An Information-Gap Theory of Feelings About Uncertainty

Russell Golman* and George Loewenstein

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Abstract

We propose a theory of feelings about uncertainty that has implications for preferences about acquiring or avoiding information as well as for preferences about exposure to uncertainty (i.e., risk or ambiguity). Our theoretical framework allows utility to depend not just on material payoffs but also on beliefs and the attention devoted to them. We use this framework to introduce the concept of an information gap – a specific uncertainty that one recognizes and is aware of. We characterize a specific utility function that describes feelings about information gaps. We suggest that feelings about information gaps are the source of curiosity as well as a second motive to manage one’s thoughts through information acquisition or avoidance. In addition, we suggest that feelings about information gaps also contribute to risk- and ambiguity preferences.

KEYWORDS: ambiguity, curiosity, information gap, motivated attention, ostrich effect, risk

JEL classification codes: D81, D83

1 Introduction

Thomas Schelling’s characterization of “The Mind as a Consuming Organ” (1987) highlights that most if not all consumption is, in fact, ‘in the mind.’ Schelling’s observation presents a challenge for the revealed preference philosophy so prevalent in economics. We cannot directly observe the objects of preference consumed in the mind. What is the mind consuming (or preferring not to consume) when we observe people succumbing to clickbait on the internet, or skipping a visit to the doctor despite unrelieved symptoms of illness, or gambling on their favorite sports teams after purchasing a high-deductible car insurance policy? Distinct behavioral economic models can describe such patterns of behavior, but economists typically shy away from attempting to

*Department of Social and Decision Sciences, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA. E-mail: rgolman@andrew.cmu.edu
explain preferences. The revealed preference philosophy dictates that preferences are taken as given and theories are meant to offer convenient representations of them. We believe, however, that psychologically grounded assumptions about how people think and feel about uncertainty allow us to develop a unified theory that makes good predictions about when people will obtain or avoid information and about when people will exhibit risk- and ambiguity aversion or lovingness.\(^1\)

The study of decision making under uncertainty grew out of expected utility theory (von Neumann and Morgenstern, 1944; Savage, 1954; Anscombe and Aumann, 1963). Recognizing that people can obtain information to deal with uncertainty, George Stigler in the 1960s used the economics of uncertainty to develop the economics of information. In Stigler’s (1961) account, information is a means to an end; it is valuable because, and only to the extent that, it is useful, i.e., it enables people to make superior decisions that raise their expected utility (cf. Hirshleifer and Riley, 1979).

Of course, people do not generally conform to expected utility theory. Psychologists developed prospect theory to account for violations of expected utility, predicting that the same person may buy insurance and lottery tickets (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), and economists developed more general theories of decision making under uncertainty that allow them to represent a range of possible preferences about risk and ambiguity, still taking such preferences as given (e.g., Gul, 1991; Klibanoff et al., 2005). Non-expected-utility theories led economists to revisit the value of information. Given exposure to some uncertainty, acquiring information about that uncertainty can be seen as resolving a lottery over the remaining uncertainty. The value of this information would be the difference between the utility of the compound lottery and the utility of the original prospect, and outside the bounds of expected utility theory, this value could be negative (Kreps and Porteus, 1978; Wakker, 1988; Grant et al., 1998; Dillenberger, 2010; Andries and Haddad, 2015).

Theories of belief-based utility have recognized that people derive utility not (only) from objective reality but from their beliefs about that reality, e.g., their anticipatory feelings.\(^2\) From this perspective, acquiring information can be seen as resolving a lottery about what the person may come to believe. Risk aversion (lovingness) over beliefs implies that people will want to avoid (obtain) information (Caplin and Leahy, 2001; Köszegi, 2003). Risk aversion over beliefs (and hence information avoidance) could develop when people hold favorable beliefs and don’t want to lose them (e.g., Benabou and Tirole, 2002; Köszegi, 2006) or when people are generally loss averse (e.g., Köszegi, 2010)). However, if risk preferences over beliefs are given as independent of those beliefs, then anticipatory feelings alone do not explain informational preferences that vary

\(^1\)Just as psychology has moved beyond behaviorism and embraced cognition, economics also has much to gain by acknowledging the inner workings of the mind (Bernheim and Rangel, 2009; Chater, 2015; Kimball, 2015).

\(^2\)See also Abelson, 1986; Geanakoplos et al., 1989; Asch et al., 1990; Yariv, 2001.
with prior belief, because with Bayesian updating ex-post beliefs cannot be expected to be better or worse than ex-ante beliefs (Eliaz and Spiegler, 2006).

Belief-based utility theories clearly do make predictions about when people will obtain information and when they will avoid it, and prospect theory clearly does make predictions about when people will seek or steer clear of risk and uncertainty, but some stylized facts still call out for explanation. First, (in contrast to the predictions of Benabou and Tirole (2002) and Köszegi (2006)) information avoidance is more common while holding unfavorable beliefs than while holding favorable beliefs (Lieberman et al., 1997; Fantino and Silberberg, 2010; Karlsson et al., 2009; Eil and Rao, 2011; Ganguly and Tasoff, 2015; Dwyer et al., 2015). Second, (and hard to reconcile with anticipatory feelings) information acquisition occurs in situations in which a person does not care what he finds out, e.g., answers to trivia questions (Berlyne, 1954; 1960). And third, information acquisition or avoidance is highly dependent on situational determinants, such as the presence of clues about the information content or awareness of related uncertainties (Loewenstein, 1994).

Stylized facts in the domain of preference under risk and ambiguity also call out for explanation. People tend to avoid exposing themselves to small monetary risks (Holt and Laury, 2002; Callen et al., 2014). The degree of risk or ambiguity aversion they exhibit depends on contextual factors, such as the presence of other risky or ambiguous options for comparison or for mental accounting (Fox and Tversky, 1995; Gneezy and Potters, 1997). At the same time, people also tend to gamble on uncertainties that they feel they have expertise about (Heath and Tversky, 1991).

