Development and Validation of an Advanced Model of a Passive Dynamic Biped Walking Robot

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1. Motivation

Mathematical modeling is an excellent tool for analyzing passive dynamic walking. There are two limitations: (1) impact-based models and simplified friction models, which cause the bipedal walking model discontinuous, and (2) lack of experimental validation. In this work, we develop a continuous model of passive dynamic walking, in which the Hunt-Crossley contact model and the LuGre friction model were used to represent the normal and tangential ground reactions continuously. A physical passive walker was built to validate the proposed mathematical model. A traditional impact-based passive walking model was also used as a reference to demonstrate the advancement of the proposed passive dynamic walking model.

2. A continuous bipedal walking model

Figure 1 shows a schematic of the passive walking model. The Hunt-Crossley contact mode, an extension of the Hertz contact model was used to include hysteretic damping in the contact forces. The friction between the foot and the surface is modeled using the LuGre friction model. The final state space model has ten states that are described in [1].

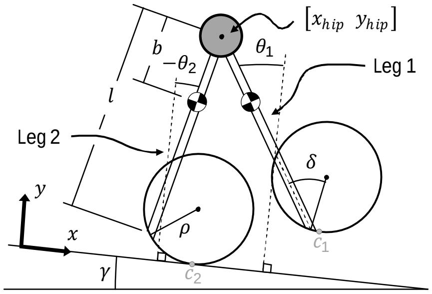

Figure 1. Model diagram

3. Results and conclusions

Figure 2 is the physical walker built with sensors attached. Figure 3 compares the proposed and impact-based models with the experimental measurements. Results show that the proposed model can predict both the trends and magnitudes of the gait parameters, while the impact-based model can only match the trend. More importantly, the proposed model avoids the theoretical problems of non-smooth systems.

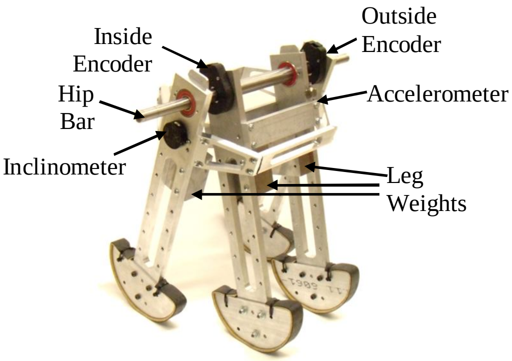

Figure 2. Physical bipedal walking robot

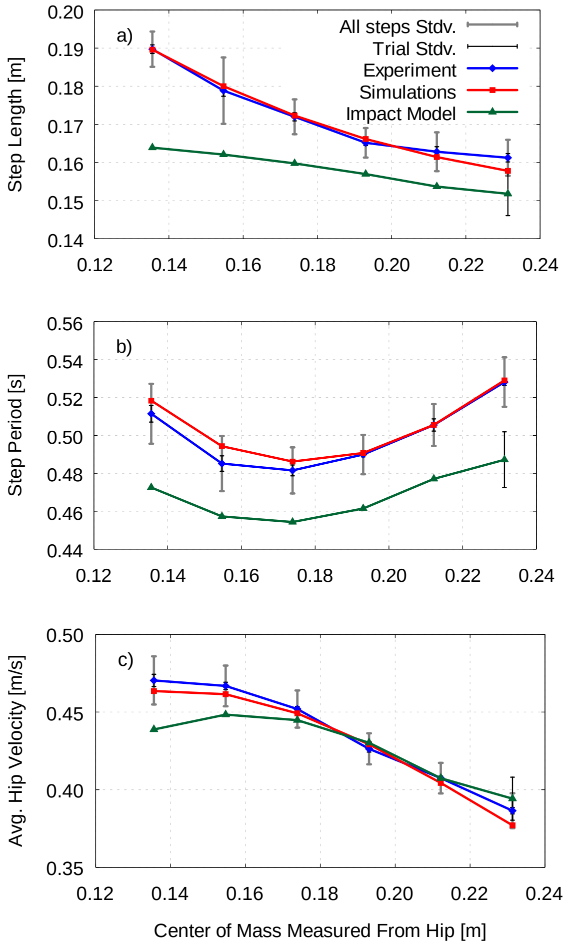

Figure 3 Comparison between experimental walker, proposed model, and impact-based model.

