A Theoretical Explanation for the Relative Leg Stiffness of Animals

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1 Motivation

It has been shown that most legged animals have a normalized "relative leg stiffness" in the vicinity of 10, regardless of size or morphology [1, 2]. The existence of this approximately universal constant of animal and human locomotion remains a mystery and any partial or complete explanation for it would likely provide basic knowledge useful for the field of legged locomotion.

2 State of the Art

One approach in the field of legged locomotion is to use theoretical models and analysis to provide basic knowledge [1]. One particular group of these models with rotary hip torque and radial leg damping has demonstrated a greater degree of robust stability and biological relevance. The previously developed Clock Torqued Spring-Loaded Inverted Pendulum [3] and Torque-Actuated Dissipative Spring Loaded Inverted Pendulum [4] both have a controlled hip torque as well as leg damping and are fully asymptotically stable. Recently, a simplified model of locomotion with hip forcing and leg damping called the Hip-actuated SLIP [5] has been developed and analyzed to study the effects of forcing and damping on the stability of locomotion. An important next step would be to use these hip actuated models to explain the near universality of relative stiffness in animal locomotion.

3 Approach

We analyze the simple Hip-actuated SLIP [5] model to determine how the stability of locomotion and the energetic cost of locomotion relates to the relative stiffness. Here, as in [1], we define the relative stiffness to be $k_{rel} = \frac{kl_0}{mg}$, where k is the stiffness of the leg, l_0 is the resting leg length, m the mass, and g is the constant acceleration due to gravity. By understanding how locomotion stability and energetics might change with respect to relative stiffness, we might better understand why a nearly universal range of relative stiffness values are selected for animal locomotion, despite a very large diversity of animal size and morphology.

4 Current Results

We have found that there exists an upper limit of acceptable values of relative stiffness for which fully stable locomotion solutions exist. As shown in Fig. 1, we find no stable locomotion when the relative stiffness is greater than 30. Further, there exists an optimal value of relative stiffness between 10 and 15, as shown in Fig. 1. When the relative stiffness deviates from the optimal point in either direction, the range of hip torque values that yield stable locomotion shrinks. Further, the energetic cost of locomotion increases when relative stiffness deviates from the optimal value.

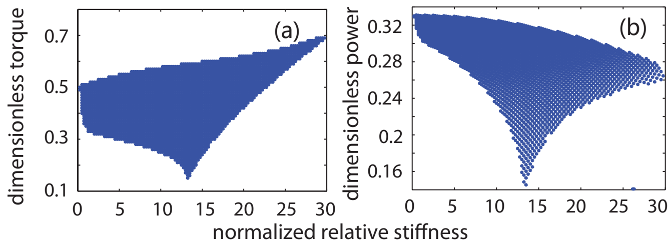

Figure 1: (a), The stable parameter region (shaded area) of Hip-actuated SLIP. (b), The average power of the fixed points within the stable parameter region. Here the average forward speed is maintained at $\sqrt{gl_0}$. The hip torque is scaled by a factor of mgl_0 and average power by $mg\sqrt{gl_0}$.

5 Best Possible Outcome

Overall, we hypothesize that a relative stiffness close to 10 may be optimal for both the energetic cost of locomotion and the underlying open-loop stability of locomotion. In previous work analyzing the Hip-SLIP model of locomotion [5], we discovered that the rotary-based hip-torque mechanism may be one explanation for a basis of robust stability of locomotion exhibited by animals and rotary-based robots. We hypothesize that this basic mechanism persists in more complex locomotion systems containing multiple forcing elements and feedback control, and so we predict that preferences for relative stiffness in animals would be similar to those discovered for the Hip-SLIP model.

References

[1] Holmes et al. *SIAM Rev.*, 2006.

[2] Blickhan et al. *J of Comp Physi A*, 1993.

[3] Seipel et al. *Regular and Chaotic Dynamics*, 2007.

[4] Ankaral et al. *Chaos*, 2010.

[5] Shen et al. *Bioinspiration & Biomimetics*, 2012.