Doctoral Thesis

# Essays on Bank Capital and Home Production

William Bednar

Carnegie Mellon University
Tepper School of Business

April 23, 2021

*Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics*

# Dissertation Committee

Ariel Zetlin-Jones
Carnegie Mellon University
Tepper School of Business

Burton Hollifield
Carnegie Mellon University
Tepper School of Business

David Childers
Carnegie Mellon University
Tepper School of Business

Pietro Bonaldi
Carnegie Mellon University
Tepper School of Business

# Abstract

Chapters 1 and 2 are focused on capital and regulation in the banking system. The financial crisis of the late 2000's demonstrated the need for a "macro" perspective of financial regulation. Systemic issues in the banking sector contributed to the resulting severe recession. This illustrated the need to think about regulation at the system level as opposed to the individual-firm level. In other words, a "macroprudential" approach as opposed to a "microprudential" approach. The result has been a renewed interest from both regulators and researchers in the aggregate effects of financial regulation. This is especially true of bank capital regulation, a primary component of the financial regulatory architecture. There are a number of variants and different thresholds that must be satisfied. However, generally, capital requirements are simply restrictions on the proportion of debt banks can use to finance themselves. Funding with more capital as opposed to debt acts as a buffer against earnings shocks. It also forces banks to have more "skin in the game", potentially reducing moral hazard problems due to deposit insurance. At the aggregate level, well-capitalized banks are better able to withstand economic downturns. This reduces the chances of cascading bank defaults or the failure of systemically important banks, events that can turn a recession into a financial crisis.

A challenge with taking a "macroprudential" approach is the observed variation in levels of bank regulatory capital. This variation exists both across banks and within individual banks and over time. A proportion of banks have capital ratios that are typically near the regulatory constraint, while capital levels at many banks put them significantly away from these constraints. Additionally, bank's capital ratios do not stay constant over time. In fact, for some banks, it can vary significantly, even from one quarter to the next. This is especially true in uncertain times, when returns and thus the level of bank equity capital are volatile, such as in recessions. This type of variation calls into question the role that capital regulation is playing in the determination bank capital ratios. Are bank's significantly away from the regulatory capital constraint being impacted by it? How can we evaluate the impacts of regulatory policy in the presence of such heterogeneity? In these two chapters, I attempt to make progress on addressing questions such as these.

In Chapter 1, my objective is twofold. First, I attempt to document key patterns in observed regulatory capital ratios, as this is a necessary first step in attempting to understanding the implications of such variation. The regulatory measure I focus on is the total-risk based capital ratio (RBCR), a constraint on how much debt financing a bank can use relative to how much risk it exposes itself to in its asset portfolio. This description implies two components of this statistics, a financing component and a portfolio component; and so, along with documenting key patterns in the RBCR itself, I do the same with each of these underlying parts as well. Second, I try to provide some way of interpreting these cross-sectional and time series differences in regluatory capital by mapping to a measure of bank risk exposure, which is the ultimate concern

of a regulator. This exercise shows that the "riskiness" of a given capital ratio is dependent on both macro and bank-specific factors, specifically interest rates and cost efficiency, and that by accounting for these factors, a useful comparison of different levels of regulatory capital, across banks and over time, can be generated.

In Chapter 2, I turn my attention to the regulation itself, and ask: how does the banking industry respond to changes in capital regulation, and what does this imply about how the burden of changing regulation is distributed between borrowers, savers, and banks? I consider this question in the context of a dynamic banking industry equilibrium framework. Heterogeneity, endogenous interest rates, and entry and exit allow for within-industry reallocation in the provision of banking services. The extent of this reallocation determines how much of the cost of the regulation is paid by bank customers. In a quantitative exercise, the model is calibrated to features of the US banking industry, and I decompose changes in surplus for banking market participants associated with a counterfactual tightening of capital requirements into the component driven by restricting bank behavior, and the component driven by the associated equilibrium reallocation in the provision bank services and endogenous industry changes that occur over the longer-run as a result of these restrictions.

Chapter 3, a joint work with Nick Pretnar, is focused on how the concept of home production and more expansive notions about how households use their time can help us understand several long-term macroeconomic trends, primarily the shift from a goods-oriented to a services-oriented economy in the United States. We construct a general equilibrium model with home production where consumers choose how to spend their off-market time using market consumption purchases. The time-intensities and productivities of different home production activities determine the degree to which variation in income and relative market prices affects both the composition of expenditure and market-labor hours per worker. For the United States, substitution effects due to relative price changes dominate income effects from wage growth in contributing to the rise in the services share and the fall in hours per worker. The model generates a measure of welfare characterized by the relative value of in-home activities for households at different wage thresholds. When considering how welfare has changed for consumers of different incomes, we find that the variance of the cross-sectional distribution has risen since the 1980s, though to a lesser extent than implied by the evolution of the income and wealth distributions. This is because improvements to technologies and the quality of the consumption stock have increased consumer welfare beyond what can easily be measured through expenditure or income.

# Contents

# 1. The Distribution of Bank Regulatory Capital

## 1.1. Introduction

For financial regulators, the concern about bank capital is not actually about bank capital. Instead, they really care about the exposure of banks to risk. This concern is primarily due to the fact that a large proportion of bank liabilities (deposits) are insured by the regulators (government). This insurance is provided to improve the stability of the banking system by reducing the prospect that bank runs occur. Banks turn short-term, liquid, safe liabilities into risky, long-term, potentially illiquid investments. By eliminating runs, which are essentially margin calls by depositors, banks can confidently transform risk on their balance sheet. This facilitates long-term investment that is valuable for an economy. However, more risk implies that there is a higher probability of bank defaults. Due to insurance, a default implies that the regulator must cover the cost of insured liabilities. The problems at one bank can also spillover to others, magnifying these potential expenses. Furthermore, the provision of insurance can decouple the interest rate paid on bank's liabilities from its default risk. This limitation in market discipline provides opportunity for moral hazard, and excess risk-taking by banks. To prevent excess risk-taking, banks are regulated.

Regulatory capital ratios then act as a measurable proxy for risk-taking behavior by banks. Capital requirements generally come in two forms: leverage constraints and risk-based capital constraints. Leverage constraints set a minimum bound on the extent to which banks can finance themselves with equity as opposed to debt. Risk-based capital requirements restrict the proportion of debt financing relative to default risk in the bank's asset portfolio. Both encourage the funding of bank investment with some minimum amount of equity capital. The reason is that equity capital acts as a buffer against volatile bank returns, as opposed to debt, which is a fixed obligation. Additionally, since more equity increases the exposure of banks owners to return volatility, this can encourage less risk-taking. The capital ratios of a bank are then a measure of the bank's risk exposure, and thus ripe for regulatory limitations.

But how well does a bank capital ratio represent the risk exposure of a bank; and the economist's answer is that, of course, it depends. For example, a bank with a low regulatory capital ratio due to a high proportion of risky assets is facing more risk in a period of high return volatility compared to when volatility is low. Or, if two banks have the same capital ratio, and one bank has higher production costs than the other, it faces more risk because it has an additional financial burden when compared with the possibility of low asset returns. Therefore, the riskiness of a given capital ratio is dependent on other factors, some that may be specific to and consistent within an individual bank over time, and others that may be consistent across banks but vary over time. Additionally, it is likely the case that the other factors that help to determine the risk to a bank are influencing the bank's choices with respect to the variables that make up the regulatory capital ratios. If a bank is earning high average returns on its risky portfolio, then it may be less likely to receive a bad shock; however, this may encourage the bank to increase risk in its portfolio, increasing exposure to the shock it does face. Alternatively, if a bank faces a low cost of raising deposits, while that lower cost may mean that a shock to asset returns is less concerning for overall profitability, it may encourage the bank to increase its leverage, again increasing its exposure to volatility. Evaluating a bank's regulatory capital structure then requires evaluating the co-determining factors that affect bank risk.

In this paper, I first ask: what is observed about bank regulatory capital ratios? I document summary statistics and key patterns related to capital ratios in a panel of US banks. The focus is on the degree of variation in regulatory capital ratios, both in the cross-section and over time. Also considered is what drives this variation in terms of key "financial" decisions made by the bank, specifically leverage, or the bank's financing choices, and portfolio risk, or the bank's investment decision. Risk-based capital is simply the ratio of the leverage ratio to portfolio risk. Key patterns in these choices, both in the cross-section and over time, are documented as well as how they relate to each other along these dimensions.

I then consider how these observations relate to a measure of bank risk. To begin this process, a simple dynamic bank problem is model, and risk defined as a probability of default or alternatively the probability of a certain degree of equity loss (below which a bank would likely default on its obligations). I then describe the other co-determining factors of the bank's problem that determine the degree of risk associated with a given capital structure, which primarily include interest rates and the bank's production costs. I attempt to quantify these in data in order to provide a mapping between bank capital and risk given these other factors.

Therefore, in the first section, I document the degree of variation in regulatory capital ratios that is observed in the cross-section and over time, and in the second section, I attempt to provide meaning to this variation in terms of what capital regulation is meant to guard against, bank risk. For example, is 20 percent capital meaningfully different than 10 percent capital? The argument put forth in this paper is that this the answer depends on the risk exposure to a given bank, to which the capital ratio is one factor. And so to assess whether or not this is a meaningful difference, one needs to first define meaningful (risk exposures), and then assess the extent to which this represents a meaningful difference along the metric defined. Here, this will depend on the context, both with respect to a given firm and with respect to a given time period, in which a particular capital ratio is observed.

This idea has important implications for capital regulation in particular, and financial regulation more generally. Primarily, restrictions on bank capital have applied uniformly across institutions: each bank had to satisfy the same minimum lower bound. This is despite the fact that some banks may be able to operate more safely at lower levels of capital. Additionally, with regulation fixed over time, the risk represented by a given minimum will even vary within a bank. Recently, however, with the advent of Basel III capital regulation, more conditionality has been added to the requirements. Banks must now satisfy cyclical buffers, additional capital is required for systemically important institutions, and banks' balance sheet positions need to be able to pass stress tests. These are tacit acknowledgments of the conditional meaning of bank capital, and the purpose of this paper is then to evaluate observed capital ratios and formalize this notion of conditionality.

The paper proceeds as follows. After a literature review, in Section 2, I evaluate observed regulatory capital ratios in data. In Section 3, a proposed mapping between regulatory capital and risk is described. In Section 4, I measure the other factors that determine bank risk and how they are related to regulatory capital in the data. In Section 5, I discuss how bank risk factors are related to each other, and what this implies about risk-taking behavior. Section 6 concludes.

### 1.1.1. Literature

This paper is primarily related to the empirical literature that looks at bank capital and the effects of capital regulation. This research area has generally three types of the papers. The first type considers what determines a bank's capital level/ratio/buffer and what role does regulation play in that determination. Early work on this topic include Dietrich and James 1983, which used evaluated the impact of a measure of regulatory pressure on changes in bank capital, and Marcus 1983, which was concerned with how interest rates affect bank capital levels. Wall and Peterson 1995 also attempts to estimate the impact of regulation on bank capital ratios and Flannery and Rangan 2008 considers what drove banks to build higher levels of capital during the 90's; while several papers use a partial adjustment model to determine if banks have capital ratio targets (Flannery and Rangan 2006), and what determines those targets (Berger et al. 2008). Lindquist 2004 evaluates bank-level determinants of capital buffers, or the difference between the observed capital ratio and the regulatory minimum, in a sample of Norwegian banks. This is followed by a similar look at capital buffers in Fonseca and González 2010, which was interested in the effects of deposits costs and market concentration on bank's excess capital. Both Gropp and Heider 2010 and Teixeira et al. 2014 find that regulation plays only a second order effect determining bank capital ratios. In the former it is done via "standard" corporate finance capital structure regressions; and in the latter, the focus is on whether or not this result is sensitive to the presence of a financial crisis. De Jonghe and Öztekin 2015 looks at how bank capital evolves over time, finding that banks are most likely to increase equity through retained earnings; and Lepetit, Saghi-Zedek, and Tarazi 2015 looks at the impact of different corporate governance structures on bank capital levels.

A prominent line of work in this area includes the simultaneous equations methods used early on by Shrieves and Dahl 1992, and later by Jacques and Nigro 1997 and Jokipii and Milne 2011. Generally, this approach involves evaluating determinants of bank capital or capital buffers and other simultaneous determined variables, primarily measures of risk. This approach was repeated with data for banks in a number of different countries or sets of countries including Sweden (Rime 2001). Germany (Heid, Porath, and Stolz 2003 and Kleff and Weber 2008), Malaysia (Ahmad, Ariff, and Skully 2008), emerging markets (Godlewski 2005), south eastern Europe ( Athanasoglou 2011) and G-10 countries (Van Roy 2005). Altunbas et al. 2007 augments this simultaneous equations approach to include a measure of cost efficiency to see how it is determined with bank capital and risk; while, Shim 2013 includes business cycle indicators to evaluate the impact of business cycles on the joint determination of these variables. Finally, Kwan and Eisenbeis 1997 evaluates a simultaneous equation model that also includes loan performance and interest rate risk to see how they evolve with levels of bank capital and cost efficiency as well.

The second type of paper includes those that look at how bank capital and regulation impacts other behaviors of a bank, prominently bank lending, or how it impacts measures of bank performance. In terms of how capital and regulation affect levels of bank lending and loan growth, Furlong 1992 and Francis, Osborne, et al. 2009 both use a speed-of-adjustment model to investigate the relationship between levels of bank capitalization and loan growth. Kishan and Opiela 2000 and Carlson, Shan, and Warusawitharana 2013 are also concerned with the relationship between bank capitalization and loan growth, the former interested in how this relationship depends on bank size and the latter interested in how this relationship is affected by controlling for local effects. Several papers are focused on the cyclical relationship between capital, regulation and lending including Kishan and Opiela 2006 and Behn, Haselmann, and Wachtel 2016, which both focus the impact of cyclical shocks to lending after the implementation of Basel II's model based regulation; and Košak et al. 2015 and Kim and Sohn 2017 are interested in how other components of the bank balance sheets, either the composition of financing in the case of Košak et al. 2015 and the quantity of liquid assets in the case of Košak et al. 2015, impact the relationship between capital and lending . More generally, Hancock, Laing, and Wilcox 1995 estimates the impact of shocks to bank capital on other balance sheet components using panel VAR, and finds capital and securities recover more quickly than lending. Peek and Rosengren 1995 uses observed enforcement actions, or regulatory interventions that occur when a bank has been determined to have violated some regulatory thresh hold in order to assess the effect of regulation on bank lending; Schwert 2018 shows that bank-dependent firms are more likely to borrow from well capitalized banks; and Aiyar et al. 2014 analyzes the impact of higher capital regulation on cross-border loan supply.

There is also a literature that focuses on the relationship between capital, regulation, and measures of bank risk which includes Delis, Tran, and Tsionas 2012, Camara, Lepetit, and Tarazi 2013, Lee and Hsieh 2013, Dias 2021, and Altunbas, Binici, and Gambacorta 2018. Additionally, Hovakimian and Kane 2000 estimates the impacts of regulation on bank risk using an option based model of deposit insurance; and Aggarwal and Jacques 2001 uses simultaneous equations approach to evaluate the effects of prompt corrective action on credit risk in bank portfolios. Laeven and Levine 2009 looks at how a bank's governance structure affects risk-taking and the relationship between risk and capital. Altman and Sabato 2005 looks at the impact of Basel reforms on portfolio choices for small and medium banks under the internal ratings-based approach; Behr, Schmidt, and Xie 2010 looks at the impact of market concentration on the relationship between regulation and risk-taking, finding that regulation is more effective at reducing risk when there is less concentration; and Baker and Wurgler 2015 investigates the "low risk anomaly", which is essentially the idea that by reducing risk, regulation increases bank's equity costs. Other work looks at the relationship between capital and measures of bank performance like stock prices (Akhigbe, Madura, and Marciniak 2012 and Demirguc-Kunt, Detragiache, and Merrouche 2013) and measures of cost or production efficiency (Barth et al. 2013 and Das and Ghosh 2006). There are several papers that look at the relationship between capital and liquidity creation including Distinguin, Roulet, and Tarazi 2013 and Horváth, Seidler, and Weill 2014. Santos and Winton 2019 look at capital affects the loan spreads banks charge, and Anginer, Demirgüç-Kunt, and Mare 2018 evaluates the relationship between a bank's capital ratio and its contribution to systemic risk.

The final set of papers focuses on aggregate relationships involving bank capital and regulation. In terms of the relationship between bank capital and other aggregate variables, Berrospide and Edge 2010 and Noss and Toffano 2016 use VAR models to evaluate the effects of shocks to aggregate bank capital on other macroeconomic variables, including aggregate loan growth. Gambacorta and Mistrulli 2004 and Gambacorta and Shin 2018 look at the role of bank capitalization in determining the aggregate effects of monetary policy shocks, or the so-called "bank capital" channel; while Gambacorta 2011 considers the effects of capitalization on real macroeconomic variables and Jordà et al. 2021 looks at how it affects the probability the probability of a financial crisis, both using long data panels. In terms of the macroeconomic effects of capital regulation, Goodhart, Hofmann, and Segoviano 2004 documents the cyclical effects associated with the implementation of regulatory policies, and Kim, Koo, and Park 2013 conducts at cross-country analysis looking at the impact of regulation on likelihood of banking crisis. A number of papers study the impact of regulatory policy on macroeconomic variables including Cerutti, Claessens, and Laeven 2017 (credit growth, cross-border flows, crises), Akinci and Olmstead-Rumsey 2018 (credit growth and house price appreciation), Boar et al. 2017 (GDP and GDP volatility), Meeks 2017 (lending, expenditure, credit spreads, and probability of housing crisis), and Bruno, Shim, and Shin 2017 (credit growth, bank capital and bond flows).

A number of other papers deal with different implications related to capital and regulation at the aggregate level. Both Duca 2016 and Irani et al. 2021 consider the role of capital regulation in driving growth in the shadow banking sector; while Utrero-González 2007 finds that capital regulation leads to increased indebtedness in certain industries. Gauthier, Lehar, and Souissi 2010 evaluates the relationship between bank capital allocation and systemic risk using data on interbank linkages; while Hoggarth, Mahadeva, and Martin 2010 documents cross-border bank capital flows during the financial crisis. Finally, Jiménez et al. 2017 uses the policy of "dynamic provisioning" in Spain, essentially a countercyclical capital regulation policy, to estimate the effects of financial regulation on credit supply.

## 1.2. Empirical Analysis of Capital Ratios

In this section, I document key patterns in regulatory capital ratios using data from the FDIC's Call Reports. The final data set is a panel containing 815,276 observations, with 17,339 unique banking institutions[1]. The panel covers 108 quarters over 27 years (1993-2019). In the sample, several well-known trends in the banking industry are occurring. The number of banks has declined from over 10,000 institutions at the beginning of the sample to roughly half that by the end. Additionally, banks are getting larger, and the industry is becoming more concentrated. The top 10 percent of banks held roughly 26 percent of bank assets in 1993. This increased to nearly 53 percent by 2019. Summary statistics related to these trends are provided in Table 1.1.

To describe the bank's regulatory capital ratio, the empirical balance sheet of bank $i$ at time $t$

[1]Commercial banks in the same holding company are combined into a single observation. As discussed in Kashyap and Stein 2000, this nests the assumption that banks can freely move funds within a holding company.

| Time Period | Number of Banks | ln(Assets) | Concentration |
|---|---|---|---|
| 1993-2019 | 7549 | 11.87 | 45.64% |
| 1993-1995 | 10338 | 11.20 | 26.25% |
| 1996-1998 | 9024 | 11.36 | 33.41% |
| 1999-2001 | 8300 | 11.52 | 40.47% |
| 2002-2004 | 7902 | 11.73 | 44.31% |
| 2005-2007 | 7574 | 11.90 | 49.29% |
| 2008-2010 | 7182 | 12.07 | 54.15% |
| 2011-2013 | 6580 | 12.20 | 55.47% |
| 2014-2016 | 5876 | 12.34 | 54.65% |
| 2017-2019 | 5163 | 12.51 | 52.75% |

**Table 1.1.:** Summary Statistics: Quarterly Averages

is defined as:

$$e_{it} + d_{it} = l_{it} + b_{it} \tag{1.1}$$

where $e_{it}$ is equity, $d_{it}$ is debt/deposits, $l_{it}$ is "risky" loans and $b_{it}$ are risk-free bonds. The following measures from the data are used: $e_{it}$ is measured using total equity capital, $d_{it}$ using total liabilities, and $l_{it}$ using risk-weighted assets (RWA). To measure bonds $b_{it}$, define $a_{it} = e_{it} + d_{it} = l_{it} + b_{it}$, where $a_{it}$ is measured using total assets. Bonds $b_{it}$ is then total assets net of risk-weighted assets, or $b_{it} = a_{it} - l_{it}$. I provide a list of data counterparts to conceptual variables in Table 1.2. Clearly, this representation is a simplification of an actual bank balance sheet. For example, there are different categories of equity capital specifically defined for regulatory purposes (i.e. Tier 1, Tier 2, etc.). Additionally, debt is composed of both demand deposits as well as wholesale debt and other borrowings. On the asset side, risk-weighted assets are actually a weighted sum actual bank assets [2], and not a category of assets at all. I abstract from these issues in order to provide a more succinct analysis of bank regulatory capital.

| Variable | Empirical Counterpart |
|---|---|
| $e_{it}$ | Total equity capital |
| $d_{it}$ | Total liabilities |
| $l_{it}$ | Risk-weighted assets |
| $a_{it}$ | Total assets |
| $b_{it}$ | Total assets - Risk-weighted assets |
| $c_{it}$ | Non-interest expense - Non-interest income |

**Table 1.2.:** Data for Variables

From this empirical balance sheet, the primary regulatory capital ratio of interest is the total

[2] The weights are approximately determined the default risk of the asset or category of assets. However, there is some question of how well these weights generate a measure of the riskiness a bank's asset portfolio.

