ESSAYS ON ASSET PRICING PUZZLES

by

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To my dad, my mum, Fabio, and Fatoş.

Abstract

My thesis is comprised of three chapters. In the first chapter, I examine the uncovered interest rate parity (UIP) puzzle in a two-country economy where agents have recursive preferences. The model rationalizes the anomaly thanks to the presence of two ingredients: preference for the early resolution of risk and stochastic volatility in consumption growth. When U.S. consumption volatility is relatively low, exchange rate variability is closely tied to shocks in U.K. consumption. This is foreign exchange risk for the U.K. investor. At the same time, the preference for the early resolution of risk drives the U.S. interest rate up when U.S. volatility is low, thus solving the puzzle.

In the second chapter, coauthored with David K. Backus, Chris Telmer and Stanley E. Zin, we investigate the UIP puzzle and its relation to monetary policy. The puzzle, according to which high interest rate currencies appreciate over time, is primarily a statement about short-term interest rates and how they are related to exchange rates. Short-term interest rates are strongly affected by monetary policy. The UIP puzzle, therefore, can be restated in terms of monetary policy. When one country has a high interest rate policy relative to another, why does its currency tend to appreciate? We represent monetary policy as foreign and domestic Taylor rules. Foreign and domestic pricing kernels determine the relationship between these Taylor rules and exchange rates. We examine different specifications for the Taylor rule and ask which can resolve the UIP puzzle. We find evidence in favor of asymmetries. If the domestic Taylor rule responds more aggressively to inflation than does the foreign Taylor rule, the excess expected return on foreign currency increases. A related effect applies to Taylor rules that respond to exchange rates and/or lagged interest rates. A calibrated version of our model is consistent with many empirical observations on real and nominal exchange rates, including the negative correlation between interest rate differentials and currency depreciation rates.

In the third chapter, I show that long-run risk — highly persistent variation in expected consumption growth — arises endogenously in a production economy with nominal frictions. The 'long-run' part comes from price stickiness. Nominal frictions in the model generate a consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. The 'risk' part comes from Epstein-Zin preferences, which result in a large risk premium being associated with variation in the conditional mean. The model provides new testable implications for long-run-risk models, and restricts the joint distribution of consumption and nominal equity and bond risk premia. A calibrated version of the model generates consumption, a risk-free interest rate, and equity risk premium behavior that are consistent with U.S. data.

Contents

Chapter 1

Uncovered Interest Rate Parity Puzzle: An Explanation based on Recursive Utility and Stochastic Volatility

1.1 Introduction

The uncovered interest rate parity (UIP) puzzle states that high interest rate currencies appreciate over time and therefore pay a positive expected excess return. This empirical finding is consistently confirmed by numerous studies (see, among others, Engel (1996) and Lewis (1995)). I show that the anomaly arises naturally in a two-country model with two ingredients: (i) Epstein-Zin preferences with a preference for the early resolution of risk, and (ii) stochastic volatility in consumption growth.

The economics underlying my results are as follows. Fama (1984) noted that the U.S. minus U.K. (real) interest rate differential can be written as

$$r_t - r_t^* = p_t + q_t \quad ,$$

where q_t is the expected rate of depreciation on the U.S. (real) exchange rate and, therefore, p_t is the expected excess return on a "carry trade" which delivers U.K. goods and receives U.S. goods. Backus, Foresi, and Telmer (2001) showed that (with conditional lognormality),

p_t and q_t can be written as

$$p_t = Var_t(\log m^*_{t+1})/2 - Var_t(\log m_{t+1})/2 \tag{1.1}$$

$$q_t = E_t \log m^*_{t+1} - E_t \log m_{t+1} \quad , \tag{1.2}$$

where m and m^* are the U.S. and U.K. real pricing kernels, respectively, and, with complete markets, the realized depreciation rate of the U.S. real exchange rate is

$$d_{t+1} = \log m^*_{t+1} - \log m_{t+1} \quad , \tag{1.3}$$

so that $q_t = E_t d_{t+1}$. Fama's well-known conditions for the resolution of the UIP puzzle are that (i) $cov(p, q) < 0$ and (ii) $var(p) > var(q)$. I show that stochastic volatility and Epstein-Zin preferences are sufficient to have both these conditions satisfied.

Why stochastic volatility? Equation (1.1) makes it clear that, at least with lognormality, there is no choice. Without variability in the conditional variances, p_t is a constant and both of Fama's condition are violated. With stochastic volatility, what is going on is as follows. When U.K. consumption volatility is relatively high then, according to equation (1.3), variations in the exchange rate will be dominated by variations in U.K. consumption. The U.K. investor views this as 'exchange rate risk' and, therefore, requires a positive risk premium in order to hold a security which is long this source of risk. A carry trade that is long U.S Dollars will, therefore, have a positive expected payoff.

Why Epstein-Zin (EZ) preferences? Because the UIP puzzle requires a connection between the conditional variance and the conditional mean of the pricing kernel. With standard time and state separable preferences, the stochastic volatility that is driving the risk premium, p_t, cannot affect the expected depreciation rate, q_t. The result is a violation of Fama's condition (i). In addition, the separation of risk aversion from intertemporal elasticity of substitution is instrumental in getting the interest rate differential to move in the right direction. Intuitively, when the conditional variance of the U.K. pricing kernel is relatively high — so that the risk premium is positive — the U.K. interest rate will be relatively low if agents have preference for the early resolution of risk.

Figure 1.1 clarifies the mechanism described above. The trees in Panels A and B show how different attidutes toward the timing in the resolution of uncertainty affect utility over states of nature and time. With early resolution of risk, an increase in conditional variance is analogous to moving the agent from the tree bifurcating early in Panel A, which has a

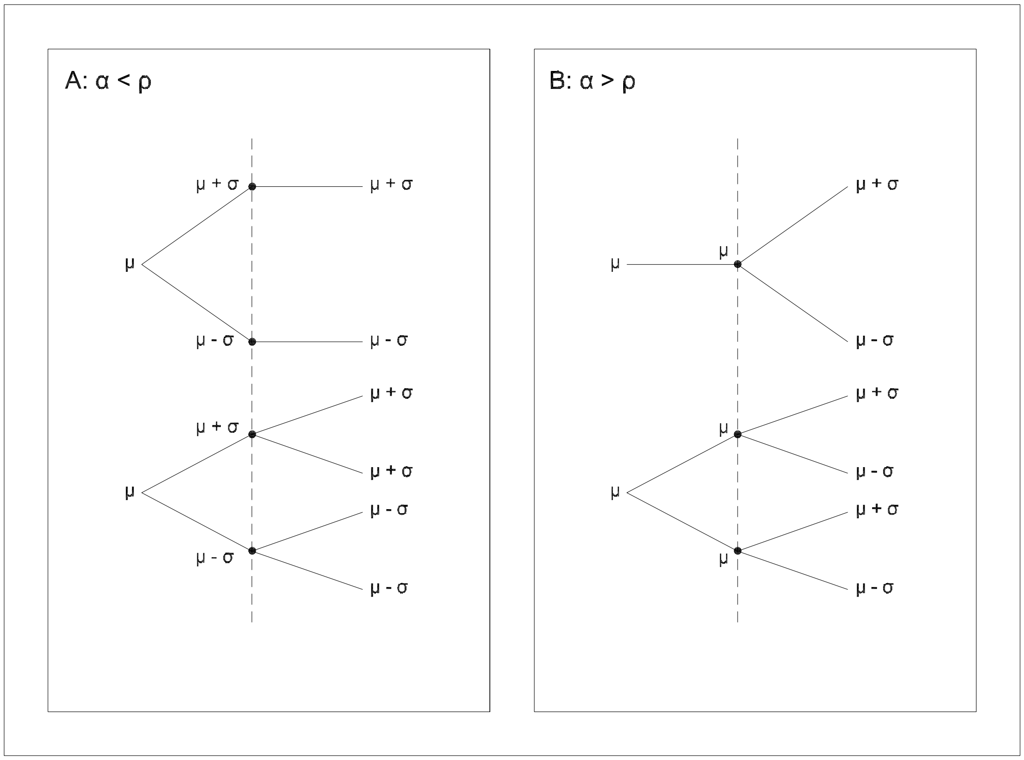

Figure 1.1: The role of the timing of the resolution of uncertainty. Panel A and B show the case of preference for the early and the late resolution of risk, respectively ($\alpha < \rho$ vs. $\alpha > \rho$). The lower trees reproduce in a more extended way the same trees in the upper part of the Figure to emphasize the differences between the two cases. The vertical dotted line represent the moment when utility is evaluated. In Panel A, where the uncertainty is resolved early, the conditional mean varies and the conditional variance is zero. In Panel B, where the uncertainty is resolved late, the conditional mean is constant and the conditional variance is positive. Given preference for the early resolution of risk, a positive shock to the conditional variance is equivalent to moving the agent from Panel A to Panel B.

non-constant conditional mean and zero conditional variance, to the tree bifurcating late in Panel B, which has a constant conditional mean and a positive conditional variance. Agents are ultimately worse off. In order to make them indifferent, one could increase the level of the conditional mean, thus leading to a decrease in the interest rate.[1]

[1] In contrast to the standard case of state and time separable utility, with recursive preferences, an increase in the conditional variance of consumption does not necessarily imply a decrease in the level of the interest rate. The usual precautionary savings effect is modified to take into account the role played by the timing of the resolution of uncertainty. The interpretation of Figure 1.1 is due to Stanley Zin.

To summarize, the story is this. If U.K. consumption volatility increases relative to that of the U.S., then the U.K. agent views foreign 'currency' investments as being riskier than does her U.S. counterpart, because U.K. consumption shocks become more strongly related to exchange rate shocks. The U.K. agent will require a risk premium. This risk premium must be manifest in either a relatively low U.K. interest rate, or an expected appreciation in the U.S. exchange rate, or a bit of both. The facts say that it must be a bit of both. EZ preferences deliver 'a bit of both' by (i) allowing stochastic volatility to affect both the first and second moments of the (log) pricing kernels, and (ii) allowing preference for the early resolution of risk to drive down the U.K. interest rate without affecting one-to-one the exchange rate.

Two related studies of the UIP puzzle which pre-date this paper are Bansal and Shaliastovich (2010) and Verdelhan (2010). Bansal and Shaliastovich analyze the anomaly with recursive preferences but emphasize the importance of long-run risk. In contrast, this paper indicates that long-run risk is not necessary for the UIP puzzle. It argues that stochastic volatility — in tandem with EZ preferences — is sufficient and, in light of equation (1.3), more directly related to the requisite risk premium. The intuition behind Verdelhan's model is similar but the economics are very different. In his paper, results are driven by the relative distance from habit consumption levels, which affects the level of risk aversion of the agents. Here instead, risk aversion is constant and a crucial role is played by stochastic volatility in consumption growth.

I calibrate the model to match U.S. monthly consumption data. The implied average level of the real interest rate is 1.0% and the cross-country correlation in consumption is consistent with what we observe in the data. The implied volatility of the depreciation rate on the U.S. real exchange rate is around 16.6%.

The rest of this paper is organized as follows. Section 1.2 introduces the model, Section 1.3 provides a solution to the UIP puzzle, Section 1.4 delivers the results and Section 1.5 offers suggestions for further research and concludes.

1.2 The Model

In this Section I describe the preferences of the agents and introduce the process followed by consumption growth in both countries.

1.2.1 Epstein-Zin Preferences

There is a representative agent in each country who chooses to maximize the recursive utility function given by Epstein and Zin (1989). The intertemporal utility functions for the U.S. and U.K. agents, U_t and U_t^* respectively, are the solution to the recursive equations:

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}$$

and

$$U_t^* = [(1-\beta^*)c_t^{*\rho^*} + \beta^*\mu_t^*(U_{t+1}^*)^{\rho^*}]^{1/\rho^*} \quad ,$$

where β and β^* characterize impatience, ρ and ρ^* measure the preference for intertemporal substitution, and the certainty equivalents of random future utility are specified as

$$\mu_t(U_{t+1}) \equiv E_t[U_{t+1}^\alpha]^{1/\alpha}$$

and

$$\mu_t^*(U_{t+1}^*) \equiv E_t[U_{t+1}^{*\alpha^*}]^{1/\alpha^*} \quad ,$$

where α and α^* measure static relative risk aversion (RRA). Both α and ρ are defined for values not greater than one. The relative magnitude of α and ρ determines whether agents prefer early resolution of risk ($\alpha < \rho$), late resolution of risk ($\alpha > \rho$), or are indifferent to the timing of resolution of risk ($\alpha = \rho$). The U.S. marginal rate of intertemporal substitution is

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho} \quad .$$

An equivalent expression can be obtained for the U.K. representative agent. Standard time and state separable utility corresponds to the case in which $\alpha = \rho$.

1.2.2 A Consumption growth process with stochastic volatility

Consumption growth $x_{t+1} \equiv \log(c_{t+1}/c_t)$ follows a heteroskedastic AR(1) process. The U.S. process evolves statistically according to

$$x_{t+1} = (1-\varphi_x)\theta_x + \varphi_x x_t + v_t^{1/2}\epsilon_{t+1}^x \quad ,$$

where

$$v_{t+1} = (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v\epsilon_{t+1}^v$$

is the process for the conditional volatility of U.S. consumption growth. Similarly, the dynamics for U.K. consumption growth $x^*_{t+1} = \log(c^*_{t+1}/c^*_t)$ satisfy

$$x^*_{t+1} = (1-\varphi^*_x)\theta^*_x + \varphi^*_x x^*_t + v^{*1/2}_t \epsilon^{x^*}_{t+1} \quad ,$$

where

$$v^*_{t+1} = (1-\varphi^*_v)\theta^*_v + \varphi^*_v v^*_t + \sigma^*_v \epsilon^{v^*}_{t+1} \quad .$$

I refer to v_t and v^*_t as stochastic volatilities: they will prove essential in the solution of the puzzle. For any t, innovations to consumption growth and stochastic volatility are serially uncorrelated and distributed according to the following multivariate Normal:

$$\begin{bmatrix} \epsilon^x_t \\ \epsilon^v_t \\ \epsilon^{x^*}_t \\ \epsilon^{v^*}_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 & & & \\ 0 & 1 & & \\ \chi_x & 0 & 1 & \\ 0 & \chi_v & 0 & 1 \end{bmatrix} \right) \quad .$$

I allow for non-zero correlation between respective innovations across countries, and define $\chi_x \equiv corr(\epsilon^x_t, \epsilon^{x^*}_t)$ and $\chi_v \equiv corr(\epsilon^v_t, \epsilon^{v^*}_t)$. The process for consumption growth requires that the volatility process be positive, which places further restrictions on the parameters.

Regardless of the specific structure of the economy, with complete financial markets the following first order conditions must hold:

$$E_t(m_{t+1}R_{t+1}) = 1 \quad , \tag{1.4}$$

and

$$E_t(m^*_{t+1}R^*_{t+1}) = 1 \quad , \tag{1.5}$$

where R_{t+1} and R^*_{t+1} are the gross domestic and foreign one period returns. The nature of this exercise is to make parametric assumptions about the processes followed by the observed domestic and foreign consumption growth, and give sufficient conditions on preference parameters to solve the UIP puzzle. A fully specified general equilibrium model is not needed for the analysis. In other words, I take observed consumption data as the competitive allocation resulting from the underlying structure of the economy.

Notice that, unlike in the model of Bansal and Yaron (2004), consumption growth does not contain long-run risk. Bansal and Shaliastovich (2010) study the UIP puzzle within the standard long-run risk framework and emphasize the importance of the contemporaneous presence of *three* ingredients: long-run risk, stochastic volatility and early resolution of risk. I argue that long-run risk in consumption growth is not needed to rationalize the puzzle, as it adds a degree of freedom to the analysis but does not capture any essential component of the anomaly.

1.2.3 The Pricing Kernel

For brevity, the following derivations are provided for the U.S. agent only. Their extensions to the U.K. agent are straightforward. The log of the equilibrium domestic marginal rate of substitution is given by

$$\log(m_{t+1}) = \log\beta + (\rho-1)x_{t+1} + (\alpha-\rho)[\log W_{t+1} - \log\mu_t(W_{t+1})] \quad , \tag{1.6}$$

where W_t is the value function. The first two terms are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution (IES). The third term in the pricing kernel is a new term coming from EZ preferences.

I work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton, and Li (2005). In particular, the value function of each representative agent, scaled by the observed equilibrium consumption level is

$$\begin{aligned} W_t/c_t &= [(1-\beta)+\beta(\mu_t(W_{t+1})/c_t)^\rho]^{1/\rho} \\ &= \left[(1-\beta)+\beta\mu_t\left(\frac{W_{t+1}}{c_{t+1}}\times\frac{c_{t+1}}{c_t}\right)^\rho\right]^{1/\rho} \quad , \end{aligned}$$

where I use the linear homogeneity of μ_t. In logs,

$$w_t = \rho^{-1}\log[(1-\beta)+\beta\exp(\rho u_t)] \quad ,$$

where $w_t = \log(W_t/c_t)$ and $u_t \equiv \log(\mu_t(\exp(w_{t+1}+x_{t+1})))$. Taking a linear approximation of the right-hand side as a function of u_t around the point $\bar{m}$, I get

$$\begin{aligned} w_t &\approx \rho^{-1}\log[(1-\beta)+\beta\exp(\rho\bar{m})] + \left[\frac{\beta\exp(\rho\bar{m})}{1-\beta+\beta\exp(\rho\bar{m})}\right](u_t-\bar{m}) \\ &\equiv \bar{\kappa}+\kappa u_t \quad , \end{aligned}$$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the "log aggregator", the linear approximation is exact with $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$.

Similarly to Gallmeyer, Hollifield, Palomino, and Zin (2007), the expression for the linearized real pricing kernel is:

$$\begin{aligned} -\log(m_{t+1}) &= -\log\beta + (1-\rho)x_{t+1} \\ &\quad -(\alpha-\rho)[(\omega_x+1)v_t^{1/2}\epsilon_{t+1}^x + \omega_v\sigma_v\epsilon_{t+1}^v - \frac{\alpha}{2}(\omega_x+1)^2 v_t - \frac{\alpha}{2}\omega_v^2\sigma_v^2] \\ &= \delta + \gamma_x x_t + \gamma_v v_t + \lambda_x v_t^{1/2}\epsilon_{t+1}^x + \lambda_v\sigma_v\epsilon_{t+1}^v \quad , \end{aligned} \tag{1.7}$$

where

$$\delta = -\log\beta + (1-\rho)(1-\varphi_x)\theta_x + \frac{\alpha}{2}(\alpha-\rho)\omega_v^2\sigma_v^2$$

$$\gamma_x = (1-\rho)\varphi_x \quad ; \quad \gamma_v = \frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 \tag{1.8}$$

$$\lambda_x = (1-\alpha) - (\alpha-\rho)\omega_x \quad ; \quad \lambda_v = -\left(\frac{\alpha}{2}\right)\left(\frac{\kappa(\alpha-\rho)}{1-\kappa\varphi_v}\right)\left(\frac{1}{1-\kappa\varphi_x}\right)^2$$

$$\omega_x = \left(\frac{\kappa}{1-\kappa\varphi_x}\right)\varphi_x \quad ; \quad \omega_v = \left(\frac{\kappa}{1-\kappa\varphi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\varphi_x}\right)^2\right] \quad .$$

Details for the derivation are provided in Appendix A.1.

The first two conditional moments of the real pricing kernel are

$$E_t \log m_{t+1} = -\delta - \gamma_x x_t - \gamma_v v_t \tag{1.9}$$

and

$$Var_t \log m_{t+1} = \lambda_v^2\sigma_v^2 + \lambda_x^2 v_t \quad . \tag{1.10}$$

The conditional mean of the pricing kernel depends both on consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility only. Note that with standard expected utility ($\alpha = \rho$), the pricing kernel collapses to

$$-\log m_{t+1} = -\log\beta + (1-\rho)x_{t+1} \quad ,$$

and its conditional moments become

$$E_t \log m_{t+1} = -\hat{\delta} - (1-\rho)\varphi_x x_t$$

and

$$Var_t \log m_{t+1} = (1-\rho)^2 v_t \quad ,$$

where $\hat{\delta} = -\log\beta + (1-\rho)(1-\varphi_x)\theta_x$. When $\alpha = \rho$, stochastic volatility is not priced as a separate risk source. Indeed, in this case, both the factor loading and the price of risk of stochastic volatility, γ_v and λ_v, collapse to zero. EZ preferences allow agents to receive a compensation for taking volatility risk, to which they would not be entitled with standard time-additive expected utility preferences.

1.3 A solution to the UIP puzzle

In this Section, I derive the risk-free interest rate, the expected depreciation rate and the foreign exchange risk premium, and provide an economic interpretation of the mechanism behind the model.

1.3.1 Risk-free Interest Rate

From equation (1.4) and the log pricing kernel in equation (1.7), the continuously compounded one-month risk-free interest rate is

$$\begin{aligned} r_t &\equiv -\log E_t(m_{t+1}) \\ &= r_0 + \gamma_x x_t + r_v v_t \quad , \end{aligned} \tag{1.11}$$

where $r_0 = \delta - \frac{1}{2}\lambda_v^2\sigma_v^2$ and

$$r_v = -\frac{1}{2}\lambda_x^2 + \gamma_v \quad . \tag{1.12}$$

The coefficient r_v governs the covariance between the risk-free rate and stochastic volatility. Without EZ preferences, it collapses to the standard precautionary savings coefficient. Section 1.4 shows that, when the model is calibrated to match U.S. consumption data, a preference for the early resolution of risk results in a negative r_v coefficient. A positive shock to volatility drives the interest rate down.

1.3.2 Expected Depreciation, Forward Premium and Risk Premium

I impose complete symmetry in the coefficients, but allow for imperfect correlation across countries. From equation (1.3) and Fama's decomposition, the expected depreciation q_t is equal to:

$$q_t = \gamma_x(x_t - x_t^*) + \frac{\alpha}{2}(\alpha - \rho)\left(\frac{1}{1-\kappa\varphi_x}\right)^2 (v_t - v_t^*) \quad . \tag{1.13}$$

The forward premium is

$$\begin{aligned} f_t - s_t &= r_t - r_t^* \\ &= \gamma_x(x_t - x_t^*) + r_v(v_t - v_t^*) \quad , \end{aligned} \tag{1.14}$$

where $f_t = \log F_t$ denote the logarithm of the one-period forward exchange rate and the first equality follows from covered interest parity. Stochastic volatility creates a link between the expected depreciation and the forward premium. Equations (1.13) and (1.14) show that with $\alpha < 0$, when the agents have preference for the early resolution of risk, a (relatively) low U.S. volatility is associated with (i) an expected appreciation of the U.S. dollar and (ii) a relatively high U.S. interest rate. This is exactly what the puzzle says: high interest rate currencies tend to appreciate.

The risk premium is defined as the expected excess return on a "carry trade" which delivers U.K. goods and receives U.S. goods. Using the processes followed by U.S. and U.K. consumption growths, we have

$$\begin{aligned} p_t &\equiv f_t - s_t - q_t \\ &= -\frac{1}{2}\lambda_x^2(v_t - v_t^*). \end{aligned} \tag{1.15}$$

Unlike the expected depreciation rate and the forward premium, the risk premium does not depend on current consumption growth, but only on stochastic volatility: relatively high U.K. stochastic volatility drives the risk premium up.

Recall that, at least with lognormality, stochastic volatility is not an option. It is a requirement. On the contrary, I argue that long-run risk is an option. To see this, note that Bansal and Shaliastovich (2010) risk premium is as follows (with my notation):

$$\hat{p}_t = -\frac{1}{2}(\lambda_x^2 + \varphi_{LR}^2 \lambda_{LR}^2)(v_t - v_t^*) \quad , \tag{1.16}$$

where φ_{LR} is the long-run risk volatility and λ_{LR} is the long-run price of risk. From equation (1.16) it is clear that, although long-run parameters enter the coefficient of the risk premium, thus affecting its level and variability, its time series depends exclusively on stochastic volatility. If we shut down the long-run risk channel, we can still explain the anomaly; if we shut down the stochastic volatility channel we cannot. The next Section further builds on the differences between the models.

1.3.3 The UIP Slope Coefficient

Simple regressions of the currency depreciation rate on the interest rate differential strongly reject UIP. If UIP is satisfied, the slope coefficient of the interest rate differential is equal to one and the intercept is equal to zero. On the contrary, results typically show evidence of a slope coefficient well below unity, and often negative. In my model, the UIP slope coefficient is equal to

$$\begin{aligned} b &= \frac{cov(p+q,q)}{var(p+q)} \\ &= \frac{\gamma_x^2 \ var(x_t - x_t^*) + \gamma_v r_v \ var(v_t - v_t^*)}{\gamma_x^2 \ var(x_t - x_t^*) + r_v^2 \ var(v_t - v_t^*)} \quad . \end{aligned} \tag{1.17}$$

With stochastic volatility and EZ preferences, the UIP slope coefficient can be negative under quite general scenarios. No long-run risk is needed. Indeed, the covariance between the risk premium and the expected depreciation is

$$cov(p_t, q_t) = -\frac{1}{4}\lambda_x^2 \alpha(\alpha - \rho)\left(\frac{1}{1-\kappa\varphi_x}\right)^2 var(v_t - v_t^*) \quad .$$

When $\alpha < 0$, it is sufficient to have a coefficient of risk aversion larger than the inverse of the elasticity of intertemporal substitution ($\alpha < \rho$) to generate a negative covariance between p_t and q_t, thus satisfying Fama's condition (i). Therefore, agents prefer the early resolution of risk.

The variances of the expected depreciation rate and of the risk premium are, respectively,

$$var(q_t) = \gamma_x^2 \; var(x_t - x_t^*) + \frac{1}{4}\left(\alpha(\alpha-\rho)\left(\frac{1}{1-\kappa\varphi_x}\right)^2\right)^2 var(v_t - v_t^*) \tag{1.18}$$

and

$$var(p_t) = \frac{1}{4}\lambda_x^4 \; var(v_t - v_t^*) \quad . \tag{1.19}$$

When the model is calibrated to match U.S. consumption data, Fama's condition (ii), $var(p_t) > var(q_t)$, is satisfied when agents have a sufficiently *strong* preference for the early resolution of risk.

1.3.4 Economic interpretation

It is the interaction between the timing of the resolution of uncertainty and the correlation in consumption growth, both within and across countries, that allows me to resolve the puzzle. To simplify the analysis and to better understand the intuition underlying the model, this Section studies the case of zero autocorrelation in consumption growth ($\varphi_x = 0$) and zero cross-country correlation in stochastic volatility ($\chi_v = 0$). This simplification allows me to isolate the effect of the timing of the resolution of uncertainty.

With complete markets, the depreciation rate of the U.S. Dollar is equal to the ratio of the U.S. to the U.K. pricing kernel (see equation (1.3)). The conditional variability of the depreciation rate is therefore

$$\begin{aligned} var_t(d_{t+1}) &= var_t(\log m_{t+1}^*) + var_t(\log m_{t+1}) - 2cov_t(\log m_{t+1}^*, \log m_{t+1}) \\ &= 2\lambda_v^2\sigma_v^2 + (1-\alpha)^2(v_t + v_t^*) - 2\chi_x v_t^{1/2} v_t^{*1/2} \quad . \end{aligned} \tag{1.20}$$

When the conditional volatility of the U.K. pricing kernel is high relative to the one of the U.S., exchange rate variability is closely tied to shocks in U.K. consumption volatility. This represents exchange risk for the U.K. investor who therefore requires a positive premium to hold a security which is long this source of risk. This is evident from the expression of the risk premium, which simplifies to

$$p_t = -\frac{1}{2}(1-\alpha)^2(v_t - v_t^*) \quad .$$

Times of relatively high U.K. volatility are associated with a positive expected excess return.

The level of risk aversion determines its size and variability (but not its sign): the higher the risk aversion, the larger and the more volatile the risk premium.

A positive risk premium is not enough to resolve the anomaly. The risk premium has to covary negatively with the expected depreciation rate or, equivalently, the interest rate differential — U.S minus U.K — has to increase whenever entering a long position in U.S. Dollars pays a positive expected excess return. This is where the joint use of EZ preferences and stochastic volatility produces its effects. Equation (1.14) becomes

$$r_t - r_t^* = -\frac{1}{2}\left((1-\alpha)^2 - \alpha(\alpha-\rho)\right)(v_t - v_t^*) \quad .$$

Two terms affect the sign of the interest rate differential. The first term depends solely on risk aversion and represent the usual precautionary savings coefficient in the standard case of time and state separable utility. The second term is a non linear interaction between risk aversion and the timing of resolution of uncertainty. In times of relatively high consumption volatility in the U.K., the anomaly can be explained when the second effect outweighs the first. A sufficient condition for this is that the representative agents show preference for the early resolution of risk: interest rates are low when consumption volatility is high. In the language of Backus, Foresi, and Telmer (2001), when agents prefer the early resolution of risk, the differences in conditional variances and conditional means of the log pricing kernels move in opposite direction (see equation (1.1) and (1.2)).

In the next Section, I relax the simplifying assumption of zero autocorrelation in consumption growth and calibrate the model to U.S. data. The economic intuition remains the same but the analysis is complicated by the presence of consumption growth in the expression for the interest rate differential. In particular, the size and variability of the risk premium now depends non-linearly on risk aversion, timing of the resolution of uncertainty and correlation in cross-country consumption growth. The results show that a *strong enough* preference for the early resolution of risk is sufficient to rationalize the UIP puzzle.

1.4 Results

1.4.1 Data and Calibration

The model is calibrated at monthly frequency and reproduces the mean, variance and first order autocorrelation of the U.S. consumption growth process specified in Bansal and Yaron (2004). This is done to emphasize that a model *without* long-run risk in consumption that

Parameter		Value
Consumption Dynamics:		
Mean of consumption growth	θ_x	0.0015
Autocorrelation in consumption growth	φ_x	0.0436
Mean of stochastic volatility	θ_v	6.35×10^{-5}
Autocorrelation in stochastic volatility	φ_v	0.987
Volatility of market variance	σ_v	6.5×10^{-6}
Correlation of consumption shocks	χ_x	0.35
Correlation of volatility shocks	χ_v	0
Preference parameters:		
Time preference parameter	β	0.9998
Risk aversion	$1-\alpha$	5
Intertemporal elasticity of substitution	$1/(1-\rho)$	2

Table 1.1: Calibrated parameter values for the baseline model.

matches the first two consumption moments of a model *with* long-run risk can nonetheless explain the basic features of the UIP puzzle (see Appendix A.2).

Table 1.1 shows the calibrated parameter values for the baseline model. The level of relative risk aversion is equal to 5 and the intertemporal elasticity of substitution is equal to 2. Both values are broadly consistent with the long-run risk literature.[2] The discount factor β is equal to 0.9998 and is used to pin down the unconditional mean of the real interest rate. The coefficient $\bar{m}$ in the log linearization of the wealth-consumption ratio is set equal to zero, thus allowing me to obtain clean expressions for the coefficients κ and $\bar{\kappa}$.

The cross-country correlation in consumption growth is equal to 0.35. This value is consistent with Brandt, Cochrane, and Santa-Clara (2006) who report correlation coefficients between +0.24 and +0.42 for annual consumption growth between the United States and other industrialized countries. The stochastic volatility processes are highly autocorrelated within countries ($\varphi_v = 0.987$) and are assumed to be independent across countries ($\chi_v = 0$). The latter captures the intuition that, in the short run, economies with the same intrinsic features can be hit by unrelated shocks.

Panel A of Table 1.2 reports the consumption growth moments implied by the baseline

[2]The long-run risk literature typically assumes an elasticity of intertemporal substitution of 1.5 (see, among others, Bansal and Yaron (2004) and Bansal and Shaliastovich (2010)). The larger EIS value used in this paper makes it easier to satisfy Fama condition (ii). See Section 1.4.2 for details.

Moment	Data	Model
Panel A: Consunption Growth Dynamics		
$E(x_t)$	1.80	1.80
$std(x_t)$	2.72	2.72
$Corr(x_t, x_{t+1})$	n.a.	0.04
$Corr(x_t, x_t^*)$	≈ 0.30	0.35
Panel B: Other Moments		
$E(r_t)$	0.86	1.01
$std(r_t)$	0.97	0.08
$Corr(r_t, r_{t+1})$	0.78	0.45
$std(d_t)$	≈ 15.00	16.56
b	≈ -1.50	-0.95

Table 1.2: Moment conditions for the baseline model. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The autocorrelation moments refer to monthly autocorrelations. Panel A reports consumption growth moments and Panel B reports other relevant moments and the UIP slope coefficient b. The empirical moments for consumption growth within country are taken from Bansal and Yaron (2004). Cross-country moments are taken from Bansal and Shaliastovich (2010) and Brandt, Cochrane and Santa-Clara (2006).

calibration. In particular, the annualized average consumption growth is equal to 1.80%, with an annualized unconditional volatility of 2.72%. The monthly first order autocorrelation in consumption growth is equal to 4.36% and the cross country correlation in consumption growth is equal to 0.35.

1.4.2 Findings and Comparative Statics

UIP Slope, Risk-free Interest Rate and Depreciation Rate.

Panel B of Table 1.2 reports the main results of the paper. Consistently with the data, the UIP slope coefficient is negative (and equal to -0.95). The annualized average level of the one-month real interest rate is 1.01%.

The volatility of the real interest rate is 0.08%, which is one order of magnitude smaller than what is observed in the data. This is a manifestation of the international asset pricing puzzle highlighted by Brandt, Cochrane, and Santa-Clara (2006). In the context of the model developed in this paper, the puzzle can be resolved in two ways. One could impose a very high cross-country correlation in consumption growth or, alternatively, calibrate the

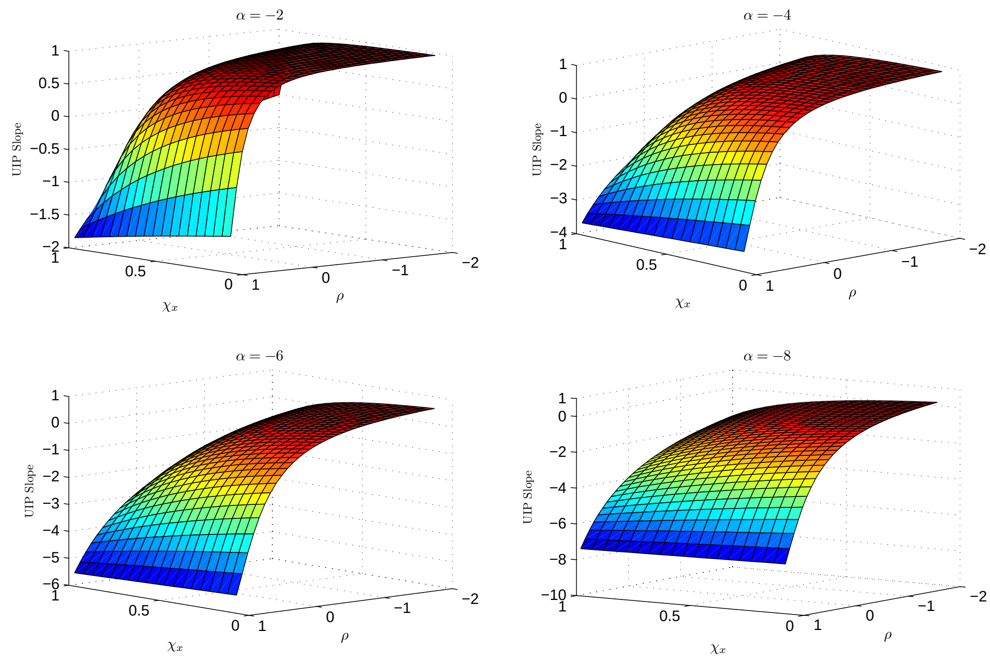

Figure 1.2: UIP slope coefficient b as a function of ρ and χ_x. Relative risk aversion $(1 - \alpha)$ is set equal to 3, 5, 7, and 9.

model to obtain domestic and foreign pricing kernels that are not very volatile, thus implying a low volatility for the equilibrium interest rate processes. In the baseline calibration, I follow the latter strategy.

The model requires a strong preference for the early resolution of risk. For a given level of risk aversion, the IES needed to obtain a negative slope coefficient increases for smaller levels of cross-country correlation in consumption growth. A large RRA reduces the need for a large IES, but at the same time dramatically increases the volatility of the depreciation rate. Figure 1.2 shows three-dimensional graphs of the UIP slope coefficient as a function of the intertemporal elasticity of substitution and the cross-country correlation in consumption growth. Consistently with the data, the annualized volatility of the depreciation rate is 16.56%.

There is a tension in the model between the UIP slope coefficient and the volatility of the depreciation rate. For given IES, a larger risk aversion coefficient facilitates the resolution of the UIP puzzle as it sharply increases the factor loading of stochastic volatility γ_v, while

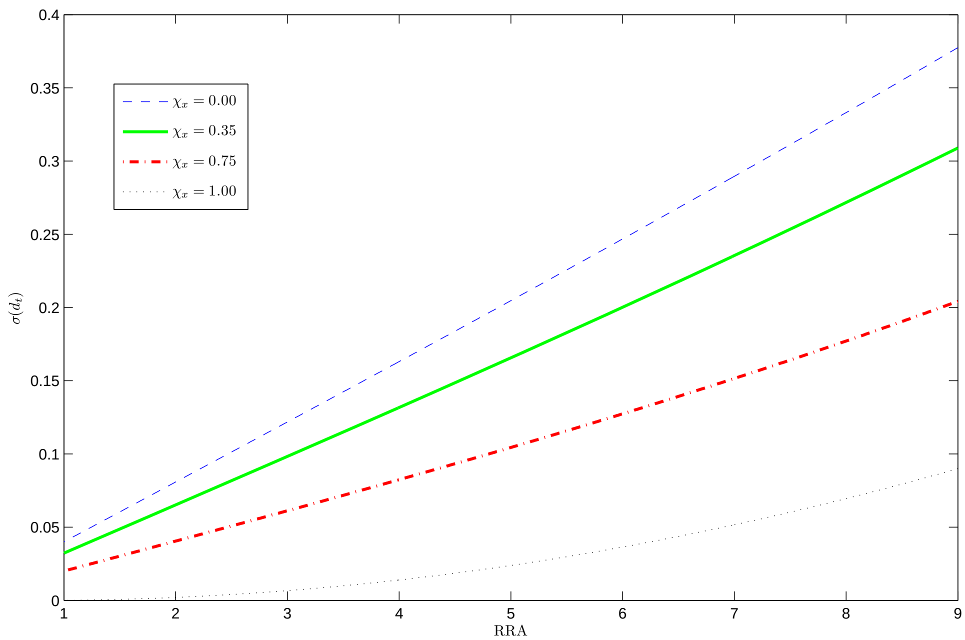

Figure 1.3: Volatility of the deprecation rate as a function of relative risk aversion for different values of cross-country correlation in consumption growth χ_x: 0.00, 0.35, 0.75, and 1.00. Annualized percentage.

its effect on the coefficient r_v is mitigated by the presence of the consumption price of risk, λ_x (see equations (1.8) and (1.12)). However, at the same time, a larger risk aversion coefficient significantly increases the unconditional volatility of the depreciation rate. To see this, recall from the usual variance decomposition formula that

$$var(d_{t+1}) = Evar_t(d_{t+1}) + varE_t(d_{t+1}) \quad . \tag{1.21}$$

The first term, $Evar_t(d_{t+1})$, which accounts for more of 99% of the variability of the depreciation rate, is highly sensitive to an increase in the level of risk aversion.[3] Figure 1.3 shows how the implied volatility of the depreciation rate changes with the level of risk aversion for different values of cross-country correlation in consumption growth. For empirically plausible values of cross-country correlation in consumption growth, a large risk aversion coefficient implies a highly volatile depreciation rate process.

[3]To see this, one need only take the unconditional expectation of equation (1.20) in Section 1.3.4.

Long-run Risk and Cross-Country Correlation in Consumption Growth

How can a model without long-run risk explain the UIP puzzle? To show this, I first assume that the *cross*-country correlation in consumption growth is very high and study the sensitivity the results to the level of *within*-country autocorrelation in consumption growth. This is obviously an unrealistic case, since data suggest low cross-country correlation in consumption growth, but with this simplification one can derive expressions that are easy to interpret and highlight the differences with the work of Bansal and Shaliastovich (2010). With χ_x close to one, I get

$$\begin{aligned} std(q_t) - std(p_t) &\approx -\frac{1}{2}\left(\lambda_x^2 - \alpha(\alpha-\rho)\left(\frac{1}{1-\kappa\varphi_x}\right)^2\right) std(v_t - v_t^*) \\ &= r_v \; std(v_t - v_t^*) \quad , \end{aligned} \tag{1.22}$$

so that Fama's condition (ii) is satisfied whenever $r_v < 0$. Figure 1.4 shows the coefficient r_v as a function of α and ρ, assuming a very high level of autocorrelation in the consumption growth. When relative risk aversion is large enough, an increasing number of values for r_v is positive. It is easy to show that, in order to avoid this possibility, the intertemporal elasticity of substitution has to be larger than one. This result is similar in spirit to the one obtained by Bansal and Shaliastovich (2010). In their specification, it is the autocorrelation in the long-run risk factor — and not the autocorrelation in consumption growth — that enters the coefficient r_v. By definition, the long-run risk process is extremely persistent and, for this reason, their model requires $IES > 1$ to deliver a negative UIP slope coefficient.

What happens when the autocorrelation in consumption growth is low? Figure 1.5 shows that, as far as $r_v < 0$ is concerned, I need not take a stand on IES being larger than one. When the coefficient of relative risk aversion is larger than one, any value of ρ will do. In sum, if *cross*-country correlation in consumption growth was high I would need only check Fama's condition (i), which is satisfied when agents prefer the early resolution of risk. Again, this is similar to what happens in Bansal and Shaliastovich (2010), with the usual caveat that in my model, cross-country correlation in consumption growth — and not cross-country correlation in long-run risk — affects the expected depreciation rate. Bansal and Shaliastovich (2010) follow Colacito and Croce (2011) and a very high cross-country correlation between the long-run risk processes. This is why they can disregard the contribution to the volatility of the depreciation rate coming from the variance of the long-run risk factors.

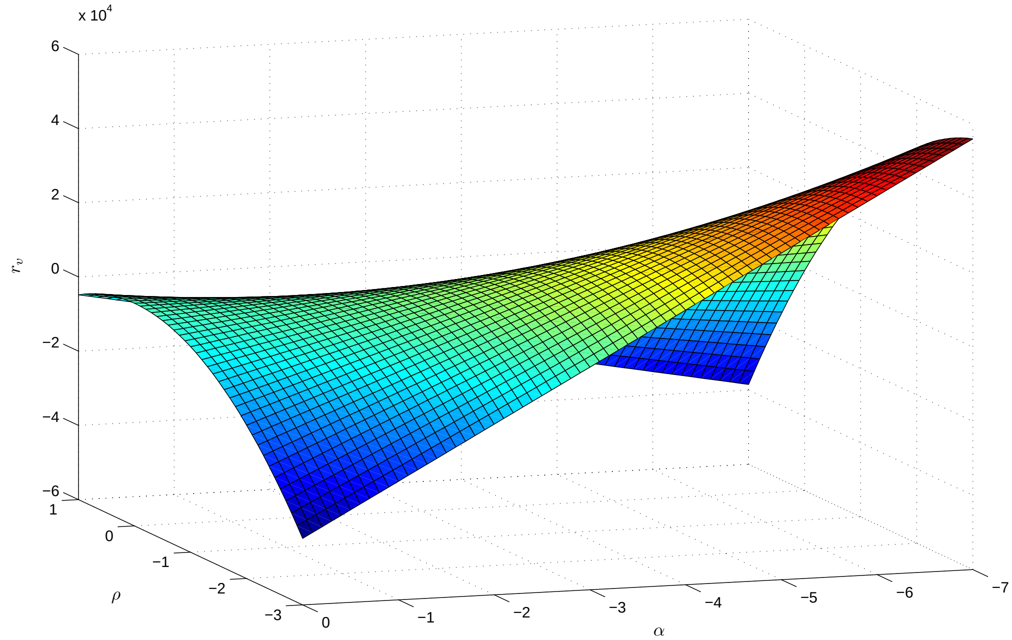

Figure 1.4: Coefficient r_v as a function of α and ρ. The autocorrelation in consumption growth φ_x is set at 0.99. Relative risk aversion is $(1-\alpha)$ and the intertemporal elasticity of substitution is $1/(1-\rho)$.

In my model, consumption growth — and not long-run risk — enters the expected depreciation rate. I need not make any assumption on the level cross-country correlation: data tell us it is in the order of 0.30-0.40. This imposes a lower bound to the value of $var(x_t - x_t^*)$ equal to 1.2-1.4 times the variance of the consumption growth process (either one, since I have assumed symmetry in the coefficients). A strong preference for the early resolution of risk lowers the variability of the depreciation rate and increases the variability of the risk premium, thus satisfying Fama's condition (ii). To see this, notice that equation (1.18) shows that the variance of the expected depreciation rate depends both on the variability of the consumption growth differential and the variability of the stochastic volatility differential, with a coefficient of proportionality equal to the square of the respective factor loading, γ_x and γ_v. Equation (1.19), on the other hand, shows that the variance of the risk premium is proportional to the price of risk of consumption growth, λ_x. A high level of IES helps satisfying Fama's condition (ii), since it lowers γ_x and γ_v while increasing λ_x.

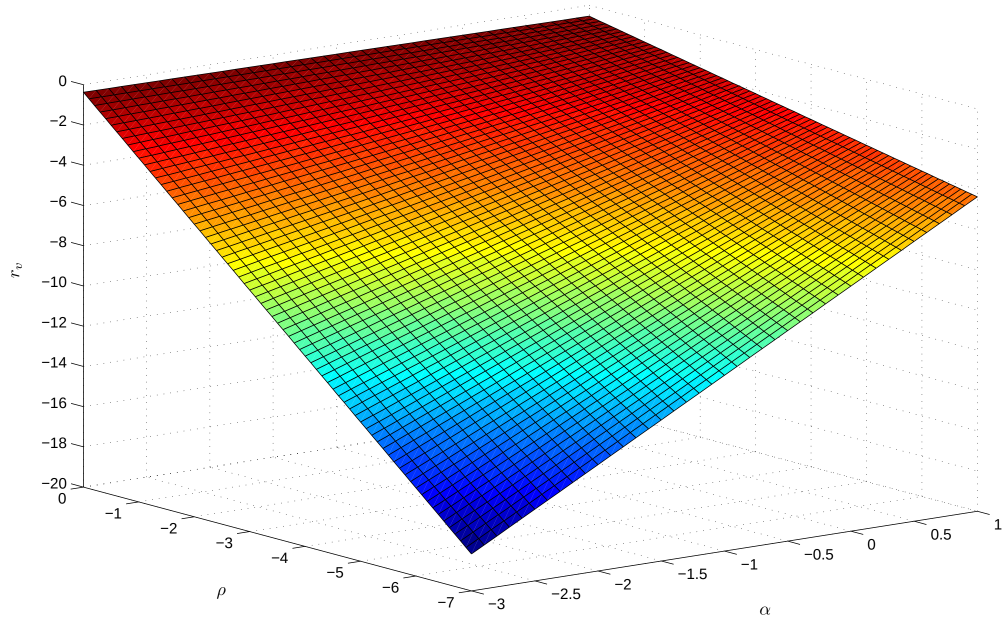

Figure 1.5: Coefficient r_v as a function of α and ρ. The autocorrelation in consumption growth φ_x is set at 4.36%. Relative risk aversion is $(1-\alpha)$ and intertemporal elasticity of substitution is $1/(1-\rho)$.