Here we propose a unified theory that accounts for these stylized facts. We follow Caplin and Leahy (2001) and Köszegi (2010) in applying expected utility theory to psychological states rather than to physical prizes, but we expand the domain of psychological states that people can have feelings about. We propose a specific utility function that takes as input beliefs and the attention devoted to them (as well as material payoffs). We incorporate Loewenstein’s (1994) insight that specific pieces of missing information –information gaps– stimulate curiosity as well as Tasoff and Madarasz’s (2009) insight that obtaining information stimulates attention and thus complements anticipatory feelings. We define an information gap as a question that a person is aware of (along with a set of possible answers), but uncertain about. To the extent that a person is thinking about (i.e., attending to) an information gap, feelings about this information gap contribute to utility. We explore the implications of our proposed utility model for information acquisition or avoidance in a companion paper (Golman and Loewenstein, 2015). In another companion paper (Golman et al., 2015) we argue that the model developed here provides a new account of risk and ambiguity aversion and seeking. We outline these analyses here.

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3Curiosity correlates with brain activity in regions thought to relate to anticipated reward (Kang et al., 2009), suggesting that information is a reward in and of itself. Similarly, making decisions while aware of missing relevant information correlates with brain activity in regions thought to relate to fear (Hsu et al., 2005).
We introduce our framework in Section 2. We present, and provide psychological motivation for, our assumptions about attention and utility in Section 3. We formally characterize our utility function with seven properties in Section 4. In Section 5.1 we outline the application of our utility model to account for preferences for information acquisition or avoidance, and in Section 5.2 we outline our argument that thoughts and feelings about information gaps can either increase or decrease preference for uncertain gambles depending on whether it is painful or pleasurable to think about the information one is missing.

2 Theoretical Framework

2.1 Cognitive States

Traditional economic theory assumes that utility is a function of consumption bundles or material outcomes, or (perhaps subjective) distributions thereof. Our basic premise is that utility depends not only on such material outcomes but also on one’s cognitive state, encompassing the attention paid to each of the issues or questions that one is aware of as well as subjective judgments about the possible answers to these questions. While people have preferences about their beliefs (and the attention paid to them), we do not treat beliefs (or attention) as choice variables. People can choose whether or not to acquire information that will influence beliefs, but we assume that one’s beliefs, given one’s information, are constrained by Bayesian inference.

While there surely is an infinite set of possible states of the world, we assume, realistically we believe, that a person can only conceive of a finite number of questions at any one time. We represent awareness with an array of ‘activated’ questions and a remaining set of ‘latent’ questions. Activated questions are those that the individual is aware of. Latent questions are those that the individual could become, but is not currently, aware of. The finite subset of questions a person is aware of (i.e., paying at least some attention to) is denoted \( Q \). We label these activated questions as \( Q_1, \ldots, Q_m \). A vector of attention weights \( w = (w_1, \ldots, w_m) \in \mathbb{R}_+^m \) indicates how much attention each activated question gets.\(^4\) These attention weights depend on three factors that we designate “importance,” “salience,” and “surprise.” We return to define and discuss these concepts in Section 3.

A question \( Q_i \) has a countable set\(^5\) of possible (mutually exclusive) answers \( \mathcal{A}_i = \{A^1_i, A^2_i, \ldots\} \).\(^6\) A person may not know the correct answer to a given question, but reasonably has a subjective belief about the probability that each answer is correct. (The subjective probabilities across different questions may well be mutually dependent.) This framework allows us to capture information

\(^4\)We can think of the (presumably infinite) set of latent questions as having attention weights of zero.

\(^5\)We use the term countable here to mean at most countable. The restriction of a countable set of answers to a countable set of possible questions does still allow an uncountable set of possible states of the world, but as awareness is finite, the precise state of the world would be unknowable.

\(^6\)We assume that there is no such thing as an answer that is disconnected from a question.
gaps, which are represented as activated questions lacking known correct answers, as depicted in Table 1.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latent</td>
<td>–</td>
<td>Unawareness</td>
</tr>
<tr>
<td>Activated</td>
<td>Unknown</td>
<td>Uncertainty</td>
</tr>
<tr>
<td></td>
<td>Known</td>
<td>Certainty</td>
</tr>
</tbody>
</table>

\[ \text{information gap} \]

Table 1: The question-answer knowledge structure.

Anticipated material outcomes, or prizes, can also be incorporated into this framework. We let \( X \) denote a countable set of prizes – i.e., material outcomes. The subjective probability over these prizes is in general mutually dependent with the subjective probability over answers to activated questions; that is, the receipt of new information often leads to revised beliefs about the likelihood of answers to many different questions as well as about the likelihood of different material outcomes. Denote the space of answer sets together with prizes as \( \alpha = A_1 \times A_2 \times \cdots \times A_m \times X \). Thus, given a state of awareness defined by the set of activated questions \( Q \), we represent a person’s cognitive state \( C \) with a subjective probability measure \( \pi \) defined over \( \alpha \) (i.e., over possible answers to activated questions as well as eventual prizes) and a vector of attention weights \( w \). We denote the set of all possible cognitive states as \( \mathcal{C} = \Delta(\alpha) \times \mathbb{R}^m_+ \) (with the notation \( \Delta(\alpha) \) referring to the space of probability distributions over \( \alpha \) with finite entropy. The restriction to distributions with finite entropy serves a technical purpose, but it should not trouble us – intuitively, it means that a person cannot be aware of an infinite amount of information, which is also the basis for our assumption that the set of activated questions is finite.). Each marginal distribution \( \pi_i \) specifies the subjective probability of possible answers to question \( Q_i \), and similarly \( \pi_X \) specifies the subjective probability over prizes.