risk-based capital ratio (RBCR). This is defined as:

$$\gamma_{it} = e_{it}/l_{it} \tag{1.2}$$

or the ratio of equity to risky assets. Risk-based capital requirements say that $\gamma_{it} \geq \underline{\gamma}$, a lower bound. Throughout the sample period, the lower bound has been 8 percent; however, banks need 10 percent to be considered "well capitalized."[3] I can then rewrite this capital ratio in order to represent it as two endogenous "financial" choices of a bank. In particular:

$$\gamma_{it} = \frac{e_{it}}{l_{it}} = \frac{e_{it}/a_{it}}{l_{it}/a_{it}} = \frac{\psi_{it}}{\phi_{it}} \tag{1.3}$$

where $\psi_{it} = e_{it}/a_{it}$ and $\phi_{it} = l_{it}/a_{it}$. Note that $e + d = a = l + b$. This means $\psi$ is the proportion of assets financed with equity, and so it represents the bank's financing choice. Lower values of $\psi$ mean that the bank is financing investment more with debt, or the bank has higher leverage. This is why, in the regulatory lingo, $\psi$ is known as the leverage ratio. Leverage ratios must satisfy their own separate requirement, meaning $\psi_{it} > \underline{\psi}$. For the leverage constraint, 4 percent is the lower bound and 5 percent is required to be considered "well-capitalized." In the denominator, $\phi$ is the ratio of "risky" loans to total assets. This represents the bank's portfolio choice, or how much risk exposure it is taking on the asset side of its balance sheet. From this decomposition, the RBCR is simply the ratio of how the bank finances itself to how it invests its available capital.

| Variable | | Average | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| $\gamma_{it}$ | overall | 0.187 | 0.112 | 0.077 | 0.828 |
| | between | | 0.109 | 0.077 | 0.828 |
| | within | | 0.063 | -0.459 | 0.899 |
| $\psi_{it}$ | overall | 0.110 | 0.045 | 0.048 | 0.364 |
| | between | | 0.045 | 0.048 | 0.364 |
| | within | | 0.027 | -0.170 | 0.404 |
| $\phi_{it}$ | overall | 0.639 | 0.142 | 0.256 | 0.954 |
| | between | | 0.129 | 0.256 | 0.954 |
| | within | | 0.077 | -0.034 | 1.327 |

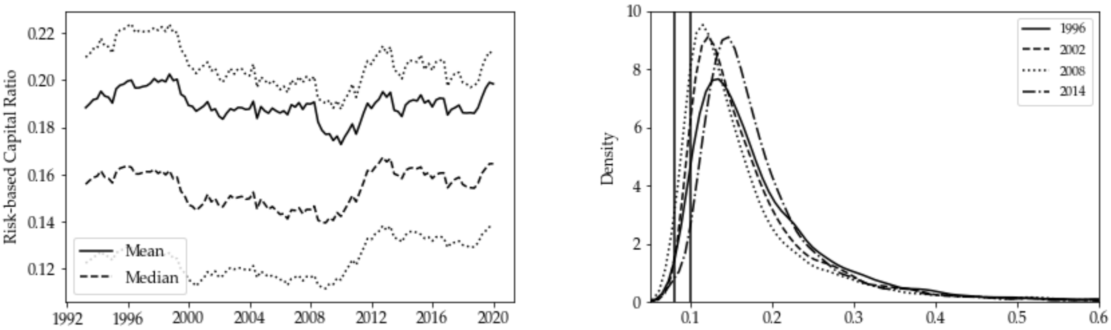
**Table 1.3.:** Panel Summary Statistics: $\gamma_{it}$, $\psi_{it}$ and $\phi_{it}$

Panel summary statistics for $\gamma$, $\psi$ and $\phi$ are provided in Table 1.3. Throughout the sample, the average RBCR ($\gamma$) is nearly 19 percent, well above the regulatory minimums. The average $\psi$ is 11 percent, meaning on average 11 percent of bank's investment portfolios were financed with equity. And the average for portfolio risk $\phi$ is 64 percent, or risk-weighted assets amount to that percentage of total assets. Additionally, there is variation in RBCRs both within individual firms over time, and across firms in each time period. Comparing the measures of "within" and "between" variation in the RBCR and its two underlying component provides some evidence

[3]Additional capital buffers requirements have been added since the advent of Basel 3, and will be discussed in a later section. See Appendix for details.

of the extent to which overall variation in the panel is being driven by ex-ante vs ex-post heterogeneity. By ex-ante heterogeneity, I mean differences at the bank level that persist over time, but may vary across banks. Consider a scenario in which a bank's ex-ante type determines a target capital structure that it achieves each period, and that types leading to different capital structures vary across banks. In this case, all of the variation in the panel would be due to between variation, as within variation is zero. In reality, capital structures do vary at the bank level over time; and the relative contribution of that bank-level, or within variation compared with the contribution of the between variation to total panel variation in $\gamma_{it}$, $\psi_{it}$, and $\phi_{it}$ demonstrates the extent to which persistent, or ex-ante heterogeneity across firms or transient, or ex-post changes within firms is the main driver of differences in regulatory capital. Here, while both dimensions contribute to total variation, the between standard deviation is larger than within standard deviation for $\gamma_{it}$ (0.109 compared with 0.063), $\psi_{it}$ (0.045 compared with 0.027), and $\phi_{it}$ (0.129 compared 0.077). This suggests that persistent cross-sectional differences, or ex-ante heterogeneity, is a significant component of the differences in regulatory capital ratios.

The left panel of Figure 1.1 shows the mean, median and inter-quartile range of RBCRs over time. The right panel shows the cross-sectional distributions at the end of four different years in the sample: 1996, 2002, 2008, 2014. Capital ratios seemed to stay flat or rise slightly throughout the 90's, and then gradually declined in the 2000's. There was a sharp increase during the financial crisis and the years immediately following. In the post-crisis period, they declined gradually until a recent uptick. In terms of cross-sectional variation, the inter-quartile range begins at around 10 percentage points. It then compresses a bit throughout the 90's and 00's to between 6 and 8 percentage points, where it remained. Additionally, for all quarters, the median is larger than the mean, indicating a right skew in the distribution. The right panel shows that the distribution drops off near the regulatory constraint in a sample of years. This is a possible indicator that the regulation is affecting bank behavior in the cross-section[4].



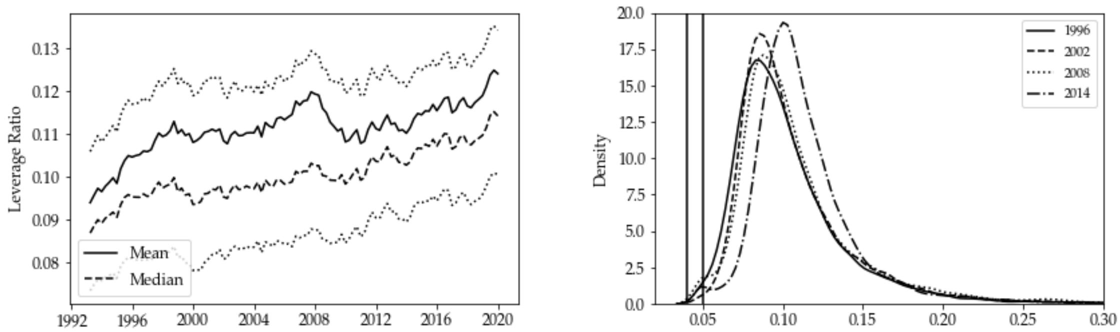

**(a)** Distributional Moments **(b)** Cross-sectional Distributions

**Figure 1.1.:** Total Risk-based Capital Ratio

Distributional moments for $\psi$ and $\phi$ are plotted in Figures 1.2 and 1.3, respectively. The trend for both has increased over time, with these trends existing throughout the cross-sectional

[4]This is only speculation as one would need a formal model to determine the precise role of the regulation.

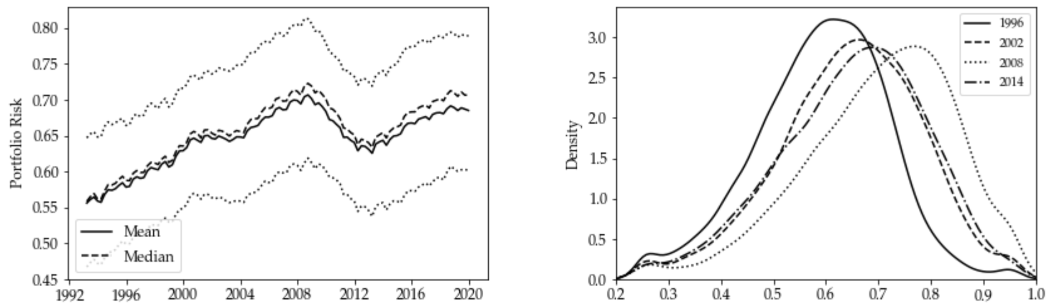
distribution. The exception to this is a decline in portfolio risk during the financial crisis. As will be discussed, this is likely due to the compression of the expected risk-premium during this time period. There is a right skew in capital ratios, likely generating the skew in risk-based capital ratios. Despite this skew, capital ratios are generally several percentage points above restrictions on the capital ratios themselves. Note that these capital buffers are only with respect to capital ratio requirements, not risk-based capital requirements. Portfolio risk has been largely symmetric over this time period, although it has skewed left towards the end of the sample.




**(a)** Distributional Moments

**(b)** Cross-sectional Distributions

**Figure 1.2.:** Leverage Ratio



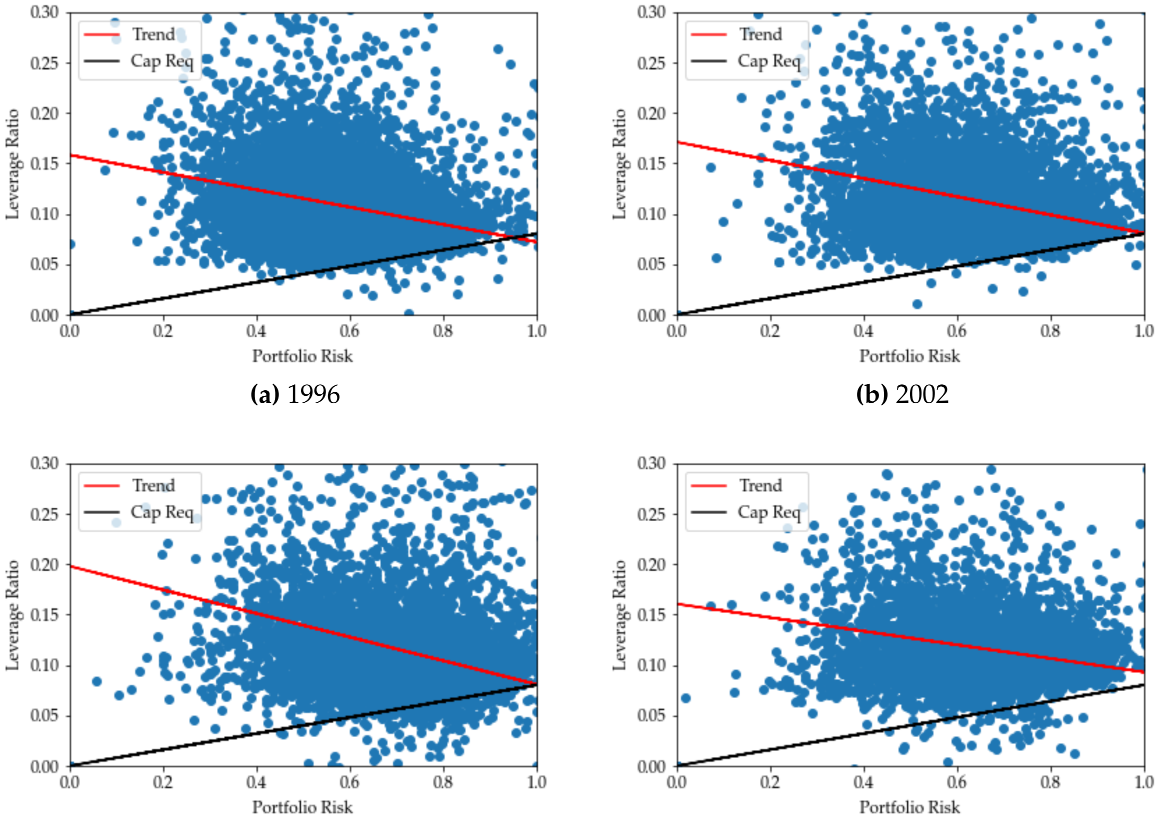

**(a)** Distributional Moments

**(b)** Cross-sectional Distributions

**Figure 1.3.:** Portfolio Risk

Figures 1.2 and 1.3 show the key features of the distributions of leverage ratios and portfolio risk. However, since $\gamma = \psi/\phi$, the joint distribution of $\psi$ and $\phi$ is also of interest. For example, one might ask the question: do banks with low regulatory capital have high leverage, high portfolio risk, or both? In Figure 1.4, I show the cross-sectional scatter plots between portfolio risk and leverage ratios. Also plotted is the trend line, and the line indicating the regulatory lower bound. Note that risk-based capital requirements indicate that $\psi_{it}/\phi_{it} \geq \underline{\gamma} = 0.08$. This line then represents: $\psi = 0.08 \times \phi$.

These plots demonstrate a pattern that is consistent throughout the sample. Portfolio risk and leverage ratios are inversely related in the cross-section. Moreover, banks close to the regulatory requirement have relatively low capital and high portfolio risk. This implies that, on average, banks do not protect against higher portfolio risk with higher proportions of equity




**(a)** 1996

**(b)** 2002

**(c)** 2008

**(d)** 2014

**Figure 1.4.:** Portfolio Risk and Leverage Ratios

financing. Instead, higher risk-taking on one side of the balance sheet is correlated with higher risk-taking on the other. While this figure only shows four specific quarters, the patter is persistent throughout the sample. The average quarterly correlation between portfolio risk and leverage ratios is -0.178. The min and max are -0.248 and -0.087 respectively.

The previous analysis provides general relationships across banks and over-time. However, while it provides a summary of the variation in individual decisions, it does not necessarily provide a way to think about the systemic implications of these decisions. The reason is that it is not accounting for the importance of an individual capital ratio or a set of capital ratios in the banking system as a whole. For example, consider a banking system with two banks, one small and one large. The large bank has a low capital ratio, signifying higher risk, and the small bank has high capital. Averaging across the banks would indicate that capital levels in the banking system are average. However, this does not account for the fact that the large bank is systemically important and the small banks is not. Based on a systemic viewpoint, overall bank capital may be evaluated as low because the systemically important institution has low capital.

To connect the individual and systemic views, first consider the total risk-based capitalization of the banking sector, defined:

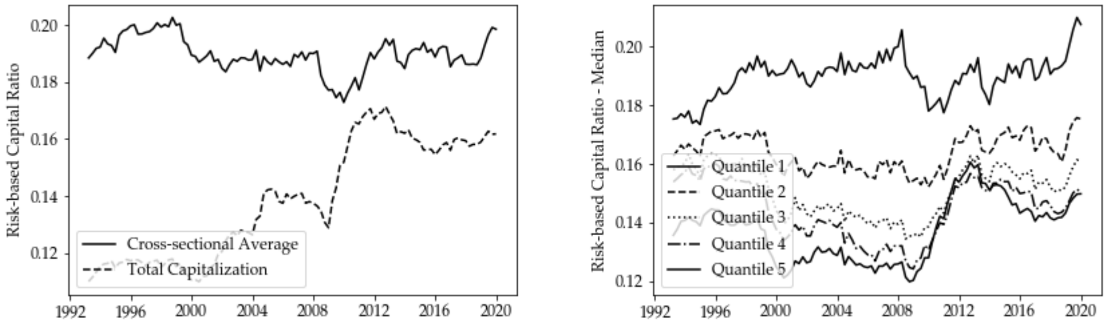
$$\Gamma_t = \left( \sum_{i=1}^{I_t} e_{it} \right) / \left( \sum_{i=1}^{I_t} l_{it} \right) \tag{1.4}$$

where $I_t$ is the number of banks operating at time $t$. This means $\Gamma_t$ is the ratio of the total equity capital to the total amount invested in risky (or risk-weighted) assets. Or in other words, it is the quantity of equity per dollar of risky assets in the bank system. The left panel of Figure 1.5 shows $\Gamma_t$, or total capitalization, with the average cross-sectional average RBCR over time. Total capitalization is less than average capitalization throughout the sample period. The gap was widest in the 90's, but began to close during the early 00's. It closed more rapidly during and just after the financial crises, and has since widened again. What accounts for the gap between aggregate and average risk-based capitalization in the banking sector? Note that total capitalization can be rewritten:

$$\Gamma_t = \frac{\sum_{i=1}^{I_t} e_{it}}{\sum_{i=1}^{I_t} l_{it}} = \sum_{i=1}^{I_t} \frac{e_{it}}{l_{it}} \frac{l_{it}}{L_t} \tag{1.5}$$

where $L_t = \sum_{i=1}^{I_t} l_{it}$. This demonstrates that total capitalization, $\Gamma_t$, is simply a weighted-average of individual capital ratios. The weight of each bank $i$ is determined by its share of risky assets. The difference between the unweighted (cross-sectional) average and this weighted average is then the covariation between individual capital ratios and the weights. In particular, the lower total capitalization relative to the bank level average indicates that banks with higher weights have lower capital ratios.

Given that $l_{it} = \phi_{it} a_{it}$, the weight of a given bank is determined by its size and its portfolio choice. If portfolios are relatively similar across the size distribution, larger banks will constitute a larger component of the aggregate measure. The gap between aggregate capitalization and average capitalization would then imply that larger banks have lower RBCRs. To illustrate this, I separate banks each quarters into quantiles by asset size. Quantile 1 is the smallest (or lowest

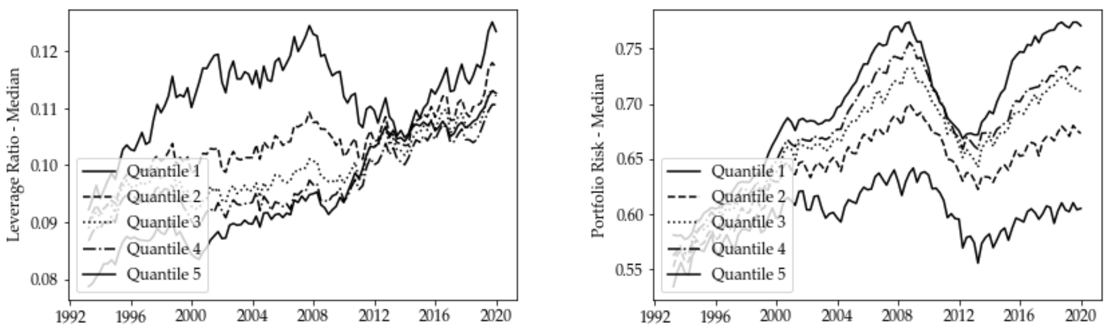



**(a)** Cross-section vs. Aggregate **(b)** RBCR: By Size Quantile

**Figure 1.5.:** Aggregate Risk-based Capitalization

total assets), while Quantile 5 is the largest. In the right panel of Figure 1.5, I plot the median RBCR by size quantile throughout the sample period. This shows that, in general, larger banks tend to have lower RBCRs. Additionally, aside for a brief period in the early 10's, the median RBCR of each quantile is higher than the one that precedes it. The quantile medians are ordered by size: the smallest banks have the highest RBCRs and the largest banks have the lowest. This implies a systematic relationship between assets size and RBCR.

In Figure 1.6, I do a similar quantile analysis for the individual components of the RBCR. As defined, these are the leverage ratio $\psi_{it}$ and portfolio risk $\phi_{it}$, where the RBCR $\gamma_{it} = \psi_{it}/\phi_{it}$. This exercise shows that a similar pattern persist as with the RBCR. Larger firms tend to have lower leverage ratios and higher portfolio risk; a pattern that is persistent across the size distribution. Therefore, the gap between average and aggregate capitalization is due to larger banks using less equity financing and investing in riskier portfolios.




**(a)** Leverage: By Size Quantile **(b)** Risk: By Size Quantile

**Figure 1.6.:** Size Bias in Individual Components

## 1.3. Mapping Regulatory Capital to Risk

Key cross-sectional and time series patterns in bank regulatory capital ratios are described. However, while it is a useful step, it does not give much in the way of interpreting these observations. For example, if one bank has a 20 percent capital ratio and another has 10 percent, one could ask whether or not this is a meaningful difference? Alternatively, if a bank's capital starts at 20 percent and drops to 10, should there be concern? Or more generally, what are the implications of what is observed regarding regulatory capital ratios in the data? To fill in this picture, it is useful to go back to what regulators actually care about, namely bank risk exposures, and risk is related to observed levels of regulatory capital. This is done using the following recursive dynamic banking framework.

### 1.3.1. A Dynamic Banking Problem

To begin each period, a bank starts with a pile of available cash called their net worth, or $n_{it-1}$. They then choose how much equity capital $e_{it}$ they want to retain in the bank, and the difference, $n_{it-1} - e_{it}$ is paid out to them as dividends. An assumption here is that the flow payout to the bank's owners/decision makers is a function of how much they receive in dividends. Specifically, there is a utility function $u(.)$ that is a function of $n_{it-1} - e_{it}$, and is increasing and concave. Future dividends are assumed to be discounted at rate $\beta \in [0,1]$. Note, that dividends can be negative if $e_{it} > n_{it-1}$. This represents a net capital injection or recapitalization at the bank. No distinction is made between a dividend payment and the actual raising of new capital, and dividends are considered to be the net of the two as it represents the net contribution of shareholders to equity capital.

While choosing how much equity to retain in the bank, the bankers also decided how much they want to raise in debt/deposits $d_{it}$. Additionally, they choose how they want to invest their funds, equity and deposits, in a combination of risky loans $l_{it}$ and risk-free bonds $b_{it}$, giving the same balance sheet as described in Equation 1.1. In what remains, I will substitute out bonds using the balance sheet constraint, and assume that the vector $[e_{it}, d_{it}, l_{it}]$ is chosen from a compact, convex set $\Theta_t$, which is determined by capital regulation and other physical constraints of the banks problem.