Autocorrelation in Consumption Growth

Empirical studies on the failure of the uncovered interest rate parity typically focus on monthly data. For consistency with the existing literature, the consumption-based asset pricing model analyzed in this paper is calibrated at monthly frequency. Unfortunately, data on monthly consumption growth is either unavailable or subject to significant errors, thus making it unsuitable for a quantitative exercise. Given the lack of reliable monthly consumption growth data, it seems interesting to analyze how the UIP slope coefficient b varies with the persistence in consumption growth. In order to do so, I fix the IES and cross-country correlation and let φ_x vary. Figure 1.6 plots the UIP slope coefficient b as a function of φ_x, for different levels of risk aversion. For $\varphi_x < 0$, the UIP slope coefficient decreases monotonically as φ_x increases. For $\varphi_x > 0$, it first rises from its minimum level reached at $\varphi_x = 0$, and then decreases again for high values of φ_x. The higher the risk aversion, the sooner the slope coefficient restarts falling. Yet this is another difference with

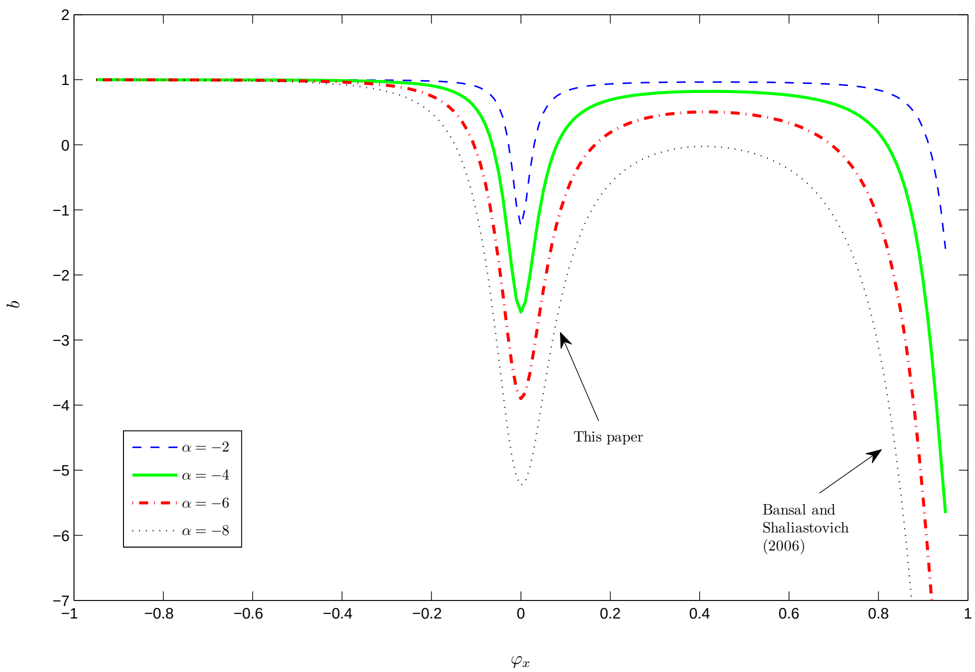

Figure 1.6: UIP slope coefficient b as a function of φ_x. Risk aversion is set at 3, 5, 7, and 9. The model of this paper focuses on the area with low autocorrelation in consumption growth ($\varphi_x \approx 0$), whereas Bansal and Shaliastovich (2010) solve the UIP puzzle for the case with a highly persistent state variable ($\varphi_x \approx 1$).

Bansal and Shaliastovich (2010). Since the long-run risk factor is modeled as a highly persistent component of consumption growth, they essentially focus on the extreme right of Figure 1.6, whereas this paper focuses on its middle part, where the autocorrelation is around zero.

The reason why $b < 0$ when $|\varphi_x|$ is small can be seen observing equation (1.17). A small persistence lowers the impact of consumption growth on the UIP slope coefficient, therefore giving more scope to the role played by stochastic volatility. Without the need for a persistent long-run risk component in consumption growth, the model can reproduce the main features of the UIP puzzle. EZ preferences, which allow me to price stochastic volatility, are all it's needed.

1.5 Conclusion

I study the economic foundations of the UIP puzzle and show that the anomaly naturally arises in a two-country model with two ingredients: stochastic volatility and a strong preference for the early resolution of risk. EZ preferences allow me to price stochastic volatility and create a wedge between the dynamics of the expected depreciation rate and the interest rate differential. This results in a UIP slope coefficient that is different from one and that can be negative for suitably chosen preference parameters.

In light of the fact that the currency risk premium is a function of higher order moments of the domestic and foreign pricing kernels (and therefore of their variances only in the case of lognormality), I argue that the puzzle can be solved without long-run risk components in consumption growth. I believe this approach has the advantage to rely less on variables that are intrinsecly hard to measure while giving a more intuitive explanation of the puzzle based on the relative level of consumption volatility across countries.

The simple calibration exercise in this paper is not a good substitute for a more rigourous simulation exercise and its quantitative implication should be further investigated. Future research will carefully address this issue and further explore the trade-offs between preference parameters, UIP slope coefficient and implied moments of interest and depreciation rates.

Finally, this paper provides an explanation of the anomaly based on purely real factors — consumption growth and stochastic volatility — and deliberately omits monetary policy. As a consequence, this model is completely mute about outstanding issues on the relative importance of real and nominal factors in the UIP puzzle (see Lustig and Verdelhan (2007), Burnside (2007) and Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006)). Current research (Backus, Gavazzoni, Telmer, and Zin (2010)) is carefully investigating this aspect.

Chapter 2

Monetary Policy and the Uncovered Interest Rate Parity Puzzle[1]

2.1 Introduction

Uncovered interest rate parity (UIP) predicts that high interest rate currencies will depreciate relative to low interest rate currencies. Yet for many currency pairs and time periods we seem to see the opposite. The inability of asset-pricing models to reproduce this fact is what we refer to as the *UIP puzzle*.

The UIP evidence is primarily about *short-term* interest rates and currency depreciation rates. Monetary policy exerts substantial influence over short-term interest rates. Therefore, the UIP puzzle can be restated in terms of monetary policy: Why do countries with high interest rate *policies* have currencies that tend to appreciate relative to those with low interest rate *policies*?

The risk-premium interpretation of the UIP puzzle asserts that high interest rate currencies pay positive risk premiums. The question, therefore, can also be phrased in terms of currency risk: When a country pursues a high-interest rate monetary policy, why does this make its currency risky? For example, when the Fed sharply lowered rates in 2001 and the ECB did not, why did the euro become relatively risky? When the Fed sharply reversed course in 2005, why did the dollar become the relatively risky currency? This

[1] This chapter is joint work with David K. Backus, Chris Telmer, and Stanley E. Zin.

paper formulates a model of interest rate policy and exchange rates that can potentially answer these questions.

To understand what we do it's useful to understand previous work on monetary policy and the UIP puzzle.[2] Most models are built upon the basic Lucas (1982) model of international asset pricing. The key equation in Lucas' model is

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1} e^{-\pi^*_{t+1}}}{n_{t+1} e^{-\pi_{t+1}}} \quad , \tag{2.1}$$

where S_t denotes the nominal exchange rate (price of foreign currency in units of domestic), n_t denotes the intertemporal marginal rate of substitution of the domestic representative agent, π_t is the domestic inflation rate and asterisks denote foreign-country variables. Equation (2.1) holds by virtue of complete financial markets. It characterizes the basic relationship between interest rates, nominal exchange rates, real exchange rates, preferences and consumption.

Previous work has typically incorporated monetary policy into Equation (2.1) via an explicit model of *money*. Lucas (1982), for example, uses cash-in-advance constraints to map Markov processes for money supplies into the inflation term, $\exp(\pi_t - \pi^*_t)$, and thus into exchange rates. His model, and many that follow it, performs poorly in accounting for data. This is primarily a reflection of the weak empirical link between measures of money and exchange rates.

Our approach is also built upon Equation (2.1). But — like much of the modern theory and practice of monetary policy — we abandon explicit models of money in favor of interest rate rules. Following the New Keynesian macroeconomics literature (*e.g.*, Clarida, Galí, and Gertler (1999)), the policy of the monetary authority is represented by a Taylor (1993) rule. Basically, where Lucas (1982) uses money to restrict the inflation terms in Equation (2.1), we use Taylor rules. Unlike his model, however, our allows for dependence between the inflation terms and the real terms, n_t and n^*_t. This is helpful for addressing the evidence on how real and nominal exchange rates co-move.

A sketch of what we do is as follows. The simplest Taylor rule we consider is

$$i_t = \tau + \tau_\pi \pi_t + \tau_x x_t \quad , \tag{2.2}$$

[2] Examples are Alvarez, Atkeson, and Kehow (2009), Backus, Gregory, and Telmer (1993), Bekaert (1994), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006), Canova and Marrinan (1993), Dutton (1993), Grilli and Roubini (1992), Tiff Macklem (1991), Marshall (1992), McCallum (1994) and Schlagenhauf and Wrase (1995).

where i_t is the nominal short-term interest rate, π_t is the inflation rate, x_t is consumption growth (analogous to the output-gap in a model with nominal frictions)", and τ, τ_π and τ_x are policy parameters. We also assume that the private sector can trade bonds. Therefore the nominal interest rate must also satisfy the standard (nominal) Euler equation,

$$i_t = -\log E_t\, n_{t+1} e^{-\pi_{t+1}} \quad , \tag{2.3}$$

where (as above) n_{t+1} is the real marginal rate of substitution. An equilibrium inflation rate process must satisfy both of these equations at each point in time, which requires inflation to solve the nonlinear stochastic difference equation:

$$\pi_t = -\frac{1}{\tau_\pi}\left(\tau + \tau_x x_t + \log E_t\, n_{t+1}\, e^{-\pi_{t+1}}\right) \quad . \tag{2.4}$$

A solution to Equation (2.4) is an endogenous inflation process, π_t, that is jointly determined by the response of monetary authority and the private sector to the same underlying shocks. By substituting such a solution back into the Euler equation (2.3), we arrive at what Gallmeyer, Hollifield, Palomino, and Zin (2007) (GHPZ) refer to as a 'monetary policy consistent pricing kernel': a (nominal) pricing kernel that depends on the Taylor-rule parameters τ and τ_π. Doing the same for the foreign country, and then using Equation (2.1), we arrive at a nominal exchange rate process that also depends on the policy parameters τ, τ_π and τ_x. Equations (2.1)–(2.4) (along with specifications for the shocks) fully characterize the joint distribution of interest rates and exchange rates and, therefore, any departures from UIP.

Given a Taylor rule such as (2.2), and its foreign counterpart, we can ask whether the implied exchange rate process in (2.1) tends to appreciate when the implied interest rate in (2.3) is relatively high. If so, then the source of UIP deviations can be associated with this Taylor rule. Moreover, we can generalize the specification of the Taylor rule in Equation (2.2) and analyze the consequences of alternative monetary policies for exchange rates. In addition, we can ask whether the Taylor rule parameters are identified by the UIP facts. Cochrane (2011) provides examples in which policy parameters and the dynamics of the shocks are not separately identified by the relationship between interest rates and inflation. Our framework has the potential for identifying monetary policy parameters from the properties of exchange rates.

We now turn from our methodology to a more specific description of our *question*.

A number of papers (*e.g.*, Backus, Foresi, and Telmer (2001), Lustig, Roussanov, and Verdelhan (2011)) have demonstrated the importance of *asymmetries* between foreign and domestic pricing kernels for explaining the UIP puzzle. Inspection of Equation (2.1) shows why. Exchange rates are all about *differences* in nominal pricing kernels, $n_{t+1}\exp(-\pi_{t+1})$ and $n^*_{t+1}\exp(-\pi^*_{t+1})$. If there are no differences, then the exchange rate is a constant. Many previous papers have come up with *statistical* models of such differences, but far fewer have come up with *economic* models. Herein lies our basic question. Are differences in monetary policies a plausible source of the asymmetries that we know we need? If the Fed's policy is described by $[\tau\,\tau_\pi\,\tau_x]$, and the ECB's by $[\tau^*\,\tau^*_\pi\,\tau^*_x]$ (from Equation (2.2)), then do the differences between the two help us understand the UIP puzzle? We find that they can.

Our main result is as follows. If the domestic country has a relatively tight monetary policy — as measured by a relatively large value for the inflation-stabilization coefficient, $\tau_\pi > \tau^*_\pi$ — then the unconditional means of domestic inflation and interest rates are relatively *low*, and the unconditional mean of the *foreign* currency risk premium is positive. Moreover, the *conditional* risk premium is more variable, relative to the case of symmetric monetary policies. The basic reason, explained in more detail in Section 2.4, is that, *ceteris paribus*, a tighter monetary policy *increases* the variability of the nominal pricing kernel. Our main result, then, is consistent with the broad set of facts that characterize tight-policy countries such as Germany, Japan and Switzerland as having low interest rates (and inflation), and issuing 'funding currencies' for the foreign currency carry trade.

Our main result is derived in a very simple setting. This setting is inadequate for addressing quantitative questions related to interest rates and exchange rates. In Section 2.5 we enhance our model, modeling foreign and domestic consumption as following long-run risk processes as in Bansal and Yaron (2004) and the exchange rate applications in Bansal and Shaliastovich (2010) and Colacito and Croce (2011). Section 2.6 presents quantitative results. We characterize conditions under which real and nominal exchange rates will resolve the UIP puzzle and show that the latter depend on the Taylor rule parameters. Our calibration satisfies the following criteria: (i) the Bilson-Fama UIP regression coefficient is negative, (ii) UIP holds unconditionally, so that the mean of the risk premium is is zero, (iii) changes in real and nominal exchange rates are highly correlated (Mussa (1986)), (iv) exchange rate volatility is high relative to inflation differentials, (v) exchange rates exhibit near random-walk behavior but interest rate differentials are highly autocorrelated, (vi) international pricing kernels are highly correlated but international aggregate consumption growth rates are not (Brandt, Cochrane, and Santa-Clara (2006)), (vii) domestic real and nominal interest rates are highly autocorrelated with means and volatilities that match data. Our calibrated values for the Taylor rule parameters satisfy conditions required for a solution to exist and, interestingly, are also in the ballpark of typical reduced-form estimates. We find $\tau_\pi = 1.4$ and $\tau_x = 1.5$, where the latter is the coefficient on consumption growth, the analog of the output gap in our setting.

Some supplementary results are reported in an Appendix. Appendix B.1 attempts to take a closer look at *exactly* how the Taylor rule affects nominal exchange rates by ignoring variation in real exchange rates. This means that $n_t = n_t^*$ and, according to Equation (2.1), relative PPP holds: $\log(S_t/S_{t-1}) = \pi_t - \pi_t^*$. We also go one step further and set $n_t = n_t^* = e^r$, thus abstracting from real interest rate variation (this doesn't really matter for nominal exchange rates and it makes the analysis easier). The resulting Euler equation for the nominal interest rate (with lognormality) is as follows.[3]

$$i_t = r + E_t\,\pi_{t+1} - \frac{1}{2}\,Var_t(\pi_{t+1}) \quad . \tag{2.5}$$

The Taylor rule (2.2) becomes $i_t = \tau + \tau_\pi \pi_t + z_t$, where z_t is a 'policy shock.' The model therefore boils down to two equations for each country — two Euler equations and two Taylor rules — along with specifications for the policy shocks. The latter must necessarily feature stochastic volatility. Otherwise the conditional variance in Equation (2.5) would be a constant and UIP would hold (up to a constant). The solution for inflation is of the form $\pi(z_t, v_t)$, where v_t is the volatility of z_t. Most of our analysis focuses on variation arising from v_t because only it affects currency risk.

What we find here is negative in nature. We find that simple Taylor rules of the form (2.2) can generate deviations from UIP, but not as large as those typically focused upon in the literature.[4] The basic reason is straightforward and, we think, informative. The Euler equation (2.5) imposes restrictions between the current interest rate and *moments* of future inflation. The Taylor rule imposes an additional, *contemporaneous* restriction between the

[3] Equation (2.5) also shows how our paper relates to the benchmark New-Keynesian setup. All that really distinguishes the two is the conditional variance term. But, for us, this is where all the action is. That is, if inflation were homoskedastic then the nominal interest rate would satisfy the Fisher equation (up to a constant), the difference equation (2.4) would be linear, and the solution for inflation would be in the same class as, say, Clarida, Galí, and Gertler (1999). What would also be true, however, is that UIP would be satisfied (up to a constant) and well-known regression (Bilson (1981), Fama (1984)) of the depreciation rate on the interest rate differential would yield a (population) slope coefficient of 1.0. Our paper would be finished before it even began. Stochastic volatility, therefore, is not a choice, it is a *requirement*. The only issue is where it comes from.

[4] Specifically, our model (without real rate variation) can generate slope coefficients from the regression of depreciation rates on interest rate differentials that are less than unity, but not less than zero.

current interest rate and current inflation. It says that a volatility shock that increases inflation by 1% must increase the interest rate by *more than* 1%. This is because $\tau_\pi > 1$, the so-called "Taylor principle" required for the inflation solution to be non-explosive. However, if inflation is a stationary, positively autocorrelated process, then its conditional mean in Equation (2.5) must increase by *less than* 1%. The only way that both can be satisfied is if the conditional variance in Equation (2.5) *decreases*. But this means that the mean and variance of the (log) pricing kernel are positively correlated, something which contradicts Fama (1984) necessary conditions for resolving the anomaly. There are two ways around this. The first is that volatility is negatively autocorrelated. This is empirically implausible. The second is that the volatility shock that affects inflation also affects the real interest rate (and the real exchange rate). This is the subject of Section 2.5.

This reasoning — spelled out in detail in Appendix B.1 — is admittedly complex. But the basic point is not. Taylor rules of the form (2.2) imply restrictions on the co-movement of the mean and variance of the pricing kernel. Getting this co-movement right is critical for resolving the UIP puzzle, so these restrictions can be binding. Models of the inflation term in Equation (2.1) that are driven by exogenous money supplies do not impose such restrictions. Neither do models in which an exogenous inflation process is used to transform real exchange rates into nominal exchange rates. The sense in which we're learning something about how the conduct of modern monetary policy relates to exchange rates is the sense in which these restrictions identify the policy parameters, τ and τ_π, and the parameters of the shock process z_t.

Our next results are more positive. While continuing to abstract from real exchange rate and interest rate variability, we examine two alternative Taylor rules relative to that in Equation (2.2). In both cases there are parameterizations of the model that admit UIP deviations similar to those observed in data. The first alternative introduces an additional variable and an asymmetry to the Taylor rule (2.2). The variable is the contemporaneous currency depreciation rate, $\log(S_t/S_{t-1})$. The asymmetry is that the foreign central bank reacts more to the exchange rate than does the domestic central bank. Or, in concrete terms, the Bank of England reacts to variation in the pound/dollar exchange rate, but the Fed does not. Such an asymmetry seems plausible. The international role of the U.S. dollar versus the pound is certainly not symmetric. A small country like New Zealand might pay closer attention to the kiwi/yen exchange rate than a large country like Japan. There is also some empirical and theoretical support for such an asymmetry (*c.f.* Benigno (2004), Benigno and Benigno (2008), Clarida, Galí, and Gertler (1998), Eichenbaum and Evans

(1995), Engel and West (2006)).

The second alternative Taylor rule we consider is based on McCallum (1994) and is emphasized in Woodford (2003). We include the lagged interest rate into Equation (2.2). Like McCallum, we find parameterizations of the model that work. Our approach extends his work by endogenizing the currency risk premium which, in his paper, is exogenous.[5] This is an important step since it constrains the sense in which the UIP anomaly is driven by endogenous equilibrium *inflation risk*. That is, in our model, a shock is realized, the Taylor rule responds to that shock, and as a result so does inflation. Whether or not this shock commands a risk premium depends on the parameters of the model. We can then ask if the way in which monetary policy reacts to shocks is consistent with risk premiums that are capable of creating sizable deviations from UIP.

The remainder of the paper is organized as follows. In Section 2.2 we provide a terse overview of existing results on currency risk and pricing kernels that are necessary for our analysis. Section 2.3 develops our baseline model of real and nominal exchange rates. Section 2.4 presents our main result on how asymmetric policies can help account for the UIP puzzle. Section 2.5 enhances the model of Section 2.3 to be more amenable to the data and Section 2.6 conducts a calibration and presents quantitative results. Section 2.7 concludes. Appendix B.1 takes a closer look at Taylor-rule mechanics as they relate to exchange rates by examining the special case of zero variability in real exchange rates.

2.2 Pricing Kernels and Currency Risk Premiums

We begin with a terse treatment of existing results in order to fix notation. The level of the spot and one-period forward exchange rates, in units of U.S. dollars (USD) per unit of foreign currency (say, British pounds, GBP), are denoted S_t and F_t. Logarithms are s_t and f_t. USD and GBP one-period interest rates (continuously compounded) are denoted i_t and i_t^*. Covered interest parity implies that $f_t - s_t = i_t - i_t^*$. Fama (1984) decomposition of the interest rate differential (forward premium) is

$$i_t - i_t^* = f_t - s_t \quad = \quad \big(f_t - E_t s_{t+1}\big) + \big(E_t s_{t+1} - s_t\big)$$

[5] Engel and West (2006) also study a model of how Taylor rules affect exchange rates. Their analysis, while focusing on a different set of questions, is related to McCallum's in that they interpret their 'policy shock' as an amalgamation of an actual policy shock and an exogenous risk premium. Our paper relates to theirs in that both derive an exchange rate process as the solution to a forward-looking difference equation. The main difference is that our deviations from UIP are endogenous.

$$\equiv \quad p_t + q_t$$

This decomposition expresses the forward premium as the sum of q_t, the expected USD depreciation rate, and p_t, the expected payoff on a forward contract to receive USD and deliver GBP. We define the latter as the *foreign currency risk premium.* We define *uncovered interest parity* (UIP) as $p_t = 0$. The well-known rejections of UIP are manifest in negative estimates of the parameter b from the regression

$$s_{t+1} - s_t = c + b\big(i_t - i_t^*\big) + \text{residuals} \quad . \tag{2.6}$$

The population regression coefficient — we'll call it the "Bilson-Fama coefficient" — can be written

$$b = \frac{Cov(q_t, p_t + q_t)}{Var(p_t + q_t)} \quad . \tag{2.7}$$

Fama (1984) noted that necessary conditions for $b < 0$ are

$$Cov(p_t, q_t) \quad < \quad 0 \tag{2.8}$$

$$Var(p_t) \quad > \quad Var(q_t) \tag{2.9}$$

Our approach revolves around the standard (nominal) pricing-kernel equation,

$$b_t^{n+1} = E_t\, m_{t+1} b_{t+1}^n \quad , \tag{2.10}$$

where b_t^n is the USD price of a nominal n-period zero-coupon bond at date t and m_t is the pricing kernel for USD-denominated assets. The one-period interest rate is $i_t \equiv -\log b_t^1$. An equation analogous to (2.10) defines the GBP-denominated pricing kernel, m_t^*, in terms of GBP-denominated bond prices, b_t^*.

Backus, Foresi, and Telmer (2001) translate Fama (1984) decomposition into pricing kernel language. First, assume complete markets so that the currency depreciation rate is

$$s_{t+1} - s_t \quad = \quad \log\left(m_{t+1}^* / m_{t+1}\right)$$

Fama (1984) decomposition becomes

$$i_t - i_t^* \quad = \quad \log E_t\, m_{t+1}^* - \log E_t\, m_{t+1} \tag{2.11}$$

$$
\begin{aligned}
q_t &= E_t \log m^*_{t+1} - E_t \log m_{t+1} && (2.12)\\
p_t &= \left(\log E_t\, m^*_{t+1} - E_t \log m^*_{t+1}\right) - \left(\log E_t\, m_{t+1} - E_t \log m_{t+1}\right) && (2.13)\\
&= Var_t(\log m^*_{t+1})/2 - Var_t(\log m_{t+1})/2 \ , && (2.14)
\end{aligned}
$$

where Equation (2.14) is only valid for the case of conditional lognormality. Basically, Fama (1984) conditions state that the means and the variances must move in opposite directions and that the variation in the variances must exceed that of the means.

Our objective is to write down a model in which $b < 0$. Inspection of Equations (2.8) and (2.14) indicate that a necessary condition is that p_t vary over time and that, for the lognormal case, the log kernels *must* exhibit stochastic volatility.

2.3 Model

Consider two countries, home and foreign. The home-country representative agent's consumption is denoted c_t and preferences are of the Epstein and Zin (1989) (EZ) class:

$$U_t = [(1-\beta)c_t^\rho + \beta \mu_t(U_{t+1})^\rho]^{1/\rho}$$

where β and ρ characterize patience and intertemporal substitution, respectively, and the certainty equivalent of random future utility is

$$\mu_t(U_{t+1}) \equiv E_t[U^\alpha_{t+1}]^{1/\alpha} \ ,$$

so that α characterizes (static) relative risk aversion (RRA). The relative magnitude of α and ρ determines whether agents prefer early or late resolution of uncertainty ($\alpha < \rho$, and $\alpha > \rho$, respectively). Standard CRRA preferences correspond to $\alpha = \rho$. The marginal rate of intertemporal substitution, defined as n_{t+1}, is

$$n_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho} . \tag{2.15}$$

We also refer to n_{t+1} as the *real* pricing kernel. The *nominal* marginal rate of substitution — the pricing kernel for claims denominated in USD units — is then

$$m_{t+1} = n_{t+1} e^{-\pi_{t+1}} \ , \tag{2.16}$$

where π_{t+1} is the (continuously-compounded) rate of inflation between dates t and $t+1$. The foreign-country representative agent's consumption, c_t^*, and preferences are defined analogously. Asterisks' are used to denote foreign variables. Foreign inflation is π_{t+1}^*.

The domestic pricing kernel satisfies $E_t(m_{t+1}R_{t+1}) = 1$ for all USD-denominated asset returns, R_{t+1}. Similarly, $E_t(m_{t+1}^* R_{t+1}^*) = 1$ for all GBP-denominated returns. The domestic pricing kernel must also price USD-denominated returns on GBP-denominated assets:

$$E_t\left(m_{t+1}\frac{S_{t+1}}{S_t}R_{t+1}^*\right) = 1 \ . \tag{2.17}$$

We assume that international financial markets are complete for securities denominated in goods units, USD units and GBP units. This implies the uniqueness of the nominal and real pricing kernels and therefore, according to Equation (2.17),

$$\frac{S_{t+1}}{S_t} = \frac{m_{t+1}^*}{m_{t+1}} = \frac{n_{t+1}^* e^{-\pi_{t+1}^*}}{n_{t+1}e^{-\pi_{t+1}}} \ . \tag{2.18}$$

Equation (2.18) must hold in any equilibrium with complete financial markets. This is true irrespective of the particular goods-market equilibrium that gives rise to the consumption allocations c_t and c_t^* that are inherent in n_t and n_t^*. Our approach is to specify c_t and c_t^* *exogenously* and calibrate them to match the joint behavior of data on domestic and foreign consumption. We are silent on the model of international trade that gives rise to such consumption allocations. Bansal and Shaliastovich (2010), Colacito and Croce (2011), Gavazzoni (2008), Verdelhan (2010) and others follow a similar approach. Hollifield and Uppal (1997), Sercu, Uppal, and Hulle (1995) and the appendix in Verdelhan (2010) — all building upon Dumas (1992) — are examples of more fully-articulated complete markets models in which imperfectly-correlated cross-country consumption is generated by transport costs. Basically, our approach is to these models what Hansen and Singleton (1983) first-order-condition-based approach was to Mehra and Prescott (1985) general equilibrium model.

Domestic consumption growth, $x_{t+1} \equiv \log(c_{t+1}/c_t)$, follows an AR(1) process with stochastic volatility u_t.

$$x_{t+1} = (1-\varphi_x)\theta_x + \varphi_x x_t + \sqrt{u_t}\epsilon_{t+1}^x$$
$$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \epsilon_{t+1}^u$$

The innovations ϵ_t^x and ϵ_t^u are standard normal and independent of each other. The analogous foreign consumption process is denoted with asterisks: $x_{t+1}^* \equiv \log(c_{t+1}^*/c_t^*)$, with volatility u_t^*. Foreign parameter values are assumed to be identical ('symmetric') to their domestic counterparts and the innovation-pairs, $(\epsilon_t^x, \epsilon_t^{x^*})$ and $(\epsilon_t^u, \epsilon_t^{u^*})$ are assumed to be correlated so that (i) cross-country consumption correlations are low, and (ii) cross-country volatility correlations are high (numerical values are calibrated below). The former is a well-documented empirical fact. The latter is less well-documented, but plays a pivotal role in many recent, related papers (*e.g.*, Bansal and Shaliastovich (2010), Colacito and Croce (2011)). In addition in gives our model the feature that the asymmetries that motivate our question — see the discussion in the introduction — are *parameter* asymmetries that load on global shocks. Lustig, Roussanov, and Verdelhan (2011) argue persuasively that the data are supportive, perhaps necessarily so, of such a structure.[6]

The final ingredients are domestic and foreign Taylor rules:

$$
\begin{aligned}
i_t &= \bar{\tau} + \tau_\pi \pi_t + \tau_x x_t & (2.19)\\
i_t^* &= \bar{\tau}^* + \tau_\pi^* \pi_t^* + \tau_x x_t^* & (2.20)
\end{aligned}
$$

For future reference we denote $\Upsilon \equiv [\tau\ \tau_\pi\ \tau_x]$ and $\Upsilon^* \equiv [\tau^*\ \tau_\pi^*\ \tau_x^*]$. Here, we make the foreign specification explicit so as to emphasize the asymmetry that is a focal point of our paper, $\tau_\pi \neq \tau_\pi^*$. For clarity, we hold $\tau_x = \tau_x^*$. Departures from this are considered in a sensitivity analysis.

The Taylor rules (2.19) and (2.20) are fairly typical in the literature, the main exception being that we use consumption growth instead of the 'output gap.' In our model, which abstracts from any frictions that can give rise to a 'gap,' the distinction is meaningless. In Appendix B.3 we extend the basic specification (2.19, 2.20) to include exchange rates and lagged interest rates.

2.3.1 Solution

What *is* a 'solution?' Since we take foreign and domestic consumption to be exogenous, it is just a stochastic process for domestic inflation and one for foreign inflation such that

[6] We represent a common factor as highly-correlated volatility processes instead of the cleaner case of $u_t = u_t^*$. We do this because we seek to isolate the effect of asymmetric monetary policy by restricting all parameter values to be symmetric *except* Taylor rule coefficients. Given this, the case of $u_t = u_t^*$ implies a *real* exchange rate risk premium of zero and a higher-order factor structure for nominal exchange rates than for real exchange rates (two versus one). Both are strongly at odds with the data.

the nominal interest rates implied by the nominal pricing kernels, (2.16), are the same as those implied by the Taylor rules, (2.19) and (2.20). A process for the nominal exchange rate follows immediately by virtue of Equation (2.18).

We proceed as follows. Starting with the domestic country, we derive an expression for the *real* pricing kernel in terms of the model's state variables, x_t and u_t. Next, we solve for domestic inflation and, therefore, obtain an endogenous expression for the domestic *nominal* pricing kernel. We can do this independently of the foreign country because (i) consumptions are exogenous, and (ii) there is no cross-country interaction in the Taylor rules (condition (ii) is relaxed in Appendix B.3). Next, we do the same things for the foreign country. Finally, we compute the nominal exchange rate as a ratio of the foreign and domestic nominal pricing kernels.

Following Hansen, Heaton, and Li (2005), we linearize the logarithm of the real pricing kernel, Equation (2.15), around zero. The result is

$$-\log n_{t+1} \quad = \quad \delta^r + \gamma_x^r x_t + \gamma_u^r u_t + \lambda_x^r \sqrt{u_t}\epsilon_{t+1}^x + \lambda_u^r \sigma_u \epsilon_{t+1}^u \quad , \tag{2.21}$$

where

$$\gamma_x^r = (1-\rho)\varphi_x \quad , \quad \gamma_u^r = \frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 \quad ,$$

$$\lambda_x^r = (1-\alpha) - (\alpha-\rho)\omega_x \quad , \quad \lambda_u^r = -(\alpha-\rho)\omega_u \quad ,$$

where $\omega_x > 0$ and $\omega_u > 0$ are positive linearization coefficients. Expressions for these, and the constant δ^r, are given along with derivations in Appendix B.5. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_x^r \ \gamma_u^r]^\top$ as real factor loadings and to $\lambda^r = [\lambda_x^r \ \lambda_u^r]^\top$ as real prices of risk. The one-period real interest rate is $r_t = -\log E_t n_{t+1} = \overline{r} + \gamma_x^r x_t + (\gamma_u^r - \frac{1}{2}(\lambda_x^r)^2)u_t$, where $\overline{r} = \delta^r - \frac{1}{2}(\lambda_u^r\sigma_u)^2$.

The Euler equation for the nominal one-period interest rate is

$$i_t = -\log E_t \, n_{t+1} e^{-\pi_{t+1}} \quad . \tag{2.22}$$

This, combined with the domestic Taylor rule (2.19), implies that a solution for endogenous inflation, $\pi(x_t, u_t)$, must solve the difference equation.

$$\pi_t = -\frac{1}{\tau_\pi}\left(\bar{\tau} + \tau_x x_t + \log E_t\, n_{t+1}\, e^{-\pi_{t+1}}\right) \quad . \tag{2.23}$$

We guess that the solution is of the form

$$\pi_t = a + a_x x_t + a_u u_t \tag{2.24}$$

for coefficients a, a_x and a_u to be determined. The unique "minimum state variable" solution is

$$\begin{aligned} a_x &= \frac{(1-\rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \\ a_u &= \frac{\frac{\alpha}{2}(\alpha-\rho)(\omega_x+1)^2 - \frac{1}{2}\Big((1-\alpha) - (\alpha-\rho)\omega_x + a_x\Big)^2}{\tau_\pi - \varphi_u} \quad , \end{aligned}$$

with the (relatively inconsequential) solution for a available on request. Putting together Equations (2.21), (2.22) and (2.24), we arrive at the nominal pricing kernel

$$-\log m_{t+1} \quad = \quad \delta + \gamma_x x_t + \gamma_u u_t + \lambda_x \sqrt{u_t}\epsilon^x_{t+1} + \lambda_u \sigma_u \epsilon^u_{t+1} \quad , \tag{2.25}$$

where

$$\delta = \delta^r + a + a_x(1-\varphi_x)\theta_x + a_u(1-\varphi_u) \quad ;$$

$$\gamma_x = \gamma^r_x + a_x\varphi_x; \quad \gamma_u = \gamma^r_u + a_u\varphi_u;$$

$$\lambda_x = \lambda^r_x + a_x; \quad \lambda_u = \lambda^r_u + a_u \quad .$$

The nominal one-period interest rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{\iota} + \gamma_x x_t + (\gamma_u - \frac{1}{2}\lambda_x^2)u_t \quad , \end{aligned} \tag{2.26}$$

where $\bar{\iota} = \delta - \frac{1}{2}(\lambda_u\sigma_u)^2$.

Analogous calculations for the foreign country yield solutions for n^*_{t+1}, π^*_{t+1}, m^*_{t+1}, r^*_t and i^*_t. We omit these calculations since they are almost identical, the only differences being that (i) the state variables, x^*_t and u^*_t are imperfectly correlated with x_t and u_t, and (ii) the foreign Taylor-rule coefficient τ^*_π is distinct from its domestic counterpart τ_π.

2.3.2 Exchange Rates

Using Equation (2.18), the nominal deprecation rate is $d_{t+1} \equiv \log(S_{t+1}/S_t) = m^*_{t+1}/m_{t+1}$. Following Section 2.2, the forward premium, expected depreciation rate and risk premium can be written

$$\begin{aligned} f_t - s_t &= i_t - i^*_t = (\iota - \iota^*) + (\gamma_x x_t - \gamma^*_x x^*_t) + (\gamma_u - \frac{1}{2}\lambda^2_x)u_t - (\gamma^*_u - \frac{1}{2}(\lambda^*_x)^2)u^*_t \\ q_t &= (\delta - \delta^*) + (\gamma_x x_t - \gamma^*_x x^*_t) + (\gamma_u u_t - \gamma^*_u v^*_t) \\ p_t &= (\bar{\iota} - \bar{\iota^*}) - (\delta - \delta^*) - \frac{1}{2}\left(\lambda^2_x u_t - (\lambda^*_x)^2 u^*_t\right) \end{aligned}$$

The Bilson-Fama regression coefficient is $b = Cov(f_t - s_t, q_t)/Var(f_t - s_t)$. A useful reference point is the special case of $\varphi_x = 0$. If the Taylor rule parameters are symmetric (*i.e.*, $\Upsilon = \Upsilon^*$), then

$$b = \frac{\gamma_u}{\gamma_u - \frac{1}{2}\lambda^2_x} \quad .$$

If volatility is positively autocorrelated ($\varphi_u > 0$), then, for given Taylor coefficients, sufficient conditions for $b < 0$ are (i) $\alpha < 0$, (ii) $\rho - \alpha > 0$ *and* large enough, so that the representative agent has a *strong* preference for the early resolution of uncertainty. For the more general case of $\varphi_x \neq 0$ and $\tau \neq \tau^*$, the same is true for the empirically relevant range of the parameter space.

This last point highlights an important feature of our setup thus far. The extent to which the Bilson-Fama coefficient is negative hinges on *preferences*. The sign of the Bilson-Fama coefficient is driven by real exchange rate behavior, not by the properties of endogenous inflation. Indeed, (i) the *real* Bilson-Fama coefficient — the slope coefficient of a regression of the real depreciation rate on the real interest rate differential — is unambiguously negative if $\alpha < 0$ and $\rho > \alpha$, and (ii) the nominal Bilson-Fama coefficient is typically greater than its real counterpart. Endogenous inflation, in other words, pushes us *toward* UIP. Exogenous inflation, in contrast, imposes no such restrictions. Papers such as Bansal and Shaliastovich (2010) and Lustig, Roussanov, and Verdelhan (2011) take the latter route and show that an exogenously calibrated inflation process is consistent with the nominal UIP deviations observed in the data. However, these papers assume — rather than endogenously derive — that inflation is driven by the same factors driving the real pricing kernel, and freely specify the sensitivity of inflation to those factors. We don't have this freedom. The Euler equation and Taylor rule, together, tell us what this sensitivity must be.

Two related points are as follows. First, this is not a general feature of Taylor-rule implied inflation. In Appendix B.3 we show that more elaborate Taylor rules — *e.g.*, incorporating exchange rates and/or lagged nominal interest rates — *can* generate a nominal Bilson-Fama coefficient that is less than its real counterpart. We choose, however, the simpler setting because it articulates our main point most clearly. Second, suppose that our model *did* generate nominal deviations from UIP that were substantially larger than real deviations. While this would support our cause — our view that monetary policy is important for exchange rate behavior — it would also rise to an important empirical tension. We know that real and nominal exchange rates behave quite similarly (Mussa (1986)), and that real and nominal Bilson-Fama regressions also look quite similar (Engel (2011) is a recent example). The most obvious way around this tension — if one wants to continue down the road in which monetary policy plays an important role — is to consider environments in which there is some feedback between nominal variables/frictions and real exchange rates. This is beyond the scope of this paper.

2.4 Asymmetric Monetary Policy

What is the effect of $\tau_\pi > \tau_\pi^*$?

To answer this, we work with the excess expected return on a forward contract that is long GBP and short USD. The definition of p_t above is the opposite. Therefore we work with $-p_t$.[7] In addition, we work with the currency risk premium in *levels*, not in logs. This is because, as has become commonplace in the literature, we'd like to evaluate our model in terms of the Sharpe ratio on a traded portfolio.

With lognormality, (logs of) expected returns in levels are expected returns in logs plus half the conditional variance. Therefore, the main object of interest is

$$-p_t + \frac{1}{2} Var_t d_{t+1} = -Cov_t(\log n_{t+1}, d_{t+1}) + Cov_t(\pi_{t+1}, d_{t+1}) \quad ,$$

[7] We do so because we find it more intuitive to say "this is the risk premium on holding *foreign* currency," as opposed to the premium on holding domestic currency. Our notational convention for p_t, from Section 2.2, follows that which is common in the literature.

where, recall, $d_{t+1} \equiv \log(S_{t+1}/S_t)$, the depreciation rate of USD. For our model, the covariance terms can be written

$$
\begin{aligned}
- p_t + \frac{1}{2} Var_t d_{t+1} &= -\lambda_u \sigma_u^2 (\lambda_u^* \eta_{u,u^*} - \lambda_u) + \lambda_x \sqrt{u_t} (\lambda_x \sqrt{u_t} - \lambda_x^* \sqrt{u_t^*} \eta_{x,x^*}) \\
&\approx \lambda_u \sigma_u^2 (\lambda_u^* - \lambda_u) - \lambda_x^2 u_t \quad , \qquad (2.27)
\end{aligned}
$$

where η_{x,x^*} and η_{u,u^*} are the cross-country correlations of consumption growth and volatility, respectively. The approximation that gives rise to the last line is valid if (i) η_{x,x^*} is close to zero, and (ii) η_{u,u^*} is close to one. The former, while obviously extreme, captures the empirical feature of "low cross-country consumption correlations." The latter (in combination with the former) captures one of our motivational facts, the Lustig, Roussanov, and Verdelhan (2011) result that carry trade profits are dominated by a global risk factor.

How do the Taylor rule parameters affect the risk premium, (2.27)? First, it is easily shown that the first term, $\lambda_u \sigma_u^2 (\lambda_u^* - \lambda_u)$, is negligible. Asymmetries in the price of risk of stochastic volatility ($\lambda_u \neq \lambda_u^*$) do not play a significant role in the FX risk premium.[8] This leaves us with λ_x. Recalling that $\lambda_x = \lambda_x^r + a_x$, and that λ_x^r (the real price of consumption growth risk) is independent of monetary policy, we are left with

$$
a_x = \frac{(1-\rho)\varphi_x - \tau_x}{\tau_\pi - \varphi_x} \quad . \qquad (2.28)
$$

If $0 < \rho < 1$ (so that the EIS is greater than 1), and the autocorrelation in consumption growth is small enough (which is empirically plausible), then a strong enough reaction of the monetary authority to consumption growth, τ_x, implies that $a_x < 0$. Inspection of (2.28) then indicates that an increase in τ_π must make a_x *less negative*, the nominal price of consumption risk, λ_x, larger, and, therefore, the risk premium on GBP *larger*. This establishes our main comparative static result.[9]

Result 1: *Asymmetric monetary policy and currency risk*

> If the correlation of cross-country consumption growth is small, and the correlation of cross-country volatility is large, and if the (symmetric) reaction to

[8]This result is standard in consumption models with stochastic volatility. It basically says that the conditional variance of consumption growth, u_t, is much larger than the conditional variance of stochastic volatility, σ_u^2.

[9]Note that if we worked with the log risk premium instead of the level, then the pivotal term in Equation (2.27) would involve the coefficient $(\lambda_x - \lambda_x^*)$, not just λ_x. Thus, the nature of our comparative static result would be unaffected, since it would hold τ_π^* fixed.

consumption growth, $\tau_x = \tau_x^*$, is large enough, then, *ceteris paribus*, a relatively tight domestic monetary policy, $\tau_\pi > \tau_\pi^*$, implies a positive unconditional risk premium on foreign currency.

2.4.1 Economic Intuition

We define a country with a *weak* price-stabilization policy as one with a relatively *low* value for the inflation coefficient, τ_π, from the Taylor rule (2.19). Such a country (in our model) will have unconditionally high inflation and nominal interest rates, and is characterized by a relatively weak response to an inflation surprise.[10] Similarly, we define a country with a *strong* employment-stabilization policy as one with a relatively *high* value for the 'output gap' coefficient, τ_x.

Our basic result is that a weak price-stabilization policy implies a riskier currency. The logic is a simple implication of Equation (2.14), an expression for the currency risk premium that applies to *any* lognormal model. We reproduce it here for clarity:

$$- \, p_t = Var_t(\log m_{t+1})/2 - Var_t(\log m_{t+1}^*)/2 \ \ . \tag{2.29}$$

Recalling that *minus* p_t is the risk premium on *foreign* currency, this expression says something that, at first blush, might seem counterintuitive. It says that the country with the *low* pricing-kernel variability is the country with the *risky* currency. Low variance ... high risk! At second blush, however, it makes perfect sense. It is a general characteristic of the "change-of-units risk" that distinguishes currency risk from other forms of risk. Change-of-units risk is a relative thing. It measures how I perceive the risk in unit-changing *relative* to how you perceive it.

To understand this, recall that the (log) depreciation rate is the difference between the two (log) pricing kernels. If they are driven by the same shocks (and if loadings are symmetric), then the conditional variances in Equation (2.29) are the same and currency risk is zero for both foreign and domestic investors. If, instead, domestic shocks are much more volatile than foreign shocks, then variation in the exchange rate and variation in the *domestic* pricing kernel are more-or-less the same thing. The domestic investor then views foreign currency as risky because its value is being dominated by the same shocks as is his

[10] The language, 'inflation surprise,' while commonplace in the literature, is obviously loose. Inflation is an endogenous variable. Better language — language that is precise in our specific setting — is that a relatively large value for τ_π translates into a stronger impulse response in the nominal interest rate to any shock that generates a positive impulse response in inflation.

marginal utility. The foreign investor, in contrast, feels *relatively* sanguine about exchange rate variation. It is *relatively* unrelated to whatever it is that is affecting his marginal utility. Hence, the risk premium on the foreign currency must be positive and the premium on the domestic currency (which, of course is the "foreign" currency for the foreign investor) must be negative. While the latter might seem counterintuitive — the foreign investor views currency as a *hedge* — it is inescapable. If GBP pays a positive premium then USD *must* pay a negative premium.[11] Remember it like this; "high variability in marginal utility means low tolerance for exchange-rate risk and, therefore, a high risk premium on foreign currency."

Now that we've established intuition for Equation (2.29) we can turn to monetary policy. Why does a weak price stabilization policy result in a *highly* variable pricing kernel? The key, interestingly, is the other policy parameter, τ_x. Recall the basic definition of the nominal pricing kernel:

$$\log m_{t+1} = \log n_{t+1} - \pi_{t+1} \tag{2.30}$$

$$\Longrightarrow Var\big(\log m_{t+1}\big) = Var\big(\log n_{t+1}\big) + Var\big(\pi_{t+1}\big) - 2Cov\big(\log n_{t+1}\,,\,\pi_{t+1}\big)\ . \tag{2.31}$$

If τ_x is large enough, then the covariance term is typically positive enough so that $Var(m_{t+1}) < Var(n_{t+1})$; the nominal pricing kernel is less variable than the marginal rate of substitution. This is not the case for any set of parameter values, but it is for the economically-interesting ones that we characterize in our calibration (Section 2.6). It is a fairly typical characteristic of most New Keynesian models and was first pointed out in our specific class of models by Gallmeyer, Hollifield, Palomino, and Zin (2007). It is also empirically-plausible in the sense that a large value for τ_x implies an endogenous inflation process that is negatively correlated with consumption growth — thus implying a positive correlation between inflation and marginal utility in Equation (2.31) — something we typically see in the data.

