Essays on Public Communication and Economic Incentives in Capital Markets

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To my parents

Abstract

This dissertation aims at enhancing our understanding of the role of information in the interaction among economic agents in capital markets. In order to thoroughly evaluate the trade-offs involved in maximizing the real efficiency and the price efficiency of capital markets, economic incentives of those who prepare, disseminate, and use information must be understood. The research questions I attempt to answer in this dissertation center around two themes: (i) how various characteristics of accounting information as mandated by accounting standards influence investors' assessment of a firm's future profitability and their trading behaviors in financial markets, and (ii) what are the incentives of investors to publicly express their stock opinions on social media. Specifically, the first two chapters examine the desirability of accounting information characteristics such as comparability, transparency, and clarity. The last chapter moves to study individual investors' voluntary expression of their polarized sentiments in a public communication network.

In the first chapter, I investigate how accounting comparability affects the monitoring role and the risk allocation role of capital markets. I develop the statistical and informational properties of accounting reports under varying degrees of comparability. A perfectly comparable accounting information system enables investors to perfectly infer the *difference* between any two firms' future cash flows although investors remain uncertain about either firm's cash flow. Comparability alleviates entrepreneurs' moral hazard problem by strengthening the price response to the relative accounting performance, but can induce excessive price risk as well as residual systematic cash flow risk. Unlike the investors (users) who earn their surplus by bearing the residual systematic risk, the entrepreneurs (preparers) do not find perfect comparability desirable. Hence, a standard setter would mandate higher comparability than preferred by preparers, but not perfect comparability.

In the second chapter, I study how public information about a firm's fundamental value affects the firm's stock price behavior and efficiency in the presence of Keynesian beauty contests when investors heterogeneously interpret such information. In an overlapping gen-

erations rational expectations model of capital markets, I show that higher-order beliefs regarding other investors' interpretations can induce investors to overweight their private interpretations. Moreover, while improving a public disclosure's clarity generally increases price efficiency, improving its transparency can be detrimental. In the latter case, the interior level of transparency that maximizes price efficiency monotonically increases in clarity. In other words, the best attainable level of price efficiency hinges on the level of clarity.

In the third chapter, I examine individuals' incentives to express their polarized sentiments and the efficiency of the subsequent aggregate action in the presence of coordination motives. I consider situations in which every agent would like to take an action that is coordinated with those of others, as well as close to a common state of nature. Agents have polarized sentiments in the sense that their beliefs about the state can be biased in opposite directions. Before the coordination game is played, agents decide whether to reveal to the others their polar type. In equilibrium, full disclosure takes place only when the population composition is expected to be more balanced; otherwise, the minority group of agents have an incentive to mimic their majority counterparts. In addition, the analysis on the aggregate action indicates that a diverse yet unbalanced population can jointly make a more efficient decision, especially when the coordination motive is strong but little is known about the underlying state.

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Introduction

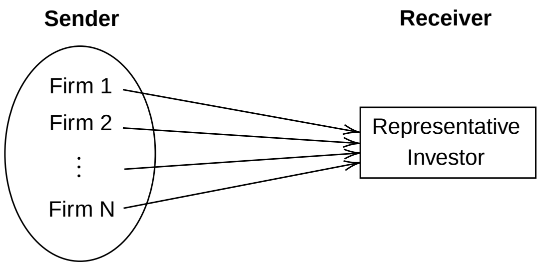
Information is considered by many the driving force of vibrant capital markets. However, the prior academic literature has not emphasized the process of information generation and how various information structures affects market participants' incentives to disseminate and utilize information. In this dissertation, I attempt to fill the gap by focusing on the role of information structures in the interaction among agents, along with its implications on the real efficiency and the price efficiency of financial markets. I use analytical models as a tool to build theoretical constructs of various qualitative characteristics of information such as comparability, understandability, and sentiment polarization, and explain empirical anomalies in accounting practices and institutions that contradict conventional wisdom.

One principal reason for developing accounting standards is to facilitate financial comparisons among enterprises under a common set of accounting methods. Indeed, the FASB's Conceptual Framework, which serves as a guidance for the development of financial accounting standards, identifies comparability as an enhancing qualitative characteristic of accounting information. Yet the current standards still allow for diverse practices for similar transactions, and preparers of financial information also frequently dispute the importance of comparability. Chapter 1 attempts to offer a consistent explanation for (i) the existence of diverse practices despite regulatory emphasis on comparability, and (ii) the different attitudes toward comparability by preparers and users of financial reporting.

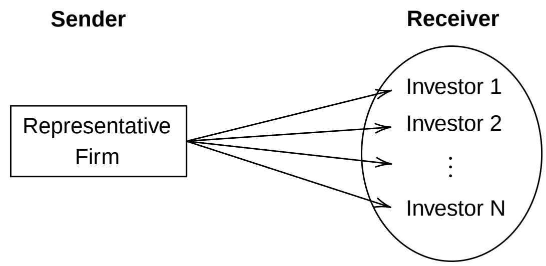
I first develop the statistical and informational properties of accounting reports under varying degrees of comparability, and then examine how accounting comparability affects monitoring and risk allocation roles of capital markets. In the presence of information externalities induced by comparable financial reports, the representative investor efficiently uses other firms' reports to price any firm in the market, as illustrated by Panel (a) of Figure 1. Comparability alleviates managers' moral hazard problem by strengthening the price response to the relative accounting performance, but can induce excessive price risk and systematic cash flow risk. Unlike the investors (users) who are better off by bearing the residual systematic risk, the entrepreneurs (preparers) do not find perfect comparability desirable. Thus, a benevolent policy maker would mandate higher comparability than preferred by preparers, but not perfect comparability.

Another enhancing qualitative characteristic of accounting information I examine in this dissertation is understandability. Prior literature has assumed public disclosure as common knowledge to market participants, and concluded that more precise public disclosure leads to more efficient capital markets. However, some economic phenomena are inherently complex and hard to understand. Hence, conventional wisdom suggests the following trade-off: excluding information about those phenomena from financial reports will make financial reports easier to understand, but will also lead to incomplete and thus potentially misleading reports. Taking a step back, in Chapter 2, I show that the above trade-off might not be relevant for the informational efficiency of stock prices in a short-horizon economy.

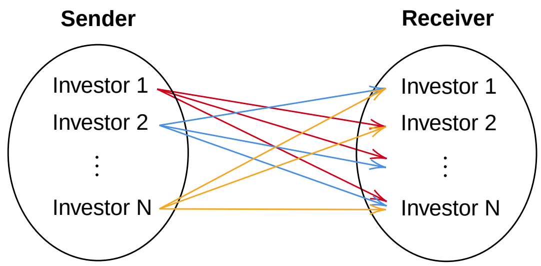
Moving from a setting of a representative receiver and multiple senders to a setting of a single sender and multiple receivers as illustrated in Panel (b) of Figure 1, an imperfect understandability induces investors to form heterogeneous but correlated interpretations of public disclosures. This results in the two dimensions of understandability of a public disclosure which I term transparency and clarity. The former regulates the total amount of information that can be processed out of the public disclosure, whereas the latter governs the degree of consensus among investors. In the context of stock markets as a Keynesian beauty contest, a metaphor originally proposed by J. M. Keynes in 1936, the public disclosure plays a dual role of conveying information about the value of the firm (fundamental role) and



(a) Chapter 1: $N \to 1$



(b) Chapter 2: $1 \to N$



(c) Chapter 3: $N \to N$

Figure 1: Communication Networks

serving as a focal point for the others' beliefs (coordination role). The opposite effects of transparency and clarity on the coordination role lead to their different impacts on price efficiency. Specifically, while more clarity generally increases price efficiency, there exists an interior level of transparency that maximizes price efficiency. When transparency and clarity of a public disclosure are of comparable magnitude, clarity is of first-order importance in improving price efficiency.

Information in capital markets does not just flow from the more informed managers to the less informed investors, but is also spread among equally well informed investors. For instance, individual investors voluntarily express their polarized opinions (e.g., bullish or bearish) about stocks on social media investing platforms like StockTwits and WallStreet-Bets. Meanwhile, trading strategies based on the sentiment extracted from social media become increasingly popular. Motivated by these two empirical observations, Chapter 3 models the degree of polarization as a payoff-relevant but value-irrelevant uncertainty and provides a consistent explanation for the seemingly puzzling phenomena that the aforementioned expression of polarized sentiments may happen even before investors take trading positions. Specifically, I show that it is coordination motives that drive investors to disclose their polarized sentiments in a public communication network as illustrated by Panel (c) of Figure 1.

As in the previous chapter, short-term investors' private information plays a dual role. Its strategic value is affected by the fraction of each group through the aggregate sentiment. Such effect is non-monotonic because a perfectly balanced population results in an unbiased aggregate sentiment and hence makes an investor's private signal least informative about the aggregate sentiment. Thus, investors will only express their sentiments when they are the majority group because doing so increases the strategic value of their private information by affecting the perceived population composition. Lastly, I also apply the framework to decision-making within organizations. Further analysis on the efficiency of the aggregate action indicates that a diverse yet unbalanced board or legislature can jointly make a more efficient decision, especially when the coordination motive is strong but little is known about the underlying state.

Chapter 1

Accounting Comparability and Relative Performance Evaluation by Capital Markets

1.1. Introduction

Over the past 40 years, reducing diverse practices and inconsistent guidance is the most frequently cited reason by the Financial Accounting Standards Board (FASB) to take on a project, and more than half of the standards are intended to enhance comparability (Jiang, Wang and Wangerin, 2018). Despite that, the current standards still allow for diverse practices and inconsistent treatment for similar transactions. FASB board members who are former CFOs or controllers are less likely than those with user backgrounds to dissent because a standard allows for exceptions or gives the management accounting alternatives. As

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noted by practitioners and researchers, while the users of financial information emphasize on accounting comparability, the preparers often dispute its importance (see, e.g., Van Riper, 1994; De Franco, Kothari and Verdi, 2011; Jiang et al., 2018; Kurt, 2020). The natural question is: What drives the different attitudes toward comparability by the two major constituents of financial reporting? Why do we still observe diverse practices despite regulatory emphasis on comparability?

One explanation for the existence of diverse accounting practices is that it allows for flexibility and is thus insisted by preparers who may benefit from opportunistic reporting. However, preparers do not always benefit from more discretion when investors rationally correct for the opportunistic reporting on average (see, e.g., Guttman, Kadan and Kandel, 2006; Gao and Zhang, 2019). In this paper, we provide a different rationale from an information-externality perspective in the sense that permitting alternative accounting methods for the same economic phenomenon diminishes *comparability* among reports (FASB, 2018, QC25; IASB, 2018, 2.29). We show that the information externalities of comparable financial reports can potentially explain why preparers are less enthusiastic about adopting common accounting practices, even in the absence of opportunistic reporting. Specifically, a too-high-level of comparability can induce excessive price risk as well as risk premium in the prices. Hence, the preparers prefer a lower level of comparability than the investors who earn their surplus through the risk premium. As a result, a policy maker influenced by both constituents would mandate a higher level of comparability than that preferred by the preparers, but not perfect comparability demanded by the investors.

We first develop the statistical and informational properties of comparability as they are decision-relevant to economic agents. Comparability is viewed as a property of the relationship between the accounting information of at least two firms (FASB, 2018, QC21; IASB, 2018, 2.25). Suppose an accounting information system measures the fundamental value of each firm with errors, we model comparability as the correlation between the measurement errors of any pair of firms under a reporting regime. From an informational perspective, comparability as defined here does not affect the precision of a stand-alone report about its reporting firm. A higher correlation between measurement errors renders the *difference* between two reports more informative about the *difference* between their reporting firms' fundamentals. Hence, comparability improves the users' knowledge about the firm-specific fundamentals relative to other firms. This informational effect manifests itself through the increased price response to a firm's own report. From this perspective, comparability is an *enhancing characteristic* of useful financial information as described by the conceptual frameworks in FASB (2018, QC19) and IASB (2018, 2.23).

Our proposed measure of comparability conceptually conforms to the argument that achieving comparability through adopting a common standard renders a higher correlation between the measurement processes of firms (Wang, 2014). It is also theoretically consistent with the notion that adopting a common standard features a larger common measurement error (Dye and Sridhar, 2008; Zhang, 2013), as the correlation between measurement errors is positively associated with the proportion of the common error among reports. While comparability improves users' knowledge about the firm-specific fundamentals, it impairs their knowledge about the average fundamental and thus the aggregate economy due to a larger common measurement error.[1]

To study the economic consequences of accounting comparability, we build our model on the overlapping generations setting in Dye (1990), Dye and Sridhar (2007) and Gao (2010), and extend it to multiple firms. Each firm is initially owned and managed by a risk-averse entrepreneur. The cash flow to each firm is jointly determined by an unobservable effort by the entrepreneur, an economy-wide shock common to all firms, and an idiosyncratic shock specific to each firm. After privately exerting the effort, the entrepreneur publicly issues an accounting report which is a noisy signal about the cash flow. Entrepreneurs then sell their

[1] We can further illustrate this informational trade-off using an analogy in which students' ability is assessed through exams. The students can be taking the same exam, or assigned to distinct and independent exams. The grades in the former case are more "comparable" because students are evaluated under the same criterion, and hence the ranking of students is more informative of their relative ability. On the other hand, the grades in the latter case are more informative about the average ability of the class because the noisiness induced by independent exams is diversified away in the average grade.

firms to risk-averse investors and consume the proceeds due to life-cycle considerations. At the last date, cash flows are realized, and investors consume the cash flows generated by the firms.

In the presence of information externalities, investors efficiently use other firms' reports to price any firm in the market. Their inference about the cash flows given their conjectures of the entrepreneurs' efforts can be decomposed into two tasks. In the first task, the investors infer each firm's idiosyncratic cash flow shock by comparing its report with the other reports. In the second task, the investors learn from all reports to infer the cash flow shock common to all firms. Comparability improves the first inference but impairs the second one. This informational property leads to various efficiency implications.

Comparability alleviates the moral hazard problem arising from the unobservability of the entrepreneurs' efforts. An entrepreneur internalizes the return to his effort to the extent that it affects the firm price through the accounting report. As the investors' inference about the common shock is disciplined by the reports of the other firms, the entrepreneur is incentivized to work hard only to differentiate his report from the others' reports. Due to the noisiness of accounting signals, the rate at which his effort increases the market price is lower than the rate at which it increases the firm value. Higher comparability alleviates this moral hazard problem by strengthening the price response to the relative accounting performance.

Despite its positive effect on the monitoring role of capital markets, perfect comparability is not considered desirable by the entrepreneurs as it induces excessive price risk and risk premium. An undiversified entrepreneur who consumes the proceeds from selling his firm bears the risk associated with the volatility of the firm price. This price risk has both an idiosyncratic and a systematic component. Each component of the price risk is positively associated with the extent to which the investors use the reports to infer its corresponding cash flow shock. Although comparability impairs the investors' inference about the common shock, it can increase the price risk due to the investors' enhanced inference about the idiosyncratic shock when the level of comparability is sufficiently high. The entrepreneurs sell at a lower price due to the risk premium which is determined by the diversified investors' residual uncertainty about the common shock. Higher comparability increases the risk premium by increasing such uncertainty about the aggregate economy.

However, maximizing comparability makes the diversified investors better off because they earn their surplus by bearing the residual systematic risk (Gao, 2010; Bertomeu and Cheynel, 2016). The risk premium equals their marginal cost of bearing the systematic risk which is higher than the average cost, and their surplus increases in the residual systematic risk. The above results combined suggest that the preparers prefer a lower level of comparability than the users. As noted by a former FASB member Van Riper (1994):

> "Though its importance is often disputed by preparers of financial information, comparability of financial information has long been a major concern of analysts and legislators."

As lower comparability is associated with diverse accounting practices, our model can potentially explain why preparers are more tolerant of diverse practices relative to users and even tend to lobby for a lower level of standardization (Jiang et al., 2018). Our model further predicts that a policy maker taking into account both constituents would mandate a higher level of comparability than that would otherwise be chosen by the preparers, but not perfect comparability because of the excessive costs on the preparers. In other words, we should still observe diverse accounting practices despite regulatory emphasis on comparability.[2]

Our analysis generates various additional empirical implications. The pricing equation that reflects the efficient use of accounting information suggests a *structural* approach to construct a regime-level empirical proxy for comparability. Our model predicts that, controlling for a firm's own report, the firm's price response to the average report in a reporting regime decreases in the level of comparability and can even become negative with sufficiently

[2]There can be many other reasons for users to prefer higher comparability. For example, comparability can perhaps facilitate more efficient information processing by investors. These possible forces not captured by our model do not contradict our results.

high comparability. This property of the pricing equation allows the price coefficient on the average report estimated with a cross section of firms to proxy for the level of comparability in this reporting regime. Intuitively, higher comparability renders the firm price more responsive to the relative accounting performance, thus resulting in the other firms' positive reports being used more as bad news when pricing a firm. This empirical proxy can be used to compare comparability across regimes at a particular time, or to study the time-series variations of comparability in a particular regime over time, for example, around policy changes.[3]

If the standard setter incorporates the preferences of both preparers and users of financial reports, the observed standards should reflect both of their preferences (Watts and Zimmerman, 1978). The model allows us to characterize the level of comparability chosen by a standard setter who takes into account both constituents.[4] Perfect comparability is not desirable because it induces excessive price risk borne by the entrepreneurs as well as excessive residual systematic cash flow risk borne by the investors. In addition, the standard setter would mandate a higher level of comparability when the idiosyncratic cash flows are less volatile. In other words, we should observe standards allowing for more diverse practice in an economy with highly heterogeneous firms. This prediction and the pricing equation can be jointly tested by using the proposed empirical proxy for comparability as implied by the pricing equation. In addition, our main results continue to hold qualitatively even when extending our model to allow for firm-wise heterogeneity in the level of comparability.

This paper contributes to the current literature in several aspects. First, it formally defines comparability and investigates the economic consequences of increasing comparability from an informational perspective. Accounting has been intensively studied as an information function (Butterworth, 1972; Liang and Zhang, 2008). Current standard setters follow the conceptual-framework approach to standard setting by first defining desirable

[3]We thank Frank Zhou for pointing it out at the 2019 Junior Accounting Theory Conference.

[4]In practice, there are many other stakeholders of financial reporting not captured by our analysis. Arguably, preparers and users are among the most important ones.

qualitative characteristics of financial information and then evaluating alternative standards against them (Bullen and Crook, 2005). However, the common notion of comparability is theoretically ambiguous (Sunder, 2010). A key feature of our theoretical definition is that we disentangle comparability from the informativeness of a stand-alone report which has been studied extensively since Feltham (1972). We examine how comparability affects the usefulness of an accounting report given other firms' reports while controlling for the informativeness of a stand-alone report. Although comparability enhances the usefulness of an accounting report given other firms' reports, we show that the standard that takes into account both constituents would not feature perfect comparability. Nevertheless, we provide a justification for the regulatory emphasis on comparability, as preparers would otherwise choose an even less comparable reporting system than what the standard setter would choose.

Second, this paper is related to the study of the monitoring role of capital markets in addressing agency issues (e.g., Holmstrom and Tirole, 1993; Dye and Sridhar, 2004, 2007). Efficient pricing by the market serves as a disciplinary mechanism for those with control rights. Consequently, how much information is available to the market affects the efficiency of their actions. We extend this line of inquiry by examining the information externalities of financial reports in the sense of Dye (1990) and Dye and Sridhar (2008). Comparability can facilitate the investors' learning of firm value through comparison between firms, thus incentivizing the entrepreneurs to exert efforts to differentiate themselves from the others. This mechanism also formally establishes the intuition in Core, Guay and Larcker (2003) that equity incentives render an *implicit* relative performance evaluation scheme. More recently, Jennings, Seo and Soliman (2020) formally document that a firm's price depends on the relative ranking of its earnings in its peer group.

Finally, this paper addresses the risk allocation effect of disclosure policies (e.g., Dye, 1990; Dye and Sridhar, 2004, 2007; Gao, 2010). Disclosure resolves uncertainty early, increasing the price risk while reducing the residual cash flow risk (Hirshleifer, 1971). In the situation where the current shareholders have to sell to the next generation of shareholders, more precise disclosure is efficient if and only if the new generation is relatively more risk averse than the old generation. Our information structure differs from the standard one in that the level of comparability does not affect the precision of a stand-alone report, but rather affects risk allocation through the information externalities. Due to the investors' diversification of the idiosyncratic risks, only the systematic risk is allocated between the two generations. A higher level of comparability shifts more systematic risk to the investors. Nonetheless, even when the entrepreneurs are relatively more risk averse than the investors, it is not efficient to allocate all the systematic risk to the investors as it also induces excessive idiosyncratic price risk borne by the undiversified entrepreneurs.

The rest of the paper is organized as follows. Section 1.2 develops the model, discusses the measure of comparability in more details, and characterizes the equilibrium given an exogenous level of comparability. Section 1.3 studies the welfare implication of increasing comparability on both constituents of financial reporting. Section 1.4 discusses the empirical implications. Section 1.5 extends our main setting and enables firm-wise heterogeneity in the level of comparability. Section 1.6 concludes.

1.2. Model

This section describes the basic setting of the economic model and the informational properties of comparability. Since the notion of comparability can only be applied to more than one firm, we model a continuum of firms, with each firm indexed by $i \in [0, 1]$.[5] Each firm is owned and managed by an entrepreneur with the same index as the firm he owns for simplicity. There is also a continuum of investors indexed by $j \in [0, 1]$ who buy the firms from entrepreneurs after the release of accounting signals, but before the cash flows are realized.

[5] The extent of the information externalities increases in the number of firms in a finite-firm setting (Dye and Sridhar, 2008), so one can view this assumption as a limiting case of the finite-firm setting.

1.2.1. Setup

Firm i's cash flow $\tilde{\theta}_i$ is determined jointly by (i) the effort a_i exerted by its entrepreneur, (ii) the economy-wide shock $\tilde{\eta}$ common to all firms, and (iii) an idiosyncratic shock $\tilde{\delta}_i$ specific to each firm. That is, given the effort a_i,

$$\tilde{\theta}_i = a_i + \tilde{\eta} + \tilde{\delta}_i, \text{ with } \tilde{\eta} \sim \mathcal{N}(0, \tau_\eta^{-1}), \tilde{\delta}_i \sim \mathcal{N}(0, \tau_\delta^{-1}), \tag{1.1}$$

where $\tau_\eta > 0$ and $\tau_\delta > 0$ are the precisions of the two cash flow shocks, respectively. Both the price and the return of the risk-free asset are normalized to be 1.

The output of the accounting system is a noisy signal about the firm's cash flow. It carries both a common error and an idiosyncratic error. The common error comes from the use of a common set of measurement methods required by the accounting standards, and the idiosyncratic error is due to diverse practices. Formally, conditional on the cash flow, firm i's accounting signal is

$$\tilde{s}_i = \theta_i + \tilde{\epsilon} + \tilde{\xi}_i, \text{ with } \tilde{\epsilon} \sim \mathcal{N}(0, \tau_\epsilon^{-1}), \tilde{\xi}_i \sim \mathcal{N}(0, \tau_\xi^{-1}), \tag{1.2}$$

where $\tilde{\epsilon}$ and $\tilde{\xi}_i$ are the common error and the idiosyncratic error, with corresponding precisions $\tau_\epsilon > 0$ and $\tau_\xi > 0$, respectively. Since the firms are otherwise symmetric, we may also use the economy-wide average signal $\tilde{\bar{s}} \equiv \int_0^1 \tilde{s}_i \mathrm{d}i = a + \tilde{\eta} + \tilde{\epsilon}$ as a sufficient statistic of all (other) firms' accounting reports, where $a \equiv \int_0^1 a_i di$ is the average effort. To disentangle comparability from the informativeness of a stand-alone report, we restrict the precision of an individual signal about its reporting firm (hereafter, *reporting precision*) to be a constant τ, with $0 < \tau < \infty$ to ensure usefulness of accounting signals, i.e.,

$$\tau^{-1} \equiv \tau_\epsilon^{-1} + \tau_\xi^{-1}. \tag{1.3}$$

In equilibrium, the investors rationally infer the unobservable effort exerted by the en-

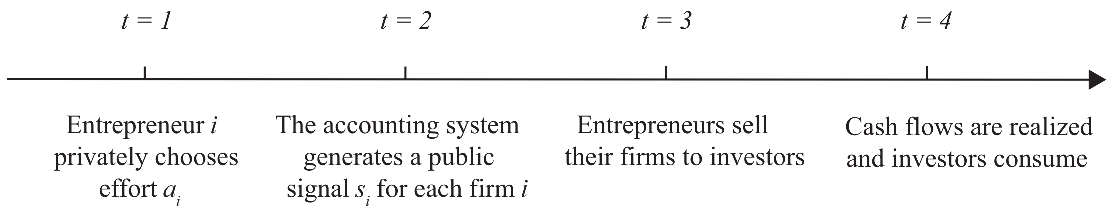


Figure 1.1: The Timeline of Events

trepreneurs. However, they are unable to perfectly deduce the underlying cash flows of the firms, as the cash flows are subject to the common shock and the idiosyncratic shocks as well. The noisy accounting reports thus help them to imperfectly estimate the extent to which the cash flow shocks have affected the terminal cash flows.

Entrepreneurs and investors are assumed to be risk averse with constant absolute risk aversion (CARA). Entrepreneur i's utility function is given by $u_i(\tilde{w}_i) = -\frac{1}{\rho_E}e^{-\rho_E \tilde{w}_i}$, where $\tilde{w}_i$ denotes his consumption financed by the proceeds from selling his firm. Investor j's utility function is given by $v_j(\tilde{w}_j) = -\frac{1}{\rho_I}e^{-\rho_I \tilde{w}_j}$, where $\tilde{w}_j$ denotes her consumption financed by the cash flows generated by the portfolio she buys from the entrepreneurs.

The timeline of the model consists of four dates, as depicted in Figure 1.1. At date $t = 1$, the entrepreneur of each firm $i \in [0, 1]$ chooses an effort a_i and personally bears a quadratic cost $\frac{1}{2}a_i^2$. The level of the effort is private information to the entrepreneur who made the choice. At date $t = 2$, the accounting system of each firm produces a public signal about the cash flow that will be generated by the firm. Investors then use all the accounting signals to make portfolio decisions. At date $t = 3$, the trading takes place. Entrepreneurs sell the firms to investors and consume the price of their firms. At the last date $t = 4$, cash flows of all firms are realized, and investors consume the cash flows generated by their portfolios.

All random variables are independent from each other and all parameters are common knowledge unless specified otherwise.

1.2.2. Measure of Comparability

Our theoretical measure of comparability is based on its statistical and informational properties. Wang (2014) defines comparability as the correlation between the measurement processes of two firms' accounting earnings. Because the measurement errors capture the informational properties of the measurement process in our informational framework, the correlation between the measurement processes of the two firms, i and i', is

$$c \equiv \mathrm{Corr}(\tilde{s}_i - \tilde{\theta}_i, \tilde{s}_{i'} - \tilde{\theta}_{i'}) = \frac{\mathrm{Cov}(\tilde{\epsilon} + \tilde{\xi}_i, \tilde{\epsilon} + \tilde{\xi}_{i'})}{\left[\mathrm{Var}(\tilde{\epsilon} + \tilde{\xi}_i)\mathrm{Var}(\tilde{\epsilon} + \tilde{\xi}_{i'})\right]^{\frac{1}{2}}} = \frac{\tau_\epsilon^{-1}}{\tau_\epsilon^{-1} + \tau_\xi^{-1}}. \tag{1.4}$$

In other words, we measure comparability as the proportion of variance from the common measurement error, $c \in [0, 1]$. Thus, controlling for the precision of an individual report as τ, we have

$$\tau_\epsilon^{-1} = c\tau^{-1},\ \tau_\xi^{-1} = (1 - c)\tau^{-1}. \tag{1.5}$$

According to the FASB (2018, QC21) and the IASB (2018, 2.25), comparability enables users to identify and understand differences among firms. Taking the difference between the accounting reports of firm i and i' renders a noisy signal of the difference between their fundamentals:

$$\tilde{s}_i - \tilde{s}_{i'} = (\tilde{\theta}_i - \tilde{\theta}_{i'}) + (\tilde{\xi}_i - \tilde{\xi}_{i'}). \tag{1.6}$$

The common measurement error is cancelled out during the comparison, and the variance of the noise is $2\tau_\xi^{-1}$ which decreases in c. That is, the amount of information conveyed through the comparison of accounting signals is positively associated with comparability.

This measure is also consistent with the notion that financial reports are more comparable through the application of a common standard because applying a common standard can be associated with a larger common measurement error (Dye and Sridhar, 2008; Zhang, 2013). However, it should be noted that comparability is not equivalent to uniformity. In our setting, comparability not only increases common measurement error, but also reduces the idiosyncratic measurement error so that "like things look alike and different things look different" (FASB, 2018, QC23; IASB, 2018, 2.27).

Limitations of the measure. First, we do not attempt to model how accounting standards map transactions into financial reports. Therefore, the proposed measure of comparability can neither be used to study the frictions in the measurement process nor to provide any operational guidance on evaluating alternative accounting rules (Gao, 2013). Specific accounting rules differ in multiple dimensions in terms of all qualitative characteristics. One qualitative characteristic may have to be diminished to maximize another qualitative characteristic (FASB, 2018, QC34; IASB, 2018, 2.38). Choosing one accounting rule over another may increase comparability but impair other qualitative characteristics, which reduces the informativeness of a report about its reporting firm. In our theoretical inquiry, we hold constant the precision of each stand-alone report in order to focus on the information-externality effects.

Second, our measure does not capture firm-wise heterogeneity in the level of comparability. The level of comparability between any two firms' reports may be affected by their reporting firms' characteristics, including to what extent their transactions are regulated by the standard or if they are in the same industry (De Franco et al., 2011). In other words, c can only be interpreted as a regime-level measure of comparability among all reports. Studying a setting with heterogenous exposures to correlated measurement errors requires carefully specifying the distribution of the exposures, which has nontrivial implications for the way investors use the reports. Nevertheless, in section 1.5, we simplify the analysis by considering an extension with only two groups of firms, with firms in each group only subject to a group-wide common measurement error besides idiosyncratic errors. Our main results remain qualitatively robust in this extended setting.

Third, our measure of comparability captures the notion that applying a common standard enhances comparability in a reduced-form manner of modelling the common measurement error. It omits other aspects of adopting a common standard. For example, applying a common disclosure regulation to all firms can economize on the information processing costs of users confronted with multiple reports (Gao, Jiang and Zhang, 2019), reduce the distortion created by lobbying (Friedman and Heinle, 2016), and impose rigidity on financial reporting thus reducing opportunistic reporting (Dye and Sridhar, 2008).

1.2.3. Equilibrium Analysis

The equilibrium in this model consists of the effort choices simultaneously made by the entrepreneur of each firm and the pricing rules applied by the investors in the capital market. We characterize a perfect Bayesian equilibrium in which all players make optimal decisions that maximize their utility given all their available information as well as their rational expectations regarding the strategic behavior of the other players, utilizing Bayes' rule to make inferences and update their beliefs. Formally, denoting by $\hat{a}_i$ the conjecture of the players about the effort level a_i^* and $P_i : \mathbb{R}^2 \to \mathbb{R}$ the pricing equation, respectively, any perfect Bayesian equilibrium must satisfy the following conditions for any $i \in [0,1]$ and for any $s_i, \bar{s} \in \mathbb{R}$:

(i) $a_i^* \in \arg\max_{I_i} \mathbb{E}[P_i] - \frac{\rho_E}{2}\mathrm{Var}(P_i) - \frac{1}{2}a_i^2$;

(ii) $P_i(s_i, \bar{s}) = \mathbb{E}[\tilde{\theta}_i | s_i, \bar{s}] - \rho_I \mathrm{Var}(\tilde{\eta} | s_i, \bar{s})$; and

(iii) $\hat{a}_i = a_i^*$.

Condition (i) pertains to the efforts simultaneously chosen by the entrepreneurs. Each entrepreneur chooses the level of the effort that maximizes his expected utility, given his rational expectations about the other entrepreneurs' efforts and the investors' pricing rule. Condition (ii) describes the pricing rule applied by the investors following firms' issuance of accounting reports.[6] Here, the risk-averse investors are only compensated for bearing

[6]The pricing equation is taken as given in this definition of equilibrium for succinctness. In fact, since there is no asymmetric information among individual investors, it is directly solved by no-arbitrage and market clearing. A similar characterization can be also found in Gao (2019). The complete proof is provided in Appendix A.

nondiversifiable risks, and the risk premium equals their marginal cost of bearing such risk. Condition (iii) ensures that all players have rational expectations regarding each other's behavior.

We derive the interrelated equilibrium outcomes of entrepreneurs' efforts and the capital market prices using backward induction.

Lemma 1.1. *Each investor holds an identical proportion of the market portfolio and is only compensated for bearing nondiversifiable risks. With conjectured levels of effort $\hat{a}_i$ and $\hat{a}$ for firm i and the economy-wide average, respectively, the pricing equation applied by the investors is:*

$$P_i(s_i, \bar{s}) = \hat{a}_i + \frac{\tau_\xi}{\tau_\delta + \tau_\xi}(s_i - \bar{s}) + \frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}(\bar{s} - \hat{a}) - \rho_I \frac{1}{\tau_\eta + \tau_\epsilon}. \tag{1.7}$$

In equilibrium, investors' conjecture coincides with the actual effort exerted by the entrepreneurs:

$$\hat{a}_i = a_i^* = \frac{\tau_\xi}{\tau_\delta + \tau_\xi}. \tag{1.8}$$

In the presence of cash flow uncertainties, investors cannot precisely infer each firm's cash flow despite their rational expectations about the entrepreneurs' efforts. They thus find the accounting reports useful for learning about the two cash flow shocks. When pricing a firm, the inference is decomposed into two tasks. First, they use the relative accounting performance, i.e., the difference between the firm-specific signal and the average signal, to learn about the idiosyncratic shock to the firm. Second, they use the average signal to learn about the common shock to all firms. Essentially, the average report plays two informational roles - it not only is informative about the aggregate economy, but also teases out the common measurement error in the individual firm's report. The latter role is similar to the efficient use of information in relative performance evaluation schemes in Holmstrom (1982).

The equilibrium effort exerted by an entrepreneur is always lower than the efficient level, that is, there is a moral hazard problem.[7] Because the effort is unobservable to the investors, the entrepreneur is incentivized to exert effort only to the extent it increases the firm price through the accounting report. Although the investors cannot distinguish cash flow shocks from the effect of the effort on the firm value, the inference about the common shock is disciplined by the average report. Hence, the entrepreneur can only jam their inference about the idiosyncratic shock. As can be seen from equation (1.7), the marginal benefit of effort to the entrepreneur is the price coefficient on the relative performance. In other words, the entrepreneur works hard to differentiate his own report from the average report. The economic return of one additional unit of effort in improving future cash flows is 1, but the price coefficient on the relative accounting performance is always lower than 1 due to the imperfection of the accounting signals. This incentive effect of the pricing equation is consistent with the conjecture in Core et al. (2003) that equity incentives of managers render an implicit relative performance evaluation scheme.

Improving comparability has different effects on investors' posterior uncertainty about the two cash flow shocks. An increase in comparability leads to a simultaneous increase of the common error and decrease of the idiosyncratic error in accounting signals (see equations (1.5)), and thus improves investors' knowledge about the idiosyncratic shock while impairing their knowledge about the common shock. More precisely, the posterior variances of the two shocks are, respectively,

$$\mathrm{Var}(\tilde{\delta}_i|s_i, \bar{s}) = \mathrm{Var}(\tilde{\delta}_i|s_i - \bar{s}) = \frac{1}{\tau_\delta + \tau_\xi}, \tag{1.9}$$

$$\mathrm{Var}(\tilde{\eta}|s_i, \bar{s}) = \mathrm{Var}(\tilde{\eta}|\bar{s} - \hat{a}) = \frac{1}{\tau_\eta + \tau_\epsilon}. \tag{1.10}$$

Higher comparability enables the investors to learn more about a specific firm relative to the other firms, but less about the overall fundamental of the aggregate economy. As a

[7] The efficient level of effort is the one that maximizes the net return of the costly effort. Denote it as a_i^{FB}, then $a_i^{\mathrm{FB}} = \arg\max_{a_i} a_i - \frac{1}{2}a_i^2 = 1$. This efficient outcome can also be achieved if there is no information asymmetry regarding the level of effort.

result, firm price depends more on the relative accounting performance and less on the average accounting performance following an increase in comparability. This prediction is consistent with the standard setter's view of relevance being a *fundamental characteristic* and comparability being an *enhancing characteristic* because for the same level of reporting precision of a stand-alone firm, comparability increases the usefulness of the focal firm's report given other firms' reports as reflected by its increased earnings response coefficient.

Remark. *When both constituents are risk neutral, perfect comparability is uniquely Pareto-optimal, i.e.,* $c^*_{rn} = 1$.

In a risk-neutral world, both entrepreneurs and investors would unanimously agree on implementing a perfectly comparable accounting standard. Indeed, in this case, the investors break even on expectation and are indifferent about the level of comparability. The entrepreneurs prefer perfect comparability because it resolves the moral hazard problem. Higher comparability increases the entrepreneurs' perceived marginal benefit from exerting effort through the strengthened price response to the relative accounting performance, and thus encourages the entrepreneur to work harder. Following Holmstrom and Tirole (1993), we refer to this as the *market monitoring effect* of comparability. This incentive effect derives from the market's efficient use of all other firms' reports as comparability does not affect the precision of a stand-alone report. In a risk-averse world, however, the information externalities of the financial reports also have effects on the risk borne by the entrepreneurs and the investors. In the next section, we show that risk aversion is a key driver of our main results that are consistent with the observed different attitudes toward the desirability of comparability by the different constituents of financial reporting.

1.3. Main Results

In equilibrium, the risk premium in firm prices equals the diversified investors' marginal cost of bearing the residual systematic cash flow risk and is higher than the average cost.

The gap between the two is the source of the investors' surplus. Thus, the investors' net certainty equivalent from bearing the risk is half of the risk premium. With rational expectations, investors' conjectured levels of effort coincide with the actual ones chosen by the entrepreneurs, which are in turn incorporated into the prices. As a result, an entrepreneur internalizes the net return of his effort in addition to the risk premium. Moreover, the risk-averse entrepreneurs also bear the price risk associated with the volatility of price which contains both an idiosyncratic and a systematic component since they cannot diversify. Formally, the *ex-ante* certainty equivalents of a representative investor and a representative entrepreneur are respectively:

$$\mathrm{CE}_I = \frac{\rho_I}{2}\frac{1}{\tau_\eta + \tau_\epsilon}, \tag{1.11}$$

$$\mathrm{CE}_E = \underbrace{\frac{\tau_\xi}{\tau_\delta + \tau_\xi} - \frac{1}{2}\left(\frac{\tau_\xi}{\tau_\delta + \tau_\xi}\right)^2}_{\text{net return of effort}} - \underbrace{\frac{\rho_E}{2}\left(\frac{1}{\tau_\delta}\frac{\tau_\xi}{\tau_\delta + \tau_\xi} + \frac{1}{\tau_\eta}\frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}\right)}_{\text{price risk}} - \underbrace{\rho_I\frac{1}{\tau_\eta + \tau_\epsilon}}_{\text{risk premium}}. \tag{1.12}$$

Comparability impairs the investors' knowledge about the common cash flow shock, leading to a higher residual systematic cash flow risk. Since investors earn a higher surplus by bearing more systematic risk, their surplus is maximized with perfect comparability.[8] That is, investors prefer the highest level of comparability, i.e., $c_I^* = 1$.

Entrepreneurs' preference for comparability is jointly determined by its effects on the three terms in equation (1.12). Despite its positive market monitoring effect on net return of effort, higher comparability results in a less precise inference about the common shock and a higher risk premium, which makes the entrepreneurs worse off. We refer to this as the *risk premium effect*. On the other hand, the *price risk effect* associated with the *ex-ante* volatility of the price can go in either direction. To see this, note that the more responsive the price to information, the more volatile it is *ex-ante*. As comparability strengthens the price response to the relative accounting performance and weakens the price response to

[8]For a more detailed discussion on risk premium and investors' surplus, see Gao (2010) and Bertomeu and Cheynel (2016).

the average accounting performance, the price volatility first decreases and then increases in comparability, and is minimized at an intermediate level of comparability. To summarize, the three effects each has a different inclination to the entrepreneurs' preferred level of comparability c_E^*:

(i) the market monitoring effect drives c_E^* toward 1;

(ii) the price risk effect drives c_E^* toward $\frac{\tau_\delta}{\tau_\eta+\tau_\delta}$; and

(iii) the risk premium effect drives c_E^* toward 0.