The bank earns interest at rate $r_{ft}$ on bonds, and pays interest at rate $r_{dt}$ on deposits. It also pays a net non-interest expense $c_{it}$, which is the net of production costs (wages, rent, etc.) and service fees. For loans, the bank earns returns at a stochastic rate $\tilde{r}_{it} \sim G_t(.)$, where $G_t(.)$ is the cumulative distribution function. This stochastic return is what makes loans "risky". Profits of the bank are then given by:

$$\pi_{it} = \tilde{r}_{it} l_{it} + r_{ft} b_{it} - r_{dt} d_{it} - c_{it} \tag{1.6}$$

and, after substituting in the balance sheet constraint, net worth evolves according to:

$$n_{it} = e_{it} + \pi_{it} = (1 + r_{ft}) e_{it} + (\tilde{r}_{it} - r_{ft}) l_{it} + (r_{ft} - r_{dt}) d_{it} - c_{it} \tag{1.7}$$

After the bank observes its returns and the environment for the following period, it has the option to default and exit. In this case, the banker receives zero in perpetuity. Denote $x_{it} \in \{0,1\}$ as the bank's exit decision, where $x_{it} = 1$ means an exit. Given this, the bank's problem can be written as follows:

$$V(n_{it-1}) = \max_{[e_{it}, d_{it}, l_{it}] \in \Theta_t} u(n_{it-1} - e_{it}) + \beta \mathbb{E}_t\left[(1 - x_{it})V(n_{it})\right] \tag{1.8}$$

$$\text{s.t.:} \quad n_{it} = (1 + r_{ft})e_{it} + (\tilde{r}_{it} - r_{ft})l_{it} + (r_{ft} - r_{dt})d_{it} - c_{it} \tag{1.9}$$

### 1.3.2. Capital and the Probability of Default

To illustrate the role of capital regulation, consider the ex-post choice $x_{it}$, which is the default choice for the bank. This is what concerns regulators. If the bank defaults, the regulator must cover bank (insured) deposits net of the asset liquidation value. A default option encourages excess risk-taking when borrowing costs are decoupled from risk. Capital regulation is set to limit this risk taking. The solution to the default decision is represented by a cutoff rule. If $V(n_{it})$ is monotonic, their exists a $\underline{n}_{it}$ below which the bank chooses to default. Given optimal choices, the default probability is then:

$$\text{Prob}(n_{it} \leq \underline{n}_{it}) = \text{Prob}((1 + r_t^f)e_{it} + (\tilde{r}_{it} - r_t^f)l_{it} + (r_t^f - r_{it}^d)d_{it} - c_{it} \leq \underline{n}_{it}) \tag{1.10}$$

Suppose the cutoff is represented as a proportion of the beginning equity: $\underline{n}_{it} = \underline{r}_{it}^e e_{it}$. This recasts this default threshold as a lower bound on the return on equity. The default probability then becomes:

$$\text{Prob}((1 + r_{ft})e_{it} + (\tilde{r}_{it} - r_{ft})l_{it} + (r_{ft} - r_{dt})d_{it} - c_{it} \leq \underline{r}_{it}^e e_{it}) \tag{1.11}$$

$$= G_t\left(r^{ft} - \frac{(r_{ft} - r_{dt}) + (1 + r_{dt} - \underline{r}_{it}^e)\psi_{it} - \bar{c}_{it}}{\phi_{it}}\right) \tag{1.12}$$

where $\bar{c}_{it} = c_{it}/a_{it}$ is the bank's net non-interest costs per dollar of assets, a measure of production efficiency. There are several things to note about this measure of bank default. First, define the default threshold as:

$$\underline{r}_{it} = r_{ft} - \frac{(r_{ft} - r_{dt}) + (1 + r_{dt} - \underline{r}_{it}^e)\psi_{it} - \bar{c}_{it}}{\phi_{it}} \tag{1.13}$$

and so the probability of default is $G_t(\underline{r}_{it})$. If $G_t(.)$ is a proper cumulative density function, then $G_t(.)$ is increasing in $\underline{r}_{it}$. This implies that changes to variables that increase the threshold also increase default risk. Consider the derivative of $\underline{r}_{it}$ with respect to $\psi_{it}$:

$$\partial \underline{r}_{it}/\partial \psi_{it} = -(1 + r_{dt} - \underline{r}_{it}^e)/\phi_{it} < 0$$

which says that the default threshold, and thus default risk, is decreasing in the proportion of equity financing. This illustrates the logic for leverage constraints, which take the form $\psi_{it} \geq \underline{\psi}$, a lower bound. Additionally, note that:

$$\partial^2 \underline{r}_{it}/(\partial \psi_{it} \partial \phi_{it}) = (1 + r_{it}^d - \underline{r}_{it}^e)/\phi_{it}^2 > 0$$

This shows that the impact of changes in capital structure on default risk depends on portfolio risk. In particular, for higher levels of portfolio risk, higher equity capital implies a stronger reduction in default risk. Alternatively, for low levels of capital, higher portfolio risk results is a stronger increase in default risk. This illustrates a logic for risk-based regulation taking the form: $\psi_{it}/\phi_{it} = \gamma_{it} \geq \underline{\gamma}$.

As shown in the simple model presented, the components that comprise the regulatory capital ratio are related to bank risk in expected ways. A higher proportion of equity capital funding reduces risk, while more risky loans increases it. Moreover, higher levels of capital financing reduce the risk associated with a given level of portfolio risk. However, note that the components of regulatory capital ratios are not the only variables that determine bank risk. Specifically, risk is also driven by returns ($r_t^f$,$r_t^d$, and $G_t$) and production efficiency $\bar{c}_{it}$. The signs of the partial derivatives for $r_t^f$, $r_{it}^d$, and $\bar{c}_{it}$ are as follows: $\partial \underline{r}_{it}/\partial r_t^f \leq 0$, $\partial \underline{r}_{it}/\partial r_{it}^d \geq 0$, and $\partial \underline{r}_{it}/\partial c_{it} \geq 0$. This says that a higher risk-free rate reduces risk (if $\phi_{it} < 1$) as it represents a higher guaranteed income from risk-free assets; and risk is increasing in the rate at which the bank has to pay either interest or production costs. Therefore, low capital may be offset by high risk-free returns or low costs in the determination of bank risk.

The $G_t$ distribution determines the probability at which a default threshold lies. However, it is difficult to say too much more here without specifying a distribution. Suppose, as is done in the following quantitative section, that $G_t = \mathcal{N}(\bar{r}_t, \sigma_t^2)$, or that the risky returns in time $t$ are normally distributed with mean $\bar{r}_t$ and variance $\sigma_t^2$. In this case, given a default threshold $\underline{r}_{it}$, risk, or the $Pr\{\tilde{r}_{it} \leq \underline{r}_{it}\}$, is decreasing in the average return $\bar{r}_t$, and increasing in the variance. The last component not yet discussed is the lower bound equity return $r_{it}^e$ that determines the default threshold. This is somewhat difficult to determine exactly because the size of a loss a bank must incur before it is in trouble of defaulting is likely determined by a number of different factors. For one, this may be impacted by the cost at which a bank can recapitalize or the loss a bank must incur before regulators step in an institute prompt corrective action. Here, I avoid these issues by taking essentially a value-at-risk approach (VAR), and specify a fixed threshold, and evaluate risk in terms of a fixed percentage equity loss, and evaluate risk in terms of the threshold generated by this and the other variables discussed.

## 1.4. Quantitative Model and Evaluation

In the previous section, I showed that while leverage and portfolio risk contribute to overall bank risk, the level of risk for a given bank in a given period depends on other factors, namely interest rates and production costs. Here, I measure these objects in order to generate a quantitative evaluation of bank capital ratios.

Starting with deposit rates, assume that $r_{it}^d$ is specific to each bank-period, and is known ex-ante or before the bank makes its funding and portfolio choices. If $d_{it}$ is measured using total liabilities, and $r_{it}^d d_{it}$ is total interest expense, the interest rate is calculated as:

$$r_{it}^d = \frac{\text{Total Interest Expense}_{it}}{\text{Total Liabilities}_{it}} \tag{1.14}$$

For the remaining components of the interest rate process, the risk-free rate $r_t^f$ and the distribution of stochastic returns $G_t$, I make the following assumptions. First, all banks face the same risk-free rate and return distribution, each of which is known ex-ante. Additionally, it is assume that $G_t = \mathcal{N}(\bar{r}_t, \sigma_t^2)$, or the stochastic returns are normally distributed with mean $\bar{r}_t$ and variance $\sigma_t^2$. Total investment income $I_{it}$ is measured as interest income plus securities gains/losses minus provisions for loan and lease losses. From the model, set:

$$I_{it} = \tilde{r}_{it} l_{it} + r_t^f b_{it} \tag{1.15}$$

Dividing by $l_{it}$, and recognizing that $\tilde{r}_{it} = \bar{r}_t + \epsilon_{it}$ where $\epsilon_{it} \sim \mathcal{N}(0, \sigma^2)$, the risk-free rate and stochastic return distribution are estimated using the cross-sectional regression:

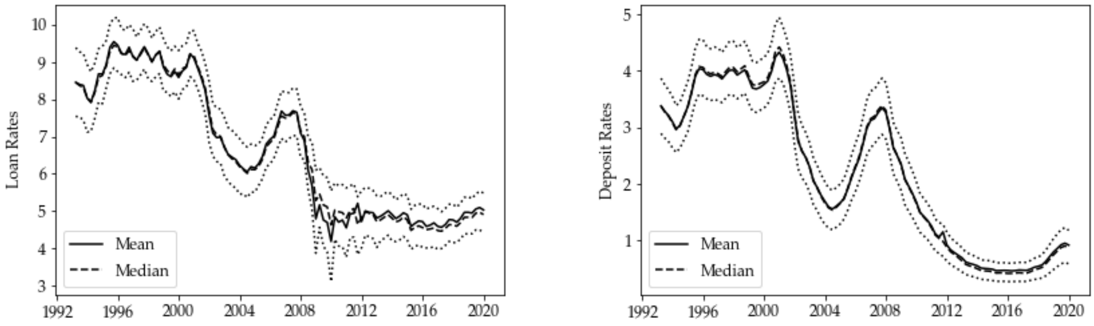
$$\frac{I_{it}}{l_{it}} = \bar{r}_t + r_t^f \left( \frac{a_{it} - l_{it}}{l_{it}} \right) + \epsilon_{it} \tag{1.16}$$

where again $l_{it}$ is risk-weighted assets and $a_{it}$ is total assets. The last factor to quantify is average production costs $\bar{c}_{it} = c_{it}/a_{it}$. Production costs, $c_{it}$, is measured as net non-interest expense, or non-interest expense minus non-interest income. Non-interest mainly includes wages and fixed capital costs, while non-interest income largely consists of service fees on deposit accounts.

| Variable | | Average | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| $\tilde{r}_{it}$ | overall | 0.070 | 0.026 | -0.139 | 0.244 |
| | between | | 0.020 | -0.138 | 0.236 |
| | within | | 0.021 | -0.166 | 0.289 |
| $r_{it}^d$ | overall | 0.024 | 0.015 | 0.001 | 0.055 |
| | between | | 0.011 | 0.001 | 0.055 |
| | within | | 0.012 | -0.024 | 0.076 |
| $\bar{c}_{it}$ | overall | 0.006 | 0.003 | -0.001 | 0.019 |
| | between | | 0.003 | -0.001 | 0.019 |
| | within | | 0.002 | -0.012 | 0.025 |

**Table 1.4.:** Panel Summary Statistics: $\tilde{r}_{it}$, $r_{it}^d$ and $\bar{c}_{it}$

Table 1.4 presents panel summary statistics for $\tilde{r}_{it}$, $r_{it}^d$ and $\bar{c}_{it}$. The interest rates have been annualized. The average annualized return on risky assets over this time period was 7 percent, and the average interest rate on bank debt was 2.4 percent. The average of bank's average costs was 0.6 percent, meaning that the net non-interest expense was 0.6 percent of total assets on average over this period. Figure 1.7 shows the mean, median, and inter-quartile range of annualized $\tilde{r}_{it}$ and $r_{it}^d$ over time. There is an overall decline in these series, which is consistent with the well-known long-run decline in interest rates. Average returns on loans hovered at around 9 percent during the 90's, fell to 6 percent by the mid-00's before going back to 8 percent just before the financial crisis. During the crisis, rates dropped to around 5 percent and have settled there. Deposit rates have followed a similar pattern, albeit at a lower level opening a net interest margin, with the following corresponding percentages: 4 percent, 1.75 percent, 3.25 percent, and < 0.5 percent. For both series, the mean and median are similar indicating mostly symmetric distributions.

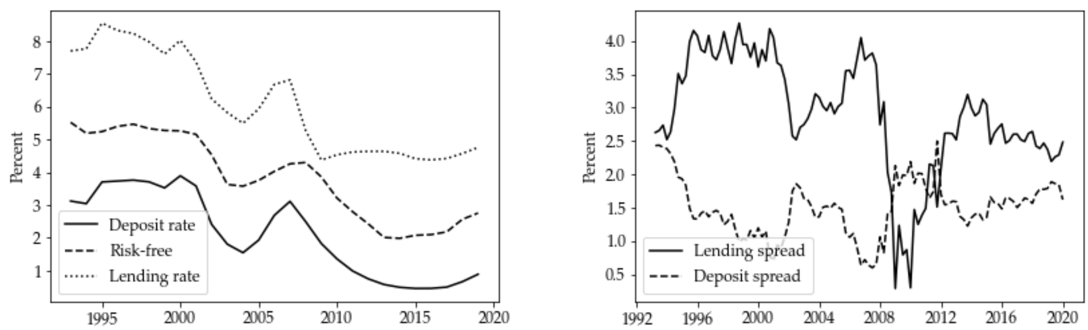



**(a)** Lending Rates

**(b)** Deposit Rates

**Figure 1.7.:** Lending and Deposit Rates

The left plot of Figure 1.8 shows the average lending rate, $\bar{r}_t$, the estimated risk-free rate $r_t^f$, and the average deposit rate: $\bar{r}_t^d = (1/I_t)\sum_{i=1}^{I_t} r_{it}^d$. This plot shows that the risk-free rate, which sits between the average lending and deposit rates, follows a similar long-run pattern[5]. Additionally, since it sits between the lending and deposit rate, it opens two important interest rate spreads: the lending spread $\bar{r}_t - r_t^f$, and the deposit spread $r_t^f - \bar{r}_t^d$. The importance of these spreads is illustrated in equation 1.7, the equation of bank net worth. In this equation, loans are multiplied by the lending spread, while deposits are multiplied by the deposit spread. for loans, the lending spread represents the average expected return premium a bank can expect to earn by investing in loans instead of risk-free bonds. The deposit spread is the premium a bank can earn by simply raising an additional dollar of deposits and investing in the risk-free asset. Therefore, these spreads represent the excess returns earned from each of these banking activities. The determinant of the spreads, and the extent to which a bank takes advantage of them bin increasing deposits (leverage) or loans (portfolio risk) likely depends on how costly it is for the bank to engage in these activities, which would be represented by the relationship between $l_{it}$, $d_{it}$, and $c_{it}$. This relationship will be discussed in a subsequent section.




**(a)** Lending Rates

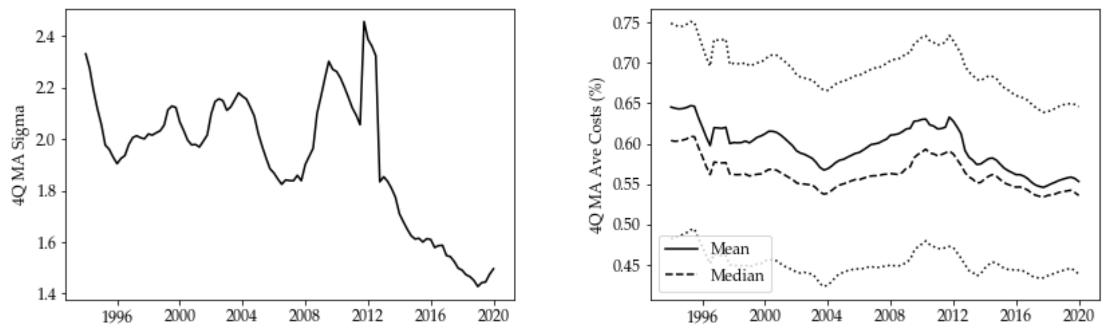
**(b)** Deposit Rates

**Figure 1.8.:** Average Rates and Spreads

[5]Additionally, if this estimated risk-free rate roughly follows the 10-year treasury yield, which might be considered the corresponding (default) risk-free asset in a bank's asset portfolio, lending credibility to the estimate.

For now, these spreads are simply plotted in the right panel of Figure 1.8. Generally, the lending spread has been larger than the deposit spread, although this relationship reversed during the financial crisis. While the deposit spread did increase, the lending spread collapsed during this period. It is interesting to note that this corresponds to a sharp decline in portfolio risk (see Figure 1.3); each of which recovers on a similar timeline as well. This, again, will be further discussed in a subsequent section on the inter-relationship between the variables that determine bank risk.

While the plot of lending rates in Figure 1.7 provides some indication of the variance of the estimated $G_t$ distribution for each period based on the plotted inter-quartile range, I can also calculate and show the variance ($\sigma_t^2$) or standard deviation ($\sigma_t$) directly. The left plot in Figure 1.9 shows the 4-quarter moving average[6] of the standard deviation $\sigma_t$ of cross-sectional risky returns over time in annualized percentage points. This standard deviation varied around 2 percentage points until the mid-2000's, and then spike somewhat during the great recession. Subsequently, it has declined after the financial crisis, reaching near 1.5 percentage points. Finally, the average, median, and inter-quartile range for average production costs is shown in the right panel of Figure 1.9[7] They are plotted as percentages of total assets. The mean and median average costs were roughly between 0.65 and 0.60 percent of total assets, with a gradual declining trend over this time period. The upper bound of the IQR was between 0.75 and 0.70, and the lower bound was between 0.50 and 0.45. The mean is larger than the median indicating a right skew in the cross-sectional distribution of average production costs over time.




**(a)** Risky Return Volatility

**(b)** Average Costs

**Figure 1.9.:** Volatility and Costs

After generating estimates of the other factors determining risk-exposures of the bank, the question becomes: how are these factors related to the regulatory capital ratios? Table 1.5 shows the correlation matrix for all of the variables that affect bank risk: $\psi_{it}$, $\phi_{it}$, $\tilde{r}_{it}$, $r_{it}^d$, and $\bar{c}_{it}$. I also included the total risk-based capital ratio: $\gamma_{it} = \psi_{it}/\phi_{it}$. As previously discussed, there is a negative overall correlation between leverage ratios $\psi_{it}$ and portfolio risk $\phi_{it}$, indicating that banks with more debt financing tend to have higher levels of risk in their portfolio. Additionally, both lending and deposit rates are negatively correlated with the components of the RBCR,

[6]There appears to be some within year cyclicality in this measure. Therefore, the moving average is plotted to give a better overall indication of the trend in volatility. The moving average is also used in the subsequent risk calculations to remove seasonal bias in the calculation.

[7]Also as 4QMA due to seasonality.

with the net relationship being a negative correlation overall. This means that higher levels of debt financing and portfolio risk are associated with lower interest rates[8]. Finally, average production costs are positively correlated with both equity financing and portfolio risk.

| | $\gamma_{it}$ | $\psi_{it}$ | $\phi_{it}$ | $\tilde{r}_{it}$ | $r^d_{it}$ | $\bar{c}_{it}$ |
|---|---|---|---|---|---|---|
| $\gamma_{it}$ | 1.000 | | | | | |
| $\psi_{it}$ | 0.826 | 1.000 | | | | |
| $\phi_{it}$ | -0.563 | -0.168 | 1.000 | | | |
| $\tilde{r}_{it}$ | -0.068 | -0.129 | -0.164 | 1.000 | | |
| $r^d_{it}$ | -0.055 | -0.112 | -0.086 | 0.677 | 1.000 | |
| $\bar{c}_{it}$ | 0.010 | 0.065 | 0.090 | 0.128 | -0.009 | 1.000 |

**Table 1.5.:** Correlation Matrix: Factors for Bank Risk

To further evaluate the relationship between regulatory capital, interest rates, and production costs, I regress the RBCR on a lag of itself and the other factors. I run similar regressions with both leverage ratios and portfolio risk as well. Table 1.6 presents the results of this analysis. For all regressions, the auto-regression coefficients are quite strong, 0.78-0.79 for $\gamma_{it}$, 0.87-0.94 for $\psi_{it}$, and 0.87-0.96 for $\phi_{it}$. This demonstrates the slow moving nature of RBCRs and its underlying components within a firm, over time. Generally, controlling for the other factors, higher risky returns are associated with higher regulatory capital ratios overall, a positive relationship with the proportion of equity financing and a negative relationship with portfolio risk. Deposit rates are negatively correlated with both RBCRs and leverage ratios, and positively correlated with portfolio risk; and the estimated risk free-rate, which acts as a time control because it is the same rate faced by all banks in a given period, is negatively associated with both leverage ratios and portfolio risk, the net effect being an overall positive relationship with risk-based capital ratios. Production efficiency, $\bar{c}_{it}$, like deposit rates, is negatively associated with $\gamma_{it}$ and $\psi_{it}$, and positively associated with $\phi_{it}$.

