Polya's Bees: A Model of Decentralized Decision Making

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How do social systems make decisions with no single individual in control? We observe that a variety of natural systems, including colonies of ants and bees and perhaps even neurons in the human brain, make decentralized decisions using common processes involving information search with positive feedback and consensus choice through quorum sensing. We model this process with an urn scheme that runs until hitting a threshold, and we characterize an inherent tradeoff between the speed and the accuracy of a decision. The proposed common mechanism provides a robust and effective means by which a decentralized system can navigate the speed-accuracy tradeoff and make reasonably good, quick decisions in a variety of environments. Additionally, consensus choice exhibits systemic risk aversion even while individuals are idiosyncratically risk neutral. This too is adaptive. The model illustrates how natural systems make decentralized decisions, illuminating a mechanism that engineers of social and artificial systems could imitate.

Introduction

Ants and bees appear to rely on decentralized-decision making for critical choices. For example, in choosing a new nest site—a decision that has huge implications for the survival of the group—decisions must be made without central control and with no single individual evaluating the total available information (*1*) or any one individual making direct comparisons of the available options (*2–5*). Even though individual agents follow simple rules that allow them to uncover very limited and local information, the colony as a whole must efficiently integrate the resulting flow of information into a high-quality, final decision (*6–10*).

Consider how a swarm of honey bees, *Apis mellifera*, chooses a new hive location (*8–14*). When a swarm abandons the old hive, it temporarily gathers at a tenuous location. Approximately 5% of the bees are scouts, and after exploring the surrounding area for possible hive sites, a scout may perform a waggle dance that indicates the location of the site they discovered (*15, 16*). The likelihood of performing a waggle dance, and its duration, depends on the quality of the site that was investigated (*9, 13, 14*). The waggle dances serve to recruit additional scouts to further investigate the "advertised" sites. The longer the dance, the more likely that new scouts will investigate the site and bring back independent evaluations. Over time, positive feedback loops are generated (*8, 13*) and once the number of scout bees at a particular site reaches a quorum threshold (of around thirty to forty bees), those scouts return to the swarm and lead it to the new site (*12*).

Leptothorax (Temnothorax) albipennis ants choosing a new nest site behave similarly (*1, 6, 17*). When a scout finds a higher quality site, it quickly returns to the old nest site and recruits a nest-mate by tandem running, a tedious process that entails the scout teaching the recruit the route to the new site (*18–21*). The speed of recruitment is tied to the quality of the site (*1, 3*), with better sites inducing quicker responses. As before, positive feedback arises when recruits

become recruiters. Finally, when the number of ants at the new site reaches a quorum threshold, the recruiting ants switch from tandem running to the much faster process of carrying their remaining nestmates from the old to the new site (*22*).

Other social organisms make collective decisions with mechanisms reminiscent of ants and bees. Social spiders coordinate their emigrations to a new nest (*23*) with silk draglines allowing positive reinforcement of existing routes much like pheromone trails of ants (*23–25*). Cockroaches are more likely to remain in shelters when other cockroaches are nearby, leading to a collective choice of a single home (*26,27*). Even bacteria share information and detect quorums, allowing for collective decisions regarding sporulation, virulence, and gene exchange (*28*).

There is some speculation that primate brains use a similar decentralized-decision mechanism. No single neuron is solely responsible for the brain's decision. In a visual discrimination task, for example, a subset of specialized neurons integrates sensory signals from other neurons and allows the brain to make a decision to trigger other neurons to initiate a motor response (*29*). Complex, decentralized information processing can be achieved with a cell-assembly, a recurrent circuit of neurons that becomes active when stimulation spreads with positive feedback (*30, 31*). Neurons are generally understood to accumulate information and fire when the stimulus hits a threshold in order to implement a decision (*32–34*). In this regard, the primate brain may function analogously to a colony of social insects (*35–37*).

Similar mechanisms may even be at work in large-scale social processes (*38*). For example, consider the choice of a personal MP3 player. Consumers who purchase such players "advertise" them when they use them (particularly if the players have some distinctive feature, for example, white ear buds). Moreover, consumers who enjoy their players are more likely to use them. Someone new to the MP3 market may observe the players that others use, and purchase based on these observations. This kind of direct marketing is often a major driver of consumer demand for new products, especially when competing brands have not yet established distinct

reputations (*39*). At some point in the evolution of the market, however, a critical mass of consumers may choose the same product and fundamentally change the market dynamics (say, by adopting a particular technological innovation, by fueling economies of scale in the production process, or by enticing suppliers or producers of complements to enter into exclusive agreements) so that only the leading product can survive in the market (*40, 41*).[1]

Any decision mechanism must trade off exploring new options versus exploiting the best option known to date. While further exploration helps identify superior options, it comes at the cost of not acting on a known option. Thus, too much exploration may lead to indecisiveness and thus harm fitness, while too little may imply the acceptance of sub-optimal choices. Given the inherent tradeoff between speed and accuracy, the diffusion model of decision making (*34*)— which does not incorporate positive feedback in the accumulation of evidence—is efficient for binary choice, but it requires agents to compare the options, or at least to inhibit activity for competing options (*35, 44*). When this is not possible, positive feedback in the process of exploration may prove useful for making reasonably good, quick decentralized decisions.

The Model

We now present a model of a two-part process for decentralized-decision making, involving search and recruitment with positive feedback in the first phase and quorum detection to trigger a consensus choice[2] (without centralized processing) in the second. First, individual agents randomly search over the set of feasible options, biased by the quality of the options revealed during previous searches. Then, a final consensus choice is triggered when a quorum of agents investigating any one particular option forms. We capture this process with an urn scheme that

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[1]Similarly, people collaborating to make a group decision also tend to share information that favors options that already have popular support while hesitating to share information that favors unpopular options (*42*), and they commonly reach a consensus choice through plurality voting (*43*).