Proposition 1.1. *When both constituents are risk averse, the investors prefer perfect comparability, i.e., $c_I^* = 1$, and the entrepreneurs prefer a strictly lower level of comparability, i.e., $0 \leq c_E^* < 1$.*

As comparability has different implications on the idiosyncratic risk and the systematic risk, the two risk-averse constituents desire different levels of comparability. While the investors prefer perfect comparability, the entrepreneurs prefer a strictly lower level of comparability. From the entrepreneurs' perspective, despite its positive effect on the net return of effort, perfect comparability induces excessive price risk as well as risk premium. As shown by Gao (2010), resolving uncertainty allocates risk from investors to entrepreneurs. Although the systematic risk is allocated between the two generations, perfect comparability maximizes the risk premium without minimizing the price risk. Increasing comparability not only allocates more systematic risk to the diversified investors through the increased uncertainty about the common shock and increases the risk premium, but also increases the idiosyncratic risk borne by the undiversified entrepreneurs through the decreased uncertainty about the idiosyncratic shock. Moreover, the entrepreneurs may not desire any comparability at all when the risk premium effect dominates in the presence of sufficiently risk-averse investors.

The different preferences for comparability by the entrepreneurs and the investors coincides with the fact that the importance of comparability is often disputed by the preparers of financial information despite the users' desire for comparability (Van Riper, 1994; Jiang et al., 2018). To capture in a parsimonious way that the standard setter is influenced by both constituents of financial reporting (Watts and Zimmerman, 1978), we assume the standard setter aims to maximize $W \equiv (1-\omega)\mathrm{CE}_E + \omega \mathrm{CE}_I$, where $\omega \in (0,1)$ is the weight placed on the investors' surplus, and denote the optimal level of comparability chosen by the standard setter as c^*.

Proposition 1.2. *The optimal comparability c^* is strictly higher than the one that would otherwise be chosen by the entrepreneurs whenever it is non-zero, i.e., $c^* > c_E^*$ if $c_E^* \in (0,1)$. Moreover, there exists a threshold $\hat{\omega} \in (\frac{2}{3}, 1)$ such that c^* is strictly lower than the perfect level, i.e., $c^* < 1$, if and only if $\omega < \hat{\omega}$. The interior c^* increases (decreases) in the weight the standard setter places on the investors (entrepreneurs).*

The optimal level of comparability chosen by the standard setter is greater than that desired by the entrepreneurs because the standard setter caters for the investors' preference for perfect comparability. In addition, as long as the standard setter places a sufficient weight on the entrepreneurs, the optimal policy would not feature perfect comparability as it induces excessive price risk as well as residual systematic cash flow risk. Indeed, the relative weight placed on the two constituents determines not only the relative importance but also the direction of the effect of the residual systematic risk on the optimal comparability. Since the investors' net certainty equivalent from bearing the risk is half of the risk premium (see equations (1.11) – (1.12)), the effect of the residual systematic risk drives the optimal comparability toward the minimum level 0 when $\omega < \frac{2}{3}$, but toward the maximum level 1 when $\omega > \frac{2}{3}$. In other words, the standard setter's inclination on the risk premium is more aligned with the entrepreneurs (investors) in the former (latter) case. Moreover, even when the effect of the residual systematic risk vanishes at $\omega = \frac{2}{3}$, the price risk effect will still render the optimal comparability lower than 1. Thus, the threshold $\hat{\omega}$ above which the optimal comparability is perfect has to be strictly higher than $\frac{2}{3}$.

Taken together, the above results potentially explain why we still observe diverse accounting practices despite regulatory emphasis on comparability. Without the regulatory intervention, the level of comparability implemented by the preparers would likely not internalize the users' preference for perfect comparability and hence be lower than the optimal level from the standard setter's perspective. To better understand the role played by different agents' risk attitudes in generating our results, we consider the following two special cases.

Corollary 1.1. *When either constituent is risk neutral, we have*

(i) $\rho_E > \rho_I = 0$*: investors are indifferent, and* $c_E^* = c^* \in (0, 1)$*; and*

(ii) $\rho_I > \rho_E = 0$*:* $c_E^* < c^* < c_I^* = 1$ *when* $c_E^* > 0$ *and* ω *is sufficiently low.*

When the investors are risk neutral, they are not compensated for bearing risks. As a result, the break-even investors are indifferent to the level of comparability, and the standard setter's optimal policy coincides with the one preferred by the risk-averse entrepreneurs. Since the existence of the price risk effect diminishes the desirability of perfect comparability, the standard setter would mandate an interior level of comparability. On the other hand, when the investors are risk averse, they prefer perfect comparability because it maximizes their surplus gained by bearing the residual systematic cash flow risk. However, the risk-neutral entrepreneurs prefer a lower level of comparability even without the price risk effect because they sell at a lower price due to the risk premium. The standard setter's optimal policy is thus determined by trading off both constituents' preferences. When the entrepreneurs prefer some level of comparability and the standard setter places a sufficient weight on the entrepreneurs, the optimal comparability admits an interior solution and is higher than that would otherwise be chosen by the entrepreneurs.

To summarize, investors' risk aversion is necessary for our model to speak to their preference for the level of comparability. In particular, to generate the joint empirical regularity that (i) preparers of financial information prefer a lower level of comparability than users, and (ii) perfect comparability is not an observed policy, the assumption that the investors are risk averse is indispensable. In other words, the higher risk premium due to the higher systematic risk left in the market by a higher level of comparability is the key friction that prevents the observed policy from featuring perfect comparability when the standard setter places sufficient weights on both constituents.

Next, we explore how the primitive variables of the model affect the optimal policy that would be chosen by the standard setter.

Corollary 1.2. *The optimal level of comparability c^*, whenever it admits an interior solution,*

(i) decreases in investors' degree of risk aversion ρ_I, i.e., $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_I} < 0$ if and only if $\omega < \frac{2}{3}$;

(ii) increases in entrepreneurs' risk aversion ρ_E, i.e., $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_E} > 0$, if and only if $\omega < \frac{2}{3}$, $\rho_I > 2\frac{1-\omega}{2-3\omega}\frac{\tau_\eta \tau_\delta^2}{\tau_\eta \tau_\delta + \tau(\tau_\eta + \tau_\delta)}$ and $\rho_E > \frac{\tau_\delta^2(2\tau_\eta - \tau)}{\tau_\eta \tau_\delta + \tau(\tau_\eta + \tau_\delta)}$;

(iii) increases in the common cash flow volatility τ_η^{-1} i.e., $\frac{\mathrm{d}c^*}{\mathrm{d}\tau_\eta} < 0$ if and only if $\rho_E > \frac{2-3\omega}{1-\omega}\rho_I$;

(iv) decreases in the idiosyncratic cash flow volatility τ_δ^{-1}, i.e., $\frac{\mathrm{d}c^*}{\mathrm{d}\tau_\delta} > 0$; and

(v) decreases in the reporting precision τ, i.e., $\frac{\mathrm{d}c^*}{\mathrm{d}\tau} < 0$, if $\omega < \frac{2}{3}$ and $\rho_E > \frac{2-3\omega}{1-\omega}\rho_I$.

Recall that the relative weight placed on the two constituents determines to which direction the residual systematic risk drives the optimal comparability. Generally, investors' risk aversion tends to decrease the optimal level of comparability because the associated risk premium effect drives the optimal comparability closer to the minimum level. However, when the weight placed on the investors is excessively high, i.e., $\omega > \frac{2}{3}$, investors' surplus takes precedence, making the effect of the residual systematic risk drive the optimal comparability closer to the maximum level instead. Hence, the optimal comparability increases in the investors' risk aversion in this case.

The effect of entrepreneurs' risk aversion depends on the original level of comparability. Recall that the price risk borne by the entrepreneurs is minimized at $c = \frac{\tau_\delta}{\tau_\eta + \tau_\delta}$. When the weight placed on the investors is not excessively high and both constituents are relatively risk averse, the market monitoring effect is dominated by the price risk effect and risk premium effect, leading the optimal comparability to lie in $\left(0, \frac{\tau_\delta}{\tau_\eta+\tau_\delta}\right)$. As the entrepreneurs become more risk averse, it increases the optimal comparability toward its upper bound $\frac{\tau_\delta}{\tau_\eta+\tau_\delta}$. Otherwise, when the optimal level of comparability lies in $\left(\frac{\tau_\delta}{\tau_\eta+\tau_\delta}, 1\right)$, the optimal comparability moves closer toward its lower bound $\frac{\tau_\delta}{\tau_\eta+\tau_\delta}$ as the entrepreneurs become more risk averse.

As discussed earlier, a higher level of comparability allocates more systematic risk to the investors. When the investors are relatively more risk tolerant than the entrepreneurs, they are efficiently allocated more risks. Hence, in this case, an increase in the common cash flow volatility τ_η^{-1} leads to an increase in the optimal level of comparability. Similarly, when the entrepreneurs are relatively more risk tolerant than the investors, they are efficiently allocated more risk with a decreased level of comparability.

Observe from equations (1.11) – (1.12) that the idiosyncratic volatility affects the optimal comparability only through the net return of effort and the idiosyncratic component of the price risk. As the idiosyncratic shock becomes more volatile, that is, as τ_δ^{-1} increases, firm price is more responsive to the relative accounting performance, which results in higher effort by the entrepreneurs but also more price risk. Hence, the optimal policy leans more toward reducing the idiosyncratic component of the price risk than resolving the moral hazard problem by introducing more idiosyncratic measurement error with a lower level of comparability.

As for the effect of an improvement in the reporting precision on the optimal comparability, we first consider the special case in which the two constituents have the same risk attitude and are equally weighted by the standard setter. In this case, the standard setter's optimal choice of comparability is independent of both the common cash flow shock and the common measurement error due to the risk allocation role of comparability. Consequently, when the reporting quality improves, the optimal allocation of the reporting precision to the idiosyncratic measurement error stays the same and it is more efficient to devote all the efforts to reducing the common measurement error. Generally, when the weight placed on the investors is not excessively high, increasing the relative risk aversion of the entrepreneurs will only strengthen the idiosyncratic part of the price risk effect. As a result, the negative effect of an improved reporting precision on the optimal level of comparability will even be more salient.

1.4. Empirical Implications

Besides rationalizing the existence of diverse accounting practices despite regulatory emphasis on comparability, our analysis generates various additional empirical implications. The pricing equation (1.7) suggests that firms are priced relatively to other firms given their relative accounting performance. More recently, Jennings et al. (2020) document that a firm's price responds positively to an improvement in the relative ranking of its earnings among its peers. Our theory thus predicts that such price response to the relative accounting performance is more pronounced in regimes with a higher level of standardization.

Rewriting the pricing equation in *Lemma 1.1*, we have

$$\begin{aligned} P_i(s_i, \bar{s}) = \hat{a}_i + \alpha s_i + \beta \bar{s} - \frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}\hat{a} - \rho_I \frac{1}{\tau_\eta + \tau_\epsilon}, \text{ where} \\ \alpha = \frac{\tau_\xi}{\tau_\delta + \tau_\xi}, \ \beta = \frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon} - \frac{\tau_\xi}{\tau_\delta + \tau_\xi}. \end{aligned} \tag{1.13}$$

The price coefficient α captures the extent to which a firm's price relies on its own accounting signal, and is henceforth referred to as the direct earnings response coefficient (the direct ERC), whereas the price coefficient β captures the extent to which a firm's price relies on the average accounting signal (or, equivalently, the peer firms' accounting signals), and is henceforth referred to as the cross earnings response coefficient (the cross ERC).

Corollary 1.3. *The direct ERC increases in comparability and the cross ERC decreases in comparability. Moreover, the cross ERC is positive (negative) if comparability is lower* *(higher) than* $\frac{\tau_\delta}{\tau_\eta+\tau_\delta}$.

Higher comparability increases the direct ERC due to the increased informativeness of the relative accounting performance. It enables the market to learn more about the idiosyncratic characteristic of each firm relative to the average firm (market portfolio). As a result, the price of the firm responds more aggressively to the firm's report when the reports are more comparable. On the other hand, achieving higher comparability impairs investors' knowledge of the common shock by introducing more common error. For example, a high average report can stem from either a favorable common shock or too much false positive due to the common measurement error. When there is more common error, the average report is used relatively more in correcting for the common error than in updating about the common shock. The cross ERC thus decreases in comparability and can even be negative when comparability is sufficiently high.

We can exploit the above feature of the pricing equation to construct a regime-level empirical proxy for comparability. The cross ERC estimated using a cross section of firms is negatively associated with the level of comparability. A limitation of this proxy is that it does not allow for firm-wise variations as in De Franco et al. (2011) and Fang, Iselin and Zhang (2021). However, an advantage is that it does not require time-series data assuming that comparability is time-invariant. Hence, this proxy can be used to compare comparability across regimes at a particular time, or to study time-series variations of comparability in a particular regime over time, for example, around policy changes. In addition, this proxy can be adopted to test some of the predictions in *Corollary 1.2*. For example, the model predicts that the optimal level of comparability decreases in the idiosyncratic cash flow volatility, implying that markets with more heterogeneous firms tend to feature more diverse accounting practices.[9]

[9] Dye and Sridhar (2008) make a similar prediction that when the level of heterogeneity is high, a flexible regime with opportunistic reporting and no common measurement error dominates a rigid regime with a common measurement error but no opportunistic reporting. We abstract from the reporting discretion issue but compare regimes with a spectrum of proportions of the common error with respect to the total reporting noise.

1.5. Extension

In this section we extend the baseline model to allow for firm-wise heterogeneity in reporting comparability. For example, financial reports of firms in the same industry may be more comparable than those in different industries. To simplify the analysis, we assume there are two groups of firms and firms in each group are subject to a group-specific common measurement error in addition to firm-specific idiosyncratic measurement errors. Indeed, we restrict the cross-group comparability to be zero while allowing the within-group comparability levels to vary across the two groups. We obtain the results corroborating the robustness of the analysis in the baseline setting, and discuss the potential of testing our predictions using firm-level empirical measures of comparability such as the ones constructed by De Franco et al. (2011) and Fang et al. (2021).[10]

Specifically, in this extended setting, the future cash flow of firm i in group $k \in \{1, 2\}$ is

$$\tilde{\theta}_{k,i} = a_{k,i} + \tilde{\eta} + \tilde{\eta}_k + \tilde{\delta}_{k,i}, \tag{1.14}$$

where $a_{k,i}$ is the effort chosen by the entrepreneur, $\tilde{\eta} \sim \mathcal{N}(0, \tau_\eta^{-1})$ is the economy-wide common cash flow shock, $\tilde{\eta}_k \sim \mathcal{N}(0, \tau_{\eta_k}^{-1})$ is the within-group common cash flow shock, and $\tilde{\delta}_{k,i} \sim \mathcal{N}(0, \tau_{\delta_k}^{-1})$ is the firm-specific idiosyncratic shock. Conditional on the cash flow, firm i's accounting signal is

$$\tilde{s}_{k,i} = \tilde{\theta}_{k,i} + \tilde{\epsilon}_k + \tilde{\xi}_{k,i}, \tag{1.15}$$

where $\tilde{\epsilon}_k \sim \mathcal{N}(0, \tau_{\epsilon_k}^{-1})$ is the within-group common error and $\tilde{\xi}_{k,i} \sim \mathcal{N}(0, \tau_{\xi_k}^{-1})$ is the firm-specific idiosyncratic error. As in the baseline model, controlling for the reporting precision, i.e., $\tau_k^{-1} \equiv \tau_{\epsilon_k}^{-1} + \tau_{\xi_k}^{-1}$, we measure within-group comparability as $c_k = \frac{\tau_{\epsilon_k}^{-1}}{\tau_{\epsilon_k}^{-1} + \tau_{\xi_k}^{-1}}$. Without loss of generality, we assume that each group is of a half measure.

Lemma 1.2. *With conjectured levels of effort $\hat{a}_{k,i}$ and $\hat{a}_k$ for firm i and group-k average,*

[10] Those firm-level empirical measures of comparability are constructed for each firm-year against the other firms in an identified peer group.

respectively, the pricing equation applied by the investors is:

$$P_{k,i}(s_{k,i}, \bar{s}_k, \bar{s}_{k'}) = \hat{a}_{k,i} + \frac{\tau_{\xi_k}}{\tau_{\delta_k} + \tau_{\xi_k}}(s_{k,i} - \bar{s}_k) + b_{kk}(\bar{s}_k - \hat{a}_k) + b_{kk'}(\bar{s}_{k'} - \hat{a}_{k'}) - \rho_I \mathbf{m}^\intercal \mathbf{\Sigma}_{\eta|s} \mathbf{m}, \quad (1.16)$$

where b_{kk}, $b_{kk'}$, $\mathbf{\Sigma}_{\eta|s}$, and $\mathbf{m}$ are constants given in Appendix A.

In equilibrium, investors' conjecture coincides with the actual effort exerted by the entrepreneurs:

$$\hat{a}_{k,i} = a^*_{k,i} = \frac{\tau_{\xi_k}}{\tau_{\delta_k} + \tau_{\xi_k}}. \quad (1.17)$$

When inferring the future cash flow of firm i in group k, investors first use the difference between the firm-specific signal and the group average signal $s_{k,i} - \bar{s}_k$ to infer about the firm's idiosyncratic shock $\tilde{\delta}_{k,i}$, and then use the average signal of each group $\bar{s}_k$ and $\bar{s}_{k'}$ to learn about the common shocks $\tilde{\eta} + \tilde{\eta}_k$. Both group average signals are useful in the second task because of the economy-wide correlation induced by the common cash flow shock $\tilde{\eta}$. Lastly, the risk premium is determined by the investors' residual uncertainties about the undiversifiable common shocks $\{\tilde{\eta} + \tilde{\eta}_1, \tilde{\eta} + \tilde{\eta}_2\}$, $\mathbf{\Sigma}_{\eta|s}$, and the mass of each group in the market portfolio $\mathbf{m}$.

Rewriting the pricing equation (1.16) following equation (1.13), the direct ERC and cross ERC for firms in group k are respectively:

$$\alpha_k = \frac{\tau_{\xi_k}}{\tau_{\delta_k} + \tau_{\xi_k}}, \ \beta_k = b_{kk} - \frac{\tau_{\xi_k}}{\tau_{\delta_k} + \tau_{\xi_k}}. \quad (1.18)$$

Since the average signal of the other group $\bar{s}_{k'}$ also affects the price of a firm in group k, from an econometrics point of view, the reports of other groups also need to be controlled in order to have unbiased estimates of the direct ERC and cross ERC. Moreover, the direct ERC α_k increases in comparability c_k and the cross ERC β_k decreases in comparability c_k. At the same time, however, the cross ERC of firms in the other group $\beta_{k'}$ increases in comparability c_k. In other words, the difference $\alpha_k - \alpha_{k'}$ monotonically increases in $c_k - c_{k'}$, and the difference $\beta_k - \beta_{k'}$ monotonically decreases in $c_k - c_{k'}$. Hence, the predictions in *Corollary 1.3* can be tested using firm-level empirical measures of comparability. For example, Chen, Kurt and Wang (2020) show that the empirical measure constructed by De Franco et al. (2011) is positively associated with the direct ERC.[11]

The certainty equivalents of a representative investor and a representative entrepreneur of each group are respectively:

$$\mathrm{CE}_I = \frac{\rho_I}{2}\mathbf{m}^\intercal \mathbf{\Sigma}_{\eta|s}\mathbf{m}, \tag{1.19}$$

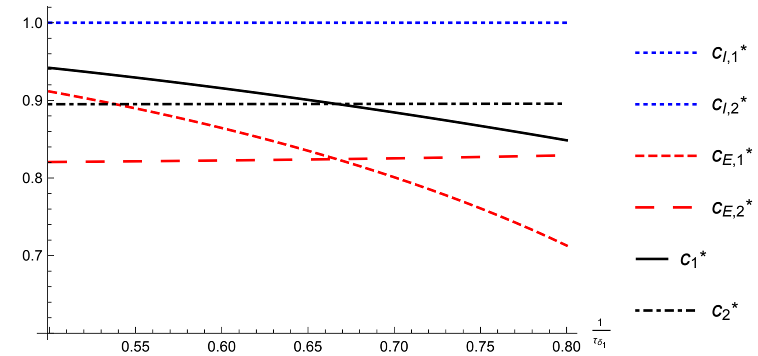
$$\mathrm{CE}_{E,k} = a_k^* - \frac{1}{2}(a_k^*)^2 - \frac{\rho_E}{2}\left(\frac{1}{\tau_{\delta_k}}a_k^* + \left(\mathbf{\Sigma}_\eta - \mathbf{\Sigma}_{\eta|s}\right)_{k,k}\right) - \rho_I \mathbf{m}^\intercal \mathbf{\Sigma}_{\eta|s}\mathbf{m}, \tag{1.20}$$

where $\mathbf{\Sigma}_\eta$ denotes the prior uncertainty about the common shocks $\{\tilde{\eta}+\tilde{\eta}_1, \tilde{\eta}+\tilde{\eta}_2\}$. Assuming that the standard setter aims to maximize $W \equiv \frac{1}{2}(1-\omega)(\mathrm{CE}_{E,1} + \mathrm{CE}_{E,2}) + \omega \mathrm{CE}_I$, let $\{c_{I,1}^*, c_{I,2}^*\}$, $\{c_{E,1}^*, c_{E,2}^*\}$, and $\{c_1^*, c_2^*\}$ be the pair of comparability that respectively maximizes CE_I, $\mathrm{CE}_{E,1} + \mathrm{CE}_{E,2}$, and W.[12] Consistent with our main results in *Propositions 1.1 – 1.2*, the investors always prefer perfect comparability for both groups of firms, whereas the entrepreneurs prefer a strictly lower level of comparability. Consequently, the optimal levels of comparability chosen by the standard setter lie in between.

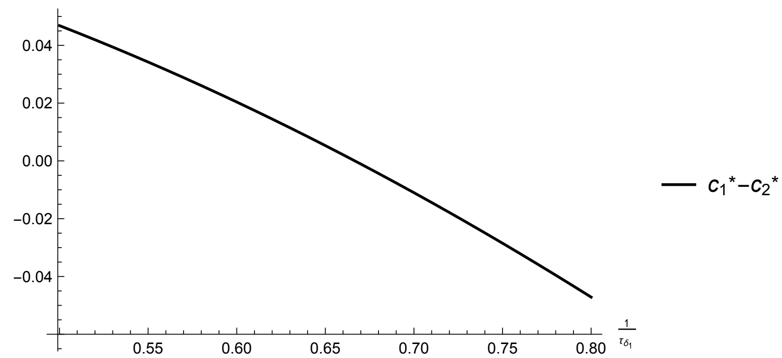
We also illustrate the robustness of our results in this extension using numerical examples as presented in Figure 1.2 due to computational complexity. We focus on the effect of idiosyncratic cash flow volatility because it is the only unambiguous and monotone comparative statics in *Corollary 1.2*. As can be seen from Figure 1.2(a), although group-1 idiosyncratic cash flow volatility affects the entrepreneurs' preferred comparability $c_{E,k}^*$ and the optimal policy c_k^* for both groups, its effects on group-1's tend to be in the opposite direction than

[11] Variations in the firm-level empirical measures are mostly cross-sectional. Hence, we need to show that the cross-sectional difference $c_k - c_{k'}$ affects the cross-sectional differences $\alpha_k - \alpha_{k'}$ and $\beta_k - \beta_{k'}$ in the same manner as c affects α and β in *Corollary 1.3*.

[12] Here, the entrepreneurs' preferred pair of comparability is defined as the one that maximizes their total surpluses rather than the Nash-equilibrium outcome because we consider the entrepreneurs as a single political constituent who influences the standard setter as a coalition. Nevertheless, solving for the pair of comparability as a Nash-equilibrium outcome yields qualitatively similar results.



(a) Constituents' preferred and optimal levels



(b) Cross-sectional difference in optimal levels

Figure 1.2: The Effects of Idiosyncratic Cash Flow Volatility on Comparability

Note: Figure 1.2(a) plots the effects of the group-1 idiosyncratic cash flow shock volatility, i.e., $\tau_{\delta_1}^{-1}$, on different constituents' preferred and the optimal levels of comparability, holding constant the group-2 idiosyncratic cash flow shock volatility. Figure 1.2(b) plots the effect on the cross-sectional difference in the two groups' optimal levels of comparability. Other parameter values are $\omega = 0.5$, $\rho_E = \rho_I = 0.5$, $\tau_\eta = \tau_{\eta_1} = \tau_{\eta_2} = 1$, $\tau_{\delta_2} = 1.5$, and $\tau_1 = \tau_2 = 1$.

those on group-2's and also much more significant in magnitude. Combining it with the observation from Figure 1.2(b), both the group-1 optimal comparability c_1^* and the cross-group difference in the optimal comparability $c_1^* - c_2^*$ decrease in group-1 idiosyncratic cash flow volatility $\tau_{\delta_1}^{-1}$. Indeed, the same holds symmetrically for group-2 idiosyncratic cash flow volatility. Hence, the empirical predictions provided by *Corollary 1.2(iv)* can potentially be tested using firm-level empirical measures of comparability as well.

1.6. Conclusion

Comparability of financial information has long been a major concern of regulators, investors, and researchers. Nevertheless, the FASB and the IASB's definitions of comparability are highly conceptual and binary. They do not make a statement about whether one accounting regime is more comparable than the other, but rather describe the characteristics of comparable information. A proper definition of comparability from an informational perspective is rather important because both FASB and IASB adopt the conceptual-framework approach to standard setting; they first define the desirable qualitative characteristics of financial information as the guideline for further deliberation of detailed standards to fulfill those characteristics. This paper attempts to fill the gap by developing the statistical and informational properties of accounting reports under varying degrees of comparability, which are arguably consistent with and faithful to the descriptions by the regulators and academics.

Our theoretical framework highlights the information externalities of comparable accounting reports and the resulting trade-off in learning about similarities and differences between firms. Such an informational trade-off leads to subsequent economic trade-offs and implies different preferences for accounting comparability by different agents, depending on whether they are the users or the preparers of financial information. As a result, although preferred by the users, perfect comparability is not what standard setters should attempt to achieve because it imposes excessive costs on the preparers. The model thus explains why we still observe diverse accounting practices despite regulatory emphasis on comparability. As accounting policies are influenced by constituents with different interests, the analysis of the "optimal level of comparability", which internalizes the interests of both entrepreneurs (preparers) and investors (users) in the model, provides empirical predictions about the determinants of accounting policies from a positive perspective (Watts and Zimmerman, 1978). However, we abstract from any institutional frictions that affect how different interests are incorporated into the standard setting process (Bertomeu and Cheynel, 2013).

Our study is subject to other caveats. We work with comparability as an informational property while assuming away a more practical problem: how standards should be set to achieve this property. Analytical research can potentially add to understanding this practical problem. For example, most accounting measurement methods are discrete in the sense that continuous accounting inputs are truncated at a recognition threshold (Gao and Jiang, 2020). It is thus an open question whether using the same threshold for all firms (uniformity) enhances or undermines comparability. We also assume away opportunistic reporting by the preparers, although the lack of comparability can be endogenously caused by opportunistic reporting. Dye and Sridhar (2008) compare two polar cases: one with common measurement error but without reporting discretion (the rigid regime) versus one without common measurement error but with reporting discretion (the flexible regime). Future research can extend the analysis to examine how more flexibility and opportunistic reporting cause lower correlation of measurement errors and lower comparability. Moreover, we assume a stylized economy in which firms do not have strategic interactions with others. In reality, firms could be substitutes because they are competitors, or complements because there are technology and demand spillovers.

Chapter 2

Heterogeneous Interpretations of Public Disclosure, Price Efficiency, and Keynesian Beauty Contests

2.1. Introduction

The role that public disclosures play in stock markets remains a fundamental and pervasive concern of accounting researchers and practitioners. Conventional wisdom seems supported by most extant theory predicting that more disclosure improves financial market quality (Goldstein and Yang, 2017). Nevertheless, Morris and Shin (2002) argue that public information can be detrimental in the sense that it can induce agents to coordinate in placing more weight on it than a social planner would. However, embedding such a notion of Keynesian beauty contests into a stock market setting with overlapping generations follow-

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ing Allen, Morris and Shin (2006) (henceforth, AMS), Gao (2008) argues that more public information unambiguously improves market efficiency. In this paper, we show that this is not necessarily true if public information is interpreted heterogeneously by investors. In particular, we examine the conditions under which more public information that is subject to heterogeneous interpretations impairs market efficiency.

There is abundant evidence indicating that investors interpret firms' disclosures heterogeneously (Barron, 1995; Bamber, Barron and Stober, 1997; Leuz and Verrecchia, 2000; Cho and Kwon, 2014). The extent of such disagreements seems to be related to the disclosure's properties.[1] As stated by Barron (1995), "[e]vidence that an announcement has been commonly interpreted also suggests that its contents have been communicated clearly." Following Barron (1995) and more recent papers such as Myatt and Wallace (2012), we denote the extent to which investors agree on the interpretation of a public disclosure as *clarity*. However, regardless of their clarity, disclosures may contain different amounts of information. That is, they may differ in *transparency*. Formally, we measure transparency as the amount of information investors can ultimately collectively extract from a public disclosure (see, e.g., Barth, Konchitchki and Landsman, 2013; Lang and Maffett, 2011; Sadka, 2011).[2]

We investigate the effects of public disclosures on price efficiency in a market populated by short-horizon investors with heterogeneous public disclosure interpretations. We model heterogeneous interpretations as private signals that are correlated conditional on the fundamental value of the firm. We decompose the noise of each private signal into a common component and an idiosyncratic component.[3] The common noise reflects the imperfection in the investors' collective knowledge and leads to a positive correlation among signals, whereas the idiosyncratic noise reflects the investors' disagreement in interpreting the public disclo-

[1] For instance, Collins, Gong and Hribar (2003) provide empirical evidence that earnings announcements containing more accruals seem to generate more disagreement among investors.

[2] For example, Barth et al. (2013) measure transparency as "the extent to which earnings and change in earnings covary contemporaneously with stock returns."

[3] Myatt and Wallace (2012) provide a micro-foundation of such an information structure based on the notion of "rational-inattention." An investor's understanding of a public signal is costly and, thus, depends on the investor's information-acquisition choice.

sure. This results in two properties of the public disclosure: transparency, measured by the precision of the common noise, and clarity, measured by the precision of the idiosyncratic noise. Our analysis reveals that while increasing clarity generally increases market efficiency, increasing transparency can decrease it. Moreover, the level of transparency that maximizes price efficiency monotonically increases in clarity, implying that the best attainable level of price efficiency hinges on the level of clarity. Thus, our results suggest that beyond a certain level of transparency, fostering clarity should take precedence in promoting market efficiency.

We embed the information structure into a competitive capital market with short-horizon investors. Investors' short horizons are characterized through a sequence of overlapping generations, each of which lives for only one period (see, e.g., He and Wang, 1995; Bacchetta and Van Wincoop, 2008; Gao, 2008). Each generation buys the shares from the previous generation and sells them to the next generation, except for the last generation, who consumes the terminal payoff of the firm. Therefore, investors in all but the last generation make their trading decisions concerned only about the interim price at which they will sell their shares.

Specifying prices as average expectations as in AMS, we first perform a preliminary analysis that intuitively illustrates the basic underlying economic trade-offs in closed form and extends to an arbitrary finite number of periods. Then, in our main analysis, we assume that risk-averse investors form rational expectations about their future payoffs using the information revealed by current and historical prices. Although intractability forces us to confine the analysis to a two-period model, we are able to confirm the existence and uniqueness of the equilibrium. We show that, although in AMS and Gao (2008) the price formed in a short-horizon economy places more weight on the public signal than an analogous long-horizon economy does, this is not necessarily the case when short-horizon investors interpret the public disclosure heterogeneously. Short-horizon investors overweight their private interpretations because they care only about the aggregate interpretation rather than the fundamental. This makes their private interpretations relatively more informative in forecasting the future price than the common prior.

Viewing the price as a weighted average of the prior and the aggregate interpretation blurred by the supply noise, transparency and clarity affect price efficiency through two channels: the weight channel and the precision channel. The former is tantamount to price sensitivity, which refers to the extent to which the price responds to the investors' collective private information. The latter is determined by both the precision of the aggregate interpretation and the price informativeness. The precision of the aggregate interpretation represents the amount of the investors' collective information about the firm's fundamental value, and the price informativeness measures the extent to which the price conveys such information.

While transparency and clarity have symmetric effects on price informativeness in a long-horizon economy, they have different effects in a short-horizon economy. Specifically, if transparency and clarity are of the same magnitude, clarity is of first-order importance in improving price informativeness in a short-horizon economy. Moreover, although in the long-horizon economy a higher transparency always leads to an improvement in price informativeness, increasing transparency in a short-horizon economy can be detrimental. Intuitively, an increase in transparency has two countervailing effects. On the one hand, it increases the informativeness of private interpretations about the fundamental value of the firm, inducing investors to rely more on their own interpretations (the *information effect*). On the other hand, it reduces the correlation among interpretations, which makes investors' private interpretations less informative about other investors' interpretations, thereby reducing investors' reliance on their private interpretations (the *coordination effect*). If transparency is already high, an increase in transparency does not increase the informativeness of private interpretations significantly due to the existence of the idiosyncratic noise; thus, the coordination effect dominates.

Transparency and clarity affect price sensitivity in a qualitatively similar manner. However, since investors use both their private interpretations and the prices to determine their demand for the risky asset in a noisy rational expectations equilibrium, the aggregate interpretation ends up being heavily weighted in prices. The resulting price sensitivity is thus higher than the level that maximizes price efficiency. Consequently, any increase in price sensitivity drags it further away from such a level and hence yields a lower price efficiency. Taking the above two channels together, although increasing clarity improves price informativeness, the price sensitivity channel can dominate and thus lead price efficiency to decrease when the level of clarity is relatively low. On the other hand, while a higher transparency leads to a more precise aggregate interpretation and a lower price sensitivity, its overall effect on price efficiency can be negative if the price informativeness channel dominates, which happens only when transparency is sufficiently high. This implies that there exists an interior level of transparency that maximizes price efficiency, and we show numerically that such a level of transparency monotonically increases in clarity.

We extend our model assuming supply noise persistence. Although persistence in liquidity trading gives rise to multiple equilibria as in Cespa and Vives (2015), our main results remain qualitatively robust. Price efficiency can decrease in transparency in the only two stable equilibria, the low and the intermediate information equilibria, when transparency is sufficiently high and clarity is sufficiently low. Moreover, the persistence of supply shocks relaxes the conditions under which price efficiency decreases in transparency. Indeed, the presence of persistence in liquidity trading reinforces investors' coordination motive and hence strengthens our main results.

The rest of the paper is organized as follows. The following section provides a review of related literature. Section 2.3 introduces the information environment and studies the iterated average expectations model. Section 2.4 characterizes the noisy rational expectations equilibrium and examines the effects of transparency and clarity on price efficiency. Section 2.5 confirms that our main results are robust to an extension with correlated supply shocks. Section 2.6 discusses our results and concludes.

2.2. Literature Review

In the economic context of what Keynes (1936) first called a *beauty contest*, Morris and Shin (2002) show that public information can have a detrimental effect on welfare if individuals also have access to private information. Applying this insight to an asset pricing setting with short-horizon investors, AMS illustrate the dual role of public information. Short-horizon investors are concerned about the price at which they expect to sell the firm, which depends on the expectations of other investors about such a price. As a result, public information is informative about the firm's fundamental value (informational role) and about the other investors' beliefs (coordination role). Such a dual role drives investors to excessively use public information, biasing the price away from the firm's fundamental value.

A few subsequent papers have examined the effects of public information on price efficiency (Gao, 2008; Chen, Huang and Zhang, 2014b; Dugast and Foucault, 2018; Banerjee, Davis and Gondhi, 2018). Gao (2008) shows that, in the asset pricing setting of AMS, more public information always increases price efficiency. While Morris and Shin (2002) decouple the coordination and informational roles of public information, Gao (2008) shows that these roles are endogenously linked in an asset pricing setting, allowing the informational role to dominate and driving the price closer to the firm's fundamental value. However, Chen et al. (2014b) find that investors endowed with asymmetrically precise private information will further overweight public information, making public information detrimental again.[4] Indeed, in the above studies, the excessive weight results from the fact that the informational and coordination roles of public information always go in the same direction. In our setting, however, increasing transparency has countervailing effects: it still provides more information about the fundamental, but it also lowers the correlation between private interpretations, making them less useful in predicting the other investors' beliefs. Such a trade-off yields a

[4]In addition, Chen et al. (2014b) show that heterogeneity in private information precision generates a multiplicity of equilibria for low precisions of public information. The existence of multiple equilibria can also emerge with a discontinuity in the marginal value of additional public information (Hellwig and Veldkamp, 2009) or with the coexistence of a competitive effect and an informational leverage effect (see, e.g., Froot, Scharfstein and Stein, 1992; Lundholm, 1988; Dow, Goldstein and Guembel, 2017).

starkly different result: if transparency is large enough to start with, increasing it further can harm price efficiency. Moreover, we show that the coordination motive can actually induce agents with rational expectations to underweight, rather than overweight, public information when it is heterogeneously interpreted by investors.

Our paper contributes to the growing literature in accounting theory examining the effects of information in coordination games. To name a few, Gao and Jiang (2018) examine banks' misreporting decisions under the threat of bank runs. Arya and Mittendorf (2016) analyze the voluntary disclosure of actions in an investment complementarity setting with public and private information. Corona, Nan and Zhang (2019) examine stress tests disclosure when the regulator may be forced to bail banks out if enough of them fail concurrently. The information structure in our model has also been used previously in the higher-order beliefs literature. For instance, Myatt and Wallace (2012) embed this information structure in a setting similar to that of Morris and Shin (2002) and show that investors pay attention to the clearest signals available. Banerjee, Qu and Zhao (2022) experimentally corroborate the importance of clarity in beauty contests, to the extent that participants may completely ignore a public signal if it is not clear enough. Finally, Liang and Zhang (2019) examine the effects of a similar information structure in a bank run setting and show that objectivity can be more important than accuracy in mitigating inefficient panic-based bank runs. We contribute to this thread of literature by examining an asset pricing setting and focusing on the effects of transparency and clarity on price efficiency. In addition, we show that our main results are reinforced if supply shocks are correlated, as in Cespa and Vives (2015), notwithstanding the multiplicity of equilibria.

Overall, our paper can be viewed as a marriage of two streams of literature. On the one hand, we borrow our setting from the short-horizon asset pricing literature launched by AMS and, specifically, with a focus on price efficiency as in Gao (2008). On the other hand, we analyze the effects of correlated private signals in a coordination game as in Myatt and Wallace (2012), Banerjee et al. (2022) and Liang and Zhang (2019). Therefore, our work extends the former by introducing a new information structure with practical significance, and augments the latter by examining price efficiency in an overlapping generations noisy rational expectations setting with short-horizon investors.

2.3. Information Environment

2.3.1. Information Structure

There is a risk-free asset and a risky asset in the economy. The risk-free asset acts as the numeraire of the economy and its return is normalized to zero. The risky asset is a firm's risky stock, the per share random payoff of which, denoted by $\tilde{v}$, is unknown to investors until the last period when it liquidates. We refer to $\tilde{v}$ as the fundamental value or liquidation value of the firm. Investors share the common prior belief that $\tilde{v}$ is normally distributed with mean $\bar{v}$ and precision τ_v. Investors' short horizons are characterized by the overlapping generations assumption. Each generation has a unit continuum of investors indexed by $i \in [0, 1]$. They are born with an endowment, live for only one period, and transfer the firm's ownership to the next generation through trading. Thus, the firm is priced repeatedly by a sequence of overlapping generations of investors until its liquidation at the end of the last period.

At the beginning of the first period, there is a public disclosure regarding the firm's liquidation value. The public disclosure is available to all generations of investors, but they cannot directly use it to assess the firm's value without first interpreting it. Each investor analyzes the public disclosure independently but imperfectly, resulting in heterogeneous interpretations.[5] Formally, each investor $i \in [0, 1]$ of generation $t \in \{1, 2, \ldots, T\}$ privately processes the public disclosure, obtaining a *private interpretation* $\tilde{s}_{ti} = \tilde{v} + \tilde{\eta} + \tilde{\epsilon}_{ti}$, where $\tilde{\eta} \sim \mathcal{N}(0, \tau_\eta^{-1})$ is an noise term that is common to all investors' private interpretations and $\tilde{\epsilon}_{ti} \overset{i.i.d.}{\sim} \mathcal{N}(0, \tau_\epsilon^{-1})$ is an investor-specific (idiosyncratic) noise. We assume information quality is well defined, i.e., τ_v, τ_η, and τ_ϵ are positive and finite, and that all random variables are

[5] See, for example, Lundholm (1988) and Holthausen and Verrecchia (1990).

independent of each other.

We denote the *aggregate interpretation* of all investors in one generation by $\tilde{y} \equiv \int_0^1 \tilde{s}_{ti} \mathrm{d}i = \tilde{v} + \tilde{\eta}$. The precision of the common noise, τ_η, measures the *transparency* of public disclosure, capturing the collective knowledge of all investors. When the public disclosure is perfectly transparent ($\tau_\eta = \infty$), the information that investors collectively know reveals the fundamental value perfectly. When the public disclosure is not transparent at all ($\tau_\eta = 0$), neither any investor's private interpretation nor the aggregate interpretation is informative about the fundamental. That is, transparency is a characteristic of public disclosure whose deficiency bounds investors' collective knowledge about the public disclosure. Even if the price perfectly conveyed all information available to investors in capital markets, a lack of transparency in a public disclosure would prevent the price from perfectly revealing the fundamental value.

