is every bit as intelligible and philosophically respectable as many other doctrines currently in favor, e.g., the doctrine that mental events are identical with brain events; the attempt to give a linguistic construal of this latter doctrine meets many of the same sorts of difficulties encountered above (see Hempel, *op. cit.*). Secondly, I think that evidence for universal determinism may not, as a matter of fact, be so hard to come by as one might imagine. It is a striking fact about our world that we never observe any genuine cases of parallelism; it always seems possible to design some sort of interaction between any two genuine empirical magnitudes. If this is correct, then a true theory T can be deterministic only if universal determinism reigns.

CONCLUDING REMARKS

After reading what I have written, I am left with the feeling that most of what I have said is either too obvious or too obscure. But, apart from my desire to appease my philosophical conscience, there are two reasons for saying it. With the exception of the papers by Montague and Hempel, I know of no place in the philosophical literature where the problems involved in formulating the doctrine of determinism are discussed in any depth or even explicitly recognized. Secondly, I hope that the foregoing will make possible a more meaningful discussion of the implications of modern science for the doctrine of determinism.

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DETERMINISM, IGNORANCE, AND QUANTUM MECHANICS *

IT has never been entirely clear whether the indeterminacies of quantum mechanics are the result of indeterminacies in nature itself or merely the expression of the limits of human knowledge about a deterministic world. I think that recent work on the foundations of the quantum theory has partially answered this question, and, although the answer is incomplete, we do at least now have an idea regarding where to look for a more complete result. The main burden of this paper is to present a simple version of what I take to be the principal argument against the thesis that the indeterminacies of quantum theory are entirely epistemic.

* To be presented in an APA symposium on Determinism, December 29, 1971; co-symposiasts will be John Earman and Kent Bendall; see this JOURNAL, this issue, pp. 729–744 and 751–761.

It seems to me that determinism has two components, about neither of which will I be very precise. Roughly, I think the world deterministic in respect of a set of quantities if at every time each quantity in the set has a single, precise value and if in addition, given the values of all quantities prior to any time, *t*, for any time after *t* only one value is possible for each quantity. Determinism requires both the determinateness of quantities and the impossibility of forks in history. These two criteria are separable, but a tradition I shall respect warrants treating them as aspects of the same notion.[1] I shall regard a theory as deterministic if all the worlds it describes are of this kind.

Given a specification of all the forces acting within and on a system of point-masses, or given the Hamiltonian function for an isolated system, classical mechanics provides a set of differential equations relating derivatives of the positions and momenta of the components of the system. Provided the functions in these equations meet certain continuity, differentiability, and boundedness conditions, an initial data set will determine a unique solution to the equations of motion, at least for a neighborhood of the initial data set. It is this feature, together with the supposed determinateness of all quantities occurring in the theory, which affords the grounds for regarding classical mechanics as the paradigm deterministic theory. The grounds may seem far from adequate. Earman has pointed out[2] that for many systems the nonlinear differential equations of classical celestial mechanics fail to determine a solution after a finite period of time. Further, classical physics is consistent with time-dependent forces and allows, mathematically at least, that two systems may develop through exactly the same states up to a certain time and diverge thereafter. If classical mechanics is nonetheless regarded as the paradigm of deterministic theories, then presumably it is because these defects are plausibly thought to result from the incompleteness of the theory. The equations of celestial mechanics may reasonably be thought to fail to determine the development of celestial systems for all time just because they omit the consideration of numerous forces that become important at short distances. Again, one might reasonably hope that all time-dependent forces could be reduced to forces determined solely

[1] Pierce and Reichenbach, for example, seem to have regarded the notion of determinism as compounded of these two aspects. Compare H. Reichenbach, *The Direction of Time* (Berkeley: Univ. of California Press, 1956), section 11, and C. S. Pierce "The Doctrine of Necessity Examined" *The Monist* (1892), reprinted in G. Dworkin, ed., *Determinism, Free Will, and Moral Responsibility* (Englewood Cliffs, N.J., Prentice-Hall, 1970).