[2]A *consensus choice* is a choice that the entire system must abide (*45*), but of course it is not necessarily a unanimous choice via "consensus sensing" (*10*).

runs until hitting a threshold.

Search and Recruitment

Agents (scouts) explore one of many possible options and then return home to recruit additional agents to further explore that option. A Poisson process for each agent governs its trips home, and the recruitment of additional scouts on each trip depends on the quality of the explored option.

We assume that there are C possible options, and that some number of agents w_c^t investigate each $c \in C$ at any time t. We refer to w_c^t as the weight on c at time t. All weights are initially set to the same, positive value w^0. Each agent is equally likely to return home at any time, and when it does, it recruits additional agents to join it and further explore the same option. The chance of an agent recruiting for option c at time t is simply proportional to the weight w_c^t. Each option has a set of immutable attributes that defines its quality, and the extent of recruitment for c depends on its quality, v_c. Agents investigating c will recruit v_c additional agents to continue exploring it when they return. We think of the number of recruits per return trip home as an ordinal measure of the quality of c.

Quorum Detection

The search process above generates a distribution of agents investigating each possible option at any given time. Given the decentralized nature of these systems, there must be some feasible trigger that ends the search process and finalizes the consensus choice. One possible solution to this problem would be to have the search probabilities converge to zero or one—that is, have all of the probability concentrated on a single option. Forcing such a unanimous decision on the system is problematic, as it may form extremely slowly, perhaps leading to a serious loss of fitness. Moreover, we have empirical evidence, at least in the case of honey bees (*46, 47*),

ants (*6*), and stickleback fish (*48, 49*), that unanimity is not what triggers a consensus choice. Instead, a final choice is made once the number of agents in favor of a particular option reaches a quorum (*45, 50*).

Based on the above arguments, we incorporate into our model a *quorum threshold*, τ, that triggers—as the final decision—any option that is being investigated by at least that number of agents. The level of the quorum threshold has important implications for the decisions that arise in the system. If the threshold is set too high, then a quorum may not be reached for a long time, resulting in prolonged inaction. If the threshold is set too low, then a quorum might be achieved for a relatively low-quality option. Thus, the optimal quorum threshold depends on a tradeoff between speed and accuracy in the decision-making process. From a normative standpoint, a good threshold allows the system to withstand various transients in the probability distribution while still remaining responsive to the acquired information in a timely manner.

The Urn Scheme

We use a simple (Polya) urn process to model this decision mechanism. This process is easy to visualize. Assign to each of the C options a unique color, and place w^0 balls of each color into an urn. The number of balls of a particular color in the urn corresponds to the number of agents investigating the associated option. Each ball has the same rate at which it may be randomly drawn from the urn. When a ball with color c is drawn, it is immediately placed back into the urn along with v_c identically-colored balls. This process continues until a threshold number of balls τ is reached. The evolution of the mix of balls over time will determine the behavior of this decision process.

Analytical Results

We can precisely characterize how the composition of balls in the urn evolves over time. Lemma 3.1 of (*51*) gives us the distribution of balls of each color at any time t (in the absence of a threshold for stopping the process).

Lemma 1. *The moment-generating function* $\phi_c(t,s) = \mathbf{E}\left[e^{s\,w_c^t}\right]$ *is given by* $\phi_c(t,s) = \left(\frac{e^{v_c(s-t)}}{e^{v_c(s-t)}-e^{v_c s}+1}\right)^{\frac{w^0}{v_c}}$ *(where s is the argument of the moment generating function).*

Proof. We can describe this Polya process with a diagonal $C \times C$ matrix with the v_c values along the diagonal and 0's elsewhere. The evolution of the number of balls of a given color is independent of the evolution of other colors (until the threshold is hit). Thus, Lemma 3.1 of (*51*) directly applies. □

In principle, this moment generating function fully characterizes the distribution of weights, w_c^t, where s is the argument of the moment-generating function. In practice, however, calculating the likelihood of hitting a threshold τ at a given time t is complicated.

An asymptotic result is simple to obtain. Suppose the threshold τ is infinite so that the Polya process can run forever. Eventually, almost all of the weight converges on the choice with the highest quality.

Theorem 1. *If there is a unique optimal choice $c^* = \arg\max_c v_c$, then*

$$\lim_{t\to\infty} \frac{w_c^t}{\sum_j w_j^t} = \begin{cases} 1 & \text{if } c = c^* \\ 0 & \text{otherwise.} \end{cases}$$

Proof. As $t \to \infty$, $\frac{w_c^t}{e^{v_c t}} \xrightarrow{\mathcal{D}} \text{Gamma}\left(\frac{w^0}{v_c}, v_c\right)$ (see Theorem 3.1 of (*51*)). □

In the infinite time limit, we recover the unanimity decision rule, and the probability of a mistake (i.e., selecting an option other than c^*) vanishes.

While the asymptotic properties of the urn process are informative, feasible decentralized systems must make decisions in finite time and require a finite threshold. For any given threshold τ, we would like to describe the probability $p_c(\tau)$ of selecting each possible choice c as well as the waiting time $T(\tau)$ until the decision is made.

We can characterize the waiting time $T_c(\tau)$ until the number of agents exploring option c would hit the threshold τ (independent of recruitment for other options). Let λ denote the intensity of the Poisson process for each agent's return home.