We find this implication of monetary policy to be interesting, irrespective of how things

[11] One should take care not to confuse this with "Siegel's Paradox," the statement that — because of the ubiquitous Jensen's inequality term — the forward rate cannot equal the conditional mean of the future spot rate, irrespective of the choice of the currency numeraire. This *is not* what is going on here. The Jensen's term is (one half of) the variance in the difference of the (log) kernels, not the difference in the variances from Equation (2.29). To make this crystal-clear, consider the case in which this entire discussion *would be* a futile exercise in Siegel's Paradox. Suppose that the log kernels are independent of one-another with constant and identical conditional variances. Then the log risk premium, p_t is zero, and the level risk premium is $-Var_t(d_{t+1})/2$, the familiar Siegel-Jensen term. The above discussion, and our model, presume no such independence nor homoskedasticity. The name-of-the-game is the covariance term, $Cov_t(d_{t+1}, m_{t+1})$ (or its foreign-kernel equivalent).

work out for exchange rates. What's intriguing is that a policy which seeks to fulfill the "dual mandate" by reacting to real economic activity will typically generate *inflation risk*; a negative (positive) correlation between consumption growth (marginal utility) and inflation. This means that securities denominated in nominal units will have expected returns that incorporate an inflation risk premium. What it also means, however, is that *nominal risk is less than real risk*; the nominal pricing kernel is less variable than its real counterpart. Sharpe ratios on nominal risky assets will therefore tend to be *less* than those on real risky assets (Hansen and Jagannathan (1991)). Of course, we don't observe data on the latter, but the implication seem interesting nevertheless.

Turning now to the policy parameter of direct interest, τ_π, we get to the heart of the story. Why does a relatively large value for τ_π translate into a larger *foreign* currency premium? Because it undoes the effect of τ_x. That is, as τ_π gets large, the variance of endogenous inflation decreases (*i.e.*, the as the central bank 'cares more' about inflation it drives the variability of inflation to zero). Thus, the extent to which $Var(m_{t+1}) < Var(n_{t+1})$ is mitigated which, because $Var(n_{t+1})$ is exogenous in our setting, must mean that $Var(m_{t+1})$ increases. Thus the foreign-currency risk premium from Equation (2.29) increases.

Summarizing, then, the economic intuition goes as follows. A procyclical interest rate rule makes the nominal economy "less risky" than the real economy. A stronger interest rate reaction to inflation *undoes* this. It makes domestic state prices more variable so that domestic residents view currency as being more risky *relative to* foreign residents. In a nutshell, 'weak' interest rate rules make for riskier currencies. This seems to accord with the data. The countries with (supposedly) strong anti-inflation stances — *e.g.*, Germany, Japan, Switzerland — have, on average, had low interest rates, low inflation and negative risk premiums.

2.5 Enhanced Model

We now turn to a calibration and quantitative assessment of our model. The specification in Section 2.3 is empirically inadequate. We know, for example, that it cannot account for volatility observed in interest rates, exchange rates and the nominal pricing kernel, at the same time as having realistic implications for the cross-country correlation of consumption growth (Brandt, Cochrane, and Santa-Clara (2006)). As a result, we follow Bansal and Yaron (2004), and the application to exchange rates of Bansal and Shaliastovich (2010), by modeling domestic consumption growth, x_{t+1}, as containing a small and persistent component (its 'long-run risk') with stochastic volatility:

$$\log(c_{t+1}/c_t) \equiv x_{t+1} = \mu + l_t + \sqrt{u_t}\,\epsilon^x_{t+1} \tag{2.32}$$

$$l_{t+1} = \varphi_l l_t + \sqrt{w_t}\,\epsilon^l_{t+1} \tag{2.33}$$

where

$$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \epsilon^u_{t+1} \tag{2.34}$$

$$w_{t+1} = (1-\varphi_w)\theta_w + \varphi_w w_t + \sigma_w \epsilon^w_{t+1} \tag{2.35}$$

Foreign consumption growth, x^*_{t+1} is defined analogously. The innovations are assumed to be multivariate normal and independent *within*-country: $(\epsilon^x, \epsilon^l, \epsilon^u, \epsilon^w)' \sim \text{NID}(0, I)$, but we allow for correlation *across* countries: $\eta_{\epsilon^j} \equiv Corr(\epsilon^j, \epsilon^{j^*})$, for $j = (x, l, u, w)$.

The process (2.32)–(2.35) looks complicated, but each of the ingredients are necessary. Stochastic volatility is necessary because without it the currency risk premium would be constant and the UIP regression parameter, b, would be 1.0. Long-run risk — by which we mean time variation in the conditional mean of consumption growth, l_t — isn't critical for exchange rates, but it is for achieving a realistic calibration of interest rates. It decouples the conditional mean of consumption growth from other moments of consumption growth, thereby permitting persistent and volatile interest rates to co-exist with relatively smooth and close-to-*i.i.d.* consumption growth. Finally, cross-country correlation in the innovations is critical for achieving realistic cross-country consumption correlations. The latter imposes substantial discipline on our calibration (*c.f.*, Brandt, Cochrane, and Santa-Clara (2006)).

We also allow for more flexibility in terms of the Taylor rules.[12]

$$i_t = \tau + \tau_\pi \pi_t + \tau_l l_t + z_t \ , \tag{2.36}$$

where z_t is a policy shock governed by

$$z_{t+1} = (1-\varphi_z)\theta_z + \varphi_z z_t + \sqrt{v_t}\epsilon^z_{t+1} \tag{2.37}$$

$$v_{t+1} = (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon^v_{t+1} \ . \tag{2.38}$$

[12] For parsimony, we use expected consumption growth, l_t, and not its current level, x_t, as is instead standard in the literature. Doing so reduces our state space by one variable. The model can readily be extended to allow for a specification that includes x_t instead of l_t.

Analogous equations, denoted with asterisks, characterize the foreign-country Taylor rules. We include the exogenous policy shocks, z_t, in order to allow for some flexibility in the distinction between real and nominal variables. Without policy shocks endogenous inflation will depend only on consumption shocks. The same will therefore be true of nominal exchange rates. We find it implausible that monthly variation in nominal exchange rates is 100% attributable to real shocks. Note that stochastic volatility in the policy shocks is a necessary condition for them to have any affect on currency risk premiums.

As before, we use the Hansen, Heaton, and Li (2005) linearization of the real pricing kernel around zero. The result is

$$-\log(n_{t+1}) = \delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t \tag{2.39}$$

$$+ \lambda_x^r \sqrt{u_t}\epsilon_{t+1}^x + \lambda_l^r \sqrt{w_t}\epsilon_{t+1}^l + \lambda_u^r \sigma_u \epsilon_{t+1}^u + \lambda_w^r \sigma_w \epsilon_{t+1}^w \tag{2.40}$$

where

$$\gamma_l^r = (1-\rho); \quad \gamma_u^r = \frac{\alpha}{2}(\alpha-\rho); \quad \gamma_w^r = \frac{\alpha}{2}(\alpha-\rho)\omega_l^2$$

$$\lambda_x^r = (1-\alpha); \quad \lambda_l^r = -(\alpha-\rho)\omega_l; \quad \lambda_v^r = -(\alpha-\rho)\omega_u; \quad \lambda_w^r = -(\alpha-\rho)\omega_w$$

Details for the derivation, together with the expressions for the constant δ^r and the linearization coefficients ω_l, ω_u, and ω_w, can be found in Appendix B.5. Following the affine term structure literature, we refer to $\gamma^r = [\gamma_l^r \ \gamma_u^r \ \gamma_w^r]'$ as real factor loadings and to $\lambda^r = [\lambda_x^r \ \lambda_l^r \ \lambda_u^r \ \lambda_w^r]'$ as real prices of risk.

The conditional mean of the real pricing kernel is equal to

$$E_t \log n_{t+1} = -(\delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t)$$

and its conditional variance is

$$Var_t \log n_{t+1} = \left(\lambda_x^r\right)^2 u_t + \left(\lambda_l^r\right)^2 w_t + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2$$

The conditional mean depends both on expected consumption growth and stochastic volatility, whereas the conditional variance is a linear function of current stochastic volatility processes only. Notice that, in the standard time and state separable utility case, volatility is not priced as a separate source of risk and the real pricing kernel collapses to the familiar:

$$-\log n_{t+1} = \delta^r + (1-\alpha) l_t + (1-\alpha)\sqrt{u_t}\epsilon^x_{t+1}$$

Next, the real short rate is

$$\begin{aligned} r_t &\equiv -\log E_t(n_{t+1}) \\ &= \bar{r} + \gamma^r_l l_t + r^r_u u_t + r^r_w w_t \end{aligned}$$

where

$$\bar{r} = \delta^r - \frac{1}{2}[(\lambda^r_u \sigma_u)^2 + (\lambda^r_w \sigma_w)^2] \ ,$$

and

$$r^r_u = \gamma^r_u - \frac{1}{2}(\lambda^r_x)^2; \qquad r^r_w = \gamma^r_w - \frac{1}{2}(\lambda^r_l)^2 \ .$$

Assuming symmetry, the expression for the expected real depreciation, q^r_t, the real forward premium, $f^r_t - s^r_t$, and the real risk premium, p^r_t, are:[13]

$$q^r_t = \gamma^r_l(l_t - l^*_t) + \gamma^r_u(u_t - u^*_t) + \gamma^r_w(w_t - w^*_t) \ ,$$

$$\begin{aligned} f^r_t - s^r_t &= \gamma^r_l(l_t - l^*_t) + r^r_u(u_t - u^*_t) + r^r_w(w_t - w^*_t) \ , \\ p^r_t &= -\frac{1}{2}\left((\lambda^r_x)^2(u_t - u^*_t) + (\lambda^r_l)^2(w_t - w^*_t)\right) \ . \end{aligned}$$

Result 2: *The real UIP slope coefficient*

If all foreign and domestic parameter values are the same, then the real UIP regression parameter, obtained by the regressing the real interest rate differential on the real depreciation rate is:

$$\begin{aligned} b^r &= \frac{Cov(f^r_t - s^r_t, q^r_t)}{Var(f^r_t - s^r_t)} \\ &= \frac{(\gamma^r_l)^2\, Var(l_t - l^*_t) + \gamma^r_u r^r_u\, Var(u_t - u^*_t) + +\gamma^r_w r^r_w\, Var(w_t - w^*_t)}{(\gamma^r_l)^2\, Var(l_t - l^*_t) + (r^r_u)^2\, Var(u_t - u^*_t) + (r^r_w)^2\, Var(w_t - w^*_t)} \end{aligned}$$

[13] Symmetry means that both the parameters governing the motion of the state variables and the preference parameters are the same across countries.

Without the presence of both stochastic volatility and EZ preferences, b^r is equal to one and, in real terms, UIP holds identically. Also, when the long-run state variables, l_t and w_t, are perfectly correlated across countries, the slope coefficient reduces to $b^r = \gamma_u^r / r_u^r$. This is the case considered by Bansal and Shaliastovich (2010).

For b^r to be negative, we require $Cov(f_t^r - s_t^r, q_t^r) < 0$. The expression above makes it evident that only stochastic volatility terms can contribute negatively to this covariance. In particular, a necessary condition for a negative real slope coefficient is that the γ^r and $r^r = (r_u^r, r_w^r)'$ coefficients have opposite sign, for at least one of the stochastic volatility processes. A preference for the early resolution of risk ($\alpha < \rho$) and an EIS larger than one ($\rho < 0$) deliver the required covariations.

2.5.1 Inflation and the Nominal Pricing Kernel

Following the technique developed above, we guess that the solution for endogenous inflation has the form

$$\pi_t = a + a_1 l_t + a_2 u_t + a_3 w_t + a_4 z_t + a_5 v_t \ ,$$

substitute it into the Euler equation (2.3), compute the moments, and then solve for the a_j coefficients by matching up the result with the Taylor rule (2.36). This gives,

$$a_1 = \frac{\gamma_l - \tau_l}{\tau_\pi - \varphi_l}; \quad a_2 = \frac{\gamma_u - \frac{1}{2}\lambda_x^2}{\tau_\pi - \varphi_u}; \quad a_3 = \frac{\gamma_w - \frac{1}{2}\lambda_l^2}{\tau_\pi - \varphi_w};$$

$$a_4 = \frac{-1}{\tau_\pi - \varphi_z}; \quad a_5 = \frac{-\frac{1}{2}a_4^2}{\tau_\pi - \varphi_v}$$

$$\begin{aligned} a &= \frac{1}{\tau_\pi - 1}[\delta - \tau + a_2(1-\varphi_u)\theta_u + a_3(1-\varphi_w)\theta_w + a_5(1-\varphi_v)\theta_v \\ &\quad - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2 + (\lambda_v\sigma_v)^2] \end{aligned}$$

where the constant term, the factor loadings and the pricing of risk of the nominal pricing kernel are

$$\delta = \delta^r + a + a_2(1-\varphi_u)\theta_u + a_3(1-\varphi_w)\theta_w + a_5(1-\varphi_v)\theta_v$$

$$\gamma_l = \gamma_l^r + a_1\varphi_l; \quad \gamma_u = \gamma_u^r + a_2\varphi_u; \quad \gamma_w = \gamma_w^r + a_3\varphi_w; \quad \gamma_z = a_4\varphi_z; \quad \gamma_v = a_5\varphi_v$$

$$\lambda_x = \lambda_x^r; \quad \lambda_l = \lambda_l^r + a_1; \quad \lambda_u = \lambda_u^r + a_2; \quad \lambda_w = \lambda_w^r + a_3; \quad \lambda_z = a_4; \quad \lambda_v = a_5$$

The linearized nominal pricing kernel is

$$\begin{aligned} -\log m_{t+1} &= -\log n_{t+1} + \pi_{t+1} \\ &= \delta + \gamma_l l_t + \gamma_u u_t + \gamma_w w_t + \gamma_z z_t + \gamma_v v_t \\ &+ \lambda_x\sqrt{u_t}\epsilon_{t+1}^x + \lambda_l\sqrt{w_t}\epsilon_{t+1}^l + \lambda_u\sigma_u\epsilon_{t+1}^u \\ &+ \lambda_w\sigma_w\epsilon_{t+1}^w + \lambda_z\sqrt{v_t}\epsilon_{t+1}^z + \lambda_v\sigma_v\epsilon_{t+1}^v \ . \end{aligned}$$

The Taylor rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices of risk. This would not be the case if the inflation process was exogenously specified. On the other hand, the factor loadings and the prices of risk of the nominal state variables, z_t and v_t, depend exclusively on the choice of the Taylor rule parameters.

The nominal short rate is

$$\begin{aligned} i_t &\equiv -\log E_t(m_{t+1}) \\ &= \bar{\iota} + \gamma_l l_t + \gamma_z z_t + r_u v_t + r_w w_t + r_v v_t \ , \end{aligned}$$

where

$$\bar{\iota} = \delta - \frac{1}{2}[(\lambda_u\sigma_u)^2 + (\lambda_w\sigma_w)^2 + (\lambda_v\sigma_v)^2] \ ;$$

$$r_u = \gamma_u - \frac{1}{2}\lambda_x^2; \qquad r_w = \gamma_w - \frac{1}{2}\lambda_l^2; \qquad r_v = \gamma_v - \frac{1}{2}\lambda_z^2 \ .$$

The nominal interest rate differential, the expected depreciation rate and the risk premium can be derived from Equations (2.11–2.14). Assuming symmetry across countries, we have

$$q_t = \gamma_l(l_t - l_t^*) + \gamma_z(z_t - z_t^*) + \gamma_u(u_t - u_t^*) + \gamma_w(w_t - w_t^*) + \gamma_v(v_t - v_t^*) \ ,$$

$$f_t - s_t = \gamma_l(l_t - l_t^*) + \gamma_z(z_t - z_t^*) + r_u(u_t - u_t^*) + r_w(w_t - w_t^*) + r_v(v_t - v_t^*) \ ,$$
$$p_t = -\frac{1}{2}\left(\lambda_x^2(u_t - u_t^*) + \lambda_l^2(w_t - w_t^*) + \lambda_z^2(v_t - v_t^*)\right) \ .$$

Result 3: *The nominal UIP slope coefficient*

If all foreign and domestic parameter values are the same, the nominal UIP slope coefficient is

$$\begin{aligned} b &= \frac{Cov(f_t - s_t, q_t)}{Var(f_t - s_t)} \\ &= \frac{\gamma_l^2\, Var(l_t - l_t^*) + \gamma_z^2\, Var(z_t - z_t^*) + \gamma_u r_u\, Var(u_t - u_t^*) + \gamma_w r_w\, Var(w_t - w_t^*) + \gamma_v r_v\, Var(v_t - v_t^*)}{\gamma_l^2\, Var(l_t - l_t^*) + \gamma_z^2\, Var(z_t - z_t^*) + r_u^2\, Var(u_t - u_t^*) + r_w^2\, Var(w_t - w_t^*) + (r_v)^2\, Var(v_t - v_t^*)} \ . \end{aligned}$$

As was the case for the real UIP slope coefficient, without EZ preferences and stochastic volatility in consumption growth, long run risk and policy shock, $b = 1$.

Discussion

The results obtained in this section rely crucially on three ingredients: EZ preferences, stochastic volatility and the choice of the Taylor rule parameters. We now analyze their impact on the UIP slope coefficient and risk premium.

Remark 1: *With EZ preferences, volatility is priced as a separate source of risk*

From the previous section, we learned that if we want to explain the UIP puzzle we need stochastic volatility. In the model with real exchange rate variability, the necessary variation for the real UIP slope, b^r, comes from consumption growth, in the form of short-run volatility, u_t, and long-run volatility, w_t.

With standard expected utility ($\alpha = \rho$), both the volatility real factor loadings γ_u^r and γ_w^r, and the real prices of risk, λ_u^r and λ_w^r, collapse to zero. Consequently, the real UIP slope coefficient is identically equal to one. EZ preferences allow agents to receive a compensation for taking volatility risk, to which they would not be entitled with standard time-additive expected utility preferences. The contemporaneous presence of both stochastic volatility and EZ preferences is needed to explain the anomaly in real terms. Without stochastic volatility in the real pricing kernel, the real currency risk premium is constant and both of Fama's condition are violated. Without EZ preferences, stochastic volatility in consumption growth is not priced at all.

Remark 2: *The role of persistence in stochastic volatility*

Similarly to the purely nominal symmetric example of Appendix B.1, the persistence of country specific volatility φ_u plays a crucial role in the determination of the sign of the UIP slope. Too see this, consider again for simplicity the case in which the long-run factor l_t and its volatility w_t are perfectly correlated across countries. Also, assume the policy shock z_t is not autocorrelated ($\varphi_z = 0$). The nominal slope coefficient simplifies to

$$b \;=\; \frac{Cov(f_t - s_t, q_t)}{Var(f_t - s_t)} = \frac{\gamma_u r_u \, Var(u_t - u_t^*) + \gamma_v r_v \, Var(v_t - v_t^*)}{r_u^2 \, Var(u_t - u_t^*) + (r_v)^2 \, Var(v_t - v_t^*)} \; .$$

For the necessary condition of $Cov(f_t - s_t, p_t) < 0$ to be satisfied, we investigate the coefficients on short-run consumption volatility and policy shock volatility. First, γ_v and r_v cannot have opposite sign. The reason is the same as in the symmetric purely nominal example of the previous section: a shock to a *nominal* state variable of inflation, z_t or v_t, together with $\tau_\pi > 1$, imply the muted response of interest rate to a *nominal* shock, relative to that of the inflation rate. Therefore, unless the policy shock volatility is negatively autocorrelated, the contribution of the nominal state variables to $Cov(f_t - s_t, p_t)$ is necessarily of the wrong sign. As was the case for the purely nominal example, introducing asymmetries across countries, or allowing for interest rate smoothing in the Taylor rules can overcome this problem.

A different mechanism is at work for short-run consumption volatility. In this case, similarly to what we have seen above for the real slope coefficient, γ_u and r_u can have different signs, provided that the agents in the economy have preference for the early resolution of risk. However, a positive autocorrelation in stochastic volatility necessarily works against it. This is a direct consequence of endogenizing inflation and deriving the GHPZ monetary policy consistent pricing kernel. To see this, recall that

$$\gamma_u = \gamma_u^r + a_2 \varphi_u \; , \qquad a_2 = \frac{\gamma_u - \frac{1}{2}\lambda_x^2}{\tau_\pi - \varphi_u} = \frac{r_u}{\tau_\pi - \varphi_u} \; .$$

Since we require γ_u and r_u to have opposite signs for the resolution of the puzzle, we *must* have $a_2 < 0$. Therefore, $\gamma_u < \gamma_u^r$, and, all other things being equal, we require a stronger preference for the early resolution ($\alpha << \rho$) of risk, relative to the one we needed for the real case.

Consequently, it is in general harder to get a negative *nominal* UIP slope rather than a negative *real* UIP slope. As we have seen, the first reason is that the contribution of the nominal state variables necessarily goes in the wrong direction, at least in our simple symmetric case with Taylor rules reacting to (expected) consumption growth and current inflation. The second reason is that, with endogenous inflation, positive autocorrelated consumption volatility makes it harder to get the required magnitudes of the factor loadings and prices of risk of short- and long-run volatility. Nonetheless, a careful choice of Taylor parameters can deliver the required Fama conditions.

2.6 Quantitative Results

We'd like our model to be able to account for the following exchange rate facts. Foremost, of course, is the negative nominal UIP slope coefficient. But other important features are (i) UIP should hold unconditionally, so that the mean of the risk premium, p_t is zero, (ii) changes in real and nominal exchange rates are highly correlated (Mussa (1986)), (iii) exchange rate volatility is high relative to inflation differentials, (iv) exchange rates exhibit near random-walk behavior but interest rate differentials are highly autocorrelated, (v) international pricing kernels are highly correlated but international aggregate consumption growth rates are not (Brandt, Cochrane, and Santa-Clara (2006)). In addition, domestic real and nominal interest rates should be highly autocorrelated with means and volatilities that match data.

We calibrate our model using a monthly frequency. We begin by tying-down as much as we can using consumption data. The parameters for domestic and foreign aggregate consumption growth are chosen symmetrically so that (i) the mean and standard deviation match U.S. data, (ii) the autocorrelation is close to zero, (iii) the cross-country correlation is 0.30, and (iv) the autocorrelation of the conditional mean, l_t, is 0.993 and its cross-country correlation is 0.90 (following, roughly, Bansal and Shaliastovich (2010), Bansal and Yaron (2004) and Colacito and Croce (2011)). The autocorrelations of the short and long-run volatilities are chosen, primarily, to match the autocorrelation in interest rates and inflation rates. The parameters of the policy shock processes, z_t and z_t^*, are set so that the shocks are independent across countries and uncorrelated across time ($i.e.$, $\varphi_z = \varphi_z^* = 0$). Finally, the level and persistence of the volatility of the policy shocks are chosen — alongside

Parameter		Value
Subjective discount factor	β	0.999
Mean of consumption growth	μ	0.0016
Long run risk persistence	φ_l	0.99
Short run volatility level	θ_u	1.75×10^{-5}
Short run volatility persistence	φ_u	0.98
Short run volatility of volatility	σ_u	2.10×10^{-6}
Long run volatility mean	θ_w	2.80×10^{-8}
Long run volatility persistence	φ_w	0.98
Long run volatility of volatility	σ_w	3.40×10^{-9}
Policy shock persistence	φ_z	0
Policy shock volatility level	θ_v	8.33×10^{-6}
Policy shock volatility persistence	φ_v	0.98
Volatility of policy shock volatility	σ_v	1.05×10^{-6}
Cross-correlation short run growh shocks ϵ^x	η_x	0.292
Cross-correlation long run growth shocks ϵ^l	η_l	1
Cross-correlation short run volatility shocks ϵ^u	η_u	0
Cross-correlation long run volatility shocks ϵ^w	η_w	1
Cross-correlation policy shocks ϵ^z	η_z	0
Cross-correlation volatility policy shocks ϵ^v	η_v	0
Risk aversion	$1 - \alpha$	10
Elasticity of intertemporal substitution	$1/(1-\rho)$	2
Taylor-rule parameter, constant	$\bar{\tau}$	-0.00035
Taylor-rule parameter, inflation	τ_π	1.4
Taylor-rule parameter, expected consumption growth	τ_x	1.5

Table 2.1: Calibrated Parameter Values

risk aversion, intertemporal substitution, and the Taylor rule parameters — to match (i) the variance of the nominal exchange rate, (ii) the mean and variance of inflation and the nominal interest rate, (iii) the autocorrelation of the interest rate differential (forward premium), and (iv) the UIP regression parameter, b. The resulting parameter values are reported in Table 2.1.

Our model's population moments, evaluated at the parameter values of Table 2.1, are reported in Table 2.2. By and large, the model performs pretty well. Endogenous inflation

Moment	Sample	Population (Model)
$E(x_t) \times 12$	1.92	1.92
$\sigma(x_t) \times \sqrt{12}$		1.50
$Corr(x_t, x_t^*)$	≈ 0.3	0.32
$E(r_t) \times 12$		1.24
$\sigma(r_t) \times 12$		0.82
$\sigma(r_t - r_t^*) \times 12$		0.13
$E(i_t) \times 12$	6.42	5.60
$\sigma(i_t) \times 12$	3.72	2.99
$\sigma(i_t - i_t^*) \times 12$		0.43
$E(\pi_t) \times 12$	4.34	4.30
$\sigma(\pi_t) \times \sqrt{12}$	1.32	1.26
$Corr(r_t, r_{t-1})$		0.99
$Corr(i_t, i_{t-1})$	0.98	0.99
$Corr(i_t - i_{t-1}, i_t^* - i_{t-1}^*)$		0.98
$Corr(\pi_t, \pi_{t-1})$	0.71	0.67
b^r		-5.89
b	≈ -2	-1.07
$\sigma(s_{t+1} - s_t) \times \sqrt{12}$	≈ 15.00	17.41

Table 2.2: Sample and Population Moments

— the focal point of our paper — matches the the sample mean, variance and autocorrelation of the U.S. data. The same applies for interest rates and the interest rate differential (the forward premium). Simulations of these variables are reported in Figures 2.1 and 2.2. Real and nominal exchange rates fit the Mussa (1986) evidence. See Figure 2.3. Nominal exchange rate variability is higher than in the data, but only slightly, at 17.41% versus 15.0%. This is good news in light of the point made by Brandt, Cochrane, and Santa-Clara (2006); the high pricing kernel variability required to explain asset prices (Hansen and Jagannathan (1991)) requires either highly correlated foreign and domestic pricing kernels, highly variable exchange rates, or some combination of the two. With standard preferences, low cross-country consumption correlations rule out the former, thus implying that observed exchange rate variability is *too small* relative to theory. Our model resolves this tension with

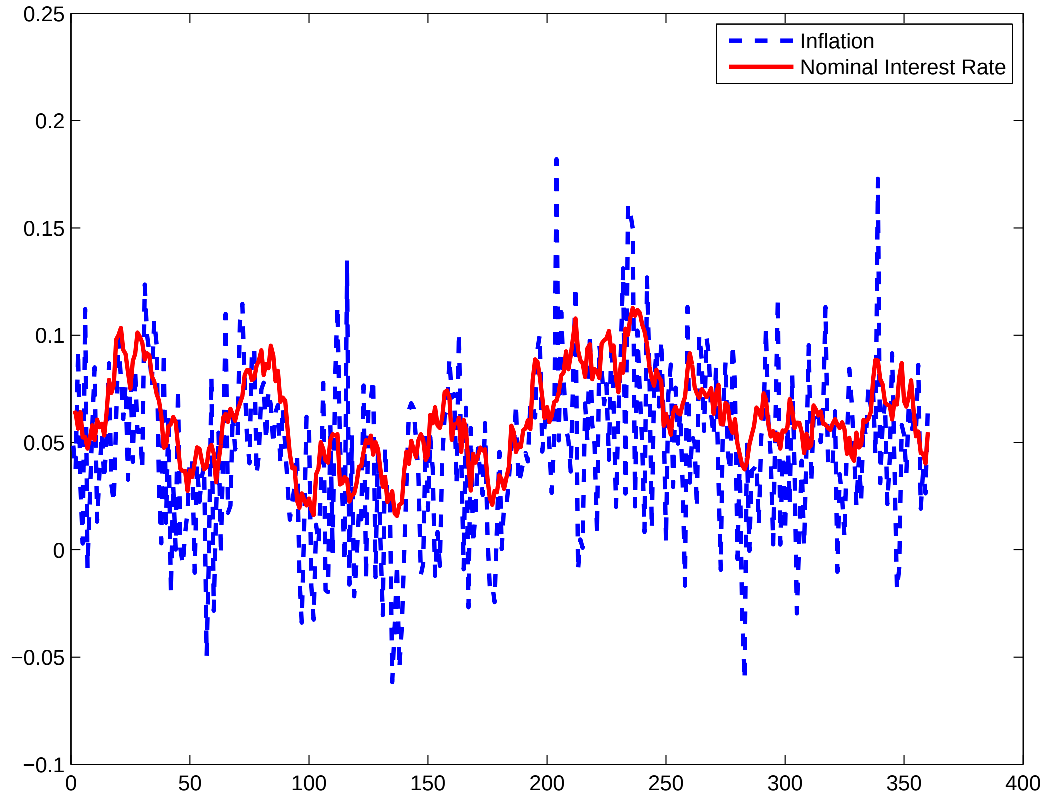

Figure 2.1: Annualized Inflation and the Nominal Interest Rate, 30-Year Simulation. Discussion in Section 2.6.

the combination of recursive preferences and high correlation in cross-country long-run risk processes. This point has been made previously by Colacito and Croce (2011). Its empirical validity is an open question.

Where our model falls somewhat short is in the magnitude of the *nominal* UIP slope coefficient. While Fama's conditions are satisfied — see Figure 2.4 and the "carry trade" graph, Figure 2.5 — we nevertheless get $b = -1.07$ whereas a rough average from the data is around $b = -2.00$. Herein lies our overall message, which echoes that of Appendix B.1. The restrictions on inflation imposed by the Taylor rule are binding in the sense that, although the slope coefficient for real variables may be strongly negative, its nominal counterpart is less so. Put differently, if the mapping between real and nominal variables is an *exogenous* inflation process, then, given our real model, a realistic nominal slope coefficient would be easy to obtain. *Endogenous* inflation, on the other hand, ties one's hands in an important manner.

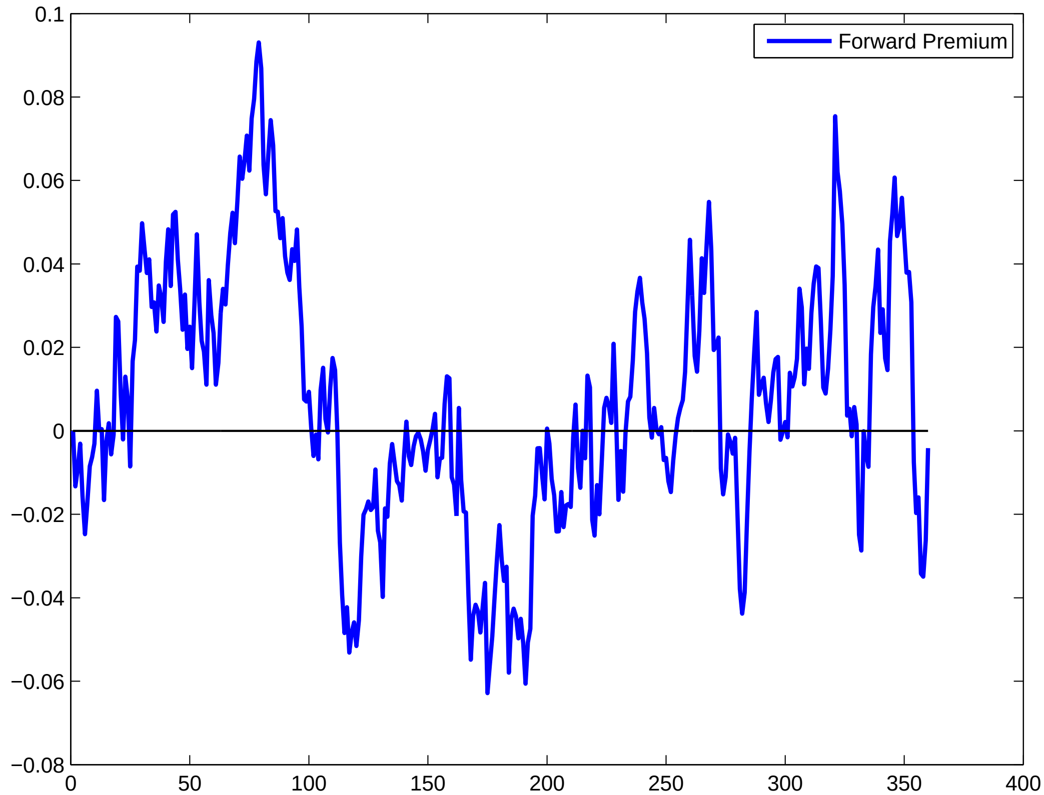

Figure 2.2: Annualized Interest Rate Differential (Forward Premium), 30-Year Simulation. Discussion in Section 2.6.

2.7 Conclusions

How is monetary policy related to the UIP puzzle? Ever since we've known about the apparent profitability of the currency carry trade people have speculated about a lurking role played by monetary policy. The story is that, for some reason, central banks find themselves on the short side of the trade, borrowing high yielding currencies to fund investments in low yielding currencies. In certain cases this has seemed almost obvious. It's well known, for instance, that in recent years the Reserve Bank of India has been accumulating USD reserves and, at the same time, sterilizing the impact on the domestic money supply through contractionary open-market operations. Since Indian interest rates have been relatively high, this policy basically *defines* what it means to be on the short side of the carry trade. This leads one to ask if carry trade losses are in some sense a *cost* of implementing Indian monetary policy? If so, is this a good policy? Is there some sense in which it is *causing* the exchange rate behavior associated with the carry trade?

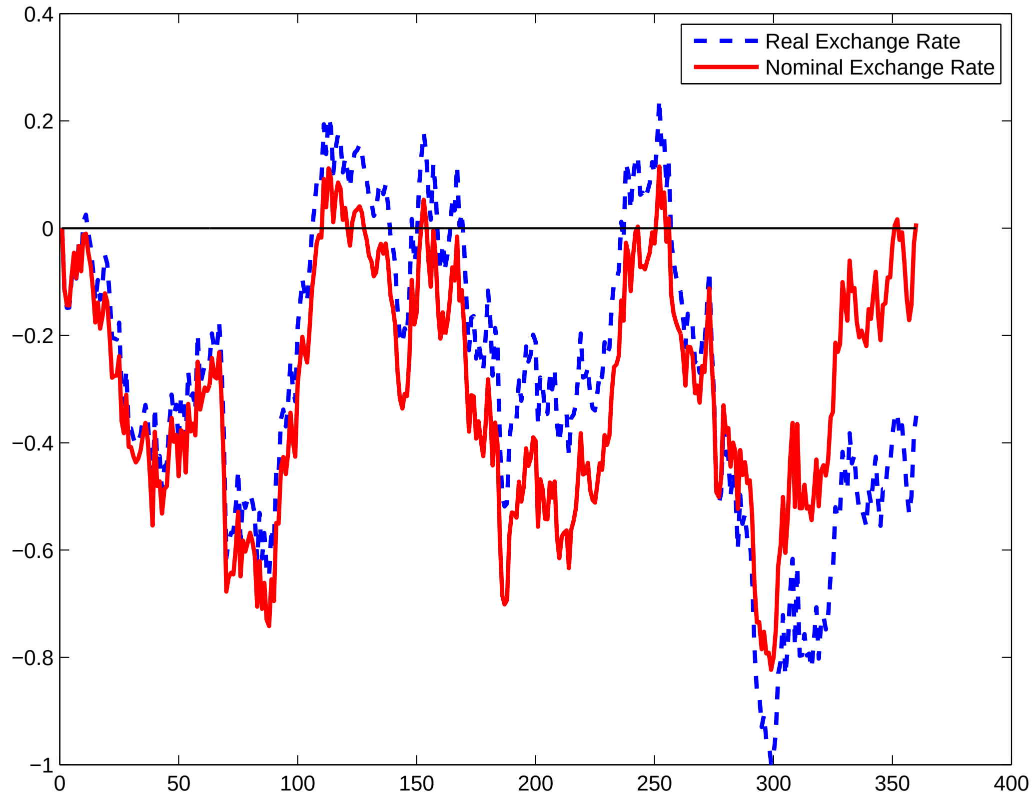

Figure 2.3: Log Real and Nominal Exchange Rate, 30-Year Simulation. Discussion in Section 2.6.

Our paper's questions, while related, are less ambitious than these speculations about India. What we've shown goes as follows. It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency *units*:

$$\frac{S_{t+1}}{S_t} = \frac{n^*_{t+1}\exp(-\pi^*_{t+1})}{n_{t+1}\exp(-\pi_{t+1})} \ .$$

It is less a tautology that we can write down sensible stochastic processes for these four variables that are consistent with the carry trade evidence.[14] Previous work has shown that such processes have many parameters that are difficult to identify with sample moments of data. Our paper shows two things. First, that by incorporating a Taylor rule for interest rate behavior we reduce the number of parameters. Doing so is sure to deteriorate the model's fit. But the benefit is lower dimensionality and parameters that are economically

[14] See, for example, Backus, Foresi, and Telmer (2001), Bakshi and Chen (1997), Bansal (1997), Brennan and Xia (2006), Frachot (1996), Lustig, Roussanov, and Verdelhan (2011), and Saá-Requejo (1994).

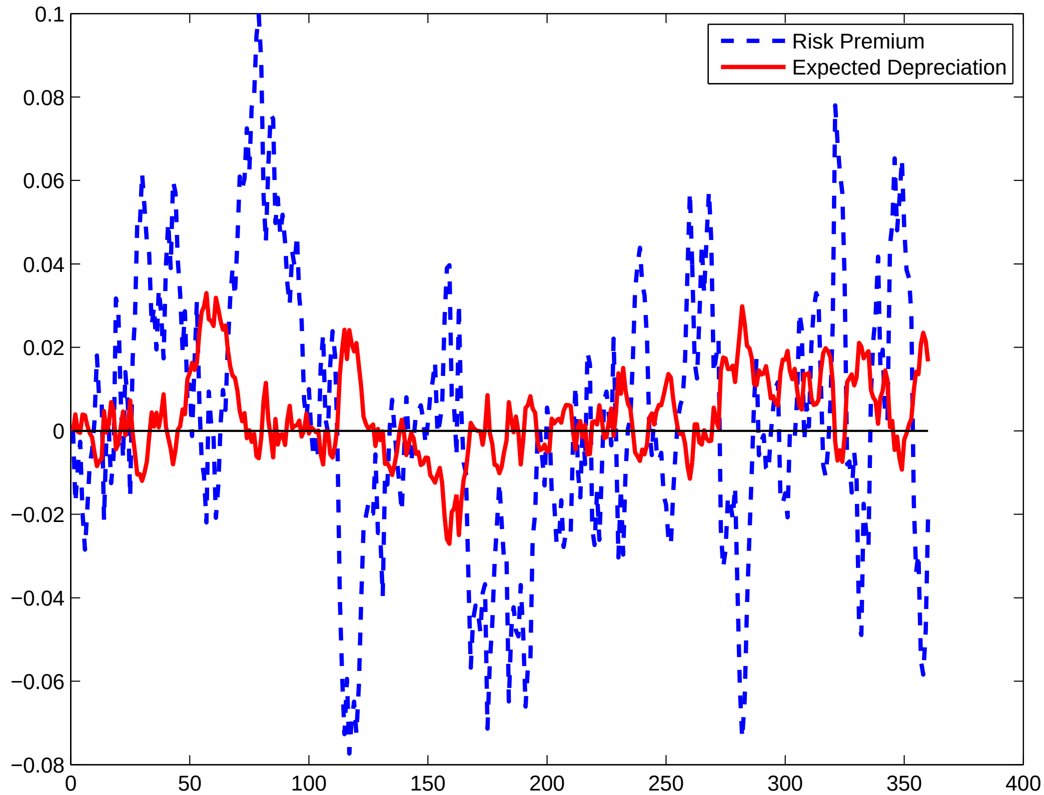

Figure 2.4: Currency Risk Premium and Expected Depreciation, 30-Year Simulation. Discussion in Section 2.6.

interpretable. Second, we've shown that some specifications of Taylor rules work and others don't. This seems helpful in and of itself. It also shows that there exist policy rules which, when combined with sensible pricing kernels, are *consistent* with the carry trade evidence. This is a far cry from saying that policy is *causing* carry trade behavior in interest rates and exchange rates, but it does suggest a connection that we find intriguing. In our models, for instance, there exist changes in the policy parameters, τ_π and τ_x, under which the carry trade profits go away.

Finally, it's worth noting that India, of course, is much more the exception than the rule. Most central banks — especially if we limit ourselves to those from OECD countries — don't have such explicit, foreign-currency related policies. However, many countries *do* use nominal interest rate targeting to implement domestic policy and, therefore, we can think about central banks and the carry trade in a *consolidated* sense. For example, in early 2004 the UK less U.S. interest rate differential was around 3%. Supposing that this was, to

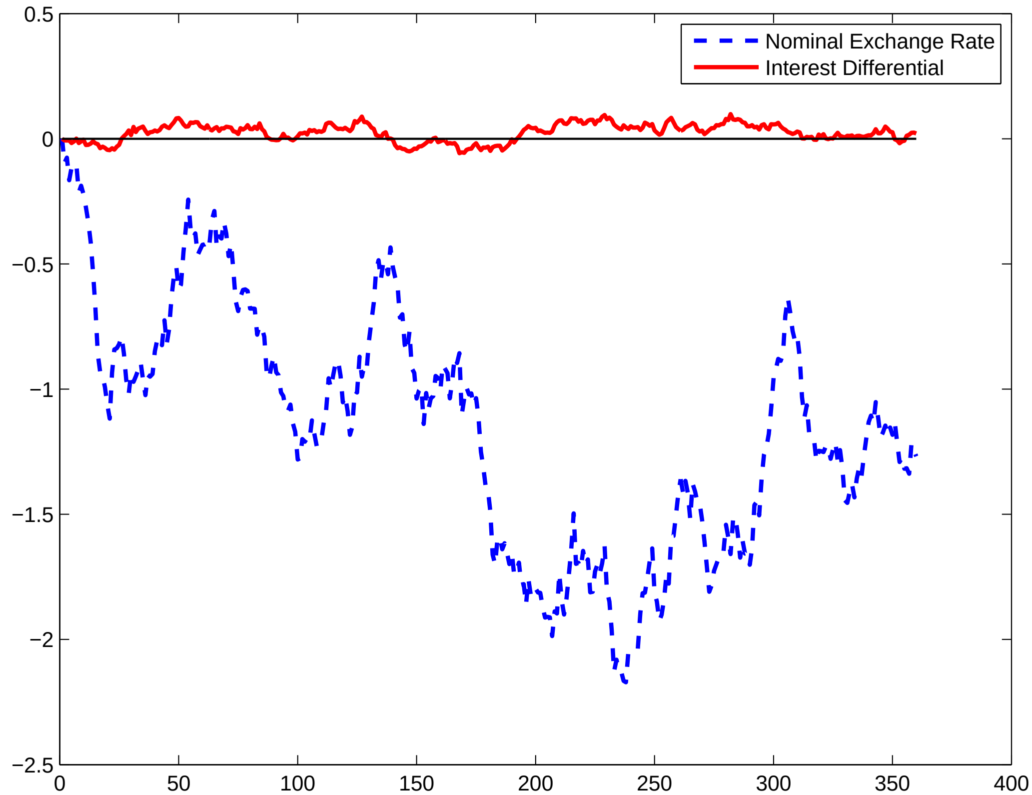

Figure 2.5: Log Nominal Exchange Rate and Interest Rate Differential, 30-Year Simulation. The interest rate differential is USD less GBP. The exchange rate is "price of GBP." So, UIP predicts that when the red line is above zero, the blue line will *increase*. The profitability of the carry trade is premised upon the opposite. While it's obviously not clear from the graph (as in an analogous graph of data), the latter tends to happen slightly more than the former. The graph also highlights the riskiness of the carry trade. Variation in nominal exchange rates is large relative to the interest differential and its components, p and q. This graph is discussed in Section 2.6.

some extent, a policy *choice*, consider the open-market operations required to implement such policies. The Bank of England would be contracting its balance sheet — selling UK government bonds — while (at least in a relative sense) the Fed would be expanding its balance sheet by buying U.S. government bonds. If the infamous carry-trader is in between, going long GBP and short USD, then we can think of the Fed funding the USD side of the carry trade and the Bank of England providing the funds for the GBP side. In other words, the consolidated balance sheets of the Fed and Bank of England are short the carry trade and the carry-trader is, of course, long. In this sense, central banks and their interest-rate policies may be playing a more important role than is apparent by just looking at their foreign exchange reserves.

Chapter 3

Nominal Frictions, Monetary Policy, and Long-Run Risk

3.1 Introduction

Bansal and Yaron (2004) show that long-run consumption risk — highly persistent variation in the conditional mean of consumption growth — can have quantitatively important implications for asset prices if investors have recursive preferences. This finding has given rise to an active empirical debate. At its heart lies a dilemma. Observed consumption growth looks *i.i.d.*, implying that any predictable variation must be small and hard to detect. Bansal and Yaron's insight was that a small component is enough. Recursive preferences do not *need* the predictable variation to be large. The dilemma, however, is that the basic story is difficult to verify. There exist many alternative models of the equity risk premium. How are we to discriminate between them and the long-run risk model if a tenet of the latter is "it's there but you can't see it?"

One approach for making progress is to use a more fully-articulated general equilibrium model to enlarge the set of testable restrictions. In such a model, consumption is an endogenous variable. So are output, capital and labor, wages, the return on capital, and so on. If consumption contains a magical, yet difficult-to-detect, ingredient then it is likely that these other variables are affected by the same ingredient. Examining them all in unison, through the lens of a model, might shed light on the plausibility of long-run consumption risk. This is the approach of Kaltenbrunner and Lochstoer (2010). They endogenize long-run risk in a real business cycle model. Their basic mechanism is the consumption smoothing motive that arises in a model with capital and investment. They show that it gives rise to high persistence in expected consumption growth. More recently, alternative mechanisms have been proposed. Kung and Schmid (2011) show that long-run risk arises in a production economy in which technological progress is determined endogenously by firms' R&D activity. Kuehn, Petrosky-Nadeau, and Zhang (2010) embed search frictions in the labor market of a real business cycle model and show that they generate high persistence in output and a large equity premium. My paper follows a similar path. I also endogenize long-run risk, but I propose a substantially different mechanism.

The mechanism emphasized here is *nominal frictions.* This paper asks if nominal frictions — and therefore monetary policy — imply consumption outcomes that display long-run risk behavior. There is, of course, no shortage of motivation for examining this mechanism. Economic theory — ranging from the classical economists, to both old and New Keynesian models, to the modern search-theoretic models of money — gives us many coherent reasons to believe in monetary non-neutrality. There is also much empirical evidence. Christiano, Eichenbaum, and Evans (1999) and Christiano, Eichenbaum, and Evans (2005) estimate the effects of monetary policy shocks using a structural vector autoregressive model and find that output, consumption, investments, real profits, real wages, and labor productivity fall in response to an exogenous monetary policy tightening. Sims (1992), Galí (1992), Bernanke and Mihov (1998), and Uhlig (2005) reach similar conclusions. Romer and Romer (1989) use an alternative identification approach and find that monetary policy shocks have large and persistent effects on production and unemployment. Closer to my paper is Rudebusch and Swanson (2012). They show that a New Keynesian model of nominal frictions can explain a fairly wide set of nominal term structure facts. My approach shares much in common. My model, while placing primary emphasis on the equity premium, also has sharp predictions for how nominal frictions affect real and nominal interest rates. But where my approach is distinctly different is in its emphasis on the distribution of consumption. I develop a general equilibrium model that formally ties all of this nominal-friction motivation to the consumption behavior emphasized by Bansal and Yaron (2004) and its descendants.

