

Essays in Finance:
Pre-borrowing: Co-existence of Cash and Debt;
Predators, Prey and Volatility on Wall Street.

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Abstract

In three essays I explore the effects of financial frictions on agents optimal decisions. In the first two essays I look jointly at liquidity and leverage decisions. First, in the theoretical framework I establish the pre-borrowing motive: the incentives for firms to issue debt earlier save proceeds in cash and then use internal liquid funds to finance investments. That strategy is a hedge against the volatility in the terms of borrowing. I explore implications of this motive in the setting of multi-period dynamic model of the firm, generating persistence in leverage and cash ratios that matches the observed data patterns. Also the model predicts that most risky firms would use revolving short term debt, which is also consistent with data. The safer firms with moderate liquidity needs are expected to rely on Long-term debt, while firms with a lot of liquidity needs and moderate risk profile will issue both short and long term debt. I test model implications on a sample of US publicly traded firms, and find that indeed, firms with more volatile market to book ratio are more likely to have positive debt and cash balances, as is predicted by the model. That finding is consistent with pre-borrowing motive, but cant be reconciled within other frameworks of precautionary demand for cash holdings. The third essay (co-authored with Richard Green) is looking at the difference in ability of traders in financial markets and demonstrates how it can translate into exaggerated fluctuations in employment on Wall Street. We show that even a very small shock to the fundamental profits can generate massive exit of traders from the market, essentially collapsing the market; while large positive shock would be required to bring market back to the large employment steady state, contributing to the growing literature studying the sources of excess volatility in financial markets.

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Chapter 1

The Pre-Borrowing Motive: A Model of Coexistent Debt and Cash Holdings.

Abstract

This paper demonstrates how costly default gives rise to the risk-averse type of behavior by firms. Firms are exposed to the risk of change in the terms of borrowing. With costly default, firms are better off hedging this risk. Hedging motivates firms to borrow earlier with long term debt and keep proceeds in cash until the funds are needed. The finding is novel in the light that the result does not rely on collateral constraints. We examine full implications of the pre-borrowing motive in the dynamic neo-classical model of the firm and characterize optimal borrowing and cash holding policies. In a calibrated version of the model cash and debt co-exist and levels are persistent in time, consistent with the data.

1.1 Introduction

In data, US publicly traded firms hold cash and have debt outstanding at the same time: about 7% of firms hold more than 20% of assets in cash while more than 20% of their assets are financed with debt (see Figure 1.1). In 2009 half of firms had Cash/Assets and Debt/Assets above 5%. Moreover, the proportion of firms that has Cash/Assets and Debt/Assets above 20% was almost 10%.¹ This is puzzling, given numerous financing frictions that make outside financing costly.

While most would agree that firms issue debt because of its tax advantage, there is less consensus on why firms hold cash. Empirically, cash holdings are positively related to the measures of cash flow volatility (see Table 1.8 and Bates, Kahle, and Stulz [2009], among others). That speaks in favor of precautionary demand for cash holdings. Theoretical models that justify holding cash for hedging purposes rely heavily on the assumption that financial constraints are binding. Yet empirically both financially constrained and financially unconstrained firms (as proxied by paying positive dividends, having positive net income or being large in size) exhibit positive relationship between cash holdings and volatility of cash flows.

This paper builds a model of precautionary demand for cash holdings that is relevant for both financially constrained and unconstrained firms. Because costly default, the only friction this paper is relying on to break the M&M type of irrelevance of liquidity policy, is arguably faced by all firms. Firms are hedging the volatility in the terms of borrowing because default costs introduce concavity in the value function of the firm.

This paper contributes to the stream of literature investigating the effect of financial frictions on corporate policy in a structural setting (Hennessy and Whited [2007], Moyen [2004], Gomes and Schmid [2010], Kuehn and Schmid [2011], among others). Most of these papers feature unrestricted access to capital markets but do not model cash and debt policies simultaneously. That is, there is no value for holding cash and issuing debt at the same time. In contrast, this paper is extending otherwise standard neoclassical model of the firm by introducing two period (Long-Term) debt in

¹Appendix 1.8 demonstrates that observed cash and debt co-existence is mostly driven by long-term debt rather than short-term debt. Figure 1.9 illustrates that cash and debt coexist in a sample for a particular year as well as for a pool of 1985-2010 data points. From Table 1.1 we can see that the proportion of firms with high levels of both cash and debt was consistently high in the past quarter-century.

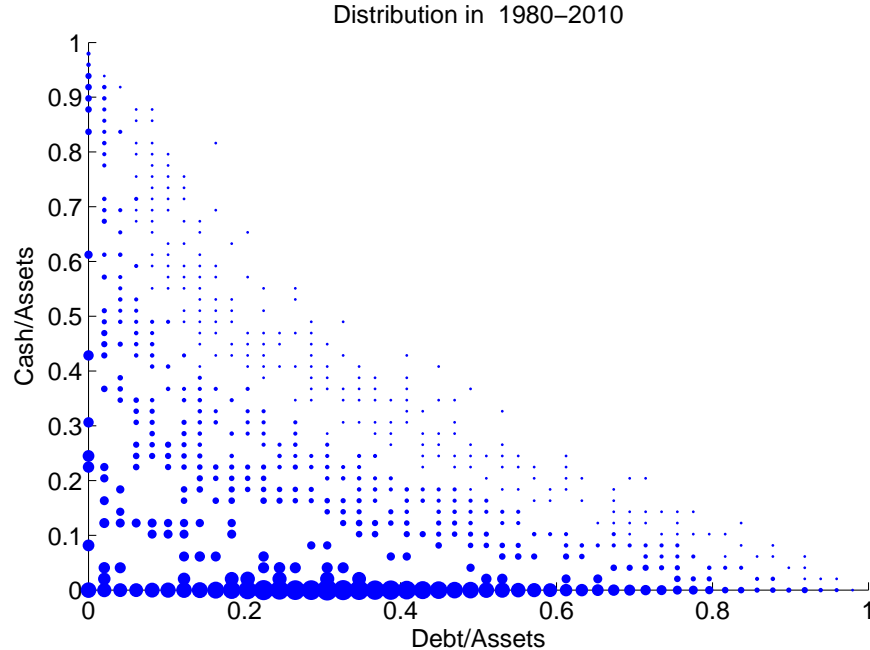


Figure 1.1: Empirical distribution of Cash and Leverage ratios. Data is taken from Compustat, it is a panel of firms in 1980-2010 excluding financial and utility companies.

addition to rolled-over one period debt, that is standard in the literature. Costly default makes the firm not indifferent between borrowing with LT debt and rolling over ST debt; the firm also might find it optimal to borrow with LT debt today and keep proceeds in cash for one period if there is debt maturing next period. In other words, by extending the maturity choice available to the firm, we introduce value to debt and cash co-existence.

Another stream of literature provided a distinct role for cash holdings by imposing collateral constraints on borrowing firms (Acharya, Almeida, and Campello [2007], Almeida, Campello, and Weishbach [2004], Han and Qiu [2007], Bolton et al. [2011], Acharya, Davydenko, and Strebulaev [2011]). In these models firms effectively have restricted access to financial markets and therefore, resemble smaller firms in data - those that are more likely to run into collateral constraints. This paper contributes to this literature by analyzing the precautionary demand for cash holdings for financially-unconstrained firm, or the firm that has unrestricted access to financial markets (the firm can always undertake positive NPV investment), in data — bigger firms that are more likely to pay dividends.

This paper is also related to the work of Riddick and Whited [2009] that study determinants of optimal cash saving and holding policy in a structural setting for an all equity financed firm. The focus of current work is on the joint borrowing through debt and liquidity management through cash holdings, while the question of optimal leverage ratio is left for further research. Gamba and Triantis [2008] investigate the value of liquidity policy given, among other frictions, the fixed cost of issuing debt. Cash and debt co-existence in the current period has value to the firm as it allows to minimize issuance costs in the future. In this paper, we directly investigate how the current state of the firm influences its cost of borrowing with debt. That is, firms face heterogenous costs of borrowing (firm-specific credit spread), conditional on current level of profitability shock and liquidity position. That allows us to investigate the optimal maturity choice of the firm. In other words, we open ‘black box’ of debt issuance costs that help Gamba and Triantis [2008] match average leverage and cash ratios empirically.

The model presented in this paper delivers coexistence of debt and cash holdings as a result of optimal financing policy for financially unconstrained firm.² The only friction that firms face is costly default. The model predicts that firms have an incentive to hedge the risk of the arrival of news and they use cash holdings to do so. Firms anticipate an investment to be made at an intermediate date. They prefer to borrow immediately and avoid exposure to risk in the terms of borrowing, which is derived from the risk of news arrival. In the static version of the model, firms hold cash on their balance sheets from date 0 to date 1 and have debt outstanding.

In the dynamic version of the model, pre-borrowing motive manifests in positive contemporaneous correlation between new debt issuances and cash holdings. Firms exploit benefits of the tax shield of debt by rolling-over two period debt, but not every two periods, as you might expect. Most firms borrow to re-finance the debt before it matures and keep proceeds in cash.

It is worth noting that this model establishes the importance of a liquidity policy for a financially unconstrained firm. This is in contrast to the widely held view that firms use cash holdings to overcome financial constraints. For example, Almeida, Campello, and Weishbach [2004] write:

If a firm has unrestricted access to external capital — that is, if a firm is financially unconstrained — there is no need to safeguard against future investment needs and

²It is financially unconstrained in the sense that it can always fund a positive NPV investment.

corporate liquidity becomes irrelevant.

In our model the objective function of the firm is concave with respect to the level of the signal about future prospects, which induces risk-averse type of behavior. The firm has unrestricted access to capital markets both prior to investment and at the date of the investment opportunity. It prefers to borrow earlier rather than later because borrowing earlier helps it to avoid exposure to the news risk.³

The key risk in this paper is news arrival. Firms face an endogenous default boundary that depends on a random signal about the future prospects of the company. In a frictionless world, firms are risk neutral with respect to that risk. The presence of default costs induces a value maximizing firm to minimize the probability of default. The firm's objective function becomes concave in the risk of default boundary. Companies optimally choose to minimize this exposure through borrowing before the arrival of the signal, thereby avoiding randomness in the default boundary. Hence, firms are acting in a risk-averse manner due to the presence of default costs, but are not directly hedging the event of default.

Froot, Scharfstein, and Stein [1993] established a somewhat similar result with respect to the risk of cash flows — they show that expected default costs are convex in cash inflow; hence, a firm would be interested in hedging it. They argue that this generates a demand for external hedging instruments by firms. Unlike cash flow, an easily verifiable variable, news is harder to verify and hence to contract upon. Therefore, it may be harder for firms to hedge news arrival with external instruments. In our paper we show first that news arrival in general is a risk that firms are willing to hedge. Second, since external hedging instruments are not readily available, we show how internal instruments (early issue of debt and cash holdings) can be used to hedge the risk of the news signal. In other words, Froot, Scharfstein, and Stein [1993] showed that firms should be hedging the volatility in the amount of external borrowing. In contrast, in our paper we show that firms should also be hedging the volatility in the terms of borrowing.

We provide empirical evidence that supports our idea. In a regression that uses controls from Bates, Kahle, and Stulz [2009] we show that the estimate of volatility in firm's market to book ratio

³In that case interest rate paid on debt is not subject to change due to news arrival.

has strong positive impact on cash holdings (see Table 1.8) for big and dividend paying firms. It is natural to think of market to book ratio as an indicator of future prospects of the firm. Hence, the variance of the market to book ratio is a good proxy for the volatility in the news about future prospects of the firm. According to our model the more volatile the signal is, the more incentives the firm has to hedge it. Therefore, the more cash holdings it will have. Data supports that claim — volatility of market to book ratio has positive impact on cash holdings.

We use solution of the dynamic model to simulate a panel of firms and show that most of simulated moments resemble that of empirical data. For example, cash holding ratios are highly persistent — 0.53 (in data) and 0.62 in simulations . Leverage ratios are persistent as well (in simulations 0.97, 0.43 in data) consistent with the idea of 'hysteresis' (see Lemmon, Roberts, and Zender [2008]). Net debt issuance is positively related to cash holdings.⁴

The paper is organized in the following way: static model establishes the pre-borrowing motive in 3-period setting. Building of that result, dynamic model shows implications for introducing two period debt in the otherwise standard neo-classical model of the firm (without investment).

1.2 Static Model

We now consider a risk-neutral firm in 3-period setting that is choosing between short and long term financing for an investment project.

1.2.1 Motive for Hedging

The firm lives for 2 periods. An investment opportunity occurs at date 1. The firm has to pay I out of either internal or external funds or both. It also has some assets in place. A firm's payoff in period 2 would be $G + v$, where G is a constant and $v \in [0, \infty)$ follows distribution with the cumulative distribution function $K(v)$. Default costs are fixed at ξ .⁵ The risk-free interest rate is normalized to be zero.

⁴Cash balances, conditional on the firm characteristics - cash flow volatility, market to book ratio, size, cash flow level and leverage ratio, are positively related to the net debt issuances with correlation coefficient of 10.6% contemporaneously and with coefficient of 3.42% with one lag.

⁵Support for v starts at 0. Hence, if the worst possible scenario happens, bond holders would receive $G - \xi$, which is assumed to be non-negative WLOG. That is the only purpose of having constant G in the final payoff.

The investment amount is assumed to be fixed to avoid the discussion of Myers [1977] debt-overhang problem. The hedging motive that arises due to endogenous default boundary in this model is different from the debt-overhang problem. As will soon be clear, the costs of default and the endogenous default boundary make both the total value of the firm and shareholder value as of period 1 concave in the news signal, thereby creating a hedging motive. Hence, the hedging motive is not related to the conflict of interest between shareholders and bondholders, as the debt-overhang problem is.

At date 1, the firm receives a signal λ about its future prospects. The updated distribution function for v becomes $F(\lambda, v)$, such that:

$$\int_{\underline{\lambda}}^{\bar{\lambda}} F(\lambda, v)g(\lambda) d\lambda = K(v), \quad (1.1)$$

where $g(\lambda)$ is the marginal distribution of λ , the information signal. Also, assume that $F_{\lambda} < 0$, $F_{\lambda\lambda} > 0$. That is, with better news, we expect that the probability that v is below some number a is a decreasing with decreasing marginal effect.

At date 1, the firm has access to an outside capital market that has infinitely many risk neutral price taking lenders. Firm is issuing unsecured bonds that are payable at date 2. The interest rate at which lenders are willing to give money to the firm is determined by setting the expected profit of lenders to zero. Since the firm has no internal funds carried from date 0, it needs to borrow the total amount of investment I at date 1. Assume that it can do so for any realization of the news signal. That is, there always exist an $r \geq 0$ such that the expected promised payoff to bond holders is at least I . Then interest rate $c(\lambda)$, is endogenously determined by:

$$I(1+c)\left[1 - F\left(\lambda, I(1+c) - G\right)\right] + \int_0^{I(1+c)-G} (x+G-\xi)f(\lambda, x) dx = I, \quad (1.2)$$

where $f(\lambda, v)$ is the marginal distribution of payoff v conditional on information λ .

The first term in 1.2 is the bond's face value times the probability of no default. The second term is the expected payment to bond holders in case of default. Hence, the interest rate sets expected payments on the bond equal to the proceeds from issuing it.

It is intuitively clear that the better the news, the better the prospects of the company, hence, the more likely the firm to repay all of its obligations in period 2. Hence, the coupon payment required by the bond market is smaller. The next lemma states this formally.

Lemma 1.2.1. *If $F(\lambda, v)$ satisfies $F_\lambda < 0$, then $c_\lambda < 0$.*

Proof can be found in the appendix.

The total expected value of the firm at $t = 1$, contingent on the realization of the signal is

$$V_1(\lambda) = E\left(G + v - \xi 1_{\{G+v \leq I(1+c(\lambda))\}} \mid \lambda\right) \quad (1.3)$$

The expectation is taken with respect to F_λ . We can re-write 1.3 to be

$$V_1(\lambda) = E(G + v \mid \lambda) - \xi F(\lambda, I(1 + c(\lambda)) - G)$$

If we now look at the value of the firm from the perspective of period 0 we will see that the dependence on news signal is integrated out of the first term in a risk-neutral way, while the second term depends on news signal non-linearly. The next proposition establishes that default costs are a convex function of the news signal.

Proposition 1.2.2. *Define $C(\lambda) = \xi F(\lambda, I(1 + c(\lambda)) - G)$. If $F(\lambda, v)$ satisfies*

$$\left(\frac{f(\lambda, v)}{1 - F(\lambda, v)}\right)_\lambda < 0 \quad (i)$$

and

$$\left(\frac{f(\lambda, v)}{1 - F(\lambda, v)}\right)_v > 0 \quad (ii)$$

then $C(\lambda)$ is a convex function.

The conditions imposed on the cumulative distribution function are fairly mild.⁶ Condition (i)

⁶Exponential distribution with mean parameter λ satisfies it. I was also able to show numerically that normal distribution with mean λ also satisfies it.

can be interpreted as decreasing marginal probability to be at the default boundary given that there is no default. It is natural to think that the better the news, the less likely the firm is to end up close to the default boundary if it has not defaulted. Condition (ii), as discussed by Froot et al. [1993], is satisfied by a wide class of distributions.

The convexity of the expected default costs $C(\lambda)$ with respect to the news signal makes the total value of the firm $V_1(\lambda)$ a concave function of the signal. Hence, the firm has an incentive to hedge the risk of news arrival.

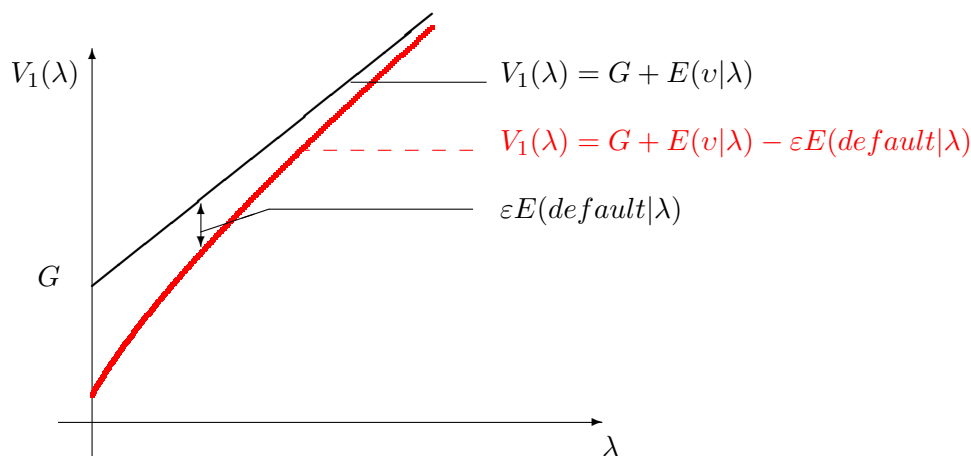


Figure 1.2: Value function of the firm. Default costs introduce a "penalty" $\varepsilon E(\text{default}|\lambda)$ to the firm valuation that is decreasing in the news signal. Hence, if we start with value of the firm that is linear in the news signal, the introduction of default costs brings concavity to the value function. Hence, firms have incentives to behave as if they are risk averse.

The convexity of cost function has a simple intuitive explanation — the better the news, the lower the expected default costs, since the company is expected to perform better. However, the marginal decrease in expected default costs is smaller with each additional piece of good news. After all, even if the news is extremely good, there is still a chance that the firm will default on its obligations so that costs never fall down to zero. That implies that the gap between the value function with and without costly default is decreasing in the news signal (see figure 1.2).

The proposition above establishes that if the firm is borrowing at the moment when it needs funds for investment, the firm is subject to the volatility in terms of borrowing (as a result of news

arrival). And since the value of the firm is concave in the news signal, the firm is interested in hedging that risk. In the next section, we will show how pre-borrowing (that is, borrowing before the news arrival), and using hoarded cash to finance the investment in period 1 reduces the exposure to the risk in terms of borrowing.

1.2.2 Pre-borrowing policy

Now let us allow the firm to choose when to borrow — at date 0 before the realization of the signal (and hence on average terms), or at date 1 after the news are revealed (and hence subject to the volatility in terms of borrowing). The face value of debt issued at date 1 is denoted by B (that is, principal plus the interest payable at date 2 is equal to B). Let D stand for the face value of debt issued at period 0. The firm is allowed to chose any combination of D and B that will deliver it enough liquid assets to pay I at date 1.

Note that B-bonds holders are assumed to have junior priority in asset distribution in case of default. The assumption of seniority of debt claims that were issued earlier is not the one driving our results. It is made for the purpose of tractability only. The same intuition goes through with say reversed priority — debt holders at period zero predict the optimal behavior by the firm for each realization of λ and incorporate that into their pricing decision. Hence, they are still making decision based on average terms and the firm is not exposed to the risk in the news signal if it decides to borrow at period 0 the whole amount of investment.

If the firm decides to borrow funds at date 0 before the news is revealed, it promises to pay an interest rate that does not depend on the news. Denote by $\delta(D)$ the proceeds from the issue of that debt:

$$\delta(D) = E^\lambda \left(\int_0^{D-G+\xi} (v + G - \xi) f(\lambda, v) dv + D \left(1 - F(\lambda, D - G + \xi) \right) \right) \quad (1.4)$$

The first term in 1.4 represents the expected value of partial recovery when the firm declares default. The second term is nominal value times the probability of no default.⁷ The firm keeps

⁷Please note that upper bound of integration in the first term in 1.4 assumes that there is some junior debt outstanding ($B > 0$) so that when $G + v = D$, default is still triggered.

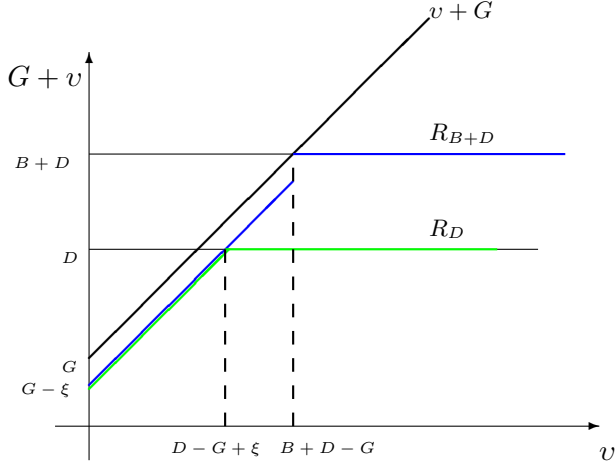


Figure 1.3: The figure above illustrates cash flows to holders of senior (D) and junior (B) bonds. $v + G$ stands for the total realized value of the firm. If that is above the total debt outstanding, then both bonds are paid in full. In case $v + G < D + B$, default is triggered and ξ is lost. Bond holders are left with $v + G - \xi$ that is first allocated to senior bond holders and the remainder, if any, goes to holders of B bonds. R_D stands for recover (full or partial) of senior debt and R_{B+D} stands for recovery of all bond holders.

$\delta(D)$ until date 1 when investment needs arise. Then it will have to borrow the remaining amount at period 1 so that the total sum of proceeds from both issues of debt is equal to investment expenditure. Denote by $P(\lambda, B)$ the proceeds from issuing debt at date 1:

$$P(\lambda, B) = \int_{D-G+\xi}^{D+B-G} (v + G - \xi - D) f(\lambda, v) dv + B[1 - F(\lambda, D + B - G)]. \quad (1.5)$$

Then we know that D and B are chosen so that the firm has enough resources to make the investment:

$$\delta(D) + P(\lambda, B) = I \quad (1.6)$$

The interest rate promised on the debt issued in period 1 depends on the news signal, and hence, the face value of debt is a function of the news signal: $B(\lambda)$. It is implicitly defined by 1.6.

The total value of the firm as of period 0 can be written as

$$V = E^\lambda \left(E \left(G + v - \xi 1_{\{G+v < B(\lambda)+D\}} | \lambda \right) \right) \quad (1.7)$$

The outer expectation is taken with respect to news signal distribution. We can re-write 1.7 to be:

$$V = E(G + v) - \xi E^\lambda(F(\lambda, B(\lambda) + D - G))$$

The next proposition shows that when the firm decides to borrow marginally more in period 0 and therefore can borrow marginally less after the news arrival, the company is expected as of date 0 to default with a smaller probability. That is, the company is better off borrowing earlier rather than later if it wants to save expected default costs.

Proposition 1.2.3. *If $F(\lambda)$ satisfies*

$$\left(\frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)_\lambda < 0 \tag{i}$$

and

$$\left(\frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)_v > 0 \tag{ii}$$

then $V_D > 0$ for D s.t. $\delta(D) < I$.

In other words, if currently the company's policy is to borrow with both long-term (D) and short-term (B) debt, the firm can increase its value by borrowing more at date 0 with long-term debt (D) and, therefore, less at date 1 with short-term debt (B).

The firm's decision to borrow slightly more before the news arrival rather than after reduces the firm's exposure to risk of the news signal. We have already seen that default costs are convex in the news signal, hence, the firm is better off hedging this risk. The risk hedging is delivered by borrowing earlier, that is in period 0 rather than in period 1.

When the firm borrows early, it holds proceeds in the form of cash from the moment of debt issue until investment needs arise. That is, we observe outstanding debt and cash holdings on the balance sheets of firms at the same time. Both cash and debt balance serve the same goal - finance investment expenditure. While debt was issued to raise sufficient funds, cash is used to transfer resources to the date of investment. This financing structure allows firms to lock in the interest rate.

Proposition 1.2.3 merely establishes the incentive for firms to issue debt earlier rather than later. In a more realistic environment, firms would balance this incentive against interest expense paid on cash funds (interest-dominated asset) for a period before investment. Firms might also consider equity issuance as a source of funding the investment needs. This will influence optimal debt issuance policy of the firm. However, as we will see in dynamic version of the model, as long as firms have an incentive to finance expenditures (outflows)⁸ with debt (tax shield, etc), firms will also have an incentive to borrow earlier to avoid exposure to the risk of news arrival.

1.3 Dynamic Model

We now consider implications of pre-borrowing motive for standard neo-classical firm that has a choice between short and long term debt financing. In every period the firm can issue both one and two period debt. That is, the environment is set up as if 3-period models are overlapping instead of being stacked next to each other. Every period there are long term and short term debt maturing and long term debt outstanding (that will mature in the next period). Hence, at the same time there are 3 types of debt. The firm in this setting is not restricted to only debt financing and can issue equity.

1.3.1 Environment

Consider a firm that has 1 unit of capital. Its idiosyncratic productivity shock, z_t follows an log-AR(1) process with correlation parameter ρ_z and variance of innovations σ_z^2 .

Denote by LB_{t+1} proceeds from issue of debt that is to be repaid in the future period. If c_{t+1}^B is the coupon rate, then in the next period creditors will get $(1 + c_{t+1}^B)LB_{t+1}$. Denote

$$B_{t+1} = (1 + c_{t+1}^B)LB_{t+1}.$$

The amount of debt B_t can be negative. In that case we can think of B_t representing cash carried

⁸Dynamic version of the model does not have investment choice for the sake of computational feasibility. The pre-borrowing idea though applies to any sort of expense or outflow that is expected to happen. For example, firms that roll-over debt need funds to refinance the debt and, hence, would have an incentive to borrow one period before the debt maturity and keep proceeds in cash.

over from the last period.

In the same way, let LD_{t+1} stand for the proceeds from issuing debt that will be payable two periods from now. The coupon on 2-period debt is due only at the maturity. So, at $t + 2$ creditors will be paid $(1 + c_{t+1}^D)LD_{t+1}$.

Equity issuance is not allowed in the model. The only source of external financing available to shareholders is debt. The focus of this work is on the optimal maturity and liquidity structure, and not on the trade-offs between equity and debt financing. Pecking order theory can be thought as mostly orthogonal to the trade-offs the firm is facing in this environment. In other words, the current model is trying to answer the question, given that firm finds it optimal, for example, to be 70% equity finance, how should it structure its debt and how much cash should it hold each period.

In case shareholders decide to declare default, assets that are in the firm are sold with a discount ε , the bankruptcy cost.

1.3.2 Equity Problem

Firm is entering the period with operating cash inflow of z_t . It has long term debt outstanding D_t and liquid assets LA_t , which are the cash carried from previous period (if any) less maturing long term debt $-B_t - D_{t-1}$. If the firm decides to continue operations, it needs to make an investment to cover for the depreciated capital. Equity distributions can be written as:

$$Eq_t = z_t - f + LB_{t+1} + LD_{t+1} + LA_t \geq 0 \quad (1.8)$$

where f is fixed per-period cost of operation.

The firm has inflow of operating profit and proceeds from issue of short and long term debt. These funds are allocated to repayment of maturing debt (if any), saving and the rest is distributed to shareholders.

Each period shareholders choose the financing structure of firms operation: how much long term debt to issue and either borrow in short term debt or save some cash for the next period. Formally, the problem can be written as:

$$V(z_t, D_t, LA_t) = \begin{cases} 0, & (1.9a) \\ (z_t - f + 1 + LA_t - \beta D_t)(1 - \varepsilon), & (1.9b) \\ \max_{\substack{B_{t+1}, D_{t+1} \geq 0 \\ \text{s.t. } Eq_t \geq 0}} \{Eq_t + \beta E(V(z_{t+1}, D_{t+1}, LA_{t+1}))\} & (1.9c) \end{cases}$$

where liquid assets within the firm next period are $LA_{t+1} = -B_{t+1} - D_t$ and β stands for time discount factor.