Lemma 2. *The waiting time $T_c(\tau)$ has the* Hypoexponential$(\lambda_0, \lambda_1, \ldots, \lambda_n)$ *distribution with*

$$\lambda_i = \left(w^0 + iv_c\right)\lambda \text{ for all } i \tag{1}$$

and

$$w^0 + nv_c < \tau \leq w^0 + (n+1)v_c, \tag{2}$$

which implies

$$n = \text{ceiling}\left(\frac{\tau - w^0}{v_c}\right) - 1. \tag{3}$$

Proof. We have an Exponential(λ) distribution for the time until a given agent returns home, and thus at any time t we have an Exponential$(w_c^t\lambda)$ distribution for the time until additional agents are recruited to explore option c. Thus, the waiting time $T_c(\tau)$ until the number of agents exploring option c hits the threshold τ is the sum of independent, exponentially-distributed variables with arithmetically increasing parameters. $\square$

The hypoexponential density function is $f(t) = \sum_{i=0}^{n} C_{i,n}\lambda_i e^{-\lambda_i t}$ with $C_{i,n} = \prod_{j\neq i}\frac{\lambda_j}{\lambda_j - \lambda_i}$. Taking n and $\lambda_1, \ldots, \lambda_n$ to be functions of c and τ (given by Equation 1 and Equation 3), this gives us the probability density function $f_{c,\tau}(t)$ for each $T_c(\tau)$.

We can use the density functions for the $T_c(\tau)$ variables to get at the quantities of interest in the system.

Theorem 2. *The waiting time $T(\tau)$ distribution and the choice probabilities $p_c(\tau)$ are determined by the $T_c(\tau)$ distributions given by Lemma 2. The time until a decision is made by the decentralized system is $T(\tau) = \min_c T_c(\tau)$. The probability that the eventual decision is for choice c is $p_c(\tau) = \Pr\left[T_c(\tau) < \min_{c' \neq c} T_{c'}(\tau)\right]$.*

Proof. The urn process runs until the first time that balls of any one color accumulate to the threshold, and the probability of selecting any given choice is simply the probability that balls of the corresponding color reach that threshold first. □

Theorem 2 characterizes the (distribution of) time(s) it takes to make a decision and the choice probabilities as functions of the quorum threshold, given any menu of possible choices. We can, of course, calculate the minimum of a set of random variables, as the theorem requires us to do, but alas there is no simple, closed-form expression for this. Given the lack of a closed-form solution, we use computation to gain additional insight about this process.

Computational Results

To explore the effects of parameter variation and the introduction of noise into the process, we run computational experiments of the proposed mechanism. While the process described above runs in continuous time, we identify discrete time steps every time an agent returns home to recruit (i.e., every time a ball is drawn from the urn). Let the index μ count the number of agents that have returned home for a visit, and denote the time when the μth agent returns home as t_μ. When there are $w = \sum_c w_c^{t_\mu}$ agents exploring the set of possible options, the expected time until the next agent returns home is $\frac{1}{w\lambda}$. Setting $\lambda = 1$, which normalizes the units of time, we have

$$\mathbf{E}[t_{\mu+1} - t_\mu \mid w] = \frac{1}{w}.$$

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We generally set $w^0 = 1$ for simplicity. Each computation reports the average time until decision $T(\tau)$ and the probability of a mistake $p_{\sim c^*}(\tau) = 1 - p_{c^*}(\tau)$ as a function of the quorum threshold τ.[3]

We see the tradeoff between speed and accuracy across varying quorum thresholds by viewing the expected time until decision and the mistake probability as parametric functions of the threshold. Allowing the threshold to vary, we have a Pareto-efficient frontier along which the speed of the decision mechanism cannot be improved without sacrificing accuracy (and vice versa).

Parameter Variation

Increasing the number of possible options C makes for a less accurate decision, but a slightly quicker one as well.[4] More options provide more opportunities for suboptimal options to accumulate a quorum, leading to more mistakes and less decision time. But then, in order to reach the same level of accuracy, the system needs a higher threshold, and this increases the time required to make the decision (as shown in Figure 1). Intuitively, more possible options make for a more difficult decision.

Increasing the quality of the optimal choice, v_{c^*}, makes the decision easier (as shown in the left plot in Figure 2).[5] As the quality of the optimal choice increases, the decision can be made faster and with less chance of error. Recruitment becomes more effective, so the agents accumulate at this option more quickly and the system achieves the quorum sooner.

Increasing the quality of a suboptimal choice, however, does not have such straightforward consequences. It has three effects: 1) it makes the decision process quicker, 2) it increases the

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[3]We use a Monte Carlo method based on 100,000 trials. These results are confirmed with numeric calculations of the exact distributions in those cases where such computations are possible (typically, thresholds up to about twenty).

[4]This is shown in the appendix in Figure S1.

[5]These Pareto frontiers are derived from simulations shown in Figure S2 and Figure S3.

probability of selecting the suboptimal choice, and 3) it lowers the cost of making a suboptimal decision. Thus, the net impact on the ultimate quality of the decision could go in either direction. The right graph in Figure 2 shows that increasing the quality of a suboptimal option hurts the accuracy of the decision that can be made within a given time.[6]

Noise and Risk Aversion

Assuming that each option has some absolute quality helped keep our model analytically tractable, but we can extend the model by allowing the quality, v_c, to be a random variable. At a conceptual level, we might think of the process of search as inherently noisy, as there could be natural variation in agents' perceptions of quality. Alternatively, we might think of the process of recruitment as inherently noisy, with variation in the ability of agents to recruit other agents. In either case, the formal treatment is the same. We can think of an option with noisier quality as riskier.[7]

We compare the attractiveness of risky and safe options (i.e., options with the same expected quality but more or less noise, respectively) in Figure 3, which shows that (for a fixed quorum threshold of 100) the safe option is more likely to be selected than a risky option and it is increasingly preferred to even riskier options. There is nothing special about the threshold of 100, and the result holds for almost all thresholds (possible exceptions being low thresholds that can be reached by a single draw of the risky option, due to recruitment having discrete increments), as shown in Figure S5.[8] Thus, the decision mechanism exhibits a systematic degree

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[6] We have assumed only an ordinal scale for choice quality (stronger recruitment indicates higher quality), so we cannot judge whether the lower cost of selecting the suboptimal choice compensates for this loss in accuracy.

