Preference Signaling with Multiple Agents

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$n$ image-conscious agents decide:
Act ($a_i = 1$) to bring about joint outcome or not ($a_i = 0$)?
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**Information environment:** An observer receives signal \( \sigma_i(a) \)

- **Actions observable:** \( \sigma_i(a) = a_i \)
- **Outcome observable:** \( \sigma_i(a) = w(a) \)
- **Another agent’s action & outcome (Peer monitoring):** \( \sigma_i(a) = (w(a), a_j) \)
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Each agent chooses $a_i$ to maximize expectation of

$$U_i(a, \theta_i) = \theta_i w(a) - k a_i + E[\theta_i | \sigma_i(a)],$$

given her type $\theta_i \sim F$ and beliefs about behavior of other agents.
Agent acts if expected net benefit is positive

With monotonic beliefs, characterized by cutoff $\theta^c$, this means:

$$EU_i(1, \theta_i, \theta^c) - EU_i(0, \theta_i, \theta^c) = B(\theta^c) - [k - \theta_iW(\theta^c)] \geq 0$$
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$W(\theta^c)$ : expected impact on the outcome, given $\theta^c$

- Complementarity: $(1 - F(\theta^c))^{n-1}w$
- Substitutability: $F(\theta^c)^{n-1}w$
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$B(\theta^c)$: expected image-benefit of taking the action, given $\theta^c$.

- Depends on information environment
- **Actions** observable:

$$B(\theta^c) = B^a(\theta^c) \equiv E[\theta|a = 1, \theta^c] - E[\theta|a = 0, \theta^c]$$
Observable Actions: baseline info environment for $B(\theta^c)$

\[ B(\theta^c) = B^a(\theta^c) = \frac{1}{2} \]
In equilibrium, cutoff type is indifferent

\[ k - \theta^* W(\theta^*) = B(\theta^*) \]

Image benefit from observable Actions ⇒ agents more motivated to act than w/ no signaling
Interior eq. under Comp. with Actions observed

\[ C(\theta^c) - \theta^c W(\theta^c) = k - \theta^c (1 - \theta^c) w \]

\[ B(\theta^c) = B^a(\theta^c) = \frac{1}{2} \]
Q1: When is the motivational effect of perfect monitoring the greatest?

When material costs are relatively:

**Low** then effect is greater under complementarity: cutoff type cares little in either case, but more likely to affect outcome under complementarity

**High** then this is reversed: cutoff type cares a lot in either case, but more likely to affect outcome under subs.
Outcome technology affects cost curve, equilibrium

\[ B(\theta^c) = B^a(\theta^c) = \frac{1}{2} \]

Cost, Benefit

Comp: \( k - \theta^c (1 - \theta^c) w \)

Subs: \( k - (\theta^c)^2 w \)
Q2: When are transparency rules or investment in monitoring particularly worthwhile?

Image benefit if only *Outcome* observed:

- **Complementarity:** 
  \[
  B_c^o(\theta_c) = \frac{F(\theta_c)(1-F(\theta_c))^{n-1}}{1-(1-F(\theta_c))^n} B^a(\theta_c)
  \]

- **Substitutability:** 
  \[
  B_s^o(\theta_c) = \frac{(1-F(\theta_c))F(\theta_c)^{n-1}}{1-F(\theta^*)^n} B^a(\theta_c)
  \]

1. When \(n\) is large: low probability affecting outcome/signal

2. For small \(n\), when agents are unlikely to be able to affect outcome/signal
   - Hard/uncommon tasks under complementarity
   - Easy/common tasks under substitutability
If only *Outcome* observed, technology also affects $B(\theta^c)$

**Complementarity:**

$B^a(\theta^c) = \frac{1}{2}$

$B^o_c(\theta^c) = (1 - \theta^c)B^a(\theta^c)$

$B^o_p(\theta^c) = \frac{1 - \theta^c}{1 + (1 - \theta^c)} B^a(\theta^c)$
If only *Outcome* observed, technology also affects $B(\theta^c)$

**Substitutability:**

\[
B^a(\theta^c) = \frac{1}{2}
\]

\[
B^p_s(\theta^c) = \theta^c B^a(\theta^c)
\]

\[
B^o_s(\theta^c) = \frac{\theta^c}{1+\theta^c} B^a(\theta^c)
\]
Q3: When is *Peer monitoring* a good substitute for perfect monitoring?

Image benefit \((n = 2)\):

- **Complementarity**: \(B_c^p(\theta^c) = (1 - F(\theta^c))B^a(\theta^c)\)
- **Substitutability**: \(B_s^p(\theta^c) = F(\theta^c)B^a(\theta^c)\)

When the information that peers have is likely to be a good complement for the info contained in the outcome.
Many interesting questions remain:

- Impact of signaling behavior in markets for goods that carry social judgments?

- When the cost is tied to the *Outcome*, differing predictions regarding *Actions* vs. *Outcomes* being observable may yield a way to distinguish self-signaling from social-signaling.
Many interesting questions remain:

- Impact of signaling behavior in markets for goods that carry social judgments?

- When the cost is tied to the *Outcome*, differing predictions regarding *Actions* vs. *Outcomes* being observable may yield a way to distinguish self-signaling from social-signaling

Thank you for your feedback!
Comp ⇒ expected material cost of cutoff type is U-shaped

Example w/ $\theta \sim U[0, 1]$, $n = 2$, $k = 0.75$, $w = 3$

$$C(\theta^c) - \theta^c W(\theta^c) = k - \theta^c (1 - \theta^c) w$$

Cost,Benefit

\[ \begin{array}{c|c|c|c}
\theta^* & 0 & 0.5 & 1 \\
\hline
\text{Cost} & 0.75 & 0.5 & 0.25 \\
\text{Benefit} & 0 & 0.25 & 0.5 \\
\end{array} \]

$\theta^*$ doesn’t care little influence
Subs $\Rightarrow$ expected material cost of cutoff type is decreasing as concern for and likelihood of effecting outcome increase.

$\theta^*$

$C(\theta^c) - \theta^c W(\theta^c) = k - (\theta^c)^2 w$

Cost, Benefit
doesn’t care, little influence

0.75

0.5

0.25

0.25

0.5

0.75

0