The formal representation of a cognitive state is depicted in Table 2. Consider, for example, a college professor deciding whether or not to look at her teaching ratings. The set of activated questions (and possible answers) might include: “How many of my students liked my teaching?” (0, 1, 2, ...); “Did they applaud on the last day of class?” (yes/no); “How good a teacher am I?” (great, good, so-so, bad, awful); “Will I get tenure?” (yes/no). Prior belief about the first question might be quite uncertain. The answer to the second question, on the other hand, might already be known with certainty. There may or may not be much uncertainty about the third and fourth questions. All of these beliefs (to the extent they are uncertain) are jointly dependent. The material outcome might be next year’s salary, which would also depend on (but not be completely

\(^7\)In most cases, we will assume that activation of questions is determined exogenously – i.e., by the environment. We don’t model growing awareness (see Karni and Vierø, 2013).

\(^8\)For any \( \tilde{A} \subseteq A_i \), we have \( \pi_i(\tilde{A}) = \pi(A_1 \times \cdots \times A_{i-1} \times \tilde{A} \times A_{i+1} \times \cdots \times A_m \times X) \).
Table 2: Representation of a cognitive state.

<table>
<thead>
<tr>
<th>Activated Questions</th>
<th>Possible Answers</th>
<th>Subjective Probabilities*</th>
<th>Attention Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td>$A_1 = {A_1^1, A_1^2, \ldots}$</td>
<td>$[\pi_1(A_1^1), \pi_1(A_1^2), \ldots]$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>$A_m = {A_m^1, A_m^2, \ldots}$</td>
<td>$[\pi_m(A_m^1), \pi_m(A_m^2), \ldots]$</td>
<td>$w_m$</td>
</tr>
<tr>
<td>Possible Prizes</td>
<td>$X = {x, x', x'', \ldots}$</td>
<td>$[\pi_X(x), \pi_X(x'), \ldots]$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Answers to different questions are not generally independent. Typically, the joint probability measure $\pi \neq \pi_1 \cdots \pi_m \cdot \pi_X$.
We assume Bayesian updating here, which means that ex ante, before one knows what one will discover, an informational action determines a distribution over subjective judgments such that the expectation of this distribution equals the prior judgment. That is, by the law of total probability, \( \sum_{A_i \in A_i} \pi_i^0(A_i) \pi_i^{A_i} = \pi_i^0 \). An informational action would decrease expected entropy because conditioning reduces entropy (see, e.g., Cover and Thomas, 1991, pg. 27). New information generates surprise (as formalized in the next section), which changes the attention weights too. Given the prior attention weight vector \( w^0 \) based on salience and importance, we let \( w^{A_i} \) denote the new attention weight vector immediately after learning \( A_i \), resulting from surprise at this discovery.

### 2.3 Preferences over (Distributions of) Cognitive States

The conventional theory of choice under risk assumes that a lottery over outcomes is evaluated according to its expected utility. Given that we may think of an informational action as creating a lottery over cognitive states, we make the natural assumptions leading to an expected utility representation in this new domain.

#### Independence Across Cognitive States

We assume that there is a complete and transitive preference relation \( \succeq \) on \( \Delta(C) \) that is continuous (with respect to an appropriate topology)\(^\dagger\) and that satisfies independence, so there exists a continuous expected utility representation \( u \) of \( \succeq \) (von Neumann and Morgenstern, 1944).

The assumption here is that when information could put a person into one of many possible cognitive states, preference is consistent with valuing each possible cognitive state independently of any other cognitive states the person might have found herself in.

This might seem to imply that the utility of a state of uncertain knowledge is equal to the expected utility of each of the possible beliefs – e.g., that being uncertain of whether the object of my desire reciprocates my affections provides the same utility as the sum of probabilities times the utilities associated with the possible outcome belief states. It need not, because (as we discuss in detail below) obtaining the information, and indeed the specific information one obtains, is likely to affect one’s attention weights. Such a change in attention can encourage or discourage a decision maker from resolving uncertainty, depending on whether the news that will be revealed is expected to be good or bad.

### 2.4 Choosing Between Sequences of Actions

The discovery of information following an initial action can change the availability or desirability of subsequent actions. For example, the information in a college professor’s teaching ratings could

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\( ^9 \) We thus denote a belief with complete certainty in \( A \times x \) as \( \pi^{A \times x} \).

\( ^\dagger \) The induced topology on \( C \) (derived from the order topology on \( \Delta(C) \)) should be a refinement of the order topology on \( C \) (see Nielsen, 1984).
help her decide whether to enroll in a teacher improvement class. A sequence of actions can be
analyzed with the convention that an action operator passes through a distribution over cognitive
states.\textsuperscript{11} Thus, we represent a sequence of actions $s$ acting on a cognitive state $(\pi, w)$ as $s(\pi, w) \in \Delta(C)$.

Choice from among a set of sequences of actions $S$, where early actions may reveal information
that will inform later actions, is represented as utility maximization: a sequence $s^* \in S$ may be
chosen by a decision maker in the cognitive state $(\pi, w)$ if $s^* \in \arg \max_{s \in S} u(s \cdot (\pi, w))$. We find
it useful to define a utility function over cognitive states, contingent on the set of sequences of
actions that may subsequently be chosen:

$$U(\pi, w | S) = \max_{s \in S} u(s \cdot (\pi, w)). \quad (1)$$

In the example of the professor’s teaching ratings, the set of available subsequent actions is to enroll
in the teacher improvement class or not to enroll in the class. Looking at the ratings resolves a
lottery over cognitive states, each of which having utility that is conditional on making the optimal
choice of one of these subsequent actions.