Shareholders apart from continuing operation (1.9c), can default on their debt obligations and walk away (1.9a). In addition to that, shareholders might decide to liquidate the firm (1.9b). That option allows shareholders to sell physical assets, repay the debt holders and walk away from the firm with what is left. This decision might be optimal even for an all equity financed firm. The firm will find it optimal to liquidate when expected future operating cash flows are less than the costs of operation.

Gomes [2001] incorporates liquidation decision explicitly in the choice set for all equity financed firms. Many papers that are dealing with defaultable debt in dynamic setting abstract from fixed costs of operation (for example, Hennessy and Whited [2007]).⁹ Some papers (Gomes and Schmid [2010], Kuehn and Schmid [2011]) that have both risky debt and fixed costs of operation, do not explicitly account for liquidation option for the firm. If shareholders are not allowed to sell assets at t but the expected cash inflow from operation is smaller than operating costs, shareholders might decide to sell assets to investors through issuing too much debt. Too much in that case would mean that shareholders guarantee default in $t + 1$. That behavior contradicts U.S. bond regulations and hence biases simulated results towards issuing too much debt.

1.3.3 Bond pricing

Recovery in case of default consists of operating cash inflow, proceeds from sale of physical assets and cash if any.

⁹ Firms that do not face fixed costs of operation would not find it optimal to liquidate because their operating profit is always positive and the only reason to stop operation would be debt overhang.

$$R(z_{t+1}) = (z_{t+1} - f + 1 - B_{t+1}\mathbf{1}_{B_{t+1}<0})(1 - \varepsilon).$$

Those funds are allocated to the three existing groups of bond holders. The earliest issued long term debt is assumed to have the highest priority.

- long term bond holders receive

$$R_1(z_{t+1}, D_t) = \min(R(z_{t+1}), D_t);$$

- long term recent bond holders receive

$$R_2(z_{t+1}, D_t, D_{t+1}) = \min[\max(0, R(z_{t+1}) - D_t), \beta D_{t+1}];$$

- short term bond holders essentially receive anything that was left after paying the two groups of long term bond holders -

$$R_3(z_{t+1}, D_{t+1}, D_t, B_{t+1}) = \min[\max(0, R(z_{t+1}) - D_t - \beta D_{t+1}), B_{t+1}].$$

Risk-neutral competitive lenders decide on price of short and long term debt by guaranteeing zero profit from lending:

$$LB_{t+1} = \beta \left[(1 + c_{t+1}^B) LB_{t+1} E_t(\mathbf{1}_{V_{t+1}>0}) + E_t(R_{3,t+1} \mathbf{1}_{V_{t+1}=0}) \right] \quad (1.10)$$

$$\begin{aligned} LD_{t+1} = & \beta^2 \left[(1 + c_{t+1}^D) LD_{t+1} E_t(\mathbf{1}_{V_{t+1}>0 \& V_{t+2}>0}) \right. \\ & \left. + E_t(R_{1,t+2} \mathbf{1}_{V_{t+1}>0} \mathbf{1}_{V_{t+2}=0}) + \frac{1}{\beta} E_t(R_{2,t+1} \mathbf{1}_{V_{t+1}=0}) \right]; \end{aligned} \quad (1.11)$$

Denote $(1 + c_{t+1}^D) LD_{t+1}$ by D_{t+1} and $(1 + c_{t+1}^B) LB_{t+1}$ by B_{t+1} , then we can re-write expression above as:

$$LB_{t+1} = \beta \left[B_{t+1} E_t(\mathbf{1}_{V_{t+1}>0}) + E_t(R_{3,t+1} \mathbf{1}_{V_{t+1}=0}) \right] \quad (1.12)$$

and

$$LD_{t+1} = \beta^2 \left[D_{t+1} E_t(\mathbf{1}_{V_{t+1}>0} \mathbf{1}_{V_{t+2}>0}) + E_t(R_{1,t+2} \mathbf{1}_{V_{t+1}>0} \mathbf{1}_{V_{t+2}=0}) + \frac{1}{\beta} E_t(R_{2,t+1} \mathbf{1}_{V_{t+1}=0}) \right]; \quad (1.13)$$

As have been pointed out by Gomes and Schmid [2010], working with face value as a choice variable (B_{t+1} and D_{t+1}) and evaluating market value of the two debts has computational advantage over specifying coupon schedule c_{t+1}^B and c_{t+1}^D explicitly. In the latter case the procedure should have included outer loop for convergence of coupon schedule on top of the inner loop for value function convergence. In the current specification, in order to price the debt we need only to evaluate two functions.

1.3.4 Calibration

The model is solved using value function iterations. Grid for z_t has 15 points and its dynamics is approximated using Tauchen [1986] method. Grid for LA_t has 41 points while D_t has 11 grid points. For each value function iteration, short and long term debt is priced fairly to obtain firm's optimal choice functions. Pricing of debt is done through evaluation of functions summarized in 1.12 and 1.13. Hence, there is only one convergence loop for the value function of shareholders.

The model is calibrated at the annual frequency. I have estimated AR(1) model for $\log(\text{Sales}/\text{Assets})$ for US COMPUSTAT non-financial, non-utilities firms on 1985-2010 data. The estimates are 0.501 for persistence parameter and 0.16 for standard deviation of innovations. The size of per-period operating costs was chosen to match average value of profitability (3.5% of total assets for US COMPUSTAT non-financial, non-utilities firms on 1985-2010 data).

Risk-free interest rate is set at 3% annual, which implies discount factor of approximately 0.97, smaller than 0.98 used by Kuehn and Schmid [2011] but larger than Bhamra, Fisher, and Kuehn [2011]'s 0.96 and Gomes [2001]'s 0.939. The default costs are the same as liquidation costs and are

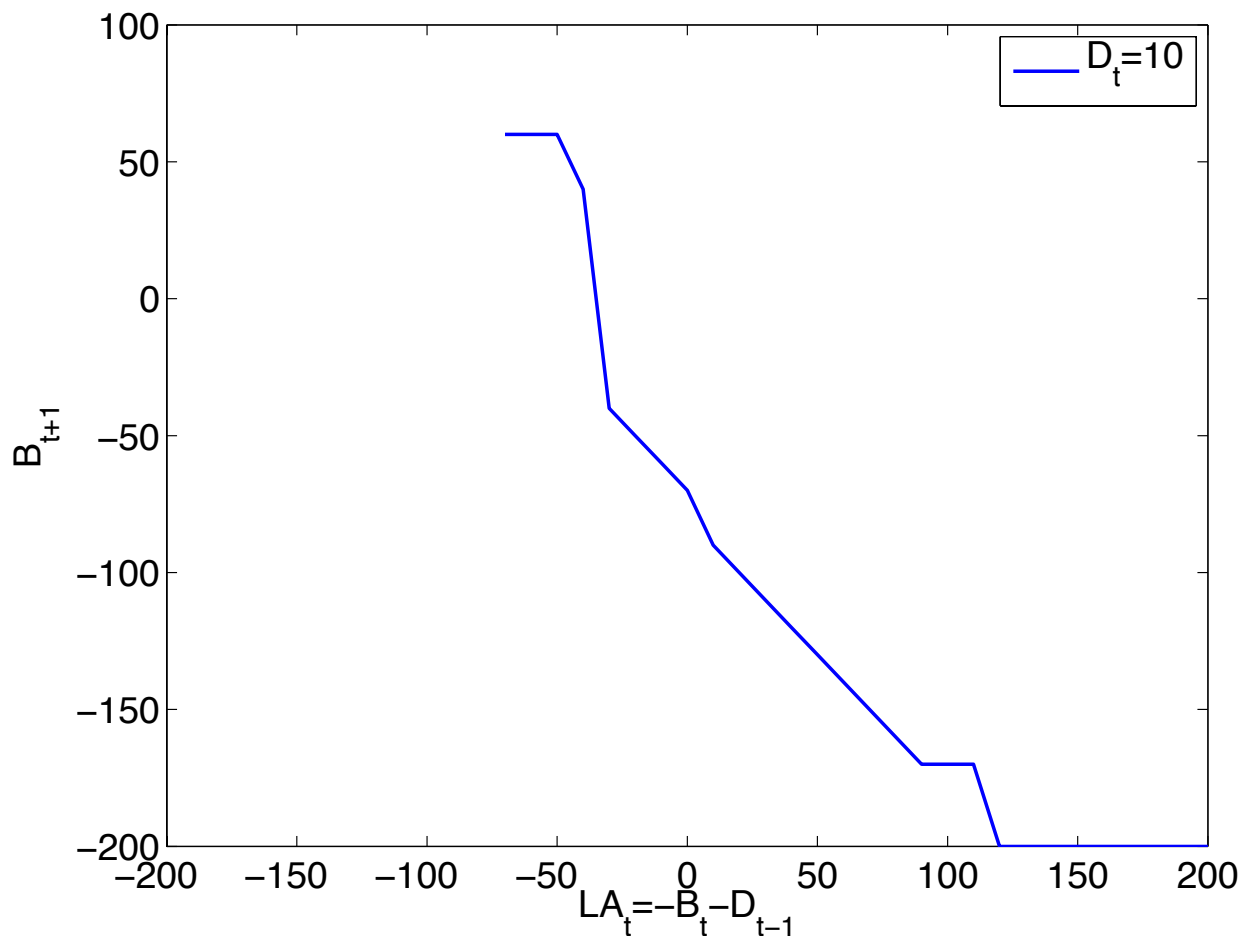


Figure 1.4: Optimal cash holdings as a function of liquid assets within the firm. The more cash the firm has at the beginning of the period, the more it saves for the next period.

set at 35% of asset value.

1.3.5 Optimal financing choice

The firm decides on debt and liquidity policy simultaneously. As diagram 1.4 illustrates, firms save more if it enters the period with more liquid assets, which is consistent with intuition that cash-rich firms carry the cushion from period to period. Firms find it optimal to decrease cash savings if the debt outstanding is too large — foreseeing the likely default next period, shareholders take cash out of the company in the current period (see Figure 1.5).

A pre-borrowing motive is illustrated in Figure 1.6. At low levels of debt outstanding, the

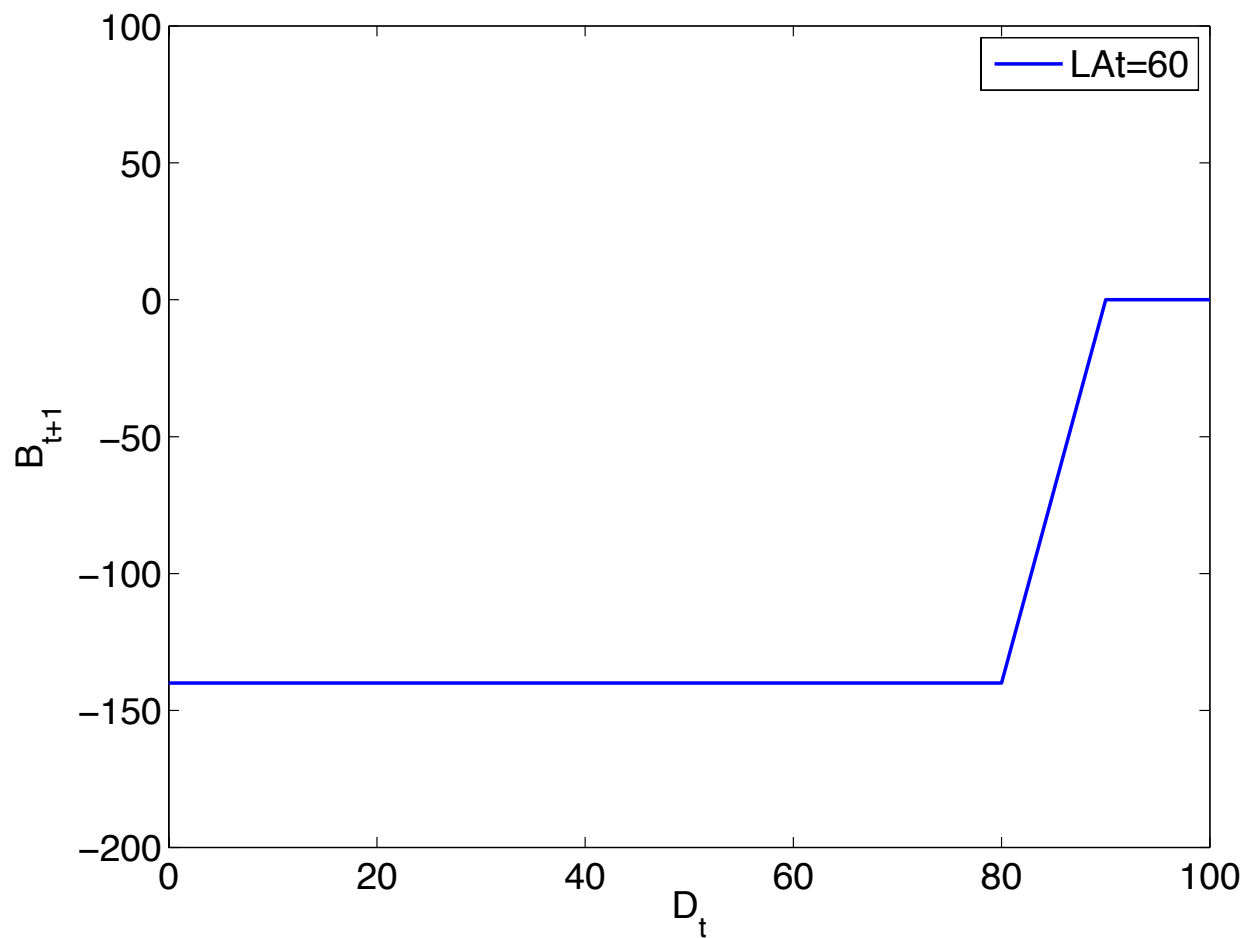


Figure 1.5: Optimal cash holdings as a function of debt outstanding. The more debt the firm has outstanding, the larger the probability it will default next period. Hence, after a certain level, shareholders find it more optimal to take liquid funds out of the firm in the current period and save no cash.

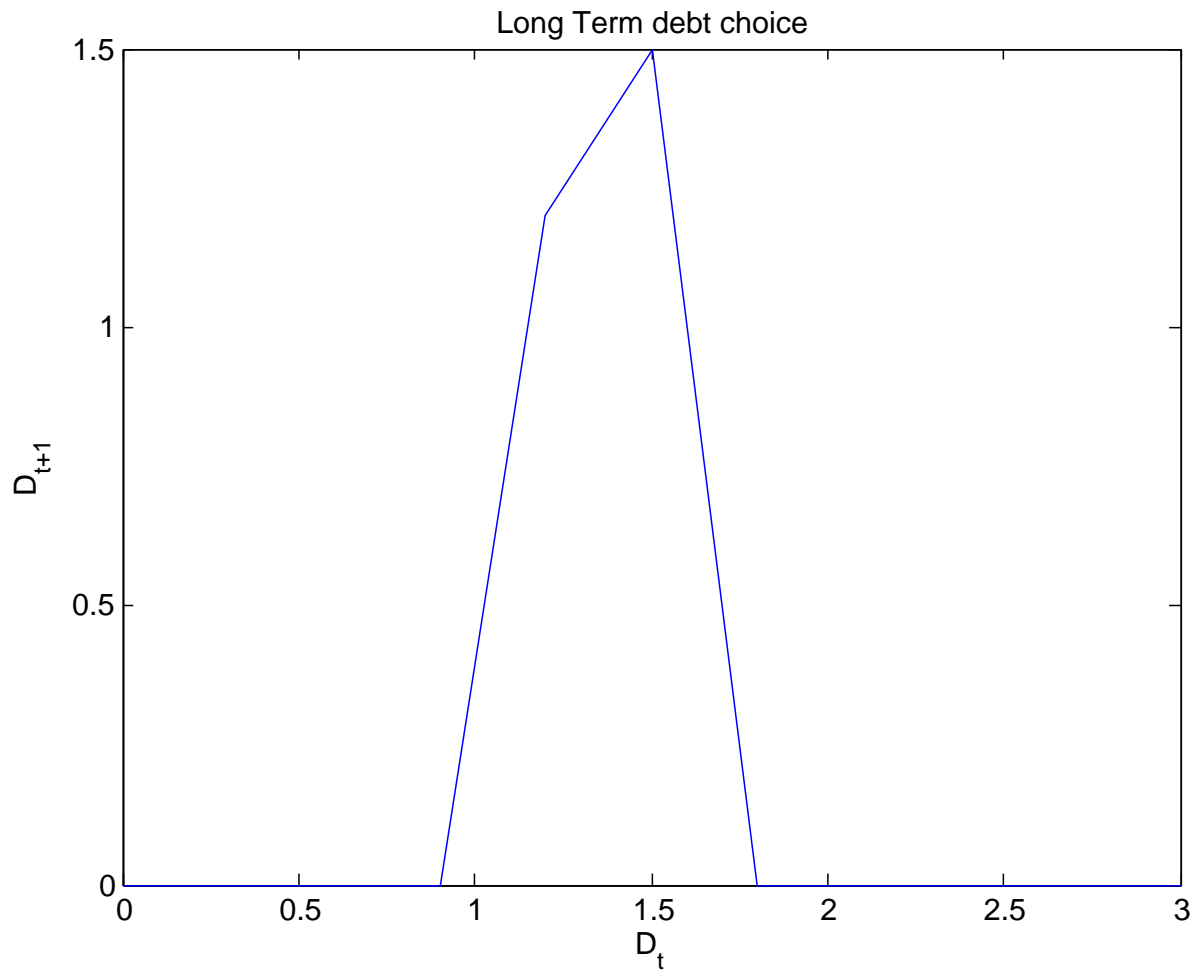


Figure 1.6: Optimal (Risky) LT borrowing.

firm has no need to borrow to cover re-payment of debt next period. As the level of long-term debt outstanding increases, firm decides to borrow more in the current period. If the level of debt maturing next period is too high though, the optimal policy is not to issue any debt and wait for the default in the next period.

The steady state distribution of firms over financing choices is summarized in Figure 1.7. As you can see, there is a mass of firms that chooses to issue long-term debt as well as save some positive amount of cash until the next period. This is consistent with observed empirical evidence — firms have significant cash holdings at the same time as having substantial amount of debt outstanding .

The model matches some of the empirical moments well. For example, cash holdings that

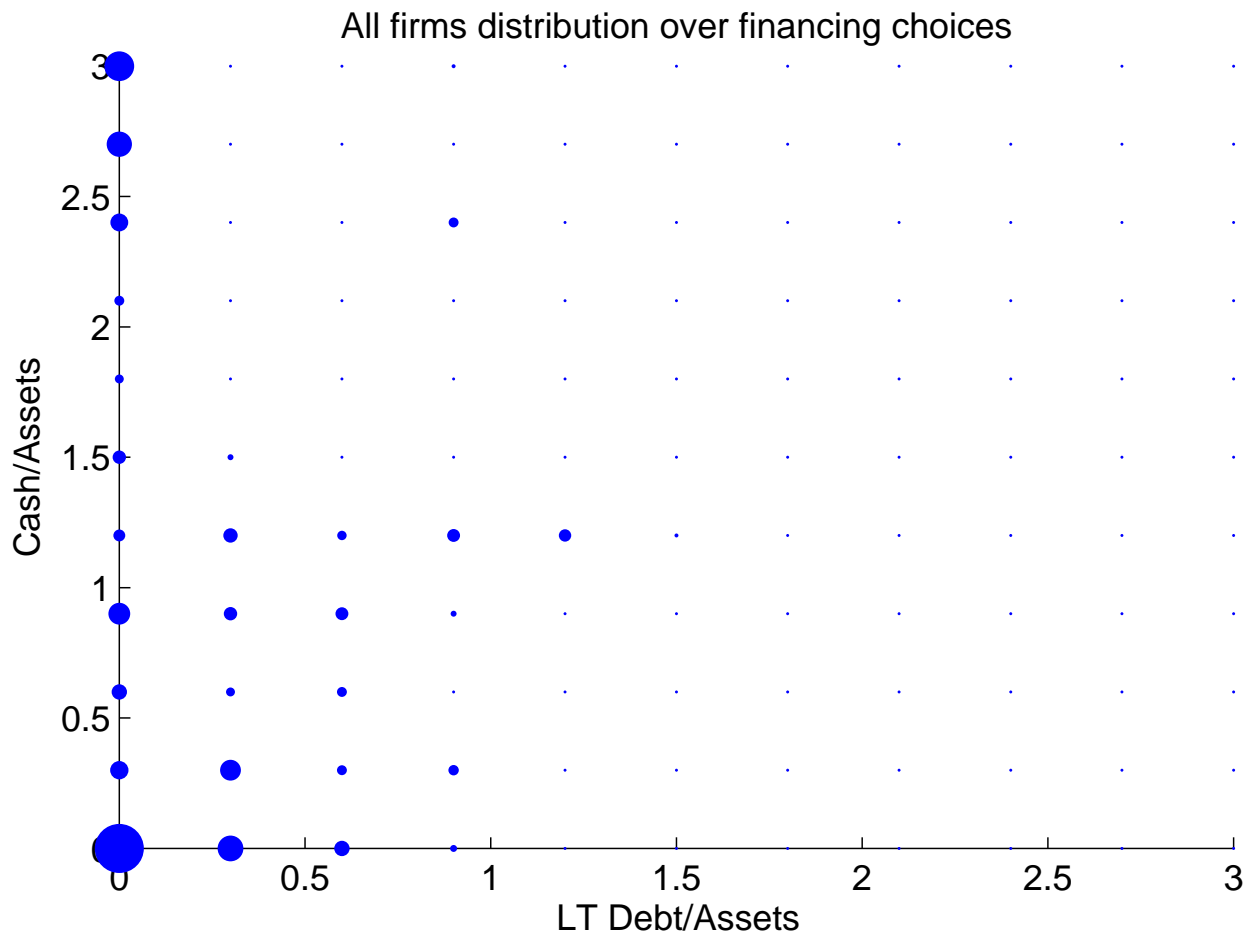


Figure 1.7: Distribution of Cash and Debt of a panel of simulated firms.

are as persistent as those observed in the data (0.62 and 0.53— estimated from a sample of US COMPUSTAT non-financial firms).

Simulated data also shows positive relationship between cash and new debt issuances, which is consistent with empirical finding by Bates, Kahle, and Stulz [2009] — the reported coefficient of new (long term) debt issuances in cash regressions is positive and statistically significant. This finding provides support for the idea that firms keep at least a part of the proceeds from debt issue in cash.

The current parameter calibration results in leverage ratios that are too high. This might be partly due to the fact that in the model the firm has access to debt of maturity up to 2 years. In practice, the array of available maturities is richer. In the current specification, the firm is issuing long term debt every period. If the model did allow for say a 5 period debt then the firm might have found it optimal to borrow say 1 or 2 periods before the maturity, not 4 periods. Therefore, with 5 year debt available there would be firms that have only one type of debt outstanding while in the current model the firm finds it optimal to have two issues of the long term debt.

Model produces very persistent leverage ratios (with correlation coefficient above 90%). That is consistent with the idea of 'hysteresis' discussed by Gomes and Schmid [2010].

1.4 Conclusions

This paper explains why financial policy can be important for a financially unconstrained firm. It also offers a precautionary explanation for coexistence of debt and cash on the balance sheets of U.S. firms.

Presence of default costs provides incentives for the firm to minimize the likelihood of default. The chance that the firm will default is a convex function of the news signal. Therefore, the firm is better off hedging the news risk. The financial policy that delivers hedging can be summarized in the following way: borrow early before the arrival of the news signal and hoard cash. When the investment needs arise finance them with the internal funds (cash). This policy allows firms to lock in the interest rate and avoid the exposure to the volatility in the terms of borrowing.

This pre-borrowing motive helps explain large levels of cash holdings observed empirically. A

calibrated version of the model produces mean cash to asset ratios that are comparable with those observed in the data. It also reproduces the positive relationship between debt issuances and cash holdings that was documented by various empirical studies.

APPENDIX

1.5 Lemma 1.2.1

Proof. For a given λ , $c(\lambda)$ is defined endogenously by the following equation:

$$I(1+c) \left[1 - F(\lambda, I(1+c) - G) \right] + \int_0^{I(1+c)-G} (x+G-\xi) f(\lambda, x) dx = H(c, \lambda) = I \quad (1.14)$$

Since the firm is assumed capable of borrowing for any realization λ , equation 1.14 has at least one solution. Depending on the particular shape of $F(\lambda, \cdot)$ the equation can be satisfied for more than single c . In that case we define $c(\lambda)$ to be the minimal c that satisfies equation 1.14.

Before using the implicit function theorem to find the sign of c_λ , simplify $H(c, \lambda)$:

$$\begin{aligned} H(c, \lambda) &= I(1+c)[1 - F(\lambda, I(1+c) - G)] + (I(1+c) - \xi)F(\lambda, I(1+c) - G) \\ &\quad - \int_0^{I(1+c)-G} F(\lambda, x) dx \\ &= I(1+c) - \int_0^{I(1+c)-G} F(\lambda, x) dx - \xi F(\lambda, I(1+c) - G) \end{aligned} \quad (1.15)$$

Now we compute the derivatives of $H(c, \lambda)$:

$$\frac{\partial H(c, \lambda)}{\partial \lambda} = - \int_0^{I(1+c)-G} F_\lambda(\lambda, x) dx - \xi F_\lambda(\lambda, I(1+c) - G) > 0 \quad (1.16)$$

$$\frac{\partial H(c, \lambda)}{\partial c} = I - IF(\lambda, I(1+c) - G) - I\xi f(\lambda, I(1+c) - G) \quad (1.17)$$

$$= I[1 - F(\lambda, I(1+c) - G) - \xi f(\lambda, I(1+c) - G)] \quad (1.18)$$

Let's look closer at expression 1.18. It is the marginal change in the market value of debt due to infinitesimal increase in the promised interest on the debt, evaluated at the default boundary. If it was negative, then there was a smaller level of c that would satisfy equation 1.14. Intuitively, it is irrational for the firm to promise the interest rate that delivers $1.18 < 0$. The debt holders would have been better off getting a smaller promised payment since the firm would default with smaller

probability. Hence, the value maximizing firm operates only with expression in 1.18 positive.

Then

$$c_\lambda = -\frac{\frac{\partial H(c,\lambda)}{\partial \lambda}}{\frac{\partial H(c,\lambda)}{\partial c}} < 0 \quad (1.19)$$

□

1.6 Proposition 1.2.2

Proof. Costs of external borrowing are

$$C(\lambda) = \xi F\left(\lambda, I(1 + c(\lambda)) - G\right)$$

Interest rate is defined by the following bond market clearing condition

$$\int_0^{I(1+c)-G} (x + G - \xi) f(\lambda, x) dx + I(1 + c)[1 - F(\lambda, I(1 + c) - G)] = I \quad (1.20)$$

Integrating by parts we obtain an alternative expression for market value of debt:

$$I(1 + c) - \int_0^{I(1+c)-G} F(\lambda, x) dx - \xi F(\lambda, I(1 + c) - G) = I \quad (1.21)$$

$$[1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)]I dc + \left[- \int_0^{I(1+c)-G} F_\lambda(\lambda, x) dx - \xi F_\lambda(\lambda, \cdot) \right] d\lambda = 0 \quad (1.22)$$

where $F(\lambda, \cdot)$ stands for $F(\lambda, I(1 + c) - G)$.

$$c_\lambda = \frac{dc}{d\lambda} = \frac{1}{I} \frac{\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx + \xi F_\lambda(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} < 0 \quad (1.23)$$

Now go back to cost function

$$C_\lambda = \xi \left[f\left(\lambda, I(1 + c(\lambda)) - G\right) I c_\lambda + F_\lambda\left(\lambda, I(1 + c(\lambda)) - G\right) \right] \quad (1.24)$$

Plug in expression for derivative of interest rate with respect to news:

$$C_\lambda = \xi \left(\frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \left[\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx \right] + \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \right)$$

Now denote

$$\widetilde{HR} = \frac{f(\lambda, I(1+c(\lambda)) - G)}{1 - F(\lambda, I(1+c(\lambda)) - G) - \xi f(\lambda, I(1+c(\lambda)) - G)}$$

So

$$C_\lambda = \xi \left(\widetilde{HR} \left[\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx \right] + \widetilde{HR} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \right)$$

1.6.1 Hazard Rate

We will show now that $\widetilde{HR}_\lambda < 0$:

$$\begin{aligned} \frac{\partial \widetilde{HR}}{\partial \lambda} &= \frac{\partial}{\partial \lambda} \left(\frac{f(\lambda, I(1+c(\lambda)) - G)}{1 - F(\lambda, I(1+c(\lambda)) - G) - \xi f(\lambda, I(1+c(\lambda)) - G)} \right) \\ &= \frac{\partial}{\partial \lambda} \left(\frac{\frac{f(\lambda, I(1+c(\lambda)) - G)}{1 - F(\lambda, I(1+c(\lambda)) - G)}}{1 - \xi \frac{f(\lambda, I(1+c(\lambda)) - G)}{1 - F(\lambda, I(1+c(\lambda)) - G)}} \right) \\ &= \frac{\partial}{\partial \lambda} \left(\frac{HR}{1 - \xi HR} \right) \\ &= \frac{HR_\lambda (1 - \xi HR) + \xi HR_\lambda HR}{(1 - \xi HR)^2} \\ &= \frac{HR_\lambda}{(1 - \xi HR)^2} \\ &= \frac{1}{(1 - \xi HR)^2} \left[\left(\frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)_\lambda + \left(\frac{f(\lambda, x)}{1 - F(\lambda, x)} \right)_{x|_{x=I(1+c(\lambda))-G}} I c_\lambda \right] \quad (1.25) \end{aligned}$$

The first term in square brackets in 1.25 is the derivative of hazard rate w.r.t. news signal. It is assumed to be negative in the proposition. Intuition behind this assumption is simple — given that firm has not defaulted we assume that firm is less likely to be on the default boundary if news are good.