[7] One potential source of noise is in the perception of the quality of options when sampled across multiple attributes. If the individual agents cannot investigate all attributes, they may produce a noisy estimate of overall quality by sampling a single (or a few) attribute(s). We can associate random sampling of attributes with noise in the overall quality of the option. If agents were to specialize in sampling particular attributes and were also more likely to recruit their own type of specialist, then the dynamics of the decision mechanism would be more complicated. Natural systems (e.g. a swarm of bees or a colony of ants) do not (to our knowledge) exhibit such behavior, but it could perhaps arise in human-engineered systems.

[8] We prove in the appendix (Proposition 1) that the probability of selecting a safe option with quality $v_{\text{Safe}} = 1$

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of risk aversion. The intuition behind this result is that the positive feedback in the search and recruitment process allows small advantages to be self-reinforcing, so an option that consistently appears relatively good fares better than one that occasionally appears either great or lackluster. In a world with natural selection, where an entire population can be decimated if a risky choice turns out badly, it may well be adaptive to use a decision mechanism that inherently favors safer choices (*52*). Moreover, our mechanism permits the system's consensus choice to be risk averse even when individual agents are risk neutral. Thus, the system could simultaneously be risk averse for systemic risk and risk neutral for idiosyncratic risk (when consensus choice is not required), which would be evolutionarily adaptive (*53*).

Discovery and Disruption

We can enrich our model by allowing agents to discover the possible options on their own and to be disrupted from search and recruitment by outside forces. We assume that a Poisson process, with intensity β_c, governs the discovery of each possible option c and that each agent has an exponentially distributed lifetime for search and recruitment with hazard rate δ of disruption. With discovery, a natural initial condition is $w^0 = 0$, i.e., the urn is initially empty and agents need to discover an option for recruitment to begin. The index μ for discrete time steps now must count all events, i.e., every time an agent discovers a possible choice or falls prey to disruption, as well as when an agent returns home to recruit. The expected time step becomes

$$\mathbf{E}[t_{\mu+1} - t_\mu \,|\, w] = \frac{1}{w(\lambda + \delta) + \sum_c \beta_c}.$$

Figure 4 shows that decision speed and accuracy is fairly robust to the introduction of option discovery and disruption of search and recruitment. Increasing the rate of discovery (for all options) speeds up the decision and reduces mistakes by limiting the sensitivity to initial

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over a risky option with quality $v_{\text{Risky}} = \left\{0, \frac{R-1}{R}; R, \frac{1}{R}\right\}$, for a quorum threshold of $\tau = R+1$, is $R\,B(R, 1+\frac{1}{R})$ (where B is the Euler beta function), which is an increasing function graphed in Figure S6.

advantage, making it easier for the optimal choice to catch up when it gets discovered later on. Increasing the rate of recruitment also speeds up the decision, but can lead to more mistakes by reinforcing initial advantages (i.e., when a suboptimal option is discovered first). Increasing the rate of disruption slows down the decision, yet it counteracts initial advantages, allowing the optimal option more time to get ahead through stronger recruitment.[9] In all cases, the quorum threshold could be adjusted to efficiently navigate the new speed versus accuracy tradeoff. The net effects, visible in Figure 4, are that increasing the rate of disruption harms the decision, whereas increasing the rate of discovery or the rate of recruitment improves the decision. Thus, even though recruitment introduces positive feedback that can reinforce suboptimal options, it speeds up the decision process enough that at higher thresholds the system can make better, quicker decisions.

Discussion

From ants (*54*) to bees (*46*) to neurons in the brain (*55*), a variety of systems productively employ decentralized-decision mechanisms. Our model of decentralized decision making abstracts beyond any one of these systems and aims to provide a deeper understanding of how such mechanisms behave.

A Polya urn scheme running until it hits a finite threshold parsimoniously captures a decentralized-decision mechanism in which agents gather local information about possible options through search and recruitment with positive feedback, and the system then makes a consensus choice when it detects a quorum in support of a particular option. Analytically, we characterized the waiting time to make a decision and the choice probabilities for any quorum threshold, and we identified an inevitable tradeoff between the speed and the accuracy of the decision. Numerical experiments showed that the system's ability to make reasonably good, quick decisions is ro-

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[9]Figure S7 shows the distinct effects on mistake probability and expected time until decision.

bust to parameter variation, noise, and disruption. Moreover, the computation reveals that the decision mechanism naturally exhibits systemic risk aversion.

Additional assumptions about the cost of waiting and the relative values of possible options would be necessary to evaluate the exact tradeoff between a decision's speed and accuracy. At the extremes, an infinite quorum threshold requires infinite waiting time, and a minimal quorum threshold corresponds to uniformly random choice, so the optimal threshold lies in between. The optimal threshold depends on the particular decision context, and there is some evidence that natural systems tune their thresholds in response to the decision context to make better tradeoffs between speed and accuracy (*17, 56*).

The fact that many natural systems independently evolved similar decentralized-decision mechanisms suggests that such mechanisms may represent a robust solution to the general problem of making good, group-level decisions in the absence of centralized control. The decentralized-decision mechanism we described here may also prove useful in the design of new social and artificial systems. Novel applications range from improving human organizations to applying such techniques to artificial systems like algorithmic search and the control of swarms of robots or networked computers.

Materials and Methods

The source code for the simulation will be made available online.