We define the desirability of a sequence of actions $s$ in cognitive state $(\pi, w)$ as

$$D(s | \pi, w) = u(s \cdot (\pi, w)) - u(\pi, w).$$\textsuperscript{12}

The desirability of taking a course of action is therefore the marginal utility relative to the trivial
‘action’ of doing nothing (not changing the cognitive state).

3 Psychological Insights

In this section we introduce a number of specific psychological insights that lead us to specify a
utility function that generates a wide range of testable predictions concerning informational phe-
nomena. These insights help us characterize the factors that influence the level of attention paid to
a question as well as to identify distinctly the valence of beliefs and the desire for clarity.

3.1 Attention

Neuroeconomic research indicates that attention shapes preference (Fehr and Rangel, 2011). At-
tention weights in our model specify how much a person is thinking about particular beliefs and,
in turn, how much those beliefs directly impact utility. We may think of beliefs as having intrinsic
value, which is then amplified by these attention weights. Our model (assuming monotonicity with

\textsuperscript{11}Analogous to the standard assumption in decision under risk, the model assumes reduction of compound distribu-
tions over cognitive states. This does not imply the traditional reduction of compound lotteries.

\textsuperscript{12}The degenerate distributions in $\Delta(C)$ correspond to individual states of knowledge. With the standard abuse of
notation, we refer to the utility of the degenerate distribution on $(\pi, w) \in C$ as $u(\pi, w)$.\textsuperscript{8}
respect to attention weights, as described in the appendix) provides a natural distinction between beliefs that have positive or negative intrinsic value: beliefs are positive specifically when more attention enhances utility and are negative in the opposite case. That is, a person likes thinking about (i.e., putting more attention weight on) positive beliefs and does not like thinking about negative beliefs.

Here we formalize the concepts of importance, salience, and surprise, all of which, we assume, contribute to attention weight. The importance $\gamma_i$ of a question $Q_i$ reflects the degree to which one’s utility depends on the answer. Thus, for example, for an egocentric, but insecure, individual, the question, “Do other people like me?” is likely to be of great importance because the answer matters to the individual. Salience, distinctly, reflects the degree to which a particular context highlights the question. If, for example, an individual hears that another person was talking about her (with no further details), the question of whether the comments were favorable or not will become highly salient. We denote the salience of question $Q_i$ as $\sigma_i \in \mathbb{R}_+$. Finally, surprise is a factor that reflects the dependence of attention on the dynamics of information revelation, and specifically on the degree to which receiving new information changes one’s beliefs. If, having believed that she was generally well-liked, our individual were to discover that the comments about her were actually unfavorable, the discovery, necessitating a radical change in her belief, would be quite surprising (and, as we presently assume, would increase her attention to the question). We denote the surprise associated with a revised belief about question $Q_i$ as $\delta_i$. We assume that the attention $w_i$ on an activated question $Q_i$ is a strictly increasing function of this question’s importance $\gamma_i$, its salience $\sigma_i$, and the surprise $\delta_i$ associated with it.

**Importance**

The importance of a question depends on the spread of the utilities associated with the different answers to that question. The degree to which an individual’s utility varies with the answers to a question depends both on the magnitude of the utility function and on the perceived likelihood of different answers. Continuing with the example of the question of how well-liked an individual is, one could distinguish two relevant traits: egocentrism – the degree to which the individual cares about being well-liked; and insecurity – the dispersion of the individual’s subjective probability distribution across possible answers. By our definition of the concept, importance should be positively related to both properties.

Given a particular prior subjective probability measure $\pi^0$ and a set $S$ of sequences of actions available to the decision maker, the importance $\gamma_i$ of question $Q_i$ is a function (only) of the likelihood of possible answers and the utilities associated with these answers, captured as

$$\gamma_i = \phi \left( \left\langle \pi_i^0(A_i), \ U^i(A_i, |S) \right\rangle \right)_{A_i \in \text{supp}(\pi_i^0)}$$
where $U$ is the utility function defined in Equation (1). Without specifying the precise form of this function $\phi$, we assume only that it (i.e., importance) increases with mean-preserving spreads of the (subjective) distribution of utilities that would result from different answers to the question, and that it is invariant with respect to constant shifts of utility. Thus, a question is important to the extent that one’s utility depends on the answer. Raising the stakes increases importance. On the other hand, if an answer is known with certainty, then by this definition nothing is at stake, so the underlying question is no longer important. While acquiring information will affect the importance of the questions being addressed, it takes time to adapt to news, so there should be some delay. We assume that the importance of a question is updated only when the new information is incorporated into a new default subjective probability measure.

Our definition of importance is, admittedly, circular. Importance depends on utility, which in turn depends on the attention weight, but importance also contributes to attention weight. There is, likely, some psychological realism to this circularity which captures the dynamic processes giving rise to obsession: attention to a question raises its importance, and the elevated importance gives rise to intensified attention. If we assume that these processes unfold instantaneously, then importance (and, in turn, attention weight and utility) will be a fixed point of this composition of functions. We can make simple comparisons of importance without going to the trouble of specifying precise values.

Salience

The salience of a question depends on a variety of exogenous contextual factors. For example, a question could be salient if it has recently come up in conversation (i.e., it has been primed) or if other aspects of the environment remind an individual about it. Alternatively, a question could be more salient to an individual if the answer is, in principle, knowable, and even more so if other people around her know the answer but she does not. Comparison and contrast generally increase a question’s salience (Itti and Koch, 2001).

Often a question may be salient despite being unimportant. Continuing the prior example, even if an individual deems others’ perceptions of her as unimportant, the question of her popularity might nonetheless be highly salient if the individual was asked, “Do you know what $x$ thinks of you?” Conversely, there are myriad questions that are important by the definition just provided, but which lack salience. There might be numerous people whose opinion of us we would care about and be unsure of, but unless something raises the issue in our mind, we are unlikely to focus on it. It seems natural to think that some degree of salience is a necessary, and sufficient, condition for attention (while some degree of importance is not). Thus, we assume that a question $Q_i$ is activated (i.e., has strictly positive attention weight $w_i > 0$) if and only if it has positive salience $\sigma_i > 0$. Further, we assume that attention weight $w_i$ has strictly increasing differences (i.e., a
positive cross-partial derivative, if we assume differentiability) in \((\gamma_i, \sigma_i)\). That is, an increase in importance produces a greater increase in attention for a more salient question.