The lack of transparency could potentially come from the measurement limitations of the original signal. For instance, take two alternative methods of reporting tradable securities on the balance sheet: historical cost and fair value. A firm that reports the aggregate value of the securities it owns at fair value provides much more precise information about the current value of its securities than reporting them at their historical cost. In other words, fair value measurement increases transparency in the sense that the information that can be potentially gleaned collectively by investors under this method is more informative about the underlying assets.

The precision of the idiosyncratic noise τ_ϵ measures the *clarity* of public disclosure. A direct interpretation of clarity is the extent to which investors form similar interpretations about the public disclosure. All investors interpret the public disclosure in the same way when it is absolutely clear ($\tau_\epsilon = \infty$). At the other extreme, if the public disclosure is not clear at all ($\tau_\epsilon = 0$), each investor's private interpretation is entirely independent of other investors' interpretations and carries infinite noise about the fundamental. Nevertheless, in contrast with the case of zero transparency, investors as a whole are still able to eliminate such a lack of clarity in the aggregate interpretation.

We view clarity as a characteristic of public disclosure that shapes the consensus among investors' interpretations. The presentation of public disclosure can render it more or less susceptible to being interpreted heterogeneously by investors. For instance, a firm may report its profit in a more aggregated or disaggregated way. A more disaggregated method can potentially increase transparency by giving more detailed information. However, each investor may make different judgments regarding the properties of each profit component and about how to aggregate them into a profit number, leading to a decrease in clarity. Indeed, as noted in the above examples, one caveat of our model is that transparency and clarity can be interdependent in the real world. Nonetheless, some accounting policies may improve the transparency of an accounting measure, decreasing clarity to a much lesser extent, or vice versa (Ijiri and Jaedicke, 1966).

The information structure we examine can also be seen as an extension of AMS and Gao (2008) where investors price the firm relying on both a public signal and uncorrelated private signals. We extend these models by assuming that private signals are correlated, leading to different economic implications. We contend that if public disclosure is subject to heterogeneous interpretations by investors, the resulting information structure is one with correlated private signals.[6]

2.3.2. Iterated Average Expectations

The average expectations specification of prices spotlights the role of higher-order beliefs, and facilitates the illustration of the effects of heterogeneous interpretations in capital markets as a beauty contest metaphor. Following AMS, the asset price formed by investors

[6]Such an information structure can also be seen as an approximation of a more nuanced setting where each investor observes a public signal $\tilde{z} = \tilde{v} + \tilde{\eta} + \tilde{\zeta}$ and is also endowed with a complementary private signal $\tilde{x}_{ti} = \tilde{\zeta} - \tilde{\epsilon}_{ti}$. Here, $\tilde{\zeta}$ represents the additional uncertainty that public information poses without being interpreted. Each investor can interpret the public signal differently by using the complementary information to obtain a signal $\tilde{s}_{ti} = \tilde{z} - \tilde{x}_{ti} = \tilde{v} + \tilde{\eta} + \tilde{\epsilon}_{ti}$. The analysis of such an alternative information structure shows that our results hold as long as $\tilde{\zeta}$ is sufficiently imprecise (available upon request).

of generation $t \in \{1, 2, \ldots, T\}$ can be expressed as

$$\begin{aligned} p_t &= \bar{\mathrm{E}}_t[p_{t+1}] = \bar{\mathrm{E}}_t[\bar{\mathrm{E}}_{t+1}[p_{t+2}]] = \cdots = \bar{\mathrm{E}}^{T-t+1}[\tilde{v}] \\ &= (1-\omega_t)\bar{v} + \omega_t y, \text{ where } \omega_t = IC^{T-t}. \end{aligned} \tag{2.1}$$

Investors in the last generation are long-horizon investors as they care only about the fundamental value of the firm. The informativeness of their private interpretations about the fundamental, measured by $I \equiv \frac{\mathrm{Cov}(\tilde{s}_i, \tilde{v})}{\mathrm{Var}(\tilde{s}_i)} = \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\tau_\epsilon^{-1}}$, determines the weight on the aggregate interpretation in the price. However, short-horizon investors in all previous generations care only about the price at which they will be selling the firm to the next generation. Hence, the weight on the aggregate interpretation is multiplied by $C \equiv \frac{\mathrm{Cov}(\tilde{s}_i, \tilde{y})}{\mathrm{Var}(\tilde{s}_i)} = \frac{\tau_v^{-1}+\tau_\eta^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\tau_\epsilon^{-1}}$, which measures the informativeness of private interpretations about the aggregate interpretation, for the number of periods to liquidation. As noted in AMS, "[a]n implication of the [Keynesian beauty contest] metaphor is that an understanding of financial markets requires an understanding not just of market participants' beliefs about assets' future payoffs, but also an understanding of market participants' beliefs about other market participants' beliefs, and higher-order beliefs." Essentially, I measures the *information value* of private interpretations, whereas C measures their *coordination value*.

If a public disclosure is interpreted heterogeneously by investors, then the aggregate interpretation of such a public disclosure is underweighted in the price relative to that in a long-horizon economy. As the time to maturity increases, the beauty contest incentive intensifies and prices become increasingly biased toward the common prior; that is, $\omega_t < \omega_{t+1}$ for any $t < T$. Generally, as long as a signal is not purely public (i.e., it carries some idiosyncratic noise), it is consistently underweighted in the price in the short-horizon economy relative to that in a long-horizon economy.

Although improving public disclosures along either dimension enhances investors' knowledge about the fundamental value, transparency and clarity have different effects on the informativeness of private interpretations about other investors' interpretations. An increase in transparency reduces the information value of investors' private interpretations, whereas an improvement in clarity enhances it. This can be more readily seen when assuming extreme values of the two precisions. If $\tau_\eta \to \infty$, investors' interpretations reduce to purely private signals and contain no coordination value in estimating other investors' interpretations. If $\tau_\epsilon \to \infty$, individual investors' interpretations reduce to a purely public signal that is common to every investor and thus perfect in estimating others' interpretations. We formalize the above observations in the following lemma.

Lemma 2.1. *While the information effect of improving the public disclosure along either dimension is positive, i.e.,* $\frac{\mathrm{d}I}{\mathrm{d}\tau_\epsilon} > 0$ *and* $\frac{\mathrm{d}I}{\mathrm{d}\tau_\eta} > 0$*, the coordination effect of increasing clarity is positive, whereas that of increasing transparency is negative, i.e.,* $\frac{\mathrm{d}C}{\mathrm{d}\tau_\epsilon} > 0$ *and* $\frac{\mathrm{d}C}{\mathrm{d}\tau_\eta} < 0$.

Understanding correlated private signals as heterogeneous interpretations of a public signal thus allows us to examine how disagreement among investors changes the effects of public disclosures on prices. In the following analysis, we refer to the weight on the aggregate interpretation in the asset price, ω_t, as *price sensitivity*, as it measures the extent to which the price responds to the aggregate interpretation.

Lemma 2.2. *For the asset price formed by investors of generation* t*, price sensitivity*

(i) monotonically increases in clarity, i.e., $\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\epsilon} > 0$*; and*

(ii) decreases in transparency if and only if transparency is sufficiently high and clarity is sufficiently low. More specifically, $\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\eta} < 0$ *if and only if* $\tau_\eta > \hat{\tau}_\eta^\omega$ *and* $\tau_\epsilon < \hat{\tau}_\epsilon^\omega$*, where* $\hat{\tau}_\eta^\omega = \frac{\tau_v \tau_\epsilon}{(T-t)\tau_v - \tau_\epsilon}$, $\hat{\tau}_\epsilon^\omega = (T-t)\tau_v$.

Recall that price sensitivity is determined by both the information value and the coordination value of private interpretations. Due to its positive information and coordination effects, an increase in clarity induces investors to rely more on their private interpretations in forming beliefs about the price at which they will be selling the firm. However, transparency has two countervailing effects on price sensitivity. On the one hand, the positive information effect of a more transparent public disclosure leads investors in the last generation to place more weight on their private interpretations, thereby increasing the weight that every previous generation places on their interpretations as well. On the other hand, the negative coordination effect of the increase in transparency induces investors in all previous generations to rely less on their private interpretations in forecasting the next generation's belief. With sufficient clarity, the negative coordination effect of increasing transparency is minimal, and hence the positive information effect dominates for all levels of transparency. However, with poor clarity, the coordination effect can dominate the information effect when the public disclosure is sufficiently transparent. As a result, price sensitivity first increases and then decreases in transparency.[7] Moreover, since the coordination effect compounds with every additional generation of investors, it is amplified by the number of periods to liquidation, resulting in a lower $\hat{\tau}_\eta^\omega$.

We now proceed to investigate how heterogeneous interpretations affect the extent to which the price reflects the fundamental value of the firm. We measure *price efficiency* (PE) as the reciprocal of the mean squared deviation of price from the fundamental following Gao (2008). That is, $\mathrm{PE} \equiv \frac{1}{\mathbb{E}[(\tilde{p}-\tilde{v})^2]}$.

Lemma 2.3. *In an average expectations specification, price efficiency*

(i) monotonically increases in clarity, i.e., $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon} > 0$*; and*

(ii) decreases in transparency if and only if transparency is sufficiently high and clarity is sufficiently low. More specifically, $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} < 0$ *if and only if* $\tau_\eta > \hat{\tau}_\eta^{\mathrm{PE}}$ *and* $\tau_\epsilon < \hat{\tau}_\epsilon^{\mathrm{PE}}$*, where* $\hat{\tau}_\eta^{\mathrm{PE}}$ *and* $\hat{\tau}_\epsilon^{\mathrm{PE}}$ *are defined in Appendix B.3.*

Viewing the price as a weighted average of the prior and the aggregate interpretation, the characteristics of public disclosures affect price efficiency through two channels: the weight

[7] To see the trade-off between the countervailing effects of transparency on the price from a mathematical perspective, note that for any $t < T$, $\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\eta} = C^{T-t-1}\left(C\frac{\mathrm{d}I}{\mathrm{d}\tau_\eta} + (T-t)I\frac{\mathrm{d}C}{\mathrm{d}\tau_\eta}\right)$. For low transparency levels, C is large and I is small, and the positive information effect dominates. As transparency increases, C decreases and I increases. Hence, the negative coordination effect becomes increasingly dominant.

and the precision of the aggregate interpretation. More precisely, plugging in the average expectations specification of prices as expressed in equation (2.1), we obtain

$$\mathrm{PE} = \frac{1}{(1-\omega)^2\tau_v^{-1} + \omega^2\tau_\eta^{-1}}. \tag{2.2}$$

All else equal, price efficiency is maximized at $\omega^* = \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}}$, which coincides with the weight on the aggregate interpretation in $\mathrm{E}[\tilde{v}|y]$. Since each individual investor does not observe the aggregate interpretation perfectly, the actual price sensitivity is smaller than ω^*, i.e., $\omega_t < \omega^*$ for all $t \in \{1, \dots, T\}$. Because idiosyncratic noises are diversified away in the aggregate interpretation, clarity affects price efficiency only through the weight channel. A higher clarity always increases price sensitivity, driving it closer to ω^*, and hence unambiguously increases price efficiency.

Transparency, on the other hand, affects price efficiency through both price sensitivity and the aggregate interpretation precision. While a higher transparency always leads to a more precise aggregate interpretation, it may also decrease price sensitivity and drag it further away from ω^*, resulting in a lower price efficiency. This happens only when the coordination effect strictly dominates the information effect, which, according to *Lemma 2.2*, can take place only for sufficiently transparent public disclosure. Therefore, price efficiency either monotonically increases in transparency, or first increases and then decreases in transparency. Furthermore, the coordination effect intensifies as the time to maturity increases, thus leading to a lower $\hat{\tau}_\eta^{\mathrm{PE}}$.

Concisely, while price sensitivity and price efficiency always increase in clarity, they can display an inverse U-shape with transparency. In such a case, a natural follow-up question is how the level of transparency that maximizes price efficiency, $\hat{\tau}_\eta^{\mathrm{PE}}$ (hereafter, the *optimal transparency*), changes in clarity.

Lemma 2.4. *For any given level of clarity, the optimal transparency, whenever it is interior, increases in clarity, i.e.,* $\frac{\mathrm{d}\hat{\tau}_\eta^{\mathrm{PE}}}{\mathrm{d}\tau_\epsilon} > 0$.

The optimal transparency is determined only by the coordination value of private interpretations. Recall that while transparency diminishes its coordination value by reducing common errors among investors, clarity enhances it by reducing idiosyncratic errors instead. Suppose the initial transparency is set at the optimal level, then in order to maintain the coordination value at its original level after an increase in clarity, transparency should also be increased. In fact, the optimal transparency increases rapidly in clarity and approaches infinity beyond a certain level of clarity. Moreover, since clarity always increases price sensitivity and price efficiency, adjusting transparency to the new optimal level following such an increase in clarity increases price sensitivity and price efficiency even further. In other words, the best attainable levels of price sensitivity and price efficiency hinge on the level of clarity.

2.4. Main Analysis

We now confine our analysis to two periods and derive equilibrium prices in a noisy rational expectations economy with independent supply shocks.[8] Investors can neither separate the two noise terms in their private interpretations nor observe how public disclosure interpretations are aggregated in the price.[9] Investors cannot communicate with each other except through the stock price. We use supply noise to summarize all forces affecting stock prices other than information. The random per capita supply noise of the firm's shares in period t follows a normal distribution, $\tilde{x}_t \sim \mathcal{N}(0, \tau_t^{-1})$, and is independent of all other random variables.

A typical investor has a CARA utility function $U(c) = -e^{-\gamma^{-1}c}$ where c is his consumption financed by selling or liquidating his holdings when he becomes old, and $\gamma > 0$ is his risk tolerance. By solving the expected utility maximization problem, the investor i's demand

[8] We relax this assumption in section 2.5 by allowing supply shocks to be correlated across periods.

[9] Kim and Verrecchia (1997) also make the similar assumption in a setting where each investor obtains private information about the error in a future public disclosure, effectively making the public disclosure meaningless without the private assessment.

for the risky asset is

$$D_i = \gamma \frac{\mathbb{E}[\tilde{w}|\mathcal{I}_i] - p}{\mathrm{Var}(\tilde{w}|\mathcal{I}_i)}, \tag{2.3}$$

where $\tilde{w}$ is the random payoff from holding stocks. $\mathbb{E}[\tilde{w}|\mathcal{I}_i]$ and $\mathrm{Var}(\tilde{w}|\mathcal{I}_i)$ represent his estimates of the mean and variance of the random payoff conditional on his information set $\mathcal{I}_i$, respectively. Short horizons induce investors in the first period to be concerned with the interim price at which they will be selling, rather than the fundamental value v. That is, $\tilde{w} = p_2$ for the first generation and $\tilde{w} = \tilde{v}$ for the second generation.

2.4.1. The Equilibrium

Definition. *A rational expectations equilibrium is defined as a pair of prices* (p_1, p_2) *that satisfies the following:*

(i) the stock market clears in both periods;

(ii) investors maximize their expected utilities, conditional on all available information, including the information gleaned from the stock price; and

(iii) investors have rational expectations in the sense that their beliefs about all random variables are consistent with the true underlying distributions.

In addition, if (p_1, p_2) are linear functions of information and supply noise, then the equilibrium is a *linear rational expectations equilibrium.*

The derivation of the equilibrium follows closely the standard procedures used in the extant literature (e.g., Gao, 2008; Chen et al., 2014b) and is hence omitted. Strong non-linearities and the interdependence between prices in the two periods preclude the availability of a closed-form characterization of equilibrium prices, but we are able to prove the existence and uniqueness of the equilibrium and characterize the equilibrium prices with the following implicit expressions.

Proposition 2.1. *There exists a unique short-horizon linear rational expectations equilibrium (p_1, p_2), characterized by*

$$p_1 = b_1\bar{v} + c_1 y - d_1 x_1, \tag{2.4}$$

$$p_2 = a_2 p_1 + b_2\bar{v} + c_2 y - d_2 x_2, \tag{2.5}$$

where

$$b_1 = 1 - c_1, \tag{2.6a}$$

$$c_1 = \tau_\eta \frac{(\tau_v + \tau_\eta)(\rho_1 + \tau_\epsilon)(\rho_1 + \rho_2 + \tau_\epsilon) + \rho_1 \tau_v \tau_\eta}{\big((\tau_v + \tau_\eta)(\rho_1 + \tau_\epsilon) + \tau_v\tau_\eta\big)\big((\tau_v + \tau_\eta)(\rho_1 + \rho_2 + \tau_\epsilon) + \tau_v\tau_\eta\big)}, \tag{2.6b}$$

$$d_1 = \frac{1}{\gamma}\frac{\tau_\eta}{\tau_v + \tau_\eta}\frac{\rho_2 + \tau_\epsilon}{\rho_2\tau_\epsilon} c_1, \tag{2.6c}$$

$$a_2 = \frac{(\tau_v + \tau_\eta)(\rho_1 + \tau_\epsilon) + \tau_v\tau_\eta}{(\tau_v + \tau_\eta)(\rho_1 + \tau_\epsilon)(\rho_1 + \rho_2 + \tau_\epsilon) + \rho_1\tau_v\tau_\eta}\rho_1, \tag{2.6d}$$

$$b_2 = \frac{\tau_v(\rho_1 + \rho_2 + \tau_\epsilon + \tau_\eta) + \rho_1\tau_\eta}{(\tau_v + \tau_\eta)(\rho_1 + \rho_2 + \tau_\epsilon) + \tau_v\tau_\eta} - a_2, \tag{2.6e}$$

$$c_2 = \frac{\tau_\eta(\rho_2 + \tau_\epsilon)}{(\tau_v + \tau_\eta)(\rho_1 + \rho_2 + \tau_\epsilon) + \tau_v\tau_\eta}, \tag{2.6f}$$

$$d_2 = \frac{1}{\gamma}\frac{\rho_1 + \rho_2 + \tau_\epsilon + \tau_\eta}{\tau_\epsilon\tau_\eta} c_2, \tag{2.6g}$$

$$\rho_1 = \left(\gamma\frac{\tau_v + \tau_\eta}{\tau_\eta}\frac{\rho_2\tau_\epsilon}{\rho_2 + \tau_\epsilon}\right)^2 \tau_1, \tag{2.6h}$$

$$\rho_2 = \left(\gamma\frac{\tau_\epsilon\tau_\eta}{\rho_1 + \rho_2 + \tau_\epsilon + \tau_\eta}\right)^2 \tau_2. \tag{2.6i}$$

Equilibrium prices are linear combinations of the prior, the aggregate interpretation, and the noisy supply. Since the prior is common knowledge and in equilibrium investors know the weights placed on each of these components, for any investor in period $t \in \{1, 2\}$, the information contained in the price is equivalent to a signal of the aggregate interpretation,

$$p_t^* = \tilde{y} - \frac{d_t}{c_t}\tilde{x}_t. \tag{2.7}$$

The extent to which the price is incrementally informative about the aggregate interpretation is measured by $\rho_t = \mathrm{Var}(p_t^*|y)^{-1}$, which we refer to as *price informativeness.*

We are mainly interested in p_1, as we want to examine the effect of investors' short horizons on pricing. The short horizon is manifested in the fact that the first generation of investors prices the firm expecting to sell it at a price p_2. That is not the case for the second generation of investors since they consume the firm's liquidation value at the end of the second period. The only purpose of p_2 is to terminate the overlapping generations cycle, allowing us to apply backward induction to solve for p_1. Nonetheless, the unavailability of a closed-form solution and the interdependence of the implicit expressions for the two prices require us to use equilibrium characteristics of p_2 to analyze the equilibrium characteristics of p_1. Such interdependence is a direct consequence of the information structure. Indeed, we extend the information structure adopted by AMS and Gao (2008) by allowing for correlation among investors' private signals. The following corollary relates our information structure to the standard ones in the extant literature more formally.

Corollary 2.1. *When $\tau_\eta \to \infty$, private interpretations are purely private, i.e., $\tilde{s}_{ti} = \tilde{v} + \tilde{\epsilon}_{ti}$, and prices coincide with those in Gao (2008). In other words, in the absence of a common noise, the two models are equivalent.*

When $\tau_\epsilon \to \infty$, private interpretations are purely public, i.e., $\tilde{s}_{ti} = \tilde{s} = \tilde{v} + \tilde{\eta}$. In other words, in the absence of idiosyncratic noises, all investors act as a representative investor and no higher-order beliefs are involved.

The interdependence between price informativeness in both periods is essentially what renders closed-form solutions unattainable (see expressions (2.6h) – (2.6i)). The dependence of ρ_1 on ρ_2, also present in AMS and Gao (2008), is not unexpected because of the first-generation investors' attempt to forecast p_2. However, the dependence of ρ_2 on ρ_1, absent in AMS and Gao (2008), emerges here as a result of the correlation among private signals. Increasing ρ_1 produces two countervailing effects on ρ_2. First, as $p1$ becomes more informative, private signals become relatively less useful in estimating the fundamental, thereby reducing the incremental informativeness of p_2 and, therefore, decreasing ρ_2. Second, as $p1$ becomes more informative, the residual uncertainty about the fundamental decreases, thereby decreasing the risk perceived by investors. Thus, investors' trading intensity increases, making p_2 more informative. Because p_1 and private interpretations are independent conditional on the fundamental, in AMS and Gao (2008), these two effects cancel each other perfectly. However, with correlated private signals, p_1 and private interpretations become correlated as well conditional on the fundamental, limiting the decrease in the residual variance. As a result, the symmetry between the two effects is lost, decreasing the second-period price informativeness, ρ_2.[10]

In order to illustrate the coordination effect due to investors' short horizons more clearly, we also characterize the unique linear rational expectations equilibrium in a benchmark setting where the investors' horizon is as long as the firm's life span. We call the equilibrium in this setting the *long-horizon equilibrium.* It constitutes a helpful benchmark to contrast the impact of correlated private signals on the equilibrium price efficiency in the presence of investors' short horizons. The formal characterization of the long-horizon equilibrium and subsequent related results are relegated to Appendix B.2 for expositional brevity.

2.4.2. Price Efficiency

In a noisy rational expectations equilibrium, the first-period price efficiency in the short-horizon economy can be expressed as

$$\mathrm{PE} = \frac{1}{(1-c_1)^2\tau_v^{-1} + c_1^2(\tau_\eta^{-1} + \rho_1^{-1})}. \tag{2.8}$$

[10]The price informativeness in the second period can be expressed more generally as $\rho_2 = \left(\gamma \frac{\beta_s}{\mathrm{Var}_{2i}(\tilde{v})}\right)^2 \tau_2$, where β_s is the loading on the private signal in $\mathbb{E}_{2i}[\tilde{v}]$. In Gao (2008), $\beta_s = \frac{\tau_\epsilon}{\rho_1+\rho_2+\tau_\epsilon+\tau_v}$ and $\mathrm{Var}_{2i}(\tilde{v}) = \frac{1}{\rho_1+\rho_2+\tau_\epsilon+\tau_v}$, resulting in $\rho_2 = (\gamma\tau_\epsilon)^2\tau_2$. However, in our setting, $\beta_s = \frac{\tau_\eta\tau_\epsilon}{\tau_v\tau_\eta+(\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon)}$ and $\mathrm{Var}_{2i}(\tilde{v}) = \frac{\rho_1+\rho_2+\tau_\epsilon+\tau_\eta}{\tau_v\tau_\eta+(\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon)}$, resulting in $\rho_2 = \left(\gamma\frac{\tau_\eta\tau_\epsilon}{\rho_1+\rho_2+\tau_\epsilon+\tau_\eta}\right)^2\tau_2$.

As in the average expectations specification, the price can be viewed as a weighted average of the prior and the aggregate interpretation. However, the aggregate interpretation in the price is blurred by the supply noise due to the existence of supply shocks. As a result, in addition to transparency, price informativeness also affects price efficiency through the precision channel. The following proposition summarizes the asymmetric effects of transparency and clarity on price efficiency.

Proposition 2.2. *Transparency and clarity have different effects on price efficiency in the short-horizon economy, both in terms of magnitude and direction. Specifically, price efficiency*

(i) decreases in clarity when clarity is sufficiently low and transparency is sufficiently high, i.e., there exists $\tau_\epsilon^\dagger, \tau_\eta^\dagger \in (0,\infty)$ such that $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\epsilon} < 0$ for $\tau_\epsilon < \tau_\epsilon^\dagger$ and $\tau_\eta > \tau_\eta^\dagger$ if and only if $\gamma^2\tau_v\tau_2 > 1$; and

(ii) decreases in transparency when transparency is sufficiently high and clarity is sufficiently low, i.e., there exists $\hat{\tau}_\eta^{\mathrm{PE}}, \hat{\tau}_\epsilon^{\mathrm{PE}} \in (0,\infty)$ such that $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^{\mathrm{PE}}$ and $\tau_\epsilon < \hat{\tau}_\epsilon^{\mathrm{PE}}$ if and only if $\gamma^2\tau_v\tau_2 > \frac{2}{3}$.

The effects of transparency and clarity on price efficiency are qualitatively similar to those stated in *Lemma 2.3*, yet there are two crucial distinctions in their underlying mechanisms across the two settings. The first distinction is that the price is blurred by the supply noise, and hence price informativeness also affects price efficiency through the precision channel.[11] In particular, as expression (2.8) indicates, a higher price informativeness results in a higher price efficiency. In this context, the following corollary states the effects of transparency and clarity on price informativeness.

Corollary 2.2. *Transparency and clarity have different effects on price informativeness in the short-horizon economy, both in terms of magnitude and direction. Specifically,*

[11] The notion of price informativeness does not exist in the average expectations specification because we shut down learning from prices. Since there is no supply shock, prices are fully informative about the aggregate interpretation. Appendix B.1 connects the two specifications by examining a hypothetical setting in which price informativeness is determined exogenously by assuming imperfect learning from prices.

(i) price informativeness always increases in clarity, i.e., $\frac{d\rho_1}{d\tau_\epsilon} > 0$*;*

(ii) price informativeness decreases in transparency when transparency is sufficiently high and clarity is sufficiently low, i.e., there exists $\hat{\tau}_\eta^\rho, \hat{\tau}_\epsilon^\rho \in (0, \infty)$ *such that* $\frac{d\rho_1}{d\tau_\eta} < 0$ *for* $\tau_\eta > \hat{\tau}_\eta^\rho$ *and* $\tau_\epsilon < \hat{\tau}_\epsilon^\rho$*; and*

(iii) if transparency is of the same or larger magnitude than clarity, the marginal effect of clarity on price informativeness dominates the marginal effect of transparency, i.e., $\frac{d\rho_1}{d\tau_\epsilon} > \frac{d\rho_1}{d\tau_\eta}$ *if* $\tau_\eta \geq \tau_\epsilon$*.*

Price informativeness summarizes how well prices reflect investors' private information. In the presence of supply noise, price informativeness is fostered by the investors' trading aggressiveness. In the long-horizon economy, the coordination motive does not exist. Transparency and clarity play symmetric roles in affecting an individual investor's demand and hence in affecting price informativeness (see *Lemma* B3). In contrast, as *Corollary 2.2* reveals, the presence of beauty contest incentives breaks the symmetry in the short-horizon economy. Because the first generation is concerned only about the second-period price, their trading aggressiveness depends on: (i) the informativeness of their private interpretations about the aggregate interpretation, and (ii) the usefulness of the aggregate interpretation in forecasting the second-period price. The former is regulated by the coordination value of private interpretations. The latter is determined by how much the second generation uses private interpretations, and thus, their information value.

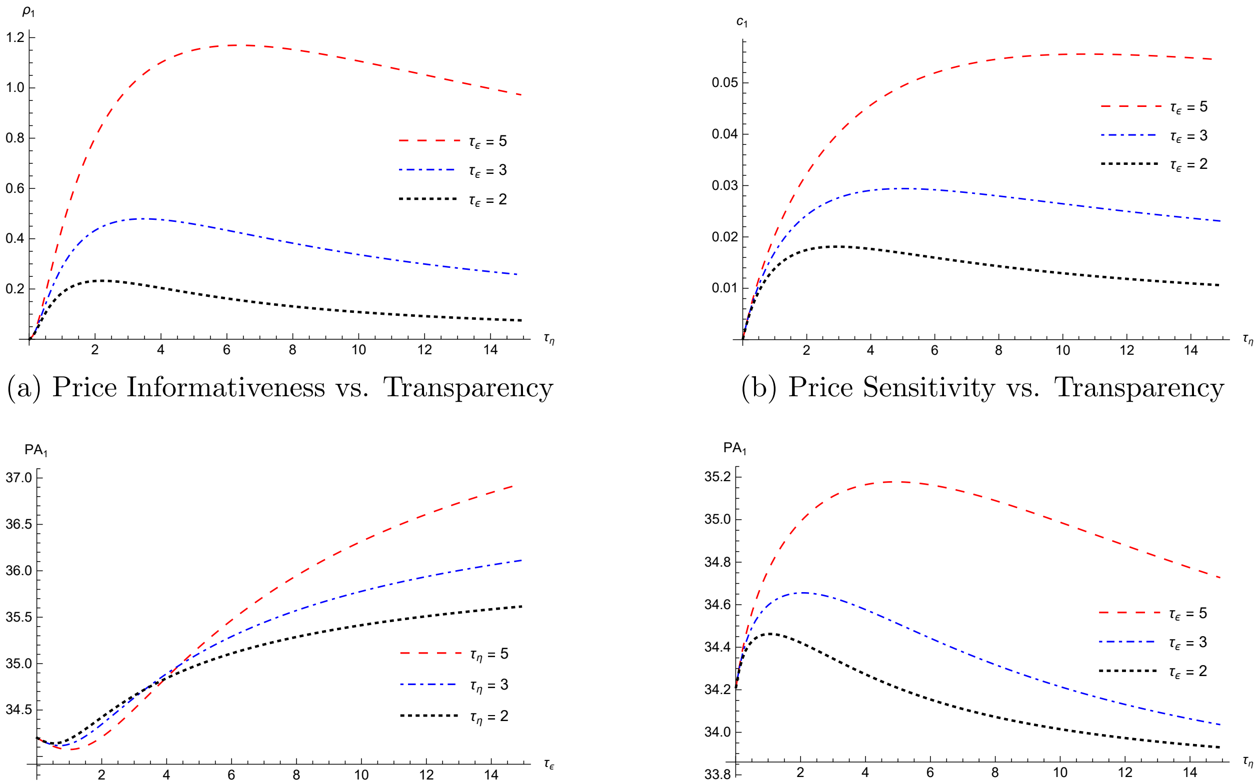
Recall from *Lemma 2.1* that clarity increases both the information value and the coordination value of private interpretations. Following an improvement in clarity, short-horizon investors are more certain about their payoffs from selling shares and hence become more aggressive in submitting their trading demands, rendering the price more efficient in reflecting the aggregate interpretation. An increase in transparency, however, increases the information value of private interpretations but decreases their coordination value. On the one hand, higher transparency makes the second generation more certain about their payoffs from liquidation. With reduced uncertainty, they become more aggressive in submitting their trading demands, rendering the second-period price more contingent on the aggregate interpretation. This is the manifestation of the positive information effect of transparency. On the other hand, a more transparent public disclosure makes the first generation more uncertain about the second generation's aggregate interpretation and thus less aggressive in trading. Such a negative coordination effect renders the first-period price less efficient in reflecting the investors' private information.

Combining the coordination effect and the information effect, the marginal effect of clarity on price informativeness dominates that of transparency when they are of comparable magnitude. This implies that, in such a case, it is more beneficial to improve clarity than transparency from a price informativeness perspective. Moreover, as shown in Figure 2.1(a), the negative coordination effect of an increase in transparency can dominate its positive information effect when clarity is sufficiently low and transparency is sufficiently high, decreasing price informativeness. Regarding the condition in *Corollary 2.2(ii)*, numerical analysis indicates that $\hat{\tau}_\epsilon^\rho$ always exists and is unique, and that if $\tau_\epsilon < \hat{\tau}_\epsilon^\rho$, then $\hat{\tau}_\eta^\rho$ also exists and is unique. In that case, the graph of ρ_1 is single-peaked and thus the aforementioned condition is in fact necessary and sufficient.[12]

The second and more important distinction on price efficiency across the two settings is that, introducing price learning renders the actual price sensitivity consistently higher than its maximizing value from the price efficiency perspective, i.e., $c_1 > c_1^* = \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\rho_1^{-1}}$.[13] This reflects the fact that in addition to their private interpretations, investors also use the information revealed by the current prices to learn about the aggregate interpretation in determining their trading demands. Consequently, any increase in price sensitivity drags it further away from c_1^*, resulting in lower price efficiency. In other words, in contrast with the

[12]This holds true for price sensitivity as well; that is, the thresholds of transparency and clarity beyond (below) which the variable of interest increases or decreases, whenever they exist, are unique. Therefore, the conditions in the following proposition are indeed necessary and sufficient.

[13]The level of price sensitivity that maximizes price efficiency in equation (2.8), c_1^*, coincides with the weight on the price-equivalent signal (as given by expression (2.7)) in $\mathrm{E}[\tilde{v}|p_1^*]$.



(a) Price Informativeness vs. Transparency

(b) Price Sensitivity vs. Transparency

(c) Price Efficiency vs. Clarity

(d) Price Efficiency vs. Transparency

Figure 2.1: Short-Horizon Equilibrium Price Efficiency

Note: The paramater values are $\gamma = 0.1$, $\tau_v = 35$, and $\tau_1 = \tau_2 = 10$. For consistency and easy comparison, we use the same parameter values for subsequent numerical examples in this section unless otherwise specified.

average expectations specification, the direction of any effect of transparency and clarity on price sensitivity is reversed on price efficiency.

Corollary 2.3. *Transparency and clarity have different effects on price sensitivity in the short-horizon economy, both in terms of magnitude and direction. Specifically, price sensitivity*

(i) always increases in clarity, i.e., $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} > 0$*; and*

(ii) decreases in transparency when transparency is sufficiently high and clarity is sufficiently low, i.e., there exists $\hat{\tau}_\eta^c, \hat{\tau}_\epsilon^c \in (0, \infty)$ *such that* $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} < 0$ *for* $\tau_\eta > \hat{\tau}_\eta^c$ *and* $\tau_\epsilon < \hat{\tau}_\epsilon^c$*.*

As the supply noise blurs the inference of the second-period price, transparency and clarity affect price sensitivity through the price informativeness in both periods and through the investors' private interpretations. Taking transparency as an example, we can decompose its effect on price sensitivity as follows,

$$\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} = \frac{\partial c_1}{\partial \tau_\eta} + \frac{\partial c_1}{\partial \rho_1}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta} + \frac{\partial c_1}{\partial \rho_2}\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\eta}.$$

The first term represents the effect of transparency on price sensitivity through the private interpretations. This effect is analogous to and stays in the same direction as the one in the average expectations specification characterized in *Lemma 2.2(ii)*. The remaining two terms represent the effect of transparency on price sensitivity through price informativeness in both periods, an effect that is absent in the average expectations specification. Considering the price as a signal of the aggregate interpretation as in expression (2.7) helps explain how price informativeness affects price sensitivity. A higher first-period price informativeness ρ_1 induces short-horizon investors to use the buying price p_1 more in learning about the aggregate interpretation, i.e., $\frac{\partial c_1}{\partial \rho_1} > 0$. In addition, a higher second-period price informativeness ρ_2 makes the aggregate interpretation more useful in forecasting the selling price p_2, i.e., $\frac{\partial c_1}{\partial \rho_2} > 0$.[14] Essentially, increases in price informativeness in both periods lead the aggregate interpretation to be weighted more in determining the price in the short-horizon equilibrium.

Since most uncertainty about the aggregate interpretation is resolved through the first-period price, the effect of transparency on price sensitivity through the second-period price informativeness is dominated by its effect through the first-period price informativeness. This effect, by *Corollary 2.2*, is consistent with the effect of transparency through private interpretations. Indeed, the effect through private interpretations dominates and drives the trend because (i) most uncertainty is resolved in the first period, and (ii) information in the price results from the aggregation of private interpretations and thus is secondary to the direct effect through private interpretations. Therefore, the overall effect remains the same qualitatively as in the average expectations specification. Figure 2.1(b) provides numerical examples illustrating the non-monotonic relationship between transparency and

[14] This is immediate by inspecting the expression of c_1 given by equation (2.6b).

price sensitivity. The intuition behind the effect of clarity on price sensitivity follows a similar argument.

It is worth noting that, in contrast to the average expectations specification, the aggregate interpretation can be overweighted in the price in the short-horizon equilibrium compared to the long-horizon equilibrium. Recall that price informativeness in either period increases price sensitivity in the short-horizon equilibrium price. Furthermore, with correlated private signals, short-horizon investors care only about the aggregate interpretation. Thus, the prior is less useful in forecasting the second-period price than it would otherwise be if investors instead cared about the fundamental as in Gao (2008). As a result, short-horizon investors rely even more on their private interpretations and the current price in forecasting their payoffs. With the endogeneity of price informativeness further reinforcing this process, the weight on the aggregate interpretation in the short-horizon equilibrium can indeed exceed that in the long-horizon equilibrium when ρ_1 is sufficiently large.[15] Numerical analysis demonstrates that this is possible when γ, τ_v, and τ_2 are sufficiently large. In stark contrast to prior literature, this observation illustrates that the coordination motive can actually induce agents with rational expectations to overweight, rather than underweight, their private signals when their private signals are correlated to each other.

To summarize, the effects of transparency and clarity on price efficiency are determined by their joint effects through the precision channel and the weight channel. Although increasing clarity improves price informativeness, the negative weight channel can dominate, thus leading price efficiency to decrease when the level of clarity is relatively low. On the other hand, when the level of transparency is sufficiently high, the positive weight channel is dominated by the negative precision channel following an increase in transparency. This results in an overall negative effect of transparency on price efficiency. In a nutshell, although improving clarity of a public disclosure generally enhances price efficiency, improving its

[15]In fact, one necessary condition for price sensitivity in the short-horizon equilibrium to be larger than that in the long-horizon equilibrium, i.e., $c_1 > c$, is for the price informativeness in the short-horizon equilibrium to be larger than that in the long-horizon equilibrium, i.e., $\rho_1 > \rho$.

transparency can be detrimental when there is a lack of clarity but sufficient transparency.

In the long-horizon benchmark, while price informativeness and price sensitivity both increase in transparency and clarity, price efficiency can also decrease in transparency (see *Lemma B5*). Nevertheless, in the short-horizon economy, the beauty contest incentive of the first generation of investors strengthens the negative effect of price informativeness and hence makes price efficiency decreasing in transparency for a much larger set of parameters.[16] Figures 2.1(c) and 2.1(d) illustrate with numerical examples where price efficiency decreases in clarity and transparency. We also confirm with numerical examples that our results on the optimal transparency (see *Lemma 2.4*) continue to hold. That is, as shown in Figure 2.2, the optimal transparency, whenever it is interior, monotonically increases in clarity, and so is the best attainable level of price efficiency. In addition, Figure 2.3(a) presents regions in which the condition $\gamma^2\tau_v\tau_2 > \frac{2}{3}$ is satisfied or violated. Figures 2.3(b) – 2.3(d) are calculated at different levels of clarity, and show regions in which price efficiency decreases in transparency if the latter is large enough. The two regions coincide with each other when clarity is sufficiently low, implying $\gamma^2\tau_v\tau_2 > \frac{2}{3}$ is indeed the necessary and sufficient condition for price efficiency to ever decrease in transparency. Note that the region in which price efficiency may decrease in transparency shrinks as clarity increases, suggesting that $\gamma^2\tau_v\tau_2 > \frac{2}{3}$ no longer guarantees the existence of τ_η such that $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0$ when τ_ϵ is large. Indeed, when $\tau_\epsilon \geq \hat{\tau}_\epsilon^{\mathrm{PE}}$, price efficiency uniformly increases in transparency.

2.5. Extension

In this section we extend the baseline model by allowing the supply shocks to be correlated, and show that all of our main results continue to hold. More specifically, the supply shocks are assumed to follow $\tilde{x}_2 = \beta\tilde{x}_1 + \tilde{u}$, where $\beta \in (0, 1)$ and $\tilde{u} \sim \mathcal{N}(0, \tau_u^{-1})$ is indepen-

[16]The necessary and sufficient condition for price efficiency to ever decrease in transparency in the long-horizon equilibrium is $\gamma^2\tau_v\tau_1 > 2$ (see *Lemma* B5) which is more stringent than that in the short-horizon equilibrium. Furthermore, even when this condition is satisfied, numerical analysis shows that the parameter region under which price efficiency decreases in transparency is much smaller than that in the short-horizon equilibrium.