### 1.4.1. Measuring Risk Using Regulatory Capital

Using the endogenous bank choices that determine the RBCR, $\psi_{it}$ and $\phi_{it}$, and the exogenous components of risk ($\bar{r}_t$, $\sigma^2_t$, $r^f_t$, $r^d_{it}$, and $\bar{c}_{it}$), bank risk exposure, or its probability of default is calculated using equation 1.12. Again, $G_t$ is assumed to be a normal distribution with mean $\bar{r}_t$ and variance $\sigma^2_t$. The gross equity return at which the bank is assumed to be in default or in trouble, $\underline{r}^e$, is assumed to be 0.95, meaning that the bank has earned a single-quarter loss equal to 5% of its book equity value. Of course, a caveat of these results is that this thresh hold may vary across banks and time. However, by keeping it fixed, it allows for a comparison of the levels of risk across these dimensions based on a fixed benchmark. Figure 1.10 shows the cross-sectional average and median default probability over time, based on this calculation. Risk was relatively low and stable throughout most of the sample, despite several noticeable long-term trends in risk-based capital ratios documented in Section 2. The exception in this

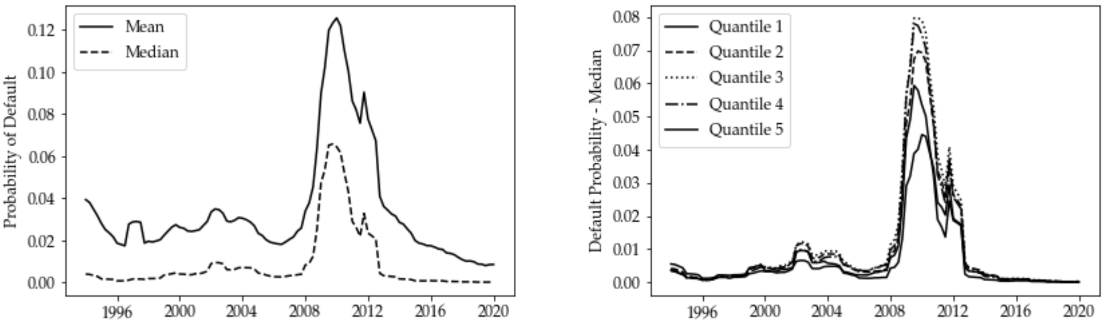
[8] The relationships between interest rates is positive, likely due to the long-term interest rate trend shown in Figure 1.8

**Table 1.6.:** Regulatory Capital and Risk Factors

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| | $\gamma_{it}$ | $\gamma_{it}$ | $\psi_{it}$ | $\psi_{it}$ | $\phi_{it}$ | $\phi_{it}$ | $\psi_{it}$ | $\phi_{it}$ |
| $\gamma_{it-1}$ | 0.790 | 0.779 | | | | | | |
| | (1895.66) | (1803.74) | | | | | | |
| $\tilde{r}_{it}$ | 0.279 | 0.275 | 0.110 | 0.109 | -0.0874 | -0.0850 | 0.109 | -0.0836 |
| | (129.91) | (126.73) | (149.41) | (144.27) | (-34.14) | (-31.69) | (167.72) | (-37.57) |
| $r_t^f$ | 0.0751 | 0.0807 | -0.0797 | -0.0812 | -0.559 | -0.630 | -0.0645 | -0.0300 |
| | (11.68) | (12.29) | (-36.76) | (-35.93) | (-74.35) | (-77.25) | (-36.42) | (-49.31) |
| $r_{it}^d$ | -0.467 | -0.467 | -0.0969 | -0.0944 | 0.568 | 0.619 | -0.100 | 0.385 |
| | (-75.17) | (-73.55) | (-46.17) | (-43.10) | (78.79) | (78.60) | (-61.41) | (68.82) |
| $\bar{c}_{it}$ | -1.951 | -1.787 | -0.437 | -0.372 | 1.827 | 1.724 | -0.653 | 1.145 |
| | (-112.18) | (-100.61) | (-73.44) | (-59.92) | (92.85) | (80.73) | (-142.99) | (92.89) |
| $\psi_{it}$ | | | | | | | | 0.0389 |
| | | | | | | | | (41.72) |
| $\psi_{it-1}$ | | | 0.881 | 0.869 | | | 0.944 | |
| | | | (2281.41) | (2105.16) | | | (3712.56) | |
| $\phi_{it}$ | | | | | | | -0.0029 | |
| | | | | | | | (-33.40) | |
| $\phi_{it-1}$ | | | | | 0.899 | 0.867 | | 0.961 |
| | | | | | (2063.80) | (1729.27) | | (3293.98) |
| Constant | 0.0372 | 0.0394 | 0.0127 | 0.0140 | 0.0694 | 0.0914 | 0.0097 | 0.0216 |
| | (138.77) | (205.72) | (146.56) | (186.58) | (173.40) | (210.17) | (95.56) | (68.67) |
| Observations | 796264 | 796264 | 796264 | 796264 | 796264 | 796264 | 796264 | 796264 |
| Fixed Effects | No | Yes | No | Yes | No | Yes | No | No |
| 3SLS | No | No | No | No | No | No | Yes | Yes |

$t$ statistics in parentheses

period of predominantly low default risk was during the financial crises, when default risk spikes. This provides come confidence in this measure of default risk as the spike coincides with an actual spike in bank defaults.




**(a)** Average and Median Default Risk

**(b)** Default Risk and Bank Size

**Figure 1.10.:** Size Bias in Individual Components

## 1.5. Conclusion

In this paper, I consider to what extent regulatory capital ratios, a metric used to restrict bank behavior, corresponds to their risk exposure. I first document key patterns observed in regulatory capital ratios, and its underlying components, in a panel of US banks. I then provided a framework for mapping capital ratios to a measure of bank risk exposure, and show that this relationship depends on other factors including interest rates and production costs. I then quantify these factors in order to determine to what extent bank risk is driven by endogenous variation in bank regulatory capital, or variation in the other circumstances faced by a given bank.

Understanding the implications of cross-sectional and time series differences in regulatory capital in terms of their representation of bank risk exposures is important for the implementation of capital regulation. The reason is that depending on the co-determining risk factors, a bank may be able to operate at more a less safely at different levels of capital in different circumstances. This implies that a lower bound regulatory capital constraint will represent different risk levels for individual banks at different points in time. This provides motivation for the more conditionally determined capital requirements, such as cyclical capital buffers and systemically important capital buffers that have been implemented since the financial crises. A promising regulatory development in this regard are stress tests which are conducted at the bank-level, and can then capture the macro and individual conditionalities. The intention of this paper is to contribute to the interpretation of levels of bank capital so that regulatory developments can be better targeted in the future.

# 2. The Incidence of Capital Regulation

## 2.1. Introduction

This paper is focused on two related questions. The first is: what key adjustments occur in the banking industry in response to a change in capital regulation? Second, how do these adjustments determine the regulatory incidence? In what follows, I consider these questions in the context of a dynamic model of the banking industry. Three model features are emphasized as important to capturing key industry adjustments. The first is endogenous determination of lending and deposit rates. This is a key feature because it is through market equilibrium that banks share the cost of a regulatory policy with its customers. The second is bank heterogeneity which generates cross-sectional differences in regulatory capital. When there is positive variation in the cross-section, some banks may be constrained by the regulation while others are not. Lending and deposits can be reallocated from constrained to unconstrained banks. The third is an endogenous industry structure, which in the long-run, allows entry and exit in response to changes in regulation. New banks may enter to take up lost output, or banks can exit due to restricted profit making opportunities.

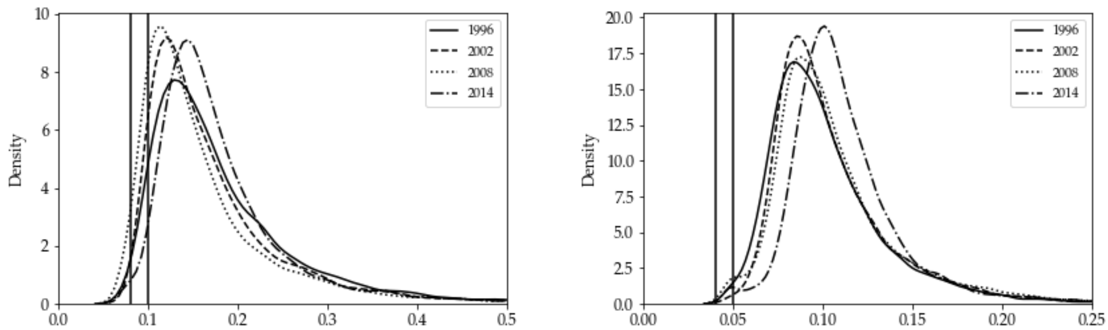
The view taken in this paper is that of banks as financial intermediaries, borrowing from savers and lending to borrowers. Savers lend to banks primarily in the form of liquid demand deposits. Banks lend to borrowers in the form of risky long-term loans. The result is a transformation of risk on bank balance sheets, a valuable service performed by the banking system. However, risk transformation also makes banking unstable. Illiquid investment financed by liquid claims can result in runs[1]. Asset return volatility can generate losses, which may ultimately lead to bank failure. Additionally, several features of the industry present opportunity for excessive risk-taking by banks. Deposit insurance guarantees the repayment of a large portion of bank debt. This (partially) disconnects borrowing costs from the risk of bank default. Also, their assets can be opaque and difficult to evaluate by outsiders. This makes it hard for financiers (savers or equity holders) to assess the portfolio risk of their financial institution.

These aspects of banking present potential opportunities for moral hazard. Therefore, in order to limit excessive risk-taking behavior, banks are regulated. Capital requirements are a key component of this regulation, and they limit risky bank behavior in several ways. Leverage constraints require banks to fund themselves with a minimum proportion of equity. A higher proportion of equity financing acts as a buffer against income volatility. It also increases the amount of exposure bank owners have to negative earnings shocks (skin-in-the-game). Alternatively, risk-based capital regulation limits the proportion of debt funding relative to bank asset risk. The result is a limit on the permitted amount of exposure to asset return volatility conditional on leverage.

[1] Diamond and Dybvig 1983

How can banks adjust to binding capital requirements? First, they can increase their equity capital. However, this is costly for bankers because it requires either a reduction in dividends paid or an increase in capital contributed. Alternatively, a bank can reduce its debt, which primarily consists of demand deposits. Conditional on a level of equity capital, lower debt reduces leverage, allowing banks to satisfy a lower-bound constraint. [2] Finally, in response to a binding risk-based requirement, a bank can change the composition of its asset portfolio. Risk-based capital requirements are represented by a minimum ratio of equity to a risk-weighted measure of assets. Risk-weighted assets are a weighted sum of the assets in a bank's portfolio. The weights for an individual asset is (ideally) proportional to that asset's default risk. Loans carry high positive weights, while cash and treasury securities have a weight of zero. This implies that in order to satisfy the regulation, a bank can shifts its investment from loans to cash or bonds. The result is lower risk-weighted assets, increasing its risk-based capital ratio in order to satisfy a minimum requirement.

These levers of adjustment imply a distribution of the burden of capital regulation between borrowers, savers, and banks. If constrained banks decide to reduce debt or deposits, the result is a reduction in demand for deposits by the banking sector. In equilibrium, deposit rates will fall, reducing welfare for savers. Alternatively, if banks that are constrained by risk-based capital regulation decide to reduce lending. This shift in their investment to cash or bonds results in a reduction in aggregate lending supply. Lending rates would then rise, reducing borrower welfare. If banks decide to increase equity capital, this represents a cost to bank shareholders, reducing their payoff. Therefore, understanding how banks respond to the regulation is needed to evaluate the incidence of the regulation. This incidence is a key metric in evaluating the overall consequences of the regulatory policy.




**(a)** Risk-based Capital Ratio **(b)** Leverage Ratio

**Figure 2.1.:** Regulatory Capital Distributions

The discussion thus far has focused on banks that are constrained by regulation. However, in reality, there is wide variation in the cross-section of bank regulatory capital ratios. Figure 2.1 shows the cross-sectional distribution for a few selected years: risk-based capital ratios in the left panel, leverage ratios in the right. Also shown are the regulatory constraints; and what

[2]In the regulatory vernacular, this is known as a leverage constraint, but is typically represented by the proportion of equity in the bank's financing.

is observed is that while a proportion of banks are near the constraints, a majority of banks are away. This calls into question the extent to which capital requirements are "binding" for the industry as a whole, and introduces an alternative channel through which the banking industry can adjust to capital regulation. If a subset of banks are constrained, and forced to limit lending or deposit-taking, unconstrained banks may increase their banking activity in response, compensating for the restricted output of constrained banks. This occurs because, as the constrained reduce their banking activity, lending rates increase while deposit rates fall, inducing a response from unconstrained bank who now face more favorable market conditions. This implies that the magnitude of the decline in aggregate lending or deposit creation, which determines the burden placed on borrowers and savers, will depend on how much within-industry reallocation occurs.

This proposed mechanism that allows for reallocation from constrained to unconstrained banks, namely the profitable opportunities provided due to restricted competition, may also present aspiring banks with sufficient incentive to enter. This presents another margin along which the banking industry can adjust that will ultimately determine the impact of the regulation: endogenous entry and exit. As banks enter, they further make-up for the shortfall in banking activity, lessening the burden on bank customers in the longer-run. Therefore, in order to properly assess the ultimate impact of the regulation, endogenous entry and exit dynamics and the resulting industry structure should be allowed to adjust in response to the regulation.

In the sections to follow, I develop a dynamic industry equilibrium framework to evaluate how the banking industry adjusts to regulation along these margins, and the cost for participants in the industry. The model is similar in style to Hopenhayn 1992, where the supply of deposits and demand for loans represented by aggregate functions. Banks use equity and deposits to finance a portfolio of loans and bonds, and face restrictions on leverage and the ratio of equity to loans. Heterogeneity is generated by idiosyncratic shocks to loan returns, and persistent differences in production costs. I distinguish between short-run and long-run stationary equilibrium. This is to disentangle the effect of short-run reallocation in response to changing prices, and the long-run impact of changes in the structure of the industry. I then define three different measures of changing aggregate surplus for borrowers, savers and banks: the first due to direct restrictions on bank behavior, the second due to the corresponding changes in short-run equilibrium, and the third to account for changes in the transition to a new long-run stationary equilibrium.

I calibrate the model to match long-run features of the US banking industry from the years 1993-2006. The key targeted moments include average regulatory capital ratios, both risk-based and the standard leverage ratio, and average interest rate margins. As is relatively standard in the literature, I use the distinction between large and small banks in the data to calibrate heterogeneity in bank production costs. The model is used to assess a counterfactual increase in capital regulation that would be binding for an average large bank but not for an average small bank. The result is a large reduction in lending and deposit-taking, with a corresponding change in rates due to the direct restriction on bank choices. This implies a large loss in surplus for all agents as well. However, this effect is mostly eliminated once interest rates are endogenized, and banking activity is reallocated across the sector. The result being, that although the surplus of borrowers and savers is still negative, value for bankers improves overall. In the long-run, firms enter, and the size of the banking industry grows as well. This induces outward shifts in both lending supply and deposit demand, resulting in further gains for bankers. It also increases the surplus of borrowers and savers, and although the surplus of borrowers is still slightly negative, savers gain enough to ultimately have positive surplus, leaving borrowers as the only (slight) loser due to the tightening regulation.

The paper proceeds as follows. After a description of the current state of the literature and a brief discussion on actual current capital regulation, I describe the model in Section 2. This includes both a description of the dynamic problem of an individual bank, as well as the definition long-run stationary equilibrium. In Section 3, I discuss the comparative statics used to evaluate the costs of the regulation, including the decomposition between the direct effect, the short-run effect, and the long-run effect. In Section 4, the model is calibrated and a counterfactual is analyzed. Section 5 concludes.

### 2.1.1. Literature

This paper is primarily related to recent literature analyzing the equilibrium effects of capital regulation. In general equilibrium, an early paper to do this quantitatively is Van denHeuvel 2008, which precedes similar work in Nguyen 2015, Van denHeuvel et al. 2016 and Begenau 2020. In these papers, banks intermediate between households and firms, and capital regulation eliminates a moral hazard problem due to deposit insurance. Van denHeuvel 2008 derives expressions for the welfare cost of regulation and the optimal policy, and estimates each using aggregate data. Begenau and Landvoigt 2018 then adds an unregulated banking sector to evaluate similar metrics, while Davydiuk 2017 evaluates optimal regulation from the Ramsey problem in the general equilibrium banking model. In these papers, banks are ex-ante homogenous (except for shadow banks in Begenau and Landvoigt 2018). This allows for aggregation of the banking sector; however, this it also generates a homogenous response with respect to regulation. Without differentiated responses, there is no partially binding capital requirements, and it is then not possible to see the reallocation in the provision of bank services within the industry, a key feature of interest in this paper.

This then connects this paper to the research looking at heterogeneous responses to bank regulation. In this literature, there are generally three different frameworks. The first are stand alone quantitative dynamic banking models used to assess the effects of capital regulation. Examples in this literature are De Nicolo and Lucchetta 2014 and Mankart, Michaelides, and Pagratis 2020. In these papers, the focus is on the role of capital regulation in dynamic problems of individual banks, and they exclude a notion of industry dynamics. The second are stand-alone industry equilibrium frameworks, which includes Zhu 2007 and Rıos-Rull, Takamura, and Terajima 2020. Zhu 2007 develops a banking model similar to Cooley and Quadrini 2001 that features endogenous industry dynamics. Banks make lending and borrowing decisions subject to regulation based on those choices. He evaluates the effects of this regulation on banking industry output and risk-taking in the stationary industry equilibrium. Similarly, Rıos-Rull, Takamura, and Terajima 2020 also develops a quantitative industry equilibrium model, but one that also features a wholesale debt choice by the bank. The wholesale borrowing cost is dependent on bank default risk, introducing a market discipline mechanism, and the focus here is on the effect of counter-cyclical regulation in the stationary equilibrium. The main difference between those two papers and this one is market pricing, which is an emphasis here. In those papers, debt, deposits in Zhu 2007 and wholesale debt in Rıos-Rull, Takamura, and Terajima 2020 are priced using arbitrage relationships, and depend on an individual bank's default risk, and lending returns are taken to be exogenous. In this paper, debt or deposit and loan rates are priced in explicit deposit and lending markets, respectively. The importance of this here is that it allows burden sharing of the costs of capital regulation between the banking sector and bank customers.

Finally, then, the third type of framework in this literature are those featuring industry dynamics and market determination of lending and deposit rates. This includes the general equilibrium frameworks in Corbae, D'erasmo, et al. 2013, Goel 2016, and Corbae and D'Erasmo 2019. Corbae, D'erasmo, et al. 2013 and Corbae and D'Erasmo 2019 have a generally similar setup. The banking industry has a leader-follower structure, with a few (Corbae, D'erasmo, et al. 2013) or one (Corbae and D'Erasmo 2019) large banks with market power. They compete with a competitive fringe, a continuum of small banks, the distribution of which evolves over time. The focus in these papers is on industry dynamics, where Corbae and D'Erasmo 2019 is augmented with a richer balance sheet in order to consider the effects of risk-based capital requirements on those dynamics. In Goel 2016, banks intermediate between households and firms, as in the previously discussed general equilibrium models. Heterogeneity generates a distribution of banks, and the focus is on the stationary general equilibrium of the model. The interest in that paper is on the effects of size-dependent regulation on this equilibrium. The focus of those papers is somewhat different than the focus here. Here, I am explicitly interested in the reallocative effects of changes in capital regulation, both within the banking industry and between banks and customers. This is a reason I simplify to a partial equilibrium setup compared to their general equilibrium frameworks. Additionally, again their are some structural differences between the banking industries in those models compared with the one in this paper that will impact those effects. For example, the presence of a dominant bank or banks in Corbae, D'erasmo, et al. 2013 and Corbae and D'Erasmo 2019, as well as the fact that deposits are largely determined as the outcome of a stochastic Markov process for banks. In Goel 2016, banks own firm equity and so there is no borrower to explicitly reallocate the impacts of constraining regulation.

This paper is also connected to the general literature on industry equilibrium frameworks. Notable works in this area include Jovanovic 1982, Hopenhayn 1992, Hopenhayn and Rogerson 1993, Ericson and Pakes 1995, Cooley and Quadrini 2001, etc. The model here is closest in structure to the model in Hopenhayn 1992. In that paper, there is a single dynamic state variable, firm productivity. Firms' choice of labor is essentially static, and so dynamic optimization is an entry/exit decision based on the realized value of productivity. Here, the combined net worth/productivity state acts in a similar fashion. However, the firm/bank has multiple choice variables, some of which are subject to potentially binding constraints, and interest is in these choices and market outcomes. This, then, is more similar to Hopenhayn and Rogerson 1993, where firms face frictional labor adjustment and interest is in labor market outcomes. Another important similarity between this and Hopenhayn 1992 is the endogeneity. There, the output price and wage are determined simultaneously with the industry structure in equilibrium, as lending and deposit rates are in this paper. Compare this with another similar model in Cooley and Quadrini 2001. In that framework, there is a firm/state or "type" specific price, the borrowing rate, which reflects a given type of firm's probability of default. In that sense, the debt of each firm type is a unique market good. This is where the role of deposit insurance is important in the banking context. When debt is insured, it no longer needs to be treated as differentiated goods in this way, and can operate in a homogenous market. This is how it is treated here as it allows for thinking of the sector as an aggregate producer of deposits/liquidity, the extent of which may vary.[3]

### 2.1.2. A Brief Primer on Capital Regulation

Although pre-existing capital regulation did exist, the modern advent of capital requirements begins with implementation of the Basel I Capital Accord. These rules were developed by the BCBS[4] in the 1980's, and were to be fully implemented in the US by the end of 1992. Although standard restrictions on bank leverage were included, the key innovation in Basel I was the implementation of risk-based requirements. The reasoning was that simply restricting leverage allowed banks to shift their portfolio in order to increase their risk and returns. The correction then was to stipulate a regulation that tied banks' required capital with the construction of their asset portfolio. The regulation stipulated broad categorizations of assets, where each grouping is assumed to have a similar level of default risk. One issue with this regulation, however, was that this categorization left open scope for arbitrage, where banks can alter their assets within categories. Basel II, the second iteration created and implemented in the mid-2000's, was then intended to improve the calculation of risk-weighted assets. The third iteration, Basel III, was a response to lessons learned from the financial crises. The risk categories were further revised, and additional required capital buffers were added for large institutions and to account for business cycles dynamics.

The capital restrictions that banks must satisfy can broadly be summarized as follows[5]. Currently, there are four different types of regulatory capital ratios. Banks must have a minimum ratio of common equity Tier-1 capital to risk-weighted assets of 4.5 percent; a minimum ratio of Tier-1 capital to risk-weighted of 6 percent; a minimum ratio of total equity capital to risk-weighted assets of 8 percent; and a minimum ratio of Tier-1 capital to total assets of 4 percent. At this point, a discussion about the different measures of equity capital is in order. Common equity Tier-1 capital, which is the narrowest measure, is primarily made up of common stock and retained earnings. Total tier-1 equity is common tier-1 capital plus certain forms of preferred stock and a few other minor instruments. Total equity capital is the sum of Tier-1 capital and Tier-2 capital, where Tier-2 capital includes a percentage of allowances for loan losses, other preferred stock, subordinated debt, and a few other minority interests.

In addition to the minimum requirements specified above, there are other capital level

[3] In Zhu 2007 and Rıos-Rull, Takamura, and Terajima 2020 debt is priced as in Cooley and Quadrini 2001, and debt pricing is a focus of the later. In Rıos-Rull, Takamura, and Terajima 2020, there are both insured deposits and wholesale debt, where only wholesale debt is priced as a differentiated good.

[4] Basel Committee on Bank Supervision.

[5] Source: FDIC's Risk Management Manual of Regulation Policies, 2.1-2.6, Capital.

designations. A bank is considered "well-capitalized" if it has a common equity Tier-1 capital ratio of at least 6.5 percent; a Tier-1 risk-based capital ratio of 8 percent; a total risk based capital ratio of 10 percent; and a Tier-1 leverage ratio of 5 percent. Also, with the implementation of the Basel III requirements, large banking institutions must have a higher Tier-1 leverage ratios. For banks with more than $250 billion in assets, they must have an additional 3 percent of Tier-1 capital to total assets, and banks with more than $700 billion in total assets are required to have an additional 2 percent on top of that. All banks are required to now have a "capital conservation buffer" related to these requirements. If a bank's buffer (the proportion beyond the minimum) for any of the risk-weighted capital ratios falls below 2.5 percent, restrictions on capital distributions (i.e. dividend payouts) are automatically put into place. In general, a violation of any of the above capital ratios may prompt an enforcement action by regulators, requiring a recapitalization by either restricting dividend payouts, forcing the issuance of new capital, or restricting asset growth.