Second term in the square brackets is the derivative of hazard rate. Many distributions, as

discussed by Froot et al. [1993], have increasing hazard ratios (derivative is positive). Proposition assumes that $F(\lambda, v)$ satisfies that property. Then it is clear that 1.25 is less than zero. That is, $\widetilde{HR}_\lambda < 0$.

1.6.2 Second Derivative of the Cost function

Let us go back to the first derivative of the expected cost of default:

$$C_\lambda = \xi \left(\widetilde{HR} \left[\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx \right] + \widetilde{HR} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \right) \quad (1.26)$$

Now differentiate expression 1.26 with respect to the news signal once again:

$$\begin{aligned} C_{\lambda\lambda} = & \xi \left(\widetilde{HR}_\lambda \left[\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx \right] + \widetilde{HR} \left[\int_0^{I(1+c(\lambda))-G} F_{\lambda\lambda}(\lambda, x) dx \right] + \widetilde{HR} F_\lambda(\lambda, \cdot) I_{c_\lambda} \right. \\ & \left. + \xi \widetilde{HR}_\lambda F_\lambda(\lambda, \cdot) + (\xi \widetilde{HR} + 1) \left\{ F_{\lambda\lambda}(\lambda, \cdot) + \underline{\underline{f_\lambda(\lambda, \cdot) I_{c_\lambda}}} \right\} \right) \end{aligned} \quad (1.27)$$

$$\begin{aligned} C_{\lambda\lambda} = & \xi \left(\underbrace{\widetilde{HR}_\lambda \left[\int_0^{I(1+c(\lambda))-G} F_\lambda(\lambda, x) dx \right]}_{>0} + \underbrace{\widetilde{HR} \left[\int_0^{I(1+c(\lambda))-G} F_{\lambda\lambda}(\lambda, x) dx \right]}_{>0} \right. \\ & \left. + \underbrace{\xi \widetilde{HR}_\lambda F_\lambda(\lambda, \cdot)}_{>0} + \underbrace{(\xi \widetilde{HR} + 1) F_{\lambda\lambda}(\lambda, \cdot)}_{>0} \right) \end{aligned} \quad (1.28)$$

$$+ I_{c_\lambda} \left(\widetilde{HR} F_\lambda(\lambda, \cdot) + (\xi \widetilde{HR} + 1) f_\lambda(\lambda, \cdot) \right) \quad (1.29)$$

Expression in the third line can be simplified to:

$$\begin{aligned} \widetilde{HR} F_\lambda(\lambda, \cdot) + (\xi \widetilde{HR} + 1) f_\lambda(\lambda, \cdot) &= \frac{f(\lambda, \cdot) F_\lambda(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} + \frac{\xi f(\lambda, \cdot) + 1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} f_\lambda(\lambda, \cdot) \\ &= \frac{1}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \left[f(\lambda, \cdot) F_\lambda(\lambda, \cdot) + (1 - F(\lambda, \cdot)) f_\lambda(\lambda, \cdot) \right] \\ &= \frac{(1 - F(\lambda, \cdot))^2}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \left[\left(\frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)_\lambda \right] < 0 \end{aligned} \quad (1.30)$$

Hence,

$$\begin{aligned}
C_{\lambda\lambda} &= \xi \left(\underbrace{\widetilde{HR}_\lambda \left[\int_0^{I(1+c)-G} F_\lambda(\lambda, x) dx \right]}_{>0} + \underbrace{\widetilde{HR} \left[\int_0^{I(1+c)-G} F_{\lambda\lambda}(\lambda, x) dx \right]}_{>0} \right) \\
&+ \underbrace{\xi \widetilde{HR}_\lambda F_\lambda(\lambda, \cdot)}_{>0} + \underbrace{(\xi \widetilde{HR} + 1) F_{\lambda\lambda}(\lambda, \cdot)}_{>0}
\end{aligned} \tag{1.31}$$

$$+ \underbrace{I c_\lambda \left(\widetilde{HR} F_\lambda(\lambda, \cdot) + (\xi \widetilde{HR} + 1) f_\lambda(\lambda, \cdot) \right)}_{>0} > 0 \tag{1.32}$$

□

1.7 Proposition 1.2.3

Proof. To start with, we have $V = V_{sh} + V_B + V_D$, that is the value of the company is split between shareholders, senior and junior bondholders. This notation refers to the period 2 values - that is, terminal values realized in period 2. Taking the expectation we can note that $E(V_D)$ represents the market value of senior debt at the moment it is issued. Denote it δ . Analogously, $E(V_B|\lambda) = P$ and we have a condition $I = \delta + P$, that is, the market value of senior and junior bond holders should constitute the required investment I .

$$E(V) = E(V_{sh}) + \delta + P = E(V_{sh}) + I$$

Naturally, the interests of shareholders are aligned with the interest of the company overall. Financial policy (how to borrow I — mainly later or earlier) does not affect the expected payoff to the bondholders since it is exactly I but might affect the welfare of shareholders

$$\begin{aligned}
\frac{\partial V}{\partial D} &= -\xi \frac{\partial}{\partial D} \left(E^\lambda [F(\lambda, B(\lambda)) + D - G] \right) \\
&= -\xi E^\lambda \left(f(\lambda, B(\lambda) + D - G) \left[1 + \frac{\partial B(\lambda)}{\partial D} \right] \right)
\end{aligned} \tag{1.33}$$

Hence, in order to determine if borrowing earlier (increasing D) is beneficial for the company

or not, need to determine the $1 + \frac{\partial B(\lambda)}{\partial D}$. Intuitively expect that expression to be less than zero when news are high — we loose money by borrowing earlier (at average terms) when news turn out to be good after all. And greater than zero when news are bad — firm saved some money by borrowing at average terms earlier. We then weight those cases by $f(\lambda, \cdot)$.

1.7.1 Deriving $\frac{\partial B(\lambda)}{\partial D}$

We will use equation $\delta + P = I$ to derive the effect of increase in early borrowing on necessary amount of later borrowing. By implicit function theorem,

$$\frac{\partial B(\lambda)}{\partial D} = -\frac{\partial \delta / \partial D + \partial P / \partial D}{\partial P / \partial B(\lambda)}$$

Start with definition of P :

$$P = \int_{D-G+\xi}^{D+B(\lambda)-G} (v + G - \xi - D) f(\lambda, v) dv + B(\lambda)[1 - F(\lambda, D + B(\lambda) - G)] \quad (1.34)$$

$$\begin{aligned} \frac{\partial P}{\partial D} &= (B(\lambda) - \xi) f(\lambda, D + B(\lambda) - G) + \left(F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G) \right) \\ &\quad - B(\lambda) f(\lambda, D + B(\lambda) - G) \\ &= F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G) \end{aligned} \quad (1.35)$$

$$\begin{aligned} \frac{\partial P}{\partial B(\lambda)} &= (B(\lambda) - \xi) f(\lambda, D + B(\lambda) - G) + 1 - F(\lambda, D + B(\lambda) - G) - B(\lambda) f(\lambda, D + B(\lambda) - G) \\ &= 1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G) \end{aligned} \quad (1.36)$$

Next, look at the definition of δ :

$$\delta = E^\lambda \left(\int_0^{D-G+\xi} (v + G - \xi) f(\lambda, v) dv + D \left(1 - F(\lambda, D - G + \xi) \right) \right)$$

$$\begin{aligned}
\frac{\partial \delta}{\partial D} &= E^\lambda \left(Df(\lambda, D - G + \xi) + 1 - F(\lambda, D - G + \xi) - Df(\lambda, D - G + \xi) \right) \\
&= E^\lambda \left(1 - F(\lambda, D - G + \xi) \right)
\end{aligned} \tag{1.37}$$

Now, we can plug those values in $\partial B(\lambda)/\partial D$:

$$\begin{aligned}
\frac{\partial B(\lambda)}{\partial D} &= -\frac{E^\lambda \left(1 - F(\lambda, D - G + \xi) \right) + F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \\
&\quad + \frac{\xi f(\lambda, D + B(\lambda) - G)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)}
\end{aligned} \tag{1.38}$$

$$1 + \frac{\partial B(\lambda)}{\partial D} = \frac{1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi) \right)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \tag{1.39}$$

Numerator of that expression has zero mean (w.r.t. λ distribution). That illustrates the intuition that on average debt issued before news arrival is no cheaper no more expensive than the debt issued after news arrival. The next step would be to look at the weights that are attached to this zero mean term. Were those weights constant, we would get zero in expectation.

1.7.2 Covariance

Recall the expression for derivative of value of the firm with respect to early issue of debt 1.33 and plug the expression that was found for change in face value of debt issued due to change in early issued debt ($1 + \partial B(\lambda)/\partial D$):

$$\begin{aligned}
\frac{\partial V}{\partial D} &= -\xi E^\lambda \left(f(\lambda, B(\lambda) + D - G) \frac{1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi) \right)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \right) \\
&= -\xi E^\lambda \left(\widetilde{HR} \left[1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi) \right) \right] \right)
\end{aligned} \tag{1.40}$$

where

$$\widetilde{HR} = \frac{f(\lambda, B(\lambda) + D - G)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \quad (1.41)$$

Since expression on square brackets in 1.40 is zero mean, we can re-write 1.40:

$$\frac{\partial V}{\partial D} = -\xi Cov^\lambda \left(\widetilde{HR}, 1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi) \right) \right) \quad (1.42)$$

In order to find the sign of covariance, need to take derivatives of expressions inside w.r.t to λ :

$$\frac{\partial}{\partial \lambda} \left(1 - F(\lambda, D - G + \xi) - E^\lambda [1 - F(\lambda, D - G + \xi)] \right) = -F_\lambda(\lambda, \cdot) > 0 \quad (1.43)$$

$$\begin{aligned} \frac{\partial \widetilde{HR}}{\partial \lambda} &= \frac{\partial}{\partial \lambda} \left(\frac{f(\lambda, B(\lambda) + D - G)}{1 - F(\lambda, B(\lambda) + D - G) - \xi f(\lambda, B(\lambda) + D - G)} \right) \\ &= \frac{\partial}{\partial \lambda} \left(\frac{\frac{f(\lambda, B(\lambda) + D - G)}{1 - F(\lambda, B(\lambda) + D - G)}}{1 - \xi \frac{f(\lambda, B(\lambda) + D - G)}{1 - F(\lambda, B(\lambda) + D - G)}} \right) \\ &= \frac{\partial}{\partial \lambda} \left(\frac{HR}{1 - \xi HR} \right) \\ &= \frac{HR_\lambda (1 - \xi HR) + \xi HR_\lambda HR}{(1 - \xi HR)^2} \\ &= \frac{HR_\lambda}{(1 - \xi HR)^2} \\ &= \frac{1}{(1 - \xi HR)^2} \left[\left(\frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)_\lambda + \left(\frac{f(\lambda, x)}{1 - F(\lambda, x)} \right)_{x|_{x=B(\lambda)+D-G}} B_\lambda(\lambda) \right] \quad (1.44) \end{aligned}$$

The proof exactly repeats the argument made in the proof of proposition 1.2.2. The sign of 1.44 depends on the sign of $B_\lambda(\lambda)$. Just as was established in Lemma 1.2.1, $B_\lambda(\lambda) < 0$ — the better the news the smaller is the interest expense on the debt. We can show that formally by applying implicit function theorem to 1.34:

$$H(\lambda, B) = \int_{D-G+\xi}^{D+B-G} (v + G - \xi - D) f(\lambda, v) dv + B[1 - F(\lambda, D + B - G)] \quad (1.45)$$

$$\begin{aligned}
\frac{\partial H(\lambda)}{\partial B} &= (B - \xi)f(\lambda, D + B - G) + 1 - F(\lambda, D + B - G) - Bf(\lambda, B + D - G) \\
&= 1 - F(\lambda, D + B - G) - \xi f(\lambda, B + D - G) > 0
\end{aligned} \tag{1.46}$$

$$\begin{aligned}
\frac{\partial H(\lambda)}{\partial \lambda} &= (B - \xi)F_\lambda(\lambda, B + D - G) - \int_{D-G+\xi}^{D+B-G} F_\lambda(\lambda, v) dv - BF_\lambda(\lambda, D + B - G) \\
&= - \int_{D-G+\xi}^{D+B-G} F_\lambda(\lambda, v) dv - \xi F_\lambda(\lambda, D + B - G) > 0
\end{aligned} \tag{1.47}$$

The sign in 1.47 is positive due to exactly the same reason as in 1.18. The optimal policy of the firm is to borrow only if the marginal benefit to debt holders from the last promised dollar of debt is positive. Hence,

$$B_\lambda(\lambda) = -\frac{\frac{\partial H(\lambda)}{\partial \lambda}}{\frac{\partial H(\lambda)}{\partial B}} < 0 \tag{1.48}$$

That makes covariance in expression 1.42 negative. And therefore, the derivative of value of the firm with respect to early issue of debt positive. \square

1.8 Empirical Evidence

Table 1.1: Proportion of firms with significant cash and debt balance.

	Year			
	1980	1995	2010	1980-2010
Cash/Assets > 5%, Debt/Assets > 5%	0.41826	0.39429	0.50617	0.42728
Cash/Assets > 20%, Debt/Assets > 20%	0.042877	0.062852	0.081735	0.072958

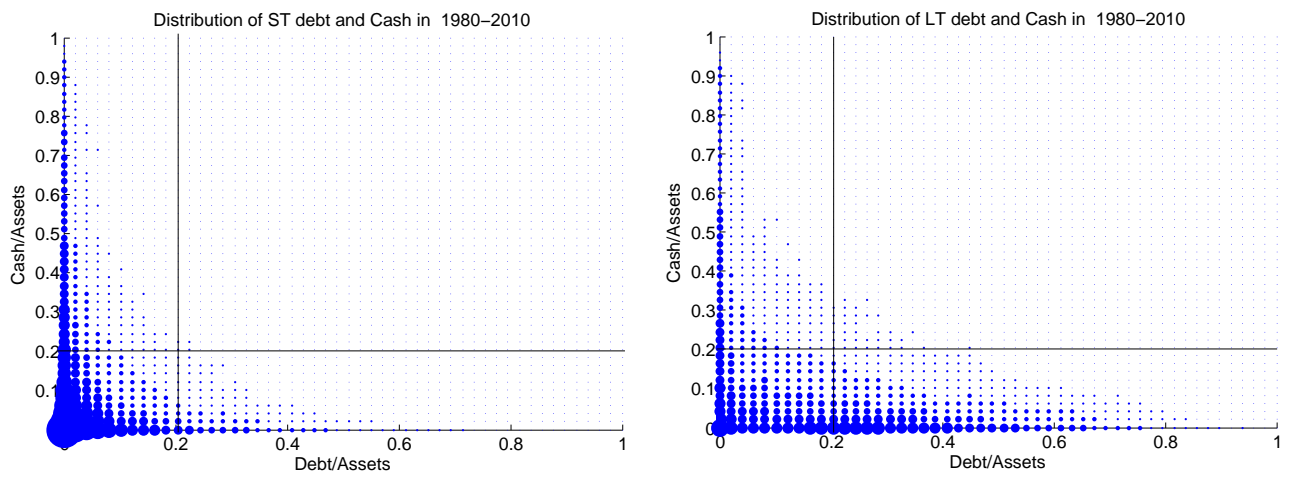


Figure 1.8: Distribution of Cash and Short Term and Long Term debt for US firms in 1980-2010. The mass of firms with co-existing cash and debt is larger for Long Term debt. Hence, Cash is more likely not to be a negative of long term debt.

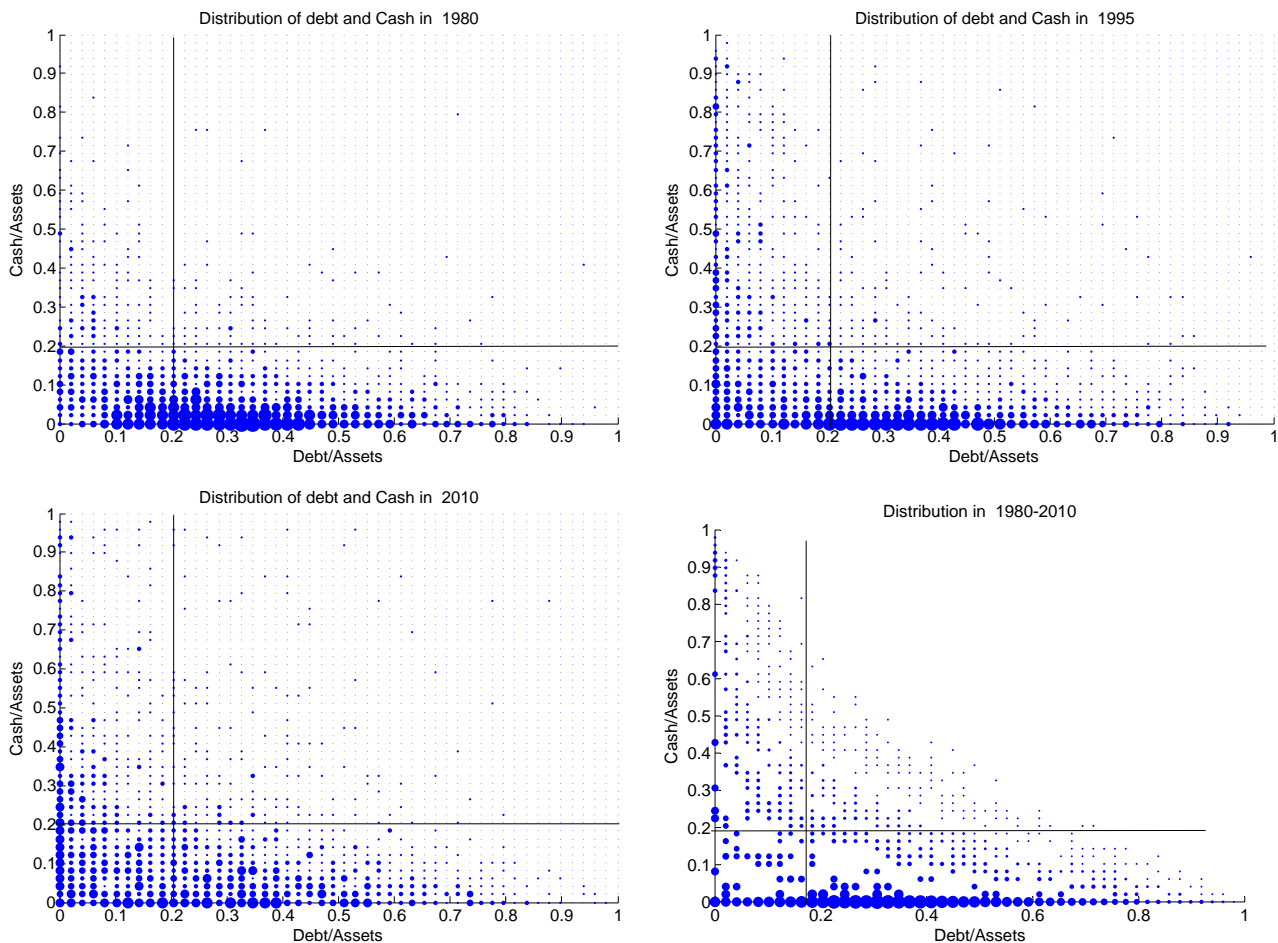


Figure 1.9: Joint distribution of Cash and Debt to Asset ratios. As you can see from the top left hand diagram, firms in 1980 were not using as much cash as in 2010 (lower left hand diagram) and fewer proportion of firms had significantly positive balances of cash and debt at the same time. The lower right hand diagram incorporates 30 years of data and emphasizes the fact there is non-zero mass of firms that have large cash and debt holdings at the same time.

Panel A. Cash Holdings and Cash Flow Risk

VARIABLES	(1) OLS all	(2) RE all	(3) Div	(4) Non-Div	(5) NI>0	(6) NI<0	(7) Big	(8) Small
Industry CF sigma	0.0104*** (0.000)	0.0020*** (0.000)	0.0026*** (0.000)	0.0131*** (0.001)	0.0030*** (0.000)	0.0197*** (0.001)	0.0041*** (0.001)	0.0118*** (0.001)
Market to Book	0.0001*** (0.000)	0.0001*** (0.000)	0.0180*** (0.001)	0.0001*** (0.000)	0.0067*** (0.000)	0.0001*** (0.000)	0.0302*** (0.001)	0.0001*** (0.000)
Real Size	-0.0113*** (0.000)	-0.0157*** (0.000)	-0.0211*** (0.000)	-0.0070*** (0.000)	-0.0116*** (0.000)	0.0164*** (0.001)	-0.0125*** (0.001)	-0.0133*** (0.002)
Cash flow/assets	-0.0098*** (0.001)	-0.0006 (0.000)	-0.0791*** (0.004)	-0.0098*** (0.001)	-0.0737*** (0.006)	-0.0066*** (0.001)	-0.2205*** (0.015)	-0.0054*** (0.001)
NWC/assets	-0.3220*** (0.003)	-0.2340*** (0.003)	-0.2575*** (0.004)	-0.3408*** (0.003)	-0.2484*** (0.003)	-0.2927*** (0.007)	-0.1688*** (0.007)	-0.2359*** (0.007)
Capex	-0.4143*** (0.007)	-0.2225*** (0.006)	-0.4430*** (0.010)	-0.4103*** (0.008)	-0.2880*** (0.006)	-0.4082*** (0.016)	-0.1932*** (0.014)	-0.3462*** (0.017)
Leverage	-0.4301*** (0.003)	-0.2971*** (0.003)	-0.2865*** (0.004)	-0.4532*** (0.003)	-0.3245*** (0.003)	-0.4663*** (0.007)	-0.1990*** (0.005)	-0.4165*** (0.008)
R&D/sales	0.0001*** (0.000)	0.0000*** (0.000)	0.0470*** (0.006)	0.0001*** (0.000)	0.7265*** (0.008)	0.0000*** (0.000)	0.0080*** (0.001)	0.0003*** (0.000)
Dividend dummy	-0.0552*** (0.001)	-0.0021 (0.002)			-0.0156*** (0.001)	-0.1220*** (0.009)	-0.0499*** (0.002)	0.0224*** (0.009)
Aquisition activity	-0.1951*** (0.008)	-0.1286*** (0.006)	-0.1336*** (0.010)	-0.2321*** (0.010)	-0.1395*** (0.006)	-0.3407*** (0.025)	-0.0906*** (0.010)	-0.1931*** (0.029)
Constant	0.3436*** (0.001)	0.2914*** (0.002)	0.2363*** (0.002)	0.3545*** (0.001)	0.2464*** (0.001)	0.4597*** (0.004)	0.2029*** (0.004)	0.3075*** (0.008)
Observations	106,647	106,647	29,603	77,044	80,698	25,949	12,807	18,411
R ²	0.331		0.383	0.303	0.404	0.225	0.315	0.198
Number of gvkey		13,815						

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Panel B. Cash Holdings and Market to Book Volatility

VARIABLES	(1) OLS all	(2) RE all	(3) Div	(4) Non-Div	(5) NI>0	(6) NI<0	(7) Big	(8) Small
M/B sigma	-0.0032*** (0.000)	-0.0021*** (0.000)	0.0015* (0.001)	-0.0032*** (0.000)	-0.0013*** (0.000)	-0.0035*** (0.000)	0.0020*** (0.001)	-0.0031*** (0.000)
Industry CF sigma	0.0112*** (0.001)	0.0027*** (0.000)	0.0034*** (0.001)	0.0148*** (0.001)	0.0041*** (0.000)	0.0274*** (0.002)	0.0054*** (0.001)	0.0182*** (0.002)
Market to Book	0.0077*** (0.000)	0.0049*** (0.000)	0.0197*** (0.001)	0.0076*** (0.000)	0.0158*** (0.000)	0.0066*** (0.000)	0.0309*** (0.001)	0.0049*** (0.000)
Real Size	-0.0111*** (0.000)	-0.0140*** (0.001)	-0.0186*** (0.001)	-0.0079*** (0.001)	-0.0097*** (0.000)	0.0204*** (0.001)	-0.0148*** (0.001)	-0.0183*** (0.003)
Cash flow/assets	-0.0052*** (0.001)	0.0011* (0.001)	-0.0662*** (0.005)	-0.0050*** (0.001)	-0.0310*** (0.010)	-0.0035*** (0.001)	-0.2267*** (0.025)	-0.0025** (0.001)
NWC/assets	-0.3613*** (0.005)	-0.2162*** (0.005)	-0.2488*** (0.008)	-0.3795*** (0.006)	-0.2446*** (0.004)	-0.2959*** (0.012)	-0.2263*** (0.012)	-0.2403*** (0.012)
Capex	-0.5432*** (0.013)	-0.2775*** (0.011)	-0.5238*** (0.021)	-0.5420*** (0.015)	-0.3629*** (0.012)	-0.5132*** (0.028)	-0.2498*** (0.025)	-0.4157*** (0.032)
Leverage	-0.3790*** (0.004)	-0.2183*** (0.004)	-0.3013*** (0.007)	-0.3867*** (0.005)	-0.3008*** (0.004)	-0.3273*** (0.009)	-0.2247*** (0.009)	-0.2865*** (0.011)
R&D/sales	0.0000*** (0.000)	0.0000*** (0.000)	0.0355*** (0.006)	0.0000*** (0.000)	0.7158*** (0.011)	0.0000*** (0.000)	0.0149*** (0.002)	0.0003*** (0.000)
Dividend dummy	-0.0687*** (0.002)	-0.0083*** (0.003)			-0.0211*** (0.002)	-0.1187*** (0.016)	-0.0483*** (0.003)	0.0348** (0.015)
Aquisition activity	-0.2770*** (0.012)	-0.1636*** (0.008)	-0.1758*** (0.016)	-0.3110*** (0.015)	-0.1954*** (0.010)	-0.3743*** (0.035)	-0.1279*** (0.017)	-0.1806*** (0.047)
Constant	0.3350*** (0.002)	0.2821*** (0.003)	0.2320*** (0.003)	0.3370*** (0.002)	0.2232*** (0.002)	0.4371*** (0.005)	0.2202*** (0.007)	0.2582*** (0.012)
Observations	44,718	44,718	10,057	34,661	32,264	12,454	5,560	7,141
R ²	0.340		0.396	0.308	0.450	0.200	0.374	0.198
Number of gvkey		7,920						

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Chapter 2

Co-existence of Cash and Debt

2.1 Introduction

These days we often hear about firms increasing their piles of cash.¹ What is more puzzling though, is that for some firms it happens while their leverage ratios are above zero. After all, you would not have a high interest credit card debt and a substantial saving account balance, would you?

Of course, in reality, debt issues might be lumpy because of fixed issuance costs, and repayment options might not be easily available. This paper presents evidence that observed debt and cash co-existence is likely to be an optimal choice rather than an occasional event: that firms that have positive cash and debt tend to do so for a number of periods rather than on random dates.

Empirical studies that investigate the determinants of cash holdings, treat leverage as given (see for example Bates, Kahle, and Stulz [2009] or Opler, Pinkowitz, Stulz, and Williamson [1999]). The opposite is true about empirical investigations of the leverage decisions (see Fama and French [2002] among others). It is interesting to see how cash and leverage ratios are jointly distributed in the cross-section of firms, since both are parts of the optimal financial policy.

This paper summarizes theoretical models that are consistent with firms optimally choosing positive leverage and cash holdings. It proceeds by testing empirical predictions, and finding empirical support for all of them. One novel model that was not studied in the literature before,

¹See, for example, <http://www.economist.com/news/economic-and-financial-indicators/21571909-corporate-cash-piles>, or <http://www.economist.com/node/16485673>.

the pre-borrowing motive (Chaderina [2012]), generates predictions about the role of volatility of the market-to-book ratio. As a firm characteristic, it is a new control in the empirical literature on cash and leverage. The paper documents effects of the volatility of the market-to-book ratio on excess cash holdings, likelihood of debt and cash co-existence, cash savings out of the net debt issuance, and on the proportion of long-term debt in the overall leverage. Consistent with the predictions implied by the pre-borrowing motive, I find that firms with more volatile market-to-book ratios tend to have larger cash balances, and they are also more likely to have excess cash and debt outstanding. On average, those firms tend to save more out of net debt issues, but they are not more likely than average firms to use long-term debt, contrary to the prediction of the pre-borrowing motive. This may be due to coarse nature of distinction between long- and short-term debt in accounting data.

This paper proceeds as following: next section summarizes motives for cash and debt co-existence. Section 3 outlines testable predictions. The notion of excess cash holdings is defined in Section 4, Section 5 summarizes the characteristics of cross-sectional distribution of cash and debt. Empirical tests are carried out in Section 6.