**Surprise**

The third factor that we posit influences attention is the surprise one experiences upon acquiring new information. Surprise reflects the degree to which new information changes existing beliefs. A natural measure of surprise was proposed in a theoretical paper by Baldi (2002) and, in an empirical follow-up investigation (Itti and Baldi, 2009), shown to predict the level of attention paid to information. Incorporating the insights from this line of research, we assume that when the answer to a particular question \(Q_j\) is learned, thereby contributing information about the answers to associated questions and causing their subjective probabilities to be updated, the degree of surprise associated with a new belief about question \(Q_i\) can be defined as the Kullback-Leibler divergence of \(\pi_i^{A_j}\) against the prior \(\pi_i^0\):

\[
\delta_i(\pi_i^{A_j} || \pi_i^0) = \sum_{A_i \in A_i} \pi_i^{A_j}(A_i) \log \frac{\pi_i^{A_j}(A_i)}{\pi_i^0(A_i)}.
\]

Surprise is positive with any new information, and is greatest when one learns the most unexpected answer with certainty. However, the feeling of surprise is not permanent. We assume that when the decision maker adapts and gets used to this new knowledge (formally, when the default subjective probability measure is reset), it is no longer surprising.

**The Belief Resolution Effect**

The impact of new information on attention is greatest when uncertainty about a question is resolved completely. Surprise immediately spikes, but in the long run fades, and the underlying question becomes unimportant because, with the answer known, there is no longer a range of possible answers. Taken together, these factors create a pattern of change in attention weight following the discovery of a definitive answer, what we call the belief resolution effect – when an answer is learned with certainty, there is an immediate boost in attention weight on it, but over time this attention weight falls to a lower level. Specifically, when the decision maker adapts and the certain belief is incorporated into the default subjective probability measure, the question then receives less attention. It is as if the brain recognizes that because a question has been answered, it can move on to other questions that have yet to be addressed. Janis (1958) recognized the belief resolution effect when he observed that surgical patients getting information about their upcoming procedures initially worry more about the surgery but subsequently experience less anxiety. The belief resolution effect allows for hedonic adaptation to good or bad news (Wilson et al., 2005; Smith et al., 2009).
3.2 Valence and Clarity

It is useful to distinguish two sources of a belief’s intrinsic value: valence and clarity. Valence refers to the value of definitive answers to questions (see Brendl and Higgins, 1996). To illustrate the concept of valence, we return to the example of a professor’s belief that she is a good (or bad) teacher as one with intrinsically positive (or, respectively, negative) valence. Smith et al. (2014) shows that valences (of beliefs about consumption) can be inferred from neural activity. Clarity refers to preferences between degrees of certainty, independent of the answers one is certain of (see Kaplan, 1991). We assume that, ceteris paribus, people prefer to have greater clarity (i.e., less uncertainty or more definitive subjective beliefs). The aversion that people feel towards uncertainty is reflected in neural responses in the anterior cingulate cortex, the insula and the amygdala (Hirsh and Inzlicht, 2008; Sarinopoulos et al., 2010). It manifests in physiological responses as well. Subjects who know to expect an electric shock, but who are uncertain whether it will be mild or intense, show more fear – they sweat more profusely, and their hearts beat faster – than subjects who know for sure that an intense shock awaits (Arntz et al., 1992). The desire for clarity is consistent with an underlying drive for simplicity and sense-making (Chater and Loewenstein, 2015).

When valence and clarity pull in opposite directions, it may be the case that people prefer a certain answer to a subjective belief that dominates it on valence or that people prefer uncertainty when it leaves space for better answers. While the preference for clarity violates Savage’s (1954) sure-thing principle, we do assume a weaker version of it:

**One-Sided Sure-Thing Principle**

For any \( \pi \in \Delta(\alpha) \), let \( \text{supp}(\pi) \subseteq \alpha \) denote the support of \( \pi \). If for all \( A \times x \in \text{supp}(\pi) \) we have \( u(\pi', w) \geq u(\pi^{A \times x}, w) \), then \( u(\pi', w) \geq u(\pi, w) \), with the latter inequality strict whenever there exist \( A' \times x' \) and \( A'' \times x'' \in \text{supp}(\pi) \) such that \( A' \neq A'' \).

The one-sided sure-thing principle asserts that people always prefer a certain answer to uncertainty amongst answers that all have valences no better than the certain answer (holding attention weight constant).

**A Measure of Uncertainty**

The assumption of a preference for clarity means that there is a preference for less uncertain subjective beliefs. A useful measure of the uncertainty about a particular question is the entropy of the subjective probability distribution over answers (Shannon 1948). The entropy of a subjective (marginal) probability \( \pi_i \) is \( H(\pi_i) = - \sum_{A_i \in \mathcal{A}_i} \pi_i(A_i) \log \pi_i(A_i) \) (with the convention that \( 0 \log 0 = 0 \)).\(^{13}\) At one extreme with maximal entropy, there are many equally likely possible

\(^{13}\)The base of the logarithm in the entropy formula is unrestricted and amounts to a normalization parameter.
answers; at the other extreme with minimal entropy 0, there is a single answer known for sure. Entropy, weighted by attention, satisfies Berlyne’s (1957) criteria for a measure of the internal conflict or dissonance in one’s cognitive state. We associate a psychological cost for beliefs with higher entropy as an instantiation of the desire for clarity.

