift errors due to noise, and a Poincare map for a single transverse section only gives a slice of the dynamics, we represent the dynamics near the periodic motion by considering a large number of Poincare sections and the mappings from one section to the next: in other words, a factorized Poincare map (an idea very closely related to that in [5, 6]).

**Inferring linear mappings.** Given nearly periodic noise-driven data, we can construct well-conditioned Poincare sections, compute the successive intersections of the trajectory with the Poincare sections, and then estimate section-to-section mappings. We mostly use linear least squares, but we have also used an approximate maximum likelihood estimations, when biased estimates due to correlated noise were a concern (see [7, 8] for more details).

4. **Current results**

**Simulating model for the whole body dynamics.** Using about 20 Poincare sections, we estimated a factorized Poincare map representation of the walk-
ing dynamics, using a 2D model of a human, with 5 segments (no foot). Because this is a dynamical model, we can now simulate the dynamics from arbitrary initial conditions near the periodic motion. The model has the property that having a too-high forward speed during mid-stance increases the next step length, as does starting with too high swing leg speed (see Figure 1). This could be interpreted as using step lengths to (partly) control forward speed, as has previously been suggested.

We performed some ‘self-imposed transient’ human experiments in which the subjects took a longer or shorter step of their own volition, and returned back to steady state. Starting from an initial condition at the middle of such a transient, we compared experimentally observed motions to the corresponding model predictions: we found that while a large fraction of the transient is explained by the model, the match is not perfect.

**Top-view step to step dynamics.** As a simpler calculation, we also computed linear mappings that capture the step to step dynamics of walking. We then estimated a linear mapping to the next foot fall position from the upper body state at mid-stance relative to current stance foot position. This mapping suggests what has been observed in the At Hof experiments, that a larger-than-usual sideways speed or sideways position during mid-stance gives rise to increased step widths and decreased step lengths: suggesting (but perhaps not exclusively) step-width control of side-to-side dynamics. A larger than usual forward speed increases the next step length, without affecting the step width significantly. (All estimates statistically significant, after assumption of model structure).

5. Best possible results

Many open problems remain. All our statistical estimates come with large error estimates (computed by bootstrap resampling), which perhaps cannot be reduced without increasing the ‘signal’ by applying external perturbations instead of relying on small internal perturbations. Of course, even small error estimates are reliable only so far as we have assumed the correct model structure and model order to fit to the data: we may be missing delay terms, significant state variables like foot angle, or even, dynamics that cannot be linearized, like dead-bands. Our eventual hope is to perform perturbation experiments that can then be used to independently perform such system identification, and perhaps compare with steady state derived dynamics. We are now fitting to the data a mechanics-based model of the human body, so we can estimate the driving joint torques or muscle forces, and then infer a mapping from body state to muscle forces; that is, inferring a controller.

**Acknowledgements.** Thanks to Phil Holmes and Shai Revzen for early conversations. Thanks to Casey Kerrigan for early pilot experiments on running a few years ago, not used in this analysis.

**References**