[2] In Part Two of his symposium paper, unpublished.

by the relative positions and eternal properties (e.g., mass, charge) of fundamental bodies. Certainly there are a variety of conceivable, consistent extensions of classical mechanics that would accomplish this internal reduction and thereby eliminate the possibility of forks in the histories of classical mechanical systems. Unfortunately, of course, none of them is true.

The quantum theory, too, provides a dynamical equation of state whenever the Hamiltonian function—or rather operator—is given, and the same sorts of apparent indeterminism that occur in the classical case because of incomplete knowledge of the forces in nature can also occur, for analogous reasons, in the quantum theory. Even supposing, however, that we had available to us a complete quantum theory in which all of the Hamiltonians for every system were known, we would still not have a deterministic theory. Rather than determining the values of primitive state variables—such as position and momentum—as functions of time, the dynamical equations of the quantum theory determine only a probability distribution over measurable quantities, "observables" as they are often called, as a function of time. That is, for each observable and Borel set of values of that observable, the state function determines the probability that a system will, on measurement, give a value of the observable lying in the set. All such probability measures show some dispersion; that is, for any state there is some observable that does not get probability unity for all Borel sets containing some particular value of the observable and get probability zero for all other Borel sets. The uncertainty relations are expressions of such dispersion properties.

The quantum theory, then, does not seem to require that all its quantities have precise values at all times. The appropriate determinist attitude toward this feature of quantum mechanics is that it, too, results from incompleteness. Determinists must hold that, if the quantum theory is true, then its observables always do have precise values even though the quantum theory does not tell us what they are, and even, perhaps, though our ignorance of such values is a necessary ignorance. There are well-known arguments that attempt to establish this view.[3] I shall sketch an argument for the contrary thesis: that, if the quantum theory is true, then the observables of some systems cannot all have precise values at any time.

With each Borel set of values of a quantum observable[4] the theory

[3] The best known is Einstein, Podolsky, and Rosen, *Physical Review*, XLVII, 777 (1935).

[4] I assume a discrete, nondegenerate spectrum.

associates a subspace of Hilbert space. If the state vector of a system lies in the subspace associated with a particular set S for an observable R, then the theory says that a measurement of R is certain to yield a value in S. The subspaces are partially ordered by the inclusion relation and every subspace has an orthocomplement. If R and S are, respectively, an observable and a Borel set associated with a subspace, let us say that the proposition "The value of R lies in S" is associated with that same subspace. If subspace M is included in subspace N then, if the probability that an ideal measurement will accord with a proposition associated with M is unity, so is the probability that an ideal measurement will accord with a proposition associated with N. A proposition associated with M has probability unity if and only if the propositions associated with the orthocomplement of M have probability zero.

If the probability of a proposition is unity, a determinist interpretation of the quantum theory ought to assign truth to that proposition, and if the probability is zero, then the proposition ought to be false. Further, if the quantum theory requires that one proposition have probability unity only if another does, then a determinist interpretation ought to allow that the first is true only if the second is. In other words, it seems entirely reasonable to require, as a condition for any deterministic extension of the quantum theory, that its complete states accord with our incomplete probability distributions where the probabilities are zero or unity. Assuming that every subspace is associated with a determinable proposition, all of this requires the existence of functions from the subspaces of any Hilbert space into [0,1] such that

(i) Every subspace receives a value in [0,1],
(ii) A subspace receives the value 1 if and only if its orthocomplement receives the value 0,
(iii) A subspace receives the value 1 only if all of the subspaces which include it also receive the value 1.

As a check on the reasonableness of our account, we can ask whether functions satisfying these three conditions exist in sufficient number that any state vector can be regarded as a partial specification of the "state" given by one or more of these functions. Recent work by Mr. Michael Friedman and myself has shown that such functions always exist and that enough of them exist to discriminate distinct subspaces of any Hilbert space.[5] For present purposes, the importance of this result is that it partially vindicates the determinist charge that quantum mechanics is incomplete and that the state

[5] "If Quanta Had Logic" *The Journal of Philosophical Logic*, I, 1, forthcoming.

vectors of quantum theory do not determine all that is true of a quantum system. One can consistently assume that the observables of quantum mechanics have values more precise than those allowed by the uncertainty principle. The question remains whether it is consistent to assume that all quantities have entirely precise values.

