Homework 9 (due November 12, 2018)

(1) Ion diffusion across membranes **(15 pts)**

A lipid membrane separates two aqueous compartments with different ion concentrations. Assume that the membrane is permeable to a single ion species with valence z. When a constant electric field E is applied between the two surfaces of the membrane, the net ion flux j is the sum of the diffusive flux j_{diff} of ions along the concentration gradient with diffusion constant D and the electrophoretic flux j_{el} of ions drifting in the electric field E with a velocity, $v_d = zeE/\zeta$.

a) Using Fick's law, $j_{diff} = -D(\mathrm{d}c/\mathrm{d}x)$, and working out j_{el} from the definition of the flux and Einstein's formula, $D = k_\mathrm{B}T/\zeta$, derive the net ion flux, *i.e.*, the Nernst-Planck formula.

b) Under equilibrium (zero net ion flux across the membrane), the *diffusive* flux balances the *electrophoretic* flux. The voltage required to maintain a difference between c_{in} and c_{out} in equilibrium is

$$U = \frac{-k_B T}{ze} \ln\left(c_{in}/c_{out}\right).$$

Show that the ion concentration ratio across the membrane follows a Boltzmann distribution in equilibrium and discuss the physical meaning of the exponent.

(2) Relaxation time and rate constants **(15 pts)**

In an isomerization, $A \rightleftarrows B$, the conformers have an energy difference, $\Delta E = E_A - E_B$.

The forward ($A \rightarrow B$) and backward ($A \leftarrow B$) rates are k_+ and k_-.

(a) Derive the time dependence, $c_B(t)$, in terms of k_+ and k_- by which the concentration of B relaxes to its equilibrium value $c_{B,\mathrm{eq}}$ from an initial value $c_{B,0}$.

(b) Calculate k_+ and k_- at room temperature for a reaction with ΔE = 2.2 kJ/mol and a relaxation time, τ = 30 s.