Examining how nominal frictions are connected to long-run risk, then, is my primary objective. But something that comes along with this is of equal importance. My model generates an endogenous process for inflation. It allows me to address questions like: What are the effects of nominal frictions on the inflation risk premium, the equilibrium consumption process, the term premium, and the equity premium? What are the effects of monetary policy? Much of the existing long-run risk literature is mute concerning these questions. It either focuses on real asset prices — thereby making it necessary to *estimate* real expected returns from nominal data — or it treats inflation as an exogenous process, which is appended to the model once real allocations and real asset prices are already determined (e.g., Bansal and Shaliastovich (2010)).

My model works as follows. It is a production economy in which the representative agent derives utility from the consumption of a basket of goods and disutility from supplying labor. Preferences are recursive as in Epstein and Zin (1989). Firms produce differentiated goods in a monopolistically competitive environment using a labor-only production function. The nominal frictions come from price stickiness, as in Calvo (1983). Production is subject to a permanent technology shock. Finally, to close the model, the monetary authority follows an interest rate rule. Two specifications that have received particular attention in the literature are considered. In the first one, the nominal interest rate is set as a function of current inflation and the current output gap, as in Taylor (1993). In the second one, the monetary authority smooths interest rates over time, as in Judd and Rudebusch (1998) and Clarida, Galí, and Gertler (2000). I refer to this rule as the one with policy inertia.

The model generates long-run consumption risk in the following sense. First, if there is no price stickiness, then output follows the dynamics of technology. I refer to this as the model *without* long-run risk, because realized and expected consumption growth have the same persistence. Second, if there is price stickiness, but no policy inertia, then a wedge arises in the dynamics of realized and expected consumption growth, resulting in a consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. I refer to this as the model with *exogenous* long-run risk, because the high persistence in expected consumption growth comes from the persistence of the exogenous technology growth process. Third, if there is price stickiness and the monetary authority smooths interest rates over time, the model generates *endogenous* long-run risk, because the inertial behavior of monetary policy induces 'momentum' in the representative agent's expectations about future consumption, resulting in a highly persistent expected consumption growth process. Note that this is a somewhat weaker result than the one obtained by Kaltenbrunner and Lochstoer (2010) who show that an *i.i.d* technology growth process is sufficient to generate endogenous long-run consumption risk. In a way that will be made clear soon, my model requires *some* of the persistence to be exogenously injected.

The price stickiness mechanism works as follows. At each point in time, only a fraction of firms can react to the technology shock affecting the economy by optimally adjusting their prices and, therefore, their production. The remaining fraction of firms, on the other hand, must keep their prices unchanged and produce a suboptimal level of output. Sluggishness in the production process induced by the nominal rigidities then reduces the persistence of realized consumption growth relative to the persistence of expected consumption growth. The larger the fraction of firms that cannot adjust prices, the lower the persistence of realized consumption growth. For levels of stickiness that are consistent with the microeconomic evidence (e.g., Bils and Klenow (2004)), this wedge is quantitatively significant, with realized consumption growth showing low persistence, as in the data, and expected consumption growth being highly persistent.

The model's nominal frictions generate real effects of monetary policy. All else being equal, a weak reaction of the monetary authority to current inflation and the output gap magnifies the effects of price rigidities on real allocations, thus reducing the persistence of realized consumption growth. While the reaction to current inflation and the output gap has a significant impact on the dynamics of realized consumption growth, interest rate inertia plays a crucial role in determining expectations about future consumption growth. Increasing the level of interest rate inertia makes firms react less aggressively in response to technology shocks, which in turn decreases the initial consumption reaction. This leads to high persistence in expected consumption growth, as production moves closer to the new steady state. High persistence in expected consumption growth then increases the amount of long-run risk in the model, resulting in a large equity premium.

An important ingredient is a *permanent* technology shock. This creates a positive correlation between realized and expected consumption growth. In the model of Bansal and Yaron (2004), shocks to realized consumption growth are independent of shocks to expected consumption growth. In the general equilibrium framework developed here, the correlation between realized and expected consumption growth depends on the nature of the shocks in the model. When the technology shock is permanent, bad times for realized consumption growth are associated with bad times for expected consumption growth. Given this, a preference for the early resolution of risk implies a large price of risk. In contrast, if the shock was transitory, bad times for realized consumption growth would be associated with good times for expected consumption growth because technology would be expected to revert to its mean. An agent with preference for the early resolution of risk would view the variation associated with expected consumption growth as a *hedge* for a bad realization of current consumption growth, and the overall price of risk would be small or even negative.

The model delivers new restrictions on the joint dynamics of real allocations and both real and nominal asset prices. These restrictions shed light on the ongoing debate over the predictability of consumption and dividend growth (e.g., Beeler and Campbell (2009) and Bansal, Kiku, and Yaron (2009)). The restrictions fall into two categories. First, asset prices can be used to test the predictability of not only consumption and dividends, as is standard in the long-run risk literature, but also wages and inflation. Second, *nominal* asset prices can be tested in their ability to predict real allocations. In the model, a high nominal interest rate and a high price-dividend ratio predict high future consumption growth, dividend growth, real wage growth, and inflation. The degree of predictability depends on the level of price stickiness and on the strength of monetary policy. Consumption, dividends, and wages are *more* predictable when nominal frictions are *small* and/or monetary policy is *strong*. When the price stickiness and the interest rate rule coefficients are consistent with empirical estimates for the U.S., the price-dividend ratio and the nominal interest rate positively predict future consumption, dividends, wages, and inflation, thus supporting the findings of Bansal, Kiku, and Yaron (2009).

A calibrated version of the model matches key features of the dynamics of U.S. consumption growth and asset returns, including a large equity premium (6.84%), a volatile return on equity (20.60%), and a low and smooth real risk-free interest rate (with an average level of 1.24% and a volatility of 1.66%, both annualized). Moreover, the equilibrium dynamics of the one-period nominal interest rate and of the inflation rate closely mimic those observed in the data. The nominal interest rate is highly persistent, and has an annualized mean of 5.56% and an annualized volatility of 2.91%. These results are obtained with a risk aversion coefficient that is consistent with the long-run risk literature and a level of price stickiness such that firms change their prices, on average, every 4.5 months. The risk aversion coefficient is lower than what is common in the long-run risk literature as a consequence of the permanent nature of the technology shock. The degree of price stickiness is a conservative measure of what has been documented at the microeconomic level. Among others, Bils and Klenow (2004) find that firms change prices every 4 to 6 months, while Nakamura and Steinsson (2008) suggests that, excluding price changes associated with sales, the average price duration is in the range of 8 to 11 months.

The rest of the paper is organized as follows. Section 3.2 describes the model, Section 3.3 inspects the mechanism at work and comments on the main theoretical results, and Section 3.4 shows quantitative results from two calibrated versions of the model, one *without* policy inertia, and one *with* policy inertia. Section 3.5 shows various sensitivity exercises, and Section 3.6 investigates additional asset pricing implications. Finally, Section 3.7 concludes.

3.2 The model

I model a production economy in which the representative agent derives utility from the consumption of a basket of goods and disutility from supplying labor for production. Firms produce a differentiated good in a monopolistically competitive environment with sticky prices. Monetary policy is conducted by means of an interest rate rule, in the spirit of Taylor (1993) and Clarida, Galí, and Gertler (2000). The nominal price rigidities in the model generate real effects of monetary policy. I will show that, under certain conditions, these real effects generate long-run risk in consumption growth.

3.2.1 Preferences

The representative agent chooses to maximize the recursive utility function given by Epstein and Zin (1989). The intertemporal utility function, V_t, over streams of consumption C_t and labor L_t is the solution to the recursive equation

$$V_t = \left\{(1-\beta)U(C_t, L_t)^{1-\psi} + \beta E_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}} \quad , \tag{3.1}$$

where β characterizes impatience, ψ^{-1} measures the elasticity of intertemporal substitution over the consumption-labor bundle, and γ measures relative risk aversion towards static gambles over the bundle. The relative magnitude of ψ and γ determines whether agents prefer early resolution of risk ($\gamma > \psi$), late resolution of risk ($\gamma < \psi$), or are indifferent to the timing of resolution of risk ($\gamma = \psi$). The intratemporal utility of consumption and labor is

$$U(C_t, L_t)^{1-\psi} = \left(\frac{C_t^{1-\psi}}{1-\psi} - \chi A_t^{1-\psi}\frac{L_t^{1+\omega}}{1+\omega}\right) \quad ,$$

where ω^{-1} is the non-compensated elasticity of substitution of labor supply, and χ determines the average time the representative agent spends at work. Scaling the disutility of labor by the technology trend A_t is necessary for the model to be consistent with balanced growth (see Uhlig (2010)). The process for A_t will be described in Section 3.2.2.[1]

The consumption good C_t is a Dixit-Stiglitz basket of the differentiated goods $C_t(j)$, for

[1]The results of the paper are not affected by the introduction of the technology trend A_t in the intratemporal utility function. Importantly, to ensure balanced growth, one need only scale the disutility of labor by the *deterministic* trend of technology and omit any stochastic component. Here, I consider the full technology process A_t for analytical convenience. See Rudebusch and Swanson (2012) for a similar specification.

$j \in [0,1]$. Specifically,

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \quad , \tag{3.2}$$

where θ is the elasticity of substitution across goods. Likewise, aggregate labor is specified as

$$L_t = \left[\int_0^1 L_t(j)^{1+\omega} dj\right]^{\frac{1}{1+\omega}} \quad , \tag{3.3}$$

where $L_t(j)$ is the labor supplied in the production of good j.

Financial markets are complete. The intertemporal budget constraint faced by representative household is

$$E_t\left[\sum_{s=0}^{\infty} M_{t,t+s} P_{t+s} C_{t+s}\right] \leq E_t\left[\sum_{s=0}^{\infty} M_{t,t+s}\left(\int_0^1 W_{t+s}(j) L_{t+s}(j) + P_{t+s}\Psi_{t+s}\right)\right] \quad ,$$

where $M_{t,t+s}$ is the nominal pricing kernel, $W_t(j)$ is nominal wage earned in the production of good j, Ψ_t measures the aggregate profits from production, and

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$$

is the nominal price of a unit of the basket of goods, where $P_t(j)$ is the price of good j.

The representative agent's maximization problem involves two steps. In the first step, the agent optimally allocates her resources across each differentiated good in a purely static fashion. As is standard in the Dixit-Stiglitz environment, this static optimization implies the following demand schedule:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t \quad .$$

In the second step, the agent solves a dynamic programming problem, arriving at the intertemporal allocation of consumption and savings. In particular,

$$\frac{W_t}{P_t} = \chi A_t^{1-\psi} C_t^{\psi} L_t^{\omega} \quad , \tag{3.4}$$

$$e^{-i_t} = E_t(M_{t,t+1}) \quad . \tag{3.5}$$

Details for the derivation can be found in Appendix C.1.1. Condition (3.4) describes the evolution of real wages, and condition (3.5) is the Euler equation for the one-period nominal interest rate i_t. The intertemporal marginal rate of substitution (the real pricing kernel), $N_{t,t+1} \equiv M_{t,t+1}(P_{t+1}/P_t)$, is given by

$$N_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\psi} \left(\frac{V_{t+1}}{E_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma} \quad . \tag{3.6}$$

It is a crucial ingredient of our model and will be discussed in Section 3.3.2.

Using (3.1), it is useful to express the scaled value function $v_t \equiv \log \frac{V_t}{C_t}$ as

$$e^{(1-\psi)v_t} \quad = \quad (1-\beta)\left(\frac{U(C_t, L_t)}{C_t}\right)^{1-\psi} + \beta e^{\left(\frac{1-\psi}{1-\gamma}\right)\log E_t\left[e^{(1-\gamma)(v_{t+1}+\Delta c_{t+1})}\right]} \quad ,$$

where $\Delta c_{t+1} \equiv \log \frac{C_{t+1}}{C_t}$ denotes (log) consumption growth. Thus, a solution to the representative agent's optimization problem can be characterized by a solution for the process v_t, which, in turn, depends on the endogenous solution for consumption growth, Δc_{t+1}.

3.2.2 Firms

Each firm $j \in [0,1]$ produces a differentiated good, but they all use an identical linear labor-only technology

$$Y_t(j) = A_t L_t(j) \quad . \tag{3.7}$$

I abstract from endogenous capital accumulation because Kaltenbrunner and Lochstoer (2010) show that, in a real business cycle model where the representative agent has recursive preferences, endogenous investment decisions can long-run risk in consumption growth, even in the absence of nominal frictions. By abstracting from capital accumulation, I isolate the effect of nominal frictions on realized consumption growth, expected consumption growth and asset prices.

The technology shock A_t evolves according to the following autoregressive process:

$$\log(A_t/A_{t-1}) \equiv \Delta a_t = (1-\varphi_a)\theta_a + \varphi_a \Delta a_{t-1} + \sigma_a \epsilon_t^a \quad . \tag{3.8}$$

The assumption of a difference stationary process for technology is common in the literature. Among others, Campbell (1994) and Rouwenhorst (1995) consider it in the context of a standard real business cycle model. More recently, based on the empirical estimates of

Nelson and Plosser (1982), Goodfriend and King (2009) adopt such a process in the context of a New Keynesian model.[2]

Producers have market power to set the price of their differentiated goods in a Calvo (1983) staggered price setting. That is, each firm may reset its price with probability $(1-\alpha)$ in any given period, independently of the time elapsed since the previous adjustment. Similarly to Yun (1996), the remaining fraction α must charge the previous period's price times an exogenous inflation target Π^*. When a producer can adjust its price optimally, the price is set to maximize the present value of expected future profits. The maximization problem of firm j can be written as

$$\max_{P_t^*(j)} E_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s} \left[P_t^*(j)(\Pi^*) Y_{t+s|t}(j) - W_{t+s|t}(j) L_{t+s|t}(j) \right] \right\} \quad ,$$

where $P_t^*(j)$ is the optimal price chosen by firm j at time t, $Y_{t+s|t}(j)$, $W_{t+s|t}(j)$, and $L_{t+s|t}(j)$ denote product, wages and labor for the firm j at time $t+s$, when the last price adjustment was at time t. The maximization problem is subject to the production function (3.7) and the product demand function

$$Y_{t+s|t}(j) = \left(\frac{P_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \quad , \tag{3.9}$$

where $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$ is the aggregate output.

As each firm that chooses a new price for its good in period t faces the same decision problem, the optimal price $P_t^*(j)$ is the same for all of them, and so in equilibrium, all prices that are chosen at time t have the common value $P_t^*(j) = P_t^*$. The Calvo price environment described above implies

$$P_t \equiv \left[\int_0^1 P_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}} = \left[(1-\alpha)(P_t^*)^{1-\theta} + \alpha(\Pi^* P_{t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad , \tag{3.10}$$

so that in order to determine the price level P_t, one need only know its initial value P_{t-1} and the single new price P_t^* that is chosen each period, without any reference about the

[2] Croce (2010), in the context of a standard real business cycle model, considers an exogenous technology growth process that contains a persistent time-varying component. He shows that such a process reproduces important features of both asset prices and macroeconomic quantities, including long-run consumption risk. In the next Section I show that, in the presence of nominal frictions, the simple autoregressive process for technology growth considered here is sufficient to deliver long-run consumption risk.

prices chosen by the firms in the past.[3]

The first order condition for the firm implies that the optimal price P_t^* satisfies

$$P_t^* = \mu \; \frac{E_t \sum_{s=0}^{\infty} \alpha^s \; M_{t,t+s} \; Y_{t+s|t} \; MC_{t+s|t}}{E_t \sum_{s=0}^{\infty} (\alpha \Pi^*)^s \; M_{t,t+s} \; Y_{t+s|t}} \quad , \tag{3.11}$$

where $\mu \equiv \theta/(\theta - 1)$ is the frictionless markup, and $MC_{t+s|t} = \frac{W_{t+s|t}}{A_{t+s}}$ is the nominal marginal cost. Details for the derivation can be found in Appendix C.1.2.

3.2.3 Monetary policy

The monetary authority sets the one-period nominal interest rate according to an interest rate rule. In particular,

$$i_t = \overline{\tau} + \tau_\pi \; (\pi_t - \pi^*) + \tau_x x_t + \tau_i \; i_{t-1} + u_t \quad , \tag{3.12}$$

where $(\pi_t - \pi^*) \equiv \log(\Pi_t/\Pi^*)$ denotes inflation deviations from its target, with $\Pi_t \equiv (P_t/P_{t-1})$. The output gap x_t is

$$x_t \equiv \log\left(\frac{Y_t}{Y_t^F}\right) \quad .$$

It is a measure of the deviation of total output Y_t from the natural level of output Y_t^F that would be produced in the economy if prices were perfectly flexible. The coefficient τ_π and τ_x measure the reaction of the monetary authority to inflation and output gap, and the constant $\overline{\tau}$ is used to determine the average level of i_t. The coefficient τ_i governs the strength of monetary policy inertia, and u_t is a zero-mean normally distributed *i.i.d.* policy shock with volatility σ_u. A similar specification for the interest rate rule is used by Judd and Rudebusch (1998), Clarida, Galí, and Gertler (2000), and Orphanides (2004).

3.2.4 Equilibrium and Solution Method

Under the constraints that consumption equals production ($C_t(j) = Y_t(j)$, for all j) and labor demand equals labor supply, the equilibrium allocations and prices simultaneously satisfy the first order conditions of the maximization problem of the representative agent (conditions (3.4), (3.5) and (3.6)), the firms' optimality condition (3.11), together with

[3] This is precisely true only in the case of specific factor markets, as in Woodford (2003).

equation (3.10) describing the evolution of the price index, and the interest rate rule (3.12).

I solve the model by first obtaining a log-linearized approximation of the equilibrium conditions and then exploiting the affine structure of the linearized model to find a solution to the approximated system. Details of the derivation can be found in Appendix C.2. After deriving the approximated equilibrium conditions of the model, I use the method of undetermined coefficients to find the equilibrium dynamics of the endogenous variables (see, among others, McCallum (1983)). In particular, I show that the output gap, inflation and the scaled value function can be written as:

$$\begin{aligned} x_t &= \overline{x} + x_a \Delta a_t + x_i \; i_{t-1} + x_u u_t \\ \pi_t - \pi^* &= \overline{\pi} + \pi_a \Delta a_t + \pi_i \; i_{t-1} + \pi_u u_t \\ v_t &= \overline{v} + v_a \Delta a_t + v_i \; i_{t-1} + v_u u_t \quad , \end{aligned} \tag{3.13}$$

where the equilibrium coefficients satisfy the equilibrium conditions and depend on the structural parameters of the economy.

3.3 Mechanism and Results

In this section, I describe the main results of the paper and inspect the mechanism at work. First, I show how the joint effect of price stickiness and monetary policy affects the equilibrium consumption growth process. Then, I describe the main features of the pricing kernel. Last, I derive closed-form expression for the price-consumption ratio and the equity premium.

3.3.1 Consumption Growth Dynamics

In the presence of price stickiness ($\alpha \neq 0$), the persistence of realized consumption growth is lower than the persistence of expected consumption growth. As this result is better understood in the case without monetary policy inertia ($\tau_i = 0$), I first describe the mechanism of the model when the monetary authority reacts only to current inflation and current output gap. Then, in following Section, I allow for monetary policy inertia ($\tau_i \neq 0$) and show how the inertial rule is instrumental in generating, endogenously, high persistence in expected consumption growth. The policy shock u_t does not play a significant role in this mechanism and will be set equal to zero for the remainder of this section (that is $u_t = 0$, for every t).

No Policy Inertia

As discussed in the Introduction, three ingredients are necessary to generate consumption dynamics and asset price behavior similar to what is obtained in the typical long-run risk context of Bansal and Yaron (2004): (i) sticky prices, which deliver low correlation in realized consumption growth but high correlation in expected consumption growth; (ii) Epstein-Zin preferences; and (iii) a permanent technology shock, so that the correlation between realized and expected consumption growth is positive. Epstein-Zin preferences are necessary to *price* long-run risk, as only a representative agent of that kind requires a compensation for being exposed to shocks to expected consumption growth, while ingredients (i) and (iii) are necessary to obtain consumption dynamics consistent to the ones assumed in long-run risk models.

In equilibrium, when there is no policy inertia ($\tau_i = 0$), realized consumption growth is equal to

$$\Delta c_t = \Delta y_t = \Delta y_t^F + \Delta x_t \quad , \tag{3.14}$$

where the second equality follows from the definition of the output gap. Understanding the dynamics of consumption therefore requires understanding the properties of the flexible output growth and the output gap. When prices are fully flexible, firms maximize profits, $P_t(j)Y_t(j) - W_t(j)L_t(j)$, subject to the technology constraint (3.7) and the demand function (3.9). It is easy to show that $\Delta y_t^F = \Delta a_t$. With flexible prices, there is no output gap, and output is always at its natural level. Therefore, if prices were flexible, equilibrium consumption growth would inherit the dynamics of the technology shock. In particular, the persistence of realized consumption growth would be equal to the persistence of expected consumption growth, that is

$$Corr(\Delta c_t^F, \Delta c_{t+1}^F) = Corr\big(E_t(\Delta c_{t+1}^F), E_{t+1}(\Delta c_{t+2}^F)\big) = \varphi_a \quad .$$

The result above makes it clear that, without nominal frictions, the model cannot recreate the requisite discrepancy in the autocorrelation dynamics of realized and expected consumption growth. In particular, the high autocorrelation in expected consumption growth required by long-run risk models necessarily implies the same high autocorrelation in realized consumption growth, which is highly at odds with the data. I refer to this model as the model *without* long-run risk.

When prices are sticky, monetary policy affects real allocations and moves realized output away from its flexible-price level. Excluding constants,

$$\Delta c_{t+1} = g_a \ \Delta a_t + (1 + x_a)\sigma_a \epsilon^a_{t+1} \tag{3.15}$$

and, therefore, expected consumption growth, $E_t \Delta c_{t+1}$, is given by

$$E_t \Delta c_{t+1} = g_a \ \Delta a_t \quad , \tag{3.16}$$

where the coefficient g_a depends on the structural parameters of the economy. From (3.15) and (3.16), I obtain the following Result.

Result 1: *Autocorrelation of realized and expected consumption growth*

> The first order autocorrelation of realized consumption growth is lower than the first order autocorrelation of expected consumption growth. In particular,
>
> $$Corr(\Delta c_t, \Delta c_{t+1}) < Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2}) = \varphi_a \quad .$$
>
> Also, the first order autocorrelation of realized consumption growth is
>
> (i) increasing in the probability $(1 - \alpha)$ of a firm being able to adjust its price optimally,
>
> (ii) increasing in the interest rate rule reaction to inflation, τ_π,
>
> (iii) increasing in the interest rate rule reaction to the output gap, τ_x.
>
> See Appendix C.5 for the proof. □

Important conclusions can be drawn from Result 1. First, the persistence of expected consumption growth is equal to the persistence of the output gap and of the technology shock (see equations (3.13) and (3.16)). In particular, it is not affected by the presence of nominal frictions. This result is a consequence of the temporary effects of monetary policy on real allocations and is specific to the case without policy inertia. I refer to this model as the *exogenous* long-run risk model, because the required high persistence in expected consumption growth necessarily comes from the persistence of the exogenous shock. In Section 3.3.1, when the monetary policy rule allows for interest rate inertia, the autocorrelation of expected consumption growth depends on the structural parameters of the economy, including the level of nominal frictions and the inertial coefficient in the interest rate rule.

Realized consumption growth is less persistent than expected consumption growth, and therefore technology, because of the presence of sticky prices. This is intuitive. Consider the case in which the economy is hit by a positive permanent technology shock at time t. The level of technology is permanently higher so that the optimal level of production goes up. As I showed before, when prices are flexible, the growth rate of production and, in equilibrium, the growth rate of consumption perfectly inherits the dynamics of the technology shock. Instead, when prices are sticky, only a fraction $(1-\alpha)$ of the firms can adjust prices optimally and produce the newly optimal level of output. The fraction of firms that cannot revise prices will face a higher demand for their products and will therefore increase production more than they would if their prices were flexible.[4] In the following period, a fraction of the firms will again be unable to set their prices optimally, and so on. The fact that at each point in time only a fraction of firms can optimally react to the shocks in the economy breaks the perfect tie between the autocorrelation dynamics of realized consumption growth and technology growth, making realized consumption growth less persistent than technology and, in light of Result 1, expected consumption growth. As the fraction of firms that are able to adjust their prices goes down – that is, as α gets larger – the persistence of realized consumption growth goes down, as more firms will be unable to react optimally to the shocks hitting the economy.

The persistence of realized consumption growth is increasing in the interest rate rule coefficients. When the interest rate rule reaction to current macroeconomic conditions is stronger, prices are smoother, and the output gap is smaller. Roughly speaking, and all else being equal, when monetary policy is strong, the component of realized consumption growth coming from changes in the output gap is less relevant (see equation (3.14)) and the autocorrelation of consumption growth will be closer to the autocorrelation of technology and, therefore, to the autocorrelation of expected consumption growth. In other words, for a *given* level of price stickiness, a weak monetary policy magnifies the effects of nominal

[4] To see this, one must consider the effects of a positive technology shock on the average real marginal cost in the economy, $MC_t/P_t = (W_t/P_t)/A_t$. The technology shock affects marginal costs through two channels: a direct channel, which decreases real marginal costs; and an indirect channel, which increases real wages W_t/P_t, and therefore marginal costs, due to higher income. Given the permanent nature of the technology shock, the indirect channel is stronger, so that the average real marginal cost goes up. On average, firms should therefore increase their prices, but only a fraction of them can do so. Therefore, agents shift their demand towards the relatively cheap goods produced by the firms that cannot revise their prices, which eventually increase production more than they would if their prices were flexible.

frictions on realized consumption growth, thus increasing the wedge with the dynamics of expected consumption growth.

In the model, nothing would prevent the monetary authority from offsetting the welfare inefficiencies caused by the nominal frictions. If the reaction to inflation and output gap is strong enough, the effects of price stickiness disappear and the equilibrium allocation coincides with the flexible price allocation, which is Pareto efficient.[5] In Appendix C.6, I discuss the case in which the presence of a shock to the disutility of labor introduces a trade-off between the optimal level of inflation and output. While I do not explicitly derive the optimal policy in that context, I show that the monetary authority does not have an outright incentive to completely offset the effects of price stickiness. Moreover, in Section 3.4, when I calibrate the model, I show that when the interest rate rule coefficients are consistent with the empirical evidence, a mild level of price stickiness is sufficient to have quantitatively significant effects on the dynamics of realized and expected consumption growth.

In the original work of Bansal and Yaron (2004), realized and expected consumption growth are affected by independent shocks. This is not possible in the context of the general equilibrium model analyzed here, since shocks affect the dynamics of all endogenous variables. Consequently, the conditional correlation between realized and expected consumption growth is determined endogenously and ultimately depends on the nature of the shocks. This leads to the following Result.

Result 2: *Consumption growth correlation*

> The conditional correlation between realized consumption consumption growth and expected consumption growth is equal to one:
>
> $$Corr_{t-1}(\Delta c_t, E_t \Delta c_{t+1}) = +1 \quad .$$
>
> See Appendix C.5 for the proof. □

Result 2 is a consequence of the permanent nature of the technology shock. When technology is hit by a positive permanent shock, the optimal level of production is permanently higher. Realized output is high and therefore realized consumption growth is high. Since in the model technology growth is positively autocorrelated, expected consumption growth

[5] Precisely, this is true only after the introduction of an employment subsidy to offset the effects of monopolistic competition, as in Galí (2008).

is also high. When the representative agent has preference for the early resolution of risk, shocks to expected consumption growth carry a large and positive price of risk and result in a large risk premium. Kaltenbrunner and Lochstoer (2010) make a similar point in the context of a standard real business cycle model.[6]

Results 1 and 2 show that, for a suitable parametrization – large autocorrelation in technology growth, a sensible level of nominal frictions, and a *not-too-strong* reaction of the monetary authority to inflation and the output gap – the model can generate consumption dynamics consistent with the long-run risk literature. These results are independent from the preferences of the representative agent and could equivalently be derived in the standard case with power utility. However, in the latter case, a highly persistent variation in expected consumption growth would have no effect on asset prices. Only when the representative agent has recursive preferences, the high variation in expected consumption growth matters because it directly affects the agent's marginal rate of substitution. The New Keynesian literature typically assumes power utility and therefore cannot say anything regarding the risk associated with variation in expected consumption growth.

While the model without interest rate inertia is capable of replicating the main features of the dynamics of consumption growth typical of the long-run risk literature, it fails in generating any endogenous persistence in the dynamics of the equilibrium variables. I address this issue in the next Section and show that a monetary policy rule with interest rate inertia induces high persistence in the dynamics of equilibrium expected consumption growth, thus delivering *endogenous* long-run risk.

Policy Inertia

When the monetary authority has the explicit desire to smooth interest rates over time, the model contains an amplification and propagation mechanism that generates *endogenous* persistence in expected consumption growth. Keeping the policy shock fixed at zero ($u_t = 0$, for every t), the equilibrium consumption growth process generalizes (3.15) and (3.16) as follows (omitting constants):

$$\Delta c_{t+1} = g_a \Delta a_t + g_i i_{t-1} + (1 + x_a)\sigma_a \epsilon^a_{t+1} \tag{3.17}$$

$$E_t(\Delta c_{t+1}) = g_a \Delta a_t + g_i i_{t-1} \quad , \tag{3.18}$$

[6]In the presence of more that one shock, the correlation need not be equal to +1. In general, a non-negative correlation is needed to obtain a large risk premium when the representative agent has preference for the early resolution of risk.

where the expressions for g_a and g_i depend on the structural parameters of the economy. Details for the derivation can be found in Appendix C.4.

From (3.17) and (3.18), one can see that the autocorrelation of expected consumption growth now depends not only on the persistence of the technology growth process, but also on the persistence of the nominal interest rate. The contribution to the persistence of expected consumption growth coming from technology growth, relative to the contribution of the nominal interest rate, depends on their relative variability, together with the relative magnitude of the g_a and g_i parameters.

In the model, interest rate inertia reduces the initial reaction of output to a technology shock. This leads to high persistence in expected consumption growth, as output is expected to move towards its new steady state. Overall, this effect manifests itself in a large g_i coefficient. Intuitively, the lagged interest rate matters more, relative to the current level of technology growth, in determining today's expectations about future consumption growth. When the model is calibrated to match the high autocorrelation in nominal interest rate observed in the data, the persistence of expected consumption growth is significantly higher than the persistence of technology growth. However, as was the case without policy inertia, this high persistence in expected consumption growth does not transfer one-to-one to the persistence of realized of consumption growth, because at each point in time price stickiness forces some firms to act suboptimally. This wedge manifests itself in the innovation term in equation (3.17).

Figures 3.1–3.4 show the impulse-response functions of the model with interest rate inertia following an (annualized) one-percent permanent technology shock and provide further support for the intuition behind the propagation mechanism at work. The autocorrelation coefficient of technology growth is set equal to $\varphi_a = 0.5$, as this value will be used in the main calibration of Section 3.4. The desire of the monetary authority to smooth the interest rate over time reduces the initial reaction of the *nominal* interest rate to a technology shock, as the equilibrium lagged interest rate level provides an anchor to the movements of the nominal rate. Because of price stickiness, a similar mechanism affects the real marginal costs, which still increase following a positive technology shock, but *not as much* as they would have increased in the absence of interest rate inertia. Given the not-as-large increase in marginal costs, firms increase their prices, but *not as much*. This results in a smaller initial increase in production, relative to the case without monetary policy inertia.

In equilibrium, this mechanism delivers: i) an initial smaller reaction of consumption; ii) an increase in the persistence of expected consumption growth, as output is expected to

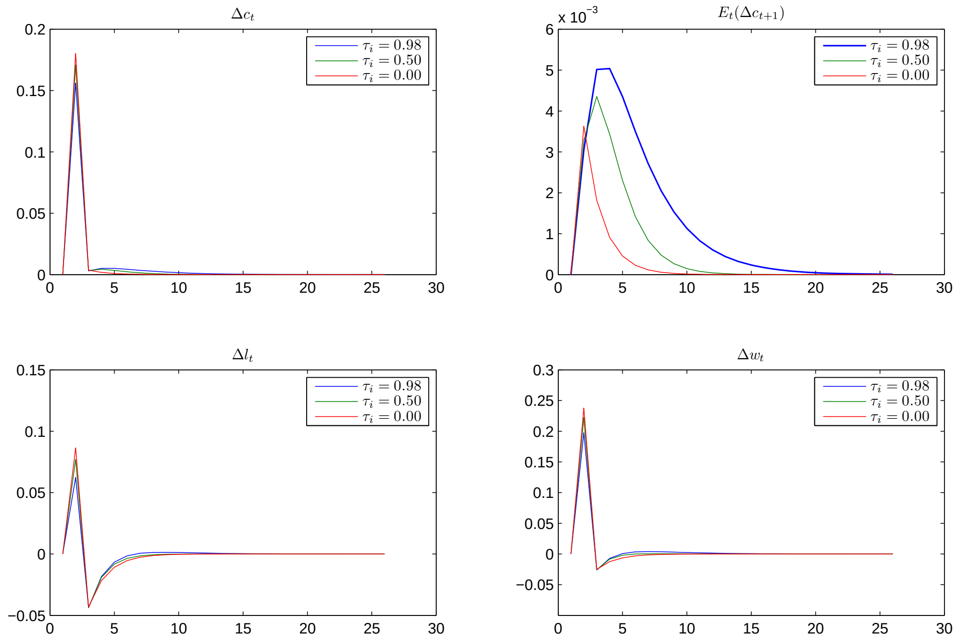

Figure 3.1: Impulse response functions to a one-percent permanent technology shock. Consumption growth is Δc_t, expected consumption growth is $E_t \Delta c_{t+1}$, labor growth is Δl_t, and real wages growth is Δw_t. The red line refers to the case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).

move to the new steady state. Put simply, the high persistence of nominal interest rates observed in the data, which in the model is captured by a high degree of inertia in the monetary policy rule, dwarfs the initial reaction of macro variables and asset prices to a technology shock. A smaller initial reaction then implies an increase in expected future growth rates, as the system moves to the new steady state.

3.3.2 The Real Pricing Kernel

The real pricing kernel N_{t+1} in equation (3.6) can be expressed, up to an approximation error, as the sum of three components. Omitting constants,

$$\log N_{t,t+1} \quad = \quad -\psi \Delta c_{t+1} \tag{3.19}$$

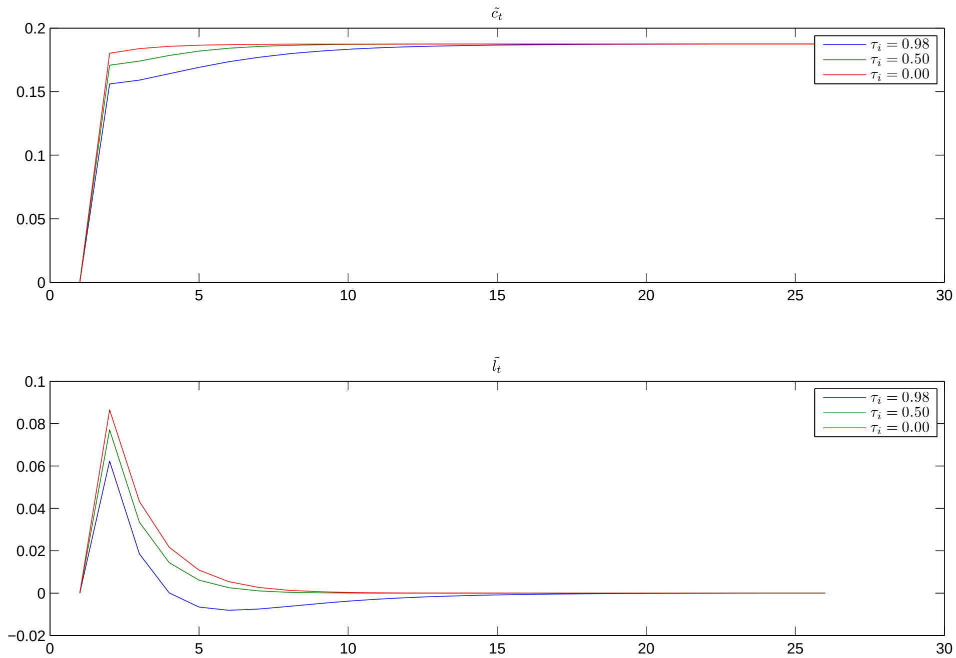

Figure 3.2: Impulse response functions to a one-percent permanent technology shock. Detrended consumption is $\tilde{c}_t$, labor is $\tilde{l}_t$. The red line refers to the case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).

$$+(\psi - \gamma) \sum_j \eta_{vv}^j (E_{t+1} - E_t) \Delta c_{t+1+j}$$
$$+(\psi - \gamma) \sum_j \eta_{vv}^j \eta_{vx} (E_{t+1} - E_t) n_{t+1+j} \quad ,$$

where $0 < \eta_{vv} < 1$ and η_{vx} are linearization coefficients. The first component, $-\psi \Delta c_{t+1}$, is standard in the asset pricing literature and captures the short-run risk in the economy. When realized consumption growth is low, the marginal rate of substitution is high and therefore any risky asset will pay a positive risk premium whenever its return covaries positively with consumption growth.

The second and third component arise when the representative agent has Epstein-Zin preferences and capture long-run risk. These last two components say that downward revisions in future expected consumption growth and labor are bad news – that is, the

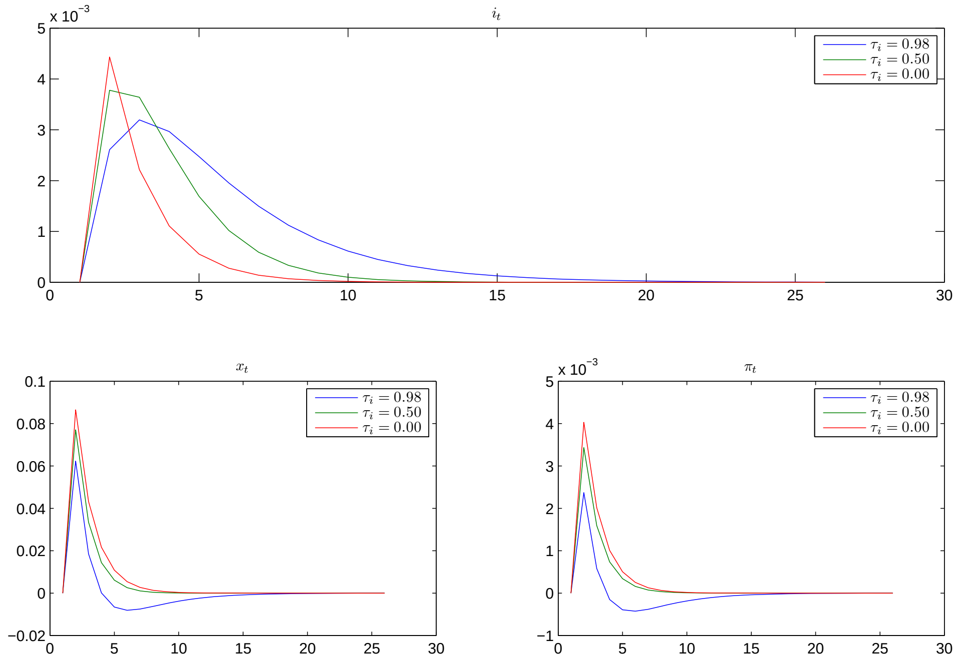

Figure 3.3: Impulse response functions to a one-percent permanent technology shock. The one-period nominal interest rate is i_t, the output gap is x_t, and inflation is π_t. The red line refers to the case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).

marginal rate of substitution is high – when agents prefer early resolution of risk ($\psi - \gamma < 0$), and good news when agents prefer late resolution of risk ($\psi - \gamma > 0$). In particular, when agents prefer early resolution of risk, risk premia on risky assets are large when good news about future consumption growth and labor are positively correlated with their returns. Therefore, a representative agent with preference for the early resolution of risk dislikes shocks both to realized and expected consumption growth. When low realized consumption growth is associated with low expected consumption growth, as is the case in the model in light of Result 2, technology shocks have a magnified effect on the marginal rate of substitution.[7]

[7] Uhlig (2007) notices that, in the data, high returns tend to predict upswings in economic activity – today's return and labor changes in the future are negative correlated – and therefore the third component in (3.19) provides a hedge against consumption news. However, when I calibrate the model, I find that the impact on the marginal rate of substitution coming from news about future labor is small compared to the effect of news to future consumption.

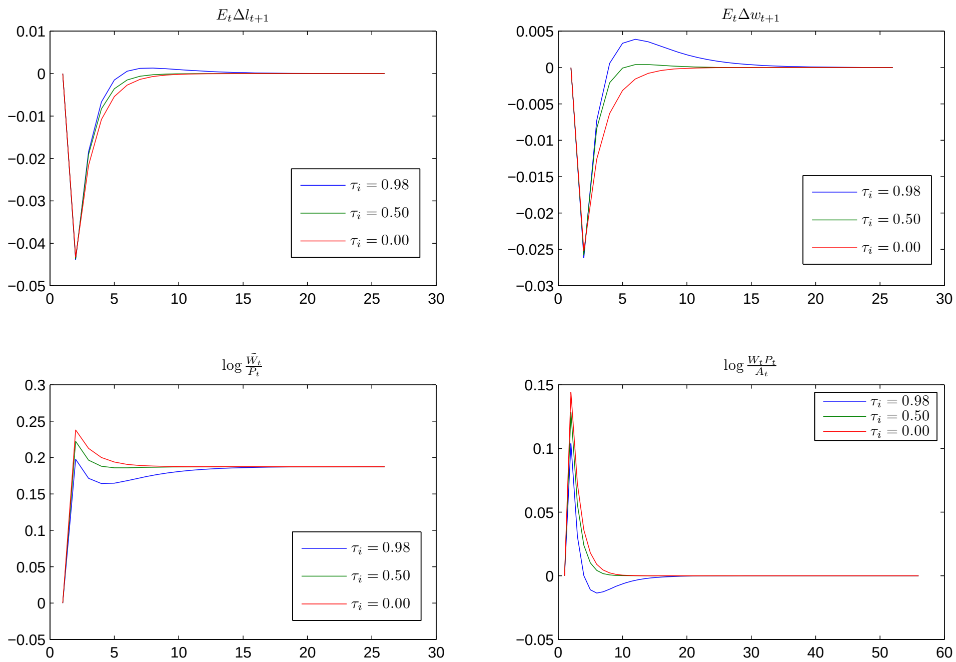

Figure 3.4: Impulse response functions to a one-percent permanent technology shock. Expected labor growth is $E_t\Delta l_{t+1}$, expected real wages growth is $E_t\Delta w_{t+1}$, detrended (log) real wage is $\log \frac{\tilde{W}_t}{P_t}$, and (log) real marginal cost is $\log \frac{W_t/P_t}{A_t}$. The red line refers to the case with no interest rate inertia ($\tau_i = 0$), the green line refers to the intermediate case of $\tau_i = 0.50$, and the blue line refers to the case of strong interest rate inertia ($\tau_i = 0.98$).

In order to gather more insight on the role of nominal frictions and monetary policy on the price of risk associated with the technology shock, one can substitute the equilibrium dynamics of consumption and labor in (3.19) to get (again, setting $u_t = 0$, for every t)

$$-\log M_{t,t+1} \quad = \quad \delta + \gamma_a \Delta a_t + \gamma_i i_{t-1} + \lambda_a \sigma_a \epsilon^a_{t+1} \quad , \tag{3.20}$$

where

$$\begin{aligned} \delta \quad = \quad & -\log\beta - (\psi-\gamma)\overline{v} + \frac{\psi-\gamma}{1-\gamma}\frac{1}{\eta_{vv}}[(1-\psi)\overline{v} - \overline{\eta_v} - \eta_{vx}\overline{x}] + I(\pi^* + \overline{\pi}) \\ + \quad & [\gamma(1+x_a) - (\psi-\gamma)v_a + I\pi_a]\,(1-\varphi_a)\theta_a \end{aligned}$$

$$
\begin{aligned}
\gamma_a &= -\left[\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{\eta_{vx}}{\eta_{vv}}+\gamma\right]x_a+\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{1-\psi}{\eta_{vv}}v_a+\left[\gamma(1+x_a)-(\psi-\gamma)v_a+I\pi_a\right]\varphi_a \\
&+ (\tau_x x_a+\tau_\pi\pi_a)(\gamma x_i-(\psi-\gamma)v_i+I\pi_i) \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_i &= -\left[\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{\eta_{vx}}{\eta_{vv}}+\gamma\right]x_i+\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{1-\psi}{\eta_{vv}}v_i \\
&+ (\tau_x x_i+\tau_\pi\pi_i+\tau_i)(\gamma x_i-(\psi-\gamma)v_i+I\pi_i) \quad ,
\end{aligned}
$$

$$
\lambda_a = \gamma(1+x_a)-(\psi-\gamma)v_a+I\pi_a \quad . \tag{3.21}
$$

Following the affine term structure literature, I refer to γ_a and γ_i as the real factor loadings of technology and interest rate. Likewise, λ_a will be referred to as the real price of risk of technology. Equation (3.20) encompasses several familiar specifications for the real pricing kernel. First, when there are no nominal frictions and preferences are of the time and state separable power utility form, the real pricing kernel collapses to the familiar

$$
-\log N_{t+1} = [\log\beta+\gamma(1-\varphi_a)\theta_a]+\gamma\varphi_a\Delta a_t+\gamma\sigma_a\epsilon^a_{t+1} \quad ,
$$

with the price of risk being equal to the coefficient of relative risk aversion. Allowing for nominal frictions, while remaining in the power utility framework, monetary policy affects the pricing kernel only through its direct effect on the output gap, that is, only through its effect on $\overline{x}$, x_a and x_i. Finally, in the more general case with nominal frictions and recursive preferences, monetary policy affects the pricing kernel through two channels: first, its direct effect on the output gap; second, its indirect effect on the equilibrium process of the scaled utility function, v_t. In particular, since the sensitivity of the scaled utility to the technology shock is positive ($v_a > 0$), when the agent prefers early resolution of risk, the price of risk λ_a will be larger than in the standard power utility case. A larger price of risk is key in the determination of a large consumption risk premium.

3.3.3 The Return on the Consumption Claim

The price S^c_t of a claim to all future consumption is $S^c_t = E_t(M_{t+1}(C_{t+1}+S^c_{t+1}))$ and its return $r^c_{t+1} = \log(R^c_{t+1}) \equiv \log((S^c_{t+1}+C_{t+1})/S^c_t)$ can be expressed as

$$
e^{r^c_{t+1}} = \left(1+\frac{S^c_{t+1}}{C_{t+1}}\right)\left(\frac{C_{t+1}}{C_t}\right)\left(\frac{C_t}{S^c_t}\right) \quad . \tag{3.22}
$$

Following Campbell and Shiller (1988), the return on the consumption claim can be approximated as

$$r_{c,t+1} = \overline{\eta_{pc}} + \eta_{pc}\ pc_{t+1} - pc_t + \Delta c_{t+1} \quad , \tag{3.23}$$

where $pc_t \equiv \log(S_t^c/C_t)$ is the log price-consumption ratio and $\overline{\eta_{pc}}$ and η_{pc} are linearization constants depending on the average level of the price-consumption ratio.