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Acknowledgments

Figures

Figure 1: Pareto frontiers of mistake probability and expected waiting time with 2 and 4 options. The optimal choice has quality $v_{c^*} = 2$, whereas the suboptimal choices have quality $v_c = 1$ for all $c \neq c^*$. As an artifact of specifying recruitment (i.e., choice quality) so precisely, there are thresholds for which the decision is both slower and less accurate than for a threshold one unit smaller. The corresponding points on the graph are clearly not on the Pareto frontier, but they are shown for completeness.
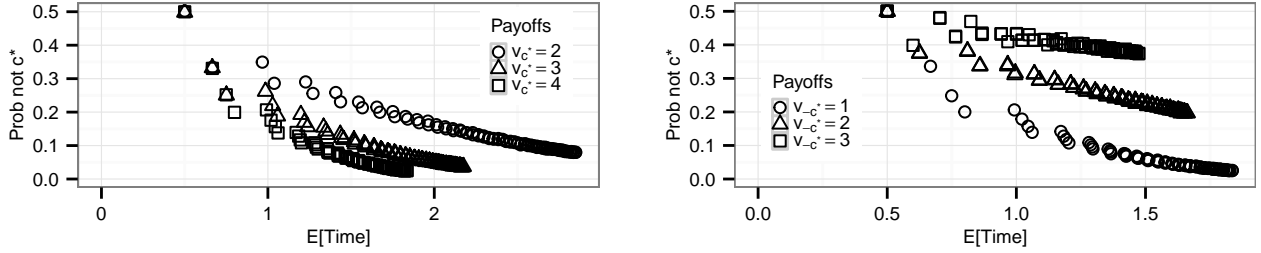
Figure 2: Pareto frontiers of mistake probability and expected waiting time with varying choice quality. Left: Varying optimal choice quality. There are $C = 2$ possible choices, and the quality of the suboptimal choice is $v_{\sim c^*} = 1$. Right: Varying suboptimal choice quality. There are $C = 2$ possible choices, and the quality of the optimal choice is $v_{c^*} = 4$.
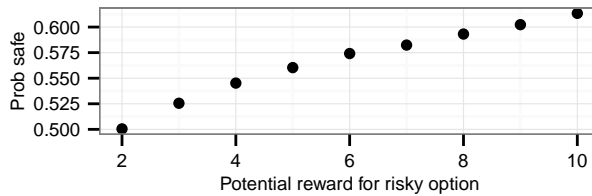
Figure 3: Probability of selecting the safe option over a risky option with the same expected quality, for a fixed quorum threshold of 100. There are $C = 2$ options. The safe option has quality $v_{\text{Safe}} = 2$. The riskiness of the risky option is indexed by the potential reward R such that the quality of the risky option is $v_{\text{Risky}} = \left\{1, \frac{R-2}{R-1}; R, \frac{1}{R-1}\right\}$, i.e., it has expected quality 2 and variance $R - 2$.
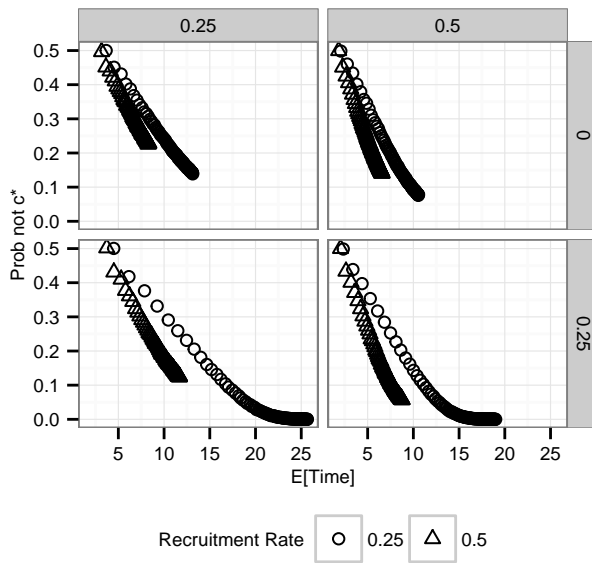
Figure 4: Pareto frontiers of mistake probability and expected waiting time with varying rates of discovery, recruitment, and disruption. There are $C = 2$ options. The optimal choice has quality $v_{c^*} = 2$, and the suboptimal choice has quality $v_{\sim c^*} = 1$. The rate of discovery β_c is the same for both options, and it varies across the columns. The agents' hazard rate of disruption δ varies across the rows. The agents' rate of recruitment λ varies within each panel.