In my framework, equity capital is treated uniformly, meaning I do not make distinctions between different types of capital. The model regulations can primarily be thought of as corresponding to the total equity capital ratio and the Tier-1 leverage ratio. Generally, this sets restrictions on equity to risk-weighted assets and equity to total assets. In the model, I distinguish between loans and bonds. Loans are the model equivalent of risk-weighted assets, bonds being the residual total assets. As mentioned, given the corresponding requirements, the minimum bounds on these ratios are 8 percent and 4 percent respectively. The quantitative model is calibrated with these restrictions in mind.

## 2.2. Dynamic Model of the Banking Industry

### 2.2.1. An Individual Bank's Problem

Time is discrete and infinite; and generally, I take an industrial organization approach to banking[6]. Under this view, Banks use equity $e$ and deposits $d$ to invest in a combination of risky loans $l$ and risk-free bonds $b$, which implies that the following balance sheet condition holds with equality each period:

$$e + d = l + b \tag{2.1}$$

Banks also produce services for its depositors (liquidity services) and its borrowers (monitoring). As is relatively standard in the bank production literature, service production is represented by the dual cost function. These production costs are a function of the quantity of loans made and the quantity of deposits taken. The standard argument (Sealey Jr and Lindley 1977) here is that total lending and deposit taking is proportional services produced. The cost function represents the cost of an optimal combination of inputs required to produce services needed for a given level of lending and deposit taking. This cost function is denoted as $c(l, d, z)$, where $c : \mathbb{R}^3 \rightarrow \mathbb{R}$ and is assumed to be continuous and increasing in all arguments. It is also assumed to be strictly convex in $l$ and $d$, and $c(0, 0, z) \geq 0$. The third argument $z \in Z = \{z_1, z_2, ...z_N\}$ is

[6] See Freixas and Rochet 2008.

a parameter govern the bank's productivity. It evolves according to a Markov process, where $\Gamma(z'|z)$ represents the transition probability from $z$ to $z'$.

Banks pay an interest cost at a rate $r_d$ on their deposits[7], and earn interest at a rate $r_f$ on bonds, and earn interest at a rate $r_l$ on loans. As will be discussed, the interest rates on loans and deposits will be endogenous, while the risk-free rate is assumed to be exogenous. Additionally, the return on loans is subject to a proportional $\epsilon$ shock. It is in this sense, loans are "risky," and $\epsilon$ is assumed to be drawn i.i.d from distribution $G(.)$ defined over the bounded interval $\left[\underline{\epsilon}, \bar{\epsilon}\right]$ . Income is earned via interest on loans and bonds, while the bank pay an interest cost on deposits and production costs. From this, I can write the bank's profit function as:

$$\pi = \epsilon r_l l + r_f b - r_d d - c(l, d, z) \tag{2.2}$$

or alternatively, after substituting in the balance sheet constraint:

$$\pi = r_f e + (\epsilon r_l - r_f) l + (r_f - r_d) d - c(l, d, z) \tag{2.3}$$

Written in this way, I see two important interest rate spreads showing up in the bank's profit function. The one multiplying loans, $\epsilon r_l - r_f$, is the loan spread, or the realized excess return the bank earns loans beyond what it can earn on a risk-free bond. This spread is a by-product of the portfolio choice problem the bank is facing and its loan issuance cost. The second multiplying deposits, $r_f - r_d$, is the bank's financing margin, or the difference risk-free assets and deposit rates. This non-zero spread arises due to positive costs of providing liquidity services. This a similar margin as the one highlighted in DeAngelo and Stulz. 2015.

The bank's objective is to maximize the stream of dividends paid to shareholders. Future dividend payments are discounted at a subjective discount rate $\beta \in (0, 1)$. Representing the model recursively, each bank enters a period with a net worth $n$ and productivity $z$. They then have the opportunity to choose the equity they want to carry into the next period $e$. Dividends are given by the difference $n - e$. The bank's preference for paying dividends is represented by the utility function $u : \mathbb{R} \to \mathbb{R}$, which is assumed to be continuous, increasing, concave. If $e > n$, the bank is raising outside equity, and faces a proportional issuance cost $\kappa$. Net worth in the subsequent period is equal to equity plus profits or:

$$n' = e + \pi = (1 + r_f) e + (\epsilon r_l - r_f) l + (r_f - r_d) d - c(l, d, z) \tag{2.4}$$

After its realization of net worth and updated productivity $z'$, the bank can exit the industry. In this case, it receives zero in perpetuity.

Finally, the bank faces both a regulatory requirement on its leverage and its risk-based capital ratio. Specifically, a regulatory policy is given by the vector $\gamma = \left[\gamma_l, \gamma_d\right]$, where the leverage constraint $\gamma_d$ says that $e/d \geq \gamma_d$, and the risk-based capital regulation $\gamma_l$ says $e/l \geq \gamma_l$. These restrictions along with non-negativity imply the following restrictions on bank choices:

$$e \geq 0, \quad 0 \leq l \leq e/\gamma_l, \quad 0 \leq d \leq (1 - \gamma_d) e/\gamma_d \tag{2.5}$$

[7]Deposits and liabilities are used interchangeably.

As in Hopenhayn 1992, there is no aggregate uncertainty from the perspective of an individual bank. Banks are atomic, and given that all information about the transition probabilities is known, banks can fully predict the industry dynamics. This means they also can predict the full sequence of interest rates. This sequence of interest rates is denoted $r = \{r_l, r_f, r_d\}$. Additionally, since I are interested in counterfactual variations in capital requirements, they are included as a component of the state space. Given an initial state $(n_0, z_0)$ and a policy $\gamma$, the bank's recursive problem is defined as follows:

$$V(n,z,r,\gamma) = \max_{e,l,d} u(n-e) + \beta\mathbb{E}\Big[\max\{0, V(n',z',r,\gamma)\}\Big] \tag{2.6}$$

subject to:

$$n'(e,l,d,\epsilon,z,r) = (1+r_f)e + (\epsilon r_l - r_f)l + (r_f - r_d)d - c(l,d,z) \tag{2.7}$$

$$e \geq 0, \quad 0 \leq l \leq e/\gamma_l, \quad 0 \leq d \leq (1-\gamma_d)e/\gamma_d \tag{2.8}$$

$$n_0, z_0 \text{ given} \tag{2.9}$$

where the expectations operator $\mathbb{E}$ is with respect to $\epsilon$, $z'$. Here, (5) is the bank's recursive value function. Note, the second max operator represents the bank's ability to exit given its current state. Again, this implies that the bank receives zero in perpetuity. The first max operator then represents the bank's optimization problem conditional on remaining in the industry. Equation (6) is the law of motion for net worth, and (7) is the various constraints on bank choices. One thing to notice about the constraints in (11) is that given a choice of $e$, the $l$ and $d$ choices are restricted to bounded convex sets. Additionally, due to the cost of equity issuance, there will be an upper bound to the desired choice of equity, so it is also bounded.

The solution to this problem is represented by the value function $V(n,z,r,\gamma)$. The Bellman equation has the conditions necessary for a contraction mapping given the restrictions on $\beta$. The value function is increasing in $n$, as is obvious by the role $n$ plays in the bank's payoff function. It is also decreasing in $z$ due to the fact that costs are increasing, and so tomorrow's net worth is decreasing. Also included in the solution to this problem are the policy function $e(n,z)$, $l(n,z)$, and $d(n,z)$, which represent the bank's balance sheet and production choices given their beginning of period net worth and productivity. The final component of the bank's policy is an exit decision, which is made at the end of the period after the realization of their net worth and updated productivity. To characterize this decision, define $x(n,z,r,\gamma)$ as:

$$x(n,z,r,\gamma) = \begin{cases} 1 & \text{if } \max\{u(n,e) + \beta\mathbb{E}\max\{0, V(n',z',r,\gamma)\}\} < 0 \\ 0 & \text{if } \max\{u(n,e) + \beta\mathbb{E}\max\{0, V(n',z',r,\gamma)\}\} \geq 0 \end{cases} \tag{2.10}$$

Generally, the bank will default on their obligations and exit the industry for some $\underline{n}(z)$, meaning $V(n,z,r,\gamma) = 0$. In this case I can define $x(.)$ as:

$$x(n,z,r,\gamma) = \begin{cases} 1 & \text{if } n \leq \underline{n}(z) \\ 0 & \text{if } n > \underline{n}(z) \end{cases} \tag{2.11}$$

where for each $z$, $n \leq \underline{z}$ implies $V(n,z,r,\gamma) < 0$.

### 2.2.2. Industry Dynamics

I can use the optimal value function $V(.)$ and set of policy functions $\{e(.), l(.), d(.), x(.)\}$ to define industry dynamics. The distribution of banks is captured by $\mu(n,z)$, where the industry begins at some $\mu_0$. The function $\mu(n,z)$ represents the counting measure of banks at state $(n,z)$. Using the law of motion for net worth in equation (5), define $\tilde{n}(n,z,\epsilon,r,\gamma)$ as:

$$\tilde{n}(n,z,\epsilon,r,\gamma) = n'(e(n,z), l(n,z), d(n,z), \epsilon, z, r) = (1+r_d)e(n,z,r,\gamma)+ \tag{2.12}$$

$$(\epsilon r_l - r_f)l(n,z,r,\gamma) + (r_f - r_d)l(n,z,r,\gamma) - c(l(n,z,r,\gamma), d(n,z,r,\gamma), z) \tag{2.13}$$

which is the law of motion after substituting in the optimal policy function. It is now a function of the bank's initial state, the idiosyncratic shock $\epsilon$, and the industry parameters $(r,\gamma)$. Denoting $\Phi(n',z'|n,z)$ as the transition function of incumbent firms, it is defined as:

$$\Phi(n',z'|n,z) = \big[1 - x(n',z',r,\gamma)\big]\Gamma(z'|z)\left[\int_\epsilon \mathbb{I}_{n' \geq \tilde{n}(n,z,\epsilon,r,\gamma)} \mathrm{d}G(\epsilon)\right] \tag{2.14}$$

where $\mathbb{I}_{n' \geq \tilde{n}(n,z,\epsilon,r,\gamma)}$ is an indicator equal to 1 if $n' \geq \tilde{n}(n,z,\epsilon,r,\gamma)$. This gives the measure of banks starting at state $(n,z)$, and ending up with productivity $z'$ and a net worth less than or equal to $n'$ in the following period. This implies that the transition of incumbent firms is:

$$\int \Phi(n',z'|n,z)\mathrm{d}\mu(n,z) \tag{2.15}$$

Entering banks start with net worth $n = \bar{n}$. They must also pay an entry cost $c^e$. If a bank decides to enter, they first pay the cost, then they draw an initial productivity $z$ from a distribution $\lambda(z)$. This implies that firms will enter as long as:

$$\sum_{z'} V(\bar{n},z',r,\gamma)\lambda(z') \geq c^e \tag{2.16}$$

If $M$ is the measure of entering firms, an object that will be determined in equilibrium, I can define the law of motion for the industry state as:

$$\mu'([-\infty,n'],z') = \int \Phi(n',z'|n,z)\mathrm{d}\mu(n,z) + M\lambda(z')\big[1 - x(\bar{n},z',r,\gamma)\big]\mathbb{I}_{n' \geq \bar{n}} \tag{2.17}$$

where $\mathbb{I}_{n' \geq \bar{n}}$ is an indicator that is equal to 1 if $n' \geq \bar{n}$.

### 2.2.3. Long-run Stationary Equilibrium

The equilibrium concept used in this paper is long-run stationary industry equilibrium. This is standard with this type of framework. Defining equilibrium first requires specifying how equilibrium "prices" are determined. The equilibrium prices here are the interest rates on loans and deposits, while the risk-free rate $r_f$ is assumed exogneous. This can be due to supply and demand for risk-free assets extends far beyond the banking industry. Alternatively, it could be related to management by a monetary authority. For the deposit rate, I assume it is determined by an aggregate industry supply curve. Similarly, the lending rate is determined by aggregate loan demand. In particular. $r_l = r_l(L)$ where $L$ is aggregate lending; $r_l = r_d(D)$ where $D$ is aggregate deposits. $r_l(.), r_d(.)$ are such that $\partial r_l/\partial L < 0$, $\partial r_d/\partial D > 0$.

In a general equilibrium context, these functions would be derived from explicit borrower and saver problems. I abstract from this to focus specifically on the industry effects. Demand for loans can be generated by investment in capital for use in a decreasing return-to-scale production technology. Deposit supply can be generated by households that value the liquidity provided by deposits. From the perspective of the industry, however, the only concern is the loan demand and deposit supply it faces[8]. The derivative assumptions on these functions simply ensure proper demand and supply relationships. From the solution to the bank's problem, I can define aggregate lending as:

$$L = \int l(n, z, r, \gamma) \mathrm{d}\mu(n, z) \tag{2.18}$$

and aggregate deposits or bank borrowing as:

$$D = \int d(n, z, r, \gamma) \mathrm{d}\mu(n, z) \tag{2.19}$$

With the demand for loans and supply of deposits defined, along with the transition dynamics, I can define the following:

**Long-run (Stationary) Equilibrium:**
Conditional on a regulatory policy $\gamma$, a long-run (stationary) equilibrium is a set

$$\Xi = \{V, x, e, l, d, r, \mu, M\}$$

such that:

(1.L) Given $r$ and $\gamma$, the functions $V$, $x$, $e$, $l$, and $d$ represent an optimal solution to the bank's problem.

(2.L) Given optimal policies and interest rates, the industry distribution satisfies:

$$\mu([-\infty, n'], z') = \int \Phi(n', z'|n, z) \mathrm{d}\mu(n, z) + M\lambda(z') \left[1 - x(0, z', r, \gamma)\right] \mathbb{I}_{n' \geq 0}$$

(3.L) Given optimal policies, the stationary distribution and $r_f$, $r = \left[r_l, r_f, r_d\right]$ is such that:

$$r_l = r_l\Big(\int l(n, z, r, \gamma) \mathrm{d}\mu(n, z)\Big)$$

$$r_d = r_d\Big(\int d(n, z, r, \gamma) \mathrm{d}\mu(n, z)\Big)$$

[8] Additionally, I abstract here from strategic market power.

(4.L) Given optimal policies and interest rates:

$$\sum_{z'} V(\bar{n}, z', r, \gamma)\lambda(z') = c^e$$

Conditions (1.L) and (3.L) represent optimization by the bank and market clearing, respectively. Condition (2.L) is the condition on industry dynamics, and says that it is at a stationary point. Finally, condition (4.L) is the free-entry condition, which says firms will enter until the expected value of entry is zero.

## 2.3. Comparative Statics

### 2.3.1. Incidence of a Regulatory Policy

Evaluating the incidence of capital regulation is a comparative static exercise. One can think of the burden caused by any particular regulation as a comparative static with respect to the non-regulation case, allowing for an evaluation of the implications of that regulation. Here, I define the notion of surplus from which I will evaluate the incidence of capital regulation. I rely on a discrete description of these changes as opposed to a variational approach to industry comparative statics[9]. The reason for this choice is the non-differentiability of the payoff and policy functions.

In a stationary equilibrium, quantities $(L, D)$ and prices $(r_l, r_d)$ are functions of a regulatory policy $\gamma$. This implies that the equilibrium objects can be written as a function of that policy:

$$L(\gamma) = \int l(n, z, r(\gamma), \gamma)\mathrm{d}\mu_\gamma(n, z) \tag{2.20}$$

$$D(\gamma) = \int d(n, z, r(\gamma), \gamma)\mathrm{d}\mu_\gamma(n, z) \tag{2.21}$$

and:

$$r_l(\gamma) = r_l(L(\gamma)) \quad r_d(\gamma) = r_d(D(\gamma)) \tag{2.22}$$

where $\mu_\gamma$ is the stationary distribution induced by $\gamma$. From this, saver and borrower surplus is a function of $\gamma$ as well:

$$S_l(\gamma) = \int_0^{L(\gamma)} (r_l(L) - r_l(\gamma))\mathrm{d}L \tag{2.23}$$

$$S_d(\gamma) = \int_0^{D(\gamma)} (r_d(\gamma) - r_l(D))\mathrm{d}D \tag{2.24}$$

Banker surplus is calculated in a similar manner; however, it requires defining the of aggregate loan supply and deposit demand curves away from the equilibrium interest rates. These are

[9] For example, see Mas-Colell, Whinston, Green, et al. 1995, ch. 10.

defined as follows:

$$\bar{L}(r_l,\gamma) = \int l(n,z,[r_l,r_f,r_d(\gamma)],\gamma)\mathrm{d}\mu^*_\gamma(n,z) \tag{2.25}$$

$$\bar{D}(r_d,\gamma) = \int d(n,z,[r_l(\gamma),r_f,r_d],\gamma)\mathrm{d}\mu_\gamma(n,z) \tag{2.26}$$

where the inverses are defined as $\bar{r}_l(L,\gamma) = \bar{L}^{-1}(r_l,\gamma)$ and $\bar{r}_d(D,\gamma) = \bar{D}^{-1}(r_d,\gamma)$. The bankers' surplus in the lending and deposit markets are then given by:

$$S_{bl}(\gamma) = \int_0^{L(\gamma)} (r_l(\gamma) - \bar{r}_l(L,\gamma))\mathrm{d}L \tag{2.27}$$

$$S_{bd}(\gamma) = \int_0^{D(\gamma)} (\bar{r}_d(D,\gamma) - r_d(\gamma))\mathrm{d}D \tag{2.28}$$

Note, here that market surplus is being used as the measure of welfare here for the banks as opposed to total valuation so that it can be comparable with the surplus of bank customers.

Suppose now there is a change in the policy from $\gamma$ to $\bar{\gamma}$. Ultimately, the industry will transition to a new stationary equilibrium. This new equilibrium is represented by new prices $(r_l(\bar{\gamma}), r_d(\bar{\gamma}))$ and quantities $(L(\bar{\gamma}), D(\bar{\gamma}))$. These then correspond to new levels of surplus $S_l(\bar{\gamma}), S_d(\bar{\gamma})$, $S_{bl}(\bar{\gamma})$ and $S_{bd}(\bar{\gamma})$. I define the burden of the change in regulation for bank customers as the percent change in surplus, or:

$$\Delta_i(\gamma,\bar{\gamma}) = \log(S_i(\bar{\gamma})/S_i(\gamma)) \tag{2.29}$$

and the burden for bankers is defined similarly.

### 2.3.2. Decomposing Changes in Surplus

The previously described policy incidence represents the change from one long-run stationary equilibrium determined by the policy $\gamma$ to another determined by the policy $\bar{\gamma}$. But what drives these long-run changes? To provide a richer description of the change in surplus, consider the following decomposition.

First, fix the interest rates and industry structure, and changing the regulatory policy. This implies that the only change in a given bank's problem is the restrictions on their choices. Aggregate lending and deposit taking can be defined from this direct effect as:

$$L^{DE}(\gamma,\bar{\gamma}) = \int l(n,z,r(\gamma),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z) \tag{2.30}$$

$$D^{DE}(\gamma,\bar{\gamma}) = \int d(n,z,r(\gamma),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z) \tag{2.31}$$

which can be substituted into the loan demand and deposit supply functions to yield interest rates:

$$r_l^{DE}(\gamma,\bar{\gamma}) = r_l(L^{DE}(\gamma,\bar{\gamma})) \quad r_d^{DE}(\gamma,\bar{\gamma}) = r_d(D^{DE}(\gamma,\bar{\gamma})) \tag{2.32}$$

with corresponding measures of surplus measures $S_l^{DE}(\gamma,\bar{\gamma})$, $S_d^{DE}(\gamma,\bar{\gamma})$, $S_{bl}^{DE}(\gamma,\bar{\gamma})$, and $S_{db}^{DE}(\gamma,\bar{\gamma})$ (see Appendix for details). The superscript $DE$ stands for direct effect, as this represents the direct effect of a change in policy without equilibrium reallocation. Here interest rates remain fixed at their previous equilibrium level, but bank's are subject to the new regulation. The behavior of unconstrained bank's at the new regulation will be unchanged as nothing about their problem has changed. The choices of banks that are now constrained will respond accordingly. This then will change the aggregate level of lending and deposit taking, and imply new equilibrium interest rates. Importantly, this does not represent a new short-run or long-run equilibrium, which is precisely the point. Here, the purpose is to decompose this direct effect from the equilibrium effects due to endogenous prices. With that said, consider the following definition of short-run equilibrium:

**Short-run Equilibrium:** Given a regulatory policy $\gamma$, and an industry distribution $\mu$, a short-run equilibrium is a set $\Xi^S = \{V^S, x^S, e^S, l^S, r^S\}$ such that:

(1.S) Given $r^S$ and $\gamma$, the functions $V^S$, $x^S$, $e^S$, $l^S$, and $d^S$ represent a solution to the bank's problem.

(2.S) Given optimal policies and $\{r_f, \mu, \gamma\}$, the interest rates $r^S = [r_l^S, r_f, r_d^S]$ satisfy:

$$r_l^S = r_l\Big(\int l^S(n,z,r^S,\gamma)\mathrm{d}\mu(n,z)\Big)$$

$$r_d^S = r_d\Big(\int d^S(n,z,r^S,\gamma)\mathrm{d}\mu(n,z)\Big)$$

Condition (1.S) says that the value and policy functions solve the bank's problem given interest rates, an industry structure, and policy. The second condition (2.S) says that given optimal policies, the market clearing conditions hold. This is consistent with standard short-run/long-run industry analysis.

Given this definition, a change in the policy from $\gamma$ to $\bar{\gamma}$, before the industry structure adjusts, corresponds to the following aggregate lending supply and deposit demand in a new short-run equilibrium:

$$L^S(\gamma,\bar{\gamma}) = \int l(n,z,r(\bar{\gamma}),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z) \tag{2.33}$$

$$D^S(\gamma,\bar{\gamma}) = \int d(n,z,r(\bar{\gamma}),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z) \tag{2.34}$$

with interest rates:

$$r_l^S(\gamma,\bar{\gamma}) = r_l(L^S(\gamma,\bar{\gamma})) \quad r_d^S(\gamma,\bar{\gamma}) = r_d(D^S(\gamma,\bar{\gamma})) \tag{2.35}$$

which also admit a corresponding set of surplus functions $S_l^S(\gamma,\bar{\gamma})$, $S_d^S(\gamma,\bar{\gamma})$, $S_{dl}^S(\gamma,\bar{\gamma})$., and $S_{dl}^S(\gamma,\bar{\gamma})$ (see Appendix for details). Here, in the transition to the new short-run equilibrium, the restrictions on banks have changed and the equilibrium prices have adjusted. However, the industry structure is still left over from the previous policy regime.