As is standard, I solve for the approximated return on the consumption claim by conjecturing a linear solution for the log price-consumption. When the policy shock u_t is shut down, the guess is

$$pc_t \quad = \quad \overline{A} + A_a \Delta a_t + A_i\ i_{t-1} \quad . \tag{3.24}$$

Substituting it in (3.23), and then using (3.22), the solution for the endogenous parameters $\overline{A}$, A_a, and A_i is

$$\begin{aligned}\overline{A} \quad &= \quad \frac{1}{1-\eta_{pc}}\Big(-\delta + \overline{\eta_{pc}} + \mu_c + \eta_{pc} A_a (1-\varphi_a)\theta_a + A_i(\overline{\tau} + \tau_x \overline{x} + \tau_\pi \overline{\pi}) \\ &+ \quad (\eta_{pc} A_a + (1 + x_a) - \lambda_a)^2 \sigma_a^2 / 2\Big) \quad ,\end{aligned}$$

$$A_a \quad = \quad \frac{g_a + \eta_{pc} A_i (\tau_x x_a + \tau_\pi \pi_a) - \gamma_a}{1 - \eta_{pc}\varphi_a} \quad , \tag{3.25}$$

$$A_i \quad = \quad \frac{g_i - \gamma_i}{1 - \eta_{pc}(\tau_x x_i + \tau_\pi \pi_i + \tau_i)} \quad . \tag{3.26}$$

The sign of the A_a coefficient determines the sensitivity of the price-consumption ratio to the technology shock. First, consider the case without monetary policy inertia ($\tau_i = 0$). The coefficient A_i collapses to zero, and the coefficient A_a becomes

$$A_a = \frac{g_a - \gamma_a}{1 - \eta_{pc}\ \varphi_a} \quad . \tag{3.27}$$

This coefficient bears similarities with the one derived by Bansal and Yaron (2004). In particular, one can show that $A_a > 0$ if and only if the intertemporal elasticity of substitution is larger than one. In response to higher expected growth, agents buy more assets, and consequently the price-consumption ratio rises. Second, A_a is increasing in the persistence of expected consumption growth which, as shown in Result 1, is equal to the persistence of the technology shock. High persistence in expected consumption growth, combined with a large elasticity of intertemporal substitution, results in a large and positive sensitivity of the price-consumption ratio to the technology shock. However, the requisite high persistence in expected consumption growth must be exogenously injected by assuming a very high autocorrelation in the technology process. Such a large autocorrelation is at odds with the data.

This unpleasant feature can be removed if one allows for interest rate inertia in the monetary policy rule. With interest rate inertia, the sign and magnitude of the A_a coefficient depend on A_i. With strong interest rate smoothing and early resolution of risk, the coefficient A_i is large and positive. That is, a high nominal interest rate *today* is associated with a large price-consumption ratio *tomorrow*. This results in a large A_a coefficient, even when technology growth is only modestly autocorrelated.

Given the lognormal structure of the model, the continuously compounded risk premium on the consumption claim is equal to

$$\begin{aligned} E_t(r^c_{t+1} - r_{f,t}) &= -Cov_t\big(\log N_{t+1} - E_t(\log N_{t+1}), r^c_{t+1} - E_t(r^c_{t+1})\big) - 0.5\, Var_t(r^c_{t+1}) \\ &= \lambda_a B_a \sigma_a^2 - 0.5\, Var_t(r^c_{t+1}) \quad , \end{aligned} \tag{3.28}$$

where $r_{f,t} = \log R_{f,t}$ is the one-period continuously compounded real risk-free rate, and $B_a = \eta_{pc} A_a + (1 + x_a)$. The size of B_a determines the size of the risk premium. Similarly to the long-run risk literature, B_a is positive, larger with early resolution of risk, and increasing in the persistence of expected consumption growth. Importantly, B_a depends on the level of interest rate inertia, through its effect on the A_a coefficient. A strong inertia delivers a large B_a coefficient, resulting in a large risk premium. The joint effect of price stickiness and interest rate inertia delivers *endogenous* long-run risk.

3.3.4 The Return on Dividends

Following Abel (1999), dividends D_t are defined as a levered claim to aggregate consumption. Omitting constants, the logarithm of dividend growth can be expressed as

$$\log(D_{t+1}/D_t) \equiv \Delta d_{t+1} = \phi_d\ (g_a\ \Delta a_t + g_i\ i_{t-1}) + \xi_d\ (1 + x_a)\ \sigma_a\ \epsilon^a_{t+1} \quad ,$$

where ϕ_d governs leverage and ξ_d affects the sensitivity of dividend volatility to consumption innovations. The solutions for the price-dividend ratio $pd_t \equiv \log(S^d_t/D_t) = \overline{A}^d + A^d_a\ \Delta a_t + A^d_i\ i_{t-1}$ and the return on dividends $r^d_{t+1} = \overline{\eta_{pd}} + \eta_{pd}\ pd_{t+1} - pd_t + \Delta d_{t+1}$ are obtained in exactly the same way as for the consumption claim.

3.3.5 Discussion

In this section, I showed that a real business cycle model with sticky prices can generate long-run risk when the representative agent has recursive preferences. In particular, I emphasized two mechanisms: i) the presence of nominal frictions breaks the link between the autocorrelation dynamics of realized and expected consumption growth; and ii) interest rate inertia in the monetary policy rule induces high persistence in the dynamics of expected consumption growth. Furthermore, I showed that the effect of price stickiness on real allocation is larger when monetary policy is weak, that is, when the monetary authority does not react very strongly to inflation and output gap. Finally, I showed how a representative agent who fears both variation in realized and expected consumption growth requires a large compensation for being exposed to such shocks.

3.4 Calibration

Section 3.3 emphasized the distinct effects of two ingredients on the equilibrium dynamics of consumption growth: price stickiness and policy inertia. Price stickiness is responsible for the reduction in the persistence of realized consumption growth, whereas policy inertia plays a crucial role in the propagation mechanism of the model, delivering high persistence in expected consumption growth. In this section, I calibrate two versions of the model to highlight the role played by each of these two ingredients. In the first version, I shut down the policy inertia channel in the interest rate rule and show that high persistence in technology growth delivers consumption growth and asset prices dynamics consistent with the data. This is the *exogenous* long-run risk model, as the required high persistence must be exogenously injected in the model. In the second version, I switch on the policy inertia channel in the interest rate rule and obtain *endogenous* long-run risk.

3.4.1 No policy inertia

Following the long-run risk literature, I calibrate the model at monthly frequency for the 1952-2006 period. The discount factor β is set to 0.999 as in Bansal, Kiku, and Yaron (2007) and the elasticity of substitution, ψ^{-1}, is equal to 1.5 as in Bansal and Yaron (2004). The Calvo parameter α is consistent with the results of Bils and Klenow (2004) and is set

Description	Parameter	Value
Discount factor	β	0.999
Elasticity of intertemporal substitution	$1/\psi$	1.5
Relative risk aversion	γ	6.5
Elasticity of labor supply	$1/\omega$	1
Degree of price rigidity	α	7/9
Elasticity of substitution of goods	θ	7
Inflation target	π^*	2.0×10^{-3}
Average technology growth	θ_a	1.5×10^{-3}
Conditional volatility of technology growth	σ_a	0.95×10^{-3}
Autocorrelation of technology growth	φ_a	0.95
Constant in the policy rule	$\overline{\tau}$	2.27×10^{-3}
Response to inflation in the policy rule	τ_π	1.1
Response to output gap in the policy rule	τ_x	0.3/12
Monetary policy inertia	τ_i	0
Leverage	ϕ_d	2.6
Sensitivity of dividends to consumption innovations	ξ_d	4.5

Table 3.1: Calibrated parameter values for the model without policy inertia ($\tau_i = 0$).

equal to 7/9, implying an average price duration of 4.5 months. I borrow the parameters governing the elasticity of substitution across goods and the elasticity of labor from the New Keynesian literature and set them equal to $\theta = 7$ and $\omega = 1$, respectively. The parameter χ is set such that the representative agent spends one third of her time endowment working. In a way broadly consistent with Taylor (1999), I set $\tau_\pi = 1.1$ and $\tau_x = 0.3/12$. Finally, I shut down the policy shock and set $u_t = 0$, for every t.[8]

Then, I choose five parameters to match five moments in the data. The parameters are: the risk aversion coefficient γ, the interest rate rule constant $\overline{\tau}$, and the three parameters governing the dynamics of technology growth (θ_a, σ_a, and φ_a). The five moments I match are the unconditional mean and volatility of consumption growth, the unconditional mean and volatility of the one-month nominal interest rate, and the Sharpe Ratio. Finally, the leverage ϕ_d and the scaling coefficient ξ_d are chosen to match the unconditional equity premium and the unconditional volatility of the return on equity. Table 3.1 shows the parameters of the calibration and Panel A of Table 3.2 shows the moments that are exactly matched by the calibration exercise.

[8] The role played by the policy shock u_t will be clear in the Section 3.4.2, where I calibrate the model allowing for inertia in the monetary policy rule.

Moment	Data	Model
Panel A: Calibrated Moments		
$E(\Delta c_t)$	1.80	1.80
$\sigma(\Delta c_t)$	2.72	2.72
$E(i_t)$	5.56	5.56
$\sigma(i_t)$	2.91	2.91
Sharpe Ratio	0.33	0.33
$E(r^d - r_f) + J.I.$	6.84	6.84
$\sigma(r_t^d) + J.I.$	20.60	20.60
Panel B: Other Moments		
$Corr(\Delta c_t, \Delta c_{t+1})$	n.a.	0.12
$\sigma(E_t \Delta c_{t+1})$	n.a.	0.63
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	n.a.	0.95
$E(\pi_t)$	4.24	4.72
$\sigma(\pi_t)$	1.02	0.74
$Corr(\pi_t, \pi_{t+1})$	0.71	0.95
$Corr(i_t, i_{t+1})$	0.98	0.95
$E(r_t)$	1.54	1.44
$\sigma(r_t)$	2.16	1.46

Table 3.2: Moment conditions of macro variables and asset prices for the model without policy inertia. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations, with the exception of the correlation between consumption growth and inflation, which is obtained after aggregating the monthly observations to annual frequency. The Jensen's inequality term is $J.I. = Var_r(r_{t+1}^d)/2$. Panel A reports the moments that were calibrated to match the data, and Panel B reports other moments. The empirical moments for consumption growth are taken from Bansal and Shaliastovich (2010). Data on nominal interest rate and inflation is from CRSP.

The persistence of the technology shock and the risk aversion coefficient are crucial magnitudes, as they determine the persistence of expected consumption growth and the Sharpe Ratio. The calibration exercise requires high persistence in technology growth ($\varphi_a = 0.95$) and a risk aversion coefficient of $\gamma = 6.5$. High persistence in technology growth is needed because the model without policy inertia cannot generate endogenous persistence. Previous studies indicate a lower correlation. Nelson and Plosser (1982) and Gomme and Rupert (2007) suggest a first order autocorrelation of technology growth in the order of 0.2-0.4.

Consistently with the discussion in Section 3.3, the risk aversion parameter implies that the representative agent has a preference for the early resolution of risk and thus fears variation in expected consumption growth. Importantly, both the persistence of expected consumption growth and the risk aversion are slightly lower than what is usually assumed in the long-run risk literature. This is a consequence of the permanent nature of the technology shock, which generates a positive covariation between realized and expected consumption growth. Such a positive covariation magnifies the effect of technology shocks on asset prices. Therefore, compared to the long-run risk literature, the model further reduces the need for a large risk aversion coefficient to generate a large equity premium.

Panel B of Table 3.2 shows important statistics of the model that were not calibrated to fit. The autocorrelation of monthly realized consumption growth is 0.12, and is much lower than the autocorrelation of expected consumption growth. This is a result of the presence of sticky prices which breaks the link between the dynamics of technology and the dynamics of realized consumption growth. When aggregated to annual frequency, realized consumption growth has a first order autocorrelation of 0.51, slightly overshooting the data. Given the absence of any amplification mechanism, the autocorrelation of expected consumption growth, inflation, and of the one-month nominal interest rate are all equal to 0.95. The one-month real interest rate has an annualized mean of 1.44% and an annualized volatility of 1.45%, and the annualized mean of the inflation rate is 4.72%, with a volatility of 0.74%.

3.4.2 Policy Inertia

When the monetary policy rule allows for interest rate inertia, the model generates endogenous long-run risk. I keep the calibration as close as possible to the parameter values used in the version of the model without policy inertia and make the following changes. First, the monetary authority has a strong desire to smooth interest rates over time ($\tau_i = 0.98$); second, I allow for a non-zero policy shock u_t. The policy shock is instrumental in obtaining sensible volatilities for interest rate and inflation. Without it, a strong inertia in the interest rate rule mechanically imposes little period-by-period changes in the interest rate, resulting in very smooth processes for both the nominal interest rate and inflation.

Similarly to the case without policy inertia, the calibration exercise consists of choosing five parameters to match five moments in the data. I choose the autocorrelation of technology growth (φ_a), the conditional volatility of technology (σ_a), the interest rate rule constant ($\overline{\tau}$), the coefficient of risk aversion (α), and the volatility of the policy shock (σ_u) to match

Description	Parameter	Value
Discount factor	β	0.999
Elasticity of intertemporal substitution	$1/\psi$	1.5
Relative risk aversion	γ	10
Elasticity of labor supply	$1/\omega$	1
Degree of price rigidity	α	7/9
Elasticity of substitution of goods	θ	7
Inflation target	π^*	2.0×10^{-3}
Average technology growth	θ_a	1.5×10^{-3}
Conditional volatility of technology growth	σ_a	0.45×10^{-3}
Autocorrelation of technology growth	φ_a	0.50
Constant in the policy rule	$\overline{\tau}$	1.12×10^{-3}
Response to inflation in the policy rule	τ_π	1.1
Response to output gap in the policy rule	τ_x	0.3/12
Monetary policy inertia	τ_i	0.98
Volatility of policy shock	σ_u	1.73×10^{-3}
Leverage	ϕ_d	3.1
Sensitivity of dividends to consumption innovations	ξ_d	4.2

Table 3.3: Parameter values for the model with policy inertia ($\tau_i \neq 0$).

the unconditional mean and volatility of consumption growth, the unconditional mean and volatility of the nominal interest rate, and the Sharpe Ratio. The calibration parameters and the resulting macroeconomic and asset pricing moments are shown in Tables 3.3 and 3.4.

The required autocorrelation of technology growth is $\varphi_a = 0.50$, with a conditional volatility $\sigma_a = 0.0045$. This is in sharp contrast with the results of the model without policy inertia, in which the absence of internal propagation required the calibration to rely on a highly persistent technology growth process. The model needs a coefficient of risk aversion equal to 10 to generate the Sharpe Ratio observed in the data. The increase in the coefficient of relative risk aversion relative to the model without policy inertia is necessary because the introduction of two transitory state variables, i_{t-1} and u_t, reduces the conditional correlation between realized and expected consumption growth.

Moment	Data	Model
Panel A: Calibrated Moments		
$E(\Delta c_t)$	1.80	1.80
$\sigma(\Delta c_t)$	2.72	2.72
$E(i_t)$	5.56	5.56
$\sigma(i_t)$	2.91	2.91
Sharpe Ratio	0.33	0.33
Panel B: Other Moments		
$Corr(\Delta c_t, \Delta c_{t+1})$	n.a.	0.11
$\sigma(E_t \Delta c_{t+1})$	n.a.	0.55
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	n.a.	0.93
$E(\pi_t)$	4.24	3.84
$\sigma(\pi_t)$	1.02	0.87
$Corr(\pi_t, \pi_{t+1})$	0.71	0.52
$Corr(i_t, i_{t+1})$	0.98	0.95
$E(r_t)$	1.54	1.24
$\sigma(r_t)$	2.16	1.66

Table 3.4: Moment conditions of macro variables and asset prices for the model with interest rate inertia and an *i.i.d* monetary policy shock. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations. Panel A reports the moments that were calibrated to match the data, and Panel B reports other moments. The empirical moments for consumption growth are taken from Bansal and Shaliastovich (2010). Data on nominal interest rate and inflation is from CRSP.

3.5 Sensitivities

In this section, a series of sensitivity exercises is performed. For consistency with the calibration section, I consider both the case without policy inertia and the case with policy inertia. Since the role played by recursive preferences and price stickiness in both versions of the model is similar, some exercises are only conducted for the case without policy inertia.

3.5.1 No policy inertia

The Role of Recursive Preferences

If preferences were of the standard power utility form, the high persistence in expected consumption growth implied by the model would have no effects on asset prices . To see

	Baseline Model	Power utility
	$\gamma = 6.5$	$\gamma = 6.5$
Moment	$\psi^{-1} = 1.5$	$\psi^{-1} = 1/6.5$
$\sigma(\Delta c_t)$	2.72	5.01
$\sigma(\pi_t)$	0.74	0.67
$E(r_t)$	1.44	7.60
$\sigma(r_t)$	1.45	5.67
$E(i_t)$	5.56	49.88
$\sigma(i_t)$	2.91	27.87
$E(r^c - r_f) + J.I.$	1.55	−1.13
$\sigma(r_t^c)$	3.95	3.83
Sharpe Ratio	0.33	
of which:	short-run risk	
	0.16	
	long-run risk	
	0.17	

Table 3.5: The role of recursive preferences. Moment conditions for the baseline calibration and for an alternative model with standard power utility ($\gamma = \psi = 6.5$). All other parameters remain unchanged. Annualized percentages. The Jensen's inequality term is $J.I. = Var_t(r_{t+1}^c)/2$. Calibration parameters are from the model without policy inertia in Table 3.1.

this, I compare the results implied by the baseline calibration with the ones that would have been obtained with standard power preferences. Keeping all other parameters unaltered, I set $\gamma = \psi = 6.5$, and analyze the impact of long-run risk on asset prices. Results are shown in Table 3.5.

Standard power utility preferences are unable to capture the riskiness associated with high variation in expected consumption growth. The excess return on the consumption claim becomes negative (equal to −1.13%) and, because of the low intertemporal elasticity of substitution, consumption growth and interest rates become more volatile. A similar effect impacts the level of interest rates, with the unconditional mean of the nominal interest rate sharply moving from 5.56% to as high as roughly 50%.

Recursive preferences, combined with early resolution of risk, magnify the risk associated with variation in expected consumption growth. Define short-run risk as the amount of risk that would arise in the standard power utility case ($\gamma = \psi$), and long-run risk as the residual. Using the expression for the price of risk of technology in (3.21), one can write

$$\begin{aligned}\text{Sharpe Ratio} &= \text{short-run risk} + \text{long-run risk} \\ &= \gamma(1 + x_a)\ \sigma(\Delta c_{t+1}) + (\gamma - \psi)v_a\ \sigma(\Delta c_{t+1})\end{aligned}$$

φ_a	$\gamma = 6.5,\ \psi = 1/1.5$	$\gamma = 6.5,\ \psi = 6.5$
0.90	1.381	−7.340
0.91	0.163	−8.946
0.92	1.964	−11.127
0.93	2.406	−14.197
0.94	3.020	−18.719
0.95	3.913	−25.789
0.96	5.303	−37.792
0.97	7.694	−60.794
0.98	12.562	−114.601
0.99	26.794	−303.584

Table 3.6: Sensitivity A_a of the price-consumption ratio to the technology shock. Comparative statics for different values of autocorrelation in technology growth, φ_a. Baseline calibration ($\gamma = 6.5$, $\psi = 1/1.5$) and standard power utility ($\gamma = \psi = 6.5$). Calibration parameters are from the model without policy inertia in Table 3.1.

$$0.33 \quad = \quad (0.16) + (0.17) \quad ,$$

where $\sigma(\Delta c_{t+1})$ denotes the unconditional volatility of realized consumption growth. Long-run risk has a large impact on asset prices and accounts for roughly 50% of the total Sharpe Ratio.

Further insights can be obtained by looking at how recursive preferences affect the sensitivity A_a of the price-consumption ratio to the technology shock. Table 3.6 shows that, with early resolution of risk, the coefficient A_a is positive and increasing in the autocorrelation of technology. Importantly, when the intertemporal elasticity of substitution is smaller than one, as is the case in the power utility calibration, the sensitivity of the price-consumption ratio to technology becomes negative. Long-run risk has a non-trivial impact on asset prices only when expected consumption growth is highly autocorrelated. In the baseline calibration, this autocorrelation is set equal to 0.95.

Figure 3.5 shows the risk premium on the consumption claim, the price of risk of technology λ_a, the sensitivity A_a of the price-consumption ratio to the technology shock, and the coefficient B_a, as I vary the autocorrelation of technology growth for three preference specifications: the baseline calibration (with early resolution of risk), the case of standard power utility and the case of preference for the late resolution of risk. The price of technology risk is increasing in the autocorrelation of technology growth, and larger when the representative agent has preference for the early resolution of risk. The risk premium is

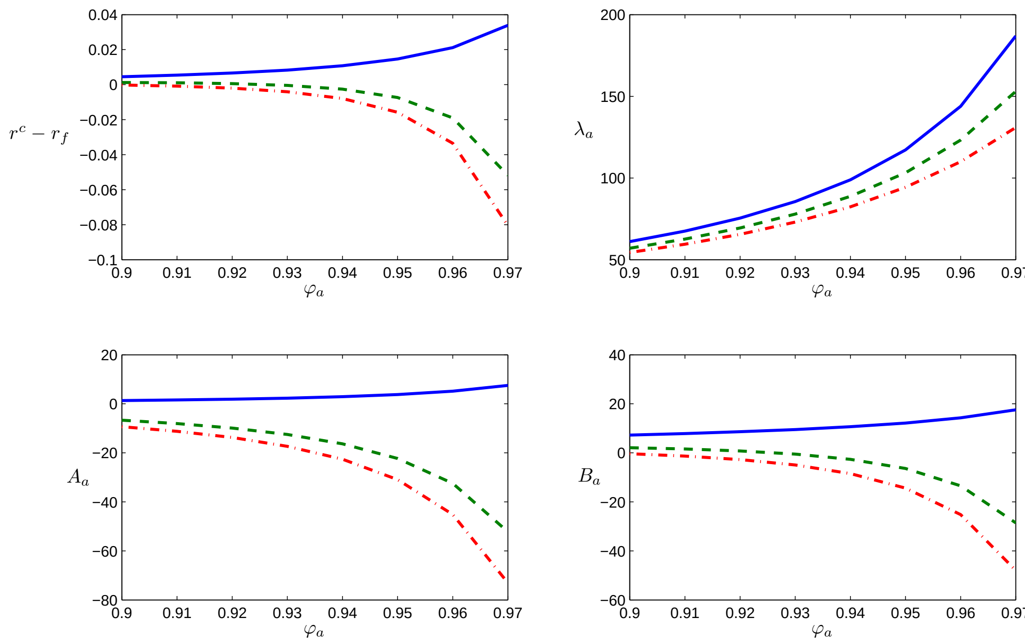

Figure 3.5: Consumption risk premium and consumption dynamics as a function of φ_a. Consumption risk premium, price of technology risk λ_a, sensitivity A_a of the price-consumption ratio to technology growth, and B_a coefficient as a function of the persistence of the technology shock, φ_a. Risk aversion γ is fixed at 6.5. The solid line refers to the calibration with $\psi = 1/1.5$, the dashed line refers to the case of power utility ($\psi = 6.5$), and the dashed-dotted line refers to the case of late resolution of risk ($\psi = 10$). All remaining calibration parameters are from the model without policy inertia in Table 3.1.

positive and increasing in the case of early resolution of risk, while it decreases in the case of power utility and late resolution of risk. A similar pattern applies for the A_a and B_a coefficients.

The Role of Price Stickiness

Price stickiness can generate consumption dynamics that are typical of long-run risk models. Intuitively, the fact that only a fraction of firms can adjust their prices at each point in time breaks the perfect tie between realized and expected output. The more the prices are sticky, the larger the wedge between the dynamics of realized and expected consumption growth. Keeping the remaining parameters unchanged, Table 3.7 shows the dynamics of consumption growth for the case of flexible prices and for three different levels of price

	Average Price Duration			
Moment	Flexible prices	2 Months	4.5 Months	9 Months
$\sigma(\Delta c_t)$	1.05	2.55	2.72	3.02
$Corr(\Delta c_t, \Delta c_{t+1})$	0.95	0.23	0.12	0.09
$\sigma(E_t \Delta c_{t+1})$	1.00	0.75	0.63	0.58
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	0.95	0.95	0.95	0.95
$E(r^c - r_f) + J.I.$	0.87	1.30	1.55	1.60

Table 3.7: Consumption growth dynamics and consumption risk premium as a function of the degree of price stickiness. Average price duration is 1 month ($\alpha = 0$, flexible prices), 2 months ($\alpha = 1/2$), 4.5 months ($\alpha = 7/9$, baseline calibration), and 9 months ($\alpha = 8/9$). The Jensen's inequality term is $J.I. = Var_t(r^c_{t+1})/2$. Calibration parameters are from the model without policy inertia in Table 3.1.

stickiness: 2, 4.5, and 9 months of average price duration.

When prices are not very sticky, consumption is more closely tied to the dynamics of technology, the output gap is small, and therefore consumption growth closely mimics the dynamics of consumption that would arise with flexible prices. On the contrary, strong frictions generate a large output gap, moving consumption away from its flexible price counterpart.

The baseline calibration, obtained with a level of price stickiness implying 4.5 months of average price duration, generates a consumption risk premium that is roughly twice the size of the one obtained with flexible prices (1.55% versus 0.87%). Interestingly, a very mild level of price stickiness is sufficient to generate a significant wedge between the dynamics of realized versus expected consumption growth. For example, when firms are allowed, on average, to reoptimize their prices every 2 months, the first order autocorrelation of realized consumption growth remains as low as 0.23. Figure 3.6 shows the consumption risk premium and the autocorrelation of realized and expected consumption growth as a function of the parameter α. As expected, the risk premium is increasing in the level of price stickiness. The autocorrelation of expected consumption growth is fixed at 0.95, while the autocorrelation of realized consumption growth is sharply decreasing in the level of price stickiness.

The Role of Monetary Policy: τ_x and τ_π

When prices are sticky, monetary policy affects the dynamics of consumption growth. Thus, a natural question to ask is how the dynamics of consumption growth are affected by changes in the interest rate rule parameters. Here, I conduct the following policy exercises. The two

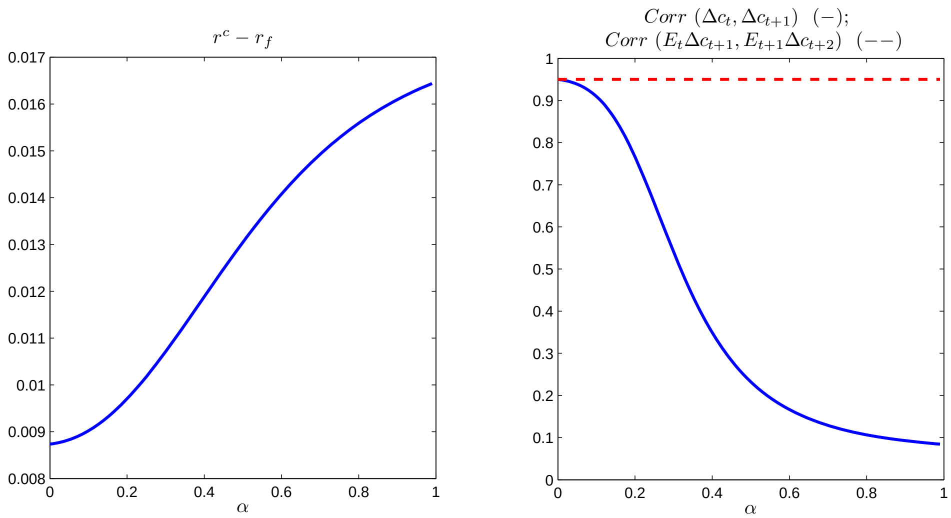

Figure 3.6: Consumption risk premium and consumption dynamics as a function of α. Consumption risk premium (on the left, solid line), autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the parameter α. Calibration parameters are from the model without policy inertia in Table 3.1.

interest rate rule coefficients, τ_π and τ_x, are, in turn, increased to the level that is necessary to obtain a reduction of 1% in the unconditional mean of the one-period nominal interest rate. All remaining parameters are unchanged.

The first exercise requires an increase of τ_π from 1.1 to 2.17, while in the second exercise τ_x has to move from 0.3/12 up to 2.55/12. Results are shown in Table 3.8. Both exercises show that a stronger monetary policy reduces the effect of nominal frictions on realized consumption growth. For a given price stickiness, stronger reactions to inflation and output gap increase the autocorrelation of realized consumption growth to the point where its dynamics become highly at odds with the data. For the exercise involving an increase in τ_π, the persistence of realized consumption growth moves from 0.12 to 0.39, while for the experiment involving an increase in τ_x, the persistence moves up to 0.50. Intuitively, a stronger monetary policy smooths the output gap and inflation, so that price stickiness matters less for realized consumption growth.

Figure 3.7 and 3.8 show the consumption risk premium and the autocorrelation of realized consumption and expected consumption growth as a function of τ_π and τ_x, respectively. A stronger monetary policy dwarfs the effect of price stickiness on asset prices and real allocations. As τ_π and τ_x get larger, the model approaches the results that would be obtained

Moment	Data	Baseline Model	$\tau_\pi = 2.17$	$\tau_x = 2.55/12$
$\sigma(\Delta c_t)$	2.72	2.72	1.62	1.43
$Corr(\Delta c_t, \Delta c_{t+1})$	n.a.	0.12	0.39	0.50
$\sigma(E_t \Delta c_{t+1})$	n.a.	0.63	0.83	0.87
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	n.a.	0.95	0.95	0.95
$E(\pi_t)$	4.24	4.72	3.32	3.25
$\sigma(\pi_t)$	1.02	0.74	0.34	0.26
$Corr(\pi_t, \pi_{t+1})$	0.71	0.95	0.95	0.95
$E(i_t)$	5.56	5.56	4.56	4.56
$\sigma(i_t)$	2.91	2.91	2.53	2.24
$Corr(i_t, i_{t+1})$	0.98	0.95	0.95	0.95

Table 3.8: Moment conditions of macro variables and asset prices. Annualized means are obtained multiplying by 12 the monthly observations. Volatilities are annualized by multiplying monthly volatilities by $\sqrt{12}$. Autocorrelations are not annualized. Two policy exercises are conducted. In both exercises, the unconditional mean of the one-month nominal interest rate is reduced by 1%. The first exercise requires τ_π to move from 1.1 to 2.17. The second exercise requires τ_x to move from 0.5/12 to 2.55/12. Calibration parameters are from the model without policy inertia in Table 3.1.

with flexible prices. However, it is interesting to notice how, for interest rate rule coefficients that are common in the literature, the model still delivers a significant effect of nominal frictions on the risk premium and the autocorrelation of consumption growth. In particular, the autocorrelation of realized consumption growth remains below 0.40 for values of τ_π and τ_x not exceeding 2.8 and 2.0/12, respectively. Similarly, the risk premium remains roughly 35% (for $\tau_\pi < 2.8$) and 43% (for $\tau_x < 2.0/12$) larger than what would have been obtained with flexible prices.

3.5.2 Policy Inertia

How does the persistence of expected consumption growth change with the inertial coefficient τ_i? Panel A of table 3.9 shows the results, together with the autocorrelation and volatility of realized consumption growth. As was expected, an increase in τ_i increases the weight of the lagged interest rate in the determination of the persistence of expected consumption growth. For $\tau_i = 0.98$, the autocorrelation of expected consumption growth is equal to 0.925, but sharply decreases as τ_i gets smaller. When there is no policy inertia, the correlation of expected consumption growth equals the correlation of the technology shock. The persistence of realized consumption growth is only marginally affected by the interest rate smoothing coefficient. The major driver of such a persistence is the level of price stick-

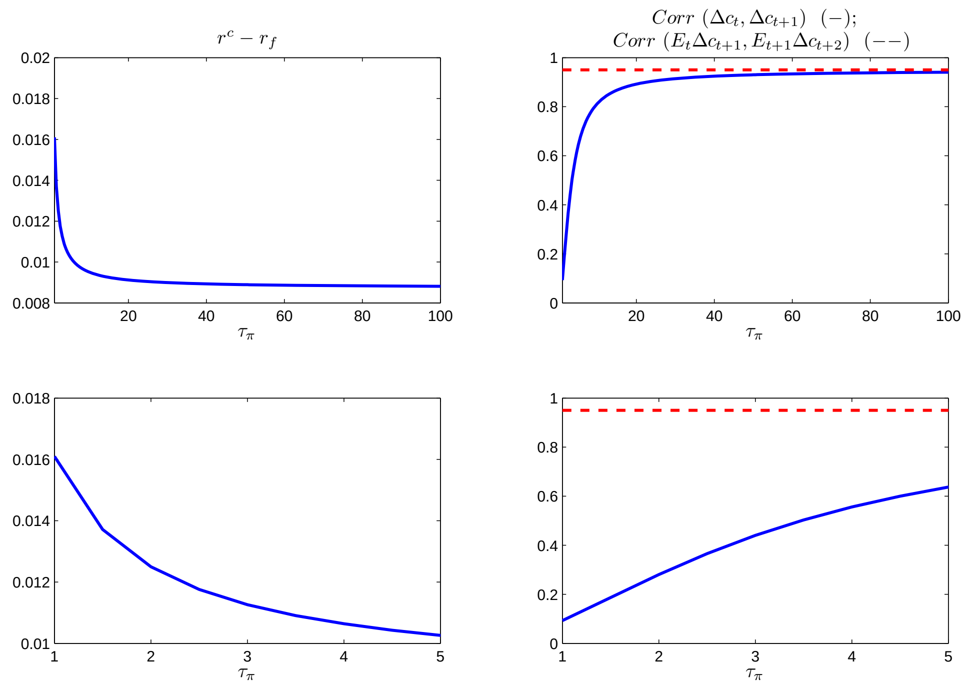

Figure 3.7: Consumption risk premium and consumption dynamics as a function of τ_π. The two top graphs show the consumption risk premium (on the left, solid line), the autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the interest rate rule sensitivity to inflation, τ_π. The two bottom graphs are a zoom of the two top graphs for values of τ_π that are common in the literature. Calibration parameters are from the model without policy inertia in Table 3.1.

iness, which creates a wedge between the dynamics of realized and expected consumption growth.

Panel B of Table 3.9 shows how the risk premium, the sensitivity of the price-consumption ratio to the lagged interest rate A_i, the price of risk of technology λ_a, the coefficient B_a, and the persistence of expected consumption growth vary for various levels of price stickiness. The persistence of expected consumption growth is increasing in the level of price stickiness, and moves from 0.63 when $\alpha = 1/9$ to 0.97 when $\alpha = 8.5/9$. A large persistence in expected consumption growth manifests itself in a large price of risk, a large B_a coefficient, and therefore a large risk premium. Moving from (roughly) one month to 18 months of average price duration, the consumption risk premium increases by 32.6%, from 0.72% to 0.95%. Interestingly, as the level of price stickiness goes up, so does the relative contribution to the risk premium coming from the lagged interest rate, as summarized by the coefficient

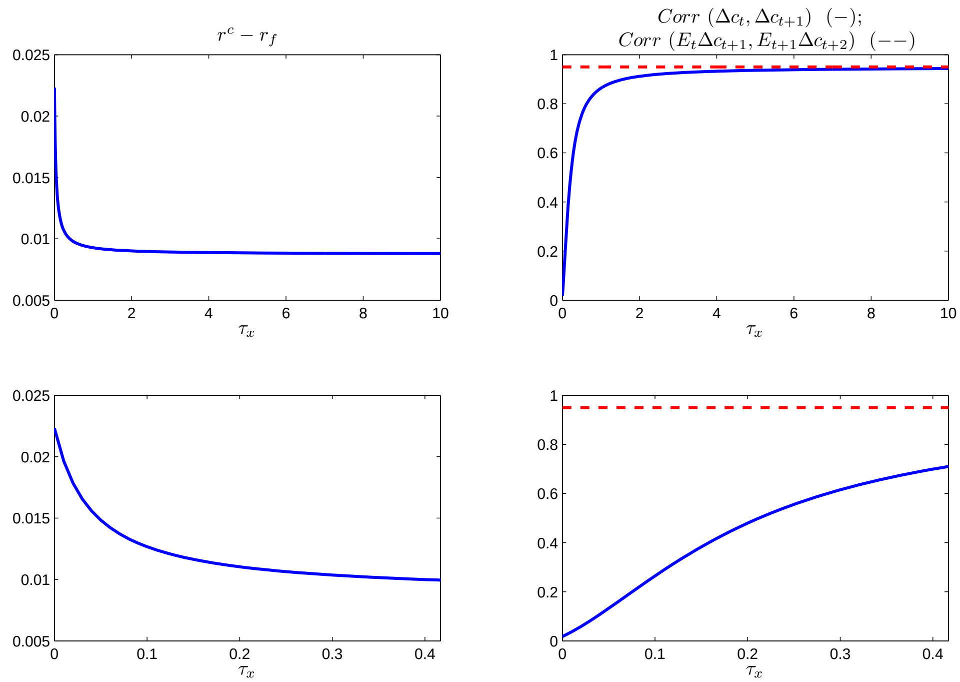

Figure 3.8: Consumption risk premium and consumption dynamics as a function of τ_x. The two top graphs show the consumption risk premium (on the left, solid line), the autocorrelation of realized consumption growth (on the right, solid line), and autocorrelation of expected consumption growth (on the right, dashed line) as a function of the interest rate rule sensitivity to inflation, τ_x. The two bottom graphs are a zoom of the two top graphs for values of τ_x that are common in the literature. Calibration parameters are from the model without policy inertia in Table 3.1.

A_i. The more the prices are sticky, the more the inertia in the interest rate rule matters for real allocations and therefore asset prices.

Figure 3.9 shows how the persistence of realized and expected consumption growth change when the level of price stickiness α varies from zero to one. In order to emphasize the role played by the τ_i coefficient, both the case with and without policy inertia are shown. As expected, the figure shows that, for both cases, the persistence in realized consumption growth is decreasing in the level of price stickiness. On the other hand, the effect of price stickiness on the persistence of expected consumption growth depends on whether monetary policy is inertial or not. Without inertia, price stickiness has no effect on the persistence of expected consumption growth. With inertia, the persistence of expected consumption growth is increasing with the level of price stickiness. Taken together, a mild level of price stickiness and a level of interest rate inertia consistent with the empirical evidence deliver a

Moment and Coefficient				
Panel A: The Role of Policy Inertia	$\tau_i = 0.98$	$\tau_i = 0.90$	$\tau_i = 0.50$	$\tau_i = 0.00$
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	0.925	0.918	0.846	0.500
$Corr(\Delta c_t, \Delta c_{t+1})$	0.114	0.109	0.105	0.102
$\sigma(\Delta c_t)$	2.720	2.739	2.941	3.122
Panel B: The Role of Price Stickiness		$\alpha = 1/9$	$\alpha = 7/9$	$\alpha = 8.5/9$
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$		0.628	0.925	0.972
$E(r^c - r_f) + J.I.$		0.721	0.934	0.949
A_i		0.434	4.637	11.645
λ_a		19.404	19.750	19.849
B_a		1.529	1.943	1.971

Table 3.9: Panel A shows the persistence of expected and realized consumption growth and volatility of realized consumption growth for various level of interest rate inertia. Panel B shows the persistence in expected consumption growth and consumption risk premium components as a function of the degree of price stickiness. The level of interest rate inertia is fixed at $\tau_i = 0.98$. The Jensen's inequality term is $J.I. = Var_t(r^c_{t+1})/2$. Calibration parameters are from the model with policy inertia in Table 3.3.

consumption growth process that is not very persistent in its realizations, but has a highly persistent conditional mean.

Long-run risk is priced only when the representative agent has recursive preferences. Following the discussion in Section 3.3, one would expect the risk premium to be high with early resolution of risk and low with late resolution of risk. Figure 3.10 confirms this intuition and shows that the consumption risk premium and the price of risk of technology are increasing in the level of the elasticity of intertemporal substitution.

3.6 Additional Asset Pricing Implications

I now focus on additional asset pricing implication of the model. The model delivers testable restrictions on the joint behavior of consumption, dividends, real wages and inflation. Some of these restrictions are novel to the framework developed in this paper. As is common in the long-run risk literature, the main source of asset price variability relative to macroeconomic aggregates is a small, predictable, and highly persistent component in consumption growth. It is therefore natural to test the model by evaluating the ability of asset prices to predict the long-run behavior of equilibrium macroeconomic variables, such as consumption, dividend,

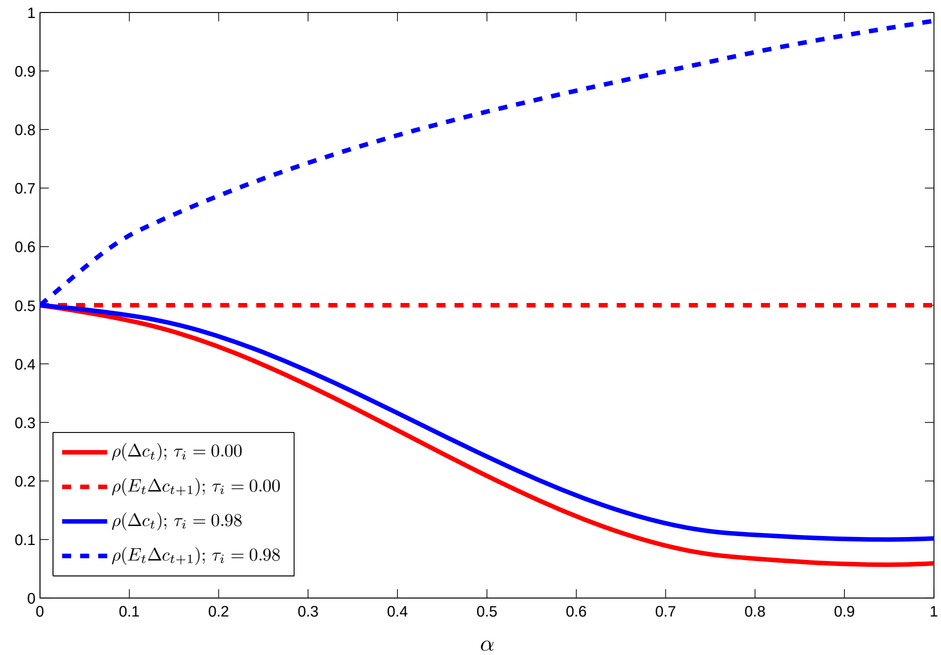

Figure 3.9: Persistence of realized and expected consumption growth as a function of the level of price stickiness α. Both the case with policy inertia and without policy inertia are considered ($\tau_i = 0.98$ and $\tau_i = 0.00$). Calibration parameters are from the model with policy inertia in Table 3.3.

real wages, and inflation. For this exercise, I consider the calibrated version of the model without policy inertia in Section 3.4.1.

Tables 3.10 to 3.13 show the results of model-implied predictive regressions. Some of these regressions, such as the ones of future consumption and dividend growth on the price-dividend ratio, are common to the the long-run risk literature in which the dynamics of consumption and dividends are exogenously specified. However, I can exploit the general equilibrium results of the model and test novel additional restrictions on the joint dynamics of allocations and prices.

The novelty of these regressions goes in two directions. First, I can test whether *nominal* asset prices and returns have predictive power for equilibrium allocations. This is not possible in standard long-run risk models, which entirely focus on the dynamics of real assets. A natural test for this exercise is the target of the monetary authority, the one-period nominal interest rate. Second, I can test the predictability power of asset prices for

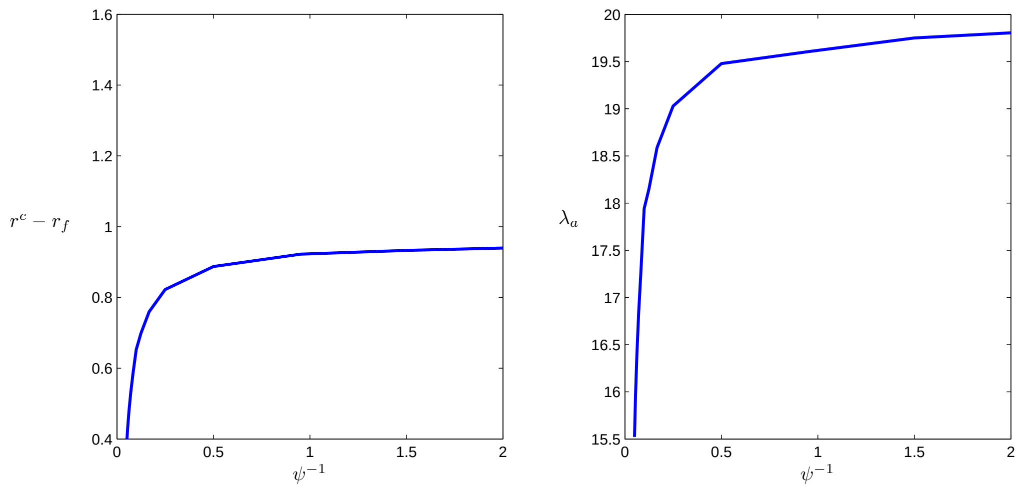

Figure 3.10: Consumption risk premium and price of technology risk λ_a as a function of the elasticity of intertemporal substitution ψ^{-1} for the model with policy inertia. All other parameters are from Table 3.3.

a larger set of macro variables, including real wages and inflation, and not limit myself to consumption and dividend growth only.[9]

3.6.1 The Predictive Power of the Price-Dividend Ratio

Tables 3.10 and 3.11 show regression coefficients, t statistics, and R^2 coefficients for predictive regressions of annual consumption, dividend, real wages growth, and inflation on the price-dividend ratio. In each of the tables, I first report the results I obtain from the data, and then compare them with what is implied by the model. In addition to the results from the baseline model, I report the results for two alternative calibrations. In Table 3.10, I consider an increase (decrease) in price stickiness, by increasing (decreasing) the probability that a firm is unable to adjust its price optimally at each point in time. In Table 3.11, I consider an alternative comparative static exercise, and study the impact of a stronger (weaker) monetary policy, by increasing (decreasing) the monetary authority reaction τ_π to inflation.

Similar to what is obtained in the long-run risk literature, the price-dividend ratio in the model positively predicts future consumption and dividend growth. At the one- (three-)

[9]In the literature, long-run risk models have usually been tested in their ability to predict future excess returns. Here, risk premia are constant as a consequence of the homoscedasticity of the technology shock.