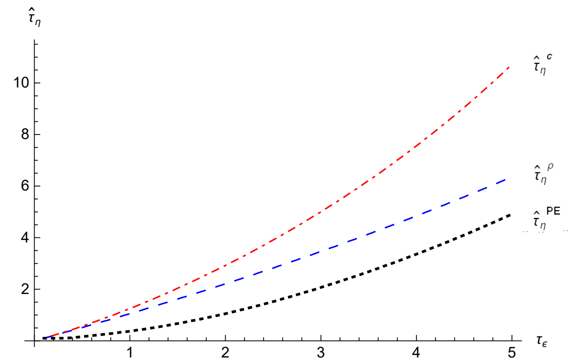
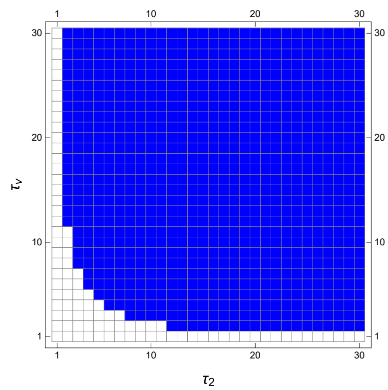
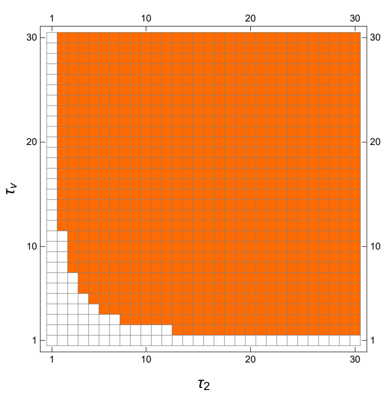


Figure 2.2: Optimal Transparency

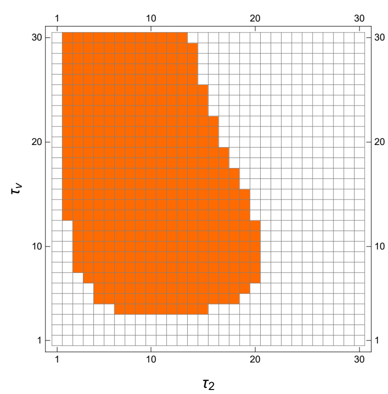
Note: This figure plots the level of transparency that maximizes price sensitivity, price informativeness, and price efficiency, respectively, from top to bottom.



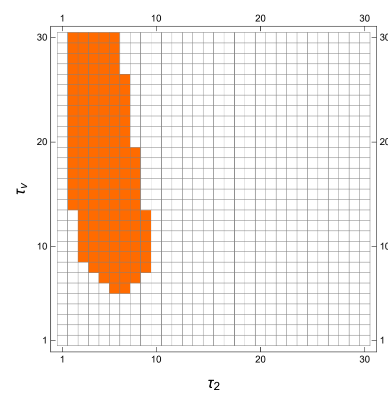
(a) $\gamma^2\tau_v\tau_2 > \frac{2}{3}$



(b) $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0,\ \tau_\epsilon = 1$



(c) $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0,\ \tau_\epsilon = 4$



(d) $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0,\ \tau_\epsilon = 7$

Figure 2.3: Region Plot on the Condition in *Proposition 2.2(ii)*

Note: By keeping $\gamma = 0.1$ and $\tau_1 = \tau_2$ for consistency but varying the combination of τ_v and τ_2, the blue shaded region represents the region in which $\gamma^2\tau_v\tau_2 > \frac{2}{3}$, and the orange shaded regions represent the regions in which $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0$ is possible with different values of τ_ϵ. Every cell is an increment of two units of precision.

dent of $\tilde{x}_1$ and the other random variables. We analytically obtain the results corroborating the robustness of the analysis in the main setting. However, the intractability of the model forces us to perform much of the analysis numerically.

A numerical examination of the model reveals that there can be multiple linear equilibria, which we refer to as the low, intermediate, and high information equilibria, respectively, ordered according to the price informativeness in the first period (see Figure 2.4). We numerically assess the stability of the equilibria and find that, in contrast to Cespa and Vives (2015), the low and intermediate information equilibria are always stable, but the high information equilibrium is always unstable.[17] Analytically, we show that for large enough transparency, only two equilibria remain. Numerically, we see that as transparency grows, the high information equilibrium vanishes, and the low and intermediate information equilibria persist. It is also possible to prove that for sufficiently low transparency, only one equilibrium survives. Numerically, we confirm that it is the low information equilibrium.

Overall, the results in the main setting are qualitatively preserved. For high transparency, the two equilibria that persist and are stable, the low and intermediate information equilibria, both display the same qualitative features as the equilibrium in our main setting.

Proposition 2.3. *In both low (LIE) and intermediate information equilibria (IIE), price efficiency increases in clarity and decrease in transparency when transparency is sufficiently high and clarity is sufficiently low.*

The comparative statics results for sufficiently high transparency and low clarity from the main model and the two surviving equilibria in this extension are summarized and compared against each other in Table 2.1. The low information equilibrium (LIE) converges to the unique equilibrium as the correlation between supply shocks approaches zero. Moreover, since $\frac{2}{2-\beta}$ is increasing in β, the parameter region under which price efficiency may decrease

[17]We perform the stability analysis by introducing a small perturbation around the equilibrium value of ψ_1 and conduct 200 rounds of numerical iterations in solving for equations (B15h) and (B15i). Whenever the numerical solver returns more than one real root, we continue with the one that is the closest to the initial value from the last iteration. The equilibrium is stable if the relative difference between the returned value and the corresponding equilibrium value is within 1×10^{-5}.

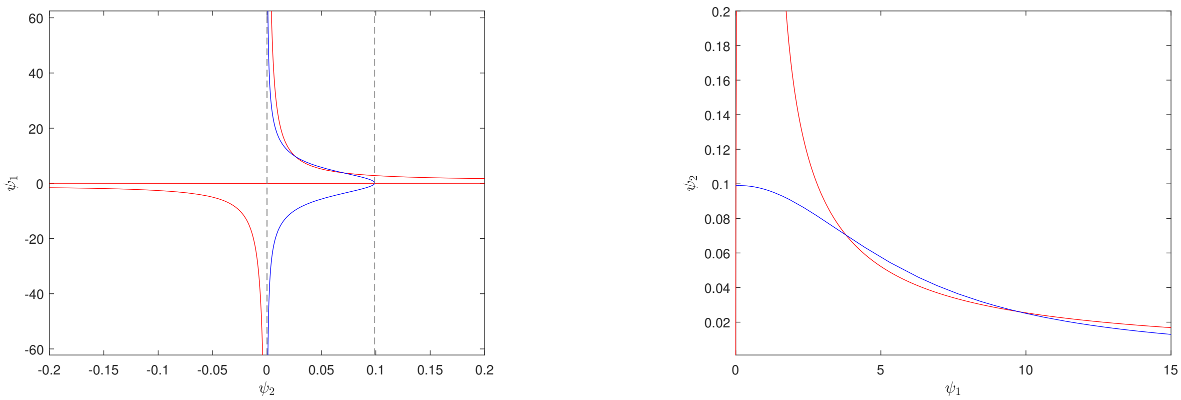


Figure 2.4: Multiple Equilibria with Correlated Supply Shocks

Note: This figure plots the proxies of first and second period price informativeness against each other. The red and the blue curves correspond to ψ_1 and ψ_2 as given by equations (B15h) and (B15i), respectively, in Appendix B.3. The left panel demonstrates that both proxies are positive in equilibrium. The right panel, taken from the upper right corner of the left panel, shows the three equilibria represented by the three intersections: the low, intermediate, and high information equilibria, respectively, from left to right. Note that the red line to the left is not vertical and does not coincide with the y-axis either. The paramater values are $\tau_v = 10$, $\tau_\eta = 100$, $\tau_\epsilon = 1$, $\tau_1 = 1$, $\gamma = 0.1$, $\beta = 0.5$, and $\tau_u = 8$. By varying the parameter combination, there can be only one or two equilibria, which are left out in this figure for the purpose of illustration.

in clarity shrinks as supply shocks become more correlated. On the other hand, since $T_1(\beta)$ is a decreasing function in β, the parameter region under which price efficiency may decrease in transparency expands with an increase in the correlation between supply shocks. In other words, albeit generating multiple equilibria, the presence of persistent liquidity trading strengthens our main results. Essentially, the correlation between supply shocks reinforces the short-horizon investors' coordination motive by reducing the remaining uncertainty about the second-period price and thus making the aggregate interpretation more useful in forecasting their payoffs.

2.6. Discussion

In contrast with most of the prior literature on asset pricing with short-term investors, we consider the effects of heterogeneous interpretations of public disclosure by decomposing the noise in investors' private signals into a common component and an idiosyncratic com-

Table 2.1: Summary of Comparative Statics Results

	$\beta = 0$ ($\tau_2 = \tau_u$)	$0 < \beta < 1$	
		LIE	IIE
$\frac{d\rho_1}{d\tau_\epsilon}$	+	+	+
$\frac{dc_1}{d\tau_\epsilon}$	+	+	- iff $\gamma^2\tau_v\tau_u > T_2(\beta)$
$\frac{dPE_1}{d\tau_\epsilon}$	- iff $\gamma^2\tau_v\tau_2 > 1$	- iff $\gamma^2\tau_v\tau_u > \frac{2}{2-\beta}$	+
$\frac{d\rho_1}{d\tau_\eta}$	-	-	-
$\frac{dc_1}{d\tau_\eta}$	-	-	-
$\frac{dPE_1}{d\tau_\eta}$	- iff $\gamma^2\tau_v\tau_2 > \frac{2}{3}$	- iff $\gamma^2\tau_v\tau_u > T_1(\beta)$	-

Note: $T_1(\beta) = \frac{\beta-3+\sqrt{(\beta+1)(\beta+9)}}{4\beta}, T_2(\beta) = \frac{1}{2\beta^2}\left(\frac{1}{2t+1} - 2t\right)$ where $t \equiv \frac{\tau_1}{\tau_u} \in (0, \frac{2\beta^3}{\beta+1+\sqrt{(9\beta+1)(\beta+1)}})$. Most of the results above are obtained by taking limits of $\tau_\eta \to \infty$ and $\tau_\epsilon \to 0$ due to computational complexity.

ponent. This information structure leads to a positive correlation among investors' signals conditional on the fundamental value of the firm, which results in the precision of the two noise components, transparency and clarity, playing qualitatively and quantitatively different roles on the equilibrium price.

This paper can be seen as a combination of the setting used in the short-horizon asset pricing literature (e.g., AMS; Gao, 2008) and the insights developed in the literature on coordination games with similar information structures (e.g., Myatt and Wallace, 2012; Banerjee et al., 2022; Liang and Zhang, 2019). It extends the former by introducing a new information structure with practical implications, and extends the latter by examining price efficiency in a noisy rational expectations setting. Introducing correlated private signals into such a setting leads to several differences in the firm's stock price behavior. We find that higher-order beliefs can induce investors to either underweight or overweight their private signals relative to the long-horizon economy. This observation is surprising in two ways. Economically, the above observation implies that, due to the semi-private nature of investors' heterogeneous interpretations, public disclosures can be underweighted as opposed to overweighted. Mathematically, it indicates that, in contrast to the prior literature, private signals can be overweighted if they are correlated. As short-horizon investors care only about the aggregate interpretation rather than the fundamental, the common prior belief becomes relatively less useful in forecasting the second-period price.

More importantly, more transparent public information can decrease rather than always increases price efficiency. This result complements the findings in Morris and Shin (2002). Both studies cast doubt on the traditional pro-transparency view by showing that more accurate public information can be bad for price informativeness or social welfare. Nevertheless, the underlying reasons behind these two similar results are strikingly different. Morris and Shin (2002) and subsequent papers claim that the detrimental effect of public information arises from the fact that the coordination motive entails the overweighting of the public signal relative to the one that would be used by the social planner. In our setting with correlated private signals, more transparent public disclosure can decrease price efficiency even when it is underweighted. When the level of transparency is sufficiently high, its negative coordination effect dominates its positive information effect. This leads investors to trade less aggressively on their private interpretations, rendering the price to aggregate less private information. The above distinctions are of key importance in deepening our understanding about the effects of heterogeneous interpretations of public disclosures and the effects of transparency of such disclosures on financial markets.

The comparative statics analyses lead to several policy implications. While improving clarity of public information generally promotes price efficiency, the optimal level of transparency that maximizes price efficiency is interior when clarity is relatively low. This prediction is consistent with the observation that monetary policies are usually vague, especially in the old days when there was not much communication among investors and thus clarity was arguably low (Goodfriend, 1986; Stein, 1989). For example, Alan Greenspan, a former Federal Reserve Chairman, sometimes set out to obfuscate his views on financial markets and monetary policies on purpose (Wessel, 1995). But with increased clarity of public information, perhaps due to the development of information technologies and the rise of social media as a communication device and financial analysts as a profession, we should observe a time-series increase in the transparency of public disclosures.

Lastly, our results emphasize the relevance of the information structure in determining how information affects equilibrium prices. When a public disclosure is interpreted heterogeneously, ideally, it should be improved in terms of both transparency and clarity to achieve the highest level of price efficiency. However, if investors' information processing capacity is constrained, priority should be given to increasing clarity and enhancing consensus among capital market participants, because the best attainable level of price efficiency is determined by clarity. Moreover, devoting all efforts to increasing transparency without improving clarity can be harmful, especially when there is little established consensus among investors. For instance, disaggregating information in the financial statements or requiring complex supplemental information to be disclosed can increase transparency but also decrease clarity. Future research may investigate the effects of such financial reporting requirements on price efficiency in detail.

Chapter 3

A Theory of Communication and Coordination in a Polarized Society

It is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance.

— John Maynard Keynes, *The General Theory of Employment, Interest, and Money*

3.1. Introduction

Social media has become increasingly relevant in modern life, and so has individuals' expression of their sentiments on social media. Financial market regulators conclude that "social media is landscape-shifting," converting the traditional two-party communication into an interactive, multi-party dialogue where users actively create content (SEC, 2012, p. 1).

This chapter is my job market paper. I appreciate helpful comments from Snehal Banerjee, Jeremy Bertomeu, James Best, Pingyang Gao, Peicong Hu, Jack Stecher, and Hao Xue. I would also like to thank seminar participants at Carnegie Mellon University, Columbia University, National University of Singapore, New York University, University of California, Berkeley, and University of Minnesota.

On one hand, individual investors regularly express their polarized opinions (e.g., bullish or bearish) about stocks on social media (Cookson and Niessner, 2020; Cookson, Engelberg and Mullins, 2021). On the other hand, trading strategies based on the sentiment extracted from social media have become increasingly popular.[1] In the more recent GameStop mania, retail traders not only spread their investment ideas, but also spurred mass coordination that caused substantial disruption in the stock price (The Wall Street Journal, 2021).[2] The aforementioned expression of polarized sentiments seems particularly puzzling as it may happen even before investors take trading positions. If the sentiments are value-relevant, why do these investors spread their investment ideas on social media, but not trade and profit from the valuable information they possess instead? If the sentiments are value-irrelevant, why do money managers, the most sophisticated institutional investors, base their trading strategies on the sentiment extracted from social media? Either way, what are the incentives for such expression of sentiments?

Besides some common behavioral intuitions for the information-sharing behavior, insiders may share privileged information to manipulate markets (Benabou and Laroque, 1992) or accelerate price discovery (Ljungqvist and Qian, 2016). However, neither vindicates the observed variations in expression of sentiments across time and individuals (see, e.g., Cookson and Niessner, 2020). In this paper, I propose that it is coordination motives that drive symmetrically informed investors to disclose their polarized sentiments. Such disclosures change the market's perceived population composition between agents of two polar types and hence affect the aggregate sentiment incorporated into the average investment.[3] Investors will only

[1] As of 2012, one third of affluent investors in the U.S. are reported to have directly relied on investment advice transmitted via social media outlets (Cogent Research, 2012). Hedge funds and high-frequency-trading firms also use sentiment indicators for market-making strategies (The Wall Street Journal, 2015).

[2] Consistent with this anecdotal incident, there is abundant empirical evidence indicating that social interactions affect investment decisions by individuals and money managers (see, e.g., Kelly and O Grada, 2000; Duflo and Saez, 2003; Hong, Kubik and Stein, 2004; Bursztyn, Ederer, Ferman and Yuchtman, 2014; Campbell, DeAngelis and Moon, 2019). Relatedly, media sentiments is also found to affect investors' trading activity and stock prices. For example, Goldman, Gupta and Israelsen (2021) observe that investors respond to news about a stock published in politically polarized newspapers by trading in the same direction as other investors who read the same newspaper, and Tetlock (2007) finds that media pessimism predicts lower market prices followed by a reversion to fundamentals.

[3] As Shiller (1992, p. 7) put it, "Investing in speculative assets is a social activity. Investors spend a

disclose their sentiments when doing so makes their forecasts of the average investment more precise. In other words, sentiments are payoff-relevant to the extent of investment complementarities, so even the most sophisticated institutional investors like money managers will base their trading strategies on the sentiment extracted from social media. Nevertheless, they are value-irrelevant to the extent of being independent from the intrinsic value of investment, so investors feel comfortable spreading their investment ideas. This novel theory complements the existing explanations for investors' information-sharing behavior and provides a single consistent explanation for the aforementioned expression of sentiments.

The analysis presented here departs from the typical assumption that the unknown state (e.g., the intrinsic value of investment or some nonstraightforward matter of fact) is the only payoff-relevant uncertainty. Specifically, this paper examines agents' decision-making with multidimensional uncertainty by injecting another uncertainty about the population composition. This additional dimension of uncertainty is non-fundamental yet affects agents' payoffs. For example, in stock markets populated with short-term investors, stock prices can deviate from the underlying fundamental by the aggregate sentiment. Hence, even a bearish investor who believes the stock is overvalued may still take long positions when he expects the bullish investors to be the dominating majority who will drive the short-term price up. Similarly, in social interactions where individuals have a preference for conformity, an authoritarian may instead advocate liberty when he expects the majority population are libertarians. In a word, the *perception* of the population composition plays a vital role in agents' decision-making process, which potentially gives rise to individuals' incentive to communicate their polarized sentiments in social interactions.

To study investors' disclosure incentives of their polarized beliefs in an economy with social interactions, I abstract from a capital market setting but instead adopt a canonical model of beauty contests, in which trading with short horizons is a special case, to capture

substantial part of their leisure time discussing investments, reading about investments, or gossiping about others' successes or failures in investing."

the coordination motives.[4] There is a finite set of agents making investment decisions. As in Angeletos and Pavan (2004), the return of the investment depends not only on a common state of nature but also on how much other agents invest. Agents thus would like to match their own actions with both the state (the *fundamental motive*) and the average action (the *coordination motive*). Each agent observes a private signal about the state. Belief polarization is operationalized as opposite sentiments across the two groups of agents, e.g., bulls and bears, while the agents are symmetrically informed about the fundamental in a statistical sense.[5] More specifically, I decompose the noise of each private signal into a common component and an idiosyncratic component.[6] The division of agents into two sentiment groups is based on the direction of their exposure to the common noise. Thus, the common noise measures the polarized sentiment across groups, and the idiosyncratic noise captures the within-group disagreement. Conditional on the realization of the state, the private signals of any two agents are positively correlated within the same group, but negatively correlated across different groups. Consequently, the private signal of one agent is asymmetrically informative about that of an other agent across the two groups. I first characterize agents' equilibrium actions given their perceived population composition, and then apply the main model to study agents' voluntary expression of their sentiments by introducing the option for agents to publicly disclose their types before the coordination game is played.[7]

[4]The beauty-contest metaphor of stock markets was originally proposed by Keynes (1936), and recently formalized by Allen et al. (2006).

[5]Prior literature shows that a unidimensional belief polarization can arise endogenously in the long-run with persuasion bias (see, e.g., DeMarzo, Vayanos and Zwiebel, 2003). More recently, Nimark and Sundaresan (2019) demonstrate that rational inattention can also lead the beliefs of ex-ante identical agents to cluster in two distinct groups at opposite ends of the belief space. Experimentally, Plous (1991) shows that people process information in a biased manner to support their initial beliefs, whereas Benoît and Dubra (2019) find a population of people to be more prone to polarization if their initial opinions have largely been based on very similar evidence.

[6]Myatt and Wallace (2012) provide a micro-foundation of such information structure with the notion of "rational-inattention"; an investor's understanding of a public signal is costly and thus depends on the investor's information-acquisition choice. See also Corona and Wu (2021) for a similar information structure to operationalize correlations in private signals.

[7]Restricting disclosure to be only about the type but not the private signal may be justified as a consequence of the communication bandwidth constraints such as character limits imposed by social media platforms and time constraints in conversation (see, e.g., Hirshleifer, 2020).

The central finding of the paper pertains to the agents' strategic disclosure (nondisclosure) of their polarized sentiments. I show that agents disclose their types only when doing so increases the value of their private information. Disclosure of types affects the informativeness of an agent's private signal about the average investment by changing the others' perceived population composition and hence their investment decisions. Whether such disclosure takes place depends not only on the strength of the coordination motive and the degree of belief polarization, but also on the expected population composition. First, it is the coordination motive that gives rise to agents' disclosure incentives.[8] Without the coordination motive, the population composition has no impact on agents' investment decisions and hence the average investment.

Second, when forming investment decisions, agents' reliance on their private signals is jointly determined by the signal informativeness about the fundamental and the average investment. We refer to the former as the *fundamental value* and the latter as the *strategic value* of private signals. While the fundamental value of an agent's private signal is independent of the population composition, its strategic value is convex in the proportion of his type. To see this, consider two extreme cases where (i) the population composition is perfectly balanced between the two types, and (ii) the population composition is extremely unbalanced and concentrated at one type. In the first case, the two types are symmetric and the aggregate investment contains no sentiment. The strategic value of private signals is the lowest as the individual sentiment is purely noise for predicting the average investment. In the second case, the individual sentiment is perfectly correlated with the aggregate sentiment, and thus results in a higher strategic value of private signals. Moreover, strong coordination motives will lead the extreme minority type to "follow the crowd" and use their private signals in an opposite way when there is more uncertainty about the sentiment than about the fundamental.

The aforementioned convexity of the strategic value of an agent's private signal with

[8] This result continues to hold even if action complementarity is replaced with action substitutability.

respect to the proportion of his type may result in nondisclosure from the minority type. Agents disclose their types only when doing so increases the value of their private information through the others' perception of the population composition. Nonetheless, disclosures from the minority type induce the majority type to rely less on their private signals due to the perceived more neutral aggregate sentiment. This decreases the strategic value of private signals of the minority type, leading them to "hide in the shadows." Since agents of the same type adopt the same disclosure strategy, a full-disclosure equilibrium, which is tantamount to a separating equilibrium, exists only when the expected population composition is relatively balanced. Otherwise, a no-disclosure equilibrium, or equivalently, a pooling equilibrium, where the posterior belief about the population composition stays unchanged as the prior, is the only equilibrium.

The model is general in that it applies to any other economic situations where agents seek not only to adapt to an unknown state of the world but also to coordinate behavior with others. The coordination motive may be interpreted as reputation concerns or preferences for conformity. Thus, the interaction between agents' use of private signals and the population composition also sheds some light on settings of collective decision-making. To this end, the paper provides further analysis on the efficiency of the average action. One key result is that a diverse yet unbalanced population can jointly make a more efficient decision, especially when the coordination motive is strong but little is known about the underlying state. Combined with results on expression of sentiments, frequent meetings within an organization may indicate a diverse organizational structure which leads to more efficient joint decisions.

3.2. Related Literature

The idea that sentiments can drive financial decision-making in uncertain environments has been at the core of capital market dynamics described by Keynes (1936), which he referred to as "animal spirits." Meanwhile, individual sentiments have been increasingly polarized in various social and political contexts since the early 1990s (see, e.g., Baldassarri and Gelman, 2008; Fiorina and Abrams, 2008). Applying these insights to a beauty contest setting, this paper makes several contributions by examining the implications of polarized sentiments on individuals' communication and coordination decisions.

First, it contributes to the recent thread of the theoretical literature that studies disclosure incentives of self-interested economic agents in settings of investment beauty contests.[9] Closely connected to the general beauty contest model (see, e.g., Morris and Shin, 2002), investment beauty contests are first examined in Angeletos and Pavan (2004, 2007). Among the others, Arya and Mittendorf (2016) examine the incentives of firms to take preemptive action and publicly disclose their investments in beauty contests. This paper complements their findings in the sense that an investor disclose his sentiment not to establish norms, but to influence how the others use their private information. The strategic disclosure incentives may also explain the mixed empirical evidence on the ability of crowdsourced information to predict financial market movements (see, e.g., Tumarkin and Whitelaw, 2001; Antweiler and Frank, 2004; Das and Chen, 2007; Garcia, 2013; Chen, De, Hu and Hwang, 2014a). Indeed, the optimistic conclusion of Condorcet's jury theorem on efficient aggregation of information is preserved only for a relatively balanced population composition.[10]

More generally, it complements the thread of literature on information transmission among traders in financial markets. I focus on the communication of sentiments among symmetrically informed agents due to coordination motives. Different from Benabou and Laroque (1992) and Goldstein, Xiong and Yang (2021), agents express their sentiments not to manipulate the market, but to increase their own prediction ability of the average action. Although several existing papers have rationalized the information-sharing behavior of in-

[9] For thorough summaries on the disclosure literature, see Verrecchia (2001), Dye (2001), and Beyer, Cohen, Lys and Walther (2010).

[10] Condorcet's jury theorem states that sincere reporting of their information by each individual is sufficient for efficient aggregation of information (Condorcet, 1785). In contrast to this background, a number of papers argue that the selfish behavior of individuals in game theoretic situations may prevent efficient aggregation of dispersed information (see, e.g., Acemoglu, Ozdaglar and ParandehGheibi, 2010; Acemoglu and Ozdaglar, 2011; Acemoglu, Dahleh, Lobel and Ozdaglar, 2011).

vestors with short-termism by arguing that information revelation can be used to accelerate price correction (Liu, 2018; Kovbasyuk and Pagano, 2020; Schmidt, 2020), to my knowledge, the current paper is the first one among the others to focus on disclosure of non-fundamental private information from a coordination perspective.

Second, this paper is also related to the literature of social learning. With the social media reducing the cost of communication to almost zero, I focus on the information content of the equilibrium disclosure strategy in a public communication setting.[11] In contrast to Galeotti, Ghiglino and Squintani (2013), information aggregated from individual disclosures in any disclosure equilibrium is robust to lying in my setting with a large number of agents. One of the most closely related papers in this thread of literature is Hagenbach and Koessler (2010), who instead focus on endogenous communication networks formed by agents with publicly observable preference heterogeneity. Despite that, some of the main results in this paper seem analogous. First, there is also asymmetric information transmission between the majority and the minority groups. Second, an agent's tendency to communicate in a public network decreases as the population becomes more unbalanced. Empirically, Bursztyn et al. (2014) conclude that both *social learning* and *social utility* channels have statistically and economically significant effects on investment decisions. While the social utility effect is captured by the coordination motive, the social learning effect differs in my model in the sense that an investor's expression of his sentiment does not directly inform the others about his investment decision, but rather affects the others' *belief* about his prospective investment.

Finally, this paper examines decision-making with multidimensional uncertainty. Following Avery and Zemsky (1998), Goldstein and Yang (2015) confirm that strategic complementarities in trading and information acquisition can arise in a setting where the asset value is affected by different fundamentals. Subsequently, they examine how public disclosures affect market efficiency when the market and the firm possess superior information along different dimensions (Goldstein and Yang, 2019). This paper instead studies how the fundamental

[11] See Mobius and Rosenblat (2014) for a complete review of the existing empirical and theoretical literatures on social learning.

uncertainty affects investors' disclosure of their private information regarding the strategic uncertainty. Further analysis on the efficiency of the aggregate decision in the presence of these two dimensions of uncertainty also sheds some light on the optimal composition of corporate boards when their members are subject to conformity bias. Prior literature has studied the optimal board composition in various contexts (see, e.g., Warther, 1998; Chemmanur and Fedaseyeu, 2018; Baldenius, Meng and Qiu, 2019). Among others, this paper arrives at a similar but more robust prediction than Malenko (2014) that a board of directors whose preferences are more diverse is desirable even in the absence of communication.

The rest of the paper is organized as follows. Section 3.3 develops the model and characterizes agents' equilibrium investment. Section 3.4 introduces an option for agents to disclose their types before investing and derives their equilibrium disclosure strategies. Section 3.5 applies the model to two additional settings and elaborates on its implications for decision making within firms and the stock market. Section 3.6 concludes.

3.3. Main Model

3.3.1. Setup

There are N agents with $N \geq 2$. Each agent $i \in \{1, 2, \ldots, N\}$ is privately informed about his type $t_i \in \{-1, 1\}$, where $t_i = 1$ with probability $\lambda \in (0, 1)$. The type affects the information structure of the agent's private signal about a common state of nature such that

$$\tilde{s}_i = \tilde{v} + t_i \tilde{\eta} + \tilde{\epsilon}_i, \tag{3.1}$$

where $\tilde{v} \sim \mathcal{N}(\mu, \sigma_v^2)$ is the state, $\tilde{\eta} \sim \mathcal{N}(0, \sigma_\eta^2)$ is the within-type common noise, and $\tilde{\epsilon}_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is the idiosyncratic noise independent and identically distributed across all agents. The opposite signs in t_i thus captures the idea of belief polarization in the sense that the private signals of any two agents from different groups are negatively correlated conditional on the realization of the state. Examples of such polarized beliefs include bulls and bears, leftists and rightists, authoritarians and libertarians, and democrats and republicans, etc. For the following analysis, we refer to type-1 and type-2 agents as agents with $t_i = 1$ and $t_i = -1$, respectively, and use them in subscripts whenever it causes no confusion. Regarding information quality, σ_v^2 measures the prior uncertainty about the state which can be based on some public information, σ_η^2 measures the across-type disagreement which serves as a proxy for the degree of polarization across groups, and σ_ϵ^2 captures the within-type disagreement. I assume information quality is well defined, i.e., σ_v^2, σ_η^2, and σ_ϵ^2 are positive and finite, and that all random variables are independent of each other.

Following Angeletos and Pavan (2004) and Arya and Mittendorf (2016), agent i's payoff function is given by

$$u_i(a, \tilde{v}) = \left((1-\omega)\tilde{v} + \omega \bar{a}_{-i}\right) a_i - \frac{1}{2} a_i^2, \tag{3.2}$$

where $\bar{a}_{-i} \equiv \frac{1}{N-1} \sum\limits_{j \neq i} a_j$ is the average action of the other agents, and $\omega \in (0,1)$ parameterizes agents' coordination motives arising from the strategic complementarity in their actions. We may also interpret such a payoff function as one with investment complementarity, where the marginal return of investment is a weighted average of the fundamental and the average investment of the others, with ω capturing the investment complementarity, and the quadratic cost may be interpreted as an investment adjustment cost from the status quo.

3.3.2. Equilibrium Analysis

To simplify the analysis, I assume there is a sufficiently large number of agents. Accordingly, the payoff function is modified to $u_i(a, \tilde{v}) = \left((1-\omega)\tilde{v} + \omega \bar{a}\right) a_i - \frac{1}{2} a_i^2$, where $\bar{a} \equiv \frac{1}{N} \sum\limits_{i=1}^{N} a_i$ is the average action. This modification is without loss of generality since a single agent's action is infinitesimal to the average action with finitely many agents.

Proposition 3.1. *Denoting agents' perceived population composition as $\hat{\lambda}$, then the equilib-*

rium action of agent i is

$$a_{1i}^* = (1-\phi_1)\mu + \phi_1 s_{1i}, \tag{3.3}$$

$$a_{2i}^* = (1-\phi_2)\mu + \phi_2 s_{2i}, \tag{3.4}$$

where

$$\phi_1 = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega(1-\hat{\lambda}))\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2}, \tag{3.5}$$

$$\phi_2 = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega\hat{\lambda})\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2}. \tag{3.6}$$

An agent's equilibrium action is a weighted average of his expectation about the state and the average action. Consequently, his reliance on the private signal is a weighted average of the signal informativeness about the underlying fundamental and the average action. That is,

$$\begin{aligned} \phi_1 &= (1-\omega)\rho_v + \omega\left(\hat{\lambda}\rho_s\phi_1 + (1-\hat{\lambda})\rho_d\phi_2\right), \\ \phi_2 &= (1-\omega)\rho_v + \omega\left(\hat{\lambda}\rho_d\phi_1 + (1-\hat{\lambda})\rho_s\phi_2\right), \end{aligned} \tag{3.7}$$

where $\rho_v \equiv \frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon}$ measures the signal informativeness about the state, and $\rho_s \equiv \frac{\sigma_v^2+\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ and $\rho_d \equiv \frac{\sigma_v^2-\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ respectively represent within-type and cross-type signal correlations. In other words, an agent's reliance on the private signal is jointly determined by its *fundamental value* and *strategic value* which are captured by the first and the second terms, respectively. While being symmetrically informed about the fundamental, agents are asymmetrically informed about the average action due to the existence of the common noise.

The extent of such asymmetry in the strategic value of agents' private signals depends on the population composition. In the special case where agents expect a perfectly balanced population, i.e., $\hat{\lambda} = \frac{1}{2}$, neither of the two types of agents expect to play a dominating role in determining the average action. All agents expect to be equally well informed about the average action, and hence place the same weight on their private signals, i.e., $\phi_1 = \phi_2 =$

$\frac{(1-\omega)\sigma_v^2}{(1-\omega)\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$. The common noise is thus cancelled out in the average action. Consequently, the average action is proportional to the fundamental, which confirms that no type has an information advantage against the other. In the more general case where $\hat{\lambda} \neq \frac{1}{2}$, the majority type of agents influence the average action to a larger extent than their minority counterparts, hence expecting to be better informed about the average action. As a result, they place a higher weight on their private signals than their minority counterparts, i.e., $\phi_1 > \phi_2$ whenever $\hat{\lambda} > \frac{1}{2}$ and $\phi_1 < \phi_2$ whenever $\hat{\lambda} < \frac{1}{2}$.[12]

Nevertheless, agents' reliance on their private signals does not always increase in the expected proportion of their own types. The following proposition summarizes how beliefs about the population composition affects agents' subsequent actions.

Proposition 3.2. *Agents' reliance on their private signals, ϕ_1 and ϕ_2, can be non-monotonic in $\hat{\lambda}$ when $\omega > \omega^T$ and $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$. More specifically, $\frac{\mathrm{d}\phi_i}{\mathrm{d}\hat{\lambda}} < 0$ for $\hat{\lambda} \in (0, \lambda_i^*)$, and $\frac{\mathrm{d}\phi_i}{\mathrm{d}\hat{\lambda}} > 0$ for $\hat{\lambda} \in (\lambda_i^*, 1)$, where $\omega^T \in [\frac{1}{3}, 1]$ and $0 < \lambda_1^* < \frac{1}{2} < \lambda_2^* < 1$ are defined in Appendix C.1.*

Otherwise, ϕ_1 monotonically increases in $\hat{\lambda}$ and ϕ_2 monotonically decreases in $\hat{\lambda}$, i.e., $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} > 0$ and $\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} < 0$ for any $\hat{\lambda} \in (0, 1)$. Moreover, when $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, there exist $0 < \lambda_1^\dagger \leq \lambda_1^* < \lambda_2^* \leq \lambda_2^\dagger < 1$ such that $\phi_1 < 0$ for $\hat{\lambda} \in (0, \lambda_1^\dagger)$ and $\phi_2 < 0$ for $\hat{\lambda} \in (\lambda_2^\dagger, 1)$.

As discussed earlier, the belief about the population composition affects agents' reliance on their private signals only by changing the strategic value of private signals. When $\hat{\lambda} = \frac{1}{2}$, the two groups are symmetric and the expected average action contains no sentiment. The private signal is least informative because the individual sentiment is purely noise for predicting the average action. Conversely, when $\hat{\lambda} \neq \frac{1}{2}$, the majority type influence the average action more, and the aggregate sentiment goes in the same direction as their individual sentiments. In other words, as $\hat{\lambda}$ increases from 0 to 1, the expected average action is first negatively and then positively affected by the sentiment. Thus, the strategic value of an

[12] In fact, since the weight on the private signal ϕ_i can be negative, this claim holds true in absolute values as well. Figure 3.1(c) provides a graphical illustration for such a case.

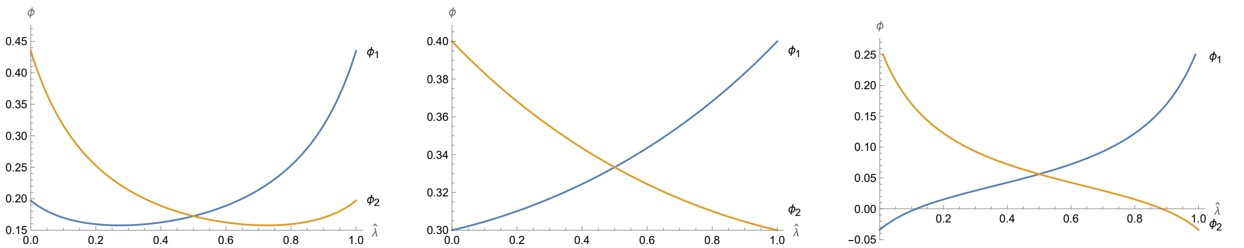
agent's private signal first decreases and then increases in the proportion of his own type.

The aforementioned non-monotonicity of the strategic value of private signals explains why agents' reliance on their private signals can be non-monotonic in the expected population composition. Intuitively, the condition $\omega > \omega^T$ ensures the coordination motive is strong enough, so that the convexity in the strategic value of private signals dominates and leads to an overall convexity of agents' reliance on their private signals, as shown in Figure 3.1(a). Indeed, this condition makes forecasting the aggregate sentiment critical and leads the extreme minority type to decrease their reliance on private signals following the supermajority type even with an increased proportion of their own type. Otherwise, the conventional wisdom prevails: agents always rely more on their private signals when they anticipate a higher proportion of their own types (see Figure 3.1(b)).

More precisely, the perceived population composition affects the expected average action both directly through the population of each type of agents and indirectly through each type's equilibrium action. Taking type-1 agents' reliance on their private signals as expressed in equation (3.7) as an example, its first-order derivative with respect to the expected proportion of their types is

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} = \omega\Bigg(\underbrace{\rho_s\phi_1 - \rho_d\phi_2}_{\text{direct effect}} + \underbrace{\hat{\lambda}\rho_s\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} + (1-\hat{\lambda})\rho_d\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}}}_{\text{indirect effect}}\Bigg). \tag{3.8}$$

While the direct effect is always positive, the indirect effect may be negative when agents are of the minority type. The indirect effect is dominated by the majority type of agents' response in their reliance on private signals. Since an increase in the proportion of the minority type neutralizes the aggregate sentiment, the private signals of the majority type become less informative about the average action. As a result, the majority type of agents rely less on their private signals, i.e., $\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} < 0$. The negative indirect effect dominates the positive direct effect only when $\omega > \omega^T$ and $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$, leading the minority type to rely less on their private signals, i.e., $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} < 0$. The latter condition also ensures that agents' reliance on their private signals is always positive. Otherwise, high uncertainty about the sentiment leads the extreme minority type to “follow the crowd” by using their signals in an opposite way. Decreased reliance in absolute terms then translates to an increase in ϕ_i (see Figure 3.1(c)).



(a) (b) (c)

Figure 3.1: The Weight of the Private Signals in Equilibrium Actions

Note: The above figures demonstrate how the weight of the private signals in equilibrium actions change in $\hat{\lambda}$. Fixing $\sigma_\eta^2 = 1$, the other parameter values are (a) $\omega = \frac{7}{8}, \sigma_v^2 = 2, \sigma_\epsilon^2 = \frac{1}{5}$, (b) $\omega = \frac{1}{2}, \sigma_v^2 = 2, \sigma_\epsilon^2 = 1$, and (c) $\omega = \frac{7}{8}, \sigma_v^2 = \frac{1}{2}, \sigma_\epsilon^2 = \frac{1}{20}$, respectively.

3.4. Expression of Sentiments

In this section, we consider the situation where agents have the option to publicly disclose their types before the coordination game is played. Formally, agents may send a message indicating their types before taking the action, i.e., $m_i \in \{t_i, \emptyset\}$.[13] In addition, agents' rational expectations about the disclosure probability of each type requires $\hat{q}_i = q_i$ in equilibrium.

The belief updating rule as described above respectively correspond to a separating and a pooling equilibria. When both types disclose with a positive probability, the underlying population of each type fully unravels with a large number of agents, which is a manifestation of the separating equilibrium.[14,15] Otherwise, agents observe disclosures from only

[13] Truthful disclosure is made possible due to the *endogenous* signaling costs created by the receiver's equilibrium choice of action rule (Crawford and Sobel, 1982; Farrell and Gibbons, 1989). For more details, see footnote 2.

[14] When the disclosure probability is strictly lower than 1, “unraveling” only happens on the aggregate level to the underlying population of each type, but not on the individual level to a particular agent's type.

[15] Alternatively, we may assume the prior belief is given by $\tilde{\lambda} \sim Beta(\alpha, \beta)$ instead of being a constant. When the total population goes to infinity, the equilibrium number of disclosures of each type goes to infinity

one type or see no disclosure at all, and hence the posterior belief does not update with the observed number of disclosures. As a result, nondisclosure from one type necessarily implies nondisclosure from the other type as well, which resembles the pooling equilibrium.

To simplify the analysis, we restrict our attention to *ex-ante* disclosure; that is, the disclosure is made *before* the realization of private signals.[16] Alternatively, we may think of an agent's type as his ideology which usually does not change over time and is more persistent than his private signals. Since our focus is on the incentives of disclosure, we also introduce a fixed cost of disclosure $\delta > 0$ so that agents' default choice is nondisclosure. The disclosure cost can be arbitrarily small and is hence omitted from the payoff function for expositional brevity.