### 3.3 A Specific Utility Function

To make precise predictions about preferences for (or to avoid) information, we consider a specific utility function incorporating the preference for clarity and the role of attention weights:

\[
u(\pi, w) = \sum_{x \in X} \pi_X(x)v_X(x) + \sum_{i=1}^m w_i \left( \sum_{A_i \in A_i} \pi_i(A_i)v_i(A_i) - H(\pi_i) \right).
\]

We represent the value of prize \(x\) as \(v_X(x)\) and the valence of answer \(A_i\) as \(v_i(A_i)\). We now describe properties (some quite strong and almost certainly not always satisfied) that characterize (and necessarily imply) this utility function (see Theorem 1 below).

### 4 Characterization of the Utility Function

#### 4.1 Properties

The utility function in Equation (2) satisfies the following seven properties.

**Independence Across Prizes**

In Section 2 we assumed independence across cognitive states. Independence might extend, as in traditional models, to material outcomes, holding beliefs constant.

**P1** Holding the rest of the cognitive state constant, the preference relation satisfies independence across prizes if \(u(\pi^A, w) = \sum_{x \in X} \pi_X^A(x)u(\pi^A \times x, w)\).

Property (P1) implies belief-dependent expected utility over lotteries that are independent of beliefs about the world. If we also were to assume belief-independent utility for prizes, then we would gain the ability to reduce compound lotteries consisting of horse races as well as roulette lotteries (Anscombe and Aumann, 1963) to single-stage lotteries. However, we believe it is often the case that utility is belief-dependent. We might say that a decision maker often has a horse in the race.

**Separability Between Questions**

Additive separability of utility between questions means that a person can place a value on a belief about a given question without needing to consider beliefs about other questions.

**P2** A utility function satisfies additive separability between questions if \(u(\pi, w) = u_X(\pi_X) + \sum_{i=1}^m u_i(\pi_i, w_i)\).\(^{14}\)

\(^{14}\)A subset of questions \(\tilde{Q} \subset Q\) can also be separable, in which case \(u(\pi, w) = \sum_{i: Q_i \in \tilde{Q}} u_i(\pi_i, w_i) + \)
Property (P2) may seem quite strong because we can imagine representations of sensible preferences that are not additively separable. For example, the value of a belief about whether a car on sale has a warranty intuitively could depend on the cost of the car in the first place (not to mention one’s desire for a new car, one’s estimation of the costs of car repairs, etc.). However, we may be able to represent these preferences as separable after all. We might suppose that these beliefs do have separable values but that they correlate with some other highly valued belief, perhaps about how good a deal one can get on the car. That is, while intuition tells us that the value of beliefs about different questions (e.g., “does she like me?” and “does she have a boyfriend?”) is often interdependent, this dependence may be mediated by the existence of additional questions (e.g., “will she go out with me?”), beliefs about which may be mutually dependent, but independently valued.

Monotonicity with respect to Attention Weights

Preferences satisfy the property of monotonicity with respect to attention weights if whenever increasing attention on a given belief enhances (or diminishes) utility, it will do so regardless of the absolute level of attention weight. At a psychological level, the interpretation of this monotonicity property is that when a belief is positive, more attention to it is always better, and when a belief is negative, more attention is always worse. In fact, the property provides a natural definition of whether a belief is positive or negative.

P3 Preferences satisfy monotonicity with respect to attention weights if for any $w, \hat{w}, \tilde{w} \in \mathbb{R}_+^n$ such that $w_i = \hat{w}_i = \tilde{w}_i$ for all $i \neq j$ and $\hat{w}_j > \tilde{w}_j > w_j$, we have $u(\pi, \hat{w}) \geq u(\pi, w)$ if and only if $u(\pi, \tilde{w}) \geq u(\pi, \hat{w})$, with equality on one side implying equality on the other, for all $\pi \in \Delta(\alpha)$.

In the case that these inequalities hold strictly, we say that $\pi_j$, the belief about question $Q_j$, is a positive belief. If they hold as equalities, we say that $\pi_j$ is a neutral belief. And, in the case that the inequalities hold in the reverse direction, then $\pi_j$ is a negative belief.

Linearity with respect to Attention Weights

The next property describes how changing the attention on a belief impacts utility. For any given attention weight, the marginal utility of a change in belief depends on what those beliefs are and how much the individual values them. The property of linearity with respect to attention weights means that, in general, the marginal utility associated with such a change in belief (assuming the utility of this belief is separable) is proportional to the attention on that belief.

P4 When the utility of question $Q_i$ is separable, linearity with respect to attention weights is

$u_{-Q}(\pi_{-Q}, w_{-Q})$ where $\pi_{-Q}$ is the marginal distribution over answers to the remaining questions and prizes and the vector $w_{-Q}$ contains the remaining components of $w$. 

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satisfied if for any \( w_i \) and \( \tilde{w}_i \in \mathbb{R}_+ \) and \( \pi'_i \) and \( \pi''_i \in \Delta(A_i) \), we have

\[
u_i(\pi'_i, \tilde{w}_i) - \nu_i(\pi''_i, w_i) = \frac{\tilde{w}_i}{w_i}(u_i(\pi'_i, w_i) - u_i(\pi''_i, w_i)).\]

Property (P4) allows us, in the case of separable utility, to assign an intrinsic value \( v \) to beliefs such that \( u_i(\pi'_i, w_i) - u_i(\pi''_i, w_i) = w_i(v_i(\pi'_i) - v_i(\pi''_i)) \). We abuse notation by referring to the valence of answer \( A_i \) as \( v_i(A_i) \), with it being defined here as the intrinsic value \( v_i \) of belief with certainty in \( A_i \). We have taken the liberty of specifying a precise relationship between attention weights and utility as a convenient simplification; it should be noncontroversial because we do not claim to have a cardinal measure of attention weight.