$\sum_{j=1}^{J}(\Delta c_{t+j})$	Data			Baseline Model			$(\alpha = 6/9)$			$(\alpha = 8/9)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.001	0.315	0.001	0.097	4.065	0.159	0.117	5.946	0.280	0.077	3.490	0.125
3	-0.012	-0.984	0.034	0.166	2.655	0.100	0.201	3.525	0.157	0.132	2.352	0.084
5	-0.018	-1.195	0.047	0.173	1.867	0.063	0.211	2.423	0.093	0.138	1.658	0.055

$\sum_{j=1}^{J}(\Delta d_{t+j})$	Data			Baseline Model			$(\alpha = 6/9)$			$(\alpha = 8/9)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.070	1.902	0.080	0.284	2.918	0.095	0.344	4.511	0.188	0.225	2.461	0.073
3	0.084	1.133	0.034	0.478	1.991	0.067	0.587	2.847	0.114	0.379	1.715	0.055
5	0.084	1.303	0.027	0.489	1.371	0.045	0.610	1.954	0.071	0.387	1.171	0.039

$\sum_{j=1}^{J}(\Delta w_{t+j})$	Data			Baseline Model			$(\alpha = 6/9)$			$(\alpha = 8/9)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.002	-0.478	0.004	0.051	1.459	0.045	0.085	3.123	0.133	0.032	0.973	0.030
3	-0.013	-0.667	0.028	0.078	0.944	0.039	0.140	2.035	0.086	0.046	0.562	0.031
5	-0.021	-0.656	0.035	0.070	0.503	0.033	0.138	1.337	0.056	0.036	0.174	0.029

$\sum_{j=1}^{J}(\pi_{t+j})$	Data			Baseline Model			$(\alpha = 6/9)$			$(\alpha = 8/9)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.008	0.658	0.007	0.114	11.993	0.605	0.300	12.299	0.618	0.031	11.967	0.603
3	0.009	0.258	0.002	0.199	5.542	0.298	0.524	5.624	0.304	0.055	5.546	0.299
5	0.003	0.063	0.000	0.215	3.802	0.172	0.561	3.755	0.174	0.060	3.825	0.173

Table 3.10: Regression coefficients, t statistics and R^2 statistics for predictive regressions of consumption, dividend, wage growth and inflation on the price-dividend ratio. The baseline model uses the coefficients reported in Table 3.1. With the exclusion of real wages, the statistics for the model are mean values for $10,000$ simulations each with 85×12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for $10,000$ simulations each with 64×12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\alpha = 6/9$, and $\alpha = 7/9$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.

year horizon, the point estimate for dividend growth is significantly different from zero and roughly four (six) times as large as in the data. The significance of the estimates decreases with the horizon. At the five-year horizon, the price-dividend ratio does not significantly predict dividend growth. The estimates for consumption growth show a similar pattern.

In the data, univariate regressions suggest that the price-dividend ratio does not predict future consumption and dividend growth (Campbell and Shiller (1988), Fama and French (1988)). In the model, the predictability depends on the level of price stickiness and on the strength on monetary policy. The predictive power of the price-dividend ratio for consumption and dividend growth is decreasing in the level of price stickiness. When firms

$\sum_{j=1}^{J}(\Delta c_{t+j})$	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.001	0.315	0.001	0.097	4.065	0.159	0.095	5.180	0.230	0.080	3.665	0.136
3	-0.012	-0.984	0.034	0.166	2.655	0.100	0.163	3.194	0.134	0.136	2.441	0.089
5	-0.018	-1.195	0.047	0.173	1.867	0.063	0.172	2.210	0.081	0.142	1.709	0.058

$\sum_{j=1}^{J}(\Delta d_{t+j})$	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.070	1.902	0.080	0.284	2.918	0.095	0.280	3.839	0.146	0.233	2.600	0.080
3	0.084	1.133	0.034	0.478	1.991	0.067	0.475	2.507	0.094	0.391	1.796	0.059
5	0.084	1.303	0.027	0.489	1.371	0.045	0.492	1.723	0.060	0.399	1.222	0.041

$\sum_{j=1}^{J}(\Delta w_{t+j})$	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.002	-0.478	0.004	0.051	1.459	0.045	0.064	2.408	0.090	0.036	1.134	0.035
3	-0.013	-0.667	0.028	0.078	0.944	0.039	0.103	1.599	0.064	0.054	0.678	0.033
5	-0.021	-0.656	0.035	0.070	0.503	0.033	0.098	1.003	0.045	0.043	0.262	0.030

$\sum_{j=1}^{J}(\pi_{t+j})$	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.008	0.658	0.007	0.114	11.993	0.605	0.055	12.307	0.617	0.078	12.010	0.606
3	0.009	0.258	0.002	0.199	5.542	0.298	0.097	5.649	0.304	0.137	5.539	0.300
5	0.003	0.063	0.000	0.215	3.802	0.172	0.104	3.797	0.175	0.149	3.801	0.173

Table 3.11: Regression coefficients, t statistics and R^2 statistics for predictive regressions of consumption, dividend, wage growth and inflation on the price-dividend ratio. The baseline model uses the coefficients reported in Table 3.1. With the exclusion of real wages, the statistics for the model are mean values for $10,000$ simulations each with 85×12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for $10,000$ simulations each with 64×12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\tau_\pi = 1.50$, and $\tau_\pi = 1.01$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.

can, on average, adjust their prices only once every nine months ($\alpha = 8/9$), the price-dividend ratio significantly predicts dividend growth only at the one-year horizon, while the point estimates for the three- and five-year horizon regressions become insignificantly different from zero. Moreover, the price-dividend ratio exhibits weaker explanatory power and, consistently with the data, only explains 8% of the variation in the one-year-ahead dividend growth and 4% of the variation in dividend growth for the next five years.

A similar decrease in the predictive power of the price-dividend ratio is obtained in the case of a weaker monetary policy. When the interest rate rule coefficient to inflation is $\tau_\pi = 1.05$, the price-dividend ratio significantly predicts future consumption and dividends only at short horizons. Moreover, the price-dividend ratio never explains more than 8% of the variation of future consumption. These results are consistent with the discussion in Section 3.4, where I showed that, for a given level of price stickiness, a weaker monetary policy magnifies the effect of nominal frictions on real allocations.

In the data, the price-dividend ratio predicts neither growth in real wages nor inflation significantly. For real wages, the model delivers similar results. Although the sign of the estimated regression coefficient is opposite to the ones obtained in the data – positive in our model, while negative in the data – none of the estimates is significantly different from zero. While at the one-year horizon the price-dividend ratio explains too much variability of the growth in real wages (4% versus virtually 0% in the data), at longer horizons the R^2 coefficients obtained from model's based regressions are comparable to the data. Similarly to what was obtained for the predictability of consumption and dividend growth, increasing the impact of the nominal friction on real allocations, either by directly increasing the level of price stickiness or by reducing the reaction of the monetary authority to inflation, reduces the predictive power of the price-dividend ratio.

In the model, and contrary to the data, a higher price-dividend ratio significantly predicts higher future inflation. At the one-year horizon, the price-dividend ratio explains 60% (!) of the variation of future inflation and, while the R^2 coefficients sharply decrease with the horizon, the point estimates remain significantly different from zero. Unlike what happens in the case of consumption, dividend and real wage growth, changing the level of nominal frictions only has a marginal impact on the predictability of future inflation.

3.6.2 The Predictive Power of the Nominal Interest Rate

Tables 3.12 and 3.13 show regression coefficients, t statistics, and R^2 coefficients for predictive regressions of annual consumption, dividend, real wages growth, and inflation on the one-month nominal interest rate. Similarly to the case of predictive regressions for the price-dividend ratio, in each of the tables, I first report the results obtained from the data, and then compare them with what is implied by the model. Again, in addition to the results from the baseline model, I report the results for two alternative calibrations. In Table 3.12, I consider and increase (decrease) in price stickiness, by increasing (decreasing) the probability that a firm is unable to adjust its price optimally at each point in time. In Table 3.13, I consider an alternative comparative static exercise and study the impact of a stronger (weaker) monetary policy, by increasing (decreasing) the monetary authority

$\sum_{j=1}^{J}(\Delta c_{t+j})$	Data			Baseline Model			($\alpha = 6/9$)			($\alpha = 8/9$)		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.073	-1.118	0.021	0.352	3.113	0.110	0.207	4.590	0.192	1.689	2.787	0.089
3	-0.178	-1.163	0.026	0.574	1.828	0.073	0.347	2.568	0.109	2.767	1.666	0.064
5	-0.157	-0.634	0.014	0.559	1.130	0.051	0.353	1.669	0.068	2.743	1.052	0.046

$\sum_{j=1}^{J}(\Delta d_{t+j})$	Data			Baseline Model			($\alpha = 6/9$)			($\alpha = 8/9$)		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.360	-0.839	0.008	1.005	2.227	0.066	0.606	3.541	0.129	4.806	1.955	0.052
3	-1.280	-1.563	0.031	1.598	1.326	0.052	1.001	2.055	0.082	7.647	1.177	0.045
5	-1.476	-1.351	0.035	1.487	0.761	0.041	0.999	1.312	0.055	7.251	0.686	0.037

$\sum_{j=1}^{J}(\Delta w_{t+j})$	Data			Baseline Model			($\alpha = 6/9$)			($\alpha = 8/9$)		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.225	-3.757	0.166	0.167	1.082	0.033	0.148	2.419	0.091	0.570	0.687	0.024
3	-0.625	-3.307	0.243	0.223	0.544	0.038	0.230	1.393	0.064	0.641	0.256	0.034
5	-1.082	-3.218	0.350	0.149	0.129	0.039	0.206	0.775	0.048	0.161	0.124	0.039

$\sum_{j=1}^{J}(\pi_{t+j})$	Data			Baseline Model			($\alpha = 6/9$)			($\alpha = 8/9$)		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.408	2.562	0.078	0.428	7.995	0.423	0.541	8.258	0.425	0.710	8.283	0.426
3	0.774	1.553	0.046	0.728	4.031	0.204	0.924	4.135	0.206	1.217	4.151	0.208
5	1.217	1.415	0.053	0.756	2.598	0.117	0.968	2.683	0.119	1.285	2.716	0.121

Table 3.12: Regression coefficients, t statistics and R^2 statistics for predictive regressions of consumption, dividend, wage growth and inflation on the one-period nominal interest rate. The baseline model uses the coefficients reported in Table 3.1. With the exclusion of real wages, the statistics for the model are mean values for $10,000$ simulations each with 85×12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for $10,000$ simulations each with 64×12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\alpha = 6/9$, and $\alpha = 7/9$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.

reaction τ_π to inflation.

In the model, the nominal interest rate predicts future consumption and dividend growth. However, the significance of the regression coefficients sharply decreases with the horizon. As was the case for the price-dividend ratio, the predictive power of the nominal interest rate is decreasing in the level of price stickiness and increasing in the strength of monetary policy. The data suggests that the nominal interest rate does not significantly predict future consumption and dividend growth, consistently with a situation of large nominal frictions and/or weak monetary policy.

In the data, the nominal interest rate positively predicts inflation and negatively pre-

$\sum_{j=1}^{J}(\Delta c_{t+j})$												
	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.073	-1.118	0.021	0.352	3.113	0.110	0.984	4.047	0.159	0.673	2.920	0.096
3	-0.178	-1.163	0.026	0.574	1.828	0.073	1.639	2.318	0.095	1.108	1.745	0.067
5	-0.157	-0.634	0.014	0.559	1.130	0.051	1.658	1.501	0.061	1.100	1.106	0.048

$\sum_{j=1}^{J}(\Delta d_{t+j})$												
	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.360	-0.839	0.008	1.005	2.227	0.066	2.862	3.035	0.102	1.921	2.063	0.057
3	-1.280	-1.563	0.031	1.598	1.326	0.052	4.690	1.797	0.069	3.082	1.248	0.048
5	-1.476	-1.351	0.035	1.487	0.761	0.041	4.630	1.132	0.048	2.934	0.738	0.039

$\sum_{j=1}^{J}(\Delta w_{t+j})$												
	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	-0.225	-3.757	0.166	0.167	1.082	0.033	0.625	1.859	0.063	0.256	0.803	0.026
3	-0.625	-3.307	0.243	0.223	0.544	0.038	0.938	1.060	0.052	0.301	0.327	0.034
5	-1.082	-3.218	0.350	0.149	0.129	0.039	0.799	0.529	0.043	0.126	0.060	0.038

$\sum_{j=1}^{J}(\pi_{t+j})$												
	Data			Baseline Model			$(\tau_\pi = 1.5)$			$(\tau_\pi = 1.05)$		
Years (J)	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2	$\hat{\beta}$	t	R^2
1	0.408	2.562	0.078	0.428	7.995	0.423	0.582	8.249	0.424	0.683	8.270	0.426
3	0.774	1.553	0.046	0.728	4.031	0.204	0.995	4.140	0.206	1.173	4.155	0.208
5	1.217	1.415	0.053	0.756	2.598	0.117	1.044	2.688	0.118	1.237	2.709	0.121

Table 3.13: Regression coefficients, t statistics and R^2 statistics for predictive regressions of consumption, dividend, wage growth and inflation on the one-period nominal interest rate. The baseline model uses the coefficients reported in Table 3.1. With the exclusion of real wages, the statistics for the model are mean values for $10,000$ simulations each with 85×12 monthly observations that are aggregated to an annual frequency. For real wages, the statistics for the model are mean values for $10,000$ simulations each with 64×12 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The last six columns report summary statistics for alternative models where $\tau_\pi = 1.50$, and $\tau_\pi = 1.01$, respectively, leaving all other parameters unchanged. With the exception of wage growth, data is monthly for the period 1926-2010. Wage growth is real compensation per hour from BLS, for the period 1947-2010.

dicts future wages growth. The model performs well when the predictability of inflation is considered. While the explanatory power of the nominal interest rate is too high, I can reproduce regression coefficients that, consistently with the data, are positive, statistically significant, and increasing with the horizon. As was the case for the regressions on the price-dividend ratio, varying the impact of the nominal friction only has a marginal effect on the predictability of future inflation.

In contrast, the model struggles to capture the negative relation between nominal interest rates and real wages. Both in the baseline calibration and in all of the alternative calibrations considered, the model cannot explain the statistically significant relation between the nominal interest rate and future wages observed in the data. This is a consequence of the simple structure of the labor market in the model, which is not able to account for the joint distribution of hours worked, real wages, and production.

3.6.3 The Term Structure of Interest Rates

Backus and Zin (1994) show that a necessary condition for the average nominal yield curve to be upward sloping is a negative autocorrelation in the nominal pricing kernel. Let $\tau_i = 0$ and $u_t = 0$, for every t. The autocovariance of the nominal pricing kernel is given by

$$Cov(\log M_t, \log M_{t-1}) = (\gamma_a + \varphi_a \pi_a)^2 Cov(\Delta a_t, \Delta a_{t-1}) + (\gamma_a + \varphi_a \pi_a)(\lambda_a + \pi_a) Cov(\Delta a_t, \epsilon_t^a) \quad ,$$

where $(\gamma_a + \varphi_a \pi_a)$ and $(\lambda_a + \pi_a)$ refer to the nominal factor loading and nominal price of risk of technology, respectively. Both the autocovariance of technology growth and the covariance of the technology growth level with its innovation are positive (see equation (3.8)). Therefore, for the autocovariance of the nominal pricing kernel to be negative, the nominal factor loading and the nominal price of risk must have opposite signs. Additionally, the price of risk must be large enough relative to the factor loading to counteract the positive autocovariance term. In the model, the factor loading and the price of risk of technology have the same sign, and therefore the average nominal yield curve is downward sloping.

One can gain more insights into the reason why the average nominal yield curve is downward sloping by considering the correlation of inflation and consumption growth. Piazzesi and Schneider (2007) argue that a surprise increase in U.S. inflation, which lowers the value of nominal bonds, was typically followed by lower consumption in the future. That is, inflation is *bad* news about future consumption. This relationship implies that long-term nominal bonds lose value precisely when agents desire consumption the most, resulting in an upward sloping average nominal yield curve. When $\tau_i = 0$ and $u_t = 0$, for every t, inflation is related to expected future consumption according to (omitting constants):

$$\pi_t = \frac{\pi_a}{g_a} \, E_t \Delta c_{t+1} \quad .$$

The coefficient π_a determines the sensitivity of inflation to technology growth. As discussed in Section 3.3, prices rise in response to a permanently higher level of technology and therefore $\pi_a > 0$. The coefficients g_a, which governs the sensitivity of expected consumption growth to technology, is also positive. This is because, following a permanent technology improvement, agents expect equilibrium consumption to be permanently higher. Therefore, in the model, inflation is *good* news about future consumption, and the average nominal yield curve is downward sloping.

Bansal and Shaliastovich (2010) develop a long-run risk model with an exogenously specified process for inflation to account for various predictability puzzles in bond and currency markets. Because of the exogeneity of the inflation process, they are free to impose a negative sensitivity of inflation to news to expected consumption growth, thus obtaining an average nominal yield curve that is upward sloping. In the general equilibrium model developed here, I do not have this freedom. When the only shock in the economy is a permanent technology shock, consumption and inflation move in the same direction, resulting in negatively sloped average nominal yield curve.

An upward sloping nominal yield curve can be obtained if the model is extended to incorporate additional shocks that generate a negative correlation between consumption and inflation. Examples of these shocks are a temporary technology shock, a price markup shock, a labor supply shock, and a government spending shock. When the effect of these additional shocks is strong enough to counteract the effect of the permanent technology shock, then the resulting nominal term structure is positively sloped. See Rudebusch and Swanson (2012) and Hsu (2011) for a treatment of the yield curve in a framework similar to the one developed in this paper.[10]

3.7 Conclusion

Where does long-run consumption risk come from? I analyze a real business cycle model in which the nominal frictions generated by price stickiness endogenously deliver an equilibrium consumption growth process that shows low persistence unconditionally, but has a highly persistent conditional mean. At each point in time, only a fixed fraction of firms can optimally adjust the level of production, resulting in a realized consumption process that is not very persistent, as we observe in the data. However, since the effects of the nominal frictions on real allocations are only transitory, expected consumption growth can be highly persistent, depending on the persistence of the state variables in the economy.

The lack of an internal propagation mechanism in the model without policy inertia re-

[10] A version of the model in which technology has both a permanent and a transitory component as in Croce (2010) can deliver both long-run consumption risk and a positively sloped average nominal term structure. Results are available upon request.

quires such a high persistence to be exogenously imposed, resulting in a counterfactually high autocorrelation in technology growth. However, when the monetary authority explicitly smooths the interest rate over time, expected consumption growth can be extremely persistent even when technology growth is only modestly autocorrelated.

The model provides a theoretical basis for identifying a new source of long-run risk in consumption growth. The general equilibrium equations in the model link real and nominal asset prices to consumption, dividends, wages, and inflation. I test this link in time-series regressions and show that the predictability of macroeconomic variables depends on the degree of price stickiness and on the strength of monetary policy. The model struggles to reproduce the predictability of future real wages observed in the data. This is not surprising, as the simple labor market considered in this paper cannot explain the joint distribution of hours worked, real wages, and production. In the future, I plan to analyze a model with a more realistic labor market in order to account for these additional macroeconomic moments.

More generally, it would be interesting to analyze the impact of alternative nominal frictions on real allocations, and to further inspect the mechanism linking those frictions to monetary policy and asset prices. Here, I focused on the role played by price stickiness, given both its widespread use in the macroeconomic literature and its analytical tractability. However, other frictions warrant further attention. For instance, staggered wage settings, sticky information, and financial frictions affecting the behavior of financial institutes, all have a non-trivial impact on real allocations. In principle, these frictions could generate consumption dynamics that are consistent with what is usually assumed in the long-run risk literature. I leave this topic for future research.

Appendix A

Uncovered Interest Rate Parity Puzzle: An Explanation based on Recursive Utility and Stochastic Volatility

A.1 Pricing Kernel Linearization

In this Appendix I linearize the pricing kernel m using the method of undetermined coefficients. Given the state variables and the log-linear structure of the model, conjecture a solution for the value function of the form,

$$w_t = \bar{\omega} + \omega_x x_t + \omega_v v_t,$$

where $\bar{\omega}$, ω_x, and ω_v are constants to be determined. Therefore

$$w_{t+1} + x_{t+1} = \bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_v v_{t+1}$$

and, using the properties of normal random variables, u_t can be expressed as

$$\begin{aligned}
u_t &\equiv \log(\mu_t(\exp(w_{t+1} + x_{t+1}))) \\
&= \log(E_t[\exp(w_{t+1} + x_{t+1})^{\alpha}]^{\frac{1}{\alpha}})] \\
&= E_t[w_{t+1} + x_{t+1}] + \frac{\alpha}{2}\mathrm{Var}_t[w_{t+1} + x_{t+1}] \\
&= \bar{\omega} + (\omega_x + 1)(1 - \varphi_x)\theta_x + \omega_v(1 - \varphi_v)\theta_v + (\omega_x + 1)\varphi_x x_t + \omega_v \varphi_v v_t \\
&\quad + \frac{\alpha}{2}(\omega_x + 1)^2 v_t + \frac{\alpha}{2}\omega_v^2 \sigma_v^2.
\end{aligned}$$

Using the above expression, I solve for the value-function parameters by matching coefficients

$$\omega_x = \kappa(\omega_x + 1)\varphi_x$$

$$\begin{aligned}
\Rightarrow \omega_x &= \left(\frac{\kappa}{1-\kappa\varphi_x}\right)\varphi_x \\
\omega_v &= \kappa[\omega_v\varphi_v + \frac{\alpha}{2}(\omega_x+1)^2] \\
\Rightarrow \omega_v &= \left(\frac{\kappa}{1-\kappa\varphi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\varphi_x}\right)^2\right] \\
\bar{\omega} &= \frac{\bar{\kappa}}{1-\kappa} + \frac{1}{1-\kappa}\left[(\omega_x+1)(1-\varphi_x)\theta_x + \omega_v(1-\varphi_v)\theta_v + \frac{\alpha}{2}\omega_v^2\sigma_v^2\right].
\end{aligned}$$

The solution allows me to simplify the term $[\log W_{t+1} - \log\mu_t(W_{t+1})]$ in the pricing kernel in equation (B.5):

$$\begin{aligned}
\log W_{t+1} - \log\mu_t(W_{t+1}) &= w_{t+1} + x_{t+1} - \log\mu_t(\exp(w_{t+1}+x_{t+1})) \\
&= (\omega_x+1)[x_{t+1} - E_t x_{t+1}] + \omega_v[v_{t+1} - E_t v_{t+1}] \\
&\quad -\frac{\alpha}{2}(\omega_x+1)^2\mathrm{Var}_t[x_{t+1}] - \frac{\alpha}{2}\omega_v^2\mathrm{Var}_t[v_{t+1}] \\
&= (\omega_x+1)v_t^{1/2}\varepsilon_{t+1}^x + \omega_v\sigma_v\varepsilon_{t+1}^v - \frac{\alpha}{2}(\omega_x+1)^2 v_t - \frac{\alpha}{2}\omega_v^2\sigma_v^2.
\end{aligned}$$

Equation (1.7) follows by collecting terms.

A.2 Consumption Growth Process Calibration

The consumption growth process of Bansal and Yaron (2004) is summarized by the following three equations:

$$l_{t+1} = \rho_l l_t + \varphi_e\sigma_t e_{t+1},$$

$$x_{t+1} = \mu + l_t + \sigma_t\eta_{t+1}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1},$$

where $(\eta_t; e_t; w_t)$ are i.i.d. mean-zero, variance-one, normally distributed innovations. Consumption growth contains a low-frequency component l_t (the long-run effect) and is heteroskedastic, with conditional variance σ_t^2. These two state variables capture time-varying growth rates and time-varying economic uncertainty. The calibrated values can be found in Table IV of Bansal and Yaron (2004).

I match the mean of the consumption growth process specified by Bansal and Yaron (2004) with the mean of the consumption growth process specified in my model:

$$E(x_t) = EE_t(x_t) = \mu \equiv \theta_x.$$

The unconditional variance of the long-run component is

$$var(l_t) = \rho_l^2 var(l_{t-1}) + \varphi_e^2 var(\sigma_t e_{t+1}).$$

The process l_t is assumed to be stationary, therefore $var(l_t) = var(l_{t-1})$, and

$$var(l_t) = \frac{\varphi_e^2\sigma^2}{1-\rho_l^2},$$

where the last equality follows from the fact that

$$var(\sigma_t e_{t+1}) = EE_t(\sigma_t^2 e_{t+1}^2) - (EE_t(\sigma_t e_{t+1}))^2 = \sigma^2.$$

The unconditional variance of the consumption growth process is

$$var(x_t) = var(l_t) + var(\sigma_t \eta_{t+1}) = \frac{\varphi_e^2 \sigma^2}{1-\rho_l^2} + \sigma^2 = 6.37 \times 10^{-5}.$$

A similar calculation gives the expression for the autocorrelation in consumption growth, i.e.

$$Autocorr(x_{t+1}, x_t) = \frac{cov(x_{t+1}, x_t)}{var(x_t)} = \frac{\rho_l \varphi_e^2 \sigma^2}{\varphi_e^2 \sigma^2 + \sigma^2(1-\rho_l^2)} = 4.36\% \equiv \varphi_x.$$

In my model, the unconditional variance of consumption growth is equal to $\frac{\theta_v}{1-\varphi_x^2}$. Given the value of φ_x obtained above, and matching the unconditional variances of the two consumption growth process, the mean θ_v of the stochastic volatility process v_t is obtained as follows

$$\theta_v = var(x_t)(1-\varphi_x^2) = 6.3553 \times 10^{-5}.$$

Finally, the autocorrelation of the stochastic volatility process is obtained by matching the relevant coefficients, i.e. $\varphi_v = \nu_1$.

Appendix B

Monetary Policy and the Uncovered Interest Rate Parity Puzzle

B.1 Abstracting from Real Exchange Rates

The crux of our question asks "how does Taylor-rule-implied inflation affect exchange rates?" In this appendix we try to clarify things further by abstracting from real exchange rate variation. We set $n_t = n_t^*$, implying that $\log(S_t/S_{t-1}) = \pi_t - \pi_t^*$, so that relative PPP holds exactly. We don't take this specification seriously for empirical analysis. We use it to try to understand exactly how the Taylor rule restricts inflation dynamics and, therefore, nominal exchange rate dynamics. As we'll see in Section 2.5, the lessons we learn carry over to more empirically-relevant models with both nominal and real variability.

We use simplest possible variant of the Taylor rule:

$$i_t = \tau + \tau_\pi \pi_t + z_t \quad , \tag{B.1}$$

where z_t is a 'policy shock' that follows the process

$$\begin{aligned} z_{t+1} &= (1-\varphi_z)\theta_z + \varphi_z z_t + \sqrt{v_t}\epsilon_{t+1}^z & \text{(B.2)} \\ v_{t+1} &= (1-\varphi_v)\theta_v + \varphi_v v_t + \sigma_v \epsilon_{t+1}^v \quad . & \text{(B.3)} \end{aligned}$$

There are, of course, many alternative specifications. A good discussion related to asset pricing is Ang, Dong, and Piazzesi (2007). Cochrane (2011) uses a similar specification to address issues related to price-level determinacy and the identification of the parameters in Equation (B.1). We begin with it for reasons of tractability and clarity. We then go on to include the nominal depreciation rate and the lagged interest rate.

In addition to $n_t = n_t^*$, we abstract from real interest rate variation by setting $n_t = n_t^* = 1$. For exchange rates, conditional on having $n_t = n_t^*$, this is without loss of generality. The (nominal) short interest rate, $i_t = -\log E_t\, m_{t+1}$, is therefore

$$\begin{aligned} i_t &= -\log E_t\, e^{-\pi_{t+1}} \\ &= E_t\, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1}) \quad . \end{aligned} \tag{B.4}$$

The Taylor rule (B.1) and the Euler equation (B.4) imply that inflation must satisfy the following difference equation:

$$\pi_t = -\frac{1}{\tau_\pi}\big(\tau + z_t + E_t\, \pi_{t+1} - \frac{1}{2} Var_t(\pi_{t+1})\big) \quad . \tag{B.5}$$

Given the log-linear structure of the model, guess that the solution has the form,

$$\pi_t = a + a_1 z_t + a_2 v_t \quad . \tag{B.6}$$

Instead of solving Equation (B.5) forward, just substitute Equation (B.6) into the Euler equation (B.4), compute the moments, and then solve for the a_i coefficients by matching up the result with the Taylor rule (B.1). This gives,

$$\begin{aligned} a &= \frac{C - \tau}{\tau_\pi} \\ a_1 &= \frac{1}{\varphi_z - \tau_\pi} \\ a_2 &= \frac{1}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} \end{aligned}$$

where

$$C \equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2 \quad .$$

More explicit derivations are given in Appendix B.2. Inflation and the short rate can now be written as:

$$\begin{aligned} \pi_t &= \frac{C-\tau}{\tau_\pi} + \frac{1}{\varphi_z - \tau_\pi} z_t + \frac{1}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} v_t \\ i_t &= C + \frac{\varphi_z}{\varphi_z - \tau_\pi}\, z_t + \frac{\tau_\pi}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)}\, v_t \\ &= C + \varphi_z a_1 z_t + \tau_\pi a_2 v_t \quad , \end{aligned}$$

and the pricing kernel as

$$\begin{aligned} -\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\epsilon^z_{t+1} + \sigma_v a_2 \epsilon^v_{t+1} \\ &= D + \frac{1}{\varphi_z - \tau_\pi}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)} v_t \\ &\quad + \frac{1}{\varphi_z - \tau_\pi} v_t^{1/2}\epsilon^z_{t+1} + \frac{\sigma_v}{2(\varphi_z - \tau_\pi)^2(\varphi_v - \tau_\pi)}\epsilon^v_{t+1} \end{aligned} \tag{B.7}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2 \quad .$$

Now consider a foreign country, say the UK. Denote all foreign variables with an asterisk. The foreign Taylor rule is

$$i_t^* = \tau^* + \tau_\pi^* \pi_t^* + z_t^* \quad .$$

with z_t^* and its volatility following processes analogous to Equations (B.2–B.3). For now, z_t and z_t^* can have any correlation structure. Repeating the above calculations for the UK and then substituting the results into Equations (2.11–2.14) we get

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_\pi a_2 v_t - \tau_\pi^* a_2^* v_t^* \\
q_t &= D - D^* + a_1 \varphi_z z_t - a_1^* \varphi_z^* z_t^* + a_2 \varphi_v v_t - a_2^* \varphi_v^* v_t^* \\
p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right)
\end{aligned}$$

where $D \equiv C + (\sigma_v a_2)^2/2$. It is easily verified that $p_t + q_t = i_t - i_t^*$.

Result 3: *Symmetry and* $\varphi_z = 0$

If all foreign and domestic parameter values are the same and $\varphi_z = \varphi_z^* = 0$, then the UIP regression parameter (2.7) is:

$$b = \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} = \frac{Cov(p_t + q_t, q_t)}{Var(p_t + q_t)} \tag{B.8}$$

$$= \frac{\varphi_v}{\tau_\pi} \tag{B.9}$$

Calculations are provided in Appendix B.2.

Discussion

The sign of $Cov(p_t, q_t)$ does not depend on φ_z. That is, $Cov(p_t, q_t)$ is essentially the covariance between the kernel's mean and its variance and, while v_t appears in both, z_t appears only in the mean. The assumption $\varphi_z = 0$ is therefore relatively innocuous in the sense that it has no effect on one of the two necessary conditions (2.8) and (2.9).

We require $\tau_\pi > 1$ for the solution to make sense. Therefore, according to Equation (B.9), $0 < b < 1$ unless $\varphi_v < 0$. The latter is implausible. Nevertheless, the UIP regression coefficient can be significantly less than unity and the joint distribution of exchange rates and interest rates will admit positive expected excess returns on a suitably-defined trading strategy.

We cannot, at this point, account for $b < 0$. But the model does deliver some insights into our basic question of how Taylor rules restrict inflation dynamics and, consequently, exchange rate dynamics. We summarize with several remarks.

Remark 3: *This is not just a relabeled affine model*

Inspection of the pricing kernel, Equation (B.7), indicates that it is basically a log-linear function of two unobservable factors. Is what we are doing just a relabeling of the class of latent-factor affine models described in Backus, Foresi, and Telmer (2001)? The answer is no and the reason is that the Taylor rule imposes economically-meaningful restrictions on the model's coefficients.

To see this consider a pricing kernel of the form

$$-\log m_{t+1} = \alpha + \chi v_t + \gamma v_t^{1/2} \varepsilon_{t+1} \tag{B.10}$$

where v_t is an arbitrary, positive stochastic process, and an analogous expression describes m^*_{t+1}. Backus, Foresi, and Telmer (2001) show that such a structure generates a UIP coefficient $b < 0$ if $\chi > 0$ and $\chi < \gamma^2/2$. The former condition implies that the mean and variance of negative the log kernel move in the same direction — this gives $Cov(p_t, q_t) < 0$ — and the latter implies that the variance is more volatile so that $Var(p_t) > Var(q_t)$.

Now compare Equations (B.10) and (B.7). The Taylor rule imposes the restrictions that χ can only be positive if φ_v is negative (because $a_2 < 0$ since $\tau_\pi > 1$) and that $\chi/\gamma = \rho_v/(2\tau_\pi(\tau_\pi - \varphi_v))$. Both χ and γ are restricted by value of the policy parameter τ_π, and the dynamics of the volatility shocks. In words, the UIP evidence requires the mean and the variance of the pricing kernel to move in particular ways relative to each other. The Taylor rule and its implied inflation dynamics place binding restrictions on how this can happen. The unrestricted pricing kernel in Equation (B.10) can account for $b < 0$ irrespective of the dynamics of v_t. Imposing the Taylor rule says that v_t must be negatively autocorrelated.

Remark 4: *Reason that negatively-correlated volatility is necessary for $b < 0$?*

First, note that $a_2 < 0$, so that an increase in volatility v_t decreases inflation π_t. Why? Suppose not. Suppose that v_t increases. Then, since $\tau_\pi > 1$, the Taylor rule implies that the interest rate i_t must increase *by more* than inflation π_t. However this contradicts the stationarity of inflation which implies that the conditional mean must increase *by less* than the contemporaneous value. Hence $a_2 < 0$. A similar argument implies that $a_1 < 0$ from Equation (B.6). The point is that the dynamics of Taylor-rule implied inflation, at least as long as the real exchange rate is constant, are driven by the *muted response of the interest rate* to a shock, relative to that of the inflation rate.

Next, to understand why $\varphi_v < 0$ is necessary for $b < 0$, consider again an increase in volatility v_t. Since $a_2 < 0$, the U.S. interest rate i_t and the contemporaneous inflation rate π_t must decline. But for $b < 0$ USD must be expected to *depreciate*. This means that, although π_t decreases, $E_t\pi_{t+1}$ must increase. This means that volatility must be negatively autocorrelated.

Finally, consider the more plausible case of positively autocorrelated volatility, $0 < \varphi_v < 1$. Then $b < 1$ which is, at least, going in the right direction (*e.g.*, Backus, Foresi, and Telmer (2001) show that the vanilla Cox-Ingersoll-Ross model generates $b > 1$). The reasoning, again, derives from the 'muted response of the interest rate' behavior required by the Taylor rule. This implies that $Cov(p_t, q_t) > 0$ — thus violating Fama's condition (2.8) — which says that if inflation and expected inflation move in the same direction as the interest rate (because $\varphi_v > 0$), then so must the USD currency risk premium. The Bilson-Fama regression (2.6) can be written

$$q_t \quad = \quad c + b(p_t + q_t) - \text{forecast error} \ \ ,$$

where 'forecast error' is defined as $s_{t+1} - s_t - q_t$. Since $Cov(p_t, q_t) > 0$, then $Var(p_t + q_t) > Var(q_t)$ and, therefore, $0 < b < 1$.

Even more starkly, consider the case of $\varphi_v = 0$ so that $b = 0$. Then the exchange rate is a random walk — *i.e.*, $q_t = 0$ so that $s_t = E_t s_{t+1}$ — and all variation in the interest rate differential is variation in the risk premium, p_t. Taylor rule inflation dynamics, therefore, say that for UIP to be a good approximation, changes in volatility must show up strongly in the conditional mean of inflation and that this can only happen if volatility is highly autocorrelated.

Remark 5: *Identification of policy parameters*

Cochrane (2011) provides examples where policy parameters like τ_π are impossible to distinguish from the parameters of the unobservable shocks. Result 3 bears similarity to Cochrane's simplest example. We can estimate b from data but, if we can't estimate φ_v directly then there are many combinations of φ_v and τ_π that are consistent with any estimate of b.

Identification in our special case, however, is possible because of the conditional variance term in the interest rate equation: $i_t = E_t\pi_{t+1} - Var_t\pi_{t+1}$. To see this note that, with $\varphi_z = 0$, the autocorrelation of the interest rate is φ_v and, therefore, φ_v is identified by observables. Moreover,

$$\frac{i_t}{E_t\pi_{t+1}} = \frac{\tau_\pi}{\varphi_v} \ ,$$

which identifies τ_π because the variables on the left side are observable.

The more general case of $\varphi_z \neq 0$ doesn't work out as cleanly, but it appears that the autocorrelation of inflation and the interest rate jointly identify φ_z and φ_v and the above ratio again identifies the policy parameter τ_π. These results are all special cases of those described in Backus and Zin (2008).

Asymmetric Taylor Rules

Suppose that foreign and domestic Taylor rules depend on the exchange rate in addition to domestic inflation and a policy shock:

$$\begin{aligned} i_t &= \tau + \tau_\pi\pi_t + z_t + \tau_3\log(S_t/S_{t-1}) && \text{(B.11)} \\ i_t^* &= \tau^* + \tau_\pi^*\pi_t^* + z_t^* + \tau_3^*\log(S_t/S_{t-1}) && \text{(B.12)} \end{aligned}$$

The asymmetry that we'll impose is that $\tau_3 = 0$ so that the Fed does not react to the depreciation rate whereas the Bank of England does. Foreign central banks reacting more to USD exchange rates seems plausible. It's also consistent with some empirical evidence in, for example, Clarida, Galí, and Gertler (1999), Engel and West (2006), and Eichenbaum and Evans (1995).

Assuming the same processes for the state variables as Equations (B.2) and (B.3) (and their foreign counterparts), guess that the inflation solutions look like:

$$\begin{aligned} \pi_t &= a + a_1z_t + a_2z_t^* + a_3v_t + a_4v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^*z_t + a_2^*z_t^* + a_3^*v_t + a_4^*v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

and collect the state variables into the vector

$$X_t^\top \equiv \left[z_t \ z_t^* \ v_t \ v_t^*\right]^\top \ .$$

Interest rates, from fishbone Euler equations with real interest rate = 0, must satisfy:

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where,

$$\begin{aligned} B^\top &\equiv \left[\ a_1\varphi_z \quad a_2\varphi_z^* \quad (a_3\varphi_v - \frac{a_1^2}{2}) \quad (a_4\varphi_v^* - \frac{a_2^2}{2})\ \right] \\ C &\equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_z^*(1-\varphi_z^*) + a_3\theta_v(1-\varphi_v) + a_4\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\left(a_3^2\sigma_v^2 + a_4^2\sigma_v^{*2}\right) \\ B^{*\top} &\equiv \left[\ a_1^*\varphi_z \quad a_2^*\varphi_z^* \quad (a_3^*\varphi_v - \frac{a_1^{*2}}{2}) \quad (a_4^*\varphi_v^* - \frac{a_2^{*2}}{2})\ \right] \\ C^* &\equiv a^* + a_1^*\theta_z(1-\varphi_z) + a_2^*\theta_z^*(1-\varphi_z^*) + a_3^*\theta_v(1-\varphi_v) + a_4^*\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\left(a_3^{*2}\sigma_v^2 + a_4^{*2}\sigma_v^{*2}\right) \end{aligned}$$

The solution for the a coefficients and the following result are provided in Appendix B.3.

Result 4: *Asymmetric reaction to exchange rates*

> If foreign and domestic Taylor rules are Equations (B.11) and (B.12), with $\tau_3 = 0$ and all remaining foreign and domestic parameter values the same, then $b < 0$ if $\tau_3^* > \tau_\pi$.

Remark 6: *Pathological policy behavior?*

Interpreted literally, $\tau_3^* > 0$ means that the Bank of England reacts to an *appreciation* in GBP by increasing the British interest rate. However, at the same time, there exist sensible calibrations of the model in which $Cov(i_t^*, \log(S_t/S_{t-1})) > 0$. This makes the obvious point that the Taylor rule coefficients must be interpreted with caution since all the endogenous variables in the rule are responding to the same shocks.

McCallum's Model

McCallum (1994), Equation (17), posits a policy rule of the form

$$i_t - i_t^* \quad = \quad \lambda\big(s_t - s_{t-1}\big) + \sigma\big(i_{t-1} - i_{t-1}^*\big) + \zeta_t \ \ ,$$

where ζ_t is a policy shock. He also defines UIP to include an exogenous shock, ξ_t, so that

$$i_t - i_t^* = E_t\big(s_{t+1} - s_t\big) + \xi_t \ \ .$$

McCallum solves the implicit difference equation for $s_t - s_{t-1}$ and finds that it takes the form

$$s_t - s_{t-1} \quad = \quad -\sigma/\lambda\big(i_t - i_{t-1}\big) - \lambda^{-1}\zeta_t + \big(\lambda + \sigma\big)^{-1}\xi_t$$

He specifies values $\sigma = 0.8$ and $\lambda = 0.2$ — justified by the policy-makers desire to smooth interest rates and 'lean-into-the-wind' regarding exchange rates — which resolve the UIP puzzle by implying a regression coefficient from our Equation (2.6) of $b = -4$. McCallum's insight was, recognizing the empirical evidence of a risk premium in the interest rate differential, to understand that the policy rule and the equilibrium exchange rate must respond to the same shock that drives the risk premium.

In this section we show that McCallum's result can be recast in terms of our pricing kernel model and a policy rule that targets the interest rate itself, not the interest rate differential. The key ingredient is a lagged interest rate in the policy rule:

$$i_t \quad = \quad \tau + \tau_\pi \pi_t + \tau_4 i_{t-1} + z_t \ \ , \tag{B.13}$$

where the processes for z_t and its volatility v_t are the same as above. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t + a_3 i_{t-1} \ \ , \tag{B.14}$$

Substitute Equation (B.14) into the pricing kernel and compute the expectation:

$$i_t = \frac{1}{1 - a_3}\Big(C + a_1 \varphi_z z_t + (a_2 \varphi_v - a_1^2/2) v_t\Big) \ \ ,$$

where

$$C \equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2$$

Match-up the coefficients with the Taylor rule and solve for the a_j parameters:

$$\begin{aligned}
a &= \frac{C}{\tau_\pi+\tau_4} - \frac{\tau}{\tau_\pi} \\
a_1 &= \frac{\tau_\pi+\tau_4}{\tau_\pi(\varphi_z-\tau_\pi-\tau_4)} \\
a_2 &= \frac{(\tau_\pi+\tau_4)^2}{2\tau_\pi^2(\varphi_z-\tau_\pi-\tau_4)^2(\varphi_v-\tau_\pi-\tau_4)} \\
a_3 &= -\frac{\tau_4}{\tau_\pi}
\end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v-\tau_\pi-\tau_4)}$$

and that matching coefficients imply

$$\frac{a_1\varphi_z}{1-a_3} = 1+\tau_\pi a_1 \quad ; \quad \frac{a_2\varphi_v - a_1^2/2}{1-a_3} = \tau_\pi a_2.$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C}{\tau_\pi+\tau_4} - \frac{\tau}{\tau_\pi} + \frac{\tau_\pi+\tau_4}{\tau_\pi(\varphi_z-\tau_\pi-\tau_4)}z_t + \\
&+ \frac{(\tau_\pi+\tau_4)^2}{2\tau_\pi^2(\varphi_z-\tau_\pi-\tau_4)^2(\varphi_v-\tau_\pi-\tau_4)}v_t - \frac{\tau_4}{\tau_\pi}i_{t-1} \\
i_t &= \frac{\tau_\pi}{\tau_\pi+\tau_4}C + \frac{\varphi_z}{\varphi_z-\tau_\pi-\tau_4}z_t + \frac{(\tau_\pi+\tau_4)^2}{2\tau_\pi(\varphi_z-\tau_\pi-\tau_4)^2(\varphi_v-\tau_\pi-\tau_4)}v_t \\
&= \frac{1}{1-a_3}\Big(C + \varphi_z a_1 z_t + (\tau_\pi+\tau_4)a_2 v_t\Big)
\end{aligned}$$

The pricing kernel is

$$-\log m_{t+1} = D + \frac{a_1\varphi_z}{1-a_3}z_t + \frac{a_2\varphi_v - a_3 a_1^2/2}{1-a_3}v_t + a_1 v_t^{1/2}\epsilon_{t+1}^z + \sigma_v a_2 \epsilon_{t+1}^v$$

where

$$D \equiv \frac{C}{1-a_3} + (\sigma_v a_2)^2/2$$

The GBP-denominated kernel and variables are denoted with asterisks. If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$), the interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$i_t - i_t^* \quad = \quad \frac{a_1\varphi_z}{1-a_3}(z_t - z_t^*) + \frac{a_2\varphi_v - a_1^2/2}{1-a_3}(v_t - v_t^*)$$

$$\begin{aligned} q_t &= \frac{a_1\varphi_z}{1-a_3}(z_t - z_t^*) + \frac{a_2\varphi_v - a_3 a_1^2/2}{1-a_3}(v_t - v_t^*) \\ p_t &= -\frac{1}{2}a_1^2(v_t - v_t^*) \end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

The nominal interest rate and the interest rate differential have the same autocorrelation:

$$\begin{aligned} Corr(i_{t+1}, i_t) &= Corr(i_{t+1} - i_{t+1}^*, i_t - i_t^*) \\ &= 1 - (1-\varphi_z)(1+\tau_\pi a_1)^2 \frac{Var(z_t)}{Var(i_t)} - (1-\varphi_v)(\tau_\pi a_2)^2 \frac{Var(v_t)}{Var(i_t)}. \end{aligned}$$

If we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned} b &= \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} \\ &= \frac{\varphi_v - \tau_4}{\tau_\pi} \end{aligned}$$

To see the similarity to McCallum's model define $\zeta \equiv z_t - z_t^*$, and subtract the UK Taylor rule from its U.S. counterpart in (B.13). Assuming symmetry, we get

$$\begin{aligned} i_t - i_t^* &= \tau_\pi(\pi_t - \pi_t^*) + \tau_4\big(i_t - i_t^*\big) + \zeta_t \\ &= \tau_\pi(s_t - s_{t-1}) + \tau_4\big(i_t - i_t^*\big) + \zeta_t\,, \end{aligned}$$

where the second equality follows from market completeness and our simple pricing kernel model. This is the same as McCallum's policy rule with $\tau_\pi = \lambda$ and $\tau_4 = \sigma$. His UIP "shock" is the same as our $p_t = -a_1^2(v_t - v_t^*)/2$, with $\varphi_z = \varphi_v = 0$. With $\varphi_v = 0$ we get the same UIP regression coefficient, $-\tau_4/\tau_\pi$. McCallum's model is basically a two-country Taylor rule model with a lagged interest rate in the policy rule and no dynamics in the shocks. Allowing for autocorrelated volatility diminishes the model's ability to account for a substantially negative UIP coefficient, a feature that McCallum's approach does not recognize. A value of $b < 0$ can only be achieved if volatility is less autocorrelated that the value of the interest rate smoothing policy parameter.

Summary

The goal of this section has been to ascertain how the imposition of a Taylor rule restricts inflation dynamics and how these restrictions are manifest in the exchange rate. What have we learned?

A good context for understanding the answer is the Alvarez, Atkeson, and Kehow (2009) (AAK) paper. The nuts and bolts of their argument goes as follows. With lognormality, the nominal interest is

$$i_t = -E_t\big(\log m_{t+1}\big) - Var_t\big(\log m_{t+1}\big)/2$$

AAK argue that if exchange rates follow a random walk then variation in the conditional mean term must be small.[1] Therefore (according to them), "almost everything we say about monetary policy

[1] *i.e.*, random walk exchange rates mean that $E_t \log(S_{t+1}/S_t) = 0$, and, from Equation (2.12), $E_t \log(S_{t+1}/S_t) = -E_t(\log m_{t+1} - \log m_{t+1}^*)$. Random walk exchange rates, therefore, imply that the *difference* between the mean of the log kernels does not vary, not the mean of the log kernels themselves. More on this below.

is wrong." The idea is that, in many existing models, the monetary policy transmission mechanism works through its affect on the conditional mean of the nominal marginal rate of substitution, m_t. But if exchange rates imply that the conditional mean is essentially a constant — so that 'everything we say is wrong' — then the mechanism must instead be working through the conditional variance.