4. References

[1] Koop, D. And Wu Q. (2013) Passive Dynamic Biped Walking Part I: Development and Validation of a Model, accepted by *ASME J. of Comp. and Nonlinear Dynamics.*

Stability Analysis of the Passive Dynamic Gait

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1. Motivation

Passive dynamic walking can be stable with the proper combination of the parameters and initial states. However, the stability region is small and has an obscured geometry. The passive walking model, discussed in [1], was used to analyze the stability with Lyapunov exponents (LEs) and the geometry of the basin of attraction was determined. A novel method was created to determine the 2D projection of the Basin Of Attraction (BOA) of the model.

2. Stability analysis via Lyapunov exponents

LEs are a valuable tool for analyzing the asymptotic behaviour of non-linear systems. They are described as the "average exponential rates of divergence or convergence of nearby orbits in phase space.". The exponents are invariant to the initial conditions, and the signs of LEs indicate the system stability, i.e., negative exponents indicates the exponential stability of the dynamic systems. LEs are calculated using the model [1] and the detailed procedure is discussed in [2]. Due to the obscured geometry of the BOA, an algorithm finding the edge of the basin of attraction is developed [2] and used here.

3. Results and conclusions

Figure 1 shows the limit cycle walking projected in the phase plane. The LEs calculated for one case is shown in Table 1. The negative sign shows that the bipedal model is exponentially stable. The geometry and the areas of BOA are shown in Figures 2 and 3. The geometry of BOA explains the difficulties in producing limit cycle walking and the size of BOA is sensitive to the location of mass center. Overall the proposed LE-based method can provide significant insights into bipedal walking stability.

4. References

[1] Koop, D. and Wu, Q. (2013) Development and Validation of an Advanced Model of a Passive Dynamic Biped Walking Robot, submitted to *Dynamic Walking 2013.*

[2] Koop, D. And Wu, Q. (2013) Passive Dynamic Biped Walking Part II: Stability Analysis of the Passive Dynamic Gait, accepted by *ASME J. of Comp. and Nonlinear Dynamics.*

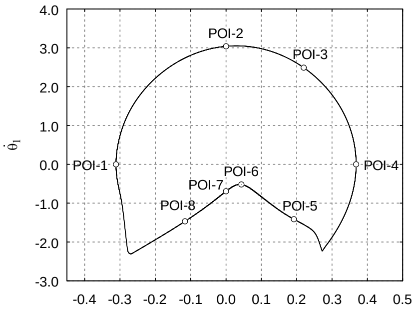

Figure 1. Limit cycle walking

Table 1. LYAPUNOV EXPONENTS.

	LE		STD	% STD
1	0	±	0	0%
2	−0.01341	±	0.00013	0.97%
3	−2.25988	±	0.00018	7.97×10^{-3}%
4	−2.25958	±	0.00013	5.75×10^{-3}%
5	−3.88372	±	0.00008	2.06×10^{-3}%
6	−69.30363	±	0.00018	2.60×10^{-4}%
7	−80.29741	±	0.00024	2.99×10^{-4}%
8	−236.70207	±	0.00022	9.29×10^{-5}%
9	−29346.903	±	0.043	1.47×10^{-4}%
10	−7413.16	±	0.20	2.70×10^{-3}%

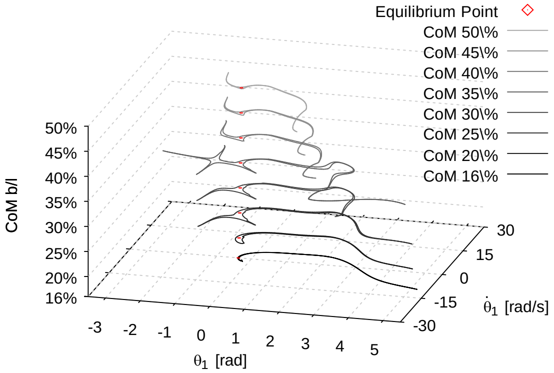

Figure 2. BOA at one of the Poincaré section.

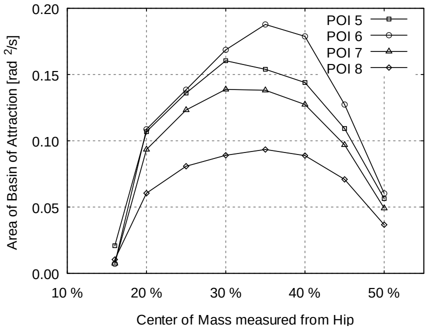

Figure 3. Areas of BOA with various mass distributions.