The final transition then is to the new long-run equilibrium with a set of surpluses:

$$\{S_l^L(\bar{\gamma}), S_d^L(\bar{\gamma}), S_{bl}(\bar{\gamma}), S_{bd}(\bar{\gamma})\}$$

Note that I can now write the change in surplus $\Delta_i(\gamma,\bar{\gamma})$ as:

$$\Delta_i(\gamma,\bar{\gamma}) = \log\left(\frac{S_i^{DE}(\gamma,\bar{\gamma})}{S_i(\gamma)}\right) + \log\left(\frac{S_i^S(\gamma,\bar{\gamma})}{S_i^{DE}(\gamma,\bar{\gamma})}\right) + \log\left(\frac{S_i(\bar{\gamma})}{S_i^S(\gamma,\bar{\gamma})}\right) \tag{2.36}$$

$$= \Delta_i^{DE}(\gamma,\bar{\gamma}) + \Delta_i^S(\gamma,\bar{\gamma}) + \Delta_i^L(\gamma,\bar{\gamma}) \tag{2.37}$$

so now the change in surplus can be decomposed into three components. The first $\Delta_i^{DE}$ is the change resulting from the direct restriction on bank behavior. Here, banks that are unconstrained under both regulations will produce at the same levels. Only the behavior of banks that are constrained by the regulation will be effected. The second, $\Delta_i^S$, is the change resulting from the transition to the new short-run equilibrium. Prices have been endogenized, allowing other banks to take advantage of restrictions placed on their competitors. This will generate a reallocative effect: bank production is reallocated from the constrained to the unconstrained. Finally, $\Delta_i^L$ is the change resulting from the transition to the new long-run equilibrium. Now the the industry composition changes, and along with it, the equilibrium. This third component is then the effect of long-run changes in industry structure due to a change in the policy. Summarizing, the burden can be decomposed into three components: the direct effect, the short-run equilibrium effect, and the long-run equilibrium effect.

### 2.3.3. Risk at the Stationary Equilibrium

As mentioned in the introduction, capital regulation is meant to limit risk-taking behavior by banks. Any burden imposed by the regulation should then be compared to the impact on risk in the industry. There are a number of ways one could define risk in the model. Generally, one would want to derive an aggregate measure of value-at-risk. However, in this model there is no aggregate uncertainty, only bank specific shocks. Therefore, the number of "failing" banks is known precisely along with their behaviors in each period. The concern for regulators is that banks default on their deposits. This is because deposits are insured and would need to be covered by the regulator. Therefore, the appropriate measure of "risk" here is the quantity or proportion of deposits defaulted on in equilibrium. Note that at the stationary equilibrium, this quantity will remain constant each period.

In equilibrium, the transition function of incumbent banks is given by:

$$\int \Phi(n', z'|n, z) \mathrm{d}\mu(n, z) \tag{2.38}$$

Weighting this by deposits and the optimal exit strategy $x(n', z', r, \gamma)$ gives:

$$\bar{D}(n', z', r, \gamma) = \int x(n', z', r, \gamma) d(n, z, r, \gamma) \Phi(n', z'|n, z) \mathrm{d}\mu(n, z) \tag{2.39}$$

where $\bar{D}(n', z', r, \gamma)$ represents the total deposits defaulted on in equilibrium. The interpretation is as follows: from $\mu(n, z, r, \gamma)$, a proportion $\Phi$ transition to state $(n', z', r, \gamma)$. These banks made deposits $d(n, z, r\gamma)$ and then default according to the rule $x(n, z, r, \gamma)$. Therefore, the incumbent banks that exit are defaulting on their liabilities from the previous period. These liabilities need to be covered by the regulator, and so this is our measure of risk. This can be expressed as a proportion of total liabilities:

$$R(\gamma) = \frac{\bar{D}(n, z, r, \gamma)}{\int d(n, z, r, \gamma) \mathrm{d}\mu(n, z)} \tag{2.40}$$

and the change in risk due to a change in capital regulation as:

$$\Delta^R(\gamma, \bar{\gamma}) = \log(R(\bar{\gamma})/R(\gamma)) \tag{2.41}$$

Given that the purpose of capital regulation is to reduce risk-taking, this measure of risk should be compared with the cost of the regulation in equilibrium. In the following section, I evaluate the burden imposed along with the change in risk for a counterfactual increase in capital regulation.

## 2.4. Quantitative Analysis

### 2.4.1. Data and Calibration

The model is calibrated using data from the US banking industry. The data comes from the FDIC's Reports on Condition and Income (Call Reports). Annual observations are the fourth quarter values reported for each bank. Banks belonging to the same holding company are also combined into a single observation. The model is set to match features of the banking industry from the period 1993-2006. [10]. Since the model is being calibrated to annual data, $\beta$ is set to 0.96. Capital requirements $\gamma$ are set to match the regulation during this time period. Specifically, the lower bound on the ratio $e/l$ was 0.08, and the lower bound on $e/(e+d)$ was 0.04. The policy in the baseline model is then set as: $\gamma = [\gamma_l, \gamma_d] = [0.08, 0.04]$.

For simplicity, I assume that the production of services for loans and deposits are separable. The cost function for each takes a convex form:

$$c^l(l) = l^{\alpha_l} \quad \text{and} \quad c^d(d) = d^{\alpha_d} \tag{2.42}$$

[10] The major change to capital regulation was the introduction of the IRB approach to risk-weighting for large banks. This was the result of the Basel II Capital Accord.

where $\alpha, \gamma > 1$. Productivity $z$ then represents the coefficients on these two independent cost functions. In other words, in each period, a bank realizes a $z = [z_l, z_d]$, a vector of multiplicative cost shocks. Their production cost in a period is then given by:

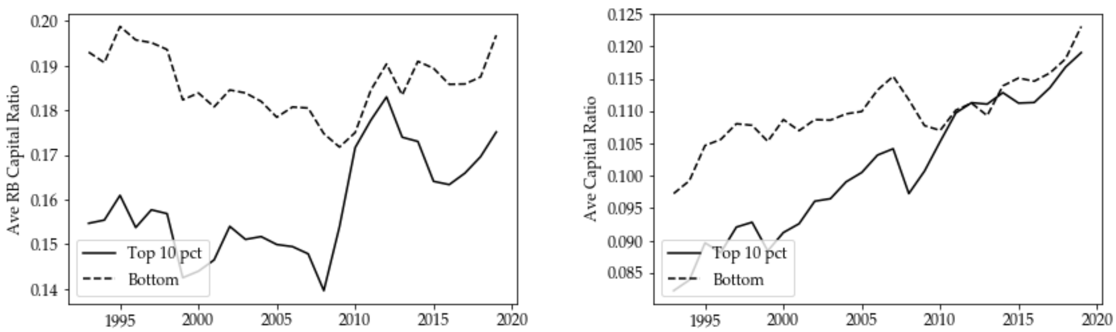
$$c(l,d,z) = \delta + z_l l^{\alpha_l} + z_d d^{\alpha_d} \tag{2.43}$$

where $\delta$ is the bank's fixed cost. For now, in order to limit the computational burden, I assume the following cost parameters $[\delta, \alpha_l, \alpha_d] = [0.00, 1.50, 1.50]$.

The Markov process governing bank productivity $z$ represents the persistent stochastic component in the model. Here, there will be two possibilities for productivity: a high cost state and a low cost state. For example for lending, there exists $z_l^H > z_l^L$, and for deposits $z_d^H > z_d^L$. If $z^i = [z_l^i, z_d^i]$, then a realization $z$ lies in the following state-space:

$$z \in Z = [z^H, z^L] \tag{2.44}$$

The values for $z$ are set to target the average capital ($e/(e+d)$) and risk-based capital ($e/l$) ratios of large and small banks, respectively. Large banks are defined as the top 10% of banks by asset size, and small banks are the remaining. Figure 2.2 shows these averages over time. The figures shows that large banks tend to have lower levels of both ratios over the sample period used. This trend does not necessarily hold in the post-financial crisis era, motivating my choice of time period. In the model, this corresponds to a lower $z$ because it corresponds to lower costs for both loans and deposits. This induces higher lending and deposit taking, and lower ratios.




**(a)** RB Capital Ratios

**(b)** Captial Ratios

**Figure 2.2.:** Bank Regulatory Capital

The setup described implies that at any point in time, banks are either efficient (low cost) or inefficient (high cost). The combinations can be interpreted in a similar way, although with slight differences, to other setups in the banking literature. For example, in De Nicolo and Lucchetta 2014 and Corbae, D'erasmo, et al. 2013, banks face persistent shocks to their deposit taking ability. These are generally referred to as funding shocks. The shock to deposit productivity here can be interpreted similarly, although instead of a direct shock to deposit flows, bank's technically have the "ability" to raise more deposits, however if in the high cost state, it may be prohibitively costly. I think of this as the bank can always open more branches to try to corral more deposits, but how many more branches need to open? Similarly, again De Nicolo and Lucchetta 2014 and Zhu 2007 have persistent shocks to bank loan returns. Here, the shock to loan productivity can be interpreted as the ability to monitor loans to a consistent level of mean returns. One also might be concerned with the additive separability of lending and deposit production. A common argument is that the lending and deposit businesses of a bank cross-subsidize each other. Here this can be controlled for by the fact that deposit and lending shocks move together. For probabilities related to the realizations of $z$, I assume that banks enter in proportion to their type. Since the low cost banks are calibrated to match the top 10% of banks, this implies that $\lambda = [0.9, 0.1]$. The transition probability is then set so that the proportion of low cost types is approximately 10% in the stationary equilibrium. To accomplish this, $\Gamma(z|z)$ is set to 0.95.

In the quantitative analysis, bank risk non-neutrality is due to expensive recapitalization In particular, they face a proportional cost of issuing equity $\kappa$, introducing concavity in their payoff function. This implies the bank's preferences are:

$$u(n, e) = \min\{n - e, (1 + \kappa)(n - e)\} \tag{2.45}$$

Here, I assume $(1 + \kappa) > \beta(1 + r_f)$. This assumption is to assure that the bank's choice of equity remains bounded, and will be true in the calibration of the quantitative model. If this were to not hold, the bank could earn a positive risk-free spread on equity, causing the bank to increase equity infinitely. When it does hold, the bank makes losses on equity retained above net worth. This causes the bank to only raise additional equity if 1) it has to because net worth is negative and it is profitable not to exit, or 2) it is constrained by capital regulation and can profitably take a loss on equity in order increase lending or deposit taking. Regardless, given the strict concavity of $\pi$ w.r.t. $l$ and $d$, at some level they become unprofitable. Then, so does raising more equity to facilitate these activities. It also represents a fixed cost of continuing as a bank if net worth is negative. This will induce positive exit in the model. Additionally, equity/retained earnings function $e(.)$ then has a familiar shape[11] in problems where there is a uni-directional cost of issuing outside equity. In particular, for some $\bar{n}(z)$, $e(n, z) = \bar{n}(z)$ for all $n > \bar{n}(z)$ for each $z$. Additionally, it is weakly increasing for $n \leq \bar{n}(z)$.

The remaining aspects of the quantitative model to describe involve the determination of interest rates. The estimated deposit rate to be targeted is calculated by dividing bank interest expense by total liabilities. This is then averaged across banks in order to generate an estimate for the average deposit rate in a given year. For lending and deposit rate targets, consider the model income earned by a bank in a given period:

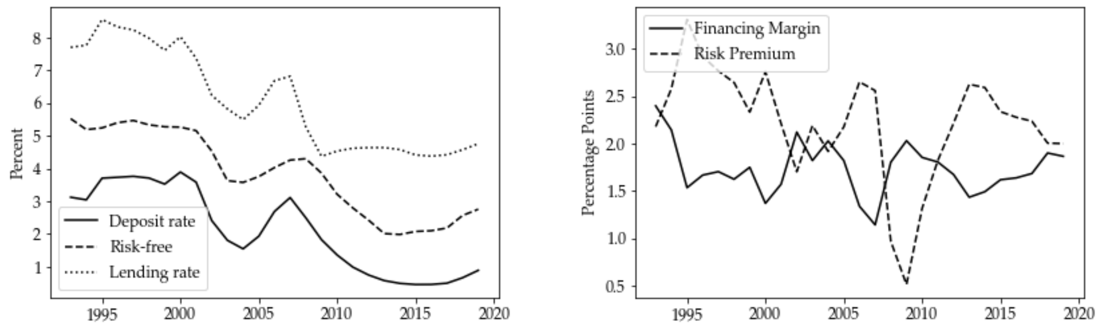
$$I = \epsilon r_l l + r_f b \tag{2.46}$$

Dividing both sides by $l$ and replacing $\epsilon = 1 + \nu / r_l$, this equation becomes:

$$(I/l) = r_l + r_f(b/l) + \nu \tag{2.47}$$

[11] See Cooley and Quadrini 2001.

Annual lending and risk-free rates are then estimated using a cross sectional regression; details are provided in the Appendix. The three estimated interest rates are plotted as time series in the left panel of Figure 2.3. I see the familiar notion of a secular decline in interest rates during this period.[12] In the model, interest rates show up as spreads. In particular, the bank's problem features a risk premium $\epsilon r_l - r_f$ and the financing margin $r_f - r_d$. These two spreads are plotted in the right panel of Figure 2.3. Both were relatively stable until 2006; then, while the financing margin remained stable, the risk-premium collapsed during the financial crises and then recovered. To calibrate the model, I use the average risk-free rate and average margins to generate interest rate moments to target. This yields a target of $\{r_l, r_f, r_d\} = \{7.27\%, 4.81\%, 3.03\%\}$.




**(a)** Interest Rates **(b)** Interest Margins

**Figure 2.3.:** Estimated Interest Rates

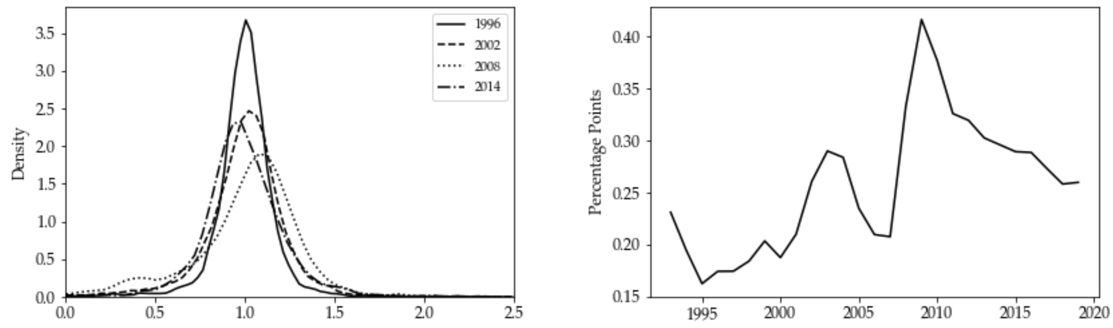
Using the same regression from the interest rate estimation, I can parameterize the distribution $G(.)$. Converting the regression residual $\nu$ back to $\epsilon = 1 + \nu / r_l$, gives a cross-sectional distribution of $\epsilon$'s for each year. This distribution is plotted for a set of select years in the left panel of Figure 2.4. In the right panel of Figure 2.4 is the standard deviation of the realized shocks. The shock volatility was relatively stable pre-financial crises as well. The model distribution is parameterized using a normal distribution i.e. $G = \mathcal{N}(1, \sigma)$, where the sigma used is the annual average. For the numerical calculation, this normal distribution is discretized and condensed into a 5-point grid. Finally, the remaining components of the model to be parameterized and calibrated are the deposit supply and loan demand functions. These function are parameterized as follows:

$$r_l(L) = \phi_l L^{\omega_l}, \quad r_d(D) = \phi_d D^{\omega_d} \tag{2.48}$$

where $\omega_l$ and $\omega_d$ govern the elasticity of loan demand and deposit supply, respectively. Here, I set $\omega_l$ and $\omega_d$ equal to -1.5 and 0.5 respectively, roughly in line with the literature. The coefficient $\phi_l$ is used the size of the banking industry in equilibrium, and $\phi_d$ is used to match the interest rate.

The calibration strategy is as follows. First, I set the interest rates equal to the targeted rates.

[12] Although not plotted, the estimated risk-free rate roughly matches the 10-year treasury yield.




**(a)** Shock Distribution

**(b)** Standard Deviation

**Figure 2.4.:** Idiosyncratic Shock Distribution

I then choose parameters all but two of the remaining parameters to match model and data moments. Finally, two free parameters, the coefficient on the deposit supply curve $\phi_d$ and the entry cost $c^e$, are set so that the equilibrium is consistent with the targeted rates. Given a parameterization, I can use the following algorithm to calculate the stationary equilibrium. Value function iteration is used to find the value and policy functions. These are all function of net worth $n$, productivity $z$, interest rates $r$ and the policy $\gamma$. I then guess a deposit rate $r_d$, which implies a lending rate $r_l$ by the entry condition. With rates and policies, the stationary distribution is calculated, which generates a quantity of aggregate deposits, and an updated deposit rate. This process is iterated on until convergence on $r_d$. Details on this algorithm are provided in the Appendix.

| Model Object | Functional Form |
|---|---|
| $u(n-e)$ | $\min\{n-e,(1+\kappa)(n-e)\}$ |
| $c(l,d,z)$ | $\delta + z_l l^{\alpha_l} + z_d d^{\alpha_d}$ |
| $G(\epsilon)$ | $\mathcal{N}(1,\sigma)$ |
| $r_l(L)$ | $\phi_l L^{\omega_l}$ |
| $r_d(D)$ | $\phi_d D^{\omega_d}$ |

**Table 2.1.:** Model Parameterization

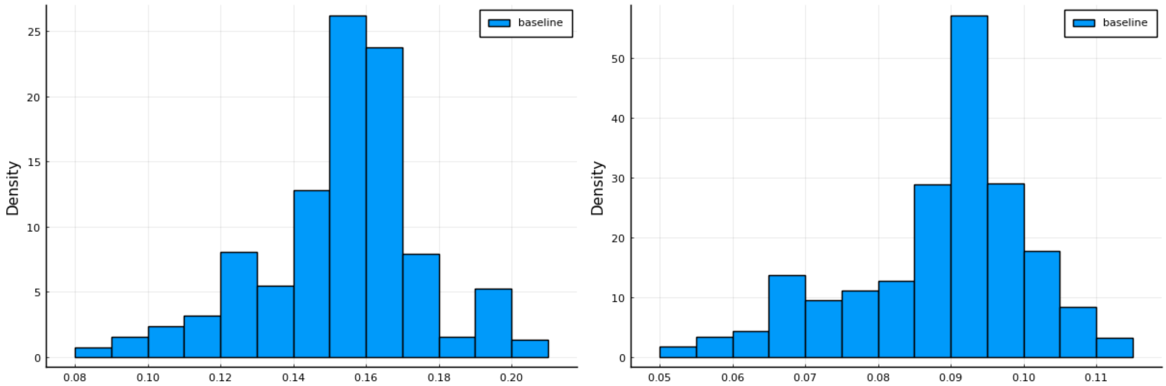
### 2.4.2. Quantitative Properties of the Model

Tables 2.1- 2.3 provide details on the calibration of the model; Table 2.4 shows how the model moments compare with their empirical counterparts. It is able to match average capital ratios to within two decimal places, and targeted interest rates to within one-tenth of one percent. Figures 2.5 - **??** shows the cross-sectional distributions of risk-based capital ratios ($e/l$) and leverage ratios ($e/(e+d)$) generated by the model in the stationary equilibrium. We see that the model generates variation in both capital ratios, as is seen in data. Risk-based capital ratios vary between 8 and 20 percent, while leverage ratios vary between 5 and 12 percent. Additionally, Figure 2.6 shows equilibrium in the lending and deposit markets. The demand for deposits and supply of loans are generated from the aggregation of individual bank demand and supply functions, respectively. In other words, these curves represent:

$$D(r_d) = \int d(n, z, \{r_l^*, r_f, r_d\}, \gamma) \mathrm{d}\mu(n, z) \tag{2.49}$$

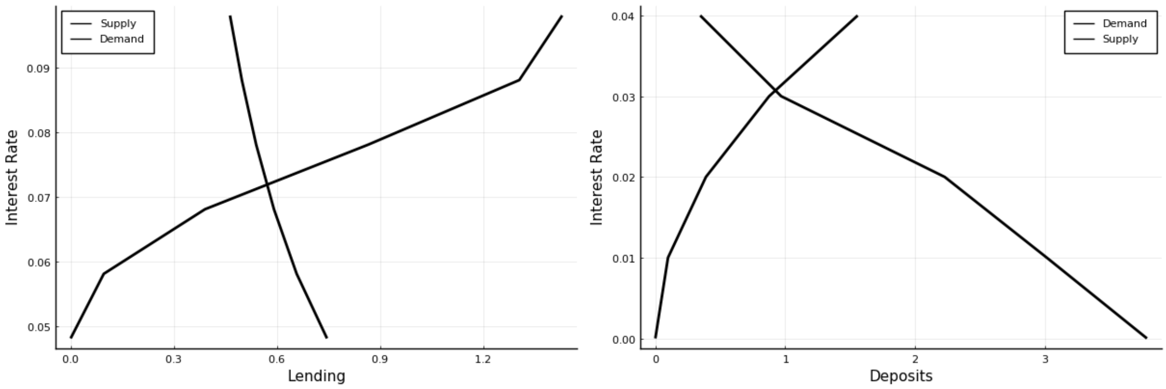
$$L(r_l) = \int l(n, z, \{r_l, r_f, r_d^*\}, \gamma) \mathrm{d}\mu(n, z) \tag{2.50}$$

where $\{r_l^*, r_d^*, d(.), l(.), \mu\}$ are the equilibrium interest rates, deposit and lending policies, and stationary distribution in the baseline model. I see the expected relationship between interest rates and aggregate banking industry behavior. Lending is increasing in the lending rate, and deposit demand is decreasing in the bank borrowing rate.