If one takes the UIP evidence seriously, this isn't quite right. The UIP puzzle *requires* variation in the conditional means (*i.e.*, it says that exchange rates *are not* a random walk).[2] Moreover, it also requires that this variation be negatively correlated with variation in the conditional variances, and that the latter be larger than the former. In terms of monetary policy the message is that the standard story — that a shock that increases the mean (of the marginal rate of substitution) *decreases* the interest rate — is wrong. The UIP evidence says that we need to get used to thinking about a shock that increases the mean as *increasing* the interest rate, the reason being that the same shock must *decrease* the variance, and by more than it increases the mean.

Now, to what we've learned. We've learned that symmetric monetary policies as represented by Taylor rules of the form (B.1) can't deliver inflation dynamics that, by themselves, satisfy these requirements. The reason is basically what we label the 'muted response of the short rate'. The evidence requires that the conditional mean of inflation move by more than its contemporaneous value. But the one clear restriction imposed by the Taylor rule — that the interest rate must move *less* than contemporaneous inflation because the interest rate must also be equal to the conditional mean future inflation — says that this can't happen (unless volatility is negatively autocorrelated).

This all depends heavily on the real interest rate being a constant, something we relax in the next section. What's going on is as follows. In general, the Euler equation and the simplest Taylor rule can be written as

$$i_t = r_t + E_t\,\pi_{t+1} - \frac{Var_t(\pi_{t+1})}{2} + Cov_t(n_{t+1}, \pi_{t+1}) \tag{B.15}$$

$$i_t = \tau + \tau_\pi \pi_t + z_t \quad . \tag{B.16}$$

The Euler equation (B.15) imposes restrictions between the current short rate and *moments* of future inflation. The Taylor rule (B.16) imposes an additional *contemporaneous* restriction between the current interest rate and current inflation. To see what this does, first ignore the real parts of Equation (B.15), r_t and the covariance term. Recalling that endogenous inflation will be a function $\pi(z_t, v_t)$, consider a shock to volatility that increases inflation by 1%.[3] The Taylor rule says that i_t must increase by *more than* 1%, say 1.2%. But, if inflation is a positively autocorrelated stationary process, then its conditional mean, $E_t\pi_{t+1}$, must increase by *less than* 1%, say 0.9%. Equation (B.15) says that the only way this can happen is if the conditional variance *decreases* by 0.2%; a volatility shock that increases π_t must decrease $Var_t\pi_{t+1}$. Therefore the mean and variance of the pricing kernel must move in the *same* direction, thus contradicting what Fama (1984) taught us is necessary for $b < 0$.

Phrased in terms of the exchange rate, the logic is equally intuitive. The increase in the conditional mean of inflation implies an expected devaluation in USD — recall that relative PPP holds if we ignore real rates — which, given the increasing interest rate implied by the Taylor rule, moves us

[2] Of course, the variation in the forecast error for exchange rates dwarfs the variation in the conditional mean (*i.e.*, the R^2 from the Bilson-Fama-regressions is very small). Monthly changes in exchange rates certainly exhibit 'near random walk' behavior, and for policy questions the distinction may be a second-order effect. This argument, however, does not affect our main point regarding the AAK paper: that exchange rates are all about *differences* between pricing kernels and its hard to draw definitive conclusions about their *levels*.

[3] A shock to z_t isn't particularly interesting in this context because it doesn't affect both the mean and variance of the pricing kernel.

in the UIP direction: high interest rates associated with a devaluing currency. Note that, if volatility were negatively autocorrelated, $E_t \pi_{t+1}$ would *fall* and the reverse would be true; we'd have $b < 0$.[4]

So, the contemporaneous restriction implied by the Taylor rule is very much a binding one for our question. This points us in two directions. First, it suggests that an interaction with the real interest rate is likely to be important. None of the above logic follows if r_t and $Cov_t(n_{t+1}, \pi_{t+1})$ also respond to a volatility shock. We follow this path in the next section. Second it points to something else that the AAK story doesn't get quite right. Exchange rate behavior tells us something about the *difference* between the domestic and foreign pricing kernels, not necessarily something about their *levels*. The above logic, and AAK's logic, is about levels, not differences. Symmetry makes the distinction irrelevant, but with asymmetry it's important. What our asymmetric example delivers is (i) inflation dynamics that, in each currency, satisfies 'muted response of the short rate' behavior, and (ii) a *difference* in inflation dynamics that gets the *difference* in the mean and the variance of the kernels moving in the right direction.

To see this, recall that $X_t^\top \equiv \begin{bmatrix} z_t & z_t^* & v_t & v_t^* \end{bmatrix}^\top$ and consider the foreign and domestic pricing kernels in the asymmetric model:

$$\begin{aligned} -\log m_{t+1} &= \text{constants} + a_1 \varphi_z z_t + a_3 \varphi_v v_t + a_1 v_t^{1/2} \epsilon_{t+1}^z + a_3 \sigma_v \epsilon_{t+1}^v \\ -\log m_{t+1}^* &= \text{constants} + A^\top \Lambda X_t + V(X_t)^{1/2} \begin{bmatrix} \epsilon_{t+1}^z & \epsilon_{t+1}^{z^*} & \epsilon_{t+1}^v & w_{t+1}^* \end{bmatrix}^\top \end{aligned}$$

where Λ is a diagonal matrix of autoregressive coefficients, and $V(X_t)$ is a diagonal matrix of conditional standard deviations. The asymmetric restriction that $\tau_3 = 0$ and $\tau_3^* \neq 0$ effectively makes this a 'common factor model' with asymmetric loadings on the common factors. A number of recent papers, Lustig, Roussanov, and Verdelhan (2011) for example, have argued persuasively for such a specification. What we've developed is one economic interpretation of their statistical exercise.[5]

More explicitly, consider the *difference* in the mean and variance of the log kernels from the symmetric and asymmetric examples of Appendix B.1. For the symmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2} a_1^2 (v_t - v_t^*) \\ q_t &= a_2 \varphi_v (v_t - v_t^*) \end{aligned}$$

whereas for the asymmetric case we have

$$\begin{aligned} p_t &= -\frac{1}{2} (a_1^2 - a_1^{*2}) v_t + \frac{1}{2} a_4^* v_t^* \\ q_t &= \varphi_v (a_3 - a_3^*) v_t - a_4^* v_t^* \end{aligned}$$

where the a coefficients are functions of the model's parameters, outlined above and in more detail in the appendix. What's going on in the symmetric case is transparent. p_t and q_t can only be negatively correlated if $\varphi_v < 0$ (since $a_2 < 0$). The asymmetric case is more complex, but it turns

[4] This intuition is also useful for understanding why we get $0 < b < 1$ with positively autocorrelated volatility. The RHS of the regression, the interest rate spread, contains *both* the mean and the variance of inflation. The LHS contains only the mean. If (negative) the mean and the variance move in the same direction, then the RHS is moving more than the LHS and the population value of b is less than unity.

[5] Note that if the conditional mean coefficients on z_t and v_t were the same across m and m^* then, contrary to AAK's assertion, monetary policy *could* affect the mean of the pricing kernel while still allowing for a random walk exchange rate. This is simply because z_t and v_t *would not appear* in the difference between the means of the two log kernels.

out that what's critical is that $(a_3 - a_3^*) < 0$. This in turn depends on the difference $(\tau_\pi - \tau_3^*)$ being negative. Overall, what the asymmetric Taylor rule does is that it introduces an asymmetry in how a common factor between m and m^* affect their conditional means. This asymmetry causes the common factor to show up in exchange rates, and it can also flip the sign and deliver $b < 0$ with the right combination of parameter values.

B.2 Additional Calculations: Symmetric Model

The short rate must satisfy both the Euler equation and the Taylor rule:

$$i_t = -\log E_t\, m_{t+1} \tag{B.17}$$

$$i_t = \tau + \tau_\pi \pi_t + z_t \ , \tag{B.18}$$

where the processes for z_t and its volatility v_t are

$$\begin{aligned} z_t &= \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v \end{aligned}$$

where ϵ_t^z and ϵ_t^v are *i.i.d.* standard normal. Given that $m_{t+1} = n_{t+1}P_t/P_{t+1}$ and $\pi_{t+1} = \log(P_{t+1}/P_t)$, set the real pricing kernel to a constant so that $m_{t+1} = \exp(-\pi_{t+1})$. Guess that the solution for endogenous inflation is:

$$\pi_t = a + a_1 z_t + a_2 v_t \ , \tag{B.19}$$

Substitute Equation (B.19) into the Euler equation (B.17) and compute the expectation. The result is

$$i_t = C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t \ , \tag{B.20}$$

where

$$C \equiv -n + a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2$$

Substiute the postulated solution (B.19) into the Taylor rule, match-up the resulting coefficients with those in Equation (B.20), and solve for the a_i coefficients:

$$\begin{aligned} a &= \frac{C-\tau}{\tau_\pi} \\ a_1 &= \frac{1}{\varphi_z - \tau_\pi} \\ a_2 &= \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} \end{aligned}$$

It's useful to note that

$$a_2 = \frac{a_1^2}{2(\varphi_v - \tau_\pi)} \ .$$

Note that this is the same as saying that

$$\frac{\partial i_t}{\partial v_t} = \tau_\pi \frac{\partial \pi_t}{\partial v_t} = \frac{\partial E_t \pi_{t+1}}{\partial v_t} - \frac{1}{2}\frac{\partial\, Var_t \pi_{t+1}}{\partial v_t}$$

Similarly, $a_1 = 1/(\varphi_z - \tau_\pi)$ is the same as saying that

$$\frac{\partial i_t}{\partial z_t} = \tau_\pi \frac{\partial \pi_t}{\partial z_t} + 1 = \frac{\partial E_t \pi_{t+1}}{\partial z_t} - \frac{1}{2}\frac{\partial Var_t \pi_{t+1}}{\partial z_t} \quad .$$

Both of these things are kind of trivial. They just say that the effect of a shock on the Taylor rule equation must be consistent with the effect on the Euler equation.
Note also that

$$C = \frac{\tau_\pi}{\tau_\pi - 1}\Big(- n - \frac{\tau}{\tau_\pi} + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2\Big)$$

Inflation and the short rate are:

$$\begin{aligned}
\pi_t &= \frac{C-\tau}{\tau_\pi} + \frac{1}{\varphi_z - \tau_\pi} z_t + \frac{1}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} v_t \\
i_t &= C + \frac{\varphi_z}{\varphi_z - \tau_\pi}\, z_t + \frac{\tau_\pi}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}\, v_t \\
&= C + \varphi_z a_1 z_t + \tau_\pi a_2 v_t
\end{aligned}$$

The pricing kernel is

$$\begin{aligned}
-\log m_{t+1} &= C + (\sigma_v a_2)^2/2 + a_1\varphi_z z_t + a_2\varphi_v v_t + a_1 v_t^{1/2}\epsilon^z_{t+1} + \sigma_v a_2 \epsilon^v_{t+1} \\
&= D + \frac{1}{\varphi_z - \tau_\pi}\varphi_z z_t + \frac{\varphi_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)} v_t \\
&\quad + \frac{1}{\varphi_z - \tau_\pi} v_t^{1/2}\epsilon^z_{t+1} + \frac{\sigma_v}{2(\varphi_z-\tau_\pi)^2(\varphi_v-\tau_\pi)}\epsilon^v_{t+1}
\end{aligned}$$

where

$$D \equiv C + (\sigma_v a_2)^2/2$$

The GBP-denominated kernel and variables are denoted with asterisks. The interest-rate differential, the expected depreciation rate, q_t, and the risk premium, p_t, are:

$$\begin{aligned}
i_t - i_t^* &= \varphi_z a_1 z_t - \varphi_z^* a_1^* z_t^* + \tau_\pi a_2 v_t - \tau_\pi^* a_2^* v_t^* \\
q_t &= D - D^* + a_1\varphi_z z_t - a_1^*\varphi_z^* z_t^* + a_2\varphi_v v_t - a_2^*\varphi_v^* v_t^* \\
p_t &= -\frac{1}{2}\left(a_1^2 v_t - a_1^{*2} v_t^* + \sigma_v^2 a_2^2 - \sigma_v^{*2} a_2^{*2}\right)
\end{aligned}$$

It is easily verified that $p_t + q_t = i_t - i_t^*$.

If we assume that all foreign and domestic parameter values are the same (*i.e.*, $\tau = \tau^*$) and if we set $\varphi_z = 0$, then the regression parameter is:

$$\begin{aligned}
b &= \frac{Cov(i_t - i_t^*\,,\, q_t)}{Var(i_t - i_t^*)} \\
&= \frac{\varphi_v}{\tau_\pi}
\end{aligned}$$

B.3 Asymmetric Taylor Rule

Taylor rules

$$\begin{aligned} i_t &= \tau + \tau_\pi \pi_t + z_t + \tau_3 d_t \\ i_t^* &= \tau^* + \tau_\pi^* \pi_t^* + z_t^* + \tau_3^* d_t \\ d_t &\equiv \log(S_t/S_{t-1}) = \pi_t - \pi_t^* \end{aligned}$$

State variables,

$$\begin{aligned} z_t &= \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v \end{aligned}$$

and the associated foreign-country processes with asterisks and with all shocks *i.i.d.*. Collect them in the state vector, X_t:

$$X_t \equiv \begin{bmatrix} z_t \ z_t^* \ v_t \ v_t^* \end{bmatrix}^\top$$

Inflation solutions:

$$\begin{aligned} \pi_t &= a + a_1 z_t + a_2 z_t^* + a_3 v_t + a_4 v_t^* \equiv a + A^\top X_t \\ \pi_t^* &= a^* + a_1^* z_t + a_2^* z_t^* + a_3^* v_t + a_4^* v_t^* \equiv a^* + A^{*\top} X_t \end{aligned}$$

Interest rates, from Euler equations with real interest rate = 0:

$$\begin{aligned} i_t &= C + B^\top X_t \\ i_t^* &= C^* + B^{*\top} X_t \end{aligned}$$

where,

$$\begin{aligned} B^\top &\equiv \left[\ a_1\varphi_z \quad a_2\varphi_z^* \quad (a_3\varphi_v - \frac{a_1^2}{2}) \quad (a_4\varphi_v^* - \frac{a_2^2}{2})\ \right] \\ C &\equiv a + a_1\theta_z(1-\varphi_z) + a_2\theta_z^*(1-\varphi_z^*) + a_3\theta_v(1-\varphi_v) + a_4\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\big(a_3^2\sigma_v^2 + a_4^2\sigma_v^{*2}\big) \\ B^{*\top} &\equiv \left[\ a_1^*\varphi_z \quad a_2^*\varphi_z^* \quad (a_3^*\varphi_v - \frac{a_1^{*2}}{2}) \quad (a_4^*\varphi_v^* - \frac{a_2^{*2}}{2})\ \right] \\ C^* &\equiv a^* + a_1^*\theta_z(1-\varphi_z) + a_2^*\theta_z^*(1-\varphi_z^*) + a_3^*\theta_v(1-\varphi_v) + a_4^*\theta_v^*(1-\varphi_v^*) - \frac{1}{2}\big(a_3^{*2}\sigma_v^2 + a_4^{*2}\sigma_v^{*2}\big) \end{aligned}$$

Taylor rules become:

$$\begin{aligned} i_t &= \tau + \tau_\pi(a + A^\top X_t) + z_t + \tau_3\big(a + A^\top X_t - a^* - A^{*\top} X_t\big) \\ &= \tau + \tau_\pi a + \tau_3(a - a^*) + \big(\tau_\pi A^\top + \iota_z^\top + \tau_3[A^\top - A^{*\top}]\big) X_t \\ i_t^* &= \tau^* + \tau_\pi^*(a^* + A^{*\top} X_t) + z_t^* + \tau_3^*\big(a + A^\top X_t - a^* - A^{*\top} X_t\big) \\ &= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) + \big(\tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*[A^\top - A^{*\top}]\big) X_t \end{aligned}$$

where $\iota_z^\top \equiv [\,1\ \ 0\ \ 0\ \ 0\,]$ and $\iota_z^{*\top} \equiv [\,0\ \ 1\ \ 0\ \ 0\,]$. Matching-up the coefficients means

$$\begin{aligned} C &= \tau + \tau_\pi a + \tau_3(a - a^*) \\ C^* &= \tau^* + \tau_\pi^* a^* + \tau_3^*(a - a^*) \end{aligned}$$

$$\begin{aligned} B &= \tau_\pi A^\top + \iota_z^\top + \tau_3(A^\top - A^{*\top}) \\ B^* &= \tau_\pi^* A^{*\top} + \iota_z^{*\top} + \tau_3^*(A^\top - A^{*\top}) \end{aligned}$$

To solve for the constants (the first two equations):

$$\begin{bmatrix} 1-\tau_\pi-\tau_3 & \tau_3 \\ -\tau_3^* & 1-\tau_\pi^*+\tau_3^* \end{bmatrix} \begin{bmatrix} a \\ a^* \end{bmatrix} = \begin{bmatrix} \tau - stuff \\ \tau^* - stuff^* \end{bmatrix}$$

where $stuff$ and $stuff^*$ are everything on the LHS of the solutions for C and C^*, except the first terms, a and a^*.

The B equations are eight equations in eight unknowns, A and A^*. Conditional on these, the C equations are two-in-two, a and a^*. The B equations can be broken into 4 blocks of 2. It's useful to write them out because you can see where the singularity lies.

$$\begin{bmatrix} (\tau_\pi+\tau_3-\varphi_z) & -\tau_3 \\ \tau_3^* & (\tau_\pi^*-\tau_3^*-\varphi_z) \end{bmatrix} \begin{bmatrix} a_1 \\ a_1^* \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi+\tau_3-\varphi_z^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^*-\tau_3^*-\varphi_z^*) \end{bmatrix} \begin{bmatrix} a_2 \\ a_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi+\tau_3-\varphi_v) & -\tau_3 \\ \tau_3^* & (\tau_\pi^*-\tau_3^*-\varphi_v) \end{bmatrix} \begin{bmatrix} a_3 \\ a_3^* \end{bmatrix} = \begin{bmatrix} -a_1^2/2 \\ -a_1^{*2}/2 \end{bmatrix}$$

$$\begin{bmatrix} (\tau_\pi+\tau_3-\varphi_v^*) & -\tau_3 \\ \tau_3^* & (\tau_\pi^*-\tau_3^*-\varphi_v^*) \end{bmatrix} \begin{bmatrix} a_4 \\ a_4^* \end{bmatrix} = \begin{bmatrix} -a_2^2/2 \\ -a_2^{*2}/2 \end{bmatrix}$$

Two singularities exist:

- *UIP holds exactly.* If $\tau_3 = 0$ (so that the Fed ignores the FX rate), $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ (complete symmetry in parameters, save τ_3 and τ_3^*) then a singularity is $\tau_3^* = \tau_\pi - \varphi_v$. As τ_3^* approaches this from below or above, the UIP coefficient goes to 1.0.

- *Anomaly resolved.* Similarly, if $\tau_3 = 0$, $\varphi_v = \varphi_v^*$ and $\tau_\pi = \tau_\pi^*$ then a singularity is $\tau_3^* = \tau_\pi$. As τ_3^* approaches from *below*, the UIP coefficient goes to infinity. As τ_3^* approaches from *above*, it goes to *negative* infinity.

The latter condition is where the UIP regression coefficient changes sign. This says that we need $\tau_3^* > \tau_3$. This may seem pathological. It says that — if we interpret these coefficients as policy responses (which we shouldn't) — the ECB responds to an appreciation in EUR by increasing interest rates *more* than 1:1 (and more than the 'Taylor principle' magnitude of $\tau_\pi > 1$).

B.4 Derivations for McCallum Model

Write the interest rate coefficients as follows:

$$\begin{aligned} i_t &= \frac{1}{1-a_3}\Big(C + a_1\varphi_z z_t + (a_2\varphi_v - a_1^2/2)v_t\Big) \\ &= c_i + c_{iz} z_t + c_{iv} v_t \end{aligned}$$

and, for reasons that will become clear, define

$$\tilde{i}_t \equiv i_t - \theta_z c_{iz} - \theta_v c_{iv}$$

The exogenous state variables obey

$$\begin{aligned} z_t &= \theta_z(1-\varphi_z) + \varphi_z z_{t-1} + v_{t-1}^{1/2}\epsilon_t^z \\ v_t &= \theta_v(1-\varphi_v) + \varphi_v v_{t-1} + \sigma_v \epsilon_t^v \end{aligned}$$

where a mean is now incorporated for z. I'm not sure if this thing is identified or not. Denote the state vector as $X_t^\top = [z_t\ v_t\ \tilde{i}_{t-1}]^\top$ so that we can write

$$X_t = (I-\Phi)\theta + \Phi X_{t-1} + V(X_{t-1})^{1/2} s_{t-1}$$

where

$$\begin{aligned} \theta^\top &= [\theta_z\ \theta_v\ c_i]^\top \\ \Phi &= \begin{bmatrix} \varphi_z & 0 & 0 \\ 0 & \varphi_v & 0 \\ c_{iz} & c_{iv} & 0 \end{bmatrix} \\ V(X_{t-1}) &= \begin{bmatrix} v_{t-1} & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ s_t^\top &= [\epsilon_t^z\ \epsilon_t^v\ 0]^\top \end{aligned}$$

The mean, variance and autocovariance of X are

$$\mu_X^\top = [\theta_z \quad \theta_v \quad C/(1-a_3)]$$

$$\begin{aligned} \Gamma_0 &= \begin{bmatrix} \frac{\theta_v}{1-\varphi_z^2} & 0 & \frac{c_{iz}\varphi_z\theta_v}{1-\varphi_z^2} \\ & \frac{\sigma_v^2}{1-\varphi_v^2} & \frac{c_{iv}\varphi_v\sigma_v^2}{1-\varphi_v^2} \\ & & \frac{c_{iz}^2\theta_v}{1-\varphi_z^2} + \frac{c_{iv}^2\sigma_v^2}{1-\varphi_v^2} \end{bmatrix} \\ \Gamma_1 &= \Phi\Gamma_0 \end{aligned}$$

Moments

- *Inflation.* Let $\pi_t = a_\pi + A_\pi^\top X_t$ where $A_\pi^\top = [a_1\ a_2\ a_3]$. Since

  $$\pi_t = a + a_1 z_t + a_2 v_t + a_3 i_{t-1} \quad ,$$

  we must have

  $$a_\pi = a + a_3\left(c_{iz}\theta_z + c_{iv}\theta_v\right) \quad .$$

  The unconditional moments are:

  $$\begin{aligned} \mu_\pi &= a_\pi + A_\pi^\top \mu_X \\ \sigma_\pi^2 &= A_\pi^\top \Gamma_0 A_\pi \\ Corr(\pi_t, \pi_{t-1}) &= A_\pi^\top \Gamma_1 A_\pi / \sigma_\pi^2 \end{aligned}$$

  I worked one out by hand as a check:

  $$\sigma_\pi^2 = (a_1\varphi_z + a_3 c_{iz})^2 \frac{\theta_v}{1-\varphi_z^2} + (a_2\varphi_v + a_3 c_{iv})^2 \frac{\sigma_v^2}{1-\varphi_v^2} + a_1^2\theta_v + (a_2\sigma_v)^2$$

The conditional moments are:

$$\begin{aligned} E_t\pi_{t+1} &= a_\pi + A_\pi^\top\big((I-\Phi)\theta + \Phi X_{t-1}\big) \\ Var_t\pi_{t+1} &= A_\pi^\top \begin{bmatrix} v_t & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_\pi \end{aligned}$$

- *Interest rate.* Let $i_t = c_i + C_i^\top X_t$, where $C_i^\top = [c_{iz}\ c_{iv}\ 0]$ and

$$\begin{aligned} C &= a + a_1\theta_z(1-\varphi_z) + a_2\theta_v(1-\varphi_v) - (a_2\sigma_v)^2/2 \\ c_i &= C/(1-a_3) \\ c_{iz} &= \varphi_z a_1/(1-a_3) \\ c_{iv} &= (\tau_\pi + \tau_4)a_2/(1-a_3) \end{aligned}$$

The moments are:

$$\begin{aligned} \mu_i &= c_i + C_i^\top \mu_X \\ \sigma_i^2 &= C_i^\top \Gamma_0 C_i \\ Corr(i_t, i_{t-1}) &= C_i^\top \Gamma_1 C_i/\sigma_i^2 \end{aligned}$$

- *Depreciation rate*: $d_t = \pi_t - \pi_t^*$. With independence across countries we have

$$\begin{aligned} \mu_\pi &= a_\pi - a_{\pi^*} + A^\top \mu_X - A^\top \mu_{X^*} \\ \sigma_d^2 &= \sigma_\pi^2 + \sigma_{\pi^*}^2 \\ Corr(d_t, d_{t-1}) &= \frac{\sigma_\pi^2}{\sigma_\pi^2 + \sigma_{\pi^*}^2} Corr(\pi_t, \pi_{t-1}) + \frac{\sigma_{\pi^*}^2}{\sigma_\pi^2 + \sigma_{\pi^*}^2} Corr(\pi_t^*, \pi_{t-1}^*) \end{aligned}$$

So — obviously, in this model where relative PPP holds exactly — we have a strong counterfactual. The autocorrelation of the depreciation rate and the inflation rate are the same. Relaxing these things may work, to some extent. Here's a start:

$$\begin{aligned} \mu_\pi &= a_\pi - a_\pi^* + A_\pi^\top \mu_X - (A_\pi^*)^\top \mu_{X^*} \\ \sigma_d^2 &= A_\pi^\top \Gamma_0 A_\pi + (A_\pi^*)^\top \Gamma_0^* A_\pi^* + Cov() \\ Corr(d_t, d_{t-1}) &= Cov(\pi_t, \pi_{t-1}) + Cov(\pi_t^*, \pi_{t-1}^*) + Cov(\pi_t, \pi_{t-1}^*) + Cov(\pi_{t-1}^*, \pi_t) \end{aligned}$$

- *Interest rate differential*: $i_t - i_t^*$.

$$i_t - i_t^* = c_i - c_{i^*} + C_i^\top X_t - C_{i^*}^\top X_t^*$$

With independence, the moments are

$$\begin{aligned} \mu_\pi &= c_i - c_{i^*} + C_i^\top \mu_X - C_{i^*}^\top \mu_{X^*} \\ Var(i_t - i_t^*) &= \sigma_i^2 + \sigma_{i^*}^2 \\ Corr(i_t - i_t^*\,,\, i_{t-1} - i_{t-1}^*) &= \frac{\sigma_i^2}{\sigma_i^2 + \sigma_{i^*}^2}\rho_i + \frac{\sigma_{i^*}^2}{\sigma_i^2 + \sigma_{i^*}^2}\rho_{i^*} \end{aligned}$$

- *UIP Coefficient.* First the expected depreciation rate, with symmetry, is

$$\begin{aligned} q_t = E_t d_{t+1} &= E_t(\pi_{t+1} - \pi^*_{t+1}) \\ &= a_\pi - a_{\pi^*} + A_\pi^\top \Phi X_t - A_{\pi^*}^\top \Phi^* X_t^* \ . \end{aligned}$$

So the covariance (with independence) is

$$\begin{aligned} Cov(i_t - i_t^*, q_t) &= Cov\left(C_i^\top X_t - C_{i^*}^\top X_t^*\ ,\ A_\pi^\top \Phi X_t - A_{\pi^*}^\top \Phi^* X_t^*\right) \\ &= C_i^\top \Gamma_0 \Phi^\top A_\pi + C_{i^*}^\top \Gamma_0^* \Phi^{*\top} A_{\pi^*} \end{aligned}$$

and the regression coefficient is

$$b = \frac{C_i^\top \Gamma_0 \Phi^\top A_\pi + C_{i^*}^\top \Gamma_0^* \Phi^{*\top} A_{\pi^*}}{Var(i_t - i_t^*)}$$

- *p and q*

$$\begin{aligned} q_t &= E_t \pi_{t+1} - E_t \pi^*_{t+1} \\ p_t &= -\frac{1}{2}\left(Var_t \pi_{t+1} - Var_t \pi^*_{t+1}\right) \\ &= -\frac{1}{2}\left(A_\pi^\top \begin{bmatrix} v_t & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} A_\pi - A_{\pi^*}^\top \begin{bmatrix} v_t^* & 0 & 0 \\ 0 & \sigma_v^{*2} & 0 \\ 0 & 0 & 0 \end{bmatrix} A_{\pi^*}\right) \end{aligned}$$

where the formulae for the conditional means is above (under the italicized heading *Inflation*).

B.5 Linearization for the Pricing Kernel

The log of the equilibrium domestic marginal rate of substitution in Equation (2.15) is given by

$$\log(n_{t+1}) = \log\beta + (\rho - 1)x_{t+1} + (\alpha - \rho)[\log W_{t+1} - \log\mu_t(W_{t+1})],$$

where $x_{t+1} \equiv \log(c_{t+1}/c_t)$ is the log of the ratio of domestic observed consumption in $t+1$ relative to t and W_t is the value function. The first two terms are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from EZ preferences.

We work on a linearized version of the real pricing kernel, following the findings of Hansen, Heaton, and Li (2005). In particular, I focus on the the value function of each representative agent, scaled by the observed equilibrium consumption level

$$\begin{aligned} W_t/c_t &= [(1-\beta) + \beta(\mu_t(W_{t+1})/c_t)^\rho]^{1/\rho} \\ &= \left[(1-\beta) + \beta\mu_t\left(\frac{W_{t+1}}{c_{t+1}} \times \frac{c_{t+1}}{c_t}\right)^\rho\right]^{1/\rho}, \end{aligned}$$

where I use the linear homogeneity of μ_t. In logs,

$$wc_t = \rho^{-1}\log[(1-\beta) + \beta\exp(\rho g_t)],$$

where $wc_t = \log(W_t/c_t)$ and $g_t \equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1})))$. Taking a linear approximation of the right-hand side as a function of g_t around the point $\bar{m}$, I get

$$\begin{aligned} wc_t &\approx \rho^{-1}\log[(1-\beta) + \beta\exp(\rho\bar{m})] + \left[\frac{\beta\exp(\rho\bar{m})}{1-\beta+\beta\exp(\rho\bar{m})}\right](g_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa g_t \end{aligned}$$

where $\kappa < 1$. Approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$, and for the general case of $\rho = 0$, the "log aggregator", the linear approximation is exact with $\bar{\kappa} = 1 - \beta$ and $\kappa = \beta$.

Given the state variables of the economy, l, u and w, and the log-linear structure of the model, we conjecture a solution for the value function of the form,

$$wc_t = \bar{\omega} + \omega_l l_t + \omega_u u_t + \omega_w w_t,$$

where $\bar{\omega}$, ω_l, ω_u and ω_w are constants to be determined. Therefore

$$wc_{t+1} + x_{t+1} = \bar{\omega} + \omega_l l_{t+1} + \omega_u u_{t+1} + \omega_w w_{t+1} + x_{t+1}$$

and, using the properties of lognormal random variables, g_t can be expressed as

$$\begin{aligned} g_t &\equiv \log(\mu_t(\exp(wc_{t+1} + x_{t+1}))) \\ &= \log(E_t[\exp(wc_{t+1} + x_{t+1})^{\alpha}]^{\frac{1}{\alpha}})] \\ &= E_t[wc_{t+1} + x_{t+1}] + \frac{\alpha}{2}\mathrm{Var}_t[wc_{t+1} + x_{t+1}]. \end{aligned}$$

Using the above expression, we solve for the value-function parameters by matching coefficients

$$\begin{aligned} \omega_l &= \kappa(\omega_l\varphi_l + 1) \\ \Rightarrow \omega_l &= \left(\frac{\kappa}{1-\kappa\varphi_l}\right) \\ \omega_u &= \kappa(\omega_u\varphi_u + \frac{\alpha}{2}) \\ \Rightarrow \omega_u &= \frac{\alpha}{2}\frac{\kappa}{1-\kappa\varphi_u} \\ \omega_w &= \kappa(\omega_w\varphi_w + \frac{\alpha}{2}\omega_l^2) \\ \Rightarrow \omega_w &= \frac{\alpha}{2}\omega_l^2\frac{\kappa}{1-\kappa\varphi_u}\ . \end{aligned}$$

The solution allows us to simplify the term $[\log W_{t+1} - \log\mu_t(W_{t+1})]$ in the pricing kernel in Equation (B.5):

$$\begin{aligned} \log W_{t+1} - \log\mu_t(W_{t+1}) &= wc_{t+1} + x_{t+1} - \log\mu_t(\exp(wc_{t+1} + x_{t+1})) \\ &= \omega_l\sqrt{w_t}\epsilon^l_{t+1} + \omega_u\sigma_u\epsilon^u_{t+1} + \omega_w\sigma_w\epsilon^w_{t+1} + \sqrt{u_t}\epsilon^x_{t+1} \\ &- \frac{\alpha}{2}(\omega_l^2 w_t + \omega_u^2\sigma_u^2 + \omega_w^2\sigma_w^2 + u_t)\ . \end{aligned}$$

Equation (1.7) follows by collecting terms. In particular,

$$\delta^r = -\log\beta + (1-\rho)\mu + \frac{\alpha}{2}(\alpha - \rho)[(\omega_u\sigma_u)^2 + (\omega_w\sigma_w)^2]$$

$$\gamma_l^r = (1-\rho); \quad \gamma_u^r = \frac{\alpha}{2}(\alpha-\rho); \quad \gamma_w^r = \frac{\alpha}{2}(\alpha-\rho)\omega_l^2$$

$$\lambda_x^r = (1-\alpha); \;\; \lambda_l^r = -(\alpha-\rho)\omega_l; \;\; \lambda_v^r = -(\alpha-\rho)\omega_u; \;\; \lambda_w^r = -(\alpha-\rho)\omega_w$$

$$\omega_l = \left(\frac{\kappa}{1-\kappa\varphi_l}\right); \;\; \omega_u = \frac{\alpha}{2}\left(\frac{\kappa}{1-\kappa\varphi_u}\right); \;\; \omega_w = \frac{\alpha}{2}\left(\frac{\kappa}{1-\kappa\varphi_w}\right)\omega_l^2$$

B.6 Moment Conditions

- Consumption growth:

$$\begin{aligned} E_t(x_{t+1}) = \mu + l_t \;, &\qquad Var_t(x_{t+1}) = u_t \;, \\ E(x_{t+1}) = \mu \;, &\quad Var(x_{t+1}) = \theta_u + Varl_t \;, \\ Cov(x_{t+1}, x_t) = \varphi_l \, Var_l \;, &\quad Corr(x_{t+1}, x_t) = \tfrac{\varphi_l \, Varl_t}{\theta_u + Varl_t} \end{aligned}$$

- Long run risk:

$$\begin{aligned} E_t(l_{t+1}) = \varphi_l l_t \;, &\quad Var_t(l_{t+1}) = w_t \;, \\ E(l_{t+1}) = 0 \;, &\quad Var(l_{t+1}) = \tfrac{\theta_w}{1-\varphi_l^2} \;, \\ Cov(l_{t+1}, l_t) = \varphi_l \, Varl_t \;, , &\quad Corr(l_{t+1}, l_t) = \varphi_l \end{aligned}$$

- Short-run volatility:

$$\begin{aligned} E_t(u_{t+1}) = (1-\varphi_u)\theta_u \;, &\quad Var_t(u_{t+1}) = \sigma_u^2 \;, \\ E(u_{t+1}) = \theta_u \;, &\quad Var(u_{t+1}) = \tfrac{\sigma_u^2}{1-\varphi_u^2} \;, \\ Cov(u_{t+1}, u_t) = \varphi_u \, Varu_t \;, &\quad Corr(u_{t+1}, u_t) = \varphi_u \end{aligned}$$

- Long-run volatility:

$$\begin{aligned} E_t(w_{t+1}) = (1-\varphi_w)\theta_w \;, &\quad Var_t(w_{t+1}) = \sigma_w^2 \;, \\ E(w_{t+1}) = \theta_w \;, &\quad Var(w_{t+1}) = \tfrac{\sigma_w^2}{1-\varphi_w^2} \;, \\ Cov(w_{t+1}, w_t) = \varphi_w \, Varw_t \;, &\quad Corr(w_{t+1}, w_t) = \varphi_w \end{aligned}$$

- Real pricing kernel:

$$\begin{aligned} E_t \log n_{t+1} &= -(\delta^r + \gamma_l^r l_t + \gamma_u^r u_t + \gamma_w^r w_t) \\ Var_t \log n_{t+1} &= \left(\lambda_x^r\right)^2 u_t + \left(\lambda_l^r\right)^2 w_t + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2 \\ E \log n_{t+1} &= -(\delta^r + \gamma_u^r \theta_u + \gamma_w^r \theta_w) \\ Var \log n_{t+1} &= \left(\lambda_x^r\right)^2 \theta_u + \left(\lambda_l^r\right)^2 \omega_u + (\lambda_u^r \sigma_u)^2 + (\lambda_w^r \sigma_w)^2 \end{aligned}$$

$$+ \quad \left(\gamma_l^r\right)^2 Var(l_t) + \left(\gamma_u^r\right)^2 Var(u_t) + \left(\gamma_w^r\right)^2 Var(w_t)$$

- Real risk free interest rate:

$$\begin{aligned} E(r_t) &= \bar{r} + r_u^r \theta_u + r_w^r \theta_w \\ Var(r_t) &= (\gamma_l^r)^2 Var(l_t) + \left(r_u^r\right)^2 Var(v_t) + \left(r_w^r\right)^2 Var(w_t) \end{aligned}$$

$$Corr(r_{t+1}, r_t) = 1 - (1-\varphi_l)\left(\gamma_l^r\right)^2 \frac{Var(l_t)}{Var(r_t)} - (1-\varphi_u)\left(r_u^r\right)^2 \frac{Var(u_t)}{Var(r_t)} - (1-\varphi_w)\left(r_w^r\right)^2 \frac{Var(w_t)}{Var(r_t)}$$

- Cross-country moments (symmetric coefficient):[6]

$$\begin{aligned} Cov(x_t, x_t^*) &= Cov(l_t, l_t^*) + \eta_{\epsilon^x} E(\sqrt{u_t}\sqrt{u_t^*}) \\ Cov(l_t, l_t^*) &= \frac{\eta_{\epsilon^l} E(\sqrt{w_t}\sqrt{w_t^*})}{1-\varphi_l^2} \\ Cov(v_t, v_t^*) &= \frac{\eta_{\epsilon^u}\sigma_u^2}{1-\varphi_u^2} \\ Cov(w_t, w_t^*) &= \frac{\eta_{\epsilon^w}\sigma_w^2}{1-\varphi_w^2} \end{aligned}$$

- Real depreciation rate:

$$E_t(d_{t+1}^r) = q_t^r \ , \quad E(d_t^r) = 0 \ ,$$

$$Var(d_{t+1}^r) = 2[\,Var(\log n_{t+1}) - Cov(\log n_{t+1}, \log n_{t+1}^*)]$$

- Inflation:

$$\begin{aligned} E(\pi_t) &= a + a_2\theta_u + a_3\theta_w + a_5\theta_v \\ Var(\pi_t) &= a_1^2 Var(l_t) + a_2^2 Var(u_t) + a_3^2 Var(w_t) + a_4^2 Var(z_t) + a_5^2 Var(v_t) \end{aligned}$$

$$\begin{aligned} Corr(\pi_{t+1}, \pi_t) &= 1 - (1-\varphi_l) a_1^2 \frac{Var(l_t)}{Var(\pi_t)} - (1-\varphi_u) a_2^2 \frac{Var(u_t)}{Var(\pi_t)} - (1-\varphi_w) a_3^2 \frac{Var(w_t)}{Var(\pi_t)} \\ &- (1-\varphi_z) a_4^2 \frac{Var(z_t)}{Var(\pi_t)} - (1-\varphi_v) a_5^2 \frac{Var(v_t)}{Var(\pi_t)} \end{aligned}$$

$$corr(x_{t+1}, \pi_t) = a_1 \frac{Var(l_t)}{Stdev(x_t) Stdev(\pi_t)}, \quad corr(x_t, \pi_t) = corr(x_{t+1}, \pi_t)\varphi_l$$

[6] The expressions for cross-country moments greatly simplify if we assume either independence or perfect correlation in the stochastic volatility processes, u_t and w_t.

- Nominal interest rate:

$$E(i_t) = \bar{\iota} + r_u\theta_u + r_w\theta_w + r_v\theta_v$$

$$Var(i_t) \quad = \quad \gamma_l^2\, Var(l_t) + \gamma_z^2\, Var(z_t) + r_u^2\, Var(u_t) + r_w^2\, Var(w_t) + \big(r_v\big)^2\, Var(v_t)$$

$$\begin{aligned} Corr(i_{t+1}, i_t) \quad &= \quad 1 - (1-\varphi_l)\gamma_l^2 \frac{Var(l_t)}{Var(i_t)} - (1-\varphi_z)\gamma_z^2 \frac{Var(z_t)}{Var(i_t)} \\ &- \quad (1-\varphi_u) r_u^2 \frac{Var(u_t)}{Var(i_t)} - (1-\varphi_w) r_w^2 \frac{Var(w_t)}{Var(i_t)} - (1-\varphi_v)\big(r_v\big)^2 \frac{Var(v_t)}{Var(i_t)} \end{aligned}$$

- Nominal depreciation rate:

$$E_t(d_{t+1}) = q_t \ , \quad E(d_t) = 0$$

$$Var(d_{t+1}) \quad = \quad 2\big[\, Var(\log m_{t+1}) - Cov(\log m_{t+1}, \log m^*_{t+1})\big]$$

Appendix C

Nominal Frictions, Monetary Policy, and Long-Run Risk

In the Appendices, I solve the model to allow for a shock to the disutility of labor χ_t and for inertia in the interest rate rule ($\tau_i \neq 0$). For parsimony, I shut down the policy shock (that is, $u_t = 0$, for every t). The solution for the model with a non-zero policy shock follows similar steps. The model of Section 3.2 is obtained by setting the labor shock χ_t equal to a constant.

C.1 Equilibrium

C.1.1 Representative agent maximization

The representative agent problem is:

$$\max V_t = \left\{(1-\beta)U(C_t, L_t)^{1-\psi} + \beta E_t[V_{t+1}^{1-\gamma}]^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}}$$

s.t.

$$E_t\left[\sum_{s=0}^{\infty} M_{t,t+s}P_{t+s}C_{t+s}\right] \leq E_t\left[\sum_{s=0}^{\infty} M_{t,t+s}\left(\int_0^1 W_{t+s}(j)L_{t+s}(j) + P_{t+s}\Psi_{t+s}\right)\right] \quad ,$$

where the consumption and utility indexes are defined in Equations (3.2) and (3.3) in the main text. The intratemporal utility function $U(C_t, L_t)$ is defined as

$$U(C_t, L_t)^{1-\psi} = \left(\frac{C_t^{1-\psi}}{1-\psi} - e^{\chi_t}A_t^{1-\psi}\frac{L_t^{1+\omega}}{1+\omega}\right) \quad , \tag{C.1}$$

where Δa_t evolves according to (3.8) in the main text and

$$\chi_t = (1-\varphi_\chi)\theta_\chi + \varphi_\chi\chi_{t-1} + \sigma_\chi\epsilon_t^\chi \quad . \tag{C.2}$$

The first order conditions are:

$$\begin{aligned}
\frac{\partial V_t}{\partial C_t} = 0 \quad &\Rightarrow \quad \frac{1-\beta}{1-\psi} C_t^{-\psi} \left[V_t^{1-\psi}\right]^{\frac{\psi}{1-\psi}} - \lambda M_{t,t} P_t = 0 \\
\frac{\partial V_t}{\partial L_t} = 0 \quad &\Rightarrow \quad \frac{1-\beta}{1-\psi} e^{\chi_t} A_t^{1-\psi} (-L_t)^{\omega} \left[V_t^{1-\psi}\right]^{\frac{\psi}{1-\psi}} + \lambda M_{t,t} W_t = 0 \\
\frac{\partial V_t}{\partial C_{t+1}} = 0 \quad &\Rightarrow \quad \beta E_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\psi}{1-\gamma}} \left[V_t^{1-\psi}\right]^{\frac{\psi}{1-\psi}} V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda M_{t,t+1} P_{t+1} = 0 \quad .
\end{aligned}$$

Rearranging, I obtain

$$\frac{W_t}{P_t} = e^{\chi_t} A_t^{1-\psi} C_t^{\psi} L_t^{\omega} \quad , \tag{C.3}$$

$$e^{-i_t} = E_t(M_{t,t+1}) \quad .$$

C.1.2 Firms

Fully flexible economy

The maximization problem of firm j under flexible prices is:

$$\max_{P_t(j)} \quad P_t(j) Y_t(j) - W_t(j) L_t(j)$$

subject to

$$\begin{aligned}
Y_t(j) &= A_t L_t(j) \\
Y_t(j) &= \left[\frac{P_t(j)}{P_t}\right]^{\frac{1}{\theta}} \quad .
\end{aligned}$$

The first order condition is

$$\frac{W_t(j)}{P_t} = \frac{1}{\mu} A_t \quad ,$$

where $\mu = \theta/(\theta-1)$ is the fixed markup applied by firms under fully flexible prices. Using the household optimality conditions, and the expression for the production technology, I can write

$$\frac{W_t(j)}{P_t} = \frac{1}{\mu} A_t = e^{\chi_t} A_t^{1-\psi} C_t^{\psi} L_t^{\omega} = Y_t^{\psi} \left(\frac{Y_t}{A_t}\right)^{\omega} \quad .$$

Therefore, the fully flexible output Y_t^F evolves according to

$$(Y_t^F)^{\psi+\omega} = \frac{1}{\mu e^{\chi_t}} A_t^{\psi+\omega} \quad ,$$

or, in logs, $\Delta y_{t+1}^F = \Delta a_{t+1} - \frac{1}{\psi+\omega} \Delta \chi_t$.