Before characterizing agents' equilibrium disclosure decisions, I illustrate with the following corollary that it is the combination of both the coordination motive and the fundamental motive that gives rise to agents' incentive to disclose their types.

Corollary 3.1. *Without coordination motive or fundamental motive, agents (strictly) prefer nondisclosure.*

When there is no coordination motive, i.e., $\omega = 0$, agents have no disclosure incentive because the average action is irrelevant to their payoffs and so is the aggregate sentiment. Since agents of both types are equally well informed about the fundamental, they rely on their private signals to the same extent when choosing their actions; that is, $\phi_1 = \phi_2 = \frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ is the same across all agents and independent of the posterior belief about the population composition $\hat{\lambda}$. When there is no fundamental motive, i.e., $\omega = 1$, agents have no disclosure incentive either because the coordination motive is so strong that they coordinate solely on the public information and ignore their private information; that is, $\phi_1 = \phi_2 = 0$ for all agents and is again independent of the expected population composition $\hat{\lambda}$.

When the coordination motive and the fundamental motive coexist, i.e., $0 < \omega < 1$,

as well, and the true realization of $\tilde{\lambda}$ is revealed to all agents.

[16] I refer interested readers to Appendix C.2 for a discussion and characterization of *ex-post* disclosure equilibrium where the disclosure of types is made *after* the realization of private signals.

agents may be incentivized to disclose their types so as to boost the others' belief about the proportion of their own types. One exception as discussed earlier is that when $\hat{q}_1\hat{q}_2 = 0$, agents expect no disclosure to be informative; hence, their posterior belief about the population composition is independent of the number of disclosures of either type n_i and solely determined by the prior probability λ. As a result, no individual agent of either type has any incentive to disclose; nondisclosure, i.e., $m_i^* = \emptyset$ and $q_1 = q_2 = 0$, can always be sustained as an equilibrium which I refer to as the *no-disclosure equilibrium* or *pooling equilibrium*.[17] For the following analysis, I therefore focus on the *disclosure equilibrium* where $q_1q_2 > 0$.

3.4.1. Equilibrium Disclosure Strategy

Consider an agent's expected payoff given the disclosures from each type:

$$\mathbb{E}[u_i|t_i;\langle n_1,n_2\rangle,\hat{q}_1,\hat{q}_2] = \mathbb{E}\left[\frac{1}{2}a_i^2\middle|t_i;\langle n_1,n_2\rangle,\hat{q}_1,\hat{q}_2\right] = \frac{1}{2}\Big(\mu^2+(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\phi_i^2\Big), \qquad (3.9)$$

where ϕ_i is defined in equations (3.5) – (3.6). By deviating from his equilibrium disclosure strategy, the agent is only able to marginally affect the other agents' posterior belief about the population composition by making one more or one fewer disclosure. He then adjusts his own action accordingly in response to the change in the other agents' action, hence the average action, following his deviation. On the other hand, his off-equilibrium disclosure decision does not affect his own posterior belief. With sufficiently many agents, the agent's posterior belief about the population composition converges to the prior probability λ by the central limit theorem.[18] More formally, suppose following his deviation from the equilibrium

[17] The no-disclosure equilibrium is not neologism-proof whenever it coexists with a disclosure equilibrium. For more details, see Farrell and Gibbons (1989) and Farrell (1993).

[18] If λ is a random variable *ex-ante* as described in footnote 15, then the left-hand-side of the IC constraints is

$$\left.\frac{\mathrm{d}\mathbb{E}_i[u_i'|t_i]}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = (\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\mathbb{E}_\lambda\left[\phi_i\left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda}\right],$$

which admits no closed-form solutions due to the complexity in the nonlinear moment function. Alternatively, we may view the conditions we derive in *Propositions 3.3* as one class of distribution whose probability disclosure, the agent's posterior belief stays unchanged at λ, but the other agents' posterior belief becomes λ', then the deviating agent's best response in terms of his reliance on the private signal ϕ_i' is

$$\begin{aligned}\phi_1' &= (1-\omega)\rho_v + \omega\left(\lambda\rho_s\phi_1(\lambda') + (1-\lambda)\rho_d\phi_2(\lambda')\right), \\ \phi_2' &= (1-\omega)\rho_v + \omega\left(\lambda\rho_d\phi_1(\lambda') + (1-\lambda)\rho_s\phi_2(\lambda')\right),\end{aligned} \tag{3.10}$$

respectively. As discussed earlier, the first term of the above expression captures the fundamental value of private signals, and is independent of the expected population composition. The second term captures the strategic value of private signals, where $\phi_i(\lambda')$ represents the other agents' action, which is given by *Proposition 3.1*, with the adjusted posterior belief λ' due to the agent's deviation from his equilibrium disclosure strategy.

The assumption of finitely many agents simplifies the analysis of the agents' disclosure decisions by rendering the effect of each agent's disclosure on the other agents' posterior belief marginal and consequently his disclosure incentive independent of the total population. That is, the incentive compatibility constraints for disclosure are

$$\begin{aligned}\left.\frac{\mathrm{d}\mathbb{E}_1[u_{1i}']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= \left(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2\right)\phi_1\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0, \\ \left.\frac{\mathrm{d}\mathbb{E}_2[u_{2i}']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= \left(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2\right)\phi_2\left.\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} < 0.\end{aligned} \tag{IC}$$

A disclosure equilibrium exists only when the above IC constraints hold for both types, which is analogous to the single-crossing condition in signaling games. The following lemma establishes how the deviating agent's action is affected by his deviation through its marginal effect on the other agents' posterior belief.

Lemma 3.1. *When $\sigma_\eta^2 < \sigma_v^2$, there exist $0 < \lambda_1^* < \lambda_1^\ddagger < \frac{1}{2} < \lambda_2^\ddagger < \lambda_2^* < 1$ such that $\left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_i^\ddagger)$, and $\left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_i^\ddagger, 1)$. Otherwise, $\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0$ and*

density is sufficiently concentrated around λ among all potential distributions that allow for the existence of a disclosure equilibrium.

$\left.\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} < 0$ *for any* $\lambda \in (0,1)$.

Similar to all agents' common belief as described in *Proposition 3.2*, the population composition affects an agent's equilibrium action only through the strategic value of his private signal. But since a deviation from an agent's equilibrium disclosure strategy changes only the others' belief about the population composition, it affects the strategic value of his private signal only indirectly through others' reliance on their private signals; that is,

$$\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = \omega\left(\lambda\rho_s\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda} + (1-\lambda)\rho_d\frac{\mathrm{d}\phi_2}{\mathrm{d}\lambda}\right). \tag{3.11}$$

In order for the minority type to rely less on their private signals following an increase in the other agents' belief about the proportion of their type, we only need the indirect effect to be negative, which is a less stringent condition compared to that in *Proposition 3.2*. The indirect effect may be further decomposed as a weighted average effect on the action of each type. When the population composition is perfectly balanced, the two types change their reliance on private signals to the same extent but in opposite directions. However, since within-type signal correlation ρ_s is always higher than cross-type signal correlation ρ_d, the indirect effect is positive. When the population composition is relatively unbalanced, the indirect effect is dominated by the majority type's response in their reliance on private signals. In particular, the majority type decreases their reliance on private signals drastically following an increase in the minority population. The condition $\sigma_\eta^2 < \sigma_v^2$ ensures that the cross-type signal correlation ρ_d is positive. As a result, the negative effect through the majority type's action translates to the minority type's decreased reliance on their private signals.

Recognizing the consequence of an agent's disclosure (nondisclosure) on his reliance of the private signal due to its impact on the others' belief, we now proceed to characterize agents' equilibrium disclosure strategy.

Proposition 3.3. *Agents of both types fully disclose in the disclosure equilibrium. More specifically, there exist* $\lambda_1^\dagger, \lambda_1^\ddagger \in (0, \frac{1}{2})$ *and* $\lambda_2^\dagger, \lambda_2^\ddagger \in (\frac{1}{2}, 1)$ *such that:*

(i) if $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*, then*

(a) when $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$*,* $m_i^* = t_i, q_1 = q_2 = 1$*;*

(b) when $\lambda \leq \lambda_1^\dagger$ *or* $\lambda \geq \lambda_2^\dagger$*, there does not exist a disclosure equilibrium;*

(ii) if $\sigma_\eta^2 < \sigma_v^2$*, then*

(a) when $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$*,* $m_i^* = t_i, q_1 = q_2 = 1$*;*

(b) when $\lambda \leq \lambda_1^\ddagger$ *or* $\lambda \geq \lambda_2^\ddagger$*, there does not exist a disclosure equilibrium;*

(iii) otherwise, $m_i^* = t_i, q_1 = q_2 = 1$*.*

As implied by the IC constraints, agents prefer to disclose their types whenever such disclosure induces them to rely more on their private signals. Intuitively, the more informative an agent's private signal is about the aggregate sentiment, the more he relies on the private signal, and the more valuable his information is. Because the marginal effect of a disclosure on agents' use of their private signals is the same to all agents of the same type, they make the same disclosure decision. Thus, there is either full disclosure or no disclosure, but not partial disclosure, from each type.

Moreover, the only disclosure equilibrium is the full-disclosure equilibrium, i.e., $q_1 = q_2 = 1$. Recall that when agents observe disclosures from only one type, the posterior belief does not update with the number of disclosures. Hence, nondisclosure from one type necessarily implies nondisclosure from the other, even though agents of the other type would have an incentive to disclose if their disclosure could affect the others' posterior belief about the population composition. Therefore, a disclosure equilibrium can only be sustained when agents of both types strictly prefer disclosure. In other words, the full-disclosure equilibrium exists only when (i) all agents use their private signals in a positive way, and (ii) agents' reliance on their private signals increases in the others' posterior belief about the proportion of their own types. Table 3.1 below provides a summary of the sign of agents' reliance on their private signals and how such reliance changes in the others' posterior belief in various cases.

Case	λ	ϕ_1	$\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}$	ϕ_2	$\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}$
(i)	$(0, \lambda_1^\dagger)$	$-$	$+$	$+$	$-$
	$(\lambda_1^\dagger, \lambda_2^\dagger)$	$+$	$+$	$+$	$-$
	$(\lambda_2^\dagger, 1)$	$+$	$+$	$-$	$-$
(ii)	$(0, \lambda_1^\ddagger)$	$+$	$-$	$+$	$-$
	$(\lambda_1^\ddagger, \lambda_2^\ddagger)$	$+$	$+$	$+$	$-$
	$(\lambda_2^\ddagger, 1)$	$+$	$+$	$+$	$+$
(iii)	$(0, 1)$	$+$	$+$	$+$	$-$

Table 3.1: Summary of the Signs of ϕ_i and $\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}$ in Different Cases for Different $\lambda' s$

Note: The indices of cases are consistent with *Propositions 3.3*. More specifically, the above cases correspond to parameter regions where (i) $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, (ii) $\sigma_\eta^2 < \sigma_v^2$, and (iii) $\omega > \frac{1}{2}$ and $\sigma_v^2 \leq \sigma_\eta^2 < \frac{\sigma_v^2+\sigma_\epsilon^2}{2\omega-1}$, or $\omega \leq \frac{1}{2}$ and $\sigma_\eta^2 \geq \sigma_v^2$, respectively.

Since the posterior belief about the population composition converges to the prior probability λ in any disclosure equilibrium, the satisfaction of the above conditions is contingent on parameter values as well as the prior probability λ. More specifically, in the first case where $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, condition (ii) is always satisfied, but condition (i) is satisfied only when the population composition is expected to be relatively balanced, i.e., $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$ (see *Proposition 3.2*). Otherwise, the strong coordination motive will induce agents of the extreme minority type to use their private signals in an opposite way as the strategic value of private signals dominates their fundamental value. This results in nondisclosure of the extreme minority type and consequently nondisclosure of the other (supermajority) type as well.

On the contrary, in the second case where $\sigma_\eta^2 < \sigma_v^2$, while condition (i) is always satisfied, condition (ii) is satisfied only when the population composition is expected to be relatively balanced, i.e., $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$ (see *Lemma 3.1*). Otherwise, the convexity of the informativeness of private signals about the average action can lead the agent of the extreme minority type to rely less on his private signal following an increase in the others' posterior belief about the proportion of his type. This results in nondisclosure of the extreme minority type and consequently nondisclosure of the other (supermajority) type as well. Lastly, in the third case, both conditions are always satisfied regardless of the expected population composition. Hence, the full-disclosure equilibrium always exists.

Indeed, both types always have an incentive to disclose their types when the population is relatively balanced. In the special case of a perfectly balanced population, i.e., $\lambda = \frac{1}{2}$, the two types of agents adjust their reliance on private signals to the same extent but in opposite directions following a change in the perceived population composition. Since within-type signal correlation is always higher than cross-type signal correlation, an agent is always induced to disclose his type in order to increase his reliance on the private signal (see equation (3.11)). When the population is relatively unbalanced, the minority type may no longer have an incentive to disclose. To see this, consider the case where the population is extremely unbalanced, i.e., $\lambda = 0$ or $\lambda = 1$. Since the majority type decreases their reliance on private signals following an increase in the minority population, less sentiment gets incorporated into the average action. When the minority type's initial reliance on their private signals are of the same sign as the cross-type signal correlation, this results in decreased strategic value of the minority type's private signals and hence their nondisclosure.

To illustrate the validity of a large number of agents as a simplifying assumption, an example of two agents is provided below. Indeed, the analysis with a large number of agents provides a sufficient but not necessary condition for the existence of a separating equilibrium.

Example $(N = 2)$**.** In a separating equilibrium, we have

$$\phi_i = \begin{cases} \phi_s = \frac{(1-\omega)\sigma_v^2}{(1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2} & \text{if } t_i = t_j, \\ \phi_d = \frac{(1-\omega)\sigma_v^2}{(1-\omega)\sigma_v^2+(1+\omega)\sigma_\eta^2+\sigma_\epsilon^2} & \text{if } t_i \neq t_j. \end{cases}$$

Recall from equation (3.9) that an agent's *ex-ante* expected utility increases in his (expected) squared reliance on the private signal, hence a separating equilibrium can be sustained if and only if

$$\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2 \quad \text{and} \quad \lambda\phi_d^2 + (1-\lambda)\phi_s^2 > \phi_2^2,$$

which is always satisfied around $\lambda = \frac{1}{2}$.

Suppose $t_i = 1$, note that $\phi_s = \phi_1(1)$. By the convexity of ϕ_1 and hence ϕ_1^2 (see *Proposition 3.2*), the relative magnitude of $\lambda\phi_s^2 + (1-\lambda)\phi_d^2$ to ϕ_1^2 then depends on that of ϕ_d^2 to $(\phi_1(0))^2$.[19] More specifically, if $\phi_d^2 \geq (\phi_1(0))^2$, then $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for any $\lambda \in (0,1)$; otherwise, there exists $0 < \lambda_{1,N=2}^T < \frac{1}{2}$ such that $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 < \phi_1^2$ for $\lambda < \lambda_{1,N=2}^T$, and $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for $\lambda > \lambda_{1,N=2}^T$. The case of $t_i = -1$ is symmetric to $t_i = 1$ about $\lambda = \frac{1}{2}$ and thus follows the similar analysis.

To conclude, pooling equilibrium is the only equilibrium when $\lambda \leq \lambda_{1,N=2}^T$ or $\lambda \geq \lambda_{2,N=2}^T$, where $0 < \lambda_{1,N=2}^T < \frac{1}{2} < \lambda_{2,N=2}^T < 1$, if (i) $\sigma_\eta^2 > \frac{\sigma_v^2+\sigma_\epsilon^2+\sqrt{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+5\sigma_\epsilon^2)}}{2}$ and $\omega > \omega_{N=2}^T$ where $\omega_{N=2}^T = \left(\sqrt{\sigma_v^4 - 2\sigma_v^2\sigma_\eta^2 + 5\sigma_\eta^4} - \sigma_v^2 - \sigma_\eta^2\right)\frac{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}{2\sigma_\eta^2(\sigma_\eta^2-\sigma_v^2)} > \frac{1}{2}$ or (ii) $\sigma_\eta^2 < \sigma_v^2$. Otherwise, separating equilibrium is the only equilibrium.

Comparing with *Proposition 3.3*, the conditions under which there exists a pooling and a separating equilibrium are the same qualitatively. Moreover, the conditions for the pooling equilibrium to be the only equilibrium are more stringent in the two-agent example than the sufficiently-many-agent example. In other words, the analysis with a large number of agents provides a sufficient but not necessary condition for the existence of a disclosure equilibrium.

3.4.2. Disclosure Region

By Jensen's inequality, agents' *ex-ante* expected payoff with disclosure is always higher than that without disclosure. In other words, with negligible disclosure costs, the full-

[19] To be more precise, it is possible that $\phi_d^2 > (\phi_1(0))^2$ yet ϕ_1^2 is concave for small values of λ. Nevertheless, $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for any $\lambda \in (0,1)$ whenever that is the case. Therefore, the concavity of ϕ_1^2 does not affect our following analysis.

disclosure equilibrium is Pareto superior to the no-disclosure equilibrium. The intuition is that information about the exact realization of population composition facilitates coordination among all agents. Accordingly, we examine how the disclosure region is affected by the information structure and the coordination motive.

Corollary 3.2. *The disclosure region may expand or shrink with the information structure and the coordination motive. More specifically,*

(i) if $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*, then* $\Delta^\dagger \equiv \lambda_2^\dagger - \lambda_1^\dagger$*, and*

(a) $\frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_v^2} > 0, \frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_\eta^2} < 0, \frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_\epsilon^2} > 0$*;*

(b) $\frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\omega} < 0$*;*

(ii) if $\sigma_\eta^2 < \sigma_v^2$*, then* $\Delta^\ddagger \equiv \lambda_2^\ddagger - \lambda_1^\ddagger$*, and*

(a) $\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_v^2} < 0, \frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_\eta^2} > 0, \frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_\epsilon^2} > 0$*;*

(b) $\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\omega} < 0$*.*

The information structure affects the disclosure region through agents' reliance on their private signals. When $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, an agent prefers to disclose his type only when he uses his signal in a positive way. Thus, whether the disclosure region expands or shrinks following a change in the information structure depends on whether such a change increases or decreases agents' reliance on their private signals. Since higher prior uncertainty increases both the fundamental value and the strategic value of private signals, it makes agents rely more on their private signals and hence more inclined to disclose their types.

Although both cross-type and within-type heterogeneities render a private signal noisier about the fundamental and hence reduces its fundamental value, they have qualitatively different effects on the disclosure region due to their different impacts on its strategic value. More cross-type heterogeneity increases within-type signal correlation, but decreases cross-type signal correlation. Consequently, when the agent is of the minority type, it decreases both the fundamental value and the strategic value of his private signal and results in a smaller disclosure region. Conversely, more within-type heterogeneity increases the strategic value of his private signal by attenuating the negative cross-type signal correlation. Moreover, this positive effect on the strategic value of private signals dominates the negative effect on the fundamental value of private signals. The overall positive effect makes agents more inclined to disclose their types and thus results in a larger disclosure region.

When $\sigma_\eta^2 < \sigma_v^2$, the effect of a change in the information structure on the disclosure region depends on how it changes the importance of the population composition in agents' investment decisions. Intuitively, as across-type or within-type heterogeneity increases, the strategic value of private signals becomes more important. Thus, the impact of disclosure on the others' actions gets larger, inducing more disclosures. Conversely, prior uncertainty about the fundamental has an opposite effect to that of both cross-type and within-type heterogeneities. Higher prior uncertainty induces agents to use their private signal more towards forecasting the fundamental, which makes the population composition less important in agents' investment decisions and hence results in fewer disclosures. This result implies that there can still be heated discussions on some social topics even as more information about the fundamental becomes publicly available.

In a nutshell, although the effects of within-type heterogeneity are qualitatively the same in part (i) and part (ii) of *Corollary 3.2*, their intuitions are quite different. Nevertheless, in both cases, a stronger coordination motive leads to a tighter disclosure region. Intuitively, as the coordination motive becomes stronger, agents are inclined to rely more on the common prior rather than their private signals in making investment decisions, which makes the aggregate sentiment and hence the population composition less important. As we see from *Corollary 3.1*, in the extreme case where there is no fundamental motive, the coordination motive is so strong that agents rely solely on the common prior belief and ignore their private information in making their investment decisions.

3.5. Additional Applications

3.5.1. Board Composition

The main model can also be applied to settings of collective decision-making by the board of directors in corporations where the coordination motive may be interpreted as pressure for conformity.[20] This section examines how board composition should be optimally chosen to maximize the efficiency of the average action. Defining action efficiency as the inverse of the expected squared distance between the average action and the fundamental, i.e., $\text{AE} = \frac{1}{\mathbb{E}[(\tilde{v}-\bar{a})^2]}$, the following proposition indicates that a perfectly balanced board is not necessarily optimal all the time, and neither is an extremely unbalanced board.

Proposition 3.4. *The efficiency of the average action is maximized at $\lambda = \frac{1}{2}$ if $\omega \leq \hat{\omega}$. Otherwise, it is maximized at $\lambda \in \{0, 1\}$ if $\sigma_v^2 \leq \hat{\sigma}_v^2$, or at $\lambda \in \{\lambda_1^{\text{AE}}, \lambda_2^{\text{AE}}\}$, where $0 < \lambda_1^{\text{AE}} < \frac{1}{2} < \lambda_2^{\text{AE}} < 1$ are defined in Appendix C.1, if $\sigma_v^2 > \hat{\sigma}_v^2$.*

Population composition affects action efficiency through the loadings on both the fundamental component and the common noise component in the average action. The conventional wisdom suggests that action efficiency should be maximized when the population composition is perfectly balanced, i.e., $\lambda = \frac{1}{2}$. The two groups of agents are symmetric and use their private signals to the same extent. The average action thus carries no common noise, i.e., $\bar{a} = \mu + \phi(\tilde{v} - \mu)$ where $\phi = \frac{(1-\omega)\sigma_v^2}{(1-\omega)\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$. However, agents' reliance on their private signals decreases in the coordination motive. In the extreme case where there is no fundamental motive but only coordination motive, agents coordinate solely on the public information and ignore their private information, which makes the average action not informative about the fundamental at all. Therefore, the seemingly intuitive result that action efficiency is maximized with a perfectly balanced population composition holds only when the coordination

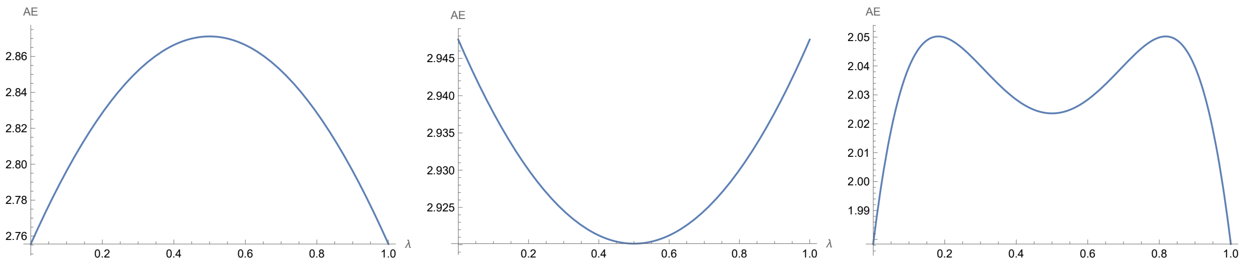
[20] According to one director, "Groupthink is one of the greatest problems boards face" (Leblanc and Gillies, 2005). Among others, Sonnenfeld (2002) conducts case studies of conformity in the boardroom, and Schwartz-Ziv and Weisbach (2013) provide empirical evidence on the rareness of dissension in boards by examining board minutes of Israeli companies. See also Malenko (2014) for a discussion on different microfoundations for conformity.

motive is sufficiently low (see Figure 3.2(a)).

Conversely, when the coordination motive is relatively high, the optimal population composition hinges on the prior uncertainty about the fundamental. Ideally, we would like to maximize the loading on the fundamental component and minimize the loading on the noise component in the average action. Yet the two loadings go hand in hand: they are both minimized with a perfectly balanced population and maximized with an extremely unbalanced one in absolute terms. When there is sufficient prior knowledge about the fundamental, investors' reliance on their private signals is relatively low. This bounds the aggregate sentiment that could potentially be incorporated into the average action, and maximizing the loading on the fundamental component becomes a major concern. Hence, in contrast to Figure 3.2(a), action efficiency is instead maximized when the population composition is extremely unbalanced, but minimized when it is perfectly balanced in Figure 3.2(b). With an extremely unbalanced population composition, the two loadings are both maximized and equal to each other, i.e., $\bar{a} = \mu + \phi(\tilde{v} \pm \tilde{\eta} - \mu)$ where $\phi = \frac{(1-\omega)\sigma_v^2}{(1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2}$.

On the other hand, great uncertainty about the fundamental increases the usefulness of private signals and induces investors to use more of their private signals. Since great uncertainty about the fundamental attenuates agents' underweighting of private signals caused by coordination motives, it is beneficial to decrease the loading on the noise component to some degree at the expense of a simultaneous decrease in the loading on the fundamental component. By symmetry, there exist two interior optimal population compositions, λ_1^{AE} and λ_2^{AE}, where the marginal benefit of decreasing the loading on the noise component equals the marginal cost of decreasing the loading on the fundamental component (see Figure 3.2(c)).

The above result on action efficiency may shed some light on the optimal board composition in corporations. For example, in the context of a takeover, a board that includes both insiders, who are biased against the takeover, and bidder representatives, who are biased in its favor, can jointly make a more efficient decision than a board in lack of variety, especially when they know little about the target's value. Nevertheless, a perfectly balanced



(a) Inverted U-Shaped (b) U-Shaped (c) Double-Humped

Figure 3.2: The Efficiency of the Average Action

Note: The above figures demonstrate how the efficiency of the equilibrium average action changes in the population composition. The parameter values are (a) $\omega = \frac{7}{32}, \sigma_v^2 = 1, \sigma_\eta^2 = \frac{1}{8}, \sigma_\epsilon^2 = 1$, (b) $\omega = \frac{17}{32}, \sigma_v^2 = \frac{1}{2}, \sigma_\eta^2 = \frac{1}{8}, \sigma_\epsilon^2 = 1$, and (c) $\omega = \frac{17}{32}, \sigma_v^2 = 1, \sigma_\eta^2 = 1, \sigma_\epsilon^2 = \frac{7}{64}$, respectively.

board between insiders and bidder representatives may not be optimal either, when board members have a strong preference for conformity. Essentially, unbalancedness encourages board members to contribute their ideas in the decision-making process, and diversity limits how their own sentiments can bias the joint decision. This prediction is consistent with the empirical evidence documented in Coles, Daniel and Naveen (2008) that firm value is non-monotonic in board structure. Combined with the results from the previous section, they jointly indicate a positive association between the efficiency of board decisions and the frequency of board meetings over which board members may freely express and communicate their polarized sentiments.

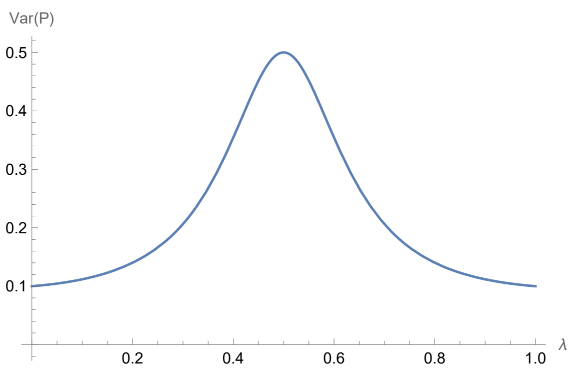
3.5.2. Price Volatility

This section extends the main model to incorporate an endogenous price. Interpreting the fundamental in the main model as the growth opportunity of a firm, the market maker then sets the price at his conditional expectation about the asset-in-place, whose value is identical to the growth opportunity, given the aggregate demand from the traders. That is,

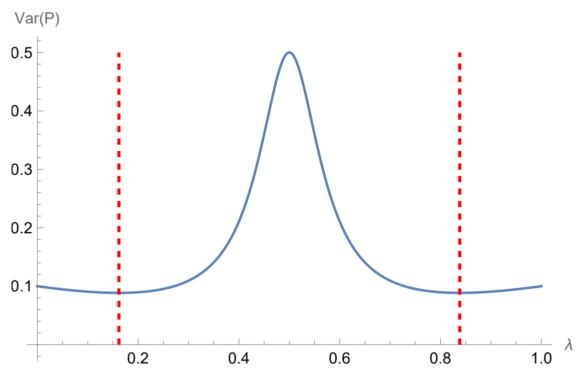
$$P = \mathbb{E}[\tilde{v}|\bar{a}] = \mu + \frac{\big(\lambda\phi_1 + (1-\lambda)\phi_2\big)\sigma_v^2}{\big(\lambda\phi_1 + (1-\lambda)\phi_2\big)^2\sigma_v^2 + \big(\lambda\phi_1 - (1-\lambda)\phi_2\big)^2\sigma_\eta^2}(\bar{a} - \mu), \tag{3.12}$$

where $\bar{a} = \mu + \lambda\phi_1(\bar{s}_1 - \mu) + (1-\lambda)\phi_2(\bar{s}_2 - \mu)$ and ϕ_i's are as defined in *Lemma 3.1*. Such a price may be justified with a separation of the asset-in-place from the growth opportunity, where the asset-in-place is publicly traded but the growth opportunity is not (see, e.g., Subrahmanyam and Titman, 1999; Foucault and Gehrig, 2008; Goldstein and Yang, 2017). Alternatively, we may interpret the agents in the main model as financial analysts who not only aim at maximizing their forecast accuracy but also want to minimize their deviation from the consensus forecast made by the other analysts due to reputation concerns, and then the price is determined by the conditional expectation given the average forecast.[21]

Proposition 3.5. *The price volatility is symmetric about and maximized at* $\lambda = \frac{1}{2}$. *In particular, when* $(2\omega-1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, $\frac{\mathrm{dVar}(P)}{\mathrm{d}\lambda} < 0$ *for* $\lambda \in \left(0, \frac{1}{2}\left(1 - \sqrt{\frac{\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right)\right)$, *and* $\frac{\mathrm{dVar}(P)}{\mathrm{d}\lambda} > 0$ *for* $\lambda \in \left(\frac{1}{2}\left(1 + \sqrt{\frac{\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right), \frac{1}{2}\right)$; *otherwise,* $\frac{\mathrm{dVar}(P)}{\mathrm{d}\lambda} > 0$ *for* $\lambda \in (0, \frac{1}{2})$.



(a) Inverted U-Shaped



(b) U-Shaped

Figure 3.3: Price Volatility

Note: The above figures demonstrate how price volatility changes in the population composition. Fixing $\sigma_v^2 = \frac{1}{2}$, $\sigma_\eta^2 = 2$ and $\sigma_\epsilon^2 = \frac{1}{20}$, the coordination motive is set at (a) $\omega = \frac{1}{2}$ and (b) $\omega = \frac{7}{8}$, respectively. Price volatility is minimized at $\lambda = 0$ and $\lambda = 1$ on the left panel, but at $\lambda = 0.16$ and $\lambda = 0.84$ (highlighted by the red dashed lines) on the right panel.

The intuition for why price volatility is highest when the population composition is perfectly balanced is straightforward. It is well known that prices are more volatile when there is more information.[22] As discussed earlier, the common noise is cancelled out in the

[21]For example, Hong, Kubik and Solomon (2000) document empirically that controlling for forecast accuracy, analysts are more likely to be terminated for forecasts that deviate from the consensus.

[22]Essentially, by law of total variance, price volatility in our setting is equal to the prior uncertainty about the fundamental minus the residual uncertainty given the average action, i.e., $\mathrm{Var}(P) = \sigma_v^2 - \mathrm{Var}(\tilde{v}|\bar{a})$.

average action at $\lambda = \frac{1}{2}$, which renders the average action fully revealing of the fundamental. The price is then set exactly equal to the fundamental and reaches its highest volatility, i.e., $\text{Var}(P) = \sigma_v^2$. This result is consistent with the empirical observation on the positive association between message volume on social media and price volatility (see, e.g., Das and Chen, 2007; Antweiler and Frank, 2004). Instead of justifying such relationship with the presence of noise traders whose actions also induce market volatility (Antweiler and Frank, 2004), my setting provides an alternative explanation by showing that a relatively balanced population composition may be an omitted variable that leads to both stronger incentives for expression of sentiments (see *Proposition 3.3*) and a higher price volatility.

A counterintuitive observation is that price volatility is not necessarily lowest when the population composition is most extreme, but rather when the two groups are somewhat unbalanced. As demonstrated in Figure 3.3, when $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, price volatility is minimized at $\lambda = \frac{1}{2}\left(1 - \sqrt{\frac{\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right) \in (0, \frac{1}{2})$ and $\lambda = \frac{1}{2}\left(1 + \sqrt{\frac{\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right) \in (\frac{1}{2}, 1)$. Recall from *Proposition 3.2* that strong coordination motive can induce agents of the minority type to use their private signals in an opposite way when the across-type heterogeneity is high. Such a negative reliance on the private signal simultaneously decreases the loading on the fundamental component and increases the loading on the noise component in the average action. Hence, it renders the average action less informative about the fundamental and leads to a lower price volatility.

3.6. Conclusion

Polarized beliefs have been pervasive in various social and political contexts (see, e.g., Baldassarri and Gelman, 2008; Fiorina and Abrams, 2008). On one hand, media and political interpreters of American politics have been promulgating a polarization narrative since the early 1990s. On the other hand, prior literature shows that belief polarization can arise endogenously in the long-run with persuasion bias (DeMarzo et al., 2003) or rational inattention (Nimark and Sundaresan, 2019). Yet there is little work examining the implications of belief polarization. This paper thus attempts to fill the gap by examining individuals' strategic expression of their polarized sentiments and the efficiency of their joint action under varying degrees of polarization in a setting with strategic complementarities.

Although sentiments are value-irrelevant in the sense of being uninformative about the common state of nature, they are payoff-relevant in the presence of strategic complementarities or conformity bias. Individuals express their polarized sentiments not aiming at manipulating the others' beliefs about the fundamental, but to influence subsequent actions of the others in a way that maximizes the value of their own information in forecasting the others' actions. In addition, the analysis on individuals' subsequent actions sheds some light on the optimal composition of corporate boards or state legislatures such as a congress and a parliament in terms of maximizing the efficiency of the aggregate decision made by their members. The result implies that a diverse yet unbalanced board or legislature can jointly make a more efficient decision, especially when the coordination motive is strong but little is known about the underlying state.

The framework set up in this paper also opens up many new opportunities for future research. One direction is to endogenize belief polarization in the presence of coordination motives, perhaps as a consequence of private communication with endogenous communication networks. Moreover, as implied in this paper, expression of polarized beliefs may have substantial implications on trading volume and price efficiency in a trading model with endogenously determined prices. Another interesting direction is to allow for disclosure about both the fundamental uncertainty, i.e., private signals, and the strategic uncertainty, i.e., polarized beliefs, and see how they interact with each other and if Pareto optimality can be achieved.

Appendix A

Proofs to Chapter 1

Proof of the Pricing Equation. The price of each firm given all available information $\mathcal{I} \equiv \{s_i\}_{i\in[0,1]}$ follows a strict factor structure. As shown by Al-Najjar (1998), strict factor structure and no-arbitrage imply that for almost every asset (or a portfolio of assets) i with payoff $\tilde{\theta}_i$,

$$\mathbb{E}[\tilde{\theta}_i|\mathcal{I}] = P_i + F$$

where P_i is the price of asset i, the constant F is the factor price, and by assumption, the loading on the factor F is 1 for all assets in our setting. Given the above relationship between price and return, the payoff of holding any portfolio is a (weakly) mean preserving spread of the payoff to the market portfolio.[1] The price of market portfolio P_m must clear the market such that

$$\frac{a + \mathbb{E}[\tilde{\eta}|\mathcal{I}] - P_m}{\rho_I \mathrm{Var}(\tilde{\eta}|\mathcal{I})} = 1,$$

which gives the risk factor $F = \rho_I Var(\tilde{\eta}|\mathcal{I})$. Hence, the risk-averse investors are only compensated for bearing non-diversifiable risks, and the price of firm i is

$$P_i = \mathbb{E}[\tilde{\theta}_i|\mathcal{I}] - F = \mathbb{E}[\tilde{\theta}_i|\mathcal{I}] - \rho_I \mathrm{Var}(\tilde{\eta}|\mathcal{I}).$$

[1] The argument resembles the two-fund separation theorem in Ross (1978) and Connor (1984).

$\square$

Proof of Lemma 1.1. Given investors' conjectured levels of effort $\hat{a}_i$ and $\hat{a}$ for firm i and the economy-wide average, respectively,

$$
\begin{aligned}
P_i(s_i, \bar{s}) &= \mathbb{E}[\tilde{\theta}_i|\mathcal{I}] - \rho_I \mathrm{Var}(\tilde{\eta}|\mathcal{I}) \\
&= \hat{a}_i + \mathbb{E}[\tilde{\delta}_i|s_i - \bar{s}] + \mathbb{E}[\tilde{\eta}|\bar{s} - \hat{a}] - \rho_I \mathrm{Var}(\tilde{\eta}|\bar{s} - a) \\
&= \hat{a}_i + \frac{\tau_\xi}{\tau_\delta + \tau_\xi}(s_i - \bar{a}) + \frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}(\bar{a} - \hat{a}) - \rho_I \frac{1}{\tau_\eta + \tau_\epsilon}.
\end{aligned}
$$

In equilibrium, $\hat{a}_i = a_i^*$ for all firms, where

$$
\begin{aligned}
a_i^* &= \arg\max_{a_i} \mathbb{E}[P_i] - \frac{\rho_E}{2}\mathrm{Var}(P_i) - \frac{1}{2}a_i^2 \\
&= \hat{a}_i + \frac{\tau_\xi}{\tau_\delta + \tau_\xi}(a_i - \bar{a}) - \rho_I \frac{1}{\tau_\eta + \tau_\epsilon} - \frac{\rho_E}{2}\left(\frac{1}{\tau_\eta}\frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon} + \frac{1}{\tau_\delta}\frac{\tau_\xi}{\tau_\delta + \tau_\xi}\right) - \frac{1}{2}a_i^2 \\
&= \frac{\tau_\xi}{\tau_\delta + \tau_\xi}.
\end{aligned}
$$

$\square$

Proof of Proposition 1.1. From equations (1.11) – (1.12), we have

$$
\frac{\mathrm{dCE}_I}{\mathrm{dc}} = \frac{1}{\tau}\frac{\rho_I}{2}\left(\frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}\right)^2 > 0, \tag{A1}
$$

$$
\begin{aligned}
\frac{\mathrm{dCE}_E}{\mathrm{dc}} &= \frac{\partial \mathrm{CE}_E}{\partial \tau_\xi}\frac{\mathrm{d}\tau_\epsilon}{\mathrm{dc}} + \frac{\partial \mathrm{CE}_E}{\partial \tau_\epsilon}\frac{\mathrm{d}\tau_\xi}{\mathrm{dc}} \\
&= \frac{1}{\tau}\Bigg\{ \underbrace{\frac{\tau_\delta^2 \tau_\xi^2}{(\tau_\delta + \tau_\xi)^3}}_{\text{market monitoring effect}} - \underbrace{\frac{\rho_E}{2}\left[\left(\frac{\tau_\xi}{\tau_\delta + \tau_\xi}\right)^2 - \left(\frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}\right)^2\right]}_{\text{price risk effect}} - \underbrace{\rho_I \left(\frac{\tau_\epsilon}{\tau_\eta + \tau_\epsilon}\right)^2}_{\text{risk premium effect}} \Bigg\},
\end{aligned} \tag{A2}
$$

where $\tau_\epsilon = \tau_\epsilon(c)$ and $\tau_\xi = \tau_\xi(c)$ are defined in equations (1.5). It follows that $\mathrm{sgn}\left(\frac{\mathrm{dCE}_E}{\mathrm{dc}}\right) =$

sgn $(\Phi(c))$ where

$$\Phi(c) \equiv 2\frac{(1-c)\tau_\delta^2}{(1-c)\tau_\delta+\tau} - \rho_E - (2\rho_I - \rho_E)\left[\frac{(1-c)\tau_\delta+\tau}{c\tau_\eta+\tau}\right]^2.$$

Let $\Phi_1(c) \equiv 2\frac{(1-c)\tau_\delta^2}{(1-c)\tau_\delta+\tau} - \rho_E$ and $\Phi_2(c) \equiv (2\rho_I - \rho_E)\left[\frac{(1-c)\tau_\delta+\tau}{c\tau_\eta+\tau}\right]^2$, then

$$\lim_{c\to 0}\Phi_1 = \frac{2\tau_\delta^2}{\tau_\delta+\tau} - \rho_E \qquad \lim_{c\to 0}\Phi_2 = (2\rho_I-\rho_E)\left(\frac{\tau_\delta+\tau}{\tau}\right)^2$$
$$\lim_{c\to 1}\Phi_1 = -\rho_E \qquad \lim_{c\to 1}\Phi_2 = (2\rho_I-\rho_E)\left(\frac{\tau}{\tau_\eta+\tau}\right)^2.$$

As a result, we have $\lim_{c\to 0}\Phi_1 > \lim_{c\to 0}\Phi_2$ if and only if $\rho_E > \frac{2(\tau_\delta+\tau)^3\rho_I - 2\tau^2\tau_\delta^2}{\tau_\delta(\tau_\delta+\tau)(\tau_\delta+2\tau)}$, and $\lim_{c\to 1}\Phi_1 < \lim_{c\to 1}\Phi_2$ for any $\rho_E, \rho_I > 0$. Hence, $c = 1$ is not desirable for the entrepreneurs. Next, we characterize the conditions when the entrepreneurs prefer an interior level of comparability. Observe that

$$\begin{aligned}
\frac{\partial\Phi_1}{\partial c} &= -2\frac{\tau\tau_\delta^2}{[(1-c)\tau_\delta+\tau]^2} < 0,\\
\frac{\partial^2\Phi_1}{\partial c^2} &= -4\frac{c\tau_\delta^3}{[(1-c)\tau_\delta+\tau]^3} < 0;\\
\frac{\partial\Phi_2}{\partial c} &= 2(\rho_E - 2\rho_I)\frac{(1-c)\tau_\delta+\tau}{(c\tau_\eta+\tau)^3}\left[\tau_\eta\tau_\delta + \tau(\tau_\eta+\tau_\delta)\right]\\
&\implies \text{sgn}\left(\frac{\partial\Phi_2}{\partial c}\right) = \text{sgn}(\rho_E - 2\rho_I),\\
\frac{\partial^2\Phi_2}{\partial c^2} &= 2(2\rho_I-\rho_E)\frac{\tau_\delta(c\tau_\eta+\tau) + 3\tau_\eta[(1-c)\tau_\delta+\tau]}{(c\tau_\eta+\tau)^4}\left[\tau_\eta\tau_\delta + \tau(\tau_\eta+\tau_\delta)\right]\\
&\implies \text{sgn}\left(\frac{\partial^2\Phi_2}{\partial c^2}\right) = \text{sgn}(2\rho_I - \rho_E).
\end{aligned}$$

When $\rho_E \geq 2\rho_I$, $c_E^* \in (0,1)$ if $\rho_E > \frac{2(\tau_\delta+\tau)^3\rho_I - 2\tau^2\tau_\delta^2}{\tau_\delta(\tau_\delta+\tau)(\tau_\delta+2\tau)}$; otherwise $c_E^* = 0$.

