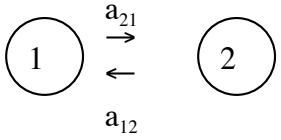
Homework 3
Modeling compartmental systems
Due: April 14, 2009

Consider the following compartmental system (**system 1**)



in which a_{21} is the rate constant for transfer to compartment 2 from 1 and a_{12} is the rate constant for transfer to compartment 1 from 2. Let $q_1(t)$ and $q_2(t)$ be the amounts of material at time t in compartments 1 and 2, respectively, and let $q_1(0)=i_1$ and $q_2(0)=i_2$.

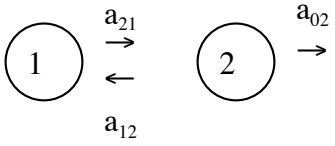
Questions (Total of 50 points)

Submit a Maple workbook and (optionally) Matlab code (as a single zip or tar file) via the dropbox.

1. What is the timescale of **system 1**? You can use Maple to calculate it or simply enter your written answer into the workbook as text.

2. Using Maple, enter the differential equations describing **system 1** without substituting values for the constants. Solve the set of equations analytically using the *dsolve* procedure.

3. Now substitute $a_{21}=15$, $a_{12}=5$, $i_1=3$, $i_2=1$ and plot the solutions for $q_1(t)$ and $q_2(t)$ on the same axes (choose an appropriate time range for the plot) [PLOT1].



4. Using Maple or Matlab, use the general solution for a first-order compartmental model with constant inputs (discussed in class) to build a model for **system 2 (above)**. Plot the solutions for $q_1(t)$ and $q_2(t)$ on the same axes over an appropriate time range (use $a_{02}=2$) [PLOT2].

5. Now add a constant input into compartment 1 at a rate of 4. Plot the solutions for $q_1(t)$ and $q_2(t)$ on the same axes over an appropriate time range [PLOT3].

6. Now assume that there is an input into compartment 1 that changes over time such that it enters compartment 1 at a rate of $20/(t+1)$. Use numerical integration to plot the solutions for $q_1(t)$ and $q_2(t)$ over an appropriate time range [PLOT4].