**(a)** Risk-based capital distribution **(b)** Leverage ratio distribution

**Figure 2.5.:** Baseline - Regulatory Capital Distributions




**(a)** Lending Market **(b)** Deposit Market

**Figure 2.6.:** Baseline - Market Equilibrium

| Parameter | Value |
| --- | --- |
| $\beta$ | 0.960 |
| $\kappa$ | 0.350 |
| $[\delta, \alpha_l, \alpha_d]$ | $[0.000, 1.500, 1.500]$ |
| $[\omega_l, \omega_d]$ | $[-1.500, 0.500]$ |
| $\bar{n}$ | 0.100 |

**Table 2.2.:** Parameters Set Outside the Model

| Parameter | Value |
| --- | --- |
| $[\gamma_l, \gamma_d]$ | $[0.08, 0.04]$ |
| $[z_l^H, z_d^H]$ | $[0.022, 0.013]$ |
| $[z_l^L, z_d^L]$ | $[0.019, 0.012]$ |
| $[\phi_l, \phi_d]$ | $[0.031, 0.032]$ |
| $\epsilon$ | $[0.570, 0.785, 1.000, 1.215, 1.430]$ |
| $g$ | $[0.067, 0.242, 0.3829, 0.242, 0.067]$ |
| $\Gamma(z\|z)$ | 0.9500 |
| $\lambda$ | $[0.100, 0.900]$ |
| $c^e$ | 0.206 |

**Table 2.3.:** Parameters Targeted to Data

| Moment | Model | Data |
| --- | --- | --- |
| Deposit Rate | 0.0308 | 0.0303 |
| Loan Rate | 0.0719 | 0.0727 |
| Large Bank RB Ratio | 0.1584 | 0.1521 |
| Small Bank RB Ratio | 0.1910 | 0.1874 |
| Large Bank Lev. Ratio | 0.0924 | 0.0926 |
| Small Bank Lev. Ratio | 0.1053 | 0.1067 |
| Proportion of Large Banks | 0.1095 | 0.1000 |
| Size of the Industry | 1.052 | 1.000 |

**Table 2.4.:** Model and Data Comparison

## 2.5. Counterfactual Policy Analysis

### 2.5.1. Tightening of Capital Regulation

In order to evaluate how the banking industry responds to changes in capital regulation, I consider a counterfactual increase in capital requirements. In particular, I compare the baseline outcomes with the model outcomes when regulation $\gamma$ is increased from $[0.08, 0.04]$ to $[0.16, 0.10]$. The reason for this choice of counterfactual is that the requirements are binding for the average low cost (large) bank, but not for the average high cost (small) bank. A comparison of the model moments between the baseline and the counterfactual economy is provided in Table 2.5.

| Moment | Baseline | Counterfactual |
|---|---|---|
| Deposit Rate | 0.0308 | 0.0312 |
| Loan Rate | 0.0719 | 0.0726 |
| Large Bank RB Ratio | 0.1584 | 0.1944 |
| Small Bank RB Ratio | 0.1910 | 0.2106 |
| Large Bank Lev. Ratio | 0.0924 | 0.1139 |
| Small Bank Lev. Ratio | 0.1053 | 0.1108 |
| Proportion of Large Banks | 0.1095 | 0.1000 |
| Size of the Industry | 1.052 | 1.139 |

**Table 2.5.:** Baseline Model vs. Counterfactual

#### 2.5.1.1. Aggregate Adjustments

The deposit rate increases from 3.08 percent to 3.12 percent and the lending rate increases from 7.19 percent to 7.26 percent. This corresponds to a 2.66 percent increase in total deposits, and a 0.66 percent decline in total lending. The change in lending is as expected. Higher restrictions on risk-based capital ratios, which is the ratio of equity to loans, implies banks may adjust by reducing lending. This would result in a reduction in loan supply, and an increase in the lending rate. The increase in deposits and the deposit rate is a bit more counter-intuitive. Given an increase in leverage constraints, the expectation would be that banks would adjust by reducing debt or deposits. Instead, deposits increase, and given the increase in leverage ratios, this also implies that equity in the banking sector is increasing as well. In fact, total equity increases by 10.51%. These changes in rates and aggregates correspond to changes in surplus for borrowers and depositors as defined in equation (26). Depositor surplus increases, borrower surplus declines slightly, and bank surplus, which is being measured by aggregate bank valuation, increases overall.

Regulatory ratios for both low cost and high cost banks increase. For the low cost banks this is unsurprising, as they would be constrained by the new regulation given their levels of regulatory capital in the baseline. Still, their regulatory capital ratios increase beyond the new regulation, and so they are still maintaining a capital "buffer" with respect to both requirements. The ratios for the high cost banks increased as well; however, not by as much as the low cost banks. In fact, leverage ratios at high cost banks are now lower on average compared with the high cost banks. Overall, none of the regulatory ratios increased in proportion with the change in the constraint, and so although banks on average have capital "buffers", these buffers have compressed with respect to the regulation. There was also some shifting in market share between low cost and high cost banks. The market share of low cost banks increased slightly in the loan market, while it fell several percentage points in the deposit market. Finally, the size of the industry increased by 8 percent. Presumably, this is due to restricted competition in lending and deposit markets, generating profitable opportunity for entering firms. This will be discussed further in the decomposition of short and long-term changes in market equilibrium.

#### 2.5.1.2. Quantitative Decomposition

I now decompose the incidence of capital regulation in the manner discussed in Section (3). This requires us to distinguish between the direct effect, the short-run effect, and the long-run effect. The direct effect fixes interest rates and changes policy to see how the industry responds. Fixing rates allows us to decouple limitations on bank behavior from responses to changing equilibrium conditions. Then, in the short-run, I allow the prices to adjust. The changes in aggregates and policy burdens is the result of endogenous reallocation within the banking sector. In the long-run, the industry structure is allowed to respond. Firms may enter or exit to take advantage of profitable opportunities. Interest rates and aggregate lending and deposits respond to these changes in the industry. The final impact on welfare for borrowers, savers and banks is the combination of these three effects.

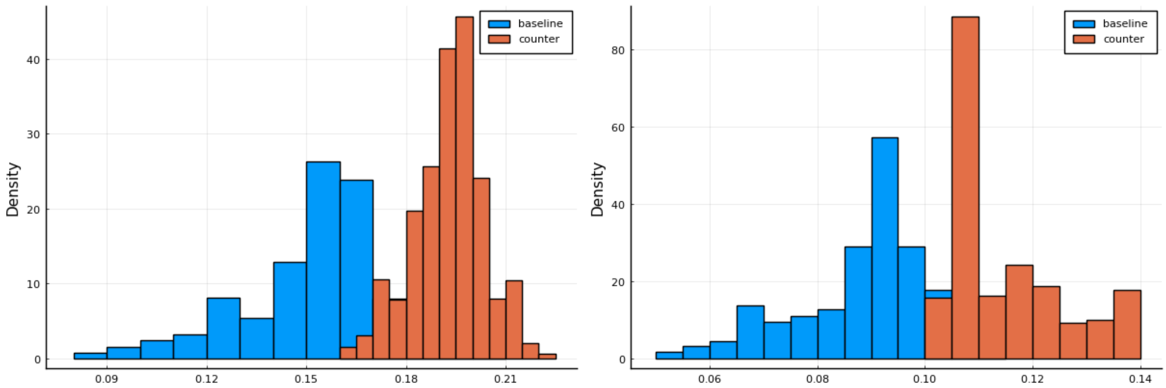
Table 2.6 shows the decomposition of welfare changes due to changing regulation. Overall, there is very little change for borrowers, and increases for savers and bankers. The direct effect is negative for all three for borrowers and savers. It represents a pure restriction on bank behavior with no change in equilibrium interest rates. The short-run effect is positive, as equilibrium prices are now adjusting. This generates reallocation in the provision of banking services across the industry. The long-run effects is also positive. Firms enter to take advantage of restricted competition. This improves outcomes overall for each segment of the industry.

| Moment | $\Delta_i^{DE}$ | $\Delta_i^{SR}$ | $\Delta_i^{LR}$ | $\Delta_i$ |
|---|---|---|---|---|
| Borrowers | -0.40 | 0.37 | 0.03 | 0.001 |
| Depositors | -7.93 | 2.21 | 5.77 | 0.052 |
| Banks Loans | 93.15 | -75.89 | -3.97 | 13.28 |
| Bank Deposits | 67.05 | 5.44 | -69.98 | 2.50 |

**Table 2.6.:** Decomposition in Change in Surplus

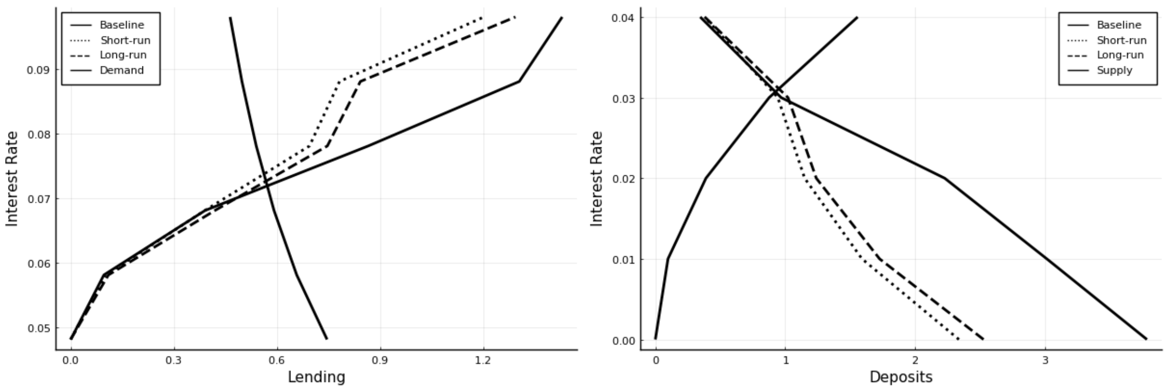
Figure 2.8 shows the shift in aggregate loan supply and deposit demand in both the short-run in response to the change in regulation. Several features of this change are apparent. In the short-run, for lower deposit rates, there is an inward shift in deposit demand. This is due to the constraints being "more" binding, or the restrictions becoming more sever for lower rates. A similar effect occurs in lending markets. For higher rates, banks are more constrained, causing an inward shift in the loan supply curve. However, these are small near the equilibrium rates.

This indicates the degree to which regulation is binding near the equilibrium. In the long-run, there are small although expected shifts in the banking industry's aggregate curves. Both the deposit demand and loan supply shift outward. This reflects the impacts of endogenous entry. Although small, the end result changes the sign of the regulatory burden on savers. In the long-run, they benefit from the tightened regulation.




**(a)** Risk-based capital distribution **(b)** Leverage ratio distribution

**Figure 2.7.:** Counterfactual - Regulatory Capital Distributions




**(a)** Lending Market **(b)** Deposit Market

**Figure 2.8.:** Counterfactual - Market Equilibrium

Ultimately, the objective of higher capital is to reduce risk in the banking sector. Here, there is no aggregate risk; all risk is idiosyncratic. Therefore, the realization of risk is observable each period. As discussed in Section (3), the relevant measure of risk is the proportion of defaulted bank liabilities. This is because policy makers are primarily concerned with moral hazard due to deposit insurance. Here, given that I are considering a counterfactual increase in the regulation, I expect risk to go down. This is precisely the result generated by the model. Measuring the change in $\Delta^R(\gamma, \bar{\gamma})$ as in Equation (38), risk declines 0.656 percent. Summarizing, welfare of borrowers declines, while the surplus of banks and savers increases. Meanwhile, risk goes down, as expected, in response to the tightening regulation.

## 2.6. Conclusion

An important consideration when evaluation the cost of financial regulation is how the burden of the regulation is distributed. The provision of bank services can be reallocated within the banking system or passed on to bank customers. In this paper, I develop a dynamic industry equilibrium model to evaluate this distribution, the incidence of the regulation. Cross-sectional heterogeneity generates variation in regulatory capital. This allows for industry reallocation in response to capital regulation. It also features endogenous industry dynamics. Changes in restrictions on incumbent bank behaviors can induce entry and exit. Both reallocation and changes in industry structure can affect how the burden of the regulation is distributed.

The model is calibrated to US banking industry data. It is set to match industry averages from 1993-2006. The model is then used to quantitatively evaluate the impact of a tighter regulation on the welfare of borrowers, savers, and banks. The direct restrictions on bank behavior reduces the welfare of all agents. Then, price adjustment to the short-run equilibrium reallocates lending and deposit-taking, generating welfare gains. In the long-run, entry and growth in the industry further adds to agent surplus.

Changes in capital regulation from the financial crises are still in the process of being implemented. Evaluating the effects of the regulation will be an ongoing area of interest for researchers and policy makers. This paper is interested in the question of how the costs of binding regulation are distributed between banks and their customers. Undoubtedly, give the evolutionary nature of the regulation, there is still much work to do in understanding the implications of this area of policy.

# 3. Home Production with Time to Consume - *joint with Nick Pretnar*

## 3.1. Introduction

Three well-documented facts are readily apparent when examining aggregate United States economic data over the last 70 years:

i. The services share of expenditure and nominal output has risen, while the share of manufactured goods has fallen.

ii. Average hours per effective full-time worker fell during mid-century before leveling off after 1970.

iii. By all widely available measures (wages, income, wealth, etc.) inequality has increased.

Much has been written separately about each of these data phenomena, yet the literature lacks an explanatory mechanism that can simultaneously reconcile the rise in the services share and the decline in average hours, while also yielding a measure of consumption/leisure inequality that reflects changing patterns in consumer behavior.

We propose a home production mechanism whereby agents simultaneously choose their allocation of goods and services consumption and their allocation of off-market time spent engaging in different activities. Each activity is separately associated with utilization of either goods or services expenditure. Our approach rests on the assumption that differential complementarities exist between the consumption of different types of market purchases and non-work time. In frameworks with only one consumption commodity and elastic labor supply, complementarities between leisure and consumption are explicitly considered. However, this is often not the case (but not always) in models where consumers derive utility from multiple consumption commodities. Under this premise we explore the fundamental question as to why household consumption allocations vary in relative prices and income.

Demand for different market purchases depends ultimately on how households spend time using those purchases in various home production activities. Gary Becker recognized this in his seminal paper on home production, "A Theory of the Allocation of Time" (Becker 1965). In a model where households choose both market purchases and how to allocate time toward their consumption, both the relative productivities and labor intensities of different home production processes determine the responsiveness of the consumption allocation to relative prices and income. We present a micro-foundational explanation, rooted in a home production model, that provides rich, interactive characterizations for how demand and off-market time utilization are simultaneously sensitive to market price and income changes. Specifically, we provide a novel explanation for why the share of spending devoted to services has risen in developed economies.[1] We estimate that substitution effects driven by variation in the relative market price of goods to services have caused a re-allocation of off-market time between tasks where different types of market commodities are used. As will be discussed, this phenomenon has also contributed to the deceleration of the decline in labor hours per worker since the mid-1970s.

The framework presented here can also help answer broader questions pertaining to how consumers adjust their behavior and spending in response to the introduction of new technologies. In particular, we explore the degree to which technological advancement has helped drive rising services consumption in the last half of the twentieth century. As Gordon (2016) discusses, most innovative labor-saving in-home appliances, such as electric laundry machines, refrigerators, and vacuum cleaners, were already in many American homes by 1950. Major technological advances in the last half of the twentieth century that most drastically affected consumer time utilization were in the realms of communication and entertainment (Gordon 2016). These encompass many products that are classified under the NIPA “services” umbrella. An open question is to what degree improvements to the consumer experience of using these new services helped contribute to the relative rise in services demand. Through the lens of our model we provide an answer to this question. Specifically, we estimate the degree to which the value-added to various off-market activities of using services grew faster or slower than that of physical, manufactured goods. Our quantitative results have broad implications for how inequality is characterized.

Moving forward, we will place our theoretical work in context with the extensive literature on structural change, home production, consumer time use, and inequality. We will then round out the introduction by defining some terms we use throughout the paper in order to distinguish where our proposed mechanism yields results that depart from the literature. Later in the paper, after presenting and analyzing a stylized version of a Beckerian model, we perform several quantitative assessments. First, we estimate the degree to which rising wages versus relative price variation are responsible for the rising services share since 1948. Second, we provide a model-based assessment of the degree to which cross-sectional welfare inequality has changed over the last 40 years, as a result of changes to the ways in which people at different income quintiles engage in consumption activities.

### 3.1.1. Literature

Our work intersects with several broad strands of literature, namely those dealing with the structural rise of the services sector, technological change, home production, off-market time use, the decline in labor hours per worker, and rising inequality. Here, we place the paper in context with others that grapple with these topics.

**Rising Services Share and Structural Change:** The literature generally posits two primary theories as to why the services share of spending has risen in developed economies. One explanation is that, as personal income has grown, so has demand for services consumption.

[1]While non-homothetic preferences are sufficient to explain variation in the expenditure basket due to rising incomes, they are not necessary. A homothetic preference structures that allows for differential complementarities between consumption and off-market time can also match the data.

Since the relative price of goods to services has fallen, then it must be that income effects play a role in driving up the relative demand for services (Caselli and Coleman 2001; Kongsamut, Rebelo, and Xie 2001; Matsuyama 2009; Herrendorf, Rogerson, and Valentinyi 2013; Uy, Yi, and Zhang 2013; Boppart 2014; Comin, Lashkari, and Mestieri 2015; Kehoe, Ruhl, and Steinberg 2018). Non-homothetic preferences represent just one mechanical way to generate an income effect that is often causally attributed to the rising services share. However, non-homothetic preferences are not necessary if the consumption of different types of products, say goods and services, is linked with separate off-market time-utilization decisions in different ways, as we show. By allowing for consumers to choose both market purchases and off-market time utilization over multiple activities, we are broadening the choice set, which generates equilibrium outcomes where, even with homothetic preferences, relative expenditure can (depending on the parameterization) still vary in income.

A partially overlapping explanation for structural change posits that differentials in capital deepening, human capital productivity, and/or total factor productivity (TFP) growth[2] leads to differences in sectoral growth rates and thus variation in relative prices and expenditure (Caselli and Coleman 2001; Ngai and Pissarides 2007; Acemoglu and Guerrieri 2008; Buera and Kaboski 2012; Autor and Dorn 2013; Comin, Lashkari, and Mestieri 2015; Herrendorf, Herrington, and Valentinyi 2015; Porzio, Rossi, and Santangelo 2020).[3] Such mechanisms can explain the rise in the share of production devoted to technologically advanced products even with homothetic preferences.

We follow guidance in Buera and Kaboski (2009) who advocate for home production models to match the structural change data. In this regard our work most closely aligns with Ngai and Pissarides (2008), though we allow for goods and services consumption each to be complementary with off-market time in different ways.[4] Ngai and Pissarides (2008) focus primarily on how the decline in hours worked per employee can be attributed to differentials in technological growth between home and market sectors. Our main result departs from that of Ngai and Pissarides (2008) and others in two important ways. First, we use a stylized model with elastic time-use and multiple off-market time-use choices that are each complementary with consumption in different ways to explicitly show that income effects can be generated *regardless* of differences in technological growth between sectors or between market output and in-home production. Second, the first result holds even if preferences are homothetic over the broad choice set. This is because wages affect both the price of off-market activities and income.

Finally, note that much of the literature on structural change considers reasons for the decline in the sectoral share of agriculture and the contemporaneous rise in manufacturing (Caselli and Coleman 2001; Kongsamut, Rebelo, and Xie 2001; Herrendorf, Rogerson, and Valentinyi 2013; Uy, Yi, and Zhang 2013; Comin, Lashkari, and Mestieri 2015; Herrendorf, Herrington,

[2] Sectoral TFP growth may also be associated with within-industry market concentration, which also contributes to a declining labor share of GDP (Autor et al. 2020).

[3] In a recent working paper Porzio, Rossi, and Santangelo (2020) use a two-sector model (agriculture and non-agriculture) to show that human capital deepening has led to a decline in agriculture's share of labor, globally. While their paper does not consider the subsequent late-twentieth century shift in labor hours from manufacturing to services in advanced economies, their mechanism is generalizable to such a setting.

[4] A secondary result of their home production formulation in which off-market time is divided between in-home labor and leisure is that as $t \to \infty$ all market hours are eventually devoted toward services. Over time the services sector eventually dominates manufacturing and agriculture due to differentials in technological change.

and Valentinyi 2015; Porzio, Rossi, and Santangelo 2020). The mechanisms proposed to explain early transitions into an industrial society are almost identical to those used to explain the post-industrial transitions from manufacturing to services: non-homothetic preferences, different sectoral rates of technological change and capital and human capital deepening. In this paper we will refer to "structural change" in the context of an already-developed economy transitioning from making and purchasing physical manufactured goods to services.

**Home Production:** The term "home production" is used to characterize a wide range of phenomena explained by models with various features. We will distinguish here between home production formulations where time use is considered directly complementary to market purchases versus those where time and market purchases are not directly combined to produce a home good. The former camp of papers generally assumes that a particular type of market purchase, say consumer durables or goods, is combined with time to yield final consumption (Becker 1965; Bernanke 1985; Locay 1990; Greenwood and Hercowitz 1991; McGrattan, Rogerson, and Wright 1993; Greenwood, Rogerson, and Wright 1995; Rupert, Rogerson, and Wright 1995; Gomme, Kydland, and Rupert 2001; Greenwood, Seshadri, and Yorukoglu 2005; Goolsbee and Klenow 2006; Ngai and Pissarides 2008; Bridgman, Duernecker, and Herrendorf 2018; Boppart and Krusell 2020; Fang, Hannusch, and Silos 2020). In most of these models, however, consumption of services is generally not associated with a corresponding time-allocation decision, as in the original Beckerian formulation.[5] Rather, only physical goods are considered home production inputs. In the latter camp of home production papers, market purchases or inventories of consumer durables are featured as inputs into some technological process that does not admit time but often features an exogenous productivity component (Gronau 1977; Graham and Green 1984; Benhabib, Rogerson, and Wright 1991; Ingram, Kocherlakota, and Savin 1997; Boerma and Karabarbounis 2019). Our formulation is related more to the former camp than the latter, though we allow both for time-use complementarities and exogenous changes to in-home productivities. Our findings will suggest that using services are time intensive, so the flexible Beckerian framework that allows for both goods and services to be complementary to different time-use decisions is important.

**Allocation of Off-market Time:** Papers on household time use typically make an effort to distinguish between time engaged in market work, work in the home (think doing chores) or human capital accumulation, and leisure activities (King, Plosser, and Rebelo 1988; Lucas Jr. 1988; Ríos-Rull 1993; Perli and Sakellaris 1998; Aguiar and Hurst 2007; Ramey and Francis 2009; Ramey 2009; Bridgman, Duernecker, and Herrendorf 2018; Kopytov, Roussanov, and Taschereau-Dumouchel 2020). The Beckerian home-production framework does not require a distinction between in-home (unpaid) work and leisure. This is because, regardless of whether the off-market activities are themselves laborious or relaxing, Becker's premise is that consumers *must spend time using* any market purchase in order to derive utility from its consumption. This is true regardless of whether the consumer decides to use a market purchase for a mundane household chore or a more pleasurable, relaxing leisure activity. We thus do not find it necessary

[5]One recent exception is the working paper by Fang, Hannusch, and Silos (2020), which most closely follows the Beckerian framework in the manner that we also employ in this paper. A more general exception is Locay (1990), where a home production model is used to demonstrate how, as markets mature, production (and thus measured economic activity) shift out of the home and toward firms.

to make a distinction between time devoted toward work in the home versus leisure. The reason for this is that either type of activity can be associated with either goods or services consumption, and we categorize time-use based on the type of consumption with which it is complementary.