2.2 Motives to Hold Cash and Issue Debt

In this section I summarize the motives existing in the literature to hold cash and issue debt. I start with models that rationalize demand for liquid assets for a given capital structure, then I discuss models of optimal capital structure. At the end, under optimal capital structure policy, I discuss what should we expect the liquidity policy to look like.

2.2.1 Cash

Transaction demand

Transaction demand for cash holdings, introduced by Keynes [1937] and formally described by Baumol [1952], balances the benefits of having a liquid asset that is suitable for making transactions against the costs of lost interest. Firms want to avoid the costs of converting assets into cash, so

they are better off having cash balances that they can slowly deplete, rather than selling assets more frequently to match the timing of expenses. The framework was extended by Miller and Orr [1966] to incorporate the uncertainty in cash inflows. The key driver of transaction demand for cash holdings is the cost of converting alternative assets into cash. Given that, it is natural to expect firms with large account receivables and other liquid assets (large Net Working Capital) to hold less cash than firms with larger share of long-term assets.

Since transaction costs that are triggered by asset conversion are thought to be lump-sum rather than proportional to the conversion amount, it is natural to expect economies of scale in liquidity management. That is, big firms will face proportionally smaller transaction costs, on top of possible benefits of diversification benefit from having multiple product lines. There is ample empirical evidence that indeed larger firms hold smaller balances of cash (among recent ones are Bates, Kahle, and Stulz [2009], Opler, Pinkowitz, Stulz, and Williamson [1999]).

Precautionary demand

From one of the first models of precautionary demand for cash holdings in Whalen [1966] we know that both transaction and precautionary demand for cash holdings can be modeled in stochastic environment. While the cash held for transaction purposes saves on expected conversion costs, the precautionary cash holdings are safe-guarding the firm from a different set of potential problems — either passing on a positive NPV opportunity or going bankrupt. The firm is effectively saving on the expected costs of accessing financial markets. Apart from standard in fixed and proportional costs of issuing debt or equity, which are in essence transaction costs, the firm might be facing potentially infinite costs of accessing financial markets in the form of limited pledgeability of funds, collateral constraints, liquidity constraints from the lenders side, etc (see Acharya, Almeida, and Campello [2007], Almeida, Campello, and Weishbach [2004], Acharya, Davydenko, and Strebulaev [2011], Hugonnier, Malamud, and Morellec [2013], Gamba and Triantis [2008] among others). In some (bad) states of the world the firm is effectively shut out of the external capital markets. Precautionary cash balances are meant to help the firm make it through those states. Though the firm might not be facing the potential infinite cost of borrowing might still benefit from having

precautionary liquid funds simply because the expected future borrowing costs it might be larger than the costs of borrowing today due to Jensen's inequality type of argument (see Chaderina [2012]).

Agency

Following the arguments of Jensen [1986], we expect entrenched managers to accumulate cash in order to pursue projects that are in their personal interest and might not be in the interest of shareholders. If those projects were to be financed with either equity or debt, the shareholders might not have approved them or the financial markets might not have supplied them. Internal liquid funds are therefore a preferred source of funding for entrenched managers. Cross-country empirical evidence suggests that stronger shareholder rights are associated with smaller cash holdings by companies: that is, the corporate governance provisions are effective at regulating managers' incentives to accumulate cash (see, for example, Dittmar et al. [2003], Ferreira and Vilela [2004]). Another set of empirical studies suggests that in US firms with weaker governance structures (higher Governance index introduced Gompers et al. [2003]) actually hold less cash (Harford et al. [2008]). That is, instead of accumulating cash, entrenched managers spend it on acquisitions and capital expenditures. We might expect that managers in the firms with stronger shareholder rights are more interested in accumulating cash — if the investment opportunity comes up that they would want to pursue, it would have been harder for them to obtain financing than for their peers in firms with weaker shareholder rights.

2.2.2 Debt

Trade-off theory of capital structure started with the model of the firm in Kraus and Litzenberger [1973] balancing the tax benefits of debt and the expected bankruptcy costs. It is, to a large extent, a set of deviations from Modigliani and Miller [1958] world that make financing policy matter. The frictions considered by various models, in addition to taxes and bankruptcy costs, include transaction costs, adverse selection and agency conflicts. Pecking order theory, as outlined in Myers [1984], is based on the same set of frictions (asymmetry of information, agency costs) as

the trade-off theory, but the crucial difference is that those costs are assumed to be large, generating corner solutions for the financing policy (see Frank and Goyal [2007] for a review of both). As noted by Fama and French [2002], the two theories have naturally similar predictions about the optimal capital structure, diverging in their prediction on effect of profitability on optimal leverage choice.

For the purposes of current study, both theories have the same implications — firms' choice of optimal optimal leverage ratio will be influenced by profitability (proxied by EBITDA), investment opportunities (Market-to-Book ratio), the severity of free-cash flow problem (GIM index), and volatility of cash inflows. This insight guided the inclusion of control variables in regressions analyzing the joint distribution of debt and cash. It should be noted though that the predicted signs of effects differ. According to trade-off models, the more profitable firms are more likely to have higher leverage since their benefit of having extra tax shields is higher (in more profitable firms the chance that debt tax shield will be redundant is lower). In pecking order theories on the other hand, since external financing costs are significantly larger than any tax benefits of debt, more profitable firms will be less levered because they will tend to use accumulated internal funds to finance investments.

2.2.3 Capital Structure and Liquidity

The focus of the current study is on the link between leverage and liquidity. In order to interpret observed cash and debt balances of firms, it is necessary to understand how the models of demand for cash can be reconciled with models of optimal capital structure.

The transaction demand for cash is not directly related to optimal capital structure of the firm. That is, it is more related to the nature of its business operations. Even though the pecking-order theory predicts that first all internal funds will be exhausted before debt is issued to finance investment, it does not mean that the firm will spend the cash balances necessary to keep operations running. So it is interesting to look at *excess cash* — cash balances in excess of cash held for transaction purposes, and see if we would expect it to co-exist with debt.

The precautionary demand for cash, on the opposite, is closely linked to the choice of capital structure. Pecking-order theory predicts that the firm should not issue debt if it has internal funds.

There are though two other ways the firm can find itself in a situation with positive debt and excess cash balances — if cash was accumulated after debt was issued, or when the debt proceeds exceed current financing needs. In the former case the firm will be losing the benefit of smaller interest expense (smaller expected deadweight costs of bankruptcy) but will avoid running into debt issuance costs in the future. In the latter case, the firm will be again paying excessive interest expense over some years, but avoiding debt issuance costs (the fixed part of it). Both cases deal with cash holdings driven by savings of financial transaction costs.² If that was indeed the driver of the co-existence, then we would expect small firms, for whom transaction costs as a percent of issue size are larger than for big firms to have positive cash and debt more often.³

Trade-off models offer more room for precautionary cash holdings and debt co-existence. Suppose the firm foresees that in some especially bad states in the future collateral constraints will be binding, that is, the firm will not be able to issue debt to finance a positive NPV project.⁴ The firm may find it optimal to issue debt now and save proceeds as cash, and then use internal funds for investment. Since small firms are more likely to be financially constrained, we would expect to see them having positive debt and excess cash balances more often than big firms.

A different prediction is coming out of pre-borrowing motive for cash holding. Consistent with trade-off model, the firm will decide on optimal leverage ratio balancing out tax benefits of debt and costs of issuing equity against the expected deadweight costs. The next question the firm would be facing is when to borrow if the investment project is coming up in the future, just as in the setting above. Even if the firm can always secure funding in the future, the interest rate it will be charged is not known as of today. It will be high in bad states and low in good states, but on average higher than the cost of borrowing today.⁵ So the firm is better off borrowing today, saving

²We typically think that fixed debt issuance costs represent not only the transaction costs of physically issuing bonds, but also account in a reduced form way for costs of asymmetric information, etc. In that case cash demand can't be deemed for transaction purposes only. The concern here is why would we expect these costs of asymmetric information to display the same economies of scale as transaction costs. Without economies of scale, it is hard to justify why the firm is saving on information asymmetry costs by combining debt issues into a single borrowing.

³I find quite the opposite to be true in data.

⁴Positive NPV project might mean a value enhancing financial transaction, for example, rolling over the debt to avoid bankruptcy.

⁵See Chaderina [2012] for technical conditions that should be satisfied for the expected default costs to be convex in information signal. Default costs are paid by shareholders since bond holders are competitive and always break even. With convexity, using the Jensen's inequality type of argument, we conclude that issuing debt at known interest rate today is cheaper than exposing the firm to volatility in the terms of borrowing.

proceeds in cash, and then investing out of internal liquid funds. The firm finds itself with positive cash and debt balances for some periods.⁶ The cash holdings are a precaution against possible increase in the cost of borrowing. The firm is effectively saving by lowering expected default costs. So the benefit is not related to financial transaction costs. Neither does it rely on lifting otherwise binding collateral constraints, as in cases discussed above. Empirically, we would expect all firms to be interested in pre-borrowing, though the story applies the most to firms with well established access to financial markets:⁷ that is, big mature firms. Hence, empirically we would expect larger firms to have positive debt and excess cash balances more often.

2.3 Testable Predictions

The firm can find itself with positive cash and debt balances if it issued debt and the ending balance of cash was positive, either because it did not spend all the proceeds, or because there was originally a positive balance of cash, or both. Or, alternatively, if debt was outstanding, and then the firm experienced positive cash inflows (positive operating profits or equity issuances) and accumulated cash.

As was discussed in the previous section, the decision to have positive cash balances and above zero leverage in both cases can be explained by:

1. transaction demand for holding cash (in which case the firm is saving on costs of converting physical assets into liquid assets), and leverage ratio justified by either trade-off models or the pecking order theories;
2. demand for cash to avoid costs of financial transactions, and leverage justified by the trade-off models;
3. precautionary demand for cash driven by collateral constraints, and positive leverage driven by trade-off considerations; or

⁶If return on cash balances within a firm is smaller than time value of money for bond holders, then the firm is losing some money over that spread. That cost thought is balanced by the benefit of smaller expected credit spread the firm has to pay for the duration of the investment project.

⁷Trade-off theory predicts higher leverage ratio for them, hence, more likely to actually benefit from smaller expected credit spread than firms that mostly rely on equity financing.

4. pre-borrowing motive — the cash held to hedge the volatility in the terms of borrowing, and again, positive leverage driven by the trade-off models.

The demand for cash holdings in the first three motives above has been extensively studied in the empirical literature (see, for example, Bates, Kahle, and Stulz [2009]). This paper looks at how consistent the various motives are with the behavior of firms that have positive leverage ratios. The fourth motive for holding cash, pre-borrowing, has not been studied empirically before, so it is interesting to see if there is empirical support for it.

In this section I identify variables that I will use to proxy for the firm characteristics driving the four motives to have positive cash and debt; and then I describe tests that will let me assess how empirically plausible each motive is.

2.3.1 Size Effect

The transaction demand for cash depends negatively on the size of the firm and the amount of other liquid assets easily convertible into cash. It is also not related to financial structure of the firm and hence applies to every firm. In order to see if there is debt and cash co-existence that is driven by frictions not related to the nature of firms operations, I compute excess cash — amount of cash that the firms are holding in excess of the transaction demand for cash.

The precautionary demand for cash holdings involves hedging the firm from either financial transaction costs or from binding into collateral constraints. Precautionary demand depends negatively on the size of the firm as well. The pre-borrowing motive has the opposite prediction about the sign of the size effect. Yet it is hard to use this prediction to test the relative performance of 2nd and 3rd vs 4th explanations of debt and cash co-existence because size was used to proxy for the scale of business operations determining the transaction demand.

2.3.2 Volatility of Market-to-Book

The pre-borrowing motive looks at another way to measure riskiness of the firm, the volatility of market-to-book. Since in the pre-borrowing motive, the driving source of risk is the information inflow, we can infer how important the information risk is for the firm from revisions in its value.

If the market value of the firm fluctuates a lot, then innovations in news are important, and are likely to generate big changes in the cost of borrowing for the firm. The more information risk is present, the more incentives the firm will have to pre-borrow in order to avoid the exposure to the change in the credit spread. Hence, we would expect to find the firm more often with positive cash and debt balances. That prediction is unique to the pre-borrowing motive, as there is no role for the book-to-market volatility in other theories of debt and cash co-existence.

2.3.3 Controls

The explanations outlined above, that are consistent with firms having positive cash and debt balances, predict higher cash balances for more risky firms (as measured by the volatility of cash flows) and for firms with better investment projects (as proxied by the market-to-book ratio). The firms with higher level of cash inflow in the current period are expected to save more cash, everything else equal. R&D expenditures, that can proxy for growth opportunities, are expected to be positively related to cash. Under the precautionary demand for cash driven by collateral constraints (3rd in the list) we would expect firms with larger capital expenditures and acquisition activity to have smaller cash balances, because fixed assets can be used as collateral, lifting otherwise binding borrowing constraints.

At the same time, we would expect more risky firms (as measured by the volatility of cash flows), as well as firms with more growth options (higher market-to-book ratio) to use less debt. Firms with higher current level of operating profit (cash inflow) are expected to either have higher leverage to utilize more of the tax shield, according to the trade-off models, or smaller leverage because they are able to finance more of their investments from internal funds according to the pecking order theories.

2.3.4 Tests

I start the empirical investigation of the four motives to have cash and debt by estimating a system of simultaneous equations for cash and leverage ratios. The variables of interest are the volatility of market-to-book and size of the firm. If the marginal effect of size on cash holdings is negative,

then we conclude that the combined effect of the first three motives (transaction demands and the demand driven by collateral constraints) is dominating the pre-borrowing motive, and that is natural to expect. Larger firms benefit from returns to scale in managing liquid assets and can afford keeping smaller cash balances. If the volatility of market-to-book ratio has a positive and significant effect on cash balances, that would show empirical support for the pre-borrowing motive to hold cash — firms that have volatile terms of borrowing are more willing to accumulate internal funds now to fund the future investments.

Since a part of demand for cash is driven by the nature of firm's operations and is not related to financial decisions, I compute the amount of excess cash that the firm holds and then investigate how observable characteristics of the firm influence the likelihood of excess cash and debt co-existence. The focus again is on the volatility of the market-to-book ratio. Under the pre-borrowing motive, I expect to see firms with more volatile market-to-book ratios to have excess cash and debt more often than an average firm.

I then turn to the investigation of how the excess cash is affected by the net debt issues. Under the pre-borrowing motive, I would expect firms with more volatile market-to-book ratio to save more out of the net debt issues than an average firm.

And the last empirical test concerns the prediction of debt maturity that the pre-borrowing motive has. Given that firms with more volatile market-to-book ratio are more exposed to the volatility in the terms of borrowing, we would expect those firms to chose longer maturity debt rather than rely on the rolled over short-term debt. In regression of the share of long-term debt on observable firm characteristics, a positive sign on the volatility of market-to-book, controlling for the overall leverage ratio, would support the prediction of the pre-borrowing motive.

2.4 Excess Cash

In order to analyze the extent to which cash and debt co-exist on the balance sheets of US firms, I estimate the amount of excess cash that is held by companies. Excess cash is defined as amount of cash that is held in excess of what is likely sufficient for operating reasons, or in other words, what the firm would hold if it was only motivated by the transaction demand for cash. For each

$t = [1980, \dots, 2010]$ I estimate the following set of cross-sectional regressions:

$$Cash_{it} = \alpha_t + \beta_t^1 SizeReal_{it} + \beta_t^2 NWC_{it} + e_{it} \quad (2.1)$$

and define $ExcessCash_{it}$ as the estimated residual from the regressions above. The real size of the firm is measured as the logarithm of total assets in 2000 prices. According to the theory of transaction demand for cash holdings, larger firms experience economies of scale in managing assets, so we should expect on average bigger firms to have a smaller share of assets held as cash for transaction purposes. Net Working Capital can be thought of as a substitute for cash as it can be more easily converted into liquid assets than physical capital units. So we should expect firms with more Net Working Capital to have less need for high cash balances for operating purposes.

In the data, the signs of coefficients appear as expected (as can be seen in Figure 2.1). Bigger firms and firms with larger net working capital tend to hold a smaller share of assets in cash.

Given that size and NWC are commonly used in the literature as proxies capturing the firm characteristics that explain transaction demand for cash holdings, the amount of Excess Cash that firms hold can be thought of as representing the portion attributed to precautionary demand and/or agency reasons. The latter will be addressed later. What is interesting to see, is how much excess cash co-exists with debt on the balance sheets of US firms.

2.5 Cash and Debt in Cross-Section

In order to investigate if there is indeed economically interesting phenomenon in the co-existence of excess cash and debt, I summarize the characteristics of firms that have more than $h\%$ of assets in excess cash and at the same time more than $h\%$ of their assets are financed with debt. I focus on several aspects — the nature of business (as described by 2 SIC industry code), size, and the persistence of being present in the $h\%$ group.

As one can see from Figure 2.2, there is a substantial mass of firms with positive excess cash and debt balances.

The mass of firms on the vertical axis represents mostly equity financed firms that have accu-

mulated substantial amount of internal funds. These are smaller than the average COMPUSTAT firm, with a high proportion of Business Services firms.⁸

Characteristics of firms that are in group 5% and 20% are presented in the Table 2.3. As one can see, firms with large balances of excess cash and positive leverage ratios are big firms, substantially larger than average COMPUSTAT firm. Transportation equipment is the most heavily represented industry. Having positive cash and debt balances seems to be a financial policy choice rather than an irregular occurrence — the unobserved firm characteristics account for more than 70% of propensity to be in those groups next year.⁹

The table also illustrates that firms in group 5% and group 20% are more risky than average COMPUSTAT firm. The two measures of idiosyncratic risk that I report are volatility of cash flows and volatility of market-to-book ratio. One can think about the volatility of market-to-book as being the composite of cash-flow risk and discount rate risk, or alternatively, following the interpretation of pre-borrowing motive in Chaderina [2012], the importance of firm-specific news. On both metrics, the firms that have excess cash and debt at the same time are on average substantially more risky.

⁸ Average size of firms with positive Excess Cash and Leverage ratio below 5% is around \$200 million 2000 dollars, while average firm in the whole sample is around \$700 million. Share of Business Services is 19.23%. The three other industries with around 14% shares are Electronics, Industrial Machinery and Petroleum Refining. The group has noticeably smaller presence of Communications and Transportation Equipment than COMPUSTAT pool overall.

⁹If co-existence of debt and cash was due to a transitory income shock — when a levered firm accumulated cash out of unexpectedly high positive cash inflow and have not invested/distributed it just yet, then co-existence would have displayed low serial correlation. The same applies to firms that have not just yet invested proceeds from debt issuance. In other words, low serial correlation would have been a sign that co-existence is more of a temporary state rather than financial policy outcome.

2.6 Empirical Tests

2.6.1 Simultaneous Equations

I examine the debt and cash co-existence with estimation of the following system of simultaneous equations:

$$\begin{aligned}\text{Cash}_{i,t} = & \alpha_0 + \alpha_1\text{Levr}_{i,t} + \alpha_2\text{Cash}_{i,t-1} + \alpha_3\text{Vol}(\text{MB})_{i,t} + \alpha_4\text{Vol}(\text{CF})_{i,t} + \\ & + \alpha_5\text{M/B}_{i,t} + \alpha_6\text{CF}_{i,t} + \alpha_7\text{Capex}_{i,t} + \alpha_8\text{R\&D}_{i,t} + \alpha_9\text{DiviD}_{i,t} + \\ & + \alpha_{10}\text{Acq.A}_{i,t} + \alpha_{11}\text{Size}_{i,t} + \alpha_{12}\text{NWC}_{i,t} + \epsilon_{i,t}\end{aligned}\tag{2.2}$$

$$\begin{aligned}\text{Levr}_{i,t} = & \beta_0 + \beta_1\text{Cash}_{i,t} + \beta_2\text{Levr}_{i,t-1} + \beta_3\text{Vol}(\text{M/B})_{i,t} + \beta_4\text{Vol}(\text{CF})_{i,t} + \\ & + \beta_5\text{M/B}_{i,t} + \beta_6\text{CF}_{i,t} + u_{i,t}\end{aligned}\tag{2.3}$$

Results are presented in Table 2.4. Firms with larger cash holdings are expected to chose larger leverage ratios, in line with the trade-off theories prediction that safer firms would use higher leverage to benefit from the extra tax shield. On the other hand firms with larger leverage ratios are not expected to chose to hold more or less cash than firms in general.

Bigger firms have significantly smaller cash balances than small firms, confirming the prediction that managing liquid assets exhibits positive returns to scale. This finding speaks in favor of the transaction demand for cash holdings, as well as the demand driven by possibly binding collateral constraints, which are more likely to bind for small firms. On the other hand, the volatility of cash inflows is negatively related to cash holdings, predicting that firms with less volatile cash inflows hold more cash. That goes against the prediction of collateral-constraints-driven demand for cash. It might be that volatility of cash inflows is a proxy for age, i.e. that more volatile firms are younger firms that did not have enough time to accumulate funds just yet. The other measure of risk, the volatility of the market-to-book ratio, is positively and statistically significantly related to cash holdings, confirming the prediction of the pre-borrowing motive that firms with more volatile terms of borrowing will be more interested in accumulating internal funds.

The volatility of the market-to-book ratio is negatively related to leverage, consistent with the

prediction of the pre-borrowing motive that firms with more volatile terms of borrowing are facing higher expected cost of debt. It is natural to expect them to use other sources of funding. The interesting question then is: would we expect more or less debt and cash co-existence for firms with higher volatility of market-to-book ratio? That is, given that firms would optimally chose smaller leverage but higher cash holdings, would those firms be more likely than the average firm to have substantial excess cash and debt?

2.6.2 Co-existence

In this section, I use the notion of excess cash (in excess of what is held for transaction purposes) rather than the overall level of cash holdings. That is because I am interested in debt and cash co-existence driven by the financial policy choice, and not by the nature of firm's operations, though it is another interesting question. In order to see how the observable firm's characteristics influence its likelihood to have positive cash and debt balances, I estimate the following relationship:

$$\begin{aligned}
 Pr(\text{group}h_{i,t} = 1) = & G(\gamma_0 + \gamma_1 \text{Vol}(M/B)_{i,t} + \gamma_2 \text{Vol}(\text{CF})_{i,t} + \gamma_3 \text{MB}_{i,t} + \gamma_4 \text{CF}_{i,t} \\
 & + \gamma_5 \text{Capex}_{i,t} + \gamma_6 \text{R\&D}_{i,t} + \gamma_7 \text{DivD}_{i,t} + \gamma_8 \text{Acq.A.}_{i,t} \\
 & + \rho \text{group}h_{i,t-1} + c_i)
 \end{aligned} \tag{2.4}$$

Recall that $\text{group}h_{i,t}$ is an indicator variable that equals 1 if the firm i in period t has leverage ratio above $h\%$ and excess cash above $h\%$ of assets, c_i is capturing firm's random effects, and G is the logit function.

See Table 2.5 for estimation results. As predicted by the pre-borrowing motive, firms with more volatile market-to-book ratios are more likely to have positive cash and debt balances. The same is true for firms with higher volatility of cash flows, which is consistent with the story of potentially binding collateral constraints. Capital expenditures do not have statistically significant association with debt and cash co-existence, while acquisition activity is strongly negatively related to debt and excess cash co-existence, again consistent with the story of occasionally binding collateral

constraints.

2.6.3 Net Debt Issuance

Next I investigate how proceeds from debt issue are related to cash holdings. According to the pre-borrowing motive, firms with more volatile market-to-book ratios would save more cash out of debt issue. Hence, we would expect positive sign on the interaction term of the volatility of market-to-book ratio and the net debt issuance.

Estimation results are presented in Table 2.6. As one can see from the first column, cash holdings are negatively, not positively related to both the net debt issuance and the net equity issuance. It means that on average in periods when firms issue debt or equity, they tend to also deplete internal funds. That would be consistent with the pecking-order theories of leverage, predicting that first all internal funds should be spend. External funds in that case were used probably because internal funds were insufficient.

The interaction term is indeed positive and statistically significant, confirming that firms more exposed to volatility in the terms of borrowing, are more likely to save more out of proceeds from the debt issue.

The second column looks at the same relationship for a subsample of big firms, defined as firms being in the largest quantile by size in a given year. Big firms save from equity and debt issues, and they have larger cash balances if their volatility of the market-to-book ratio is high. Though it seems that their propensity to save out of the net debt issuance is not affected by the level of the market-to-book volatility.

2.6.4 Composition of Debt

The pre-borrowing motive to hold cash predicts that the firm with higher volatility of market-to-book ratio will use long-term debt rather than rolled-over short-term debt. The same prediction comes from the story of binding collateral constraints. In order to test that empirically, I look at how firm characteristics are related to the share of long-term debt in the total debt outstanding

for all firms.¹⁰

See Table 2.7 for empirical results. The first two columns contain model estimates using random- and fixed-effect estimators, respectively. On average, volatility of market-to-book ratio has no statistically significant effect on the amount of excess cash. The same is true for big firms, as well as for risky firms (firms with volatility of market-to-book above 2). Safe firms (volatility of market-to-book below 2) exhibit on average a negative relationship between excess cash and volatility of market-to-book. That goes against the prediction of pre-borrowing motive. It might be the case that the observed relationship is due to heterogeneity of bonds classified as long-term on the balance sheet. That is, firms with higher volatility of market-to-book might indeed be using longer maturity bonds, but that might mean fewer 2 year bonds and more 4 year bonds, all of which are accounted as long-term bonds. Hence, the observed neutrality of market-to-book volatility. While safe firms are more likely to use short-term debt, it might be the case that high volatility of market-to-book for them is a proxy for access to commercial paper market. That is, if larger volatility of market-to-book is associated with even better access to financial markets, firms might chose to rely on good reputation and use rolled-over short-term debt.

2.6.5 Agency Motive to Hold Cash

Entrenched managers, following the logic of Jensen [1986], would prefer having excess cash because cash can be directed to projects that do not require shareholders' approval. One way to proxy for the entrenchment of managers is to look at a Governance index (Gompers et al. [2003]). Empirical tests above were silent about the demand for cash holdings that is driven by agency conflict because a Governance index is available only for a sub-set (1500) of publicly traded firms. To see if results of empirical tests above are affected by explicitly taking into account the agency conflict within a firm, I include a Governance index as a control variable. We have to be mindful, though, of possible changes to parameter estimates caused by sample bias because Governance index is available only for largest firms.

Estimation results are presented in Table 2.8 and Table 2.9. The first two columns in Table 2.8

¹⁰The best way to test that prediction would be to use the value-weighted time to maturity of all debt, not just the ratio of long-term debt, because of apparent heterogeneity in the debt that is classified as long-term.

represent estimates of the simultaneous equations model in section 2.6.1 but for the subsample of firms that have information on Governance index. As one can see, unlike average firms, big firms tend to have smaller leverage if they have larger cash holdings, holding everything else equal. That is possibly because they have other tax shields in place (e.g. depreciation on a big pool of physical assets, etc). Volatility of the market-to-book ratio is positively related to cash holdings, but for big firms it is also positively related to the leverage ratio. The volatility of cash flows is positively related to cash holdings, again unlike firms in general. That observation is hard to explain in the framework of occasionally binding collateral constraints because big firms are the ones to be least likely to run into those. Pre-borrowing motive, on the other hand, predicts a positive relationship between cash and volatility of cash flows, independent of firm size. Size is still negatively related to cash holdings, confirming the benefits of scale in managing liquid assets. Interestingly, net working capital is negatively related to cash holdings, as predicted by the transaction theory of demand for cash, and unlike for firms in general sample.

Now that we have established what the model estimates are for the smaller sample of large firms, we can see how these estimates are affected by controlling for the entrenchment of managers. Columns 3 and 4 in Table 2.8 represent model estimates when Governance index is included as an additional explanatory variable. The results are not substantially different from the estimates in first two columns. Volatility of market-to-book ratio still affects cash holdings positively, size is negatively related to cash holdings, firms with more volatile cash flows hold more cash, and cash is negatively associated with leverage. The negative and statistically significant sign on Governance index itself tells us that the stronger the manager (higher value of index), the smaller cash balances the firm has. That would be the case if strong managers are more likely to get projects approved so they were not that desperate to keep cash.

The last column of Table 2.8 presents results of maximum likelihood model from section 2.6.2 controlling for entrenchment of managers.¹¹ We see that firms with larger volatility of market-to-book ratio are more likely to have cash and debt at the same time, consistent with the pre-borrowing motive. The same is true for firms with larger volatility of cash flows, consistent with both pre-

¹¹I do not report results of likelihood model estimated on a smaller sample without Governance index because parameter estimates are not fundamentally different from those reported for the full sample.

borrowing and occasionally binding collateral constraints, as well as the transaction demand for cash. Stronger managers are on average less likely to chose positive cash and debt balances.

The first two columns in Table 2.9 report estimates of excess cash on net debt issuance. The first column does not control for entrenchment of managers, but presents model estimates for a smaller sample of firms. Since those are big firms, we again observe a positive relationship between the net debt issuance and excess cash, and between the volatility of market-to-book ratio and the excess cash, but it appears that the firms with higher volatility of market-to-book are not saving more or less than average firms out of net debt issuance. The same holds true once we include the governance index in the regression. The second column reports a negative relationship between managers' strength and excess cash holdings. The third column contains parameter estimates from regressing a fraction of long-term debt on observable firm characteristics, done for a smaller subsample of firms that have data on the Governance index. The results are fundamentally as reported before — volatility of market-to-book appears to be neutral with respect to choice of long-term vs short-term debt, controlling for the overall leverage level. The same is true once we include governance index (column 4). It is interesting to note that stronger managers seem to prefer longer maturity debt.

