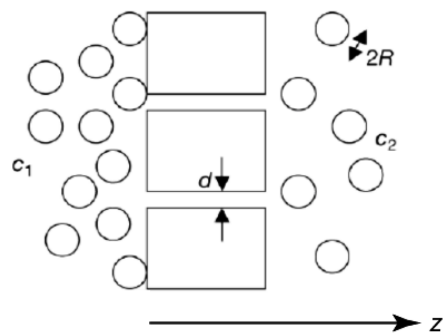
Homework 10 (due November 26, 2019)

(1) Osmosis **(20 points)**

A semipermeable membrane with pores of size d separates two solutions (concentrations $c_1 > c_2$) of particles with radius R. Since $d < 2R$, particles cannot penetrate the membrane.



(a) How does the pressure difference, $\Delta p = p_1 - p_2$, depend on the concentration difference, $\Delta c = c_1 - c_2$?

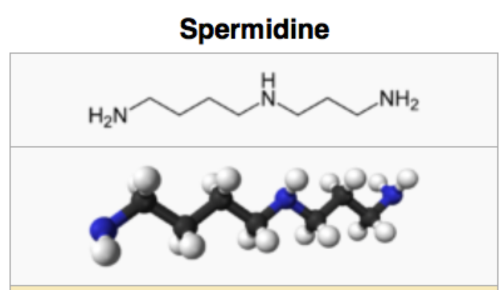
(b) Show in arguments based upon a molecular picture that gives rise to a depletion force why this pressure difference develops.

(c) Under which conditions do you expect pressure variations *along the pores* of the membrane? Explain where those pressure variations derive from.

(d) Referring to the situation pictured on the right, redraw the geometry and attach sketches of the pressure profiles $p(z)$ in the two compartments and across the membrane. Label the pressures to the right and left of the membrane, for two situations:
i) in equilibrium – no osmotic flow occurs through membrane;
ii) out of equilibrium – the overall *hydrostatic* pressure is equal on both sides of the membrane, and osmotic flow occurs to get the system into equilibrium.

(2) Membrane Potential and Peptide Binding **(10 points)**

Consider a phospholipid bilayer at $T = 300$ K that consists of 90% neutral and 10% singly charged, anionic lipid immersed in a physiological salt solution with a concentration, $c_{NaCl} = 140$ mM. The average surface area per lipid in the bilayer is $A_{lipid} = 0.7$ nm^2, and the two lipids are uniformly distributed within the bilayer.



(a) Calculate
  (i) the two-dimensional charge density, σ, of the membrane and
  (ii) the surface potential, Ψ_0, of the bilayer surface.

(b) Determine, in units of k_BT, the binding energy of the plant growth regulator spermidine, a trivalent cationic peptide, when bound to the membrane surface.

(c) At which distance from the membrane surface does a dissolved spermidine molecule experience an attraction equal to k_BT?