A Synthesis Method for Compact Nonlinear Springs with Custom Torque-Deflection Profiles and Bandwidth for Series Elastic Actuators

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1. Motivation and State of the Art

Historically, series elastic actuators (SEAs) used linear springs as the torque transmitting element in their drivetrains [1]. Use of linear springs requires tradeoffs between desired actuation bandwidth and measurable torque resolution. Nonlinear transmission elements overcome this limitation, enabling variable torque resolution and bandwidth through their deflection range. Many have recognized the benefits of variable stiffness elements in SEA drivetrains, which has led to multiple actively controlled variable stiffness actuator designs [2] as well as passive mechanisms that change stiffness during actuation [3][4]. The latter devices rely on cams to nonlinearly elongate a metal, linear spring to achieve a desired torque-deflection profile. However, these devices have a large form factor, which limits their use in existing SEA drivetrains.

2. Own Approach

We present a synthesis method for compact nonlinear springs with user defined torque-deflection profiles and bandwidth that use a nonlinear rubber element and cam profile to achieve a desired output torque (Fig. 1). Using rubber as the spring element is advantageous over metal, as rubber springs can achieve higher stiffness at smaller sizes, have a large elongation range before plastic deformation, and can be easily manufactured with custom form factors.

Spring designs result from a two part optimization. The first part optimizes the maximum spring deflection of a user defined torque-deflection profile to achieve a desired rise time by solving the motor dynamics equation for a fixed load

$$J_m \ddot{\theta}_m = \tau_{max} - \tau_{des}(\theta_m) \qquad (1)$$

where J_m, θ_m, and τ_{max} are motor inertia, angle, and maximum motor torque, and τ_{des} is the desired output torque at a given spring deflection. We optimize a cam shape profile to achieve this desired torque profile via the Covariance Matrix Adaption - Evolution Strategy (CMA-ES) [5] such that

$$\vec{\tau}_{des} = \vec{r}_{cam} \times \vec{F} \qquad (2)$$

where r_{cam} is the instantaneous cam radius at a given deflection and F is the force generated in the rubber element due to its stretch λ along the cam profile. To determine the nonlinear relationship between applied uniaxial tension and resulting stretch, we perform tensile tests on rubber samples and fit a polynominal hyperelasticity model that is then used during cam optimization (Fig. 2).

3. Current Results

The proposed method produces feasible, compact nonlinear spring designs (Fig. 3) that closely match the desired torque-deflection profile in simulation (Fig. 4). We are currently creating physical springs via rapid prototyping to experimentally verify the physical spring's bandwidth and ability to match our desired torque profile. To further decrease spring size and increase the rubber's tensile strength, we plan to use pressure casting during spring manufacturing to eliminate air bubbles introduced in the two-part rubber mixing process.

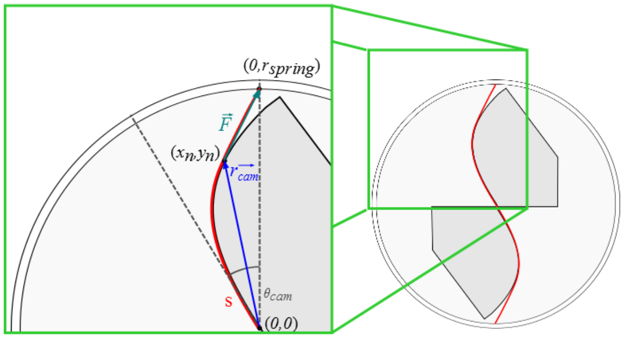

Fig. 1: Nonlinear spring concept.

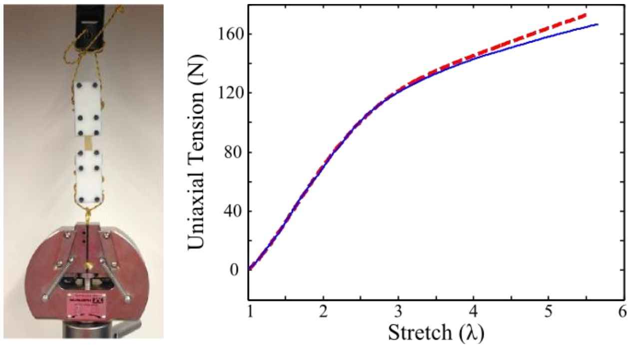

Fig. 2: L: Tensile test setup. R: Hyperelasticity model. Solid: Tensile data. Dashed: Empirical fit.

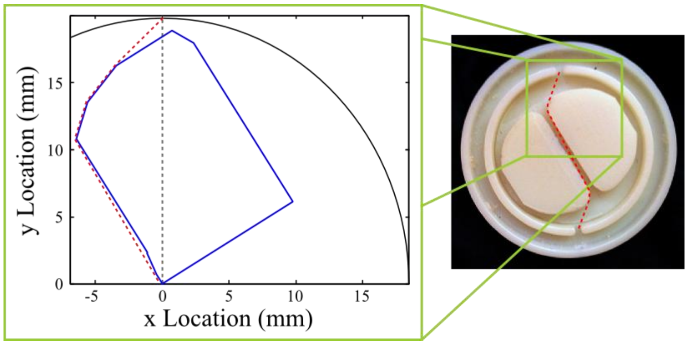

Fig. 3: L: 11 point cam profile (blue) that encodes exponentially stiffening spring for torques [0, 2.5] Nm with 10ms bandwidth. R: Prototype.

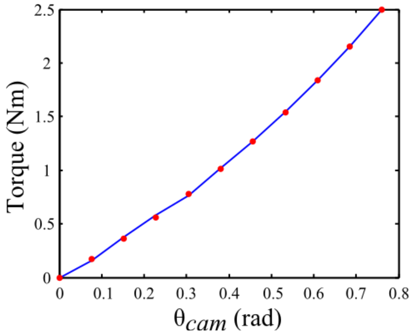

Fig. 4: Resulting torque profile of optimized cam shape. Dotted: Desired torque-deflection pairs. Solid: Torque generated by nonlinear spring.

4. Best Possible Outcome

Ideally, this research will result in a synthesis method for compact nonlinear springs that will enable roboticists to create custom, compact nonlinear springs for series elastic actuators. Such springs have the potential to improve performance and form factor of series elastic actuators used in robotics.

Acknowledgements

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