Overall, controlling for the entrenchment of managers does not change main results — cash is positively related to volatility of market-to-book ratio, and firms with more volatile market-to-book ratio are more likely to have positive cash and debt balances at the same time.

2.7 Conclusion

Big publicly traded firms hold positive amount of both cash and debt at the same time. This appears to be a policy decision rather than an irregular occurrence. Firms with more volatile market-to-book ratios, as well as firms with larger volatility of cash inflows, are more likely to have co-existent debt and cash, consistent with the prediction of pre-borrowing motive. On average, firms with larger volatility of market-to-book ratio tend to save a greater share of net debt issues. The fraction of the long-term debt in the overall leverage ratio appears to be unrelated to volatility of the market-to-book ratio, perhaps because of the heterogeneity in maturity of bonds classified as long-

term. These results are not affected by controlling for agency motive to hold cash (entrenchment of managers).

APPENDIX

2.8 Data

I am using the COMPUSTAT panel of firms from 1979 to 2010 excluding financial firms ($6000 < SIC < 6999$) and utilities ($4900 < SIC < 4999$). I also exclude firm year observations that have negative book value of equity (Book leverage is above 1), negative book value of assets ('at < 0') or negative sales ('sale < 0'). Companies that are not incorporated in US ('fic! = "USA"') are excluded as well. Total I have 212346 firm-year observations. The variables of primary interest are Cash (Cash and Short-term investments 'che'), Total Assets ('at'), Short-Term Debt (Debt in Current Liabilities - Total 'dlc') and Long-Term Debt (Long-Term Debt - Total 'dltt'). I define the leverage ratio as (Short-Term Debt + Long-Term Debt)/Total Assets. I will use Cash and Debt levels scaled by Total Assets throughout the analysis below.

I use EBITDA/Total Assets as a measure of operating cash flows ('ebitda'/ 'at'). Investment is computed as capital expenditures plus acquisitions less sale of property, plant and equipment over total assets (('capex+aqc-sppiv)/at'). The measure of cash flow volatility is obtained by taking a standard deviation of three consecutive preceding years of operating cash flow data. Market-to-Book is book value of assets minus book value of equity plus market value of equity over total assets (('at -lt-pstkl+txditc+dcvt)+csho*prcc_f)/at'). Net working capital is defined as working capital less cash and short-term investments over total assets (('wcap-che)/at'). Capital expenditures are normalized by total assets ('capx/at'). Leverage is calculated as a ratio of total debt to total assets (('dltt+dlc)/at'). R&D expenditures are scaled by sales ('xrd/sale'). The dividend dummy is set to 1 if company has paid out positive dividends in the current period ('dvc > 0'). Acquisition activity is computed as fraction of total assets ('aqc/at'). Net equity issues are calculated as difference between equity sales and equity repurchases scaled by total assets (('sstk-prstk)/at'). Net debt issues are long-term debt issues less long-term debt reductions (('dltis-dltr)/at').

Volatility of Market-to-Book ratio is computed using monthly stock prices ('prccm'), the number of shares outstanding ('csho') and annual book equity measure, as described above. So for each firm and every given year, a Volatility of Market-to-Book is computed as standard deviation of 12 data points - monthly stock price times the current number of shares outstanding, divided by end of year book value of equity.

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Cash/Total Assets	Excess Cash									
	1980	1985	1990	1995	2000	2005	2010			
Real Size	-0.0178***	-0.0216***	-0.0204***	-0.0274***	-0.0226***	-0.0298***	-0.0291***			
NWC/assets	-0.0129***	-0.0000***	-0.0006***	-0.0003*	0.0002*	0.0001***	-0.0001**			
Constant	0.0924***	0.1173***	0.1134***	0.1466***	0.1872***	0.2307***	0.2342***			
Observations	4,715	5,465	5,237	6,975	6,749	5,269	3,763			
R^2	0.089	0.077	0.072	0.074	0.053	0.092	0.112			

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.1: Estimation of Excess Cash. For each year $t = [1980, \dots, 2010]$, I estimate the cross-sectional regression of Cash on firm size and amount of net working capital. The residuals of these regressions are the amount of Excess Cash that firm holds, in excess of what is necessary for transaction reasons.

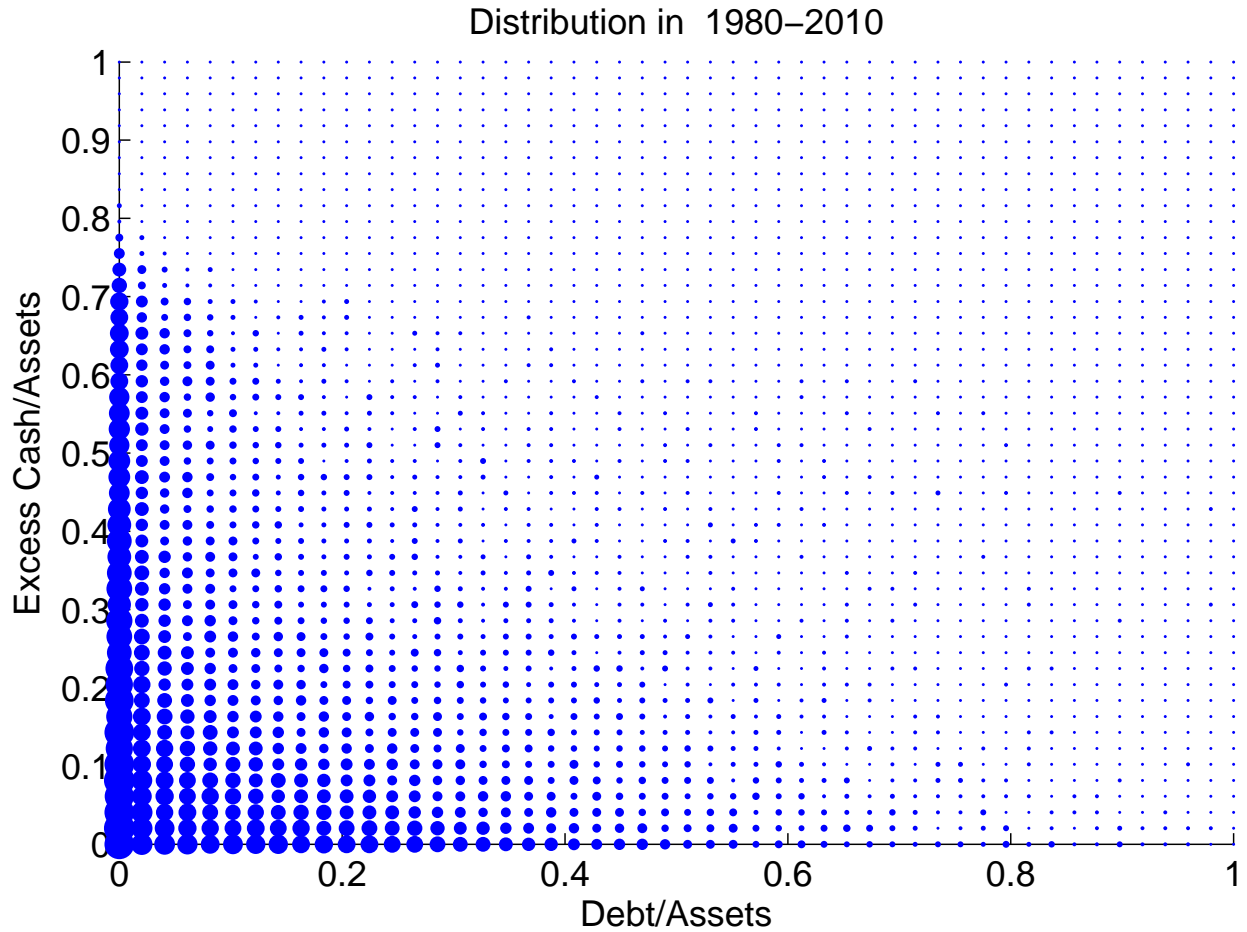


Figure 2.2: Distribution of Excess Cash and Debt of US non-financial, non-utility firms in 1980-2010. Excess Cash is defined as cash holding in excess of what firms are holding for transaction purposes. The size of the circle represents the frequency of such a firm-year observation. As one can see, there is a substantial mass of firms that have Excess Cash and Debt.

	Average Size	Industry Composition	Persistence	CF Volatility	MB Volatility
All	\$702.6	Communications 15.69% Transportation Equipment 9.59% Chemicals 7.85% Business Services 5.25%		1.4	3.83
5%	\$1577.5	Communications 6.60% Transportation Equipment 21.56% Chemicals 9.33% Business Services 7.96%	0.713	2.52	6.82
20%	\$2172.9	Communications 4.70% Transportation Equipment 37.27% Chemicals 1.59% Business Services 3.90%	0.774	5.05	11.09

Figure 2.3: Characteristics of firms with more than 5% of assets as excess cash and leverage ratio of more than 5%, and 20%, correspondingly. The industry composition is based on 2-digit SIC codes. I have reported top four represented industries - Communications (SIC code 48), Transportation Equipment (SIC code 37) - cars and airplanes, Business Services (SIC code 73), including IT, and Chemicals (SIC code 28). Please note that classification is not perfect, as Apple Inc, for example, is not included in Electronics but is classified as Machinery (SIC code 3571). Size is total assets in millions of US 2000 dollars. The persistence is estimated using panel logit random effect model, where the dependent variable is equal to one for firms with more than 5% (20%) of assets financed with debt and having more than the same fraction of assets as excess cash, controlling for observable firm characteristics including volatility of cash flows and volatility of market-to-book ratio. CF volatility is a standard deviation of 3 consecutive observations of operating profit over total assets. Market-to-Book Volatility is the standard deviation of 12 monthly market-to-book observations within given year.

Panel A. Cash and Debt		
VARIABLES	(1) Cash	(2) Levr
Leverage	-0.0004	
$Cash_{t-1}$	0.7376***	
M/B sigma	0.0001**	-0.0421***
CF sigma	-0.0002***	-0.0045
Market-to-Book	0.0000	0.0036***
Cash flow/assets	-0.0004***	-0.3790***
Capex	-0.0016***	
R&D/sales	0.0000	
Dividend dummy	-0.0168***	
Acquisition activity	-0.3062***	
Real Size	-0.0047***	
NWC/assets	0.0000	
Cash		0.8362**
$Levr_{t-1}$		0.0981***
Constant	0.0577***	0.3254***
Observations	46,423	46,423
R^2	0.603	0.426

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.4: Cash and Debt and Volatility of Market-to-Book ratio. Equations are estimated treating Cash and Leverage as endogenous variables and using all other variables (including lagged Cash and Leverage ratios) as instruments. Interestingly, firms with higher cash holdings tend to chose higher leverage ratios, but not the other way around. Larger firms tend to have smaller cash holdings, consistent with transaction as well as constraints-driven demand for cash. Volatility of Market-to-Book is positively related to cash holdings, confirming the prediction of pre-borrowing motive that firms with larger volatility of terms of borrowing tend to accumulate internal funds. Consistent with predictions of collateral constraints-driven demand, firms with larger capital expenditures and larger acquisition activity have smaller balances of cash holdings, most likely because they will have more tangible assets that can be used as collateral.

Panel B. Likelihood of Cash and Debt Co-existence		
	(1)	(2)
	group5	group20
M/B sigma	0.0035***	0.0056***
CF sigma	0.0025**	0.0044***
Market-to-Book	0.0000	0.0002**
Cash flow/assets	-0.0014**	-0.0008
Capex	-0.1698	-0.0745
R&D/sales	0.0002	0.0002
Dividend dummy	-0.4165***	-0.6613***
Acquisition activity	-2.2456***	-1.8336***
Constant	-3.8426***	-6.3550***
Observations	47,523	47,523
Number of gvkey	7,876	7,876

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.5: Likelihood of debt and cash co-existence and firm characteristics. The dependent variable is the probability of excess cash being above 5%(20%) of assets and having leverage ratio above 5% (20%). The logit function, $G(x) = \frac{1}{1+e^{-x}}$, is used to model probability. Positive and significant effect of Market-to-Book volatility on likelihood of debt and cash co-existence speaks in favor of pre-borrowing motive for cash holdings. We also see that firms with more volatile cash flows are more likely to have positive leverage and excess cash at the same time, consistent with the story of occasionally binding collateral constraints.

Panel C. Cash Holdings and Net Debt Issuance		
VARIABLES	(1) All	(2) Big
Net Debt Issuance	-0.0020***	0.2205***
Vol(MB)* Net Debt Issuance	0.0001***	-0.0025
Net Equity Issuance	-0.0007**	0.1020***
M/B sigma	0.0002***	0.0011***
CF sigma	-0.0001	0.2474***
Market-to-Book	0.0000**	0.0115***
Cash flow/assets	-0.0001	-0.0042
Capex	-0.0075***	-0.2260***
R&D/sales	0.0000**	0.0051***
Dividend dummy	0.0004	-0.0087**
Aquisition activity	-0.1282***	-0.2195***
Leverage	0.0000	-0.1078***
Constant	-0.0199***	-0.0098
Observations	39,418	4,764
Number of gvkey	7,342	948

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.6: Cash Holdings and Net Debt Issuance. The dependent variables is excess cash. Models are estimated using random effects estimator. For the full sample, excess cash holdings are negatively associated with both net debt issuance and net equity issuance, suggesting that firms not only use all proceeds but also deplete internal funds. Firms with higher volatility of market-to-book ratio on average tend to save more cash out of net debt issuance, confirming the predictions of pre-borrowing motive. For a sub-sample of big firms (second column), excess cash is positively related to both debt and equity issuance, suggesting that big firms indeed save proceeds and spend funds over time. Firms with higher volatility of market-to-book ration, though, don't seem to save more than an average large firm. Firms are defined as big if they are in the largest quantile according to their assets size in a given year.

Panel D. LT debt and Risk					
VARIABLES	(1) RE all	(2) FE all	(3) Big	(4) Safe	(5) Risky
M/B sigma	-0.0001	0.0001	-0.0008	-0.0232***	-0.0005
CF sigma	-0.0005***	-0.0000	0.0256	-0.0006***	-0.0003**
Market-to-Book	-0.0000**	-0.0000**	-0.0170***	-0.0000*	-0.0000
Cash flow/assets	0.0001	0.0000	0.1248**	0.0001	0.0011**
Capex	0.0540***	0.0288**	0.0602	0.0666***	0.1302***
R&D/sales	0.0000	-0.0000	0.0018	0.0000	0.0000
Dividend dummy	0.0918***	0.0262***	-0.0143*	0.0844***	0.2289***
Aquisition activity	0.2253***	0.1963***	-0.0116	0.2400***	0.1524**
Cash	-0.0913***	-0.0191	0.0348	-0.1139***	0.0352
Leverage	-0.0001	-0.0001	0.1575***	-0.0001	0.0016*
Constant	0.6118***	0.6555***	0.8182***	0.6404***	0.4764***
Observations	39,849	39,849	5,803	35,127	4,624
R^2		0.004			
Number of gvkey	7,233	7,233	1,037	6,826	2,379

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.7: Debt composition and Risk. Dependent variable is the ratio of long-term debt to the total debt outstanding. The first two columns contain model estimates using random and fixed effect estimators, correspondingly. The third column reports estimates for a sub-sample of big firms (in the largest quantile according to total assets in a given year). The last two columns report estimates for subsamples of safe (volatility of market-to-book ratio below 2) and more risky firms (volatility of market-to-book ratio above 2). Except for safe firms, volatility of market-to-book ratio is not significantly related to the choice of long-term vs short-term debt, at least as proxied by the share of long-term debt. The apparent neutrality might be due to heterogeneity of bonds classified as long-term debt.

Panel E. Agency Costs					
VARIABLES	(1) Cash	(2) Levr	(3) Cash	(4) Levr	(5) group5
Leverage	-0.1904*		-0.1933*		
$Cash_{t-1}$	0.5084***		0.5058***		
M/B sigma	0.0018*	0.0093***	0.0019**	0.0093***	0.0992***
CF sigma	0.0619***	0.0191	0.0613***	0.0195	1.3144***
Market-to-Book	0.0133***	-0.0093***	0.0131***	-0.0092***	0.0583
Cash flow/assets	-0.1360***	-0.4190***	-0.1367***	-0.4195***	-3.0432***
Capex	-0.4382***		-0.4435***		-1.7414
R&D/sales	-0.0000		-0.0000		-0.0140
Dividend dummy	-0.0262***		-0.0239***		-0.7894***
Aquisition activity	-0.3220***		-0.3220***		-2.4803**
Real Size	-0.0151***		-0.0146***		
NWC/assets	-0.1543***		-0.1536***		
Cash		-0.3953***		-0.3966***	
$Levr_{t-1}$		0.0081***		0.0081***	
Governance Index			-0.0022***		-0.1131***
Constant	0.1785***	0.3383***	0.1976***	0.3384***	-2.3064***
Observations	5,137	5,137	5,137	5,137	5,296
R^2	0.653	0.180	0.653	0.180	
Number of gvkey					1,870

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.8: Agency Costs. The table represent models estimates controlling for entrenchment of managers. Since Governance index data is available only for a small subsample of firms, I first re-estimate models on that small subsample, and then add Governance index to see if results change. First two columns represent estimates of debt and cash simultaneous equations from section 2.6.1. The third and fourth column are model estimates on the same subsample controlling for entrenchment of managers. As one can see, results are fundamentally the same. Also, it appears to be the case that stronger managers hold less cash, possibly because they are more likely to get approval of projects an don't need to accumulate internal funds. The last column represent the estimates of likelihood for the firm to have more than 5% of assets as excess cash and more than 5% leverage ratio. The model is estimated on a smaller subsample controlling for entrenchment of manages. The results are again in line with results from the full sample and without controlling for governance.

Panel E. Agency Costs				
VARIABLES	(1)	(2)	(3)	(4)
	ExcessCash	ExcessCash	LT_frac	LT_frac
Net Debt Issuance	0.2172***	0.2143***		
Vol(MB)*Net Debt Issuance	0.0020	0.0028		
Net Equity Issuance	0.0491**	0.0514**		
M/B sigma	0.0028***	0.0028***	-0.0009	-0.0008
CF sigma	0.1628***	0.1591***	-0.1103***	-0.1085***
Market-to-Book	0.0159***	0.0153***	-0.0257***	-0.0254***
Cash flow/assets	-0.1430***	-0.1366***	0.3383***	0.3352***
Capex	-0.3435***	-0.3604***	0.0920	0.1053
R&D/sales	0.0073***	0.0072***	0.0017**	0.0017**
Dividend dummy	-0.0313***	-0.0270***	0.0150	0.0109
Aquisition activity	-0.2380***	-0.2418***	0.0267	0.0306
Leverage	-0.1805***	-0.1779***	0.3976***	0.3972***
Governance Index		-0.0066***		0.0046**
Cash			0.0552*	0.0586*
Constant	0.0422***	0.1004***	0.6964***	0.6559***
Observations	4,290	4,290	4,741	4,741
Number of gvkey	1,671	1,671	1,715	1,715

*** p<0.01, ** p<0.05, * p<0.1

Figure 2.9: Agency Costs. The first column represent model from section 2.6.3 on a smaller subsample of firms that have data on Governance index. The second column represents estimates of the same model controlling for entrenchment of managers. The results are not affected by inclusion of Governance Index as a control variable. Excess cash is positively related to net debt issuance and net equity issuance. Firms with more volatile market-to-book ratio hold on average more excess cash, but it looks that those firms are not more likely to save more (or less) out of net debt issues. The third and fourth columns are estimates of the long-term debt fraction. Controlling for entrenchment of managers does not change the neutrality of volatility of market-to-book ratio on the composition of debt.

Chapter 3

Predators, Prey and Volatility on Wall Street (with Richard Green)

Abstract

Much of the activity in sectors of financial services such as sales and trading or the hedge-fund industry is zero-sum in nature. Along with providing transactional and diversification services, participants in this industry also prey upon each other. High-ability traders (predators) capture surplus at the expense of less able and experienced traders (prey). We show that in a dynamic model of such an industry these features amplify real shocks. In the model, traders are randomly matched each period. The presence of more low-ability traders protects others from predation by high-ability traders, leading to equilibrium at a high level of employment. Shocks to aggregate profits may trigger exit by those lower ability, which in turn exposes those of intermediate ability to greater risk of predation and motivating their exit. This leads to more dramatic contractions in financial activity. Depending on parameter values, the industry may remain stuck at the low employment level. In other cases it may, once profits recover, begin rapid growth towards the high employment level until this growth is interrupted by another bad shock. Thus, our relatively simple model generates boom-bust cycles reminiscent of Wall Street employment.

3.1 Introduction

We explore the relationship between several aspects of competition and employment on Wall Street, and we argue that these give rise to “predator-prey” dynamics in employment and trading activity. These dynamics amplify underlying real shocks to the business. Predator-prey models have long been used in biology to describe fluctuations in the populations of interdependent species, such as rabbits and foxes. Our model illustrates how such dynamics can arise in a system where much of the competition is zero-sum in nature. Traders in financial markets not only create value by providing transactional services that improve risk sharing and reallocate capital to high return activities. They also extract value from each other through trades based on information or by exploiting temporary anomalies in prices.

In our model traders differ in ability. New entrants to the industry learn their type through experience. If they learn that they have above average ability, they become predators, extracting from each trading encounter an unequal fraction of the surplus created whenever they are paired with lower ability types. When a larger fraction of the population of surviving traders are high-ability, other things equal, exit by incumbent low-ability traders increases. This further increases the vulnerability of any remaining low-ability traders to predation, encouraging more exit, and so forth. We explore the conditions under which this can generate collapses in the population and trading activity in the industry, and the extent to which employment swings exaggerate the underlying dynamics in the shared surplus from trade.

We interpret the model in terms of employment cycles, but it could apply generally to other measures of trading activity and volume. If the trading that goes on is largely or predominantly zero-sum in nature, then our model would predict cascade-like crashes. In some cases these crashes are followed by periods of gradual rebuilding. In others they lead to a permanently lower level of activity.

Our model is aimed at understanding several features of the financial services industry.

First, financial activity shows dramatic contractions over the business cycle. Figure 1 shows employment in three service sectors, all involving high levels of education and compensation, over

the last decade, covering the two most recent recessions.¹ The drop in employment in the securities business is quite dramatic in both of the past two downturns. Table 1 list the percentage drop in employment from peak to trough, and the number of months from peak to trough, in each of the last two recessions for a range of financial services, business services, all service providers, and all goods providers. Services show less dramatic contractions than goods providers. Securities trading stands out among the highly professionalized services as having had dramatic contractions in both downturns, and these contractions, despite their severity, were completed relatively quickly. Figure 2 plots demeaned growth rates for employment in securities and manufacturing for New York State since 1990. In the early portion of the time period, the growth rates for manufacturing show more volatility than for the securities industry. The peaks and troughs for manufacturing tend to lie outside of those for securities. In the later decade, this relative volatility appears to be reversed.

Second, even in periods of contraction, Wall Street continues to hire “new blood.” This is evident from the employment statistics published on the web sites of leading business schools. For example, in 2008 and 2009, just under 50% of Wharton’s graduating MBAs took jobs in financial services. This actually exceeded the percentage entering that industry in 2005 and 2006. Layoffs have resumed more recently, as firms anticipate the effects of increased regulation on trading profits. Yet in November of 2011 the *Wall Street Journal* reported²: “With many firms trying to reduce pay by cutting highly paid staff, business students intent on a Wall Street career are continuing to find opportunities, although some schools are reporting a slowdown in interviews and less robust hiring than before the 2008 crisis.”

A natural explanation for continued entry in the presence of layoffs is learning. New entrants need to work in a job if they and their employers are to assess their suitability for it. Since compensation in finance for successful workers is very high, the real option value for new entrants is high even when their ex-ante expected initial productivity in the sector is relatively low. It is also higher than that of experienced workers for whom the match is unsuitable.

Third, much of the the surplus high-ability employees generate in finance is at the expense of

¹While longer series are available for more aggregated industry sectors (i.e., financial services generally), this period allows us to consider employment at the more specific level of “Securities, Commodities and Investment,” which more closely corresponds to the trading services that are the focus of our model.

²“Wall Street Pay Hits a Wall,” November 28, 2011, *Wall Street Journal*

their less able competitors. That is, much of the competition is zero-sum in nature. This is evident in the high volume of trade in financial securities relative to the exogenous demand from outside the sector for transactional services. For example, volume in the foreign exchange markets vastly exceeds the demand for foreign exchange transactions generated by international trade of goods and services. According to the “FX Volume Survey” of the New York Fed, average annual volume in foreign exchange is between 140 and 200 trillion USD.³ Gross international trade flows are close to 20 trillion USD per year.

Similarly, volume of commodity contracts traded on exchanges grew by nearly 100% in the three-years 2005-2008, and by nearly 50% in the three years in the period 2008-2010. This is beyond any conceivable growth in the underlying demand for hedging transactional or hedging services by producers and consumers of the underlying physical commodities. Indeed, the total volume of silver mined throughout history is estimated to be slightly less than 47 billion ounces, while the turnover of exchange traded silver in 2010 was approximately 90 billion ounces.⁴

Our setting combines these elements in a dynamic model. Each period surviving traders, knowing their ability level, decide whether to exit. These decisions are conditioned on knowledge of the aggregate surplus available through trade in the coming period. New entrants, who are uncertain about their ability, join the industry. Given the resulting mix of populations, traders are randomly matched. Traders of the same type who encounter each other share the surplus their trade generates equally. Traders of unequal ability share the gains to trade asymmetrically. Thus, for a given total population and aggregate surplus available, high-ability types expect to do much better if the low-ability types and new entrants comprise a larger proportion of the population.

We formulate the dynamic programming problem faced by the different types and solve for steady-state outcomes. There are often both high-population equilibria and low-population equilibria, because of the positive participation externalities created by entry and retention of low-ability traders. As with fish that form schools or birds that form flocks, the presence of many other similar traders reduces the chances any one of them will encounter a predator, and thus increases their continuation values. Thus, if a low-ability incumbent anticipates the survival of a large number of

³See <http://www.newyorkfed.org/FXC/volumesurvey/>.

⁴See the report “Commodities Trading, March 2011,” at <http://www.TheCityUK.com>.

his peers, he in turn will choose not to exit. If he anticipates exit by his peers, he optimally does likewise.

Indeed, our model illustrates how small shocks to fundamentals can trigger a chain reaction in employment and trading activity. A shock that triggers exit by the lowest ability traders leaves those with intermediate ability exposed to greater risk of predation, even when the shock leaves their profits positive holding fixed the composition of the population. We analytically characterize situations where effects on the composition of the population amplify real shocks, and through simulation illustrate the cycles than can result.

These behaviors are examples of the sort of multiplicity of equilibria that often arise in models characterized by what Chatterjee and Cooper [1989] call “positive participation externalities.” Such externalities play an important role in models of networks (Katz and Shapiro [1985]), search in employment (Diamond [1982]), and market microstructure (Pagano [1989]). The particular externality of interest in our setting is different in that a high proportion of a particular segment of the population protects such individuals from opportunistic behavior of others, rather than the density of the population itself being of common benefit.

Our simulations of the model reveal a wide range of possible dynamic behaviors. With relatively low volatility to the fundamental shock, and a high initial proportion of low-ability types, the population is likely to grow steadily to the high-population steady state. With more volatility, shocks will occur that trigger exit by the incumbent low-ability traders, and the industry then remains stuck in the low-employment equilibrium. In other cases, the population can initially contract to a point where it becomes attractive for low-ability traders to remain active, followed by population growth. The population can then collapse when a sufficiently negative shock to profitability occurs. A period of low employment then follows, until a recovery in aggregate profits causes the population to begin rebuilding, at first quickly and then more gradually, towards the high-employment state. This growth may at any point be interrupted by a negative shock sufficient to cause sudden return to the low-employment state. Thus, our relatively simple model generates boom-bust cycles that are quite suggestive of employment patters in financial services.

The paper is organized as follows. The next section describes the model. Section 3 illustrates

the forces at work for the case where the gains to trade are deterministic. This is sufficient to describe the amplification mechanism. We can also show in this setting that exit by the lowest ability forces out those of intermediate ability, causing collapses in the population. Section 4 provides simulation results when the shocks to profits are stochastic. We conclude in Section 5. Proofs for all formal results are gathered in the Appendix.

3.2 Model Structure

Consider an environment with overlapping generations of workers in the financial sector, whom we will hereafter refer to as “traders.” Individual traders in any given cohort are infinitesimal in size. A new generation of traders of measure M is born each period. Older traders die off between periods at rate $1 - \beta$, and can also decide to exit. Alternatively, and perhaps more realistically, we can view their employers as deciding to terminate their employment.

Traders are endowed with differences in ability or aptitude for the job, denoted θ . Higher-ability traders have an advantage in trading with lower-ability traders. For any entering cohort, a mass $f_0(\theta)$ of the population is θ -ability, but individual traders do not learn their type until after they have experience on the job. Let N_t denote the total (measure of the) population of traders at date t , after exit choices have been made.