Each observable is associated in the quantum theory with an operator mapping the Hilbert space into itself, and for any operator it will be recalled that there are a distinguished class of vectors, the eigenvectors of the operator, which the operator maps onto multiples of themselves. The multiplying factor is the eigenvalue, the value that would be obtained were the observable to be measured on a system in the eigenstate. Every measured value for an observable of a system is an eigenvalue of the corresponding operator, even if the system is not in an eigenstate of the operator before measurement. For a large and fundamental class of operators, the normalized eigenstates form an orthonormal basis for the Hilbert space; that is, they are a set of vectors of unit length such that each vector lies in the orthocomplement of each of the others, and every vector in the Hilbert space can be written as a linear combination of those in the set. If the determinist thesis were correct and every observable possessed of a precise value, then for each orthonormal basis of the Hilbert space there would be distinguished one vector in the basis, namely the vector whose eigenvalue is the precise value of the observable. Provided, of course, that every orthonormal basis corresponds to an observable quantity. This gives us a fourth and final requirement for the functions from any Hilbert space into [0,1]:

(iv) For every orthonormal basis, one and only one vector in the basis receives the value 1.

One of the consequences of a theorem due to A. Gleason[6] is that on Hilbert spaces of dimension 3 or greater there exist no functions satisfying these four conditions. Since there are always a great many functions satisfying the first three conditions, the assumption of precise values for all observables should take the blame.

Our argument has assumed that, for every orthonormal basis for a Hilbert space, there is a corresponding physical quantity that can, somehow and in some cases, be measured or computed from measurements. But of course this may simply be taking mathematical pleasantries too seriously. Such quantities as are actually measurable or computable are associated with an ortho-

[6] "Measures on Closed Subspaces of a Hilbert Space," *Journal of Mathematics and Mechanics*, VI (1957): 885–894.

normal basis in the way we have described, but only mathematical simplicity warrants our assumption of the converse association. So the issue is not entirely settled, because condition (iv) may be unreasonably strong. And while we are suspecting things, we should suspect the first condition as well; for, if it is unnecessary that every orthonormal basis correspond to a real observable, it is equally unnecessary that every subspace be associated with a Borel set of values for a real observable. Revised, our four conditions come to this:

(*i) Every subspace associated with a Borel set of values for a real observable receives a value in [0,1].
(*ii) A subspace receives the value 1 if and only if its orthocomplement receives the value 0.
(*iii) A subspace receives the value 1 only if all of the subspaces which contain it receives the value 1.
(*iv) For every orthonormal basis corresponding to a real observable one and only one vector in the basis receives the value 1.

Since we have no mathematical characterization of real observables, the issue can be decided in the negative only by providing an example of a quantum-mechanical system and actually determinable quantities which are inconsistent with these four conditions. In effect, Kochen and Specker have done just that.[7] They show that there are 117 different observables—each a different coordinate decomposition of the spin angular momentum of orthohelium in its lowest orbital state—such that no function exists assigning the value 1 to one and only one vector in each associated orthonormal basis and 0 to all the rest. The quantities they use are not presently measurable, but that seems to be entirely for technical reasons.

Altogether, I think it almost conclusively established that the quantum theory is not compatible with that aspect of determinism which would require that all physical quantities have precise values at all times.[8] This, of itself, does not entail anything regarding the possibility or impossibility of forks in the history of a quantum-mechanical world. Their impossibility is at least suggested by the dynamical equations of the quantum theory, e.g., the Schrödinger equation. The contrary view, that quantum-mechanical histories contain forks, is more than suggested by the fact that, within the

[7] S. Kochen and E. Specker, "The Problem of Hidden Variables in Quantum Mechanics," *Journal of Mathematics and Mechanics*, XVII (July 1967): 59–88.

[8] An important argument against this conclusion is implicit in S. Gudder, "On Hidden Variable Theories," *Journal of Mathematical Physics*, XI (February 1970), and explicit in B. van Fraassen "Semantic Analysis of Quantum Logic," forthcoming.

limits of measurement error, we do always obtain precise values for any quantity we measure. Hence a system may, when measured, move from a state in which a given quantity has no precise value to a state in which that quantity does in fact have a precise value, even though no such state transformation accords with the Schrödinger equation.