Supplementary Materials
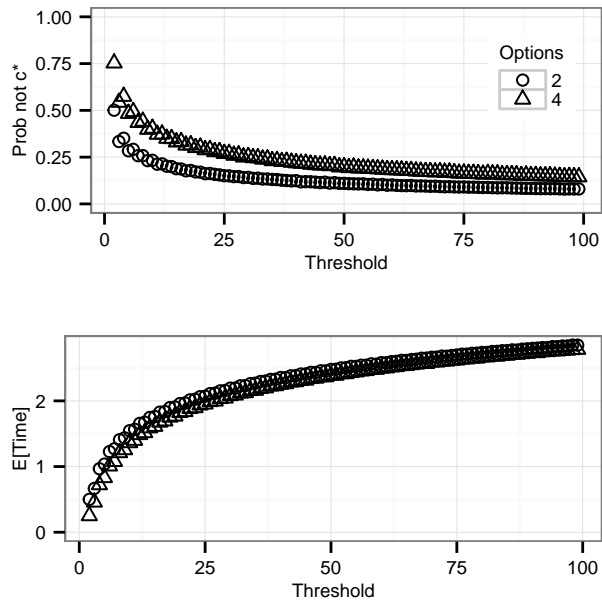
Figure S1: Mistake probability and expected waiting time, as a function of the quorum threshold with 2 and 4 possible options. The optimal option has quality $v_{c^*} = 2$, whereas suboptimal options have quality $v_{\sim c^*} = 1$.
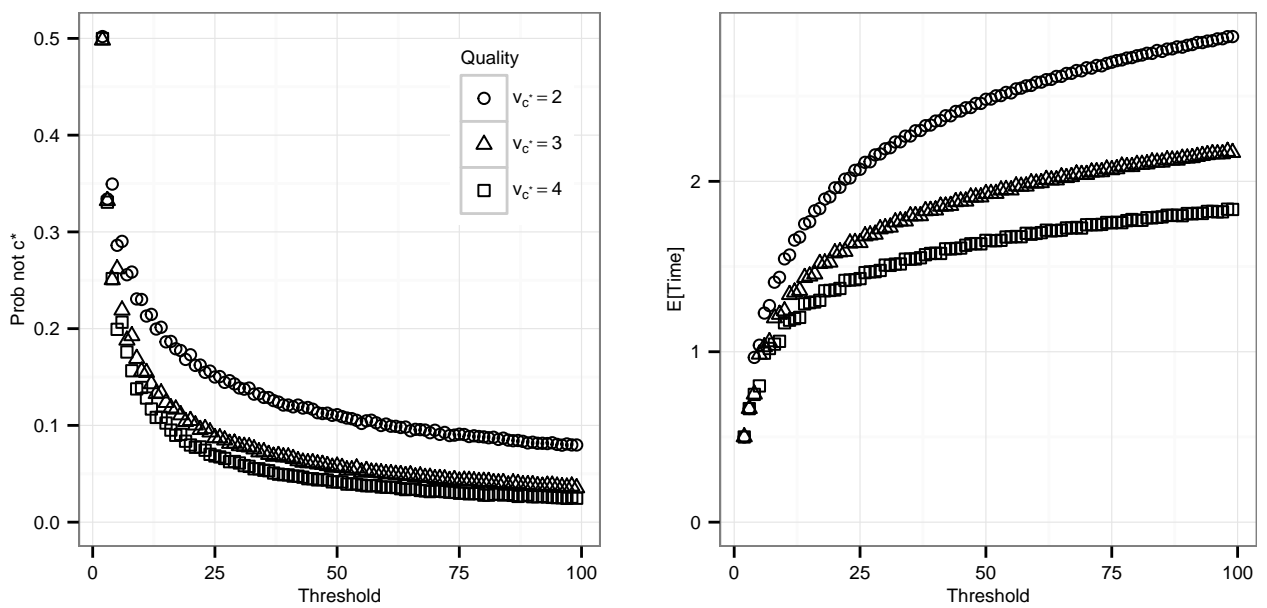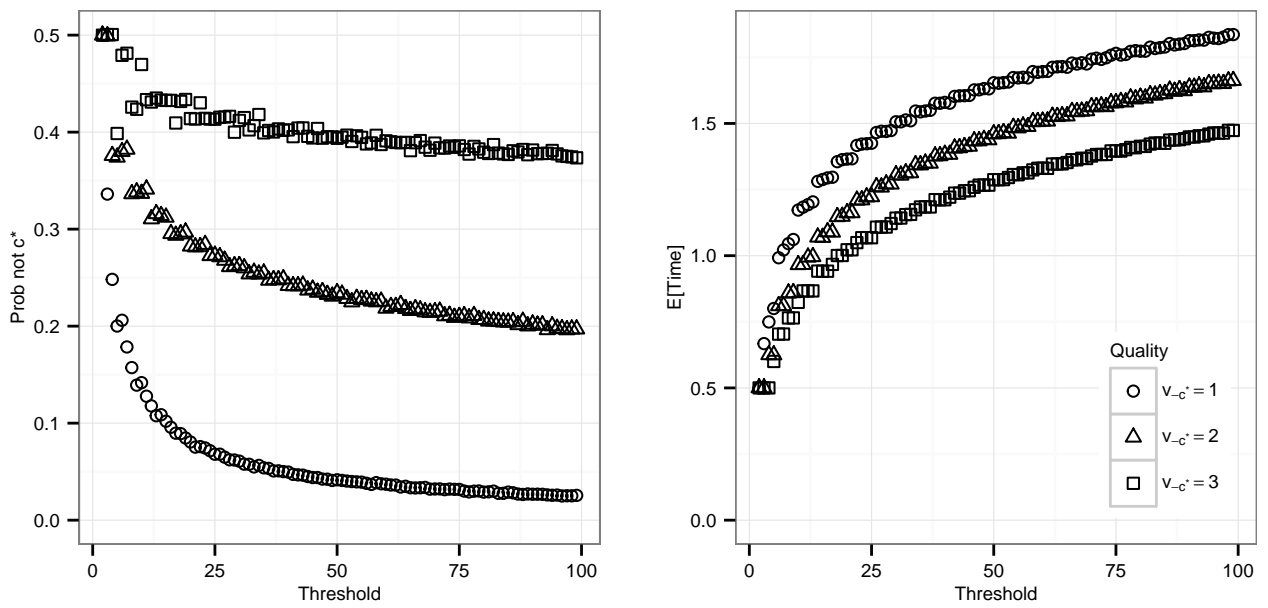