We denote as $f_t(\theta)$ be the density of the population at date t with ability level θ , where $\theta \in \{\theta_L, \theta_H\}$. At each date, each trader is randomly matched with a counter-party, and they divide a surplus $\pi(z_t, N_t)$, which can depend on the size of the population and a random shock. The division of the gains to trade in any trading encounter depends on the ability of the two traders in the following way: if a type θ_i meets a type θ_j , the i -ability trader receives a share $\frac{1}{2} + (\theta_i - \theta_j)$ of the surplus.

Each trader also faces a per-period fixed cost of operating, denoted as c . Note that we allow $\theta_i - \theta_j > \frac{1}{2}$. The lower-ability traders may anticipate being net losers when encountering a counter-party of superior skill, but if the higher-ability counter-parties are a sufficiently small portion of the population, or are expected to be in the future, it may still pay for the low-ability agents to participate.

3.2.1 Payoffs

In this section we specify the periodic payoffs to the continuing traders, and to the new entrants. We then formulate their value functions.

Traders are randomly matched with counter-parties in any given period. The per-period payoff to a continuing type- θ trader, then, is:

$$\int_{\theta_L}^{\theta_H} f_t(q)\pi(z_t, N_t) \left[\frac{1}{2} + (\theta - q) \right] dq = \pi(z_t, N_t) \left[\frac{1}{2} + (\theta - \bar{\theta}_t) \right], \quad (3.1)$$

where $\bar{\theta}_t = \int_{\theta_L}^{\theta_H} f_t(\theta)\theta d\theta$.

The expected periodic payoff for a new entrant weights the chance that he has ability θ with density $f_0(\theta)$. The new entrant's expected payoff simplifies to:

$$\pi(z_t, N_t) \left[\frac{1}{2} + (\bar{\theta}_0 - \bar{\theta}_t) \right], \quad (3.2)$$

where $\bar{\theta}_0 = \int_{\theta_L}^{\theta_H} f_0(\theta)\theta d\theta$.

These expressions make clear that expected payoff to any one trader is decreasing in the average ability of the population. The lowest ability types are the most disadvantaged in competition with other traders, yet their exit, by raising average quality, makes things worse for the next lowest type and so on. This can lead to collapses in employment in response to an adverse shock, z_t .

3.2.2 Exit

Three state variables determine the decisions to continue or exit: the shock to per-period profits, z_t , the population carried forward from the previous period, N_t , and distribution of types, f_{t-1} , the latter being an object of infinite dimension. The shock is exogenous, and the population and distribution of types are endogenous. Each incumbent trader h chooses either to stay or exit the market. In making these choices they take as given the exit decisions of others, and thus, because each individual is of zero measure, they treat the aggregate quantities N_t and f_t as given.

An incumbent with ability θ stays in the market if his/her periodic profit and continuation value exceed the outside alternative, which is normalized to be zero. The value function is thus

given by:

$$v(\theta; z_t, N_{t-1}, f_{t-1}) = \max \left\{ \pi(z_t, N_t) \left[\frac{1}{2} + (\theta - \bar{\theta}_t) \right] - c + \beta E_t[v(\theta; z_{t+1}, N_t, f_t)], 0 \right\}. \quad (3.3)$$

In the results reported below, we treat the arrival of a new generation of M workers as exogenous and deterministic to simplify the analysis. While this simplification allows us to focus on the phenomena of most interest, it could easily be generalized to allow for endogenous entry, or stochastic amounts of new entry.⁵

3.2.3 Laws of motion

The endogenous state variables evolve as follows. The population of traders combines the new entrants with the incumbents who survive exogenous attrition at rate β , and who decide not to exit:

$$N_t = M + \int_{\theta_L}^{\theta_H} p_t(\theta) \beta f_{t-1}(\theta) N_{t-1} d\theta, \quad (3.5)$$

where $p_t(\theta) = 1$ if the agent with ability type θ finds it optimal to stay in the market in period t , and 0 otherwise. The density of traders with ability level θ , in turn, is

$$f_t(\theta) = \frac{M f_0(\theta) + p_t(\theta) \beta f_{t-1}(\theta) N_{t-1}}{N_t}. \quad (3.6)$$

3.2.4 Equilibrium

All agents behave competitively, and treat the state variables as independent of their own decisions. Equilibrium requires that each of the policy functions is a best response to the others, given rational expectations about the evolution of the state variables.

⁵The value function for new entrants, of course, is simply the expectation across the distribution of ability types, using the density $f_0(\theta)$.

$$v^0(z_t, N_{t-1}, f_{t-1}) = \int_{\theta_L}^{\theta_H} f_0(q) v(q; z_t, N_{t-1}, f_{t-1}) dq, \quad (3.4)$$

which must be above the entry costs to be consistent with voluntary entry by the newly arriving generation of traders. With endogenous entry, agents that decide to continue in the market deter the entry of newcomers. Hence, an agent of a given type benefits from the presence of other agents a similar type remaining in the market not only because of lower chances of trading with higher type, but also because of smaller number of entrants. Overall, endogenous entry leads to periods of greater contraction until natural attrition reduces the population sufficiently to trigger new entry.

3.2.5 Remarks

We would generally expect that the profitability per match would be declining in the size of the population, simply as a result of competitive forces. If there is persistence in shocks to profitability, then, positive shocks should increase the size of the industry, and negative shocks reduce it. The size of M and the state of the incumbent population, N_t and f_t , control the speed with which the system adjusts to unexpected shocks to profitability. If the measure of potential entrants is small, it might take multiple periods for new entry to eliminate rents to incumbent traders as a result of a positive and persistent shock to z_t . Similarly, if the existing population is large and has a large fraction of low ability types, large scale exit will take place in response to a negative and persistent shock, leading to a sudden drop in population. With continued bad news this decrease will slow, because fewer low-ability types will be present and adjustment must occur through natural attrition at the rate of $(1 - \beta)$.

Finally, since new entrants do not know their type, we can certainly have situations where the expected benefit to new entry is positive while negative payoffs for the low-ability types are leading to exit. Hence, we observe periods with simultaneous entry and exit.

Some things are simple to demonstrate simply through recursive argument. Since the periodic payoffs for the higher-ability types dominate those of the lower-ability types, the following must hold.

Lemma 3.2.1. *For any set of the state variables, if $\theta_i > \theta_j$ and the value functions exist,*

$$v(\theta_i; z_t, N_{t-1}, f_{t-1}) \geq v(\theta_j; z_t, N_{t-1}, f_{t-1}).$$

Accordingly, if any lower types remain, then no higher types will exit.

3.3 The Deterministic Case

To gain some intuition regarding the behavior of the model, we start by considering as a functional form for the per-period profit $\pi(z_t, N_t) = zN_t^{-\gamma}$ where N_t is the total population and z is constant.

Since profits fall with more competition, $\gamma > 0$. We further assume that

$$\frac{1}{2} \frac{z}{M^\gamma} - c > 0, \quad (3.7)$$

so that if all incumbents exit, the new generation earns positive rents. They will also have a positive valuation since they can always exit costlessly after one period.

High-Population Equilibrium

If there exists an equilibrium steady state where the endogenous quantities are independent of time, then $N_t = N$ and $f_t(\theta) = f(\theta)$. If traders exit only due to exogenous attrition, which affects all ability levels in the same way, then clearly $f(\theta) = f_0(\theta)$. The new entrants each period simply replace those dying off, and $M = (1 - \beta)N$. This determines the steady-state population:

$$N = \frac{M}{1 - \beta}. \quad (3.8)$$

If this is an equilibrium, it will clearly be the maximum (constant) population the industry can support.

The value functions for each type will just be the discounted infinite stream of periodic payoffs. If all types remain in the industry in equilibrium, the value of continuing for each type must be positive, or equivalently, their periodic profits must be positive. If they are positive for the lowest-ability traders ($\theta_i = \theta_L$), they must necessarily be so for the higher-ability traders by Lemma 3.2.1. Evaluating the profit for the lowest ability level, at $N = \frac{M}{1 - \beta}$ and $f(\theta) = f_0(\theta)$ for all θ , implies the following restriction on the parameters:

$$\frac{z(1 - \beta)^\gamma}{M^\gamma} \left(\frac{1}{2} + (\theta_L - \bar{\theta}_0) \right) - c \geq 0 \quad (3.9)$$

Notice in this expression that it will be more difficult to support a high population of traders the greater the portion of the population endowed with high ability, which implies higher $\bar{\theta}_0$, and the more dramatic the asymmetry in ability, $\theta_L - \bar{\theta}_0$.

Lower-Employment Equilibria

An alternative steady-state equilibrium might be one where all traders with ability below a certain level, θ^* exit after experience in the industry. If the population and the distribution of types are constant, $N_t = N$ and $f_t(\theta) = f(\theta)$, then the number of new entrants must just replace the lower-ability types who exit and the higher-ability types who die off. Define the following:

$$F(\theta^*) = \int_{\theta_L}^{\theta^*} f(\theta) d\theta$$

and

$$F_0(i^*) = \int_{\theta_L}^{\theta^*} f_0(\theta) d\theta.$$

The measure of the survivors must be $(1 - F(\theta^*))\beta N$, so the new entrants are

$$M = N - (1 - F(\theta^*))\beta N.$$

The density of the population with ability θ , for $\theta \geq \theta^*$ must be

$$f(\theta) = \frac{f_0(\theta)M + f(\theta)\beta N}{N}. \quad (3.10)$$

while for $\theta < \theta^*$, we have:

$$f(\theta) = \frac{f_0(\theta)M}{N}. \quad (3.11)$$

The expressions above specify the functional equation for $f(\theta)$ and N . On simplification we obtain the population in terms of exogenous variables:

$$N(\theta^*) = \frac{M(1 - \beta F_0(\theta^*))}{1 - \beta}. \quad (3.12)$$

The proportions of each type are

$$f(\theta) = \frac{f_0(\theta)}{1 - \beta F_0(\theta^*)}, \quad (3.13)$$

for $\theta \geq \theta^*$, and for the lower-ability types exiting each period,

$$f(\theta) = \frac{f_0(\theta)(1 - \beta)}{1 - \beta F_0(\theta^*)}, \quad (3.14)$$

It is evident from inspection that the population when all ability levels continue, $\frac{M}{1-\beta}$, is higher than the population given in equation (3.12). Since $F_0(\theta^*)$ and β are both positive fractions, $1 - \beta F_0(\theta^*) < 1$. Similarly, (3.13) and (3.14) imply $f(\theta) > f_0(\theta)$ for the higher-ability traders, while for those of lower ability their share of employment will be lower than their share of the entering cohorts. As in any steady state equilibrium, the valuations for the traders of each type must simply be their (constant) periodic expected payoffs, discounted as a perpetual stream using probability of survival,

$$\frac{1}{1 - \beta} \left[\frac{z}{N(\theta^*)^\gamma} \left(\frac{1}{2} + (\theta - \bar{\theta}(\theta^*)) \right) - c \right] \quad (3.15)$$

where $\bar{\theta}(\theta^*) = \int_{\theta_L}^{\theta_H} f(\theta)\theta d\theta$ is the average ability in the population, calculated using the densities in (3.13) and (3.14). Accordingly, for our conjectured equilibrium to be an equilibrium steady-state, we must have the above expression non-negative for $\theta \geq \theta^*$ and negative for $\theta < \theta^*$.

Since the average ability level $\bar{\theta}(\theta^*)$ will be increasing in the cutoff level, θ^* , for a given set of parameters it is quite possible to have these conditions holding for various levels of θ^* , including the high-population case where $\theta^* = \theta_L$. In such cases, the model will have multiple equilibria.

3.3.1 Population Cascades

A fall in aggregate profits, z , can render the high-population equilibrium infeasible. Since the average ability rises when low-ability traders exit, the drop in z can lead to a chain reaction, where even though the fall in profits leaves traders with moderate ability in a position to make profits when others remain, by driving out the weakest participants it renders those with intermediate ability more vulnerable to predation and thus causes them to exit as well. We illustrate this for the case of three types in Section XXXX.

We can illustrate the intuition, however, of the amplifying effect of the changing composition of the surviving population by considering any interior equilibrium. Suppose θ^* is an interior cutoff

point at an equilibrium steady state. Then, since the value function for this type is zero, the periodic profit must be zero, and we know that

$$\frac{z}{N(\theta^*)^\gamma} \left(\frac{1}{2} + (\theta - \bar{\theta}(\theta^*)) \right) = c \quad (3.16)$$

Taking the total differential of this expression yields

$$\begin{aligned} \frac{1}{N^\gamma} \left(\frac{1}{2} + [\theta^* - \bar{\theta}(\theta^*)] \right) dz - \frac{\gamma z}{N^{\gamma+1}} \left(\frac{1}{2} + [\theta^* - \bar{\theta}(\theta^*)] \right) dN \\ + \frac{z}{N^\gamma} \frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} d\theta^* = 0 \end{aligned} \quad (3.17)$$

$$(3.18)$$

Multiplying both sides by $\frac{N^\gamma}{z}$ and rearranging yields:

$$\frac{\frac{dN}{N}}{\frac{dz}{z}} = \frac{1}{\gamma} \left(1 + \frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} \frac{z}{\frac{1}{2} + [\theta^* - \bar{\theta}(\theta^*)]} \frac{d\theta^*}{dz} \right). \quad (3.19)$$

Substitute using the zero-profit condition for the trader with ability θ^* , and we get

$$\frac{\frac{dN}{N}}{\frac{dz}{z}} = \frac{1}{\gamma} \left(1 + \frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} \frac{z^2}{cN^\gamma} \frac{d\theta^*}{dz} \right). \quad (3.20)$$

The elasticity consists of two terms. The first simply reflects the amount of direct competition. As z falls N must also fall in an amount that reflects the parameter that governs the extent of congestion in the market, γ . The second term amplifies or mollifies this effect depending on whether

$$\frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} > 0,$$

since $\frac{z^2}{cN^\gamma} > 0$, and the cutoff ability rises as profits fall, $\frac{d\theta^*}{dz} < 0$.

Although it may not be the case if the surviving population is small relative to new entry, we would expect that generally $\theta^* - \bar{\theta}(\theta^*) < 0$ —average ability must exceed the ability of the worst

surviving trader. In such a case, if

$$\frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} > 0,$$

then the difference is becoming less negative: the pool is changing so that the worst trader is less far away from the average trader. This will be the case for the when the equilibrium distribution of ability is relatively flat and uniform in shape. As more of the distribution is dropped from the pool, the surviving traders all have roughly equal density, but they are concentrated within a smaller portion of the original interval over which the population is distributed. Then, the marginal trader is less far away from the average as ability rises. The second term in the elasticity is negative, and the composition effects of the drop in the surplus in this case tend to mitigate or mollify the drop in the surplus.

The response of average ability will amplify the effects of a drop in profitability if the distance between the ability of the worst survivor and the average grows (becomes more negative) as lower ability traders drop out. This will occur when the exiting traders have relatively high density, and thus a big effect on the mean. This will be the case when the distribution of types is discrete, or multi-modal, but it can also occur if the distribution is highly skewed. Then there are a small number of highly skilled traders who have a big impact on the mean, and large numbers of less able traders.

To see this analytically, we can use Now using 3.13 and 3.14 to express the average ability in terms of the distribution of types in the entering cohorts:

$$\bar{\theta} = \int_{\theta_L}^{\theta^*} \frac{1 - \beta}{1 - \beta F_0(\theta^*)} \theta f_0(\theta) d\theta + \int_{\theta^*}^{\theta_H} \frac{1}{1 - \beta F_0(\theta^*)} \theta f_0(\theta) d\theta \quad (3.21)$$

Now consider the derivative of $\bar{\theta}$ with respect to θ^* .

$$\begin{aligned} \bar{\theta}' &= \frac{\beta f_0(\theta^*)}{(1 - \beta F_0(\theta^*))^2} \left[\int_{\theta_L}^{\theta^*} (1 - \beta) \theta f_0(\theta) d\theta + \int_{\theta^*}^{\theta_H} \theta f_0(\theta) d\theta \right] \\ &\quad + \frac{1}{1 - \beta F_0(\theta^*)} \left[(1 - \beta) \theta^* f_0(\theta^*) - \theta^* f_0(\theta^*) \right] \\ &= \frac{\beta f_0(\theta^*)}{1 - \beta F_0(\theta^*)} (\bar{\theta}(\theta^*) - \theta^*). \end{aligned} \quad (3.22)$$

It then follows that

$$\frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} = \frac{1}{1 - \beta F_0(\theta^*)} (1 - \beta F_0(\theta^*) - \beta f_0(\theta^*)(\bar{\theta} - \theta^*)). \quad (3.23)$$

Taking a Taylor series expansion of $F_0(\bar{\theta})$ around θ^* , can rewrite the above equation as:

$$\frac{\partial [\theta^* - \bar{\theta}(\theta^*)]}{\partial \theta^*} = \frac{1}{1 - \beta F_0(\theta^*)} \left(1 - \beta F_0(\bar{\theta}) + \frac{1}{2} \beta f_0'(\theta^*)(\bar{\theta} - \theta^*)^2 + \dots \right). \quad (3.24)$$

Recall, the changes in average ability amplify the effects of a fall in surplus on the population if this expression is negative. The quadratic term has a sign that depends on $f_0'(\theta^*)$. This will be negative if θ^* is above the mode (or a mode) of the distribution: that is, if the proportion of low-ability types is falling off quickly as the cutoff ability rises. The first term, $1 - \beta F_0(\bar{\theta})$ is clearly positive, but it will be smaller the larger the fraction of types that lies below the average ability in the surviving population. If the original distribution of types is highly skewed to the right, as with a lognormal or an exponential distribution, this will be the case. In that situation a small number of very effective predators will be extracting most of the surplus from the broader population of market participants.

Figure 3 illustrates the set of equilibria for a specific example, where there are two interior equilibria. In this example, $\gamma = 1$. In each period $M = 1$ agents enter with ability level $\theta \in [0, \infty]$ distributed log normally with mean $\mu = 2.3$ and standard deviation $\sigma = 1.5$. Per-period operating costs are 0.5 and 5% of agents exit the market for exogenous reasons. The left-hand top diagram of the figure shows the density of incoming agents' ability. The right-hand top diagram represents the relationship between θ^* , the cut-off level of ability, average ability $\bar{\theta}$ and the losses of marginal trader due to difference in ability $\theta^* - \bar{\theta}$. At low levels of θ^* , the losses of the marginal trader increase with cut-off level of ability, so that in this range the effects of a drop in the surplus are amplified by the effect on average ability. With a drop in the level of z , the marginal trader would require a higher per-person share of overall surplus to cover those extra losses, meaning z/N goes up. In other words, N must fall more than z .

The plot at the bottom right shows the equilibria. The plot on the bottom left magnifies the

area close to zero to detail the behavior there. The curves in the plot represent the profits of the marginal trader for different levels of θ^* . The curved line are the profits, and the other line is the indicator function for when the profits are positive. The two intersect with the profits of the marginal trader are zero. The outer two of these intersections represent equilibria. Traders with ability below θ^* all would lose money if they entered, while traders with higher ability all make money. (At the third intersection, this is reversed, so it is not an equilibrium.) The intersection that occurs with a high value of θ^* is an equilibrium with a small population with very high ability. Here the losses to the marginal trader, $\theta^* - \bar{\theta}$, are reduced as the cutoff ability rises. The worst traders become more like the average survivor as the population contracts.

At the high-population equilibrium, the slope of $\theta^* - \bar{\theta}$ is negative, and as z falls, the marginal survivor moves further from the average because of the relatively high density of the departing marginal traders. This renders them more vulnerable to predation, leading more agent's to exit. This is the behavior we refer to as a “cascade” in the population. Specifically, in this example, $z = 47$. At the high-population equilibrium there are $N = 19.7618$ agents present in the market. When the surplus drops to $z' = 46.5$ (1.06% decrease), number of agents drops to $N' = 19.0522$ (3.59% decrease). That is, the percentage change in N is about 3 times more than z .

Co-Existence of Steady-State Equilibria: Positive and Negative Congestion

We begin by considering the case of two ability levels, $i \in \{H, L\}$, which is sufficient to illustrate some of the tensions at work in the model. Figure 4 shows the combinations of z , which governs profitability, and asymmetry in ability given by $(\theta_H - \theta_L)$, that support the different steady-state equilibria. Other parameters are held fixed. The fraction of new entrants with superior ability is $f_0^H = 0.4$. The survival probability for older generations is $\beta = 0.9$, the periodic operating cost is set to $c = 0.5$, $\gamma = 1$, and the measure of the new generation is $M = 0.25$. The figure labels the areas where the low- and high-population steady states exist. The area labeled “both” can sustain either steady state. In the unlabeled areas, neither equilibrium exists.

To sustain the high-population equilibrium, lower values of $\theta_H - \theta_L$ and higher values of z are required. This illustrates the offsetting effects of the two externalities in the model that affect the

survival of low-ability traders.

- There is a negative externality due to crowding.
- There is a positive externality due to the dilution of predators in the population.

The first effect, associated with increased competition, is a standard negative congestion externality. Each new entrant or surviving low-ability trader raises N , which reduces the profits available per individual trade for everyone. Because the agents have zero measure in this model, this takes the extreme form of each agent acting as if he has no effect at all on the aggregate. This standard externality does not depend on differential ability. It is evident in the model with one type from Section 3.1.

The negative effect of more traders on profits per trade may be offset by a positive externality that low-ability traders confer on each other (and on new entrants). This is analogous to herding by large herbivores or schooling by bait fish to avoid predation. If all low-ability traders remain each period, and that is common knowledge, then low-ability agents' profits may be non-negative, because they know there is a relatively high probability that they will be matched with other low-ability agents in trading encounters. If, instead, all the low-ability agents exit each period, and that is common knowledge, then each low-ability trader knows she is much more likely to encounter predators in trade, and she anticipates negative expected profits.

The above arguments clarify the positive congestion externality at work in the model—the expected profit per person may be positively related to the number of agents staying in the market, *despite* the negative dependence of $\pi(z, N)$ on the population size, N .

The high-ability agents also benefit from the presence of a larger proportion of low-ability traders. Their expected profit depends negatively on the probability, f_t^H , that they are matched with a counterparty of similar ability. If asymmetric ability can be viewed as the result of investments made by traders in education and technology, then these investments can be self-defeating. Greater asymmetry of ability benefits more talented traders given the population of counterparties available, but eventually drives potential prey out of the market. For sufficiently high θ , as is clear from Figure 4, the high-participation equilibrium becomes unsustainable. This argument bears

some similarity to Glode and Lowery [2012], where financial expertise is viewed as an arms race. There, however, counterparties refuse to trade, despite gains to doing so, due to adverse selection.

Which effect dominates for the high-ability traders—the larger population overall depressing profits through greater competition, or the greater availability of potential prey—depends on the speed with which profits fall as the population grows. This is governed by the parameter γ in the model. As $\gamma \rightarrow 0$, the increases in population have less and less effect on the unconditional expected profits per match (that is, averaging across possible types), which are just $\frac{z}{2N\gamma}$. In this case, the beneficial effect of a lower f_t^H in the high-participation equilibrium dominates for the high-ability types. On the other hand, as γ becomes very large, the effect of the larger population dominates. The following lemma formalizes these arguments.

Lemma 3.3.1. *If both steady states exist, then for γ sufficiently close to zero the high-ability traders earn more in the high-population equilibria, while for sufficiently large γ they earn less.*

Population Collapses

Since the payoff of any one type of trader depends on the average ability of the population, as well as on the size of the population overall, exogenous changes to aggregate profits, z_t , can trigger collapses in the population. A small change that causes the lowest ability traders to exit can have a domino effect. Since it shifts the average ability of the population, it renders continuation untenable for higher ability types, by reducing the opportunities they have to gain at the expense of others and exposing them to more predation by the most able and experienced traders in the population.

Table 2 illustrates this using the steady state equilibria in a numerical example with three types. In this example, $\beta = 0.95$, $c = 0.25$, $\gamma = 1$, and $M = 0.25$. The proportions of types in the entering cohorts are $f_0^L = 0.4$, $f_0^M = 0.3$, and $f_0^H = 0.3$, for low to high ability, respectively. Ability levels are $\theta_L = 0.6$, $\theta_M = 0.75$, and $\theta_H = 1.7$. The first two columns of the body of the table show the average ability, $\bar{\theta}(i^*)$, and population $N(i^*)$ assuming in the steady-state equilibria exist, for equilibria where all types participate, only the high- and middle-ability types participate, and where only the high-ability types participate. The remaining columns show the periodic payoffs to

each type of trader.

The parameter controlling the level of profits, $z = 10$, is set in this example so that the profits of the lowest ability traders is exactly zero in the full participation steady state. As the first row of the table shows, both the higher ability types are also making profits in this situation, so full participation is an equilibrium. The low-employment outcome is also an equilibrium. The bottom row shows that if only the high-ability types participate, they make positive profits, but neither the low- or mid-ability levels have positive payoffs. The middle row shows that it is not an equilibrium for the high and middle types to participate, if the low types do not. In this situation the mid-ability traders have negative profits, despite the fact that the top row shows at the same level of z they continue to have positive payoffs as long the lowest-ability traders participate, and despite the lower overall level of competition (N falls from 5.0 to 3.1).

Lowering z slightly below 10, then, has the effect of forcing out the lowest-ability traders, so that full-participation is no longer an equilibrium. While the drop in z does not of itself lead to negative payoffs for the mid-ability traders, the response of the lowest ability traders does, and thus forces their exit as well. In this way, a relatively small real shock can be amplified in its effect on the employment of traders.

Figure 5 illustrates the range of steady-state outcomes that can be supported. The shaded areas in the plot correspond to combinations of θ_H and z for which there are steady-state equilibria, holding the other parameters at the values in the example above. Notice that in some cases, a fall in z would move the industry from high employment to middle levels of employment. In other cases, a lower z is only consistent with the lowest employment steady state. The star on that diagram, on the boundary of the high-population area in the middle of the plot, indicates the combination of θ_H and z used in the numerical example in table 2.

3.4 Dynamics with Stochastic Profits

In this section we consider the evolution of the population when the shock to profits per match, z_t , evolves stochastically. We assume z_t follows an $AR(1)$ process with autocorrelation ρ and conditional variance σ . In particular, we assume that $z_t = \mu e^{\varepsilon_t}$ and $\varepsilon_t = \rho\varepsilon_{t-1} + e_t$, where e_t is iid

normal with zero mean and variance σ , and μ plays the role of a scaling parameter.⁶

Our purpose here is to illustrate, first, that the qualitative features of the deterministic case in the previous sections survive in a dynamic, stochastic setting. Often, multiple equilibria arise that support either high or low population levels.

Second, we show that a shock to the system that in one situation may have little effect on the path of population growth for the industry can, in another situation cause the population to fall dramatically. Not only does the high-population equilibrium suddenly cease to be viable. The drop in expected profits to any one low-ability trader falls by more than the magnitude of the real shock, because he or she becomes more exposed to predation by the high-ability traders. This triggers low-ability incumbents to exit, which in turn leaves traders of intermediate ability facing a higher probability of predation. The result is a collapse in population where all exit but the traders with the highest ability.

Consideration of the stochastic case also reveals a range of behaviors that are not apparent from the deterministic model.

When the fraction of new entrants with high ability is relatively large, the industry can become “trapped” in the low-population equilibrium. The percentage of high-ability traders in the industry jumps when low-ability traders leave in response to a negative shock. Even when aggregate profitability recovers, the population may remain stuck at a lower level because the threat of predation deters continuation by the relatively small number of low-ability new entrants who arrive each period. It can be difficult to accumulate the critical mass of prey needed to make continuation viable simply through the replenishment provided by routine new entry. This may help to explain why employment growth and subsequent contractions in financial services tend to be triggered by financial innovation. The new businesses lead to an increase in employment that refreshes the population by bringing in large numbers of new and undifferentiated workers. Even though they soon become differentiated by ability, the high population can be sustained unless and until a negative shock to fundamentals causes a dramatic “shake out.” The associated activities then continue as a

⁶The values of μ and c , the per period operating cost, jointly determine the average profitability. For instance, the expected periodic payoff (with symmetric ability) is $\frac{1}{2} \frac{\mu}{N_t^\gamma} - c$. In other words, for one \$1 of revenue, the expected cost is $\frac{2cN_t^\gamma}{\mu}$.

mature business with much lower employment even when profitability returns.

For other sets of parameters (when f_0^H is relatively low), the industry can alternate between high and low population outcomes. Following a favorable shock, the population begins to build towards the high-population steady state. This growth can then be interrupted by a downturn that triggers a drop to the low-population outcome, where the economy remains until a sufficiently favorable shock initiates, again, a period of growth. Thus, our relatively simple setting can replicate the phenomenon of steady growth interrupted by periodic busts with sudden contractions in industry employment. Varying the operating cost parameter, c , leads to variation in the average time in the boom and bust phases.

Solving for an equilibrium along a sample path away from the steady states requires an assumption about what equilibrium the incumbent agents think they will be in going forward, since there might be multiple equilibria for any set of parameters and state variables. In a given period t the expected profit of a low-ability agent can be positive given that all low-ability agents continue, and negative if all low-ability agents are exiting the system. In that situation we favor the high-population outcome—if low-ability agents can stay, they remain in the industry.