A proper determinist reply is that we ought not to neglect the effect of the measuring apparatus nor treat the system measured as isolated when it is not. We are immediately faced with what has come to be known as *the* problem of measurement in quantum mechanics. Suppose R is a measurable quantity, O a microscopic system of some kind, and M an apparatus for measuring R for systems of the appropriate kind. Then if system O begins in an arbitrary state, and M in some neutral state, the system $O + M$ must be carried into some state in which the component system, M, is in an eigenstate of the "apparatus observable," i.e., in which its pointer points at some definite value, and the state of the component system, O, must be correlated with that of M. Further, the transformation of the system $O + M$ must accord with the dynamical equations of the quantum theory. Assuming they exist,[9] the transformations of the state of the system $O + M$ must leave that system in a superposition of correlated states:

$$\text{(I)} \qquad {}^{O+M}\Psi = \Sigma c_i {}^{O}\varphi_i \otimes {}^{M}\psi_i \qquad \text{where } \Sigma_i |c_i|^2 = 1$$

and yet leave M in a mixture of pure states:

$$\text{(II)} \qquad \Sigma_j p_j {}^{M}\psi_j \qquad \text{where } \Sigma_j p_j = 1$$

Now the usual interpretation of the coefficients p_j is that they are epistemic probabilities; systems in mixed states are really in some pure state or other, and the coefficients express our uncertainty about which state it is. But this leads immediately to an inconsistency, since, supposing M is really in some unknown pure state, ${}^{M}\psi_k$, then the state of the system $O + M$ must in general really be different from (I) above.

There are several responses to this and related arguments, each involving fundamentally different attitudes toward the quantum

[9] E. Wigner answers the implicit question in the negative, and A. Fine has given the same answer to a slightly different but related question. Both assume that the final state of $O + M$ must be a mixture of states in each of which M is in a pure state. Jauch answers the question affirmatively by assuming, instead, that the final state of $O + M$ must be one in which M is in a mixed state. See E. Wigner, "The Problem of Measurement," *American Journal of Physics*, XXXI (1963); A. Fine, "Insolubility of the Quantum Measurement Problem," *Physical Review* D, II (Dec. 15, 1970); J. Jauch, *Foundations of Quantum Mechanics* (Reading, Mass.: Addison-Wesley, 1968), ch. 11.

theory. One may conclude with Fine (*op. cit.*) that measurement results contradict the quantum theory or with von Neumann[10] that the theory is consistent with observations but does not apply to human observers. But the response that is most interesting for the issue of determinism is contained in a forthcoming paper of van Fraassen.[11] Noting that the preceding argument requires the ignorance interpretation of mixed states, he suggests an alternative conception. The dynamical equations are viewed as determining the time development not of the actual world, but of an ensemble of possible worlds, one for each eigenstate of an appropriate observable. The history of any given possible world may contain forks, points at which it splits into two or more distinct possible worlds. The probability coefficients p_j in the mixed state of the measuring apparatus at the completion of the measurement are just the probabilities that the corresponding possible worlds are actual. There is no contradiction between equation (I) above and the system having an actual state ${}^{O}\varphi_k \otimes {}^{M}\psi_k$; for the former is not the state of the actual world, but of an ensemble of possible worlds. I do not know whether van Fraassen's treatment of the problem of measurement will ultimately prove adequate, but I hope so. It would be pleasing if the quantum theory evidenced uniformity enough to entail the failure of both aspects of determinism and if, to reciprocate, the failure of the two aspects of determinism should explain so many of the puzzling features of the quantum theory.

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LAPLACIAN DETERMINISM AND OMNITEMPORAL DETERMINATENESS *

MANY discussions of the thesis of Laplacian determinism overlook an important component of claim within it. The component of claim to which I refer may be phrased as follows: *The character of the actual world, the structure of the total course of events it comprises, consists in a full temporal continuum of*

[10] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton, N.J.: University Press, 1955), ch. VI.

[11] B. van Fraassen, "Measurement in Quantum Mechanics as a Consistency Problem," forthcoming.

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