**Label Independence**

Intuitively, the value of a belief should depend on how an individual values the possible answers and on how probable each of these answers is, and these factors (controlling for attention weight of course) should be sufficient to determine the utility of any (uncertain) belief. In particular, the value of a belief should not depend on how the question or the answers are labeled.

**P5** Label independence is satisfied if, when the utility of questions \( Q_i \) and \( Q_j \) are separable, a bijection \( \tau : A_i \rightarrow A_j \), such that \( v_i(A_i) = v_j(\tau(A_i)) \) and \( \pi_i(A_i) = \pi_j(\tau(A_i)) \), implies that \( v_i(\pi_i) = v_j(\pi_j) \).

**Reduction of Compound Questions**

The intuition behind the assumption of label independence also seems to suggest that the utility of a belief perhaps should not depend on the way the question giving rise to the belief is asked, i.e., on whether a complicated question is broken up into pieces. We should recall, however, that the activation of a particular question directs attention to the belief about this question. Thus, in general, the utility of a belief will not be invariant to the question being asked. Still, it may be the case that utility remains invariant when a compound question is broken into parts as long as the attention on each part is weighted properly. If utility remains invariant upon setting attention weights on conditional questions to be proportional to the subjective probabilities of the hypothetical conditions, then we say that the utility function satisfies the reduction of compound questions property. Figure 1 demonstrates the reduction of a compound question with appropriate attention weights on each subquestion.

**P6** A separable utility function satisfies the reduction of compound questions property if whenever there is a partition \( \zeta \) of the answers \( A_i \) (to question \( Q_i \)) into \( \zeta = \{A_{i_1}, \ldots, A_{i_n}\} \) and a bijection \( \tau : \zeta \rightarrow A_j \) into the answers to some question \( Q_j \) such that for any \( h \in [1, n] \) and any \( A_i \in A_{i_h} \),

\[
v_i(A_i) = v_j(\tau(A_{i_h})) + v_{i_h}(A_i)\]

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Ruling Out Unlikely Answers Increases Clarity

A final property operationalizes the preference for clarity. Controlling for the valence of one’s beliefs, by considering situations in which one is indifferent between different possible answers to a question, there should be a universal aversion to being uncertain about the answer to an activated question. As a building block toward quantifying the uncertainty in a subjective belief, we assert here that when an unlikely (and equally attractive) answer is ruled out, uncertainty decreases (and thus the utility of that uncertain belief increases).

P7 Ruling out unlikely answers increases clarity if, when the utility of question $Q_i$ is separable and all answers to this question have the same valence, i.e. $v_i(A_i) = v_i(A'_i)$ for all $A_i$ and $A'_i \in A_i$, then for any $\pi$ where without loss of generality $\pi_i(A^h_i)$ is weakly decreasing in $h$ and for any $\pi'$ such that $\pi'_i(A^h_i) \geq \pi_i(A^h_i)$ for all $h \in [1, \bar{h}]$ (with at least one inequality strict) and $\pi'_i(A^\bar{h}i) = 0$ for all $h > \bar{h}$, for some $\bar{h}$, we consequently have $v_i(\pi'_i) > v_i(\pi_i)$.

4.2 Utility Representation Theorem

Theorem 1 If the properties P1-P7 are satisfied, then

$$u(\pi, w) = \sum_{x \in X} \pi_X(x)v_X(x) + \sum_{i=1}^m w_i \left( \sum_{A_i \in A_i} \pi_i(A_i)v_i(A_i) - H(\pi_i) \right).$$

Proof Linearity with respect to attention weights allows us to pull an attention weight on question $Q_i$ outside of the utility $u_i(\pi_i, w_i) = w_i v_i(\pi_i)$ (using a neutral belief to calibrate $v_i$). A partition of
\[ A_i \] into singletons \( A_{i_k} \) such that \( v_i(A_i) = v_{i_k}(A_i) \) allows us, by reduction of the compound question, to determine that the function \( F(\pi_i) = v_i(\pi_i) - \sum_{A_i \in A_i} \pi_i(A_i)v_i(A_i) \) does not depend on \( v_i(A_i) \) for any \( A_i \in A_i \). Moreover, \(-F(\cdot)\) satisfies Shannon's (1948) axioms (continuity, increasing in the number of equiprobable answers, and reduction of compound questions) characterizing the entropy function \( H(\pi_i) = -\sum_{A_i \in A_i} \pi_i(A_i) \log \pi_i(A_i) \).

5 Phenomena

5.1 Information Acquisition and Avoidance

We can apply our utility function to decisions about information acquisition or avoidance. We develop our analysis in a companion paper (Golman and Loewenstein, 2015), and we provide a broad outline here of its implications. The desire for information, in our model, can be decomposed into three distinct motives: recognition of the instrumental value of the information; curiosity to fill the information gap(s); and motivated attention to think more or less about what could be discovered. The instrumental value of information arises from its impact on subsequent actions. As in the standard account of informational preferences, it is defined as the difference between the expected utility of subsequent actions conditional on having the information and the utility expected in the absence of the information. Curiosity arises from the expected reduction in uncertainty upon acquiring information. It is defined as the expected utility of revised beliefs, given prior levels of attention. The magnitude of curiosity depends on the attention devoted to each information gap that stands to be addressed. Motivated attention arises from the surprise upon acquiring information. It is defined as the expected utility from increased attention on whatever happens to be discovered, conditioning on all possible outcomes. Motivated attention is a motive to acquire information that’s expected to be good and to avoid information that’s expected to be bad.