Figure S2: Mistake probability and expected waiting time as a function of the quorum threshold, varying the quality of the optimal choice. There are $C = 3$ options, and the quality of the suboptimal choices are $v_c = 1$ for $c \neq c^*$.
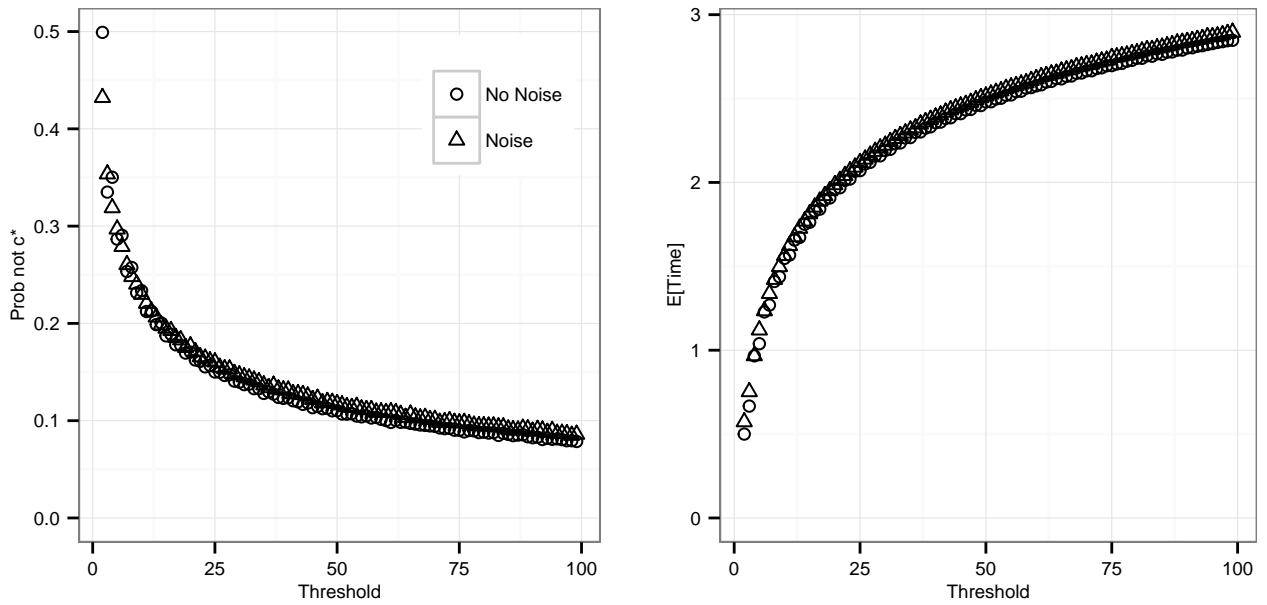
Figure S3: Mistake probability and expected waiting time as a function of the quorum threshold, varying the quality of the suboptimal choice. There are $C = 2$ options and the quality of the optimal choice is $v_{c^*} = 4$.