When $\rho_E < 2\rho_I$,

(i) if $\rho_E > \frac{2(\tau_\delta+\tau)^3\rho_I - 2\tau^2\tau_\delta^2}{\tau_\delta(\tau_\delta+\tau)(\tau_\delta+2\tau)}$: then $c_E^* \in (0,1)$; and

(ii) if $\rho_E \leq \frac{2(\tau_\delta+\tau)^3\rho_I - 2\tau^2\tau_\delta^2}{\tau_\delta(\tau_\delta+\tau)(\tau_\delta+2\tau)}$: if $\Phi_1(c) \leq \Phi_2(c)$ $\forall c \in [0,1]$, then $c_E^* = 0$; and if $\Phi_1(c) =$

$\Phi_2(c)$ has two interior solutions $0 < c_E^1 < c_E^2 < 1$ where c_E^1 is the local minimum and c_E^2 is the local maximum, then $c_E^* = 0$ or c_E^2.

$\square$

Proof of Proposition 1.2. Given the standard setter's objective function, let

$$\Lambda(c) \equiv \frac{\mathrm{d}W}{\mathrm{d}c} = (1-\omega)\frac{\mathrm{dCE}_E}{\mathrm{d}c} + \omega\frac{\mathrm{dCE}_I}{\mathrm{d}c}. \tag{A3}$$

Whenever c_E^* is interior, evaluating the first-order derivative at $c = c_E^*$ yields

$$\Lambda(c_E^*) = \omega \left.\frac{\mathrm{dCE}_I}{\mathrm{d}c}\right|_{c=c_E^*} > 0,$$

implying $c^* > c_E^*$ since CE_I is monotonically increasing in c.

Next we derive the condition under which c^* admits an interior solution. Rewriting the certainty equivalents of the two constituents as functions of c, suppose to the contrary that $c^* = 1$, then for any $c \in (c_E^*, 1)$, we have

$$\begin{aligned}
&(1-\omega)\mathrm{CE}_E(1) + \omega\mathrm{CE}_I(1) > (1-\omega)\mathrm{CE}_E(c) + \omega\mathrm{CE}_I(c) \\
\iff \quad & \omega > \frac{\mathrm{CE}_E(c) - \mathrm{CE}_E(1)}{\mathrm{CE}_E(c) - \mathrm{CE}_E(1) + \mathrm{CE}_I(1) - \mathrm{CE}_I(c)} \\
\iff \quad & \omega \geq \hat{\omega} \equiv \sup_{c \in (c_E^*, 1)} \frac{\mathrm{CE}_E(c) - \mathrm{CE}_E(1)}{\mathrm{CE}_E(c) - \mathrm{CE}_E(1) + \mathrm{CE}_I(1) - \mathrm{CE}_I(c)}
\end{aligned}$$

since $\mathrm{CE}_E(c) - \mathrm{CE}_E(1) > 0$ and $\mathrm{CE}_I(1) - \mathrm{CE}_I(c) > 0$ by the proof of *Proposition 1.1*.

Further note that by L'Hôpital's rule, we have

$$\lim_{c\to 1^-} \frac{\mathrm{CE}_E(c) - \mathrm{CE}_E(1)}{\mathrm{CE}_E(c) - \mathrm{CE}_E(1) + \mathrm{CE}_I(1) - \mathrm{CE}_I(c)} = \lim_{c\to 1^-} \frac{\mathrm{CE}_E'(c)}{\mathrm{CE}_E'(c) - \mathrm{CE}_I'(c)} = \frac{\mathrm{CE}_E'(1)}{\mathrm{CE}_E'(1) - \mathrm{CE}_I'(1)}.$$

Since $\mathrm{CE}_E'(1) < 0$ and $\mathrm{CE}_I'(1) > 0$, we conclude that $\hat{\omega} \in (0, 1)$.

To see that $\hat{\omega} > \frac{2}{3}$, we plug in $\omega = \frac{2}{3}$ and evaluate $\Lambda(1)$, which yields $\Lambda(1)|_{\omega=\frac{2}{3}} < 0$.

Now we proceed to examine how the interior c^* changes with respect to ω. When $c^* \in (0,1)$, we have

$$\left.\frac{\partial \Lambda}{\partial \omega}\right|_{c=c^*} = \left.\frac{\mathrm{dCE}_I}{\mathrm{d}c}\right|_{c=c^*} - \left.\frac{\mathrm{dCE}_E}{\mathrm{d}c}\right|_{c=c^*} > 0$$

since $\left.\frac{\mathrm{dCE}_I}{\mathrm{d}c}\right|_{c=c^*} > 0$ and $\left.\frac{\mathrm{dCE}_E}{\mathrm{d}c}\right|_{c=c^*} < 0$. Further note that $\left.\frac{\partial \Lambda}{\partial c}\right|_{c=c^*} < 0$ by local concavity. Therefore, by implicit function theorem,

$$\frac{\mathrm{d}c^*}{\mathrm{d}\omega} = -\left.\frac{\frac{\partial \Lambda}{\partial \omega}}{\frac{\partial \Lambda}{\partial c}}\right|_{c=c^*} > 0.$$

□

Proof of Corollary 1.1. The proof follows immediately from the proof of *Propositions 1.1* and *1.2*.

□

Proof of Corollary 1.2. Plugging in equations (1.11) – (1.12) to the standard setter's objective function, we have

$$W = (1-\omega)\left\{\frac{\tau_\xi}{\tau_\delta+\tau_\xi} - \frac{1}{2}\left(\frac{\tau_\xi}{\tau_\delta+\tau_\xi}\right)^2 - \frac{\rho_E}{2}\left(\frac{1}{\tau_\delta}\frac{\tau_\xi}{\tau_\delta+\tau_\xi} + \frac{1}{\tau_\eta}\frac{\tau_\epsilon}{\tau_\eta+\tau_\epsilon}\right) - \frac{2-3\omega}{2(1-\omega)}\rho_I\frac{1}{\tau_\eta+\tau_\epsilon}\right\}.$$

Comparing to expression (1.12) and following the proof of *Proposition 1.1*, we have $\operatorname{sgn}\left(\frac{\mathrm{d}W}{\mathrm{d}c}\right) = \operatorname{sgn}\left(\Gamma(c)\right)$, where

$$\Gamma(c) \equiv 2\frac{(1-c)\tau_\delta^2}{(1-c)\tau_\delta+\tau} - \rho_E - \left(\frac{2-3\omega}{1-\omega}\rho_I - \rho_E\right)\left[\frac{(1-c)\tau_\delta+\tau}{c\tau_\eta+\tau}\right]^2,$$

and that when $c^* \in (0,1)$, if $\rho_E \geq \frac{2-3\omega}{1-\omega}\rho_I$, then c^* is the only critical point; otherwise, c^* is the only critical point or $c^* = c_2$ where $0 < c_1 < c_2 < 1$ are the two critical points. By implicit function theorem,

$$\frac{\mathrm{d}c^*}{\mathrm{d}x} = -\left.\frac{\frac{\partial\Gamma}{\partial x}}{\frac{\partial\Gamma}{\partial c}}\right|_{c=c^*} \implies \operatorname{sgn}\left(\frac{\mathrm{d}c^*}{\mathrm{d}x}\right) = \operatorname{sgn}\left(\left.\frac{\partial\Gamma}{\partial x}\right|_{c=c^*}\right) \text{ for } x \in \{\rho_I, \rho_E, \tau_\eta, \tau_\delta, \tau\},$$

since $\left.\frac{\partial\Gamma}{\partial c}\right|_{c=c^*} < 0$ by local concavity. In particular,

$$\begin{aligned}
\left.\frac{\partial\Gamma}{\partial\rho_I}\right|_{c=c^*} &= -\frac{2-3\omega}{1-\omega}\left[\frac{(1-c^*)\tau_\delta+\tau}{c^*\tau_\eta+\tau}\right]^2 \implies \operatorname{sgn}\left(\frac{\mathrm{d}c^*}{\mathrm{d}\rho_I}\right) = \operatorname{sgn}\left(\omega-\frac{2}{3}\right);\\
\left.\frac{\partial\Gamma}{\partial\rho_E}\right|_{c=c^*} &= \frac{\tau_\eta+\tau_\delta}{c^*\tau_\eta+\tau}\left(\frac{\tau_\delta}{\tau_\eta+\tau_\delta}-c^*\right)\left[\frac{(1-c^*)\tau_\delta+\tau}{c^*\tau_\eta+\tau}+1\right].
\end{aligned}$$

Note that $\operatorname{sgn}\left(\Gamma(c^*)-\Gamma\left(\frac{\tau_\delta}{\tau_\eta+\tau_\delta}\right)\right) = \operatorname{sgn}\left(\frac{2-3\omega}{1-\omega}\rho_I - 2\frac{\tau_\eta\tau_\delta^2}{\tau_\eta\tau_\delta+\tau(\tau_\eta+\tau_\delta)}\right)$, then

(i) if $\omega \geq \frac{2}{3}$, then $c^* > \frac{\tau_\delta}{\tau_\eta+\tau_\delta}$;

(ii) if $\omega < \frac{2}{3}$, then when $\rho_I < \frac{2(1-\omega)}{2-3\omega}\frac{\tau_\eta\tau_\delta^2}{\tau_\eta\tau_\delta+\tau(\tau_\eta+\tau_\delta)}$, we have $c^* > \frac{\tau_\delta}{\tau_\eta+\tau_\delta}$. Otherwise, plugging in $\rho_I = \frac{2(1-\omega)}{2-3\omega}\frac{\tau_\eta\tau_\delta^2}{\tau_\eta\tau_\delta+\tau(\tau_\eta+\tau_\delta)}$, we have $\left.\frac{\partial\Gamma}{\partial c}\right|_{c=\frac{\tau_\delta}{\tau_\eta+\tau_\delta}} = -\frac{\tau(\tau_\eta+\tau_\delta)^4\left((\tau(\tau_\eta+\tau_\delta)+\tau_\eta\tau_\delta)\rho_E+\tau_\delta^2(\tau-2\tau_\eta)\right)}{(\tau(\tau_\eta+\tau_\delta)+\tau_\eta\tau_\delta)^4}$, implying $\operatorname{sgn}\left(\frac{\tau_\delta}{\tau_\eta+\tau_\delta}-c^*\right) = -\operatorname{sgn}\left(\left.\frac{\partial\Gamma}{\partial c}\right|_{c=\frac{\tau_\delta}{\tau_\eta+\tau_\delta}}\right) = \operatorname{sgn}\left(\rho_E - \frac{\tau_\delta^2(2\tau_\eta-\tau)}{\tau(\tau_\eta+\tau_\delta)+\tau_\eta\tau_\delta}\right)$.

To conclude, if $\omega \geq \frac{2}{3}$, then $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_E} < 0$; if $\omega < \frac{2}{3}$, then when $\rho_I < \frac{2(1-\omega)}{2-3\omega}\frac{\tau_\eta\tau_\delta^2}{\tau_\eta\tau_\delta+\tau(\tau_\eta+\tau_\delta)}$, $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_E} < 0$; when $\rho_I > \frac{2(1-\omega)}{2-3\omega}\frac{\tau_\eta\tau_\delta^2}{\tau_\eta\tau_\delta+\tau(\tau_\eta+\tau_\delta)}$, $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_E} < 0$ for $\rho_E < \frac{\tau_\delta^2(2\tau_\eta-\tau)}{\tau(\tau_\eta+\tau_\delta)+\tau_\eta\tau_\delta}$ and $\frac{\mathrm{d}c^*}{\mathrm{d}\rho_E} > 0$ for $\rho_E > \frac{\tau_\delta^2(2\tau_\eta-\tau)}{\tau(\tau_\eta+\tau_\delta)+\tau_\eta\tau_\delta}$.

$$\begin{aligned}
\left.\frac{\partial\Gamma}{\partial\tau_\eta}\right|_{c=c^*} &= 2\left(\frac{2-3\omega}{1-\omega}\rho_I-\rho_E\right)\left[\frac{(1-c^*)\tau_\delta+\tau}{c^*\tau_\eta+\tau}\right]^2\frac{c^*}{c^*\tau_\eta+\tau}\\
&\implies \operatorname{sgn}\left(\frac{\mathrm{d}c^*}{\mathrm{d}\tau_\eta}\right) = \operatorname{sgn}\left(\frac{2-3\omega}{1-\omega}\rho_I-\rho_E\right),\\
\left.\frac{\partial\Gamma}{\partial\tau_\delta}\right|_{c=c^*} &= 2(1-c^*)\left\{\tau_\delta\frac{(1-c^*)\tau_\delta+2\tau}{\left[(1-c^*)\tau_\delta+\tau\right]^2}+\left(\rho_E-\frac{2-3\omega}{1-\omega}\rho_I\right)\frac{(1-c^*)\tau_\delta+\tau}{(c^*\tau_\eta+\tau)^2}\right\}\\
&\implies \frac{\mathrm{d}c^*}{\mathrm{d}\tau_\delta} > 0 \quad \text{when } \rho_E \geq \frac{2-3\omega}{1-\omega}\rho_I.
\end{aligned}$$

When $\rho_E < \frac{2-3\omega}{1-\omega}\rho_I$, $\text{sgn}\left(\frac{\mathrm{d}c^*}{\mathrm{d}\tau_\delta}\right) = \text{sgn}\left(\frac{\partial}{\partial \tau_\delta}\frac{\mathrm{d}W}{\mathrm{d}c}\Big|_{c=c^*}\right)$ since $\frac{\partial}{\partial c}\frac{\mathrm{d}W}{\mathrm{d}c}\Big|_{c=c^*} < 0$.

$$\begin{aligned}
\frac{\partial}{\partial c}\frac{\mathrm{d}W}{\mathrm{d}c}\Big|_{c=c^*} &= (1-\omega)\tau\Bigg\{\frac{\tau_\delta^2[2(1-c^*)\tau_\delta-\tau]}{[(1-c^*)\tau_\delta+\tau]^4} - \rho_E\left[\frac{\tau_\delta}{[(1-c^*)\tau_\delta+\tau]^3} + \frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3}\right] \\
&\qquad + \frac{2-3\omega}{1-\omega}\rho_I\frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3}\Bigg\} < 0 \\
&\implies \rho_E > \frac{\frac{2-3\omega}{1-\omega}\rho_I\frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3} + \frac{\tau_\delta^2[2(1-c^*)\tau_\delta-\tau]}{[(1-c^*)\tau_\delta+\tau]^4}}{\frac{\tau_\delta}{[(1-c^*)\tau_\delta+\tau]^3} + \frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3}} > \frac{\rho_E\frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3} + \frac{\tau_\delta^2[2(1-c^*)\tau_\delta-\tau]}{[(1-c^*)\tau_\delta+\tau]^4}}{\frac{\tau_\delta}{[(1-c^*)\tau_\delta+\tau]^3} + \frac{\tau_\eta}{(c^*\tau_\eta+\tau)^3}} \\
&\implies \rho_E > \tau_\delta\frac{2(1-c^*)\tau_\delta-\tau}{(1-c^*)\tau_\delta+\tau} \\
\frac{\partial}{\partial \tau_\delta}\frac{\mathrm{d}W}{\mathrm{d}c}\Big|_{c=c^*} &= (1-\omega)\frac{\tau(1-c^*)}{[(1-c^*)\tau_\delta+\tau]^4}\Big([(1-c^*)\tau_\delta+\tau]\rho_E - \tau_\delta[(1-c^*)\tau_\delta-2\tau]\Big) \\
&> (1-\omega)\frac{\tau\tau_\delta(1-c^*)}{[(1-c^*)\tau_\delta+\tau]^3} > 0 \\
&\implies \frac{\mathrm{d}c^*}{\mathrm{d}\tau_\delta} > 0 \quad \text{when } \rho_E < \frac{2-3\omega}{1-\omega}\rho_I. \\
\frac{\partial\Gamma}{\partial\tau}\Big|_{c=c^*} &= 2\left(\rho_E - \frac{2-3\omega}{1-\omega}\rho_I\right)\frac{(1-c^*)\tau_\delta+\tau}{(c^*\tau_\eta+\tau)^3}[c^*(\tau_\eta+\tau_\delta)-\tau_\delta] - 2\frac{(1-c^*)\tau_\delta^2}{[(1-c^*)\tau_\delta+\tau]^2} \\
&\overset{\text{FOC}}{=} \frac{\left(\rho_E - \frac{2-3\omega}{1-\omega}\rho_I\right)\left[\frac{(1-c^*)\tau_\delta+\tau}{c^*\tau_\eta+\tau}\right]^2\left[2\frac{c^*(\tau_\eta+\tau_\delta)-\tau_\delta}{c^*\tau_\eta+\tau}+1\right] - \rho_E}{(1-c^*)\tau_\delta+\tau}.
\end{aligned}$$

Note that $\left[\frac{(1-c)\tau_\delta+\tau}{c\tau_\eta+\tau}\right]^2\left[2\frac{c(\tau_\eta+\tau_\delta)-\tau_\delta}{c\tau_\eta+\tau}+1\right] \leq 1$ with the upper bound achieved at $c = \frac{\tau_\delta}{\tau_\eta+\tau_\delta}$. Hence, $\frac{\mathrm{d}c^*}{\mathrm{d}\tau} < 0$ if $\omega < \frac{2}{3}$ and $\rho_E > \frac{2-3\omega}{1-\omega}\rho_I$. □

Proof of Corollary 1.3. The proof follows immediately after plugging in equations (1.5). □

Proof of Lemma 1.2. Recall that the price of each firm given all available information $\mathcal{I} \equiv \{s_{k,i}\}_{i\in[0,\frac{1}{2}],k\in\{1,2\}}$ follows a strict factor structure. The risk premium is determined by the non-diversifiable risk of market portfolio, $\tilde{\eta} + \frac{1}{2}(\tilde{\eta}_1 + \tilde{\eta}_2)$. Denote $\mathbf{\Sigma}_\eta$ and $\mathbf{\Sigma}_s$ as the 2×2 variance-covariance matrix of $\{\tilde{\eta}+\tilde{\eta}_1, \tilde{\eta}+\tilde{\eta}_2\}$ and $\{\tilde{\bar{s}}_1, \tilde{\bar{s}}_2\}$, respectively, and $\mathbf{\Sigma}_{\eta s}$ as the 2×2 covariance matrix of the two sets of variables. Then given investors' conjectured levels of effort $\hat{a}_{k,i}$ and $\hat{a}_k$ for firm i and the group k average, respectively,

$$\begin{aligned} P_{k,i}(s_{k,i},\bar{s}_k,\bar{s}_{k'}) &= \mathbb{E}[\tilde{\theta}_{k,i}|\mathcal{I}] - \rho_I \mathrm{Var}(\tilde{\eta} + \frac{1}{2}(\tilde{\eta}_1 + \tilde{\eta}_2)|\mathcal{I}) \\ &= \hat{a}_{k,i} + \frac{\tau_{\xi_k}}{\tau_{\delta_k} + \tau_{\xi_k}}(s_{k,i} - \bar{s}_k) + b_{kk}(\bar{s}_k - \hat{a}_k) + b_{kk'}(\bar{s}_{k'} - \hat{a}_{k'}) - \rho_I \mathbf{m}^\intercal \mathbf{\Sigma}_{\eta|s}\mathbf{m}, \end{aligned}$$

where $\mathbf{m}$ is a 2×1 vector with $\frac{1}{2}$ in each element that denotes the mass of each group, and the coefficients on the average signals and the residual uncertainties about the undiversifiable common shocks $\{\tilde{\eta} + \tilde{\eta}_1, \tilde{\eta} + \tilde{\eta}_2\}$ are:

$$(b_{ij})_{2\times 2} \equiv \mathbf{\Sigma}_{\eta s}\mathbf{\Sigma}_s^{-1} = \frac{1}{B}\begin{pmatrix} \tau_{\epsilon_1}\big((\tau_\eta + \tau_{\eta_1})(\tau_{\eta_2} + \tau_{\epsilon_2}) + \tau_{\eta_2}\tau_{\epsilon_2}\big) & \tau_{\eta_1}\tau_{\eta_2}\tau_{\epsilon_2} \\ \tau_{\eta_1}\tau_{\eta_2}\tau_{\epsilon_1} & \tau_{\epsilon_2}\big((\tau_\eta + \tau_{\eta_2})(\tau_{\eta_1} + \tau_{\epsilon_1}) + \tau_{\eta_1}\tau_{\epsilon_1}\big) \end{pmatrix},$$

$$\mathbf{\Sigma}_{\eta|s} \equiv \mathbf{\Sigma}_\eta - \mathbf{\Sigma}_{\eta s}\mathbf{\Sigma}_s^{-1}\mathbf{\Sigma}_{\eta s}^\intercal = \frac{1}{B}\begin{pmatrix} (\tau_\eta + \tau_{\eta_1})(\tau_{\eta_2} + \tau_{\epsilon_2}) + \tau_{\eta_2}\tau_{\epsilon_2} & \tau_{\eta_1}\tau_{\eta_2} \\ \tau_{\eta_1}\tau_{\eta_2} & (\tau_\eta + \tau_{\eta_2})(\tau_{\eta_1} + \tau_{\epsilon_1}) + \tau_{\eta_1}\tau_{\epsilon_1} \end{pmatrix},$$

where $B = (\tau_\eta + \tau_{\eta_1} + \tau_{\eta_2})\tau_{\epsilon_1}\tau_{\epsilon_2} + \tau_{\eta_1}\tau_{\eta_2}(\tau_{\epsilon_1} + \tau_{\epsilon_2}) + \tau_\eta(\tau_{\eta_1}\tau_{\eta_2} + \tau_{\eta_1}\tau_{\epsilon_2} + \tau_{\eta_2}\tau_{\epsilon_1})$.

The rest of the proof follows in a similar way to the proof of *Lemma 1.1* and is hence omitted. □

Appendix B

Proofs to Chapter 2

B.1. Noisy Price Interpretation

Instead of random supply, we now assume imperfect price interpretation to prevent prices from being fully revealing. That is, when forming beliefs about the aggregate interpretation, each investor i interprets the information in the price as $q_{ti} = p_t^* + \tilde{\xi}_{ti}$, where $\tilde{\xi}_{ti} \overset{i.i.d.}{\sim} \mathcal{N}(0, \tau_\xi^{-1})$ is the additional noise of interpretation.[1]

Lemma B1. *There exists a unique short-horizon linear rational expectations equilibrium* $(\dot{p}_1, \dot{p}_2)$ *with noisy price interpretation, characterized by*

$$\dot{p}_1 = \dot{b}_1 \bar{v} + \dot{c}_1 y, \tag{B1}$$

$$\dot{p}_2 = \dot{a}_2 p_1 + \dot{b}_2 \bar{v} + \dot{c}_2 y, \tag{B2}$$

where

$$\dot{b}_1 = 1 - \dot{c}_1, \tag{B3a}$$

[1] Similar to private interpretations of public disclosures, noisy price interpretation is supported by bounded rationality. See Mondria, Vives and Yang (2021) for related discussions and theoretical foundations.

$$\dot{c}_1 = \tau_\eta \frac{(\tau_v + \tau_\eta)(\tau_\epsilon + \tau_\xi)(\tau_\epsilon + 2\tau_\xi) + \tau_v \tau_\eta \tau_\xi}{\Big((\tau_v + \tau_\eta)(\tau_\epsilon + \tau_\xi) + \tau_v \tau_\eta\Big)\Big((\tau_v + \tau_\eta)(\tau_\epsilon + 2\tau_\xi) + \tau_v \tau_\eta\Big)}, \tag{B3b}$$

$$\dot{a}_2 = \frac{(\tau_v + \tau_\eta)(\tau_\epsilon + \tau_\xi) + \tau_v \tau_\eta}{(\tau_v + \tau_\eta)(\tau_\epsilon + \tau_\xi)(\tau_\epsilon + 2\tau_\xi) + \tau_v \tau_\eta \tau_\xi} \tau_\xi, \tag{B3c}$$

$$\dot{b}_2 = \frac{\tau_v(\tau_\epsilon + \tau_\eta + 2\tau_\xi) + \tau_\eta \tau_\xi}{(\tau_v + \tau_\eta)(\tau_\epsilon + 2\tau_\xi) + \tau_v \tau_\eta} - \dot{a}_2, \tag{B3d}$$

$$\dot{c}_2 = \frac{\tau_\eta(\tau_\epsilon + \tau_\xi)}{(\tau_v + \tau_\eta)(\tau_\epsilon + 2\tau_\xi) + \tau_v \tau_\eta}. \tag{B3e}$$

This equilibrium reconciles the average expectations specification and the noisy rational expectations equilibrium with noisy supplies. By inspecting equations (B3), when there is no learning from price, i.e., $\tau_\xi \to 0$, the prices coincide with the average expectations. Compared to the equilibrium characterized in *Proposition 2.1*, everything else stays the same except that the ρ's are exogenously fixed at τ_ξ.[2] Thus, the key distinction between the pricing equations in the two specifications is whether price informativeness is endogenously determined through the supply noise or exogenously fixed by the imperfectness of learning from prices.

B.2. Long-Horizon Equilibrium

Lemma B2. *The unique long-horizon equilibrium is described by*

$$p = b\bar{v} + cy - dx_1, \tag{B4}$$

where

$$b = 1 - c, \tag{B5a}$$

$$c = \frac{\tau_\eta(\rho + \tau_\epsilon)}{(\tau_v + \tau_\eta)(\rho + \tau_\epsilon) + \tau_v \tau_\eta}, \tag{B5b}$$

[2]For simplicity, we intentionally choose the supply $x_t = 0$ for $t \in \{1, 2\}$ because it represents a constant term in the price equation and does not affect the other coefficients.

$$d = \frac{1}{\gamma}\frac{\rho + \tau_\epsilon + \tau_\eta}{\tau_\epsilon \tau_\eta} c, \tag{B5c}$$

$$\rho = \left(\gamma \frac{\tau_\epsilon \tau_\eta}{\rho + \tau_\epsilon + \tau_\eta}\right)^2 \tau_1. \tag{B5d}$$

Lemma B3. *Transparency and clarity have symmetric effects on price informativeness in the long-horizon economy. Moreover, increasing either one always improves price informativeness. Specifically,*

(i) price informativeness monotonically increases in clarity, i.e., $\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} > 0$*;*

(ii) price informativeness monotonically increases in transparency, i.e., $\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta} > 0$*; and*

(iii) when transparency and clarity are of the same magnitude, their marginal effects on price informativeness are the same, i.e., $\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} = \frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta}$ *if* $\tau_\epsilon = \tau_\eta$*.*

Proof. By the implicit function theorem,

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} = \frac{2(\gamma\tau_\eta)^2(\rho+\tau_\eta)\tau_\epsilon}{(\rho+\tau_\epsilon+\tau_\eta)^3 + 2(\gamma\tau_\epsilon\tau_\eta)^2\tau_1}\tau_1 > 0, \tag{B6}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta} = \frac{2(\gamma\tau_\epsilon)^2(\rho+\tau_\epsilon)\tau_\eta}{(\rho+\tau_\epsilon+\tau_\eta)^3 + 2(\gamma\tau_\epsilon\tau_\eta)^2\tau_1}\tau_1 > 0. \tag{B7}$$

Comparing equation (B7) with (B6), we have

$$\operatorname{sgn}\left[\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} - \frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[\tau_\eta(\rho+\tau_\eta) - \tau_\epsilon(\rho+\tau_\epsilon)\right] = \operatorname{sgn}\left[\tau_\eta - \tau_\epsilon\right].$$

□

Lemma B4. *Transparency and clarity both increase price sensitivity to the aggregate interpretation in the long-horizon economy, i.e.,* $\frac{\mathrm{d}c}{\mathrm{d}\tau_\epsilon} > 0$ *and* $\frac{\mathrm{d}c}{\mathrm{d}\tau_\eta} > 0$*.*

Proof. Note that

$$\frac{\partial c}{\partial \tau_\epsilon} = \frac{\partial c}{\partial \rho} = \left(\frac{\tau_\eta}{(\tau_v + \tau_\eta)(\rho+\tau_\epsilon) + \tau_v\tau_\eta}\right)^2 \tau_v > 0,$$

$$\frac{\partial c}{\partial \tau_\eta} = \left(\frac{\rho+\tau_\epsilon}{(\tau_v+\tau_\eta)(\rho+\tau_\epsilon)+\tau_v\tau_\eta}\right)^2 \tau_v > 0.$$

Therefore,

$$\begin{aligned}
\frac{\mathrm{d}c}{\mathrm{d}\tau_\epsilon} &= \frac{\partial c}{\partial \tau_\epsilon} + \frac{\partial c}{\partial \rho}\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} > 0,\\
\frac{\mathrm{d}c}{\mathrm{d}\tau_\eta} &= \frac{\partial c}{\partial \tau_\eta} + \frac{\partial c}{\partial \rho}\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta} > 0.
\end{aligned}$$

□

Lemma B5. *Transparency and clarity have different effects on price efficiency in the long-horizon economy, both in terms of magnitude and direction. Specifically, price efficiency*

(i) increases in clarity when transparency is sufficiently high, i.e., there exists $\hat{\tau}_\eta^\ddagger \in (0,\infty)$ *such that* $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon} > 0$ *when* $\tau_\eta > \hat{\tau}_\eta^\ddagger$*; and*

(ii) decreases in transparency when transparency is sufficiently high and clarity is sufficiently low, i.e., there exists $\hat{\tau}_\eta^{\mathrm{PE}}, \hat{\tau}_\epsilon^{\mathrm{PE}} \in (0,\infty)$ *such that* $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} < 0$ *for* $\tau_\eta > \hat{\tau}_\eta^{\mathrm{PE}}$ *and* $\tau_\epsilon < \hat{\tau}_\epsilon^{\mathrm{PE}}$ *if and only if* $\gamma^2\tau_v\tau_1 > 2$.

Proof. Note that $\mathrm{PE}^{-1} = (1-c)^2\tau_v^{-1} + c^2(\tau_\eta^{-1}+\rho^{-1})$ where $c > c^* = \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\rho^{-1}}$ is defined in equation (B5). We have

$$\begin{aligned}
\mathrm{sgn}\left[\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon}\right] &= -\mathrm{sgn}\left[\frac{\mathrm{dPE}^{-1}}{\mathrm{d}\tau_\epsilon}\right] = \mathrm{sgn}\left[2\big(\tau_v^{-1} - c(\tau_v^{-1}+\tau_\eta^{-1}+\rho^{-1})\big)\frac{\mathrm{d}c}{\mathrm{d}\tau_\epsilon} + \left(\frac{c}{\rho}\right)^2\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon}\right],\\
\mathrm{sgn}\left[\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta}\right] &= -\mathrm{sgn}\left[\frac{\mathrm{dPE}^{-1}}{\mathrm{d}\tau_\eta}\right] = \mathrm{sgn}\left[2\big(\tau_v^{-1} - c(\tau_v^{-1}+\tau_\eta^{-1}+\rho^{-1})\big)\frac{\mathrm{d}c}{\mathrm{d}\tau_\eta} + c^2\left(\tau_\eta^{-2} + \rho^{-2}\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta}\right)\right],
\end{aligned}$$

where $\frac{\mathrm{d}c}{\mathrm{d}\tau_\epsilon} > 0$, $\frac{\mathrm{d}c}{\mathrm{d}\tau_\eta} > 0$, $\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon} > 0$ and $\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta} > 0$. Define $\rho^\infty \equiv \lim_{\tau_\eta\to\infty}\rho = (\gamma\tau_\epsilon)^2$, then

$$\begin{aligned}
\lim_{\tau_\eta\to\infty}\mathrm{sgn}\left[\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\tau_\eta\to\infty}\mathrm{sgn}\left[2\big(\tau_v^{-1} - c(\tau_v^{-1}+\tau_\eta^{-1}+\rho^{-1})\big)\frac{\mathrm{d}c}{\mathrm{d}\tau_\epsilon} + \left(\frac{c}{\rho}\right)^2\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\epsilon}\right]\\
&= \mathrm{sgn}\left[2\tau_\epsilon\frac{\gamma^2\tau_1\big((\rho^\infty+\tau_\epsilon)^3 + \tau_v((\rho^\infty)^2+\tau_\epsilon^2)\big) - \rho^\infty\tau_v}{(\rho^\infty)^2(\rho^\infty+\tau_v+\tau_\epsilon)^3}\right] = 1,
\end{aligned}$$

$$\lim_{\tau_\eta\to\infty}\operatorname{sgn}\left[\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta}\right]=\lim_{\tau_\eta\to\infty}\operatorname{sgn}\left[\frac{\tau_\eta^2}{\tau_\epsilon^2}\left(2\big(\tau_v^{-1}-c(\tau_v^{-1}+\tau_\eta^{-1}+\rho^{-1})\big)\frac{\mathrm{d}c}{\mathrm{d}\tau_\eta}+c^2\left(\tau_\eta^{-2}+\rho^{-2}\frac{\mathrm{d}\rho}{\mathrm{d}\tau_\eta}\right)\right)\right]$$
$$=\operatorname{sgn}\left[(\rho+\tau_\epsilon)\frac{\begin{array}{r}\tau_v\big((\rho^\infty)^2(\rho^\infty-\tau_\epsilon)+2\rho^\infty\tau_\epsilon^2(\gamma^2\rho^\infty\tau_1-1)+2\gamma^2\tau_\epsilon^4\tau_1\big)\\+(\rho^\infty+\tau_\epsilon)^2\big((\rho^\infty)^2+2\gamma^2\tau_\epsilon^2\tau_1(\rho^\infty+\tau_\epsilon)\big)\end{array}}{(\rho^\infty\tau_\epsilon)^2(\rho^\infty+\tau_v+\tau_\epsilon)^3}\right],$$
$$\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}}\operatorname{sgn}\left[\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta}\right]=\operatorname{sgn}\left[\frac{2-\gamma^2\tau_v\tau_1}{\gamma^2\tau_v^3\tau_1}\right]=\operatorname{sgn}\left[2-\gamma^2\tau_v\tau_1\right].$$

As a result of continuity of $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon}$, there exist some $\hat{\tau}_\eta^\ddagger\in(0,\infty)$ such that $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon}>0$ when $\tau_\eta>\hat{\tau}_\eta^\ddagger$. Similarly, by continuity of $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta}$, there exist some $\hat{\tau}_\eta^{\mathrm{PE}},\hat{\tau}_\epsilon^{\mathrm{PE}}\in(0,\infty)$ such that $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta}<0$ for $\tau_\eta>\hat{\tau}_\eta^{\mathrm{PE}}$ and $\tau_\epsilon<\hat{\tau}_\epsilon^{\mathrm{PE}}$ if and only if $\gamma^2\tau_v\tau_1>2$. Moreover, numerical analysis demonstrates that price efficiency increases in clarity even when transparency is extremely low, that $\hat{\tau}_\epsilon^{\mathrm{PE}}$ always exists and is unique, and that $\hat{\tau}_\eta^{\mathrm{PE}}$ also exists and is unique when $\tau_\epsilon<\hat{\tau}_\epsilon^{\mathrm{PE}}$. These results suggest that price efficiency always increases in clarity and that the condition in *Lemma B5(ii)* is in fact necessary and sufficient. □

B.3. Proofs

Proof of Lemma 2.2. For $t=T$, $\omega_T=I$ and increases in both τ_ϵ and τ_η. For $t<T$,

$$\begin{aligned}\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\epsilon}&=C^{T-t}\frac{\mathrm{d}I}{\mathrm{d}\tau_\epsilon}+(T-t)IC^{T-t-1}\frac{\mathrm{d}C}{\mathrm{d}\tau_\epsilon}>0,\\ \frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\eta}&=C^{T-t}\frac{\mathrm{d}I}{\mathrm{d}\tau_\eta}+(T-t)IC^{T-t-1}\frac{\mathrm{d}C}{\mathrm{d}\tau_\eta}\\ &=\left(C-(T-t)\frac{\tau_\epsilon^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\tau_\epsilon^{-1}}\right)C^{T-t-1}\frac{\mathrm{d}I}{\mathrm{d}\tau_\eta}<0\\ &\iff C<(T-t)\frac{\tau_\epsilon^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\tau_\epsilon^{-1}}\\ &\iff \tau_\eta>\hat{\tau}_\eta^\omega=\frac{\tau_v\tau_\epsilon}{(T-t)\tau_v-\tau_\epsilon},\ \tau_\epsilon<\hat{\tau}_\epsilon^\omega=(T-t)\tau_v.\end{aligned}$$

□

Proof of Lemma 2.3. The price efficiency is given by $\mathrm{PE} = \frac{1}{(1-\omega_t)^2\tau_v^{-1}+\omega_t^2\tau_\eta^{-1}}$.

$$\begin{aligned}
\frac{\mathrm{dPE}}{\mathrm{d}\tau_\epsilon} &= 2\frac{\tau_v\tau_\eta^2}{\left(\omega_t{}^2\tau_v + (1-\omega_t)^2\tau_\eta\right)^2}(1-C^{T-t+1})\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\epsilon} > 0,\\
\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} &= \frac{\tau_v}{\left(\omega_t{}^2\tau_v + (1-\omega_t)^2\tau_\eta\right)^2}\left(\omega_t{}^2\tau_v + 2\tau_\eta{}^2(1-C^{T-t+1})\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\eta}\right) < 0\\
&\iff \frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_\eta} < -\frac{\omega_t{}^2\tau_v}{2\tau_\eta{}^2(1-C^{T-t+1})}\\
&\iff 2(1-C)(T-t+1) > 1 + \frac{1}{1-C^{T-t+1}}.
\end{aligned}$$

Let $M(C) = 1 + \frac{1}{1-C^{T-t+1}} - 2(1-C)(T-t+1)$, where $C = \frac{\tau_\epsilon}{\tau_\epsilon + \frac{\tau_v\tau_\eta}{\tau_v+\tau_\eta}} \in (\frac{\tau_\epsilon}{\tau_v+\tau_\epsilon}, 1)$. Notice that $M'(C) > 0$ and $M(1) > 0$. $M(\frac{\tau_\epsilon}{\tau_v+\tau_\epsilon}) \geq 0$ if and only if $2\frac{\tau_v}{\tau_v+\tau_\epsilon}(T-t+1) \leq \frac{2-\left(\frac{\tau_\epsilon}{\tau_v+\tau_\epsilon}\right)^{T-t+1}}{1-\left(\frac{\tau_\epsilon}{\tau_v+\tau_\epsilon}\right)^{T-t+1}}$. Therefore, $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} > 0$ for any τ_η if and only if $\tau_\epsilon \geq \hat{\tau}_\epsilon{}^{\mathrm{PE}}$, where $\hat{\tau}_\epsilon{}^{\mathrm{PE}}$ is the unique solution to

$$2\frac{\tau_v}{\tau_v + \hat{\tau}_\epsilon^{\mathrm{PE}}}(T-t+1) - \frac{2-\left(\frac{\hat{\tau}_\epsilon^{\mathrm{PE}}}{\tau_v+\hat{\tau}_\epsilon^{\mathrm{PE}}}\right)^{T-t+1}}{1-\left(\frac{\hat{\tau}_\epsilon^{\mathrm{PE}}}{\tau_v+\hat{\tau}_\epsilon^{\mathrm{PE}}}\right)^{T-t+1}} = 0. \tag{B8}$$