Putting the three motives together, our model makes many predictions about when, and the degree to which, information will be sought or avoided. When anticipated answers are neutral or even potentially positive, information should be sought. The strength of the desire for this information should increase with the number of attention gaps that can be addressed, the attention paid to them, and the valence of the possible outcomes. However, when anticipated outcomes are sufficiently negative, information would be avoided. This “ostrich effect” when anticipating bad outcomes is consistent with a growing body of empirical evidence (see, e.g., Karlsson et al., 2009; Eil and Rao, 2011). In addition, the belief-resolution effect in our model leads to a novel prediction: individuals who discount the future less should be less likely to exhibit the ostrich effect and more likely to acquire information despite anticipated bad news.
5.2 Risk and Ambiguity Preference

Section 5.1 outlines how the model we have developed allows us to describe a desire to acquire or to avoid information that encompasses motives (namely, curiosity and motivated attention) that have been largely disregarded in the economics literature. We can apply this same model to an entirely new domain: preferences about wagers that depend on missing information. Risk and ambiguity aversion are complex topics, and we develop these applications in depth in a companion paper (Golman et al., 2015). Here, we provide a broad outline of how the model can be applied in these domains.

Decision making under risk and under ambiguity both expose decision makers to information gaps. Imagine a choice between a gamble and a sure thing. Deciding to play the gamble naturally focuses attention on the question: what will be the outcome of the gamble? Of course, deciding to not play the gamble does not stop an individual from paying some attention to the same question (or, if not choosing the gamble means it will not be played out, the related question: what would have been the outcome of the gamble?) but playing the gamble makes the question more important, and that brings about an increase in the attention weight on the question. If the individual is aware of this effect, which it seems natural to assume, then whether it encourages risk taking or risk aversion will depend on a second factor: whether thinking about the information gap is pleasurable or aversive. When thinking about the missing information is pleasurable, then the individual will be motivated to increase attention on the question, which entails betting on it. Conversely, when thinking about the missing information is aversive, the individual will prefer to not bet on it. This may help to explain why, for example, people generally prefer to bet on their home teams than on other teams, especially in comparison to a team their home team is playing against (Babad and Katz, 1991). A preference for betting on uncertainties that one likes thinking about shares much overlap with, but is distinguishable from, a preference for betting on uncertainties that one has expertise about (Heath and Tversky, 1991).

Decision making involving uncertainties that are ambiguous is similar to the case with known risks, but with an additional wrinkle: with ambiguity, there are additional information gaps. In a choice between a sure thing and an ambiguous gamble, for example, a second relevant question (in addition to the one above about the outcome of the gamble) is: what is the probability of winning with the ambiguous gamble? (And there may be additional relevant questions that could inform someone about this probability, so even a Bayesian capable of making subjective probability judgments would be exposed to these information gaps.) Again, betting on the ambiguous gamble makes these questions more important and thus will increase the attention weight on them. So, desire to play the gamble will be increasing with the degree to which thinking about the gamble is pleasurable. To the extent that abstract uncertainties are not pleasurable to think about, this model provides a novel account of standard demonstrations of ambiguity aversion, including those first
6 Conclusion

In much of the economics literature, preferences about information have been viewed as derivative from risk preferences. We take a complementary perspective, considering feelings about information gaps as primitive and viewing preferences about risk and ambiguity along with preferences about information as derivative of these feelings.

Thoughts and feelings about information gaps underlie two additional motives for information acquisition or avoidance over and above the usefulness of information for improving future decisions. People may obtain information purely to satisfy curiosity. Loewenstein (1994) proposed an information-gap account of curiosity, which provides insight about its situational determinants. There are many things that people don’t know and that don’t bother them, but awareness of specific pieces of missing information can prompt an unreasonably strong desire to fill these gaps. Our theory embraces the information gap concept and provides a new formal definition of an information gap. Our utility function assumes that people want to fill information gaps ceteris paribus (i.e., they desire clarity or dislike uncertainty), and this is a universal motive for information acquisition rather than avoidance. We identify this motive as curiosity. We hypothesize that information avoidance derives from a second motive, a desire to avoid increasing attention on a negative anticipated outcome. More generally, we suggest that individuals have an inclination to acquire (or avoid) information whenever they anticipate that what they discover will be pleasurable (or painful). Our fundamental assumption is that obtaining information tends to increase attention to it (as in Gabaix et al., 2006; Tasoff and Madarász, 2009) to the extent that it is surprising. This leads to the implication that people will seek information about questions they like thinking about and will avoid information about questions they do not like thinking about.

Research has shown that missing information has a profound impact on decision making under risk and ambiguity. For example, Ritov and Baron (1990) studied hypothetical decisions concerning whether to vaccinate a child, when the vaccine reduces the risk of the child dying from a disease but might itself be harmful. When the uncertainty was caused by salient missing information about the risks from vaccination – a child had a high risk of being harmed by the vaccine or no risk at all but it was impossible to find out which – subjects were more reluctant to vaccinate than in a situation in which all children faced a similar risk and there was no salient missing information. We suggest that ambiguity aversion in this scenario stems from the unpleasantness of thinking about this missing information (see also Frisch and Baron, 1988). This account of risk- and ambiguity preference is conceptually different from, and has different testable implications from, the usual account of risk aversion involving loss aversion and the usual account of ambiguity
aversion involving vague probabilities. Our fundamental assumption relating risk- and ambiguity preference to feelings about information gaps is that exposure to risk or ambiguity attracts attention to the underlying information gaps because it makes them more important. This leads to the implication that people will be averse to risk and ambiguity when they do not like thinking about the uncertainty and will seek risk and ambiguity when they like thinking about the uncertainty.

In this paper we introduce the concept of an information gap and use it to develop a unified theory of feelings about uncertainty. We outline our applications of this theory to preferences for information acquisition or avoidance and preferences about risk and ambiguity, and we refer the reader to companion papers (Golman and Loewenstein, 2015; Golman et al., 2015) that give these issues the attention they deserve.

\footnote{For example, low-stakes risk aversion (Rabin, 2000) could be attributed to the discomfort of thinking about uncertainties.}
References


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