Figure S4: Mistake probability and expected waiting time as a function of the quorum threshold in noisy and noiseless environments. There are $C = 2$ options. In the noisy condition, $v_{c^*} = \{1, 25\%; 2, 50\%; 3, 25\%\}$ and $v_{\sim c^*} = \{0, 25\%; 1, 50\%; 2, 25\%\}$. In the noiseless condition, $v_{c^*} = 2$ and $v_{\sim c^*} = 1$. The decision mechanism is robust to noise.
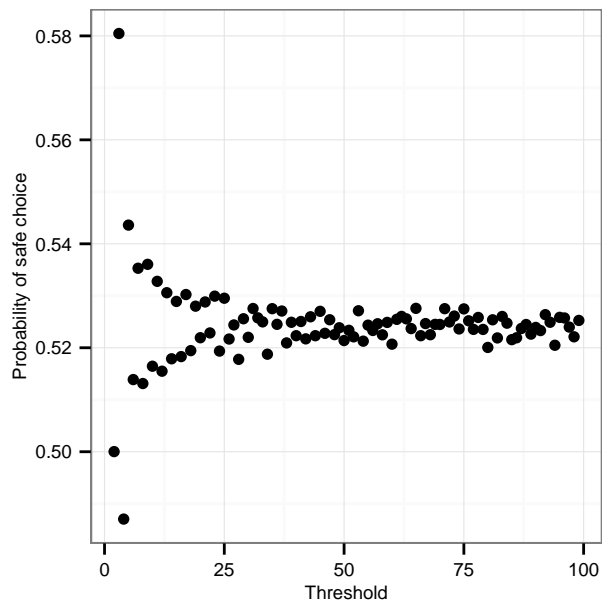
Figure S5: Probability of selecting a safe option with quality $v_{\text{Safe}} = 2$ over a risky one with quality $v_{\text{Risky}} = \{1, 50\%; 3, 50\%\}$, as a function of the quorum threshold.
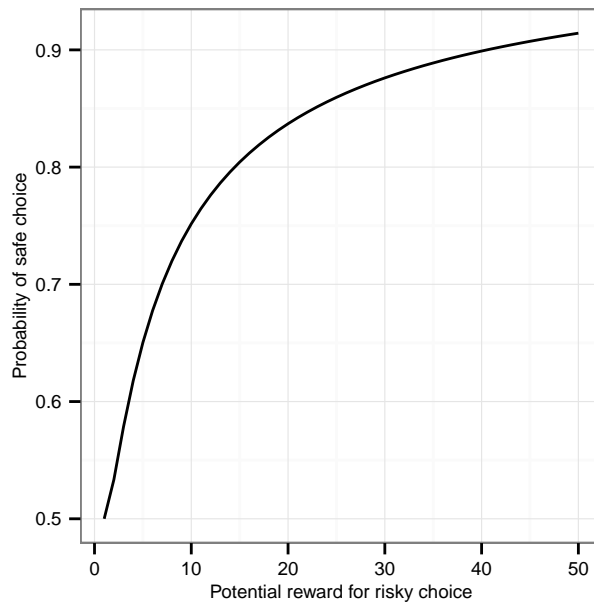
Figure S6: Probability of selecting a safe option with quality $v_{\text{Safe}} = 1$ over a risky one with quality $v_{\text{Risky}} = \left\{ 0, \frac{R-1}{R}; R, \frac{1}{R} \right\}$, for a quorum threshold of $\tau = R + 1$. The exact expression is $R\, B(R, 1 + \frac{1}{R})$, where B is the Euler beta function.
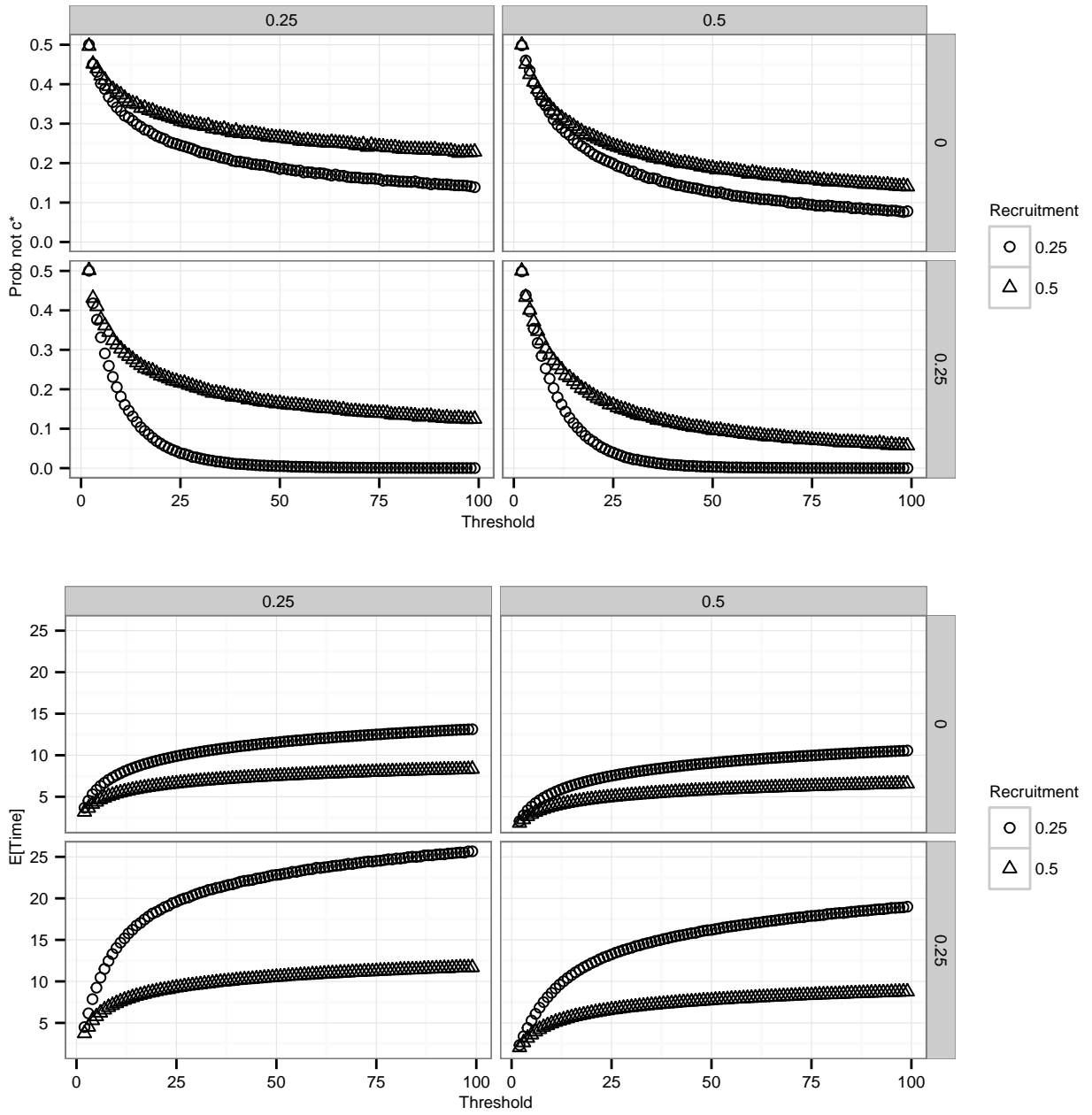
Figure S7: Mistake probability and expected waiting time as a function of the quorum threshold, with varying rates of discovery, recruitment, and disruption. There are $C = 2$ options. The optimal choice has quality $v_{c^*} = 2$, and the suboptimal choice has quality $v_{\sim c^*} = 1$. The rate of discovery β_c is the same for both options, and it varies across the columns. The agents' hazard rate of disruption δ varies across the rows. The agents' rate of recruitment λ varies within each panel.

An Ancillary Result

Proposition 1. *Let there be $C = 2$ possible choices: a safe choice with quality $v_{\text{Safe}} = 1$ and a risky choice with quality $v_{\text{Risky}} = \left\{0, \frac{R-1}{R}; R, \frac{1}{R}\right\}$. For a quorum threshold of $\tau = R + 1$, the probability of selecting the safe choice is $p_{\text{Safe}}(R + 1) = R\, B(R, 1 + \frac{1}{R})$, where B is the Euler beta function.*

Proof. When there are ω agents investigating the safe choice (and just 1 agent investigating the risky choice), the probability that the next recruit is for the safe choice is $\frac{\omega R}{\omega R + 1}$. Thus, the probability of achieving a quorum for the safe choice is

$$p_{\text{Safe}}(R + 1) = \prod_{\omega=1}^{R} \frac{\omega R}{\omega R + 1}$$
$$= \frac{R!}{\prod_{\omega=1}^{R} \omega + \frac{1}{R}} = R\, B(R, 1 + \frac{1}{R}).$$

(We can use an identity expressing the beta function in terms of gamma functions.) □