We allow for only two or three levels of ability in the model, in order to keep the number of state variables manageable. Because of this, the model does not generate realistic short-term dynamics. Along a given sample path, except for sudden collapses, the population grows or falls in a manner similar to the deterministic case away from the steady states. If the system starts out at a low level, given the anticipated path of z_t , then the low-ability types stay and the population grows steadily through new entry until the incumbent population is large enough that natural attrition exactly equals new entry—a steady state. Alternatively, if the population is initially too high given the anticipated path of z_t , low-ability traders will exit. Then, one of two things will happen. Natural attrition of the remaining high-ability types may exceed the mass of new entrants, leading to a gradual fall in the population until it is small enough that attrition is fully replaced with new entrants. Or, the low types all exit, and the system hits the steady state right away.

This illustrates our central point in a stark manner, as we will show in our simulations by comparing the results with asymmetric ability to what occurs when all traders share a common

ability level, θ . With symmetric ability in our model, all the fluctuations in the shock, z_t , are absorbed by profits and the population simply grows or declines smoothly towards a steady state. When there is asymmetric ability, there are dramatic fluctuations in population in response to small changes in fundamentals. As our simulations with three types show, there are chain reactions where a shock pushes the lowest ability types out of the population, leaving those of intermediate ability exposed to greater predation by the most talented competitors, and in turn pushing them out.

Table 3 lists the parameter values we employ in three scenarios for which we simulate the model using two types. The first two differ only in the scaling parameter, which governs average profitability. The third scenario has higher continuation cost, lower survival probability, and a lower incidence of high ability among new entrants. The third scenario leads to a lower fraction of high-ability traders in the low-population steady state.

Figures 6-10 illustrate the range of behaviors the model can generate with only two levels of ability. Figures 7 and 8 employ the same parameters. Each row of subfigures is associated with different starting values for the state variables N_t and f_t^H , the population and fraction of high-ability agents, respectively. The three plots in each row show, on the left, the population and the profits to the low-ability types along the sample path for z_t , plotted on the right. The middle figure plots the population and profits for the same parameters and starting values, but with symmetric ability $\theta_H = \theta_L$.

The difference between the first two scenarios lies with the scaling parameter, μ . For a given population, expected profits are higher, the larger is μ . Thus, the agents in the model simulated in Figure 6 face higher expected aggregate profits than in Figures 7 and 8. This is evident in the sample paths for z_t on the right-hand side of the figures. As a result of this, the population in Figure 6 converges smoothly to a high-population steady state. Notice that these steady states are at higher population levels than those associated with symmetric ability reflecting the positive externalities associated with congestion for the low-ability types. Of course, although they support higher populations, the equilibria with asymmetric ability result in lower profits per low-ability agent than the equilibrium with symmetric ability.

Figures 7 and 8, where there is lower unconditionally expected aggregate profits, illustrate how the population along some sample paths can collapse in response to a negative shock, when suddenly the higher-population equilibrium path ceases to be viable. In the model with symmetric ability convergence to the steady state is smooth, since in that case there is only one steady state. With asymmetric ability, the drop in the population can be much bigger than the associated drop in profits. Consider, for example, Case III, depicted in the bottom row of Figure 8. The negative shock that triggers the drop in population occurs at $t = 49$, and involves a fall in z_t of 36%. The population at this point, N_t , falls by 53%. In the next period z_t partially recovers, and rises by 35%, but the population barely responds, rising by only 0.03%, as it is close to the low-employment steady state. It remains at this low level, because the refreshment coming from the new entry of low-ability agents is never sufficient to make continuing in the industry viable for the low-ability traders. The industry remains stuck in the low-population equilibrium even when the fundamentals recover.

For the parameters we have considered so far, the term $\frac{1}{2} + (\theta_L - \bar{\theta})$ in the low-ability agents' payoffs is negative in the low-employment steady state. Thus, no matter how large the realization of the positive shock, z_t , profits remain negative. The low-population equilibrium thus becomes absorbing.

In the industry simulated in Figure 9 the fraction of new entrants with high ability is lower than in the cases previously considered ($f_0^H = 0.2$ instead of 0.4). There is also higher turnover of the incumbent population through natural attrition ($\beta = 0.85$ instead of 0.90). As a result, the replenishment through new entry involves more low-ability traders, leading to a smaller ratio of predators to prey. The industry now shows cycles of growth and crashes. The growth phase is at first rapid, and then more gradual as N_t approaches the high-population steady state. These periods of expansion are interrupted by crashes of short duration until the aggregate shock, z_t , recovers. For these parameters, $\frac{1}{2} + (\theta_L - \bar{\theta}) > 0$, so for high values of z_t , expected trading profits will exceed the continuation costs, encouraging the low-ability agents to continue in the industry.

The size of the continuation cost, as is evident in Figure 10, governs the time spent on average in the low-population, or “bust,” states. When c is low, so that profits associated with a given

shock are more likely to be positive, the industry recovers much more quickly. This can be seen in the simulation of the left of the figure, where the continuation cost is set to 1.00. In the middle figure, these costs are 1.30, and the system spends most of the time in the lower population steady state. In the panel on the right, the cost is set to the intermediate level of 1.15.

With only two types of agents, a drop in population that moves the industry from a path to one steady state to a path towards another is necessarily a large, discrete jump simply due to the discrete nature of the distribution of types. With more than two ability types, exit by the low-ability types in response to a negative shock imposes a negative externality on those of intermediate ability, who in turn may exist in a chain reaction to the shock to fundamentals.

Figure 11 illustrates how this can occur in the dynamic setting. The parameters are those listed in Table 4. Each row in the figure involves a different sample path the shock, and different scaling parameters for the distribution of z_t . The initial values for the state variables is the same across all three rows ($N = 0.5$, $f^L = 0.6$, $f^M = 0.3$, $f^H = 0.1$). The first two rows of the figure show familiar behaviors. In the top row, the population steadily converges to the high-population steady state, where all three types remain profitable. In the middle row, a negative shock drives out both the low- and middle-ability types, because profitability for the mid-ability traders depends positively on the presence of lower-ability traders. The bottom row shows that in some cases a negative shock drops the population to an intermediate level, while in others it can cause a collapse in which only the highest ability agent's survive in the industry.

3.5 Conclusion

The zero-sum nature of competition in the financial sector may contribute to the dramatic fluctuations in employment in that industry. In our model, traders of low ability benefit from the presence of other low-ability traders. It reduces the chances they will encounter predators, who are able to extract from them most of or more than the aggregate surplus available in any one trading encounter. This gives rise to multiple steady-state equilibria, with both high and low employment. Negative shocks to aggregate profitability can make the high-employment equilibrium disappear, after which the population gradually rebuilds to a high-employment state. Negative

shocks to fundamentals that would leave traders with intermediate levels of skill profitable holding fixed the composition of the population, will lead them to exit if trading becomes unprofitable for those of lower ability, leaving the intermediate types exposed to greater risks of predation. Thus, even our very simple setting, with finite types and uncomplicated dynamics for fundamentals, can generate the types of cyclical behavior and dramatic contractions typical of the “boom and bust” employment cycles on Wall Street.

APPENDIX

Proof of Lemma 3.2.1: Suppose there is a terminal date for the industry, T , and for $t < T$, $i > j$, $v(\theta_i; z_{t+1}, N_t, f_t) \geq v(\theta_j; z_{t+1}, N_t, f_t)$ for all possible values of the state variables. Then $E_t[v(\theta_i; z_{t+1}, N_t, f_t)] \geq E_t[v(\theta_j; z_{t+1}, N_t, f_t)]$, and

$$\begin{aligned} & \pi(z_t, N_t) \left[\frac{1}{2} + (\theta_i - \bar{\theta}_t) \right] - c + \beta E_t[v(\theta_i; z_{t+1}, N_t, f_t)] \\ & \geq \pi(z_t, N_t) \left[\frac{1}{2} + (\theta_j - \bar{\theta}_t) \right] - c + \beta E_t[v(\theta_j; z_{t+1}, N_t, f_t)] \end{aligned} \quad (\text{A-1})$$

Since $\theta_i > \theta_j$. If the left-hand side and right-hand side are both positive, then both types will survive and it follows that $v_t^i \geq v_t^j$. If the left-hand side is positive and the right hand side is zero or negative, then type θ_i survives, while type θ_j exits, and the same result follows. If both sides are negative, then both types exit, and both value functions equal zero.

One period prior to the terminal date T , the value functions clearly satisfy the inequality of interest, because they consist only of the final, periodic payoff. The value functions for the infinite horizon case, as limits to the value functions as the horizon recedes, then satisfy the inequalities as long as these limits exist. \square

Proof of Lemma ??: For the case where profits are positive at the maximum steady-state population, the arguments in the text show that at any $p_t < p^* = 1$. All incumbents would benefit by choosing $p_t^i = 1$. There is no possible population level that can be sustained that is higher than $\frac{M}{1-\beta}$, since at this level new entry exactly offsets natural attrition.

When

$$\frac{z(1-\beta)^\gamma}{2M^\gamma} - c < 0,$$

a steady state will be at that population level where periodic profits are zero: N^* as given by equation ?? . The profit function is monotonically declining and continuous at in N_t . At $N_t = 0$, profits are positive by assumption (see (3.7)). With $\gamma > 0$, profits must be negative for sufficiently large populations. It follows that N^* is unique. Suppose there is a steady-state equilibrium population lower than N^* . Profits will be positive at this population level. Since exit is costless in this model,

the value function must be non-negative. Thus, any and all incumbents will prefer to continue, and set $p_t = 1$, which implies the population will grow and the conjectured population is not a steady state. Suppose the population level exceeds N^* . Then periodic profits will be negative. If this is a steady state, the value function must simply be the discounted value of this infinite stream of negative profits. Thus, all incumbents would prefer to exit, a contradiction. \square

Proof of Lemma 3.3.1: By equations (3.8) and (3.12), the populations in the two steady states are related as

$$N^* = N(1 - \beta(1 - f_0^H))$$

where N is the high-population level and N^* is the low-population steady-state outcome. Neither depends on γ .

The profits for the high-ability agents in the high-population steady state, less their profits in the low-population steady state are:

$$\frac{z}{(N)^\gamma} \left[\frac{1}{2} + (\theta_H - \bar{\theta}_0) \right] - \frac{z}{(N^*)^\gamma} \frac{1}{2} \quad (\text{A-2})$$

Using the relation between the two populations, this can be simplified to:

$$\frac{z}{(N)^\gamma} \frac{1}{2} \left(\left[1 - \frac{1}{(1 - \beta(1 - f_0^H))^\gamma} \right] + (\theta_H - \bar{\theta}_0) \right)$$

The sign of this expression clearly depends on the term in brackets. The difference between θ_H and $\bar{\theta}_0$ is clearly positive, and does not depend on γ . The term in square brackets is positive as $\gamma \rightarrow 0$. Since both β and f_0^H are fractions, however, this expression goes to $-\infty$ as $\gamma \rightarrow \infty$, and will determine the sign of the term in brackets. \square

	% Decrease		Months	
	2001	2008	2001	2008
Securities, Commodities & Investment	10.96	8.67	32	22
Commercial Banking	0.51	4.41	7	22
Insurance	0.55	4.25	35	21
Funds, Trusts & Others	7.13	5.36	20	26
Legal Services	NA	5.95	NA	38
Accounting & Bookkeeping	11.34	10.07	22	35
Architecture & Engineering	4.70	12.34	24	31
Computer Systems	18.36	2.15	30	13
Management & Technical Consulting	3.10	2.35	17	20
All Service Providers	0.45	3.87	6	24
All Goods Producers	11.64	21.24	35	38

Table 1: Employment change in different sectors from peak to trough.

Types Participating	Population	Average Ability	Profits for Types		
	N	$\bar{\theta}$	H	M	L
$\{H, M, L\}$	5.00	0.975	2.20	0.30	0.00
$\{H, M\}$	3.10	1.205	2.96	-0.10	-0.59
$\{H\}$	1.68	1.592	3.38	-2.29	-3.19

Table 2: Profits at different levels of participation.

Parameter	Scenario 1	Scenario 2	Scenario 3
Impact of population on profits (γ)	1.00	1.00	1.00
Continuation cost (c)	0.50	0.50	1.15
Measure of new entrants (M)	0.25	0.25	0.25
Survival probability (β)	0.90	0.90	0.85
Conditional volatility of shock (σ)	0.20	0.20	0.20
Autocorrelation of shock (ρ)	0.60	0.60	0.60
Advantage to ability ($\theta_H - \theta_L$)	0.7	0.7	0.7
Scaling parameter for shock (μ)	6.00	5.00	5.00
Proportion of high-ability types among entrants(f_0)	0.40	0.40	0.20

Table 3: Parameter values for simulations with two levels of ability.

Parameter	Scenario I	Scenario II	Scenario III
Advantage to ability $\begin{pmatrix} \theta_H \\ \theta_M \\ \theta_L \end{pmatrix}$	$\begin{pmatrix} 1.3 \\ 0.65 \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 1.5 \\ 0.65 \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 1.3 \\ 0.65 \\ 0.6 \end{pmatrix}$
Impact of population on profits (γ)	1.00	1.00	1.00
Continuation cost (c)	1.15	1.15	1.15
Measure of new entrants (M)	0.25	0.25	0.25
Survival probability (β)	0.85	0.85	0.85
Conditional volatility of shock (σ)	0.20	0.20	0.20
Autocorrelation of shock (ρ)	0.60	0.60	0.60
Proportion of types by ability among entrants $\begin{pmatrix} f_0^H \\ f_0^M \\ f_0^L \end{pmatrix}$	$\begin{pmatrix} 0.3 \\ 0.2 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3 \\ 0.2 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3 \\ 0.2 \\ 0.5 \end{pmatrix}$
Scaling parameter for shock (μ)	10.00	7.00	5.00

Table 4: Parameter values for simulations with three levels of ability.

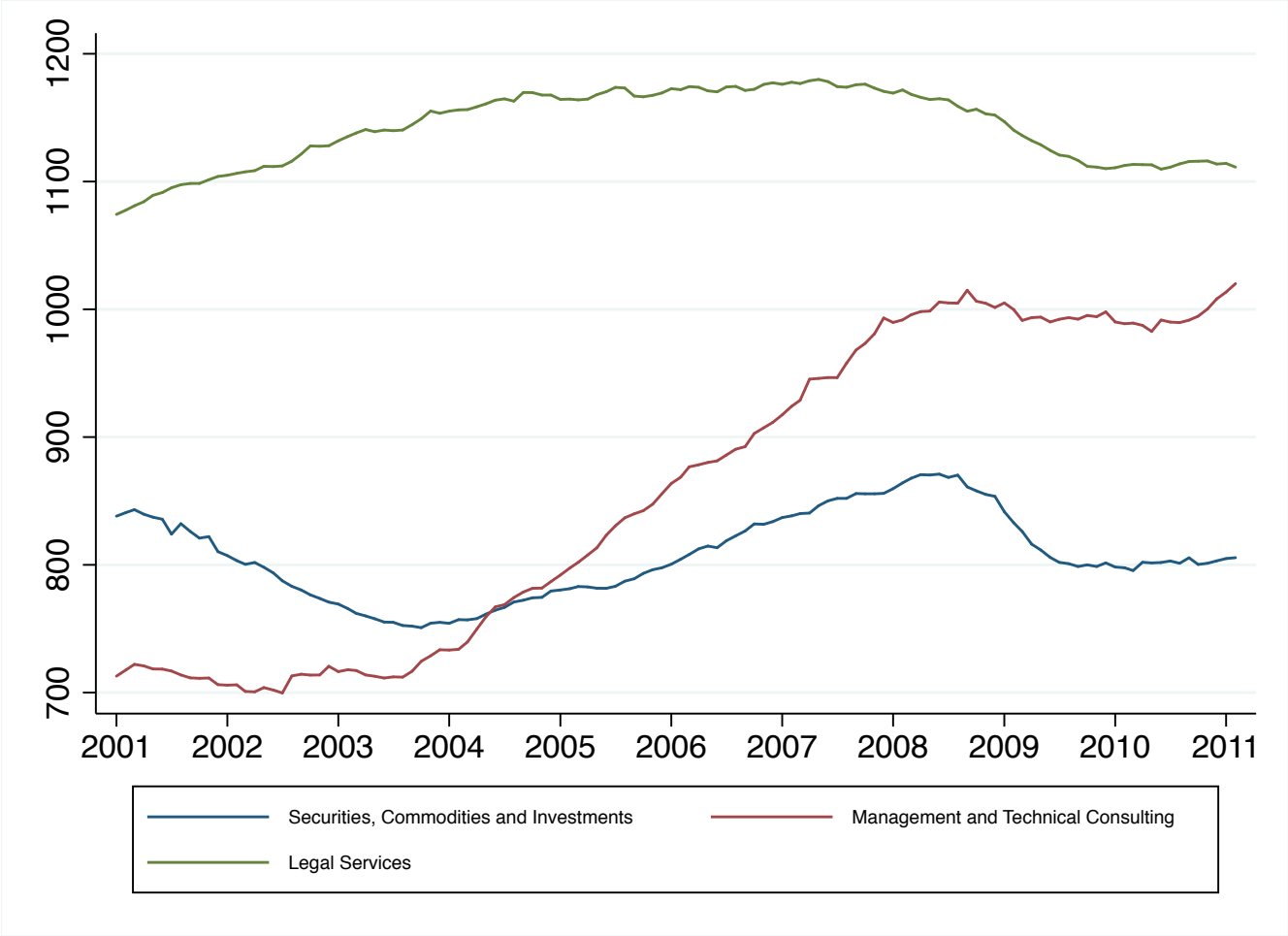


Figure 1: Employment, in thousands of employees. Graph is based on a monthly data from January, 2001 to February, 2011. Industry classification and employment statistics are taken from U.S. Bureau of Labor Statistics.

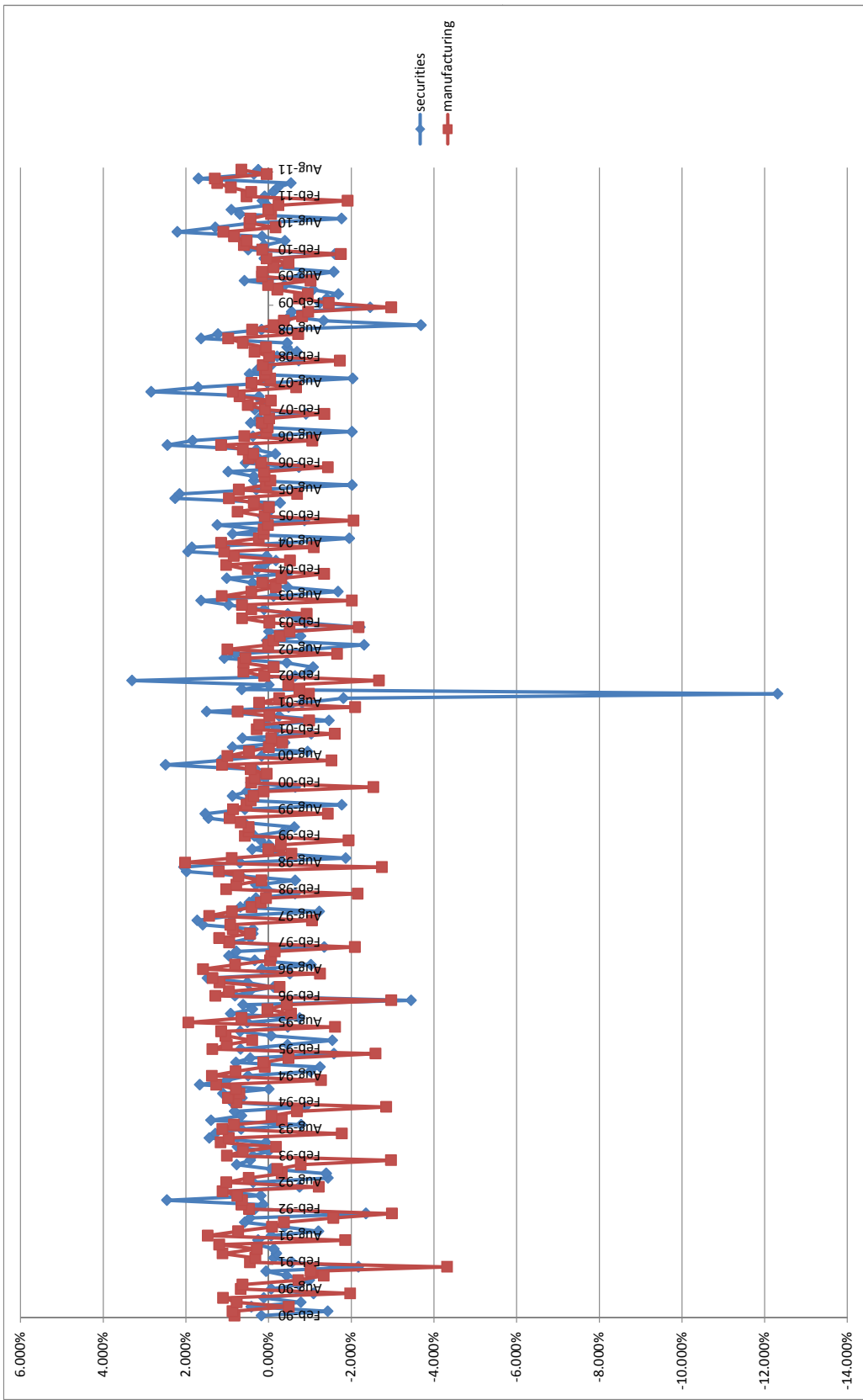


Figure 2: Monthly growth rates in employment less their time series average for manufacturing and the securities industry in New York State, January 1990 to August 2011.

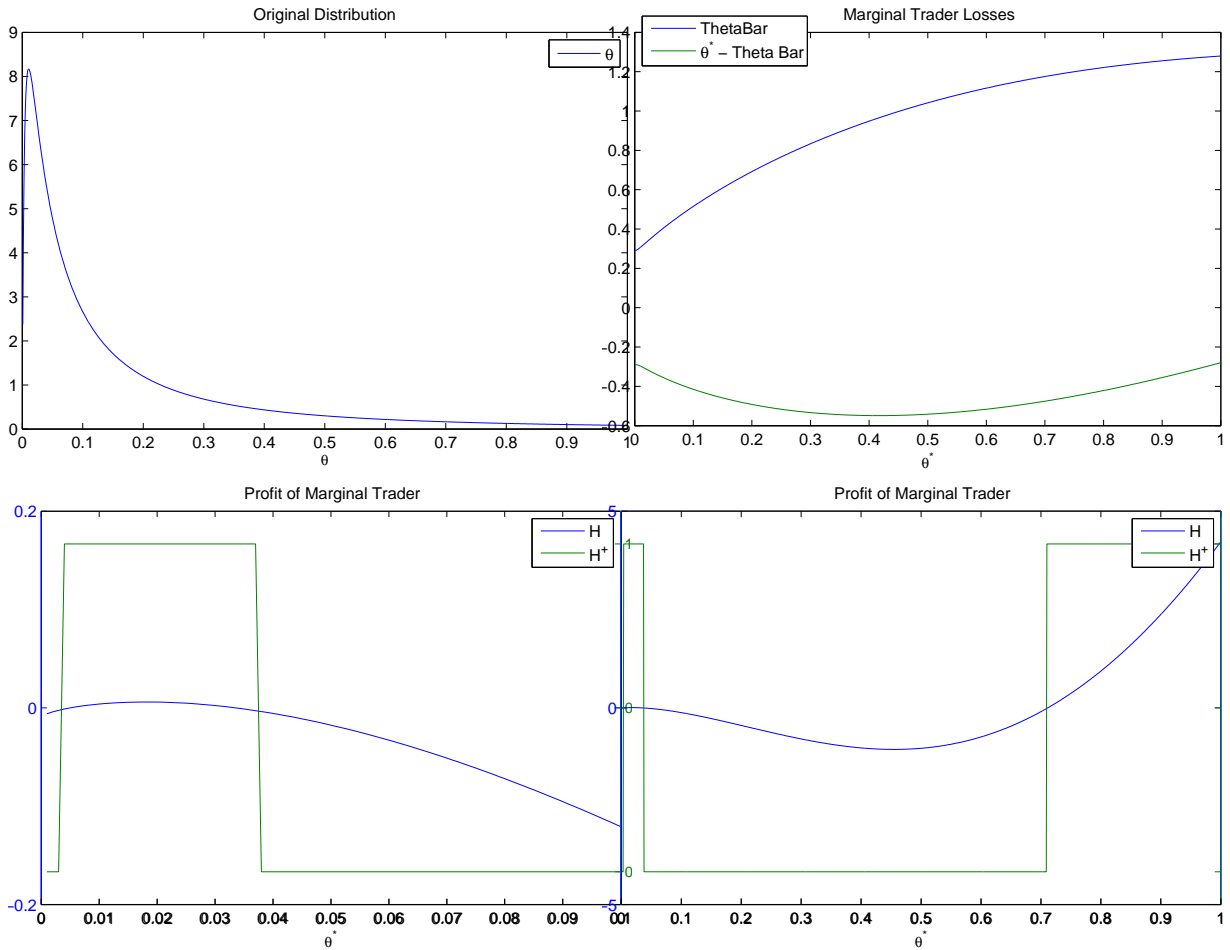


Figure 3: The top left-hand diagram represents the density of incoming agents, $f_0(\theta)$. The right hand top diagram represent the relationship between the cut-off level θ^* , the average ability level $\bar{\theta}$, and the losses of marginal investor due to differences in ability, $\theta^* - \bar{\theta}$. The effects of a shock are amplified where $\theta^* - \bar{\theta}$ is decreasing. The second row of plots represents the profits of the marginal trader (curved line) and an indicator function for when those profits are positive or negative. For the system to be in a steady state equilibrium, $H(\theta^*) = 0$, and the two curves must intersect. The left-hand figure magnifies the portion of the right-hand figure close to zero. There are two possible equilibria—one with cut-off level of ability next to zero (Big Population equilibrium) and one with θ^* close to 0.7 (Small Population equilibrium). In the latter case, only very skilled agents find it profitable to stay in the market. The third row of plots illustrates what happens to the system if income drops by 1%. The cut-off levels for both equilibria increase, and in case of Big Population equilibrium, N drops more than z .

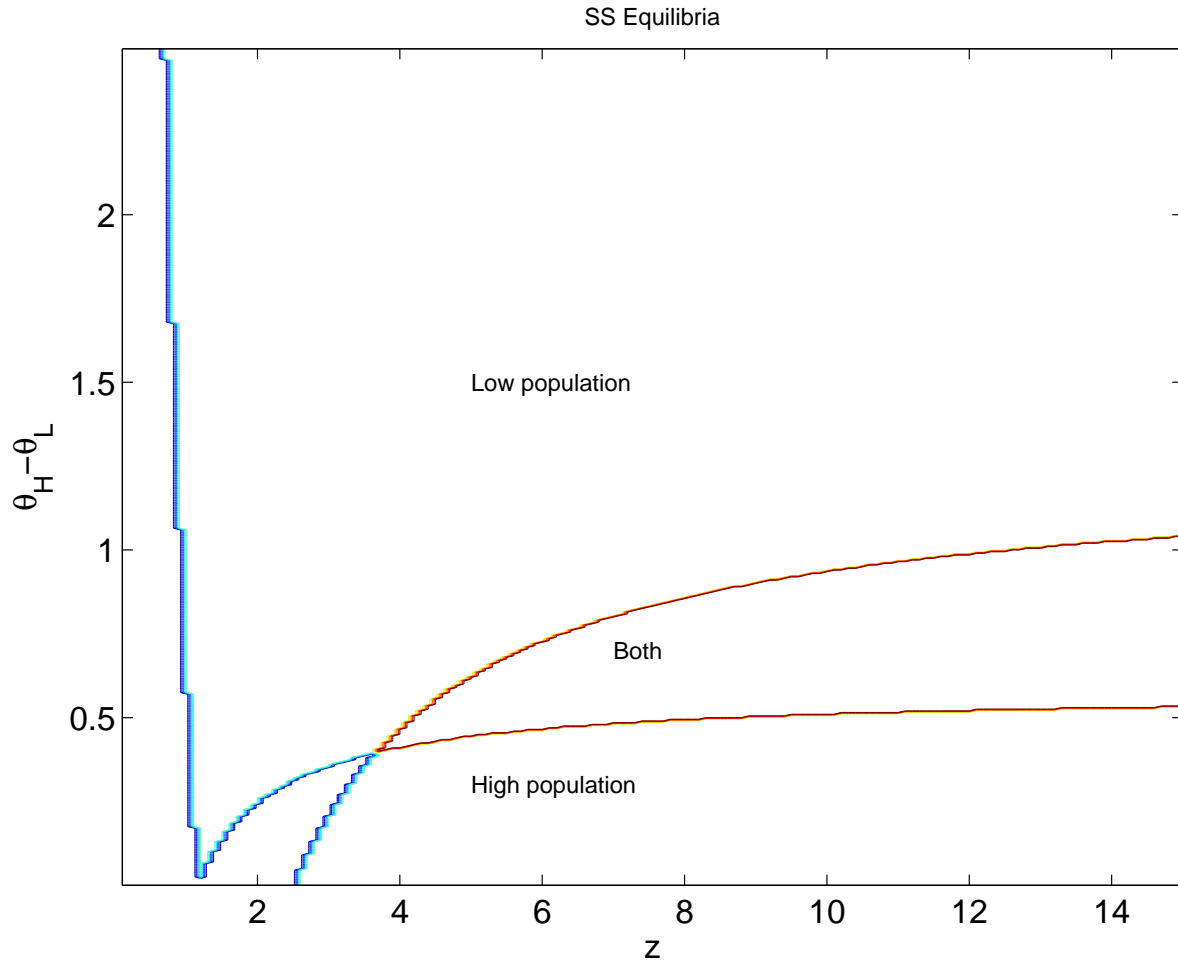


Figure 4: For $f_0^H = 0.4$, $\beta = 0.9$, $c = 0.5$, $M = 0.25$ the graph represents possible sustainable steady state equilibria. “High Population” area corresponds to pairs of $(\theta_H - \theta_L, z)$ that support steady state equilibrium where both types of traders survive. In the “low-population” area, there is a steady state in which the low-ability traders all exit. The intersection, labelled “Both,” contains parameter values where both steady-state equilibria are sustainable.