Calvo Pricing

The maximization problem of firm j is

$$\max_{P_t(j)^*} \quad E_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s} \left[P_t(j)^* (\Pi^*)^s Y_{t+s|t}(j) - W_{t+s|t}(j) L_{t+s|t}(j) \right] \right\}$$

$$
\begin{aligned}
s.t. \qquad & Y_{t+s|s}(j) = A_{t+s}L_{t+s|s}(j) \\
& Y_{t+s|t}(j) = \left(\tfrac{P_t^*(\Pi^*)^s}{P_{t+s}}\right)^{-\theta} Y_{t+s} \quad .
\end{aligned} \tag{C.4}
$$

The first order condition is

$$
\begin{aligned}
& E_t\left[\textstyle\sum_{s=0}^{\infty}\alpha^s M_{t,t+s}Y_{t+s|t}(j)\left(P_t(j)(\Pi^*)^s - \mu\tfrac{W_{t+s|t(j)}}{A_{t+s}}\right)\right] = 0 \\
\Rightarrow \quad & E_t\left[\textstyle\sum_{s=0}^{\infty}\alpha^s M_{t,t+s}Y_{t+s|t}(j)\left(P_t(j)(\Pi^*)^s - \mu e^{\chi_t}A_{t+s}^{1-\psi}P_{t+s}Y_{t+s}^{\psi}\tfrac{Y_{t+s|t(j)}^{\omega}}{A_{t+s}^{1+\omega}}\right)\right] = 0 \quad ,
\end{aligned}
$$

where the second equality uses (C.3). Rearranging, I get expression (3.11) in the main text, where I used the fact that, in equilibrium, $P(j) = P_t^*$ for any j. Replacing (C.4) , I can write the first order condition as

$$
\left[P_t^*\left(\frac{P_t^*}{P_t}\right)^{-\theta}Y_t\right]H_t = \left[\mu e^{\chi_t}Y_t^{1+\omega+\psi}\left(\frac{P_t^*}{P_t}\right)^{-\theta(1+\omega)}\left(\frac{P_t}{A_t^{\psi+\omega}}\right)\right]F_t \quad ,
$$

where the expression for H_t and F_t are given by

$$
H_t = \sum_{s=0}^{\infty}(\alpha\Pi^*)^s M_{t,t+s}\frac{Y_{t+s}}{Y_t}\left(\frac{P_t(\Pi^*)^s}{P_{t+s}}\right)^{-\theta}
$$

$$
F_t = \sum_{s=0}^{\infty}\alpha^s M_{t,t+s}\frac{e^{\chi_{t+s}}}{e^{\chi_t}}\left(\frac{Y_{t+s}}{Y_t}\right)^{1+\omega+\psi}\left(\frac{A_t}{A_{t+s}}\right)^{\psi+\omega}\left(\frac{P_t(\Pi^*)^s}{P_{t+s}}\right)^{-\theta(1+\omega)}\frac{P_{t+s}}{P_t} \quad .
$$

The expressions for H_t and F_t can be more conveniently expressed recursively as

$$
\begin{aligned}
H_t &= 1 + \alpha\Pi^* E_t\left[M_{t,t+1}\frac{Y_{t+1}^F}{Y_t^F}\left(\frac{X_{t+1}}{X_t}\right)\left(\frac{\Pi_{t+1}}{\Pi^*}\right)^{\theta}H_{t+1}\right] \\
F_t &= 1 + \alpha\Pi^* E_t\left[M_{t,t+1}\frac{Y_{t+1}^F}{Y_t^F}\left(\frac{X_{t+1}}{X_t}\right)^{1+\omega+\psi}\frac{e^{\chi_{t+1}}}{e^{\chi_t}}\left(\frac{\Pi_{t+1}}{\Pi^*}\right)^{1+\theta(1+\omega)}F_{t+1}\right] \quad ,
\end{aligned}
$$

where I used the definition $X_t = Y_t/Y_t^F$.

C.1.3 Interest Rate Rule

The monetary authority sets the one-period nominal interest rate according to:

$$
i_t = \overline{\tau} + \tau_\pi(\pi_t - \pi^*) + \tau_x x_t + \tau_i \; i_{t-1} \quad .
$$

C.1.4 System

The equilibrium system of equations is:

$$
i_t = -\log E_t[M_{t,t+1}] \tag{C.5}
$$

$$
\left(\frac{P_t^*}{P_t}\right)^{1+\theta\omega}H_t = X_t^{\omega+\psi}F_t \tag{C.6}
$$

$$\frac{P_t^*}{P_t} = \left[\frac{1}{1-\alpha}\left(1-\alpha\left(\frac{\Pi^*}{\Pi_t}\right)^{1-\theta}\right)\right]^{\frac{1}{1-\theta}} \tag{C.7}$$

$$H_t = 1 + \alpha\Pi^* E_t\left[M_{t,t+1}\frac{Y_{t+1}^F}{Y_t^F}\left(\frac{X_{t+1}}{X_t}\right)\left(\frac{\Pi_{t+1}}{\Pi^*}\right)^{\theta} H_{t+1}\right] \tag{C.8}$$

$$F_t = 1 + \alpha\Pi^* E_t\left[M_{t,t+1}\frac{Y_{t+1}^F}{Y_t^F}\left(\frac{X_{t+1}}{X_t}\right)^{1+\omega+\psi}\frac{e^{\chi_{t+1}}}{e^{\chi_t}}\left(\frac{\Pi_{t+1}}{\Pi^*}\right)^{1+\theta(1+\omega)} F_{t+1}\right] \tag{C.9}$$

$$M_{t,t+1} = \beta\left(\frac{Y_{t+1}^F}{Y_t^F}\right)^{-\psi}\left(\frac{X_{t+1}}{X_t}\right)^{-\psi}\left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma}\left(\frac{P_{t+1}}{P_t}\right)^{-1} \tag{C.10}$$

$$i_t = \overline{\tau} + \tau_\pi(\pi_t - \pi^*) + \tau_x x_t + \tau_i\, i_{t-1} \quad , \tag{C.11}$$

together with the law of motions for the shocks Δa_t and χ_t.

C.2 Log-Linear Approximation

Combine equation (C.6) and (C.7) to get

$$\left[\frac{1}{1-\alpha}\left(1-\alpha\left(\frac{\Pi^*}{\Pi_t}\right)^{1-\theta}\right)\right]^{\frac{1+\theta\omega}{1-\theta}} H_t = X_t^{\omega+\psi}F_t \quad .$$

Taking logs,

$$\frac{1+\theta\omega}{1-\theta}\log\left[\frac{1}{1-\alpha}\left(1-\alpha e^{(1-\theta)(\pi^*-\pi_t)}\right)\right] + h_t = (\omega+\psi)x_t + f_t \quad . \tag{C.12}$$

A loglinear approximation of the above expression around $\pi_t = \overline{m_\pi}$ yields

$$\frac{1+\theta\omega}{1-\theta}[D_\pi + F_\pi(\pi_t - \overline{m_\pi})] + h_t = (\omega+\psi)x_t + f_t \quad ,$$

where $\overline{m_\pi} = E(\pi_t)$, and

$$D_\pi = \log\left(\frac{1-\alpha e^{-(1-\theta)(\overline{m_\pi}-\pi^*)}}{1-\alpha}\right) \quad ; \quad F_\pi = \frac{\alpha(1-\theta)e^{-(1-\theta)(\overline{m_\pi}-\pi^*)}}{1-\alpha e^{-(1-\theta)(\overline{m_\pi}-\pi^*)}} \quad .$$

Next, loglinearize the value function. Let $v_t = \log\frac{V_t}{C_t}$ and use the homogeneity of the certainty equivalent $E_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}$ to write

$$\begin{aligned} e^{(1-\psi)v_t} &= (1-\beta)\left(\frac{U(C_t,L_t)}{C_t}\right)^{1-\psi} + \beta e^{\left(\frac{1-\psi}{1-\gamma}\right)\Psi_t} \\ &= (1-\beta)\left[\frac{1}{1-\psi} - \frac{1}{\mu(1+\omega)}e^{(\psi+\omega)x_t}\right] + \beta e^{\left(\frac{1-\psi}{1-\gamma}\right)\Xi_t} \end{aligned}$$

where $\Xi_t \equiv \log E_t\left[e^{(1-\gamma)(v_{t+1}+\Delta y^F_{t+1}+\Delta x_{t+1})}\right]$. A loglinear approximation of the above expression around $x_t = \overline{m_x}$ and $\Psi_t = \overline{m_\Psi}$, yields

$$v_t = \frac{1}{1-\psi}\left\{\overline{\eta_v} + \eta_{vx}x_t + \eta_{vv}\log E_t[e^{(1-\gamma)(v_{t+1}+\Delta y^F_{t+1}+\Delta x_{t+1})}]\right\} \quad , \tag{C.13}$$

where $\overline{m_x} = E(x_t)$, $\overline{m_\Psi} = E(\Xi_t)$, and

$$\eta_{vx} = -\frac{(1-\beta)(\psi+\omega)}{\mu(1+\omega)D_v}e^{(\psi+\omega)\overline{m_x}}, \quad \eta_{vv} = \frac{(1-\psi)}{(1-\gamma)}\frac{\beta}{D_v}e^{\left(\frac{1-\psi}{1-\gamma}\right)\overline{m_\Psi}}, \quad \overline{\eta_v} = \log D_v - \eta_{vx}\overline{m_x} + \eta_{vv}\overline{m_\Psi}$$

$$D_v = (1-\beta)\left[\frac{1}{1-\psi} - \frac{1}{\mu(1+\omega)}e^{(\psi+\omega)\overline{m_x}}\right] + \beta e^{\left(\frac{1-\psi}{1-\gamma}\right)\overline{m_\Psi}}$$

Next, loglinearize the expression for H_t. First, combine (C.8) and (C.10) to get

$$e^{h_t} = 1 + \alpha\beta E_t\left[\left(\frac{Y^F_{t+1}}{Y^F_t}\right)^{1-\psi}\left(\frac{X_{t+1}}{X_t}\right)^{1-\psi}\left(\frac{\Pi^*}{\Pi_{t+1}}\right)^{1-\theta}\left(\frac{V_{t+1}}{E_t[V^{1-\gamma}_{t+1}]^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma}H_{t+1}\right] \quad .$$

Note that

$$\begin{aligned}\frac{V_{t+1}}{E_t(V^{1-\gamma}_{t+1})^{\frac{1}{1-\gamma}}} &= \frac{e^{v_{t+1}+\Delta y^F_{t+1}+\Delta x_{t+1}}}{e^{\frac{1}{1-\gamma}\log E_t\left[e^{(1-\gamma)}v_{t+1}+\Delta y^F_{t+1}+\Delta x_{t+1}\right]}} \\ &= \frac{e^{v_{t+1}+\Delta y^F_{t+1}+\Delta x_{t+1}}}{e^{\frac{1}{1-\gamma}\frac{1}{\eta_{vv}}[(1-\psi)v_t-\overline{\eta_v}-\eta_{vx}x_t]}} \quad ,\end{aligned}$$

where the second equality follows from (C.13). Therefore, I can write

$$h_t = \log[1+\alpha\beta e^{\Omega_t}] \quad ,$$

where

$$\Omega_t \equiv \log E_t\left[e^{(1-\gamma)(\Delta y^F_{t+1}+\Delta x_{t+1})+(1-\theta)(\pi^*-\pi_{t+1})+(\psi-\gamma)v_{t+1}+h_{t+1}}\right] - \frac{\psi-\gamma}{1-\gamma}\frac{1}{\eta_{vv}}[[(1-\psi)v_t-\overline{\eta_v}-\eta_{vx}x_t] \quad .$$

A loglinear approximation of the above expression around $\Omega_t = \overline{m_\Omega}$ yields

$$\begin{aligned} h_t &= \overline{\eta_h} + \eta_{hv}v_t + \eta_{hx}x_t \\ &+ \eta_{hh}\log E_t[e^{(1-\gamma)(\Delta y^F_{t+1}+\Delta x_{t+1})+(\theta-1)(\pi_{t+1}-\pi^*)+(\psi-\gamma)v_{t+1}+h_{t+1}}] \quad ,\end{aligned} \tag{C.14}$$

where $\overline{m_\Omega} = E(\Omega_t)$ and

$$\eta_{hh} = \frac{\alpha\beta e^{\overline{m_\Omega}}}{1+\alpha\beta e^{\overline{m_\Omega}}}, \quad \eta_{hv} = -\frac{\eta_{hh}}{\eta_{vv}}\frac{(\psi-\gamma)(1-\psi)}{1-\gamma}, \quad \eta_{hx} = \frac{\eta_{hh}}{\eta_{vv}}\frac{(\psi-\gamma)\eta_{vx}}{1-\gamma}$$

$$\overline{\eta_h} = \log(1+\alpha e^{\overline{m_\Omega}}) - \eta_{hh}\overline{m_\Omega} + \frac{\eta_{hh}}{\eta_{vv}}\frac{(\psi-\gamma)\overline{\eta_v}}{1-\gamma} \quad .$$

Last, I obtain a loglinearized expression for F_t. First, combine (C.9) and (C.10) to get

$$e^{f_t} = 1 + \alpha\beta E_t \left[\left(\frac{Y_{t+1}^F}{Y_t^F}\right)^{1-\psi} \left(\frac{X_{t+1}}{X_t}\right)^{1+\omega} \frac{e^{\chi_{t+1}}}{e^{\chi_t}} \left(\frac{\Pi^*}{\Pi_{t+1}}\right)^{-\theta(1+\omega)} \left(\frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma} F_{t+1} \right] \quad .$$

In a way similar to what was done for the expression for H_t, one can write

$$f_t = \log[1 + \alpha\beta e^{\Upsilon_t}] \quad ,$$

where

$$\begin{aligned} \Upsilon_t \quad &\equiv \quad \log E_t \left[e^{(1-\gamma)\Delta y_{t+1}^F + (1+\omega+\psi-\gamma)\Delta x_{t+1} - \theta(1+\omega)(\pi^* - \pi_{t+1}) + (\psi-\gamma)v_{t+1} + f_{t+1}} \right] \\ &- \quad \frac{\psi-\gamma}{1-\gamma} \frac{1}{\eta_{vv}} [[(1-\psi)v_t - \overline{\eta_v} - \eta_{vx} x_t] \quad . \end{aligned}$$

A loglinear approximation of the expression above around $\Upsilon_t = \overline{m_\Upsilon}$ yields

$$\begin{aligned} f_t \quad &= \quad \overline{\eta_f} + \eta_{fv} v_t + \eta_{fx} x_t \\ &+ \quad \eta_{ff} \log E_t [e^{(1-\gamma)\Delta y_{t+1}^F + (1+\omega+\psi-\gamma)\Delta x_{t+1} + \theta(1+\omega)(\pi_{t+1} - \pi^*) + (\psi-\gamma)v_{t+1} + g_{t+1}}] \quad , \end{aligned} \tag{C.15}$$

where $\overline{m_\Upsilon} = E(\Upsilon_t)$ and

$$\eta_{ff} = \frac{\alpha\beta e^{\overline{m_\Upsilon}}}{1 + \alpha\beta e^{\overline{m_\Upsilon}}}, \quad \eta_{fv} = -\frac{\eta_{ff}}{\eta_{vv}} \frac{(\psi-\gamma)(1-\psi)}{1-\gamma}, \quad \eta_{fx} = \frac{\eta_{ff}}{\eta_{vv}} \frac{(\psi-\gamma)\eta_{vx}}{1-\gamma}$$

$$\overline{\eta_f} = \log(1 + \alpha e^{\overline{m_\Upsilon}}) - \eta_{ff} \overline{m_\Upsilon} + \frac{\eta_{ff}}{\eta_{vv}} \frac{(\psi-\gamma)\overline{\eta_v}}{1-\gamma} \quad .$$

The loglinearized system consists of equations (C.12)-(C.15), together with the Euler condition (C.5).

C.2.1 Solution to the Loglinear System

I guess that the solution to the system is affine in the state variables, that is

$$\begin{aligned} x_t \quad &= \quad \overline{x} + x_a \Delta a_t + x_\chi \chi_t + x_i i_{t-1} \\ \pi_t - \pi^* \quad &= \quad \overline{\pi} + \pi_a \Delta a_t + \pi_\chi \chi_t + \pi_i i_{t-1} \\ h_t \quad &= \quad \overline{h} + h_a \Delta a_t + h_\chi \chi_t + h_i i_{t-1} \\ f_t \quad &= \quad \overline{f} + g_a \Delta a_t + f_\chi \chi_t + f_i i_{t-1} \\ v_t \quad &= \quad \overline{v} + v_a \Delta a_t + v_\chi \chi_t + v_i i_{t-1} \quad . \end{aligned} \tag{C.16}$$

Substituting the guess above in Equations (C.11) to (C.15), and using the Euler equation (C.5) and the expression for the nominal pricing kernel in (C.10), one can obtain the solution for the coefficients governing the dynamics of the endogenous variables x_t, π_t, h_t, f_t, and v_t by matching coefficients. The solution involves a fixed point problem, as the linearization points are endogenously determined. See Hsu and Palomino (2011) for details.

C.3 The pricing kernel

The pricing kernel of the economy is

$$
\begin{aligned}
M_{t,t+1} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}}\right)^{\psi-\gamma}(\Pi_{t+1})^{-1}\\
&= \beta\left(\frac{Y^F_{t+1}}{Y^F_t}\right)^{-\gamma}\left(\frac{X_{t+1}}{X_t}\right)^{-\gamma}\left(\frac{e^{(\psi-\gamma)v_{t+1}}}{e^{\frac{\psi-\gamma}{1-\gamma}\frac{1}{\eta_{vv}}[(1-\psi)v_t-\overline{\eta_v}-\eta_{vx}x_t]}}\right)e^{-\pi_{t+1}}
\end{aligned}
$$

Let $M^*_{t,t+1} = \begin{cases} N_{t,t+1} & \text{if } I=0; \\ M_{t,t+1} & \text{if } I=1. \end{cases}$ Then,

$$
\begin{aligned}
-\log M^*_{t,t+1} &= -\log\beta + \gamma\Delta y^F_{t+1} + \gamma\Delta x_{t+1} - (\psi-\gamma)v_{t+1}\\
&+ \frac{\psi-\gamma}{1-\gamma}\frac{1}{\eta_{vv}}[(1-\psi)v_t - \overline{\eta_v} - \eta_{vx}x_t] + I(\pi_{t+1})
\end{aligned}
$$

Substituting the guesses for the endogenous variables and rearranging, I can write the expression for the pricing kernel more succinctly as

$$
-\log M^*_{t,t+1} = \delta + \gamma_a\Delta a_t + \gamma_\chi\chi_t + \gamma_i i_{t-1} + \lambda_a\sigma_a\epsilon^a_{t+1} + \lambda_\chi\sigma_\chi\epsilon^\chi_{t+1} \quad ,
$$

where

$$
\begin{aligned}
\delta &= -\log\beta - (\psi-\gamma)\overline{v} + \frac{\psi-\gamma}{1-\gamma}\frac{1}{\eta_{vv}}[(1-\psi)\overline{v} - \overline{\eta_v} - \eta_{vx}\overline{x}] + I(\pi^* + \overline{\pi})\\
&+ [\gamma(1+x_a) - (\psi-\gamma)v_a + I\pi_a](1-\varphi_a)\theta_a
\end{aligned}
$$

$$
\begin{aligned}
\gamma_a &= -\left[\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{\eta_{vx}}{\eta_{vv}} + \gamma\right]x_a + \left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{1-\psi}{\eta_{vv}}v_a + [\gamma(1+x_a) - (\psi-\gamma)v_a + I\pi_a]\varphi_a\\
&+ (\tau_x x_a + \tau_\pi\pi_a)(\gamma x_i - (\psi-\gamma)v_i + I\pi_i) \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_\chi &= -\left[\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{\eta_{vx}}{\eta_{vv}} + \gamma\right]x_\chi + \left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{1-\psi}{\eta_{vv}}v_\chi + [\gamma(1+x_a) - (\psi-\gamma)v_a + I\pi_a]\varphi_a\\
&+ (\tau_x x_\chi + \tau_\pi\pi_\chi)(\gamma x_i - (\psi-\gamma)v_i + I\pi_i) - \frac{\gamma}{\psi+\omega}(\varphi_\chi - 1) \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_i &= -\left[\left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{\eta_{vx}}{\eta_{vv}} + \gamma\right]x_i + \left(\frac{\psi-\gamma}{1-\gamma}\right)\frac{1-\psi}{\eta_{vv}}v_i\\
&+ (\tau_x x_i + \tau_\pi\pi_i + \tau_i)(\gamma x_i - (\psi-\gamma)v_i + I\pi_i) \quad ,
\end{aligned}
$$

$$
\begin{aligned}
\lambda_a &= \gamma(1+x_a) - (\psi-\gamma)v_a + I\pi_a \quad ,\\
\lambda_\chi &= \gamma\left(-\frac{1}{\psi+\omega} + x_\chi\right) - (\psi-\gamma)v_\chi + I\pi_\chi \quad .
\end{aligned}
$$

C.4 Consumption Growth, Price-Consumption Ratio, and Risk Premium

Here I solve for the equilibrium consumption growth process, the price-consumption ratio, and the risk premium for the model with interest rate inertia. Using (3.14) in the main text and the guess for the output gap, realized consumption growth can be written as

$$\begin{aligned}\Delta c_{t+1} = \Delta y_{t+1} &= \Delta y^F_{t+1} + \Delta x_{t+1} \\ &= \mu_c + g_a \Delta a_t + g_i i_{t-1} + (1+x_a)\sigma_a \epsilon^a_{t+1} \quad ,\end{aligned}$$

where

$$\mu_c = (1+x_a)(1-\varphi_a)\theta_a + x_i(\overline{\tau} + \tau_x \overline{x} + \tau_\pi \overline{\pi}) \quad ,$$

$$g_a = (1+x_a)\varphi_a - x_a + x_i(\tau_x x_a + \tau_\pi \pi_a) \quad ,$$

$$g_i = x_i(\tau_x x_i + \tau_\pi \pi_i + \tau_i - 1) \quad .$$

The price-consumption ratio is $pc_t = \overline{A} + A_a \Delta a_t + A_\chi \chi_t + A_i i_{t-1}$, where

$$\begin{aligned}\overline{A} &= \frac{1}{1-\eta_{pc}}\big(-\delta + \overline{\eta_{pc}} + \mu_c + \eta_{pc} A_a (1-\varphi_a)\theta_a + A_i(\overline{\tau} + \tau_x \overline{x} + \tau_\pi \overline{\pi}) \\ &+ (\eta_{pc} A_a + (1+x_a) - \lambda_a)^2 \sigma_a^2 /2\big) \quad ,\end{aligned}$$

$$A_a = \frac{g_a + \eta_{pc} A_i (\tau_x x_a + \tau_\pi \pi_a) - \gamma_a}{1-\eta_{pc}\varphi_a} \quad ,$$

$$A_i = \frac{g_i - \gamma_i}{1-\eta_{pc}(\tau_x x_i + \tau_\pi \pi_i + \tau_i)} \quad .$$

The risk premium on the consumption claim is

$$\begin{aligned}E_t(r^c_{t+1} - r_{f,t}) + 0.5\, Var_t(r^c_{t+1}) &= -Cov(\log N_{t+1} - E_t(\log N_{t+1}), r^c_{t+1} - E_t(r^c_{t+1})) \\ &= \lambda_a B_a \sigma_a \quad ,\end{aligned}$$

where $B_a = \eta_{pc} A_a + (1+x_a)$.

C.5 Proofs of Results

The system in (C.16) does not have a closed form solution (see Bekaert, Cho, and Moreno (2010)), so I rely on a numerical analysis to show that $x_a > 0$, $g_a > 0$, $\frac{\partial x_a}{\partial \alpha} > 0$, $\frac{\partial x_a}{\partial \tau_\pi} < 0$, and $\frac{\partial x_a}{\partial \tau_x} < 0$. See Figure C.1 for the results.

Proof of Result 1: Using (3.16) in the main text, the first order autocorrelation of expected con-

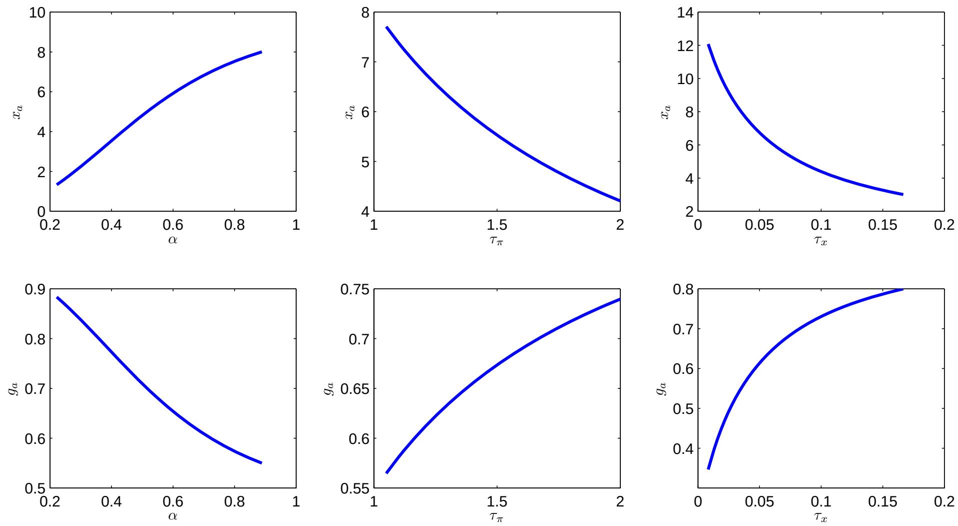

Figure C.1: Coefficients x_a and g_a as a function of the Calvo parameter α, and the interest rate rule coefficients τ_π and τ_x. All other parameters are from the model without policy inertia in Table 3.1.

sumption growth is

$$\begin{aligned} Corr(E_t\Delta c_{t+1}, E_{t+1}\Delta c_{t+2}) &= \frac{Cov(E_t\Delta c_{t+1}, E_{t+1}\Delta c_{t+2})}{Var(E_t\Delta c_{t+1})} \\ &= \frac{Cov(g_a\Delta a_t, g_a\Delta a_{t+1})}{g_a^2\, Var(\Delta a_t)} \\ &= \frac{g_a^2\ \varphi_a\ \ Var(\Delta a_t)}{g_a^2\ \ Var(\Delta a_t)} = \varphi_a \quad . \end{aligned}$$

Using (3.15) in the main text, the first order autocorrelation of realized consumption growth is

$$\begin{aligned} Corr(\Delta c_t, \Delta c_{t+1}) &= \frac{Cov(\Delta c_t, \Delta c_{t+1})}{Var(\Delta c_t)} \\ &= \frac{g_a^2\ \varphi_a\ \ Var(\Delta a_t) + (1+x_a)\ g_a\ \sigma_a^2}{g_a^2\ \ Var(\Delta a_t) + (1+x_a)^2\ \sigma_a^2} \\ &= \varphi_a + \frac{(1+x_a)\ \sigma_a^2\ (g_a - \varphi_a(1+x_a))}{g_a^2\ \ Var(\Delta a_t) + (1+x_a)^2\ \sigma_a^2} \\ &= \varphi_a - \frac{(1+x_a)\ \sigma_a^2\ x_a}{g_a^2\ \ Var(\Delta a_t) + (1+x_a)^2\ \sigma_a^2} < \varphi_a \quad , \end{aligned} \tag{C.17}$$

since $x_a > 0$. Now, define

$$b(\alpha) \equiv \frac{(1+x_a)\ \sigma_a^2\ x_a}{g_a^2\ \ Var(\Delta a_t) + (1+x_a)^2\ \sigma_a^2} \quad .$$

Substituting $g_a = (1 + x_a)\varphi_a - x_a$ and $Var(\Delta a_t) = \frac{\sigma_a^2}{1-\varphi_a^2}$, after some algebra, I get

$$\frac{1}{b(\alpha)} = \left(\frac{1+x_a}{x_a} + \frac{x_a}{1+x_a}\right)\frac{1}{1-\varphi_a^2} - \frac{2\varphi_a}{1-\varphi_a^2} \tag{C.18}$$

Express $\frac{\partial\left(\frac{1+x_a}{x_a}\right)}{\partial\alpha}$ and $\frac{\partial\left(\frac{x_a}{1+x_a}\right)}{\partial\alpha}$ in terms of $\frac{\partial(x_a)}{\partial\alpha}$:

$$\frac{\partial\left(\frac{1+x_a}{x_a}\right)}{\partial\alpha} = \frac{x_a\frac{\partial(1+x_a)}{\partial\alpha} - (1+x_a)\frac{\partial(x_a)}{\partial\alpha}}{x_a^2} = -\frac{\frac{\partial(x_a)}{\partial\alpha}}{x_a^2} \tag{C.19}$$

$$\frac{\partial\left(\frac{x_a}{1+x_a}\right)}{\partial\alpha} = \frac{(1+x_a)\frac{\partial(x_a)}{\partial\alpha} - x_a\frac{\partial(1+x_a)}{\partial\alpha}}{(1+x_a)^2} = \frac{\frac{\partial(x_a)}{\partial\alpha}}{(1+x_a)^2} \tag{C.20}$$

Now, differentiate (C.18) w.r.t. α and substitute (C.19) and (C.20):

$$\begin{aligned}
\frac{\partial\frac{1}{b(\alpha)}}{\partial\alpha} &= \left(\frac{\partial\left(\frac{1+x_a}{x_a}\right)}{\partial\alpha} + \frac{\partial\left(\frac{x_a}{1+x_a}\right)}{\partial\alpha}\right)\frac{1}{1-\varphi_a^2} \\
&= \left(-\frac{\frac{\partial(x_a)}{\partial\alpha}}{x_a^2} + \frac{\frac{\partial(x_a)}{\partial\alpha}}{(1+x_a)^2}\right)\frac{1}{1-\varphi_a^2} \\
&= \frac{\partial\,(x_a)}{\partial\alpha}\underbrace{\left(\frac{1}{(1+x_a)^2} - \frac{1}{x_a^2}\right)}_{<0}\frac{1}{1-\varphi_a^2}
\end{aligned}$$

Hence, $Sign\left(\frac{\partial\frac{1}{b(\alpha)}}{\partial\alpha}\right) = -Sign\left(\frac{\partial(x_a)}{\partial\alpha}\right)$ and $Sign\left(\frac{\partial b(\alpha)}{\partial\alpha}\right) = Sign\left(\frac{\partial(x_a)}{\partial\alpha}\right) > 0$. Therefore, from (C.17), we have that $\frac{\partial Corr(\Delta c_t,\Delta c_{t+1})}{\partial\alpha} < 0$. Similar derivations show that $\frac{\partial Corr(\Delta c_t,\Delta c_{t+1})}{\partial\tau_\pi} > 0$ and $\frac{\partial Corr(\Delta c_t,\Delta c_{t+1})}{\partial\tau_x} > 0$. □

Proof of Result 2: The conditional correlation between realized and expected consumption growth is

$$\begin{aligned}
Corr_{t-1}(\Delta c_t, E_t(\Delta c_{t+1})) &= \frac{Cov_{t-1}(\Delta c_t, E_t(\Delta c_{t+1}))}{\sigma_{t-1}(\Delta c_t)\ \sigma_{t-1}\big(E_t(\Delta c_{t+1})\big)} \\
&= \frac{(1+x_a)\ g_a\ \sigma_a^2}{|1+x_a|\ |g_a|\ \sigma_a^2} = +1 \quad ,
\end{aligned}$$

since $x_a > 0$ and $g_a > 0$. □

C.6 A Model with a Labor Supply Shock

Consider a model in which the intratemporal utility function $U(C_t, L_t)$ is given by (C.1), where the shock to the disutility of labor evolves according to (C.2). The labor supply shock introduces a wedge that distorts the static relationship between real wages and the marginal rate of substitution between consumption and labor (equation C.3). This distortion makes the problem faced by the

Moment	Data	Model
Panel A: Calibrated moments		
$E(\Delta c_t)$	1.800	1.800
$\sigma(\Delta c_t)$	2.720	2.720
$Corr(\Delta c_t, \Delta c_{t+1})$	n.a.	0.118
$E(i_t)$	6.420	6.420
$\sigma(i_t)$	3.720	3.720
Sharpe Ratio	0.340	0.340
Panel B: Other moments		
$\sigma(E_t \Delta c_{t+1})$	n.a.	0.635
$Corr(E_t \Delta c_{t+1}, E_{t+1} \Delta c_{t+2})$	n.a.	0.931
$E(\Delta c_t^F)$	n.a.	1.800
$\sigma(\Delta c_t^F)$	n.a.	1.790
$Corr(\Delta c_t^F, \Delta c_{t+1}^F)$	n.a.	-0.020
$E(\Delta c_t^E)$	n.a.	1.800
$\sigma(\Delta c_t^E)$	n.a.	1.040
$Corr(\Delta c_t^E, \Delta c_{t+1}^E)$	n.a.	0.950
$E(\pi_t)$	4.340	4.780
$\sigma(\pi_t)$	1.320	0.708
$Corr(\pi_t, \pi_{t+1})$	0.600	0.945
$Corr(i_t, i_{t+1})$	0.980	0.940
$E(r_t)$	0.862	1.351
$\sigma(r_t)$	0.969	1.466

Table C.1: Moment conditions of macro variables and asset prices for the model with an *i.i.d.* labor supply shock of Appendix C.6. Means are annualized by multiplying by 12 the monthly observation. Volatilities are annualized by multiplying by $\sqrt{12}$ the monthly observation. The Sharpe Ratio is defined as the ratio between the annualized excess return and the annualized volatility of the return on the dividend claim. The autocorrelation moments refer to the monthly autocorrelations. Panel A reports consumption moments, and Panel B reports other moments of interest. The empirical moments for consumption growth are taken from Bansal and Yaron (2004). Data on nominal interest rate and inflation is from CRSP.

monetary authority non-trivial in the sense that it introduces a trade-off between optimal inflation and optimal output.

Strictly speaking, this distortion is not *bad*, in the sense that, while it moves away the equilibrium allocation from the one that would be obtained with flexible prices and constant disutility of labor, it does not introduce any additional welfare inefficiencies other than the ones generated by price stickiness. However, as pointed out by Chari, Kehoe, and McGrattan (2007), in the model studied in this paper, like in the model of Smets and Wouters (2007), one cannot identify what structural shock is the actual *source* of such a wedge. For instance, a similar distortion could be obtained with the introduction of a time-varying price markup shock, which instead is a *bad* shock in the sense that it moves the competitive equilibrium allocation away from the efficient allocation, even further away than what is generated by price stickiness only. Here, I use a labor supply shock for analytical tractability of the firms' maximization problem (see Appendix C.1.2), and analyze the case in which the monetary authority erroneously *believes* that the labor wedge comes from a *bad* shock and therefore has an incentive to offset it.

I conduct the following exercise. I tweak the calibration in Section 3.4 to account for the additional variability in the model coming from the introduction of an *i.i.d.* labor supply shock with an annualized volatility of 1.73%. Compared to the baseline calibration of Table 3.1, I increase the Calvo parameter α to 8/9 and the interest rate rule coefficient to inflation τ_π to 1.5. This is due to the fact that the introduction of the labor shock increases the overall variability in the economy. While I do not attempt to provide a quantitative measure of the welfare costs associated with different monetary policy reactions to inflation and output, I show that the equilibrium that would be obtained with flexible prices is no longer the efficient allocation and that therefore the monetary authority does not have an obvious incentive to achieve it.

The efficient output Y_t^E is defined as the output that would be produced in the case of flexible prices and constant disutility of labor. From Appendix C.1.2, efficient output growth evolves following the dynamics of technology, that is $\Delta y_t^E = \Delta a_t$, while flexible output growth is $\Delta y_{t+1}^F = \Delta a_{t+1} - \frac{1}{\psi+\omega}\Delta\chi_t$. Notice that the monetary authority cannot influence the dynamics of χ_t, so that the efficient allocation cannot be achieved. Here, I do not perform a quantitative analysis of the welfare losses associated with different interest rate rules (see Woodford (2003) for a general treatment and Levin, Lopez-Salido, Nelson, and Yun (2008) for the specific case of recursive preferences). Rather, the results in Table C.1 show that the monetary authority does not have an obvious interest in offsetting the effects of price stickiness. Indeed, the dynamics of flexible consumption growth largely differ from the dynamics of efficient consumption growth. In particular, flexible consumption growth is more volatile than efficient consumption growth, with an annualized volatility of 1.79% (versus 1.04%), and also shows very little autocorrelation (-0.02) compared to the efficient allocation.

Bibliography

Abel, A. B. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics 43*(1), 3–33.

Alvarez, F., A. Atkeson, and P. J. Kehow (2009). Time-varying risk, interest rates, and exchange rates in general equilibrium. *Review of Economic Studies 76*(3), 851–878.

Ang, A., S. Dong, and M. Piazzesi (2007). No-arbitrage Taylor rules. Working Paper 13448, National Bureau of Economic Research.

Backus, D. K., S. Foresi, and C. I. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*(1), 279–304.

Backus, D. K., F. Gavazzoni, C. Telmer, and S. E. Zin (2010). Monetary policy and the uncovered interest parity puzzle. Working Paper 16218, National Bureau of Economic Research.

Backus, D. K., A. W. Gregory, and C. I. Telmer (1993). Accounting for forward rates in markets for foreign currency. *The Journal of Finance 48*(5), 1887–1908.

Backus, D. K. and S. E. Zin (1994). Reverse engineering the yield curve. Working Paper 4676, National Bureau of Economic Research.

Backus, D. K. and S. E. Zin (2008). Notes on identification of Taylor rules. Unpublished manuscript, New York University.

Bakshi, G. S. and Z. Chen (1997). Equilibrium valuation of foreign exchange claims. *The Journal of Finance 52*(2), 799–826.

Bansal, R. (1997). An exploration of the forward premium puzzle in currency markets. *Review of Financial Studies 10*(2), 369–403.

Bansal, R., D. Kiku, and A. Yaron (2007). Risks for the long run: Estimation and inference. Working Paper, Duke University.

Bansal, R., D. Kiku, and A. Yaron (2009). An empirical evaluation of the long-run risks model for asset prices. Working Paper 15504, National Bureau of Economic Research.

Bansal, R. and I. Shaliastovich (2010). A long-run risks explanation of predictability puzzles in bond and currency markets. Working Paper, Duke University.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance 59*(4), 1481–1509.

Beeler, J. and J. Y. Campbell (2009). The long-run risks model and aggregate asset prices: An empirical assessment. Working Paper 14788, National Bureau of Economic Research.

Bekaert, G. (1994). Exchange rate volatility and deviations from unbiasedness in a cash-in-advance model. *Journal of International Economics 36*(1-2), 29–52.

Bekaert, G., S. Cho, and A. Moreno (2010). New Keynesian macroeconomics and the term structure. *Journal of Money, Credit and Banking 42*(1), 33–62.

Benigno, G. (2004). Real exchange rate persistence and monetary policy rules. *Journal of Monetary Economics 51*(3), 473–502.

Benigno, G. and P. Benigno (2008). Exchange rate determination under interest rate rules. *Journal of International Money and Finance 27*(6), 971–993.

Bernanke, B. S. and I. Mihov (1998). Measuring monetary policy. *The Quarterly Journal of Economics 113*(3), 869–902.

Bils, M. and P. J. Klenow (2004). Some evidence on the importance of sticky prices. *Journal of Political Economy 112*(5), 947–985.

Bilson, J. F. O. (1981). The speculative efficiency hypothesis. *The Journal of Business 54*(3), 435–51.

Brandt, M. W., J. H. Cochrane, and P. Santa-Clara (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics 53*(4), 671–698.

Brennan, M. J. and Y. Xia (2006). International capital markets and foreign exchange risk. *Review of Financial Studies 19*(3), 753–795.

Burnside, C. (2007). The forward premium is still a puzzle. Working Paper 13129, National Bureau of Economic Research.

Burnside, C., M. Eichenbaum, I. Kleshchelski, and S. Rebelo (2006). The returns to currency speculation. Working Paper 12489, National Bureau of Economic Research.

Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Campbell, J. and R. Shiller (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies 1*(3), 195–228.

Campbell, J. Y. (1994). Inspecting the mechanism: An analytical approach to the stochastic growth model. *Journal of Monetary Economics 33*(3), 463–506.

Canova, F. and J. Marrinan (1993). Profits, risk, and uncertainty in foreign exchange markets. *Journal of Monetary Economics 32*(2), 259–286.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007). Business cycle accounting. *Econometrica 75*(3), 781–836.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999). Monetary policy shocks: What have we learned and to what end? In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1 of *Handbook of Macroeconomics*, Chapter 2, pp. 65–148. Elsevier.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (1998). Monetary policy rules in practice: Some international evidence. *European Economic Review 42*(6), 1033–1067.

Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature 37*(4), 1661–1707.

Clarida, R., J. Galí, and M. Gertler (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics 115*(1), 147–180.

Cochrane, J. H. (2011). Determinacy and identification with Taylor rules. *Journal of Political Economy 119*(3), 565–615.

Colacito, R. and M. M. Croce (2011). Risks for the long run and the real exchange rate. *Journal of Political Economy 119*(1), 153–181.

Croce, M. M. (2010). Long-run productivity risk: A new hope for production-based asset pricing. Working Paper, University of North Carolina.

Dumas, B. (1992). Dynamic equilibrium and the real exchange rate in a spatially separated world. *Review of Financial Studies 5*(2), 153–80.

Dutton, J. (1993). Real and monetary shocks and risk premia in forward markets for foreign exchange. *Journal of Money, Credit and Banking 25*(4), 731–54.

Eichenbaum, M. and C. L. Evans (1995). Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *The Quarterly Journal of Economics 110*(4), 975–1009.

Engel, C. (1996). The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance 3*, 123–192.

Engel, C. (2011). The real exchange rate, real interest rates, and the risk premium. Unpublished manuscript, University of Wisconsin.

Engel, C. and K. D. West (2006). Taylor rules and the deutschmark: Dollar real exchange rate. *Journal of Money, Credit and Banking 38*(5), 1175–1194.

Epstein, L. and S. E. Zin (1989). Substitution risk aversion and the temporal behavior of consumption and asset returns, a theoratical framework. *Econometrica 57*, 937–969.

Fama, E. (1984). Forward and spot exchange rates. *Journal of Monetary Economics 14*, 319–338.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of Financial Economics 22*(1), 3–25.

Frachot, A. (1996). A reexamination of the uncovered interest rate parity hypothesis. *Journal of International Money and Finance 15*(3), 419–437.

Galí, J. (1992). How well does the IS-LM model fit postwar U.S. data. *The Quarterly Journal of Economics 107*(2), 709–38.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Gallmeyer, M. F., B. Hollifield, F. Palomino, and S. E. Zin (2007). Arbitrage-free bond pricing with dynamic macroeconomic models. *Federal Reserve Bank of St. Louis Review 89*(4), 305–326.

Gavazzoni, F. (2008). Uncovered interest rate parity puzzle: An explanation based on recursive utility and stochastic volatility. Working Paper, Carnegie Mellon University.

Gomme, P. and P. Rupert (2007). Theory, measurement and calibration of macroeconomic models. *Journal of Monetary Economics 54*(2), 460–497.

Goodfriend, M. and R. G. King (2009). The great inflation drift. Working Paper 14862, National Bureau of Economic Research.

Grilli, V. and N. Roubini (1992). Liquidity and exchange rates. *Journal of International Economics 32*(3-4), 339–352.

Hansen, L. P., J. Heaton, and N. Li (2005). Consumption strikes back?: Measuring long-run risk. Working Paper 11476, National Bureau of Economic Research.

Hansen, L. P. and R. Jagannathan (1991). Implications of security market data for models of dynamic economies. *Journal of Political Economy 99*(2), 225–62.

Hansen, L. P. and K. J. Singleton (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy 91*, 249–268.

Hollifield, B. and R. Uppal (1997). An examination of uncovered interest rate parity in segmented international commodity markets. *Journal of Finance 52*, 2145–2170.

Hsu, A. (2011). Does fiscal policy matter for bond risk premia? Working Paper, University of Michigan.

Hsu, A. and F. Palomino (2011). A simple solution method for dynamic equilibrium models with time-varying volatility. Working Paper, University of Michigan.

Judd, J. P. and G. D. Rudebusch (1998). Taylor's rule and the Fed, 1970-1997. *Federal Reserve Bank of San Francisco Economic Review 3*, 3–16.

Kaltenbrunner, G. and L. A. Lochstoer (2010). Long-Run Risk through Consumption Smoothing. *Review of Financial Studies 23*(8), 3190–3224.

Kuehn, L.-A., N. Petrosky-Nadeau, and L. Zhang (2010). An equilibrium asset pricing model with labor markets search. Working Paper, Carnegie Mellon University.

Kung, H. and L. Schmid (2011). Innovation, growth and asset prices. Working Paper, Duke University.

Levin, A. T., J. D. Lopez-Salido, E. Nelson, and T. Yun (2008). Macroeconometric equivalence, microeconomic dissonance, and the design of monetary policy. *Journal of Monetary Economics 55, Supplement*(0), S48–S62.

Lewis, K. K. (1995). Puzzles in international financial markets. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of International Economics*, Volume 3 of *Handbook of International Economics*, Chapter 37, pp. 1913–1971. Elsevier.

Lucas, R. J. (1982). Interest rates and currency prices in a two-country world. *Journal of Monetary Economics 10*(3), 335–359.

Lustig, H., N. Roussanov, and A. Verdelhan (2011). Common risk factors in currency markets. *Forthcoming on the Review of Financial Studies*.

Lustig, H. and A. Verdelhan (2007). The cross-section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*, 89–117.

Marshall, D. (1992). Phd thesis. Carnegie Mellon University.

McCallum, B. T. (1983). On non-uniqueness in rational expectations models: An attempt at perspective. *Journal of Monetary Economics 11*(2), 139–168.

McCallum, B. T. (1994). A reconsideration of the uncovered interest rate parity relationship. *Journal of Monetary Economics 33*, 105–132.

Mehra, R. and E. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*, 145–161.

Mussa, M. (1986). Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications. *Carnegie-Rochester Conference Series on Public Policy 25*(1), 117–214.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics 123*(4), 1415–1464.

Nelson, C. R. and C. R. Plosser (1982). Trends and random walks in macroeconmic time series: Some evidence and implications. *Journal of Monetary Economics 10*(2), 139–162.

Orphanides, A. (2004). Monetary policy rules, macroeconomic stability, and inflation: A view from the trenches. *Journal of Money, Credit and Banking 36*(2), 151–75.

Piazzesi, M. and M. Schneider (2007). Equilibrium yield curves. In *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pp. 389–472. National Bureau of Economic Research, Inc.

Romer, C. D. and D. H. Romer (1989). Does monetary policy matter? A new test in the spirit of Friedman and Schwartz. In *NBER Macroeconomics Annual 1989, Volume 4*, NBER Chapters, pp. 121–184. National Bureau of Economic Research, Inc.

Rouwenhorst, K. G. (1995). Asset pricing implications of real business cycle models. In T. F. Cooley (Ed.), *Frontiers of Business Cycle Research*, pp. 294–329. Princeton, New Jersey: Princeton University Press.

Rudebusch, G. D. and E. T. Swanson (2012). The bond premium in a dsge model with long-run real and nominal risks. *American Economic Journal: Macroeconomics 4*(1), 105–43.

Saá-Requejo, J. (1994). The dynamics and the term structure of risk premium in foreign exchange markets. Unpublished manuscript, INSEAD.

Schlagenhauf, D. E. and J. M. Wrase (1995). Liquidity and real activity in a simple open economy model. *Journal of Monetary Economics 35*(3), 431–461.

Sercu, P., R. Uppal, and C. V. Hulle (1995). The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity. *Journal of Finance 50*, 1309–1319.

Sims, C. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review 36*(5), 975–1000.

Smets, F. and R. Wouters (2007). Shocks and frictions in us business cycles: A bayesian DSGE approach. *American Economic Review 97*(3), 586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Taylor, J. B. (1999). A historical analysis of monetary policy rules. In *Monetary Policy Rules*, NBER Chapters, pp. 319–348. National Bureau of Economic Research, Inc.

Tiff Macklem, R. (1991). Forward exchange rates and risk premiums in artificial economies. *Journal of International Money and Finance 10*(3), 365–391.

Uhlig, H. (2005). What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics 52*(2), 381–419.

Uhlig, H. (2007). Leisure, growth and long run risk. Working Paper, University of Chicago.

Uhlig, H. (2010). Easy EZ in DSGE. Working Paper, University of Chicago.

Verdelhan, A. (2010). A habit-based explanation of the exchange rate risk premium. *The Journal of Finance 65*(1), 123–146.

Woodford, M. (2003). *Interest and prices.* Princeton University Press.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics 37*(2-3), 345–370.