DISSERTATION PROPOSAL

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“Column Elimination for Integer Programs”

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The goal of this thesis is to study a recently developed approach to solving integer programs by using iteratively strengthened relaxations, which we call column elimination.

In the first chapter, we give preliminaries, including an overview of column elimination.

In the second chapter, we consider an algorithm portfolio to improve the performance of column elimination for solving graph coloring instances. The performance of column elimination for graph coloring instances depends on an implementation decision called the variable ordering. We give experimental results on several methods, which show that dynamically allocating time in an online approach can improve the performance of column elimination.

In the third chapter, we develop a column elimination algorithm for solving the Capacitated Vehicle Routing Problem (CVRP). We introduce new methodologies to column elimination, which include a subgradient descent algorithm for solving a Lagrangian reformulation of the linear program relaxation of the model, variable fixing, and cutting planes. We show that column elimination is competitive with a state-of-the-art branch-and-cut-and-price method for solving a benchmark set of CVRP instances.

In the fourth chapter, we generalize and formalize column elimination. We introduce the general model that column elimination can solve, and show how three major combinatorial optimization problems fit into this modeling framework. We close an instance of the Vehicle Routing Problem with Time Windows that has 1,000 locations, and several instances of a generalization of the graph coloring problem, called the multicoloring problem.

In the fifth chapter, we study a novel method for relaxing integer programs by adding extra variables. We call these variables artificial variables, and analyze the relaxations created by different families of artificial variables. We give a column elimination procedure for solving an integer program by iterating between solving a relaxation that includes artificial variables and removing artificial variables in the optimal solution. We give the performance of column elimination on the MIPLIB benchmark set of integer program instances, showing that it can accelerate a commercial solver on some instances.

In the sixth chapter, we consider using column elimination to solve a vehicle routing problem with real world data. The problem is called the Pickup and Delivery Problem with Time Windows and Multiple Stacks, where the term multiple stacks refers to each truck having several compartments that can hold different goods. The goal is to apply column elimination to this problem to show how it can handle more challenging constraints.