Otherwise, $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} > 0$ for $\tau_\eta < \hat{\tau}_\eta^{\mathrm{PE}}$ and $\frac{\mathrm{dPE}}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^{\mathrm{PE}}$ where $\hat{\tau}_\eta^{\mathrm{PE}}$ is the unique solution to

$$2(T-t+1)\left(1-\frac{\tau_\epsilon}{\tau_\epsilon+\frac{\tau_v\hat{\tau}_\eta^{\mathrm{PE}}}{\tau_v+\hat{\tau}_\eta^{\mathrm{PE}}}}\right)\left(1-\left(\frac{\tau_\epsilon}{\tau_\epsilon+\frac{\tau_v\hat{\tau}_\eta^{\mathrm{PE}}}{\tau_v+\hat{\tau}_\eta^{\mathrm{PE}}}}\right)^{T-t+1}\right)+\left(\frac{\tau_\epsilon}{\tau_\epsilon+\frac{\tau_v\hat{\tau}_\eta^{\mathrm{PE}}}{\tau_v+\hat{\tau}_\eta^{\mathrm{PE}}}}\right)^{T-t+1} - 2 = 0. \tag{B9}$$

□

Proof of Lemma 2.4. The optimal transparency in the sense of maximizing ω_t, $\hat{\tau}_\eta^\omega = \frac{\tau_v\tau_\epsilon}{(T-t)\tau_v-\tau_\epsilon}$, whenever it is interior, increases in τ_ϵ. By inspecting equation (B9), the optimal transparency in the sense of maximizing price efficiency, $\hat{\tau}_\eta^{\mathrm{PE}}$, is determined only through C. Applying the implicit function theorem, we have $\frac{\mathrm{d}\hat{\tau}_\eta^{\mathrm{PE}}}{\mathrm{d}\tau_\epsilon} = -\frac{\frac{\mathrm{d}C}{\mathrm{d}\tau_\epsilon}}{\frac{\mathrm{d}C}{\mathrm{d}\hat{\tau}_\eta}} > 0$. □

Proof of Proposition 2.1 (Existence and Uniqueness). From equation (2.6i) we have

$$\rho_1 = \gamma\tau_\epsilon\tau_\eta\left(\frac{\tau_2}{\rho_2}\right)^{\frac{1}{2}} - (\rho_2 + \tau_\epsilon + \tau_\eta); \tag{B10}$$

then ρ_2 in equilibrium solves the following equation:

$$R(\rho_2) = \left(\gamma\frac{\tau_v + \tau_\eta}{\tau_\eta}\frac{\rho_2\tau_\epsilon}{\rho_2 + \tau_\epsilon}\right)^2\tau_1 - \gamma\tau_\epsilon\tau_\eta\left(\frac{\tau_2}{\rho_2}\right)^{\frac{1}{2}} + \rho_2 + \tau_\epsilon + \tau_\eta = 0. \tag{B11}$$

The slope of $R(\rho_2)$ is represented by

$$R'(\rho_2) = 2\left(\gamma\frac{\tau_v + \tau_\eta}{\tau_\eta}\frac{\tau_\epsilon}{\rho_2 + \tau_\epsilon}\right)^2\frac{\rho_2\tau_\epsilon}{\rho_2 + \tau_\epsilon}\tau_1 + \frac{1}{2}\gamma\tau_\epsilon\tau_\eta\left(\frac{\tau_2}{{\rho_2}^3}\right)^{\frac{1}{2}} + 1 > 0. \tag{B12}$$

Therefore, $R(\rho_2)$ is monotonically increasing in ρ_2. Moreover, since $\lim_{\rho_2\to 0} R(\rho_2) = -\infty < 0$ and $\lim_{\rho_2\to\infty} R(\rho_2) = \infty > 0$, there always exists one unique equilibrium solution for ρ_2. Because there is a one-to-one correspondence from ρ_2 to ρ_1, there always exists a unique equilibrium satisfying equations (2.6). □

Proof of Corollary 2.1. When $\tau_\eta \to \infty$,

$$\begin{aligned}
\lim_{\tau_\eta\to\infty}\rho_2 &= (\gamma\tau_\epsilon)^2\tau_2 \equiv \rho_2^\infty, \\
\lim_{\tau_\eta\to\infty}\rho_1 &= \left(\gamma\tau_\epsilon\frac{\rho_2^\infty}{\rho_2^\infty + \tau_\epsilon}\right)^2\tau_1 \equiv \rho_1^\infty, \\
\lim_{\tau_\eta\to\infty}c_1 &= \frac{(\tau_\epsilon + \rho_1^\infty)(\tau_\epsilon + \rho_1^\infty + \rho_2^\infty) + \tau_v\rho_1^\infty}{(\tau_v + \tau_\epsilon + \rho_1^\infty)(\tau_v + \tau_\epsilon + \rho_1^\infty + \rho_2^\infty)}.
\end{aligned}$$

□

Proof of Proposition 2.2. Note that $\mathrm{PE}_1^{-1} = (1 - c_1)^2\tau_v^{-1} + c_1^2(\tau_\eta^{-1} + \rho_1^{-1})$ where $c_1 > c_1^* = \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_\eta^{-1}+\rho_1^{-1}}$ is defined in equation (2.6). We have

$$\mathrm{sgn}\left[\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\epsilon}\right] = \mathrm{sgn}\left[2\big(\tau_v^{-1} - c_1(\tau_v^{-1} + \tau_\eta^{-1} + \rho_1^{-1})\big)\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} + \left(\frac{c_1}{\rho_1}\right)^2\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon}\right],$$

$$\operatorname{sgn}\left[\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[2\big(\tau_v^{-1} - c_1(\tau_v^{-1} + \tau_\eta^{-1} + \rho_1^{-1})\big)\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} + c_1^2\left(\tau_\eta^{-2} + \rho_1^{-2}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right)\right],$$

where $\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon} > 0$, $\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta} > 0$, $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} > 0$ and $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^c$ and $\tau_\epsilon < \hat{\tau}_\epsilon^c$ by *Corollary 2.3*. Observe that

$$\begin{aligned}
&\lim_{\tau_\eta\to\infty} \tau_v^{-1} - c_1(\tau_v^{-1} + \tau_\eta^{-1} + \rho_1^{-1}) = -\frac{\tau_\epsilon(\rho_2^\infty + \tau_\epsilon)}{\rho_1^\infty(\rho_1^\infty + \tau_v + \tau_\epsilon)(\rho_1^\infty + \rho_2^\infty + \tau_v + \tau_\epsilon)},\\
&\lim_{\tau_\eta\to\infty} c_1 = \frac{(\rho_1^\infty + \tau_\epsilon)(\rho_1^\infty + \rho_2^\infty + \tau_v + \tau_\epsilon) - \tau_v\tau_\epsilon}{(\rho_1^\infty + \tau_v + \tau_\epsilon)(\rho_1^\infty + \rho_2^\infty + \tau_v + \tau_\epsilon)},\\
&\lim_{\tau_\eta\to\infty} \frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon} = 2\gamma^2\tau_\epsilon(\rho_2^\infty)^2\tau_1\frac{\rho_2^\infty + 2\tau_\epsilon}{(\rho_2^\infty + \tau_\epsilon)^3},\\
&\lim_{\tau_\eta\to\infty} \left(1 + \left(\frac{\tau_\eta}{\rho_1}\right)^2\right)\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta} = 2(\gamma\tau_\epsilon)^2\rho_2^\infty\tau_1\frac{2\gamma^2\tau_\epsilon^3(\rho_1^\infty + \rho_2^\infty + \tau_\epsilon)\tau_2 - \tau_v\rho_2^\infty(\rho_2^\infty + \tau_\epsilon)}{(\rho_1^\infty)^2(\rho_2^\infty + \tau_\epsilon)^3}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[2\big(\tau_v^{-1} - c_1(\tau_v^{-1} + \tau_\eta^{-1} + \rho_1^{-1})\big)\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} + \left(\frac{c_1}{\rho_1}\right)^2\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon}\right]\\
&= \operatorname{sgn}\left[\frac{4 - 4\gamma^2\tau_v\tau_2}{\gamma^6\tau_v^5\tau_1\tau_2^2}\right] = \operatorname{sgn}\left[1 - \gamma^2\tau_v\tau_2\right],\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{1}{\tau_\epsilon^2}\left(2\big(\tau_v^{-1} - c_1(\tau_v^{-1} + \tau_\eta^{-1} + \rho_1^{-1})\big)\tau_\eta^2\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} + c_1^2\left(1 + \left(\frac{\tau_\eta}{\rho_1}\right)^2\right)\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right)\right]\\
&= \operatorname{sgn}\left[\frac{4 - 6\gamma^2\tau_v\tau_2}{\gamma^6\tau_v^5\tau_1\tau_2^2}\right] = \operatorname{sgn}\left[2 - 3\gamma^2\tau_v\tau_2\right].
\end{aligned}$$

As a result of continuity of $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\epsilon}$, there exist some $\hat{\tau}_\eta^\dagger, \hat{\tau}_\epsilon^\dagger \in (0, \infty)$ such that $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\epsilon} < 0$ for $\tau_\eta > \hat{\tau}_\eta^\dagger$ and $\tau_\epsilon < \hat{\tau}_\epsilon^\dagger$ if and only if $\gamma^2\tau_v\tau_2 > 1$. Similarly, by continuity of $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta}$, there exist some $\hat{\tau}_\eta^{\mathrm{PE}}, \hat{\tau}_\epsilon^{\mathrm{PE}} \in (0, \infty)$ such that $\frac{\mathrm{dPE}_1}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^{\mathrm{PE}}$ and $\tau_\epsilon < \hat{\tau}_\epsilon^{\mathrm{PE}}$ if and only if $\gamma^2\tau_v\tau_2 > \frac{2}{3}$. □

Proof of Corollary 2.2. Define

$$f_1(\rho_1, \rho_2; \gamma, \tau_v, \tau_\eta, \tau_\epsilon, \tau_1, \tau_2) \equiv \left(\gamma\frac{\tau_v + \tau_\eta}{\tau_\eta}\frac{\rho_2\tau_\epsilon}{\rho_2 + \tau_\epsilon}\right)^2\tau_1 - \rho_1,$$

$$f_2(\rho_1,\rho_2;\gamma,\tau_v,\tau_\eta,\tau_\epsilon,\tau_1,\tau_2) \equiv \left(\gamma\frac{\tau_\epsilon\tau_\eta}{\rho_1+\rho_2+\tau_\epsilon+\tau_\eta}\right)^2\tau_2-\rho_2.$$

Note that in equilibrium, $f_1(\rho_1,\rho_2)=f_2(\rho_1,\rho_2)=0$. Applying the implicit function theorem, we have

$$\begin{pmatrix}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon}\\ \frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\epsilon}\end{pmatrix} = -\begin{pmatrix}\frac{\partial f_1}{\partial\rho_1} & \frac{\partial f_1}{\partial\rho_2}\\ \frac{\partial f_2}{\partial\rho_1} & \frac{\partial f_2}{\partial\rho_2}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial f_1}{\partial\tau_\epsilon}\\ \frac{\partial f_2}{\partial\tau_\epsilon}\end{pmatrix} = \frac{1}{\frac{\partial f_1}{\partial\rho_1}\frac{\partial f_2}{\partial\rho_2}-\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\rho_1}}\begin{pmatrix}\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\tau_\epsilon}-\frac{\partial f_2}{\partial\rho_2}\frac{\partial f_1}{\partial\tau_\epsilon}\\ \frac{\partial f_2}{\partial\rho_1}\frac{\partial f_1}{\partial\tau_\epsilon}-\frac{\partial f_1}{\partial\rho_1}\frac{\partial f_2}{\partial\tau_\epsilon}\end{pmatrix},$$

$$\begin{pmatrix}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\\ \frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\eta}\end{pmatrix} = -\begin{pmatrix}\frac{\partial f_1}{\partial\rho_1} & \frac{\partial f_1}{\partial\rho_2}\\ \frac{\partial f_2}{\partial\rho_1} & \frac{\partial f_2}{\partial\rho_2}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial f_1}{\partial\tau_\eta}\\ \frac{\partial f_2}{\partial\tau_\eta}\end{pmatrix} = \frac{1}{\frac{\partial f_1}{\partial\rho_1}\frac{\partial f_2}{\partial\rho_2}-\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\rho_1}}\begin{pmatrix}\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\tau_\eta}-\frac{\partial f_2}{\partial\rho_2}\frac{\partial f_1}{\partial\tau_\eta}\\ \frac{\partial f_2}{\partial\rho_1}\frac{\partial f_1}{\partial\tau_\eta}-\frac{\partial f_1}{\partial\rho_1}\frac{\partial f_2}{\partial\tau_\eta}\end{pmatrix}.$$

Note that $\frac{\partial f_1}{\partial\rho_1}<0$, $\frac{\partial f_1}{\partial\rho_2}>0$, $\frac{\partial f_2}{\partial\rho_1}<0$, $\frac{\partial f_2}{\partial\rho_2}<0$ and $\frac{\partial f_1}{\partial\tau_\epsilon}>0$, $\frac{\partial f_2}{\partial\tau_\epsilon}>0$, $\frac{\partial f_1}{\partial\tau_\eta}<0$, $\frac{\partial f_2}{\partial\tau_\eta}>0$, so $\frac{\partial f_1}{\partial\rho_1}\frac{\partial f_2}{\partial\rho_2}-\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\rho_1}>0$.[3] Thus, $\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon}>0$ and

$$\begin{aligned}
\operatorname{sgn}\left[\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right] &= \operatorname{sgn}\left[\frac{\partial f_1}{\partial\rho_2}\frac{\partial f_2}{\partial\tau_\eta}-\frac{\partial f_2}{\partial\rho_2}\frac{\partial f_1}{\partial\tau_\eta}\right]\\
&= \operatorname{sgn}\big[2(\gamma\tau_\epsilon\tau_\eta)^2\big(\tau_\epsilon(\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon)-2\tau_v\rho_2(\rho_2+\tau_\epsilon)\big)\tau_2\\
&\qquad -\tau_v\rho_2(\rho_2+\tau_\epsilon)(\rho_1+\rho_2+\tau_\epsilon+\tau_\eta)^3\big],\\
\operatorname{sgn}\left[\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon}-\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right] &= \operatorname{sgn}\left[\frac{\partial f_1}{\partial\rho_2}\left(\frac{\partial f_2}{\partial\tau_\epsilon}-\frac{\partial f_2}{\partial\tau_\eta}\right)-\frac{\partial f_2}{\partial\rho_2}\left(\frac{\partial f_1}{\partial\tau_\epsilon}-\frac{\partial f_1}{\partial\tau_\eta}\right)\right]\\
&= \operatorname{sgn}\Big[\rho_2\Big(\tau_v\tau_\epsilon^2+\big(\tau_\eta^2+\tau_v(\tau_\epsilon+\tau_\eta)\big)\rho_2\Big)\Big((\rho_1+\rho_2+\tau_\epsilon+\tau_\eta)^3+2(\gamma\tau_\epsilon\tau_\eta)^2\tau_2\Big)\\
&\qquad +2(\gamma\tau_\epsilon\tau_\eta)^2\tau_\epsilon(\tau_\eta-\tau_\epsilon)(\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon+\tau_\eta)\tau_2\Big]\\
&= 1 \text{ if } \tau_\epsilon\le\tau_\eta,\\
\operatorname{sgn}\left[\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\epsilon}-\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\eta}\right] &= \operatorname{sgn}\left[\frac{\partial f_2}{\partial\rho_1}\left(\frac{\partial f_1}{\partial\tau_\epsilon}-\frac{\partial f_1}{\partial\tau_\eta}\right)-\frac{\partial f_1}{\partial\rho_1}\left(\frac{\partial f_2}{\partial\tau_\epsilon}-\frac{\partial f_2}{\partial\tau_\eta}\right)\right]\\
&= \operatorname{sgn}\Bigg[-2\left(\gamma\frac{1}{\tau_\eta}\frac{\rho_2\tau_\epsilon}{\rho_2+\tau_\epsilon}\right)^2\frac{\tau_v+\tau_\eta}{\rho_2+\tau_\epsilon}\Big(\tau_v\tau_\epsilon^2+\big(\tau_\eta^2+\tau_v(\tau_\epsilon+\tau_\eta)\big)\rho_2\Big)\tau_1\\
&\qquad -(\tau_\epsilon-\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon+\tau_\eta)\Bigg]
\end{aligned}$$

[3]Details on the derivations are omitted for brevity and are available upon request.

$$= -1 \text{ if } \tau_\epsilon \geq \tau_\eta.$$

Define $\rho_1^\infty \equiv \lim_{\tau_\eta\to\infty} \rho_1 = \left(\gamma \frac{(\gamma\tau_\epsilon)^2\tau_2}{\gamma^2\tau_\epsilon\tau_2+1}\right)^2 \tau_1$ and $\rho_2^\infty \equiv \lim_{\tau_\eta\to\infty} \rho_2 = (\gamma\tau_\epsilon)^2\tau_2$. Then

$$\begin{aligned}
\lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right] &= \operatorname{sgn}\left[2\gamma^2\tau_\epsilon^3(\rho_1^\infty + \rho_2^\infty + \tau_\epsilon)\tau_2 - \tau_v\rho_2^\infty(\rho_2^\infty + \tau_\epsilon)\right] \\
&= \operatorname{sgn}\left[\gamma^2\tau_\epsilon\tau_2\left(\frac{2\gamma^4\tau_\epsilon^3\tau_1\tau_2}{(\gamma^2\tau_\epsilon\tau_2+1)^2} + 2\tau_\epsilon - \tau_v\right) + 2\tau_\epsilon - \tau_v\right], \\
\lim_{\substack{\tau_\eta\to\infty \\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}\right] &= \operatorname{sgn}\left[-\tau_v\right] = -1.
\end{aligned}$$

By continuity of $\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta}$, there exist some $\hat{\tau}_\eta^\rho, \hat{\tau}_\epsilon^\rho \in (0,\infty)$ such that $\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^\rho$ and $\tau_\epsilon < \hat{\tau}_\epsilon^\rho$. □

Proof of Corollary 2.3. First note that $\frac{\partial c_1}{\partial \tau_\epsilon} > 0$, $\frac{\partial c_1}{\partial \rho_1} > 0$, and $\frac{\partial c_1}{\partial \rho_2} > 0$ by inspecting equation (2.6b). Next we prove that $\frac{\partial c_1}{\partial \tau_\eta} < 0$ for sufficiently small τ_ϵ and sufficiently large τ_η.

$$\begin{aligned}
\operatorname{sgn}\left[\frac{\partial c_1}{\partial \tau_\eta}\right] &= \operatorname{sgn}\Big[(\tau_v+\tau_\eta)(\rho_1+\tau_\epsilon)^2\big((\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon)+\tau_v\tau_\eta\big)^2 \\
&\quad + \tau_\epsilon(\tau_v\tau_\eta)^2\big((\tau_v+\tau_\eta)(\rho_1+\tau_\epsilon)+\tau_v\tau_\eta\big) \\
&\quad - \tau_v\tau_\epsilon\tau_\eta\big((\tau_v+\tau_\eta)(\rho_1+\rho_2+\tau_\epsilon)+\tau_v\tau_\eta\big)\big(2(\tau_v+\tau_\eta)(\rho_1+\tau_\epsilon)+\tau_v\tau_\eta\big)\Big], \\
\lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{\partial c_1}{\partial \tau_\eta}\right] &= \operatorname{sgn}\big[\,((\rho_1^\infty+\tau_\epsilon)(\rho_1^\infty+\rho_2^\infty+\tau_v+\tau_\epsilon))^2 \\
&\quad - \tau_v\tau_\epsilon\big(\rho_2^\infty\tau_v + 2(\rho_1^\infty+\rho_2^\infty+\tau_v+\tau_\epsilon)(\rho_1^\infty+\tau_\epsilon)\big)\big] \\
&= \operatorname{sgn}\left[\left(\left(\frac{\rho_1^\infty}{\tau_\epsilon}+1\right)(\rho_1^\infty+\rho_2^\infty+\tau_v+\tau_\epsilon)\right)^2\right. \\
&\quad \left. - \tau_v\left(\rho_2^\infty\tau_v + 2(\rho_1^\infty+\rho_2^\infty+\tau_v+\tau_\epsilon)\left(\frac{\rho_1^\infty}{\tau_\epsilon}+1\right)\right)\right], \\
\lim_{\substack{\tau_\eta\to\infty \\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\partial c_1}{\partial \tau_\eta}\right] &= \operatorname{sgn}\left[-\tau_v^2\right] = -1.
\end{aligned}$$

Since $\frac{\partial c_1}{\partial \rho_2}$, $\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\epsilon}$, and $\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\eta}$ are very small compared to the other terms as shown below, $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon}$

and $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta}$ go in the same direction as the first two terms, respectively, i.e.,

$$\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} = \frac{\partial c_1}{\partial \tau_\epsilon} + \frac{\partial c_1}{\partial \rho_1}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\epsilon} + \frac{\partial c_1}{\partial \rho_2}\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\epsilon} > 0,$$
$$\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} = \frac{\partial c_1}{\partial \tau_\eta} + \frac{\partial c_1}{\partial \rho_1}\frac{\mathrm{d}\rho_1}{\mathrm{d}\tau_\eta} + \frac{\partial c_1}{\partial \rho_2}\frac{\mathrm{d}\rho_2}{\mathrm{d}\tau_\eta} < 0 \text{ when } \tau_\eta \to \infty, \tau_\epsilon \to 0.$$

In fact, we can show that $\lim_{\substack{\tau_\eta \to \infty \\ \tau_\epsilon \to 0}} \left(\frac{\tau_\eta}{\tau_\epsilon}\right)^2 \frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} = -\frac{1}{\tau_v} < 0$. Therefore, by continuity of $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta}$, there exist some $\hat{\tau}_\eta^c, \hat{\tau}_\epsilon^c \in (0, \infty)$ such that $\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\eta} < 0$ for $\tau_\eta > \hat{\tau}_\eta^c$ and $\tau_\epsilon < \hat{\tau}_\epsilon^c$. □

Proof of Proposition 2.3. We first characterize the linear rational expectations equilibria $(\hat{p}_1, \hat{p}_2)$ with correlated supply shocks,

$$\hat{p}_1 = \hat{b}_1 \bar{v} + \hat{c}_1 y - \hat{d}_1 x_1, \tag{B13}$$

$$\hat{p}_2 = \hat{a}_2 p_1 + \hat{b}_2 \bar{v} + \hat{c}_2 y - \hat{d}_2 x_2, \tag{B14}$$

where

$$\hat{b}_1 = 1 - \hat{c}_1, \tag{B15a}$$

$$\hat{c}_1 = \tau_\eta \frac{(\tau_v + \tau_\eta)(\psi_1^2\tau_1 + \tau_\epsilon)(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon) + \tau_v\tau_\eta(\psi_1^2\tau_1 + \beta\frac{\psi_1}{\psi_2}\tau_\epsilon)}{\big((\tau_v + \tau_\eta)(\psi_1^2\tau_1 + \tau_\epsilon) + \tau_v\tau_\eta\big)\big((\tau_v + \tau_\eta)(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon) + \tau_v\tau_\eta\big)}, \tag{B15b}$$

$$\hat{d}_1 = \frac{1}{\gamma}\frac{\tau_\eta}{\tau_v + \tau_\eta}\frac{\psi_2(\psi_2 - \beta\psi_1)\tau_u + \tau_\epsilon}{\psi_2(\psi_2 - \beta\psi_1)\tau_u\tau_\epsilon}\hat{c}_1, \tag{B15c}$$

$$\hat{a}_2 = \frac{\big((\tau_v + \tau_\eta)(\psi_1^2\tau_1 + \tau_\epsilon) + \tau_v\tau_\eta\big)\big(\psi_1^2\tau_1 - \beta\psi_1(\psi_2 - \beta\psi_1)\tau_u\big)}{(\tau_v + \tau_\eta)(\psi_1^2\tau_1 + \tau_\epsilon)(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon) + \tau_v\tau_\eta(\psi_1^2\tau_1 + \beta\frac{\psi_1}{\psi_2}\tau_\epsilon)}, \tag{B15d}$$

$$\hat{b}_2 = \frac{\tau_v(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon + \tau_\eta) + \tau_\eta(\psi_1^2\tau_1 - \beta\psi_1(\psi_2 - \beta\psi_1)\tau_u)}{(\tau_v + \tau_\eta)(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon) + \tau_v\tau_\eta} - \hat{a}_2, \tag{B15e}$$

$$\hat{c}_2 = \frac{\tau_\eta(\psi_2(\psi_2 - \beta\psi_1)\tau_u + \tau_\epsilon)}{(\tau_v + \tau_\eta)(\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon) + \tau_v\tau_\eta}, \tag{B15f}$$

$$\hat{d}_2 = \frac{1}{\gamma}\frac{\psi_1^2\tau_1 + (\psi_2 - \beta\psi_1)^2\tau_u + \tau_\epsilon + \tau_\eta}{\tau_\epsilon\tau_\eta}\hat{c}_2, \tag{B15g}$$

$$\psi_1 = \gamma\frac{\tau_v + \tau_\eta}{\tau_\eta}\frac{\psi_2(\psi_2 - \beta\psi_1)\tau_u\tau_\epsilon}{\psi_2(\psi_2 - \beta\psi_1)\tau_u + \tau_\epsilon}, \tag{B15h}$$

$$\psi_2 = \gamma \frac{\tau_\epsilon \tau_\eta}{\psi_1^2 \tau_1 + (\psi_2 - \beta\psi_1)^2 \tau_u + \tau_\epsilon + \tau_\eta}. \tag{B15i}$$

Plugging in $\tau_\eta \to \infty$ to equations (B15h) – (B15i) generates two solutions:

(i) $\psi_{1L}^{\infty} = \frac{(1+\beta)\gamma^2\tau_\epsilon\tau_u+1-\sqrt{((1+\beta)\gamma^2\tau_\epsilon\tau_u+1)^2-4\beta(\gamma^2\tau_\epsilon\tau_u)^2}}{2\beta\gamma\tau_u}, \psi_2^{\infty} = \gamma\tau_\epsilon$; and

(ii) $\psi_{1I}^{\infty} = \frac{(1+\beta)\gamma^2\tau_\epsilon\tau_u+1+\sqrt{((1+\beta)\gamma^2\tau_\epsilon\tau_u+1)^2-4\beta(\gamma^2\tau_\epsilon\tau_u)^2}}{2\beta\gamma\tau_u}, \psi_2^{\infty} = \gamma\tau_\epsilon$.

Note that $\rho_1 = \psi_1^2\tau_1$. Following the same proof strategy of *Corollary 2.2* through *Proposition 2.2*, in the low information equilibrium,

$$\begin{aligned}
\lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_{1L}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{1}{\tau_\epsilon}\frac{\mathrm{d}\psi_{1L}}{\mathrm{d}\tau_\epsilon}\right] = 1,\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_{1L}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\tau_\eta^2}{\tau_\epsilon^2}\frac{\mathrm{d}\psi_{1L}}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[-\gamma^3\tau_v\tau_u\right] = -1,\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}c_{1L}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{1}{\tau_\epsilon}\frac{\mathrm{d}c_{1L}}{\mathrm{d}\tau_\epsilon}\right] = \operatorname{sgn}\left[\frac{2+2\beta\gamma^2\tau_v\tau_u}{\tau_v^2}\right] = 1,\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}c_{1L}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\tau_\eta^2}{\tau_\epsilon^2}\frac{\mathrm{d}c_{1L}}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[-\frac{1}{\tau_v} - \beta\gamma^2\tau_u\right] = -1,\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_{1L}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\left(\tau_v^{-1} - c_{1L}(\tau_v^{-1}+\tau_\eta^{-1}+\rho_{1L}^{-1})\right)\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} + c_{1L}^2\psi_{1L}^{-3}\tau_1^{-1}\frac{\mathrm{d}\psi_{1L}}{\mathrm{d}\tau_\epsilon}\right]\\
&= \operatorname{sgn}\left[\frac{(\beta\gamma^2\tau_v\tau_u+1)((\beta-2)\gamma^2\tau_v\tau_u+2)}{(\gamma^2\tau_v\tau_u)^2\gamma^2\tau_v^3\tau_1}\right] = \operatorname{sgn}\left[2-(2-\beta)\gamma^2\tau_v\tau_u\right],\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_{1L}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\tau_\eta^2}{\tau_\epsilon^2}\left(2\left(\tau_v^{-1} - c_{1L}(\tau_v^{-1}+\tau_\eta^{-1}+\rho_{1L}^{-1})\right)\frac{\mathrm{d}c_{1L}}{\mathrm{d}\tau_\eta} + c_{1L}^2\left(\tau_\eta^{-2} + 2\frac{\rho_{1I}}{\psi_{1I}}\frac{\mathrm{d}\psi_{1L}}{\mathrm{d}\tau_\eta}\right)\right)\right]\\
&= \operatorname{sgn}\left[-2\frac{(\beta\gamma^2\tau_v\tau_u+1)(2\beta(\gamma^2\tau_v\tau_u)^2+(3-\beta)\gamma^2\tau_v\tau_u-2)}{(\gamma^2\tau_v\tau_u)^2\gamma^2\tau_v^3\tau_1}\right]\\
&= \operatorname{sgn}\left[\frac{\beta-3+\sqrt{(\beta+1)(\beta+9)}}{4\beta} - \gamma^2\tau_v\tau_u\right].
\end{aligned}$$

Similarly, in the intermediate information equilibrium,

$$\begin{aligned}
\lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_{1I}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\tau_\eta\to\infty} \operatorname{sgn}\left[\frac{\mathrm{d}\psi_{1I}}{\mathrm{d}\tau_\epsilon}\right] = 1,\\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}\rho_{1I}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\tau_\eta^2\frac{\mathrm{d}\psi_{1I}}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[-\frac{\tau_1+\beta^2\tau_u}{\beta^3\gamma^3\tau_u^3}\right] = -1,
\end{aligned}$$

$$
\begin{aligned}
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}c_{1I}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}c_{1I}}{\mathrm{d}\tau_\epsilon}\right] = \operatorname{sgn}\left[\gamma^2\tau_v\tau_u - \left(\frac{\beta}{\beta+1} - \frac{2(\beta+1)\tau_1+\beta^2(3\beta+1)\tau_u}{\beta^2(\beta+1)(2\tau_1+\beta^2\tau_u)\tau_u}\tau_1\right)\right], \\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{d}c_{1I}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\tau_\eta^2\frac{\mathrm{d}c_{1I}}{\mathrm{d}\tau_\eta}\right] = \operatorname{sgn}\left[-\left(\frac{\tau_1+\beta^2\tau_u}{\tau_1+\beta^2\tau_u(\gamma^2\tau_v\tau_u+1)}\right)^2\tau_v\right] = -1, \\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_{1I}}{\mathrm{d}\tau_\epsilon}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\left(\tau_v^{-1} - c_{1I}(\tau_v^{-1}+\tau_\eta^{-1}+\rho_{1I}^{-1})\right)\frac{\mathrm{d}c_1}{\mathrm{d}\tau_\epsilon} + c_{1I}^2\frac{\rho_{1I}}{\psi_{1I}}\frac{\mathrm{d}\psi_{1I}}{\mathrm{d}\tau_\epsilon}\right] = 1, \\
\lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\frac{\mathrm{dPE}_{1I}}{\mathrm{d}\tau_\eta}\right] &= \lim_{\substack{\tau_\eta\to\infty\\ \tau_\epsilon\to 0}} \operatorname{sgn}\left[\tau_\eta^2\left(2\left(\tau_v^{-1} - c_{1I}(\tau_v^{-1}+\tau_\eta^{-1}+\rho_{1I}^{-1})\right)\frac{\mathrm{d}c_{1I}}{\mathrm{d}\tau_\eta} + c_{1I}^2\left(\tau_\eta^{-2} + 2\frac{\rho_{1I}}{\psi_{1I}}\frac{\mathrm{d}\psi_{1I}}{\mathrm{d}\tau_\eta}\right)\right)\right] \\
&= \operatorname{sgn}\left[-\frac{\tau_1+2\beta^2\tau_u}{\tau_1}\left(\frac{\tau_1+\beta^2\tau_u}{\tau_1+\beta^2\tau_u(\gamma^2\tau_v\tau_u+1)}\right)^2\right] = -1.
\end{aligned}
$$

$\square$

Appendix C

Proofs to Chapter 3

C.1. Proofs

Proof of Proposition 3.1. Taking the FOC yields $a_i^* = (1-\omega)\mathbb{E}_i[\tilde{v}] + \omega\mathbb{E}_i[\bar{a}^*]$. To solve for the linear equilibrium, we thus assume $a_i^* = \mu + \phi_i(s_i - \mu)$. Hence,

$$\bar{a} = \mu + \frac{N_1}{N_1+N_2}\phi_1(\bar{s}_1 - \mu) + \frac{N_2}{N_1+N_2}\phi_2(\bar{s}_2 - \mu). \tag{C1}$$

Then

$$\begin{aligned} \mathbb{E}_{1i}[\bar{a}^*] &= \mu + \frac{\hat{\lambda}\phi_1(\sigma_v^2+\sigma_\eta^2) + (1-\hat{\lambda})\phi_2(\sigma_v^2-\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}(s_{1i}-\mu), \\ \mathbb{E}_{2i}[\bar{a}^*] &= \mu + \frac{\hat{\lambda}\phi_1(\sigma_v^2-\sigma_\eta^2) + (1-\hat{\lambda})\phi_2(\sigma_v^2+\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}(s_{2i}-\mu), \end{aligned} \tag{C2}$$

where $\hat{\lambda} \equiv \mathbb{E}[\frac{N_1}{N_1+N_2}|\langle n_1, n_2\rangle; q_1, q_2]$. Plugging $\mathbb{E}_i[\tilde{v}] = \mu + \frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}(s_i - \mu)$ and equation (C2) in the FOC, the coefficient array (ϕ_1, ϕ_2) is determined by the following system of simultaneous equations obtained from matching coefficients:

$$\begin{cases} \phi_1 = (1-\omega)\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2} + \omega\frac{\hat{\lambda}\phi_1(\sigma_v^2+\sigma_\eta^2)+(1-\hat{\lambda})\phi_2(\sigma_v^2-\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}, \\ \phi_2 = (1-\omega)\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2} + \omega\frac{\hat{\lambda}\phi_1(\sigma_v^2-\sigma_\eta^2)+(1-\hat{\lambda})\phi_2(\sigma_v^2+\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}. \end{cases} \tag{C3}$$

□

Proof of Proposition 3.2.

$$\phi_1(0) = \phi_2(1) = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2)} > 0 \text{ iff } \sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2 > 0$$

$$\phi_1(1) = \phi_2(0) = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2)} = \frac{(1-\omega)\sigma_v^2}{(1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2} > 0$$

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat\lambda} = \frac{8\omega^3(1-\omega)(\sigma_v^2\sigma_\eta^2)^2}{\big((\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat\lambda(1-\hat\lambda)\sigma_v^2\sigma_\eta^2\big)^2} F_1(\hat\lambda) \quad \text{where}$$

$$F_1(\hat\lambda) \equiv \left(\hat\lambda + \frac{\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2}\right)^2 - \frac{(\sigma_v^2-\sigma_\eta^2)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)}{4\omega^2\sigma_v^2\sigma_\eta^4}$$

If $\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2 < 0$, then $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat\lambda} > 0$. Moreover, $\phi_1 < 0$ for $\hat\lambda \in (0, \lambda_1^\dagger)$, and $\phi_1 > 0$ for $\hat\lambda \in (\lambda_1^\dagger, 1)$ where

$$\lambda_1^\dagger \equiv -\frac{\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2}. \tag{C4}$$

Otherwise, if $\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2 \geq 0$, i.e., $\omega \leq \frac{1}{2}$ or $\omega > \frac{1}{2}$ and $(2\omega-1)\sigma_\eta^2 \leq \sigma_v^2 + \sigma_\epsilon^2$, then $\phi_1(\hat\lambda) \geq 0$ for all $\hat\lambda \in [0,1]$, and

(i) if $(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2 - 2\omega\sigma_v^2\big) + 4\omega^2\sigma_v^2\sigma_\eta^2 \geq 0$, then $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat\lambda} > 0$; and

(ii) otherwise, $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat\lambda} < 0$ for $\hat\lambda \in (0, \lambda_1^*)$, and $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat\lambda} > 0$ for $\hat\lambda \in (\lambda_1^*, 1)$ where

$$\lambda_1^* \equiv \frac{\sqrt{(\sigma_v^2-\sigma_\eta^2)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)} - \sigma_v\big(\sigma_v^2+(1-2\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)}{2\omega\sigma_v\sigma_\eta^2}. \tag{C5}$$

The condition for this case further reduces to $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$ and $\omega > \omega^T$ where

$$\omega^T \equiv \left(3\sigma_v^2 + \sigma_\eta^2 - \sqrt{(\sigma_v^2-\sigma_\eta^2)(9\sigma_v^2-\sigma_\eta^2)}\right)\frac{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}{8\sigma_v^2\sigma_\eta^2} \in \left(\frac{1}{3}, 1\right). \tag{C6}$$

Since $\phi_2(\hat{\lambda})$ is symmetric to $\phi_1(\hat{\lambda})$ about $\hat{\lambda} = \frac{1}{2}$, the proof for ϕ_2 is omitted.[1] Instead, the expressions for $\lambda_2^\dagger$ and λ_2^* are specified as follows:

$$\lambda_2^\dagger \equiv \frac{\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2} \tag{C7}$$

$$\lambda_2^* \equiv \frac{\sigma_v\big(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2\big) - \sqrt{(\sigma_v^2 - \sigma_\eta^2)(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}}{2\omega\sigma_v\sigma_\eta^2}. \tag{C8}$$

□

Proof of Corollary 3.1. The proof follows immediately by plugging in $\omega = 0$ and $\omega = 1$, respectively, into equations (3.5) – (3.6), and observing that ϕ_i's are independent of $\hat{\lambda}$ in both cases. □

Proof of Lemma 3.1. Taking the first-order derivative of ϕ_1' expressed in equation (3.10) with respect to λ' yields

$$\begin{aligned}
\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= \omega\frac{\lambda\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda}(\sigma_v^2+\sigma_\eta^2) + (1-\lambda)\frac{\mathrm{d}\phi_2}{\mathrm{d}\lambda}(\sigma_v^2-\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2} \\
&= \frac{2\omega(1-\omega)\sigma_v^2\sigma_\eta^2}{\big((\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\lambda(1-\lambda)\sigma_v^2\sigma_\eta^2\big)^2} G(\lambda) \quad \text{where} \\
G(\lambda) &\equiv 4\omega^2\sigma_v^2\sigma_\eta^2 F_1 - \left((1-\omega)(\sigma_v^2+\sigma_\eta^2) + \sigma_\epsilon^2 + 4\omega^2\lambda(1-\lambda)\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}\right) \\
&\qquad \big((1-\omega)(\sigma_v^2+\sigma_\eta^2) + \sigma_\epsilon^2 + 2\omega\lambda\sigma_v^2\big)
\end{aligned}$$

$$G(0) = \omega(\sigma_\eta^2 - \sigma_v^2)\big((1+\omega)\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)$$

[1] More formally,

$$\begin{aligned}
\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} &= \frac{8\omega^3(1-\omega)(\sigma_v^2\sigma_\eta^2)^2}{\big((\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2\big)^2} F_2(\hat{\lambda}) \quad \text{where} \\
F_2(\hat{\lambda}) &\equiv -\left(\hat{\lambda} - \frac{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}{2\omega\sigma_\eta^2}\right)^2 + \frac{(\sigma_v^2-\sigma_\eta^2)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)}{4\omega^2\sigma_v^2\sigma_\eta^4}.
\end{aligned}$$

$$\implies \operatorname{sgn}(G(0)) = \operatorname{sgn}\left(\sigma_\eta^2 - \sigma_v^2\right)$$

$$G(1) = \omega(\sigma_\eta^2 + \sigma_v^2)\big((1+\omega)\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big) > 0$$

$$\frac{\mathrm{d}G}{\mathrm{d}\lambda} = \frac{2\omega\sigma_v^2}{\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2} G^1(\lambda) \quad \text{where}$$

$$G^1(\lambda) \equiv (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2 - 4\omega(1-\lambda)\sigma_\eta^2\big) - 2\omega(1-2\lambda)\sigma_\eta^2\big((1-\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2\big) - 4\omega^2\lambda(2-3\lambda)\sigma_v^2\sigma_\eta^2$$

$$G^1(0) = (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2 - 4\omega\lambda\sigma_\eta^2\big) - 2\omega\sigma_\eta^2\big((1-\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2\big)$$

$$G^1(1) = (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2\big) + 2\omega\sigma_\eta^2\big((1-\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2\big) + 4\omega^2\sigma_v^2\sigma_\eta^2 > 0$$

$$\frac{\mathrm{d}G^1}{\mathrm{d}\lambda} = 4\omega\sigma_\eta^2 G^2(\lambda) \quad \text{where } G_2(\lambda) = \big(3\omega(2\lambda-1)+2\big)\sigma_v^2 + (2-\omega)\sigma_\eta^2 + 2\sigma_\epsilon^2$$

$$G^2(0) = (2-3\omega)\sigma_v^2 + (2-\omega)\sigma_\eta^2 + 2\sigma_\epsilon^2$$

$$G^2(1) = (2+3\omega)\sigma_v^2 + (2-\omega)\sigma_\eta^2 + 2\sigma_\epsilon^2 > 0$$

1. If $G^2(0) \geq 0$, i.e., $\sigma_\eta^2 \geq \frac{(3\omega-2)\sigma_v^2 - 2\sigma_\epsilon^2}{2-\omega}$, then $\frac{\mathrm{d}G^1}{\mathrm{d}\lambda} \geq 0$ and
   (a) if $G^1(0) < 0$, i.e., $\sigma_\eta^2 > \frac{\sqrt{9\sigma_\epsilon^4 + 8(\sigma_v^2+\sigma_\epsilon^2)(17\sigma_v^2 - \sigma_\epsilon^2)} - 3\sigma_\epsilon^2}{4}$ and $\omega > \frac{\left(5\sigma_\eta^2 - \sigma_v^2 - \sqrt{(\sigma_\eta^2-\sigma_v^2)(17\sigma_\eta^2-\sigma_v^2)}\right)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)}{4\sigma_\eta^2(\sigma_v^2+\sigma_\eta^2)}$, then $G(\lambda) > 0$ for all $\lambda \in (0,1)$;
   (b) otherwise, $G^1(\lambda) \geq 0$ for all $\lambda \in (0,1)$, and
      i. if $G(0) < 0$, i.e., $\sigma_\eta^2 < \sigma_v^2$, then $G(\lambda) < 0$ for $\lambda \in (0, \lambda_1^\ddagger)$, and $G(\lambda) > 0$ for $\lambda \in (\lambda_1^\ddagger, 1)$, where $\lambda_1^\ddagger$ is the unique solution to $G(\lambda_1^\ddagger) = 0$ on $(0,1)$.
      ii. otherwise, $G(\lambda) > 0$ for all $\lambda \in (0,1)$.
2. Otherwise, if $G^2(0) < 0$, i.e., $\sigma_\eta^2 < \frac{(3\omega-2)\sigma_v^2 - 2\sigma_\epsilon^2}{2-\omega}$, then $G^1(\lambda) > 0$ for all $\lambda \in (0,1)$, and we go back to case 1b.