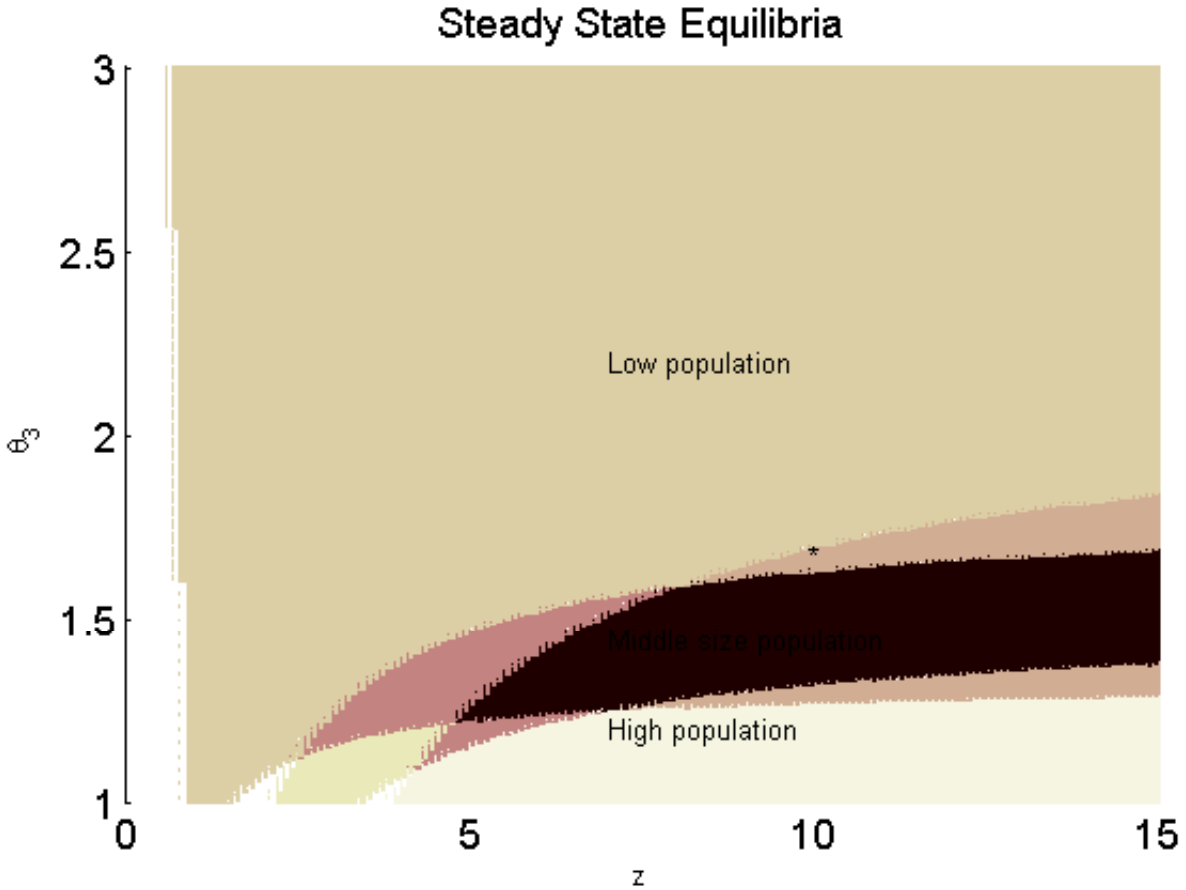


Figure 5: For $f_0 = (0.4, 0.3, 0.3)$, $\beta = 0.95$, $c = 0.25$, $M = 0.25$, $\theta_L = 0.6$ and $\theta_M = 0.75$ the graph represents possible sustainable steady state equilibria. “High population” area corresponds to the pairs of (θ_H, z) that support steady state equilibrium where all types of traders survive. In the “Low-population” area, there is a steady state in which the low and middle ability traders all exit. “Middle size population” refers to the case when only low ability traders exit while middle and high ability traders find it optimal to continue. Darker areas represent parameter values that support multiple equilibria. The central, darkest areas contains parameter values that support all three possible equilibria. The top areas and the area immediately below on the right are where only low-population and high-population equilibria are possible. These are of interest to our analysis as even smallest changes in z can trigger the change from high to low population, without passing through mid-level equilibria. White areas correspond to situations when none of the steady state equilibria are sustainable.

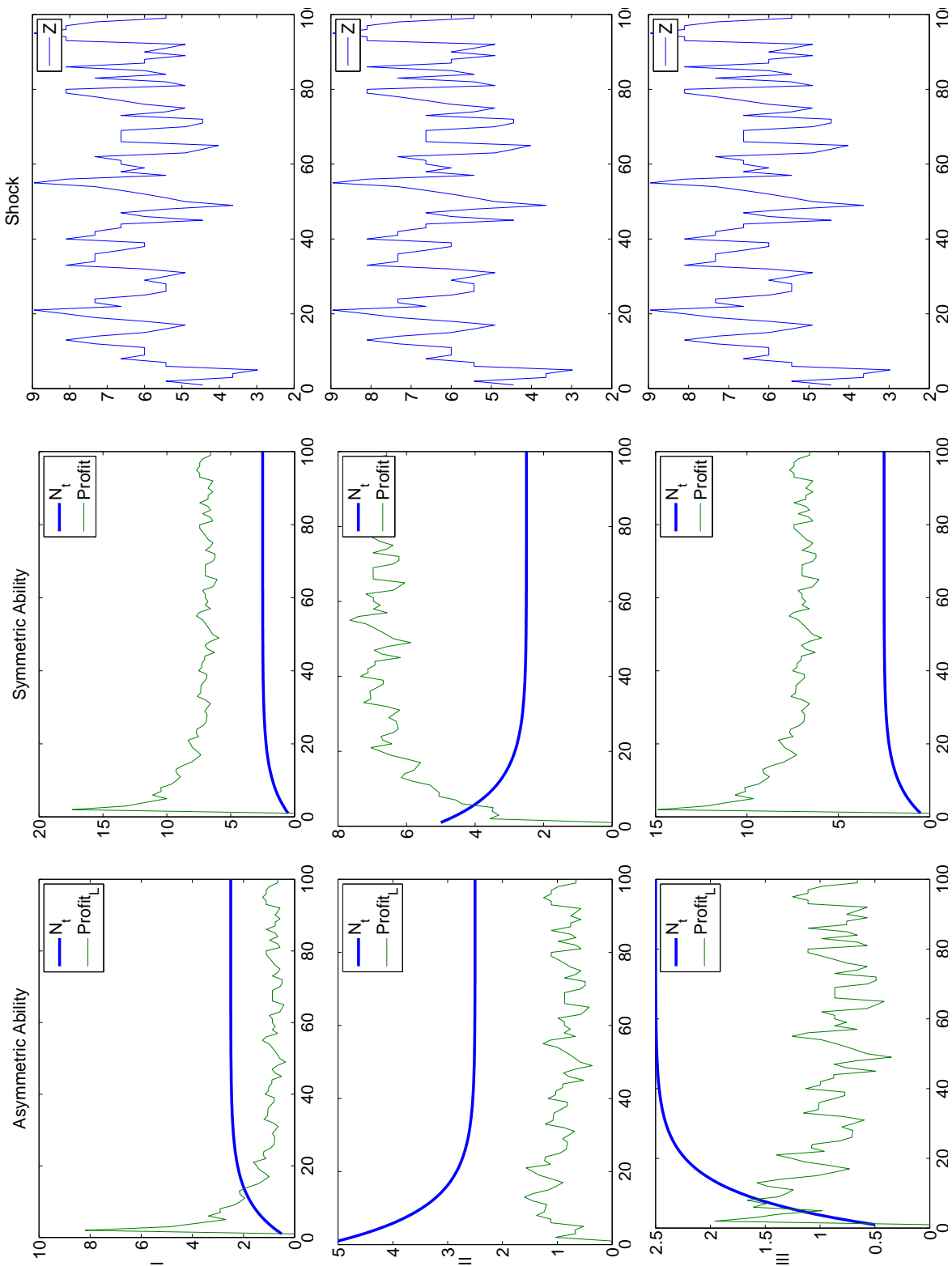


Figure 6: Scenario 1. For parameter values listed under Scenario 1 of Table 3, each row of subfigures plots the equilibrium outcomes along a sample path for the shock, z_t , which is plotted on the right-hand side. The left-hand figures plot the population and periodic profits of the low-ability types under asymmetric ability, while the center plots show the same quantities when when ability is symmetric ($\theta_H = \theta_L$). The first row corresponds to the starting values I — low proportion of skilled traders ($f_t^H = 0.08$) and small number of agents in the system ($N_t = 0.5$). Second and third row are looking at starting values II ($f_t^H = 0.08$, $N_t = 5$) and III ($f_t^H = 0.8$, $N_t = .5$)

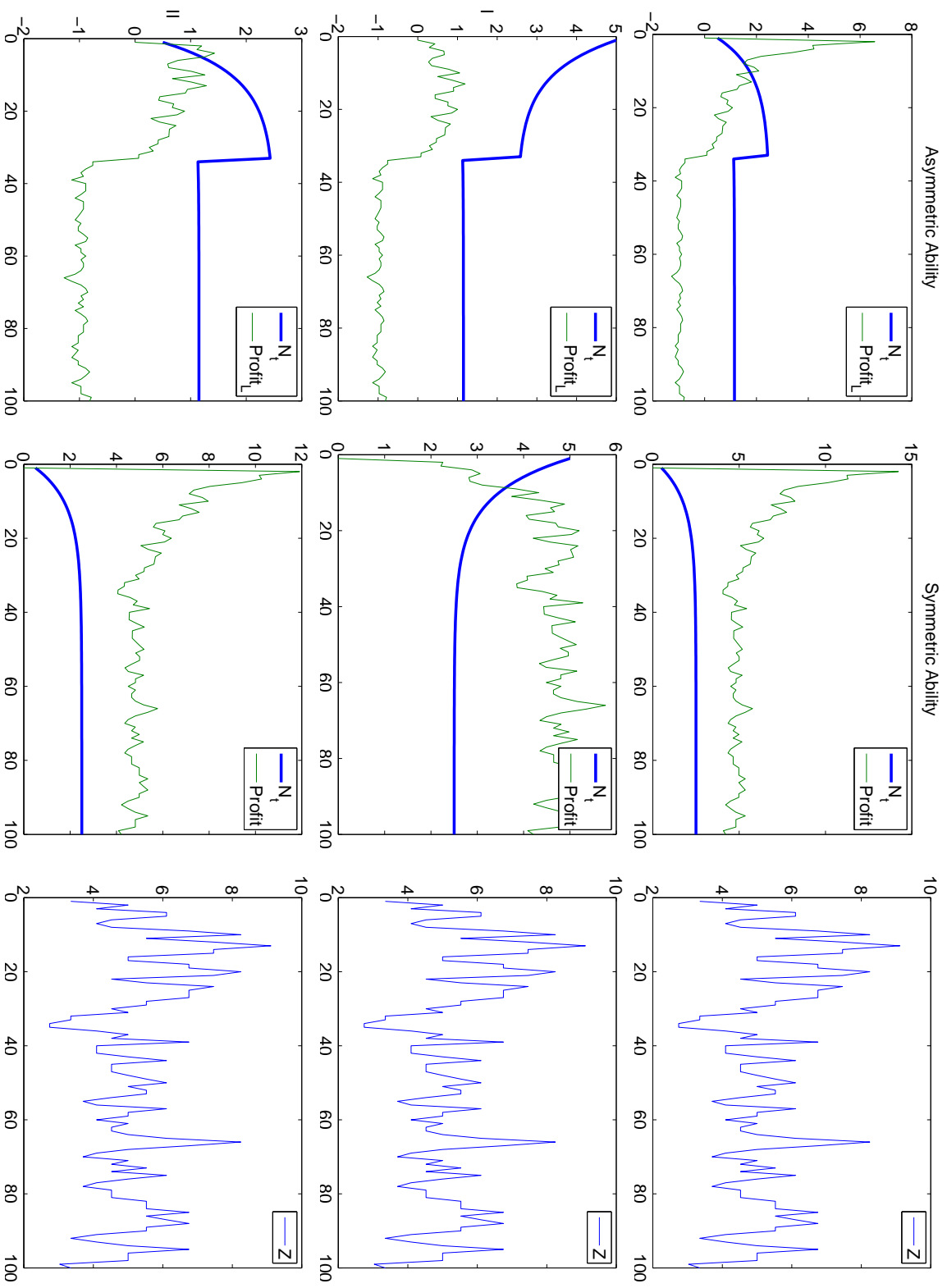


Figure 7: Scenario 2a. For parameter values in column two of Table 3, each row of subfigures plots the equilibrium outcomes along a sample path for the shock, z_t , which is plotted on the right-hand side. The left-hand figures plot the population and periodic profits of the low-ability types under asymmetric ability, while the center plots show the same quantities when ability is symmetric ($\theta_H = \theta_L$). The first row corresponds to the starting values I — low proportion of skilled traders ($f_t^H = 0.08$) and small number of agents in the system ($N_t = 0.5$). Second and third row are looking at starting values II ($f_t^H = 0.08$, $N_t = 5$) and III ($f_t^H = 0.8$, $N_t = .5$)

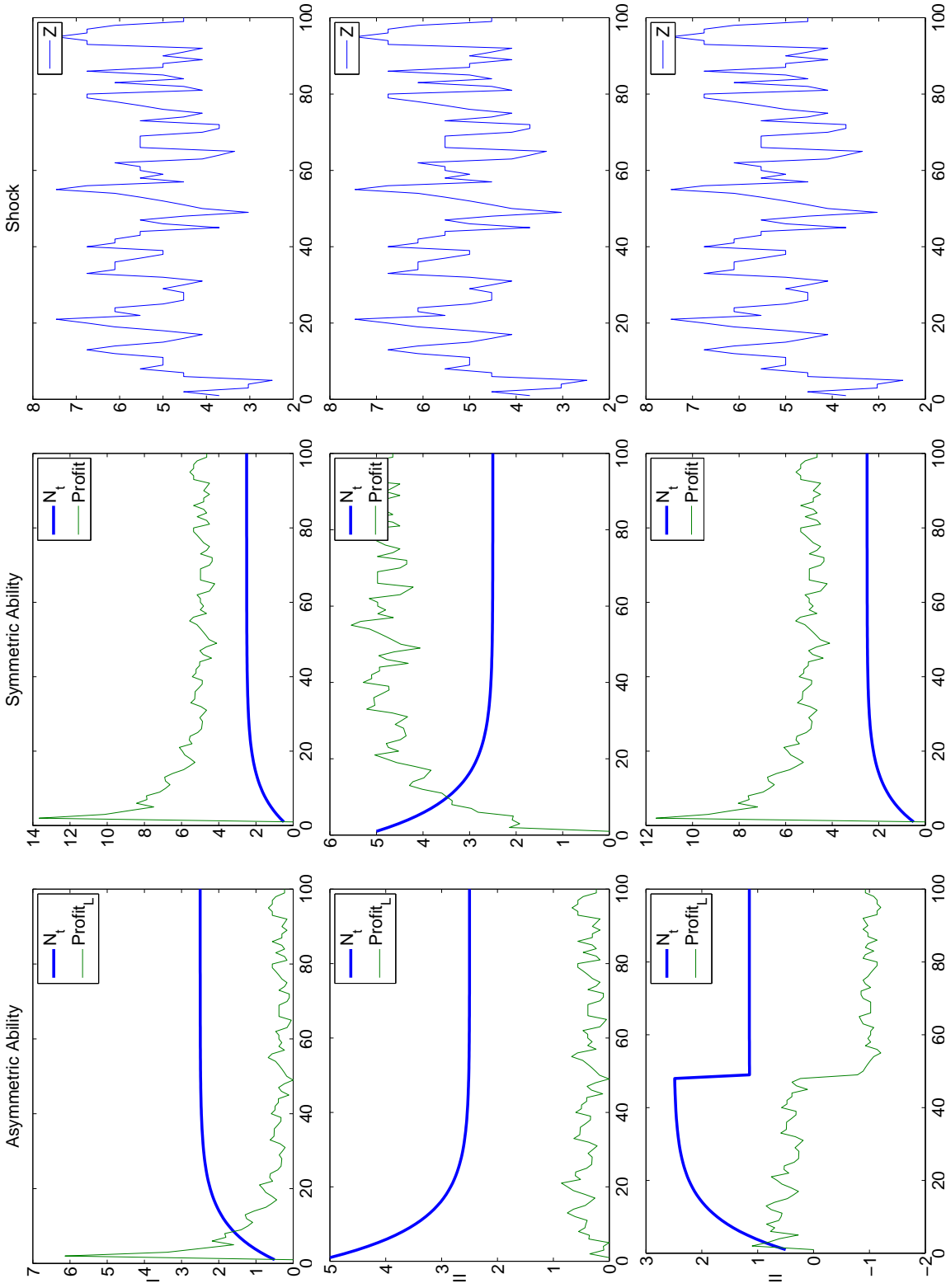


Figure 8: Scenario 2b. For parameter values in column two of Table 3, each row of subfigures plots the equilibrium outcomes along a sample path for the shock, z_t , which is plotted on the right-hand side. The left-hand figures plot the population and periodic profits of the low-ability types under asymmetric ability, while the center plots show the same quantities when ability is symmetric ($\theta_H = \theta_L$). The first row corresponds to the starting values I — low proportion of skilled traders ($f_t^H = 0.08$) and small number of agents in the system ($N_t = 0.5$). Second and third row are looking at starting values II ($f_t^H = 0.08$, $N_t = 5$) and III ($f_t^H = 0.8$, $N_t = .5$)

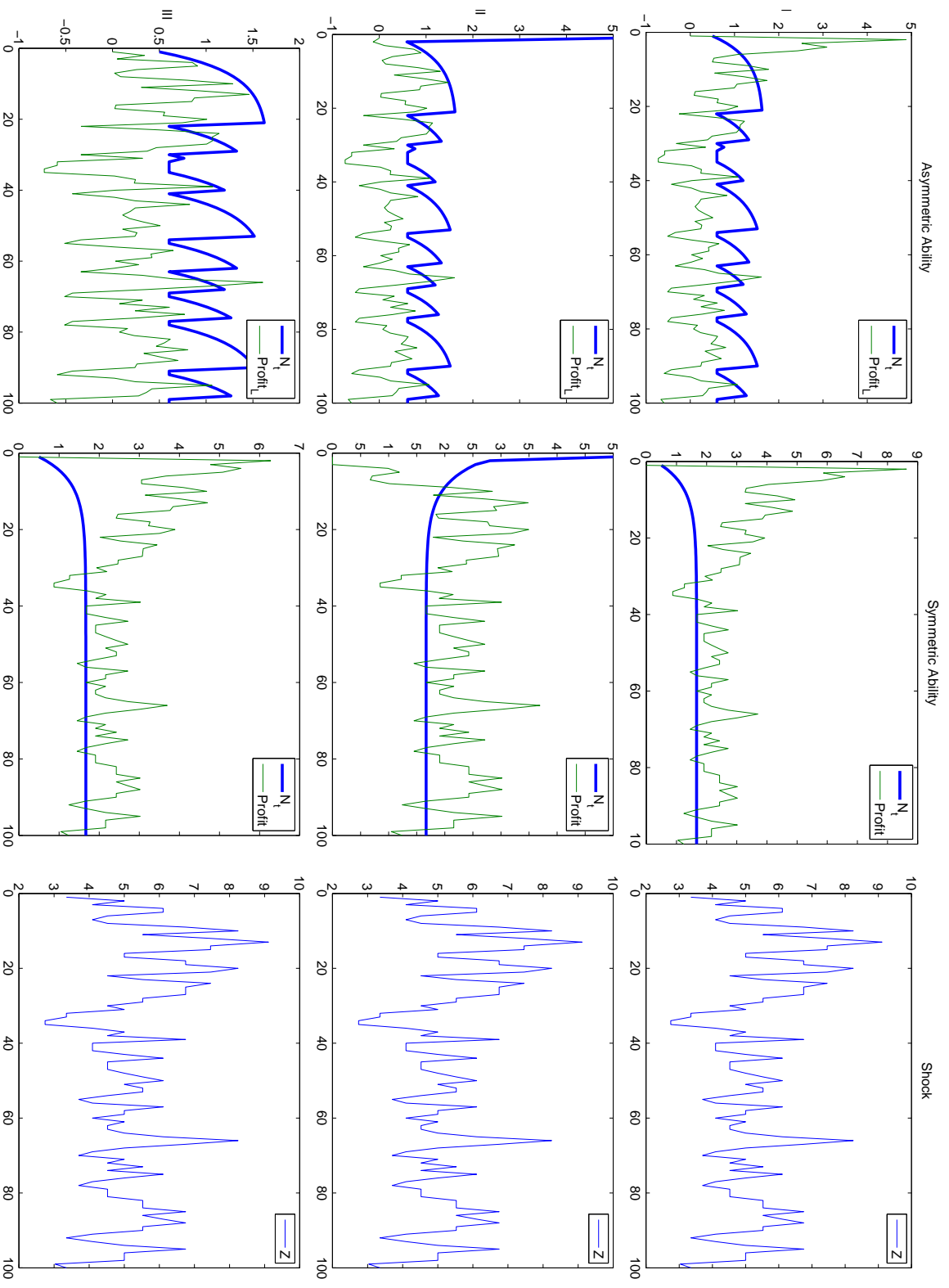


Figure 9: Scenario 3. The simulations illustrate settings where the system recovers from bad shocks relatively fast spending time in both equilibria. The first row corresponds to the starting values I — low proportion of skilled traders ($f_t^H = 0.08$) and small number of agents in the system ($N_t = 5$). Second and third row are looking at starting values II ($f_t^H = 0.08$, $N_t = 5$) and III ($f_t^H = 0.8$, $N_t = .5$)

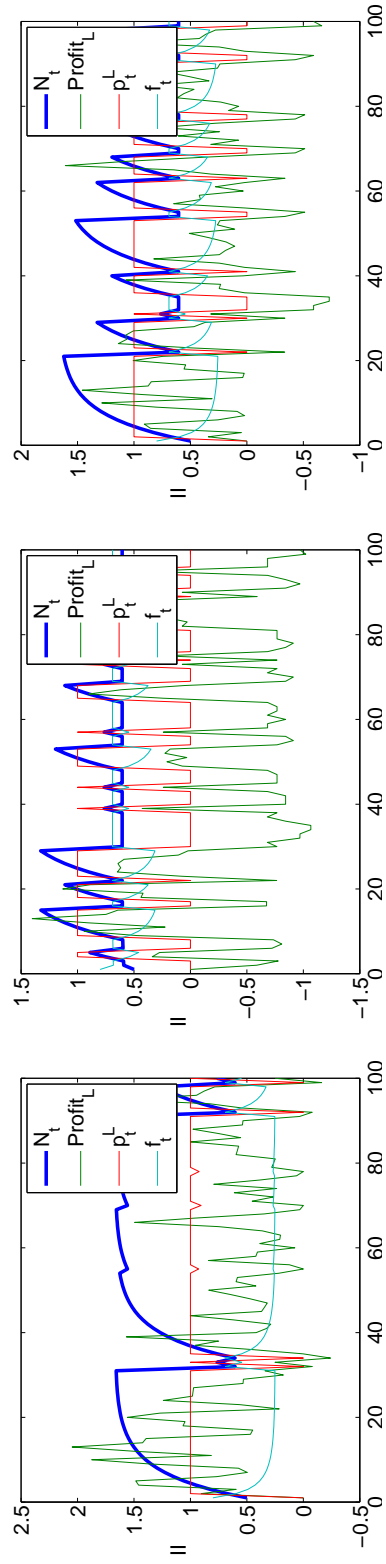


Figure 10: Variation in time spent in low-population equilibria: The different simulations illustrate the effect of varying the continuation cost, c , fixing the other parameters at the values in Scenario 3 in Table 3.

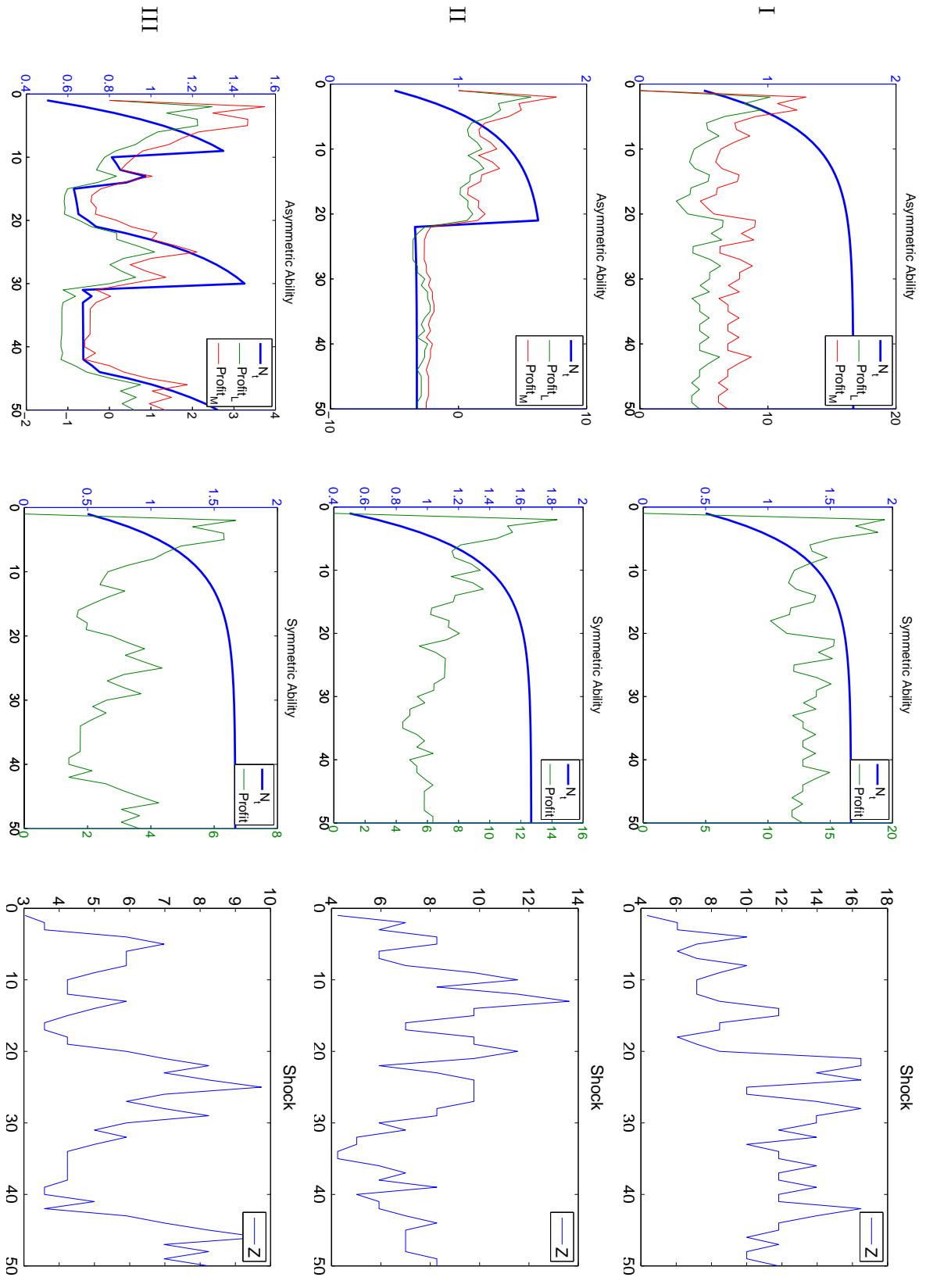


Figure 11: Each row of subfigures above corresponds to the system specification from Table 3. The last column plots the sample path of income shock z_t . Middle column represent the equilibrium path of the system with symmetric ability of all traders ($\theta_H = \theta_M = \theta_L$). The first column illustrates the evolution of the system in which some agents are more skilled than others. As you can see from comparing first and second column, differences in ability amplify the effect of income shock on the number of traders. In all three scenarios, fluctuations in z_t in symmetric case transmit only in the volatility of profit while the number of agents in the system is not affected. With asymmetric ability, the same shock dynamics results in dramatic changes in the number of traders. On each plot, the profit is on the left hand axis, the number of traders N_t is on the right hand axis.

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