Homework 4 (due Oct. 3, 2019)

(1) Gaussian Distribution **(15 pts)**

A random variable x has a Gaussian distribution around its mean, x_0:

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}}.$$

a) Using the standard integral, $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, show that $\text{var}(x) = \langle x^2 \rangle - x_0^2 = \sigma^2$.

b) Consider two Gaussian distributions, $P_1(x)$ and $P_2(x)$, with same values of their mean, $x_{0,1} = x_{0,2}$, but different variances such that $\text{var}_2(x) = 2 \cdot \text{var}_1(x)$.

Sketch the two distributions in a graph and calculate the ratio of their amplitudes at x_0.

(2) Maxwell-Boltzmann Distribution **(15 pts)**

The distribution of the speed of molecules in an Ideal Gas is given by the Maxwell-Boltzmann distribution,

$$P(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T} \right)^{3/2} \cdot \exp\left[-\frac{mv^2}{2k_B T} \right],$$ which has its mean at

$$\langle v \rangle = \sqrt{\frac{8}{\pi} \frac{k_B T}{m}}.$$

(a) Using the standard integral $\int_{-\infty}^{\infty} x^4 e^{-ax^2} dx = \frac{3}{4a^2} \sqrt{\frac{\pi}{a}}$, calculate $\text{var}(v) = \langle v^2 \rangle - \langle v \rangle^2$.

(b) Calculate the *most probable* speed.

(c) Draw a sketch of $P(v)$ and use it to argue why – unlike the Gaussian distribution – it differs from the root-mean-square speed.