To conclude, if $\sigma_\eta^2 < \sigma_v^2$, then $\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_1^\ddagger)$, and $\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_1^\ddagger, 1)$; otherwise, $\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0$ for any $\lambda \in (0,1)$. Moreover, since $G(\lambda_1^*) < 0$ and $G(\frac{1}{2}) > 0$, $\lambda_1^* < \lambda_1^\ddagger < \frac{1}{2}$.

The analysis for $\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}$ is similar and thus omitted. $\square$

Proof of Proposition 3.3. Suppose $\hat{q}_1\hat{q}_2 > 0$, then the expected population composition is given by $\hat{\lambda} = \mathbb{E}[\frac{N_1}{N_1+N_2}|\langle n_1, n_2\rangle; q_1, q_2] = \frac{\frac{n_1}{q_1}}{\frac{n_1}{\hat{q}_1}+\frac{n_2}{\hat{q}_2}}$. I first show that the incentive compatibility constraints can indeed be determined by the sign of $\frac{\mathrm{d}\mathbb{E}_i[u_i']}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}$ where λ and λ' are the other agents' equilibrium belief and off-equilibrium belief following a deviation, respectively.

Consider agent i whose type is $t_i = 1$ and equilibrium disclosure is $m_i^* = t_i = 1$. Let $\hat{\lambda} = \frac{\frac{n_1}{q_1}}{\frac{n_1}{\hat{q}_1}+\frac{n_2}{\hat{q}_2}}$, then by deviating to $m_i = \emptyset$, $\hat{\lambda}' = \frac{\frac{n_1-1}{\hat{q}_1}}{\frac{n_1-1}{\hat{q}_1}+\frac{n_2}{\hat{q}_2}}$.[2] Let $\delta \equiv \delta_0(\hat{\lambda}-\hat{\lambda}')$ where $\delta_0 > 0$ can be arbitrarily small, since $\hat{\lambda} > \hat{\lambda}'$ and $\lim\limits_{N\to\infty}\hat{\lambda} = \lim\limits_{N\to\infty}\hat{\lambda}' = \lambda$, no-deviation of agent i from $m_i^* = 1$ to $m_i = \emptyset$ requires

$$\lim_{N\to\infty}\mathbb{E}_1[u_{1i}] - \mathbb{E}_1[u_{1i}'] \geq \delta \iff \lim_{\hat{\lambda}'\to\hat{\lambda}^-}\frac{\mathbb{E}_1[u_{1i}] - \mathbb{E}_1[u_{1i}']}{\hat{\lambda}-\hat{\lambda}'} \geq \delta_0 \iff \frac{\mathrm{d}\mathbb{E}_1[u_{1i}']}{\mathrm{d}\hat{\lambda}'}\bigg|_{\hat{\lambda}'=\hat{\lambda}=\lambda} > 0.$$

Similarly, for agent i whose type is $t_i = -1$ and equilibrium disclosure is $m_i^* = t_i = -1$, no deviation to $m_i = \emptyset$ requires

$$\lim_{N\to\infty}\mathbb{E}_2[u_{2i}] - \mathbb{E}_2[u_{2i}'] \geq \delta \iff \lim_{\hat{\lambda}'\to\hat{\lambda}+}\frac{\mathbb{E}_2[u_{2i}] - \mathbb{E}_2[u_{2i}']}{\hat{\lambda}-\hat{\lambda}'} < 0 \iff \frac{\mathrm{d}\mathbb{E}_2[u_{2i}']}{\mathrm{d}\hat{\lambda}'}\bigg|_{\hat{\lambda}'=\hat{\lambda}=\lambda} < 0.$$

Otherwise, the agent prefers nondisclosure.

The only disclosure equilibrium is full-disclosure equilibrium where $m_i^* = t_i$ and $q_1 = q_2 = 1$. Notice that such a disclosure equilibrium requires $\phi_1\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\phi_2\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ (see IC constraints). Combining the results from *Proposition 3.2* and *Lemma 3.1*, we have

[2]Suppose disclosure is not constrained to be truthful, i.e., lying is also allowed, denote the disclosure probability as $q_{ij} > 0$ where i is the type of agent, and j is his disclosed type, then $\hat{\lambda} = \frac{\frac{\hat{q}_{22}n_1-\hat{q}_{21}n_2}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}}{\frac{(\hat{q}_{22}-\hat{q}_{12})n_1+(\hat{q}_{11}-\hat{q}_{21})n_2}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}}$. By deviating from $m_i^* = 1$ to $m_i = \emptyset$ or $m_i = -1$,

$$\hat{\lambda}' = \frac{\frac{\hat{q}_{22}(n_1-1)-\hat{q}_{21}n_2}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}}{\frac{(\hat{q}_{22}-\hat{q}_{12})(n_1-1)+(\hat{q}_{11}-\hat{q}_{21})n_2}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}} \text{ and } \hat{\lambda}'' = \frac{\frac{\hat{q}_{22}(n_1-1)-\hat{q}_{21}(n_2+1)}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}}{\frac{(\hat{q}_{22}-\hat{q}_{12})(n_1-1)+(\hat{q}_{11}-\hat{q}_{21})(n_2+1)}{\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21}}},$$

respectively. Suppose $\hat{q}_{11}\hat{q}_{22}-\hat{q}_{12}\hat{q}_{21} > 0$, then $\hat{\lambda} > \hat{\lambda}' > \hat{\lambda}''$ and $\lim\limits_{N\to\infty}\hat{\lambda} = \lim\limits_{N\to\infty}\hat{\lambda}' = \lim\limits_{N\to\infty}\hat{\lambda}'' = \lambda$, implying $m_i = \emptyset$ is a strictly dominated strategy whenever $\frac{\mathrm{d}\mathbb{E}_i[u_i']}{\mathrm{d}\hat{\lambda}'}\Big|_{\hat{\lambda}'=\hat{\lambda}=\lambda} \neq 0$.

(i) if $(2\omega-1)\sigma_\eta^2 > \sigma_v^2+\sigma_\epsilon^2$, then $\phi_i \frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_i^\dagger)$, $\phi_i \frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_i^\dagger, 1)$, and hence there exists a full-disclosure equilibrium iff $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$;

(ii) if $\sigma_\eta^2 < \sigma_v^2$, then $\phi_i \frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_i^\ddagger)$, $\phi_i \frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_i^\ddagger, 1)$, and hence there exists a full-disclosure equilibrium iff $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$;

(iii) otherwise, $\phi_1 \frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\phi_2 \frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$, and hence there always exists a full-disclosure equilibrium.

□

Proof of Corollary 3.2. For the first part where $\Delta^\dagger = \lambda_2^\dagger - \lambda_1^\dagger$, the proof follows immediately by inspecting equations (C4) and (C7). For the second part where $\Delta^\ddagger = \lambda_2^\ddagger - \lambda_1^\ddagger$, note that $\lambda_1^\ddagger + \lambda_2^\ddagger = \frac{1}{2}$, and hence

$$\operatorname{sgn}\left(\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}x}\right) = -\operatorname{sgn}\left(\frac{\mathrm{d}\lambda_1^\ddagger}{\mathrm{d}x}\right) \text{ for } x \in \{\sigma_v^2, \sigma_\eta^2, \sigma_\epsilon^2, \omega\}.$$

Recall from the proof of *Lemma 3.1* that $\frac{\partial G}{\partial \lambda} > 0$. Therefore, by implicit function theorem,

$$\begin{aligned} \operatorname{sgn}\left(\frac{\mathrm{d}\lambda_1^\ddagger}{\mathrm{d}x}\right) &= \operatorname{sgn}\left(-\frac{\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial \lambda}}\Bigg|_{\lambda=\lambda_1^\ddagger}\right) \\ \Longrightarrow \operatorname{sgn}\left(\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}x}\right) &= \operatorname{sgn}\left(\frac{\partial G}{\partial x}\bigg|_{\lambda=\lambda_1^\ddagger}\right). \end{aligned}$$

In particular, since $\sigma_\eta^2 < \sigma_v^2$ and $\lambda_1^\ddagger \in (0, \frac{1}{2})$, we have

(i) $\frac{\partial G}{\partial \sigma_v^2}\Big|_{\lambda=\lambda_1^\ddagger} < 0$: Note that $\frac{\partial^2 G}{\partial (\sigma_v^2)^2} < 0$, and hence,

$$\frac{\partial G}{\partial \sigma_v^2}\bigg|_{\lambda=\lambda_1^\ddagger} < \frac{\partial G}{\partial \sigma_v^2}\bigg|_{\sigma_v^2=\sigma_\eta^2,\ \lambda=\lambda_1^\ddagger} = -\omega(2\sigma_\eta^2 + \sigma_\epsilon^2) < 0.$$

(ii) $\frac{\partial G}{\partial \sigma_\eta^2}\Big|_{\lambda=\lambda_1^\ddagger} > 0$: Note that $\frac{\partial^2 G}{\partial(\sigma_\eta^2)^2} > 0$ and hence,

$$\frac{\partial G}{\partial \sigma_\eta^2}\bigg|_{\lambda=\lambda_1^\ddagger} > \frac{\partial G}{\partial \sigma_\eta^2}\bigg|_{\sigma_\eta^2=0,\ \lambda=\lambda_1^\ddagger} = \omega\left((1-\omega)\sigma_v^2 + \sigma_\epsilon^2\right) > 0.$$

(iii) $\frac{\partial G}{\partial \sigma_\epsilon^2}\Big|_{\lambda=\lambda_1^\ddagger} > 0$: Note that $\frac{\partial^2 G}{\partial(\sigma_\epsilon^2)^2} > 0$ and hence,

$$\frac{\partial G}{\partial \sigma_\epsilon^2}\bigg|_{\lambda=\lambda_1^\ddagger} > \frac{\partial G}{\partial \sigma_\epsilon^2}\bigg|_{\sigma_\epsilon^2=0,\ \lambda=\lambda_1^\ddagger} = \omega\frac{\begin{array}{c}(\sigma_v^2+\sigma_\eta^2)^2(\sigma_\eta^2-(1-2\lambda_1^\ddagger)\sigma_v^2) \\ -\,4\omega^2\lambda_1^\ddagger(1-\lambda_1^\ddagger)\sigma_v^2\sigma_\eta^2((1-2\lambda_1^\ddagger)\sigma_v^2+\sigma_\eta^2)\end{array}}{(\sigma_v^2+\sigma_\eta^2)^2} > 0.$$

In addition, $\lambda_1^\ddagger > \lim\limits_{\sigma_\epsilon^2\to\infty}\lambda_1^\ddagger = \frac{1}{2}\left(1-\frac{\sigma_\eta^2}{\sigma_v^2}\right)$.

(iv) $\frac{\partial G}{\partial \omega}\Big|_{\lambda=\lambda_1^\ddagger} < 0$: Note that $\frac{\partial^3 G}{\partial\omega^3} > 0$ and hence,

$$\frac{\partial G}{\partial \omega}\bigg|_{\lambda=\lambda_1^\ddagger} < \max\left\{\frac{\partial G}{\partial \omega}\bigg|_{\omega=0,\ \lambda=\lambda_1^\ddagger}, \frac{\partial G}{\partial \omega}\bigg|_{\omega=1,\ \lambda=\lambda_1^\ddagger}\right\} = 0.$$

□

Proof of Proposition 3.4. Notice that

$$\begin{aligned}\mathbb{E}[(\tilde{v}-\bar{a})^2] &= \big(\lambda\phi_1+(1-\lambda)\phi_2-1\big)^2\sigma_v^2 + \big(\lambda\phi_1-(1-\lambda)\phi_2\big)^2\sigma_\eta^2 \\ \implies \operatorname{sgn}\frac{\mathrm{dAE}}{\mathrm{d}\lambda} &= -\operatorname{sgn}\frac{\mathrm{d}\mathbb{E}[(\tilde{v}-\bar{a})^2]}{\mathrm{d}\lambda} = \operatorname{sgn}(1-2\lambda)(\lambda-\lambda_1^{\mathrm{AE}})(\lambda-\lambda_2^{\mathrm{AE}}),\end{aligned}$$

where $\lambda^{\mathrm{AE}} = \frac{1}{2}\left(1 \pm \sqrt{\frac{\left(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\right)\Delta}{\omega^2\sigma_v^2\sigma_\eta^2\left((3-\omega)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)-2\omega\sigma_\eta^2\right)}}\right)$ and $\Delta = (\sigma_v^2+\sigma_\epsilon^2)\Big(-(1-\omega)^2\sigma_v^2 - 2\sigma_\eta^2 + (2\omega-1)\sigma_\epsilon^2\Big) + \sigma_\eta^2\Big(\omega(5-\omega)\sigma_v^2 + (1-\omega)(2\omega-1)\sigma_\eta^2 + 2\omega(3-\omega)\sigma_\epsilon^2\Big)$.

Further observe that $\Delta \le 0$ if and only if $\omega \le \hat{\omega}$ where

$$\hat{\omega} \equiv \frac{2(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)}{2\sigma_v^2+3\sigma_\eta^2+3\sigma_\epsilon^2+\sqrt{(\sigma_\eta^2+\sigma_\epsilon^2)(8\sigma_v^2+\sigma_\eta^2+9\sigma_\epsilon^2)}} \in \left(\frac{1}{3},1\right).$$

In this case, AE is inverted U-shaped in λ, with $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} > 0$ for $\lambda \in (0, \frac{1}{2})$, and $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} < 0$ for $\lambda \in (\frac{1}{2}, 1)$.

When $\omega > \hat{\omega}$, $\lambda^{\mathrm{AE}} \in (0,1)$ if and only if $\sigma_v^2 > \hat{\sigma}_v^2$ where

$$\hat{\sigma}_v^2 = \frac{\big(\omega(5-\omega)-2\big)\sigma_\epsilon^2 - 2(1-\omega)^3\sigma_\eta^2 + \omega\sqrt{\big(2(1-\omega)^2\sigma_\eta^2 + (1+\omega)\sigma_\epsilon^2\big)^2 - 8\omega(1-\omega)^2\sigma_\eta^2\sigma_\epsilon^2}}{2(1-\omega)^2}.$$

In this case, AE is double-humped in λ, with $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} > 0$ for $\lambda \in (0, \lambda_1^{\mathrm{AE}}) \cup (\frac{1}{2}, \lambda_2^{\mathrm{AE}})$, and $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} < 0$ for $\lambda \in (\lambda_1^{\mathrm{AE}}, \frac{1}{2}) \cup (\lambda_2^{\mathrm{AE}}, 1)$. Otherwise, AE is U-shaped in λ, with $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} < 0$ for $\lambda \in (0, \frac{1}{2})$, and $\frac{\mathrm{dAE}}{\mathrm{d}\lambda} > 0$ for $\lambda \in (\frac{1}{2}, 1)$. $\square$

Proof of Proposition 3.5. By equation (3.12), we have

$$\begin{aligned}
\mathrm{Var}(P) &= \frac{\big(\lambda\phi_1 + (1-\lambda)\phi_2\big)^2\sigma_v^4}{\big(\lambda\phi_1 + (1-\lambda)\phi_2\big)^2\sigma_v^2 + \big(\lambda\phi_1 - (1-\lambda)\phi_2\big)^2\sigma_\eta^2} = \frac{\sigma_v^2}{1 + \big(\frac{\lambda\phi_1 - (1-\lambda)\phi_2}{\lambda\phi_1 + (1-\lambda)\phi_2}\big)^2 \frac{\sigma_\eta^2}{\sigma_v^2}} \\
\implies \mathrm{sgn}\frac{\mathrm{dVar}(P)}{\mathrm{d}\lambda} &= -\mathrm{sgn}\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\lambda\phi_1 - (1-\lambda)\phi_2}{\lambda\phi_1 + (1-\lambda)\phi_2}\right)^2 \\
&= \mathrm{sgn}(1-2\lambda)\left(\sigma_v^2 + (1 - 2\omega(1-2\lambda(1-\lambda))\sigma_\eta^2 + \sigma_\epsilon^2\right)
\end{aligned}$$

Solving for $\sigma_v^2 + (1 - 2\omega(1-2\lambda(1-\lambda))\sigma_\eta^2 + \sigma_\epsilon^2 = 0$ yields $\lambda = \frac{\omega\sigma_\eta^2 \pm \sqrt{\omega\sigma_\eta 2\left(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\right)}}{2\omega\sigma_\eta^2}$. Moreover, notice that

$$\mathrm{Var}(P)|_{\lambda=\frac{1}{2}} - \mathrm{Var}(P)|_{\lambda=0} = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\eta^2} = \frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2 + \sigma_\eta^2} > 0.$$

The rest of the proof follows immediately by noting that $\frac{\omega\sigma_\eta^2 \pm \sqrt{\omega\sigma_\eta 2\left(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\right)}}{2\omega\sigma_\eta^2} \in (0,1)$ if and only if $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$. $\square$

C.2. Ex-Post Disclosure

Suppose disclosure is made *after* the realization of private signals, then the left-hand side of the IC constraints for disclosure becomes

$$\left.\frac{\mathrm{d}\mathbb{E}_i[u_i']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = a_i \left.\frac{\mathrm{d}a_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = \big(\mu + \phi_i(s_i - \mu)\big)(s_i - \mu)\left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda}. \tag{C9}$$

There exists a disclosure equilibrium only when the above expression satisfies the single-crossing condition with a positive probability for both types, which is a less stringent condition compared to the *ex-ante* IC constraints due to the large population.

As implied by the above equation, how an agent's disclosure choice influences his expected utility is not only determined by the direction of its impact on his reliance on the private signal as in the *ex-ante* disclosure, but is also affected by the realization of his private signal. One special case of the *ex-post* disclosure is when the prior mean of the fundamental is unbiased, i.e., $\mu = 0$. In this case, an agent's equilibrium action is proportional to his private signal. The more an agent relies on his private signal, the further away his action is from zero, and hence the larger his expected utility is. Consequently, the *ex-post* disclosure equilibrium replicates the *ex-ante* disclosure equilibrium, except for those agents whose private signal is realized at $s_i = 0$ as their equilibrium actions are always zero whatsoever.

Proposition C1. *When $\mu = 0$, agents' ex-post equilibrium disclosure decisions are almost identical to their ex-ante disclosure decisions as described in Proposition 3.3. In particular, those agents whose private signal is realized at $s_i = 0$ never disclose.*

Proof. Plugging in $\mu = 0$ to equation (C9) gives us

$$\left.\frac{\mathrm{d}\mathbb{E}_i[u_i']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = s_i^2\ \phi_i \left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda}, \tag{C10}$$

the sign of which is the same as the *ex-ante* disclosure case except for agents whose private signal is realized at $s_i = 0$. Since $\mathrm{Prob}(s_i \neq 0 | t_i) = 1$, the only disclosure equilibrium is full-disclosure equilibrium where $m_i^* = t_i$ iff $s_i \neq 0$ and $q_1 = q_2 = 1$. The rest of the proof follows immediately from *Proposition 3.3*. $\square$

When $\mu \neq 0$, the *ex-post* disclosure is contingent on the realization of private signals. Disclosure (nondisclosure) delivers more information beyond just the agent's type when the total number of agents is small. Assuming a large number of agents is a way to circumvent this issue and simplify the analysis in the sense that the *ex-post* disclosure strategy is summarized as a disclosure probability on the aggregate level for each type so that the observed disclosures are only used to calculate the underlying population of each type but not update about the disclosing agent's private signal. In addition, the law of large numbers makes the division of the number of disclosures of each type by its associated disclosure probability equal to the true underlying population of its corresponding type.

The following proposition characterizes the *ex-post* disclosure equilibrium in the case of $\mu > 0$ without loss of generality.

Proposition C2. *When $\mu > 0$, agents of both types partially disclose in the disclosure equilibrium. More specifically, there exist $\lambda_1^\dagger, \lambda_1^\ddagger \in (0, \frac{1}{2})$ and $\lambda_2^\dagger, \lambda_2^\ddagger \in (\frac{1}{2}, 1)$ such that:*

(i) if $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, then

(a) when $\lambda < \lambda_1^\dagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (0, -\frac{\mu}{\phi_1})$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (-\infty, -\frac{\mu}{\phi_2}) \cup (0, \infty), q_1 = \Phi_1(-\frac{\mu}{\phi_1}) - \Phi_1(0), q_2 = \Phi_2(-\frac{\mu}{\phi_2}) - \Phi_2(0) + 1$;

(b) when $\lambda = \lambda_1^\dagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (0, \infty)$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (-\infty, -\frac{\mu}{\phi_2}) \cup (0, \infty), q_1 = 1 - \Phi_1(0), q_2 = \Phi_2(-\frac{\mu}{\phi_2}) - \Phi_2(0) + 1$;

(c) when $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$, $m_i = 1$ iff $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty), q_i = \Phi_i(-\frac{\mu}{\phi_i}) - \Phi_i(0) + 1$;

(d) when $\lambda = \lambda_2^\dagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (-\infty, -\frac{\mu}{\phi_1}) \cup (0, \infty)$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (0, \infty), q_1 = \Phi_1(-\frac{\mu}{\phi_1}) - \Phi_1(0) + 1, q_2 = 1 - \Phi_2(0)$;

(e) when $\lambda > \lambda_2^\dagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (-\infty, -\frac{\mu}{\phi_1}) \cup (0, \infty)$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (0, -\frac{\mu}{\phi_2}), q_1 = \Phi_1(-\frac{\mu}{\phi_1}) - \Phi_1(0) + 1, q_2 = \Phi_2(-\frac{\mu}{\phi_2}) - \Phi_2(0)$;

(ii) if $\sigma_\eta^2 < \sigma_v^2$, then

(a) when $\lambda < \lambda_1^\ddagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (-\frac{\mu}{\phi_1}, 0)$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (-\infty, -\frac{\mu}{\phi_2}) \cup (0, \infty)$, $q_1 = \Phi_1(0) - \Phi_1(-\frac{\mu}{\phi_1})$, $q_2 = \Phi_2(-\frac{\mu}{\phi_2}) - \Phi_2(0) + 1$;

(b) when $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$, $m_i = t_i$ iff $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty)$, $q_i = \Phi_i(-\frac{\mu}{\phi_i}) - \Phi_i(0) + 1$;

(c) when $\lambda > \lambda_2^\ddagger$, $m_{1i} = 1$ iff $s_{1i} - \mu \in (-\infty, -\frac{\mu}{\phi_1}) \cup (0, \infty)$, $m_{2i} = -1$ iff $s_{2i} - \mu \in (-\frac{\mu}{\phi_2}, 0)$, $q_1 = \Phi(-\frac{\mu}{\phi_1}) - \Phi_1(0) + 1$, $q_2 = \Phi_2(0) - \Phi_2(-\frac{\mu}{\phi_2})$;

(d) when $\lambda = \lambda_1^\ddagger$ or $\lambda = \lambda_2^\ddagger$, there does not exist a disclosure equilibrium;

(iii) otherwise, $m_i = t_i$ iff $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty)$, $q_i = \Phi_i(-\frac{\mu}{\phi_i}) - \Phi_i(0) + 1$.

Proof. Since the probabilities of $\big(\mu + \phi_i(s_i - \mu)\big)(s_i - \mu) > 0$ and $\big(\mu + \phi_i(s_i - \mu)\big)(s_i - \mu) < 0$ are both strictly positive for any ϕ_i, the only disclosure equilibrium is partial-disclosure equilibrium. Recall that such a disclosure equilibrium requires $\frac{\mathrm{d}\mathbb{E}_{1i}[u'_{1i}]}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\frac{\mathrm{d}\mathbb{E}_{2i}[u'_{2i}]}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$. Denoting by $\Phi_1(\cdot)$ (resp. $\Phi_2(\cdot)$) the cumulative distribution function of a normal distribution with mean $v + \eta$ (resp. $v - \eta$) and variance σ_ϵ^2, the rest of the proof follows immediately after *Proposition 3.2* and *Lemma 3.1*. □

When $\mu \neq 0$, the only disclosure equilibrium is partial-disclosure equilibrium, i.e., $q_1, q_2 < 1$.[3] Nevertheless, the disclosure decision is deterministic for each agent and depends on the realization of their private signals. That is, the notion of disclosure probability is only on the aggregate level for each type but not on the individual level. To be more precise, with the objective of taking a more extreme action, when an agent receives an optimistic signal, i.e., $s_i - \mu > 0$, he prefers to disclose his own type when (i) his action is positive and such a disclosure induces him to use more of his signal, *or* (ii) his action is negative and such a

[3]If lying is allowed, then the partial-disclosure equilibrium will become a full-disclosure equilibrium with partial truth-telling in the sense that q_i will be the probability of truthful disclosure and $1 - q_i$ will be the probability of lying (see footnotes 13 and 2). The only exception is those agents whose private signal is not informative at all, i.e., $s_i = \mu$, and those agents whose private signal leads them to take a zero-valued action, i.e., $s_i = \mu - \frac{\mu}{\phi_i}$. Their equilibrium actions do not respond to changes in the other agents' posterior belief, and hence they strictly prefer nondisclosure in the presence of a disclosure cost. Nevertheless, since these two events happen with a probability measure 0, this exception does not affect our prediction of full disclosure from both types.

disclosure induces him to use less of his signal. Conversely, when he receives a pessimistic signal, i.e., $s_i - \mu < 0$, he prefers to disclose his own type when (iii) his action is positive and such a disclosure induces him to use less of his signal, *or* (iv) his action is negative and such a disclosure induces him to use more of his signal.

Similar to the case where $\mu = 0$, the satisfaction of the above conditions is contingent on parameter values and potentially also on the expected population composition. In the first case where $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon$, an increase in the others' posterior belief about the proportion of his type induces an agent to use more of his private signal. Thus, he prefers to disclose whenever his signal is optimistic and induces a positive action, and whenever his signal is pessimistic and induces a negative action. When the population composition is expected to be relatively unbalanced, i.e., $\lambda \leq \lambda_1^\dagger$ or $\lambda \geq \lambda_2^\dagger$, agents of the supermajority type disclose when the realization of their private signals is optimistic or extremely pessimistic, i.e., $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty)$. Meanwhile, their minority counterparts use their private signals in an opposite way and thus disclose only with moderately optimistic private signals instead, i.e., $s_i - \mu \in (0, -\frac{\mu}{\phi_i})$. Otherwise, agents of both types disclose with optimistic or extremely pessimistic private signals.

In the second case where $\sigma_\eta^2 < \sigma_v^2$, all agents use their private signals in a positive way. Nevertheless, the convexity of the informativeness of private signals about the average action can lead the agent of the extreme minority type to rely less on his private signal following an increase in the others' posterior belief about the proportion of his type. Whenever this is the case, an agent prefers to disclose only when his signal is optimistic but induces a negative action, and when his signal is pessimistic but induces a positive action. Consequently, when the population composition is expected to be relatively unbalanced, i.e., $\lambda < \lambda_1^\ddagger$ or $\lambda > \lambda_2^\ddagger$, agents of the super-majority type disclose when the realization of their private signals is optimistic or extremely pessimistic, i.e., $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty)$, whereas their minority counterparts do so only with moderately pessimistic private signals instead, i.e., $s_i - \mu \in (-\frac{\mu}{\phi_i}, 0)$. Otherwise, agents of both types disclose with optimistic or extremely pessimistic private signals. Moreover, note that when $\lambda = \lambda_i^{\ddagger}$, the equilibrium actions of the minority type do not respond to changes in the other agents' posterior belief. Agents of the minority type hence strictly prefer nondisclosure in the presence of a disclosure cost, which results in nondisclosure of their majority counterparts as well. In other words, there is a discontinuity in disclosure probabilities at $\lambda = \lambda_i^{\ddagger}$ where disclosure probabilities of both types suddenly drop to $q_i = 0$ and no-disclosure equilibrium is the only equilibrium.

Lastly, in the third case, all agents always use their private signals in a positive way, and such reliance on private signals monotonically increases in the others' posterior belief about the proportion of their types. As a result, agents of both types disclose with optimistic or extremely pessimistic private signals, i.e., $s_i - \mu \in (-\infty, -\frac{\mu}{\phi_i}) \cup (0, \infty)$.

The above intuition holds for both cases where $\mu > 0$ and where $\mu < 0$, but the disclosure choice of each agent might be different depending on whether the prior mean is positive or negative due to changes in relative positions of private signals and actions to zero. Thus, the equilibrium disclosure probabilities of each type also differ between the two cases. In particular, the disclosure probability of one type is generally larger when the aggregate signal of that type goes in the same direction as the prior. For example, the disclosure probabilities of type 1 under the two cases $q_{1,\mu>0} > q_{1,\mu<0}$ when their aggregate signal $\bar{s}_1 = v + \eta$ is positive, and vice versa. The only exception happens in the second case where the disclosure probability of the extreme minority type becomes smaller when their aggregate signal goes in the same direction as the prior.

Comparing *Proposition C2* with *Proposition C1*, we find that the sign of the prior mean leads to a stark contrast in the disclosure equilibrium. In particular, a nonzero prior mean gives rise to a partial-disclosure equilibrium instead of the full-disclosure equilibrium. Interpreting the prior as a belief based on some public information about the fundamental, a zero prior translates to an unbiased public disclosure in the sense that it does not shift agents' posterior belief to either direction. Hence, the private signal realization does not alter an agent's disclosure incentive. On the other hand, a positive (negative) public disclosure shifts the posterior belief upwards (downwards). The induced posterior belief can be consistent or contradictory to each agent's private signal depending on its realization, which gives rise to different disclosure incentives even among agents of the same type. Consequently, partial disclosure arises in equilibrium. More importantly, the disclosure probabilities of the two types are positive and continuous, which results in unraveling of the population composition, almost everywhere with a nonzero prior mean.

A zero prior can be thought of as a limiting case of the more general case with a nonzero prior. The only diversion is at $\lambda = \lambda_i^\dagger$ when $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$: *Proposition C2* predicts $q_i = 1 - \Phi_i(0)$ for the minority type and $q_j = 1$ for the majority type, whereas *Proposition C1* prescribes that there is nondisclosure of either type. The reason for such divergence lies in the sign of action which in turn translates to the sign of μ. When $\lambda = \lambda_i^\dagger$, agents of the minority type do not use their private signals at all and equation (C9) reduces to

$$\left.\frac{\mathrm{d}\mathbb{E}_i[u_i']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} = \mu(s_i - \mu)\left.\frac{\mathrm{d}\phi_i'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda}.$$

Therefore, while a zero prior leads agents of the minority type to strictly prefer nondisclosure, a prior approaching zero from either direction still provides some agents with the incentive to disclose.

One caveat of the above analysis, however, is that it relies on two strong assumptions beyond rational expectations. First, there is a mediator who commits to providing agents with the actual disclosure probabilities of the two types *after* disclosures are made.[4] This assumption guarantees an unbiased posterior belief about population composition with a large number of agents. Otherwise, each agent will use his private signal to conjecture the disclosure probabilities. As a result, agents' conjectured disclosure probabilities are not only different from each other, but also biased away from the actual ones in general.[5] This will

[4]We may think of the mediator as the social media platform whose algorithm is able to infer the true identity of each agent based on the other available information.

[5]See Morgan and Stocken (2008) for a similar prediction that ignoring constituents' strategic behavior yields biased estimators in polls. Nevertheless, in our setting, there potentially exists an exception where the disclosure probabilities of the two types equal to each other so that a naïve estimator ignoring strategic

prevent the existence of linear equilibria in the investment stage in general and renders the model untractable. Second, the agents are naïve in the sense that they cannot uncover the realizations of the fundamental and the common noise from the actual disclosure probabilities of the two types provided by the mediator. In other words, they only use the actual disclosure probabilities to update about the underlying population composition, but not to learn about the state of nature.

Corollary C1. *When $\mu \neq 0$, the equilibrium ex-post disclosure probability can be non-monotonic in λ when $\sigma_\eta^2 < \sigma_v^2$. More specifically,*

(i) if $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$ and $\omega > \omega^T$, then $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ for $\lambda \in (0, \lambda_1^*)$, $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} < 0$ for $\lambda \in (\lambda_1^*, \lambda_1^\ddagger)$, $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ for $\lambda \in (\lambda_1^\ddagger, 1)$, and $\frac{\mathrm{d}q_2}{\mathrm{d}\lambda} < 0$ for $\lambda \in (0, \lambda_2^\ddagger)$, $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ for $\lambda \in (\lambda_2^\ddagger, \lambda_2^*)$, $\frac{\mathrm{d}q_2}{\mathrm{d}\lambda} < 0$ for $\lambda \in (\lambda_2^*, 1)$;

(ii) if $\frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2} \leq \sigma_\eta^2 < \sigma_v^2$ or $\omega \leq \omega^T$, then $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} < 0$ for $\lambda \in (0, \lambda_1^\ddagger)$, $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ for $\lambda \in (\lambda_1^\ddagger, 1)$, and $\frac{\mathrm{d}q_2}{\mathrm{d}\lambda} < 0$ for $\lambda \in (0, \lambda_2^\ddagger)$, $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ for $\lambda \in (\lambda_2^\ddagger, 1)$.

Otherwise, q_1 monotonically increases in λ and q_2 monotonically decreases in λ, i.e., $\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} > 0$ and $\frac{\mathrm{d}q_2}{\mathrm{d}\lambda} < 0$ for any $\lambda \in (0, 1)$.

Proof. I present the proof of the monotonicity (non-monotonicity) of q_1 with respect to λ. The monotonicity (non-monotonicity) of q_2 can be derived in a similar manner, and is thus omitted.

When $\sigma_\eta^2 < \sigma_v^2$,

$$\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} = \begin{cases} -\varphi_1(-\frac{|\mu|}{\phi_1})\frac{|\mu|}{\phi_1^2}\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda} & \text{if } \lambda < \lambda_1^\ddagger, \\ \varphi_1(-\frac{|\mu|}{\phi_1})\frac{|\mu|}{\phi_1^2}\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda} & \text{if } \lambda > \lambda_1^\ddagger. \end{cases} \tag{C11}$$

When $\sigma_\eta^2 \geq \sigma_v^2$,

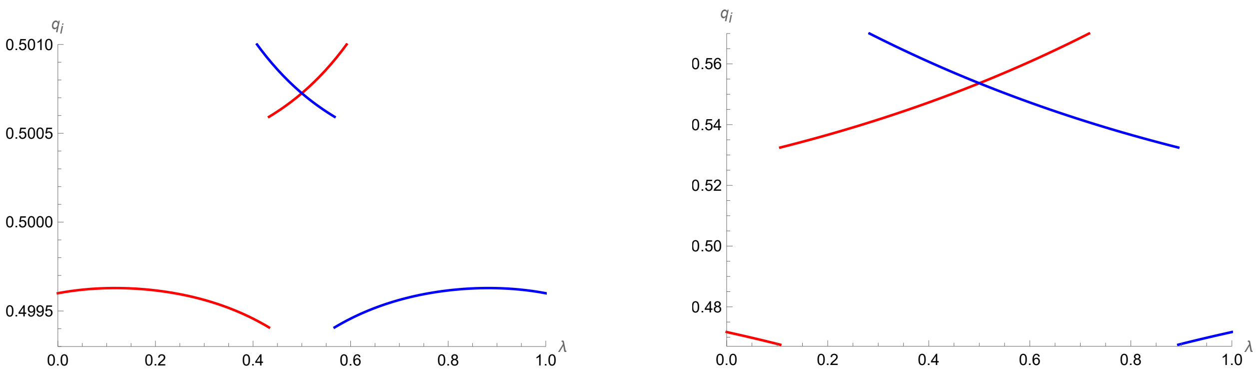
$$\frac{\mathrm{d}q_1}{\mathrm{d}\lambda} = \varphi_1(-\frac{|\mu|}{\phi_1})\frac{|\mu|}{\phi_1^2}\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda}, \tag{C12}$$

disclosure $\hat{\lambda} = \frac{n_1}{n_1+n_2}$ is unbiased. Indeed, there exists a λ for each moderately valued (v, η) combination such that $q_1 = q_2$. To see this, note that when $v = \eta = 0$, $q_1 = q_2$ at $\lambda = \frac{1}{2}$; changing the value of (v, η) within certain range shifts the disclosure probabilities around and results in a different value of λ at which the two disclosure probabilities equal to each other (see Figure C.1).

where $\varphi_1(\cdot)$ is the probability distribution function of a normal distribution with mean $v+\eta$ and variance σ_ϵ^2, and is thus always positive. The rest of the proof follows immediately from *Proposition 3.2*. $\square$

Generally, the disclosure probability of each type increases in the proportion of the corresponding type. This makes intuitive sense because an agent is more willing to disclose when there are more people of his type, which increases the value of his private information and induces him to rely more on it.

The intuition for the non-monotonicity of disclosure probabilities with respect to the proportion of their corresponding types is more convoluted. When $\sigma_\eta^2 < \sigma_v^2$, all agents use their private signals in a positive way. Nevertheless, when the population composition is expected to be relatively unbalanced, i.e., $\lambda < \lambda_1^\ddagger$ or $\lambda > \lambda_2^\ddagger$, the convexity of the informativeness of private signals about the average action can induce agents of the minority type to rely less on their private signals when the others expect a higher proportion of their type. Whenever this is the case, only those agents whose private signals are optimistic (pessimistic) but induce a negative (positive) action are willing to disclose in the case of a negative (positive) prior. The higher their reliance on the optimistic (pessimistic) signals, the more positive (negative) their actions are likely to be, and hence there are fewer agents disclosing. As a result, a higher reliance on private signals of the minority type induced by an increase in the proportion of that type ends up leading to a lower disclosure probability of the corresponding type. Figure C.1(a) provides a demonstration of non-monotonic disclosure probability of the extreme minority type due to their non-monotonic reliance on private signals (see *Proposition 3.2*). In contrast, the disclosure probability of the extreme minority type decreases in the proportion of the corresponding type in Figure C.1(b) given their monotonic reliance on private signals.



(a) $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$ and $\omega > \omega^T$

(b) $\frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2} \leq \sigma_\eta^2 < \sigma_v^2$ or $\omega \leq \omega^T$

Figure C.1: Equilibrium *Ex-Post* Disclosure Probability

Note: The above figures demonstrate how the equilibrium ex-post disclosure probabilities of the two types of agents change in λ non-monotonically as described in *Proposition C1*. q_1 and q_2 are plotted in red and blue, respectively. Fixing $|\mu| = 0.5$ and $v = \eta = 0$, the other parameter values are (a) $\omega = \frac{27}{32}, \sigma_v^2 = 2, \sigma_\eta^2 = \frac{5}{8}, \sigma_\epsilon^2 = \frac{3}{2}$, and (b) $\omega = \frac{1}{2}, \sigma_v^2 = \frac{9}{4}, \sigma_\eta^2 = 2, \sigma_\epsilon^2 = 2$, respectively.

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