**Decline in Market Hours:** The decline in market hours per worker in the U.S. and other developed economies is well-established, though theories explaining the decline are wide-ranging (Barro 1984; Ngai and Pissarides 2008; Ramey and Francis 2009; Mankiw 2010; Gordon 2016; Jones 2016; Aguiar et al. 2021; Boerma and Karabarbounis 2019; Boppart and Krusell 2020; Fenton and Koenig 2020; Kopytov, Roussanov, and Taschereau-Dumouchel 2020). Naturally, if a key mechanism affects the total allocation of off-market time, through say home production or consumption/leisure complementarities as in Ngai and Pissarides (2008), Boppart and Krusell (2020), and Kopytov, Roussanov, and Taschereau-Dumouchel (2020), market hours would be impacted as well. Recent work by Boppart and Krusell (2020) does not attempt to rationalize why labor hours have fallen, but outline the parameter constraints under which a standard, separable consumption/leisure utility function can match the data.

In a recent working paper Kopytov, Roussanov, and Taschereau-Dumouchel (2020) take an alternative approach, proposing a theory that the decline in market hours can be attributed to the declining implicit price of leisure activities due to technological advancements and quality improvements of recreational goods and entertainment services. Their results suggest that further study is warranted regarding the link between time-allocation and consumption when accounting for the particular kinds of products being consumed and used. One of our aims is to understand the effects of such linkages in detail.

**Inequality:** As Díaz-Jiménez, Glover, and Ríos-Rull (2011) point out, an ideal measure of inequality would ultimately account for differences in how households enjoy consumption and off-market activities, though "enjoyment" is difficult (perhaps impossible) to measure. Home production models, however, can be constructed so as to yield a stylized expression for the price of off-market activities consumers engage in. Several papers have used home production models to measure inequality. Gronau (1986), Jenkins and O'Leary (1996), Gottschalk and Mayer (2002), and Frazis and Stewart (2011) conclude that inequality from income is greater than inequality from so-called "extended income," which accounts for returns to home production. More recent working papers by Boerma and Karabarbounis (2019) and Boppart and Ngai (2021), however, find that inequality is typically under-reported when using widely available data measures that do not in some way account for home production.

Piketty, Saez, and Zucman (2018) note that gaps in how micro data are collected and macro data are reported make it difficult to accurately quantify the distributional implications of economic growth. While data issues must be considered, both micro and macro data reflect only the value of measured economic activity that results, mostly, from activities taking place outside the household (expenditures at stores, labor income from a workplace, capital income from firm dividends, etc.). Thus, while Piketty, Saez, and Zucman (2018) provide a comprehensive overview of how inequality has evolved with regards to *data-measurable* economic activities, we argue that an even more complete picture of inequality accounts for changes to the value of in-home activities, in addition to those data-measurable economic activities. As Becker originally noted, the value of in-home activities can be decomposed into direct costs associated with purchasing inputs from the marketplace (goods and services) and the indirect costs associated with ultimately using those inputs in various home production processes. Since the outputs of home production and their corresponding values to households are inherently latent (unobservable), model structure (and thus modeling assumptions) are required in order to understand and measure them. That is what we provide. When comparing top income earners and bottom income earners, we find that, while inequality as measured by the in-home value (not the market value) of final consumption has increased over the last 40 years, it has not increased as quickly as inequality measured from income and wealth data.

### 3.1.2. Definitions

Throughout this paper we will make several references to technical terms used in the literature in often model-specific contexts. In this section we define those terms and briefly discuss the context in which we will refer to them. It is particularly necessary to succinctly define what constitute "substitution" and "income" effects because, in the literature, these terms are used to describe various phenomena which manifest themselves in often model-dependent ways. For example, what constitutes an income effect in a model with inelastic time-use may actually be masking an underlying substitution effect which a richer model could capture.

**Classic Substitution Effect:** changes in the allocation of market purchases across multiple commodities as resulting from a change in relative prices, holding the utility level fixed.

**Classic Income Effect:** changes in the consumption allocation, possibly including the distribution of expenditure, due either to variation in income or price inflation. This comprises a parallel shift in the budget set and can result both from variation in income and prices, since changes to prices may affect the overall affordability of the current bundle.

**Pure Income Effect:** the phenomenon by which variation in income, either cross-sectionally, over time, or both, affects the relative consumption of goods and services, holding prices fixed. In a model where consumers choose amongst multiple consumption commodities but supply labor inelastically, pure income effects are equivalent to classic income effects. In a model with elastic labor, pure income effects can determine the distribution of expenditure across different commodities either through classic substitution- or classic income-effect channels, since wages simultaneously comprise the price of off-market time and affect income. Heretofore, the structural change literature documenting the rise in the services share of U.S. expenditure has mostly considered models where the pure income effect and classic income effect are synonymous. Without elastic time use non-homothetic preferences are one way to generate the observed changes in relative consumption.[6]

**Inferiority:** the phenomenon by which demand for either market purchases or off-market time-use declines in absolute terms due to an absolute increase in income.

**Classic $c/\ell$ Model:** a classic consumption/leisure model with a single consumption commodity and a single, elastic leisure choice. In such models, a rise in wages induces both

[6] For example, Kongsamut, Rebelo, and Xie (2001), Matsuyama (2009), and Herrendorf, Rogerson, and Valentinyi (2013) use variations of Geary (1950) and Stone (1954) preferences with non-zero subsistence terms, Boppart (2014) uses a more flexible PIGL specification from Muellbauer (1975) and Muellbauer (1976), while Foellmi and Zweimuller (2008) construct a quadratic utility function that yields non-linear Engel curves.

classic income and substitution effects, where one or the other may dominate, depending on preferences.

**Classic $c/\ell$ Income Effect:** rising wages lead to less time working and more time spent engaging in leisure, though on the whole, rising wages shift the budget constraint out and consumers experience a more preferable $(c, \ell)$ bundle. In a model with multiple consumption commodities and multiple off-market time-utilization decisions, consumers may increase the time they spend on certain activities more than others. If different activities are complementary with different market commodities in different ways, we will show that the re-allocation of time can also affect the allocation of expenditure.

**Classic $c/\ell$ Substitution Effect:** rising wages are associated with an increase in the opportunity cost (the price) of leisure, so consumers substitute leisure time for additional consumption which they fund by working more. In this situation leisure is an inferior good, since income rises but leisure time falls. In a model with multiple off-market time-utilization decisions, some time-use choices may be inferior while others may still be normal even though the classic $c/\ell$ substitution effect dominates.

## 3.2. Model Economy with Beckerian Home Production

Time is discrete and indexed by $t$. There is a unit-mass of infinite-lived households $i \in [0, 1]$, each of which consists of $N_t$ members. The population grows at rate $g_{Nt}$. Households buy goods $q_{igt}$ and services $q_{ist}$ on the market at prices $P_{gt}$ and $P_{st}$. $q_{igt}$ is assumed to contain both the service flows of consumer durables and new durable and non-durable purchases of physical, manufactured wares, while $q_{ist}$ encodes intangible services purchased on the market. $P_{gt}$ is the combined price of both the net present value of durables currently owned by the household but that were purchased in previous periods and the value of new durable and non-durable goods purchased this period. We thus can think of the household as renting the service flows of durables back from itself at the market re-sale price.[7] Households also allocate off-market time to two tasks $n_{igt}$ and $n_{ist}$ and supply labor $\ell_{it}$ earning wages $w_{it} = \eta_{it} w_t$, where $\eta_{it}$ is a household-specific labor productivity and $w_t$ is the average, economy-wide wage-per-hour-worked, $w_t = \int_0^1 w_{it} \mathrm{d}i$. Households have final utility over the outputs $c_{ijt}$ of home production activities $j \in \{g, s\}$, each associated with a separate market purchase. Households can also save by investing $\iota_{it}$ in market capital $k_{it}$, which is assumed non-negative in the initial period $k_{i0} \geq 0$, depreciates at rate $\delta$, and yields net return $r_t$.

There are three representative firms, each of which separately produce new goods $\overline{Q}_{gt}$, services $\overline{Q}_{st}$, and investment capital $I_t$.[8] $\overline{Q}_{gt}$ contains new durables and non-durables purchased, but not the service flows.[9] The producers of goods and services utilize capital $K_{jt}$ and labor $L_{jt}$

[7]The impact of durable service flows on broader expenditure shares is mostly ignored in the structural change literature. Ngai and Pissarides (2007), Herrendorf, Rogerson, and Valentinyi (2013), Boppart (2014), and Kehoe, Ruhl, and Steinberg (2018) do not account for durable service flows affecting household behavior in their analyses. Since our primary model mechanism driving variation in expenditure shares involves home production, we contend it is necessary to account for durable service flows, especially when making quantitative statements later in this paper.

[8]Capital letters will denote aggregates.

[9]We use bars to distinguish between goods variables associated with flows, as opposed to stocks. Note that $\overline{Q}_{st} = Q_{st}$ by definition.

as inputs in Hicks-neutral Cobb-Douglas production technologies: $\overline{Q}_{jt} = A_{jt}K_{jt}^{\alpha_j}L_{jt}^{1-\alpha_j}$. As in Acemoglu and Guerrieri (2008), we allow the intensity of capital $\alpha_j$ to vary across sectors. As in Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Boppart (2014) we allow the total factor productivities (TFP) to differ across sectors as well. These will fluctuate according to stochastic processes we parameterize in Section 3.5. Note that when labor is inelastically supplied and $\alpha_j = \alpha$ for all sectors, the ratio of total factor productivities is just the inverse of the price ratio (Herrendorf, Rogerson, and Valentinyi 2014; Herrendorf, Rogerson, and Valentinyi 2018). The investment producer only uses capital $K_{It}$, transforming it one-to-one to investment, $I_t = K_{It}$. All firms face the same average wage $w_t$ and capital rental rate $r_t$. Producers of services sell at price $P_{st}$, while goods producers sell at price $\overline{P}_{gt}$ which is the price of only new items produced in period $t$, and may, but need not, equal $P_{gt}$. Finally, the economy is assumed to be closed with all prices generated endogenously to support market clearing of goods, services, capital, and labor. In the forthcoming exposition we will focus on the household's decision process in detail, demonstrating how wage variation impacts the consumption allocation generally for models with multiple, elastic off-market time use decisions and consumption, regardless of whether preferences are homothetic.

### 3.2.1. Households

The household decision process takes a Becker (1965) form. Consumers derive periodic flow utility $u(c_{igt}, c_{ist})$ directly from the outputs of two home production processes. We denote these outputs by $c_{ijt}$, indexing them by the type of market commodity with which they are associated $j \in \{g, s\}$. Preferences are time-separable with $\beta$ governing the degree of time preference:

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{igt}, c_{ist}) \tag{3.1}$$

Expectations are taken over sequences of future prices, which are affected by real fluctuations in firm TFP's and the productivities of home production processes, which we now describe.

Market purchases $q_{ijt}$ along with off-market time $n_{ijt}$ are inputs into home production processes or activities that yield final consumption, $c_{ijt}$.[10] Note that many activities are associated with using both goods and services, simultaneously. Engaging with technologies, like computers and smart phones, requires using physical hardware ($q_{igt}$) and intangible software ($q_{ist}$). We will thus allow $u(\cdot, \cdot)$ to have a structure such that the experiences associated with using both types of products, $c_{igt}$ and $c_{ist}$, are complementary with each other.[11]

Let $f_j$ be some constant returns to scale production function. Final consumption is produced using time and either goods or services according to

$$c_{ijt} = z_{ijt} f_j(q_{ijt}, n_{ijt}), \qquad \forall j \in \{g, s\} \tag{3.2}$$

[10]Throughout the paper, we will refer to quantities, $q_{ijt}$, as "purchases" or "products" to distinguish them from "consumption" or "consumption activities/experiences." $c_{ijt}$. Such "purchases," of course, implicitly include the rents the household pays to itself when it uses elements of its stock of durable goods. We consider "consumption" to be an activity that is accomplished using both purchases and/or products along with time.

[11]See Pollak and Wachter (1975) and Barnett (1977) for a discussion of the impossibility of estimating joint in-home production models.

$z_{ijt}$ allows for variation over time in the TFP of home process $j$. We like to think of this value as in-home "folk knowledge" which varies in ways unexplained by our model. While $z_{ijt}$ may seem important now, we will soon show that it has no impact on our main result: consumption allocations will vary in income and over time even if preferences and home production functions are homogeneous of degree one and $z_{ijt}$ is always constant.

Let $\overline{n}$ denote the total time available to the household, and assume that all households face the same time constraints. Their total time allocation must satisfy $\ell_{it} + n_{igt} + n_{ist} \leq \overline{n}$. Capital investments evolve according to $k_{i,t+1} \leq k_{it}(1-\delta) + \iota_{it}$. The standard budget constraint with market expenditure and investment on the left and labor plus capital income on the right is $\sum_{j\in\{g,s\}} P_{jt}q_{ijt} + \iota_{it} \leq w_{it}\ell_{it} + r_t k_{it}$. Letting $R_t = 1 - \delta + r_t$ be the gross rate of return on capital investments, we substitute out household labor supply and flow investment using the time allocation constraint and the law of motion for capital investments to write a dynamic version of the Becker (1965) budget constraint

$$\sum_{j\in\{g,s\}} (P_{jt}q_{ijt} + w_{it}n_{ijt}) \leq w_{it}\overline{n} + R_t k_{it} - k_{i,t+1} \tag{3.3}$$

Consumers thus choose $\{q_{igt}, q_{ist}, n_{igt}, n_{ist}, k_{i,t+1}\}_{t=0}^{\infty}$ to maximize (3.1) subject to (3.2) and (3.3).

Given prices, allocations of market purchases and time-use must satisfy the infra-marginal rates of technical substitution ($MRTS_j$) between market purchases $q_{ijt}$, including durable service flows, and time $n_{ijt}$, and the marginal rate of substitution ($MRS_{it}$) for the two types of market products, $q_{igt}$ and $q_{ist}$:

$$MRTS_j(q_{ijt}, n_{ijt}) \equiv \frac{\partial f_j}{\partial q_{ijt}} \Big/ \frac{\partial f_j}{\partial n_{ijt}} = \frac{P_{jt}}{w_{it}}, \qquad \forall i, \forall j \in \{g, s\} \tag{3.4}$$

$$MRS_{it}(q_{igt}, q_{ist}) \equiv \left(\frac{z_{igt}}{z_{ist}}\right)\left(\frac{\partial u}{\partial c_{igt}} \Big/ \frac{\partial u}{\partial c_{ist}}\right)\left(\frac{\partial f_g}{\partial q_{igt}} \Big/ \frac{\partial f_s}{\partial q_{ist}}\right) = \frac{P_{gt}}{P_{st}} \tag{3.5}$$

The condition in (3.4) is the usual one, describing the real quantity of additional market products needed to maintain a constant level of home production $\underline{c} = c_{ijt}$ given a one unit decrease in time devoted to process $j$. The condition in (3.5) simply describes the trade-off between buying goods versus services. This trade-off can be decomposed into three separate components that describe both how relative productivities and changes to the ways in which consumers use market purchases in various activities affect their equilibrium decisions:

i. $\frac{z_{igt}}{z_{ist}}$ is the relative productivity of using goods versus services in home production activities.

ii. $\frac{\partial u}{\partial c_{igt}} \big/ \frac{\partial u}{\partial c_{sit}}$ is the marginal rate of substitution between different experiences or activities associated with using goods versus services.

iii. $\frac{\partial f_g}{\partial q_{igt}} \big/ \frac{\partial f_s}{\partial q_{ist}}$ is the productivity-neutral ratio of the two separate marginal products of $f_j$ with respect to $q_{ijt}$ for the two separate home production processes.

(i) is exogenous, while (ii) and (iii) are endogenous objects. In our quantitative exercises we will use a parameterized version of (3.5) to understand the degree to which variation in (i) is driving our results. We find, empirically, that declines in (ii) and (iii) over time are almost exclusively responsible for declining $MRS_{it}$ which is discussed further in Section 3.5.4.3.

As this decomposition demonstrates, the relative value of experiences/activities associated with using goods and services (ii) is not necessarily equal to the measured market value of goods and services purchased, $\frac{P_{gt}}{P_{st}}$. This is because the market value encodes both the relative value of those experiences, $\left(\frac{\partial u}{\partial c_{igt}} / \frac{\partial u}{\partial c_{sit}}\right)$, and the relative efficiency by which market purchases can be transformed into final consumption experiences, which is $\left(\frac{z_{igt}}{z_{ist}}\right)\left(\frac{\partial f_g}{\partial q_{igt}} / \frac{\partial f_s}{\partial q_{ist}}\right)$. This measure of relative efficiency contains both exogenous (i) and endogenous components (iii). The latter describes the ratio of marginal products, which will decline as durable goods contained in $q_{igt}$ deepen within the household.

The degree to which relative expenditure varies in wages $w_{it}$ will thus depend on the structures of both $u$ and $f_j$, though we are not limited to considering *only* non-homothetic, composite preferences over $(q_{igt}, q_{ist}, n_{igt}, n_{ist})$.

**Claim 1:** *In models with inelastic time-use, the composition of the market expenditure basket will vary in income if and only if preferences are non-homothetic. However, when time allocations are chosen by the consumer, non-homothetic preferences are sufficient but not necessary in order to generate market expenditure baskets that vary in income.*

Claim 1 outlines the main premise of our theoretical argument: structural change models throughout the literature require non-homothetic preferences *because* rich complementarities between time-use and market purchase decisions are not explicitly modeled. Thus, non-homothetic preferences with inelastic time use can effectively mask the intratemporal impact that both the intensive margin of labor and off-market time use considerations have on the composition of market expenditure. This is because $w_{it}$ is simultaneously the price of off-market time while also impacting income. Thus, allocations of $(q_{igt}, q_{ist})$ may vary in $w_{it}$ (and thus income) due to both classic substitution and income effects, regardless of whether preferences are homothetic, since consumers adjust their market purchasing behavior in response to changes in the opportunity cost of time. As we will show in Section 3.3, these margins depend on both the elasticity of substitution between in-home activities, $c_{igt}$ and $c_{ist}$, and the relative time-intensities of the different home production processes, $f_g$ and $f_s$. Further, variation in the market basket can occur independent of variation in $z_{ijt}$ between processes and over time. In the upcoming sections we will prove that this claim is true by exploring both the properties of a general model with no parametric restrictions on preferences and the specific properties of a stylized parametric model with homothetic preferences.

### 3.2.2. A Non-parametric Measure of Household Consumption Experiences

In this section we present empirical evidence suggesting that changes to consumers' consumption experiences have contributed to declining relative market expenditure, and thus, through general equilibrium effects, relative prices. Specifically, we argue that changes to the relative quality consumers experience when using physical, manufactured goods versus intangible services are driving long-run demand changes. This exercise helps address notorious difficulties in measuring how the quality, as experienced by the end user, of market products has changed (Boskin et al. 1998; Nordhaus 1998).

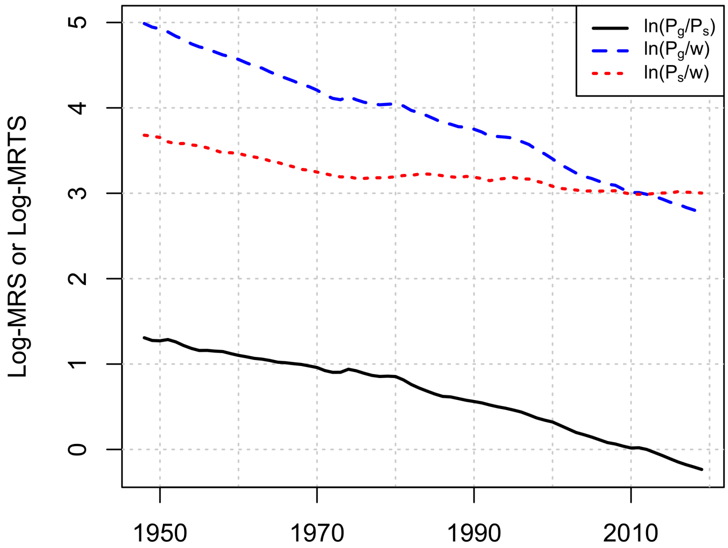



**Figure 3.1.:** The black line is equivalent to the blue line minus the red line: $\ln(P_{gt}/P_{st}) = \ln(P_{gt}/w_t) - \ln(P_{st}/w_t)$. Most of the decline in equilibrium $MRS_t$ (relative prices) since 1948 can be attributed to declines in $MRTS_g$ as proxied by $P_{gt}/w_t$. All variables are real \$2012.

Assume for this section only a representative consumer, so we can drop $i$ subscripts.[12] Assume (as always) all households are price-takers. We can use the $MRTS_j$ conditions in (3.4) to write an alternative decomposition of the $MRS_t$ condition in (3.5):

$$MRS_t(q_{gt}, q_{st}) \equiv \frac{P_{gt}}{P_{st}} = \frac{P_{gt}/w_t}{P_{st}/w_t} \equiv \frac{MRTS_g(q_{gt}, n_{gt})}{MRTS_s(q_{st}, n_{st})} \tag{3.6}$$

Exploiting the fact that along a general equilibrium path relative prices fully characterize equilibrium $MRS_t$ and thus the ratio of marginal rates of technical substitution, we can use (3.6) to understand whether changes to activities associated with using goods or services have most contributed to long run structural change.[13] These price ratios thus represent different components of the equilibrium marginal value the household derives from spending time using its market purchases.

Figure 3.1 presents the decomposition in (3.6) using aggregate data.[14] Relative prices (and thus $MRS_t$) fell by 78.6% over the period 1948-2019, while $P_g/w_t$ which proxies for $MRTS_g$ fell

[12]We will alternate between a representative-consumer environment and one with heterogeneous agents throughout the remainder of the paper.

[13]This is true under the assumption that our model accurately describes the general equilibrium data-generating process, of course.

[14]Market prices for goods and services are computed using aggregate data from NIPA Tables 1.1.3 through 1.1.5 for non-durable goods and services. Data for consumer durables are taken from the BEA Fixed Asset Tables 1.1 and 1.2. We construct a goods ($g$) price index that accounts for the value of non-durables, the value of new durables purchased, and the value of durable service flows. Price-index construction procedures are detailed in Online Technical Appendix A.1. For aggregate wages, we turn to NIPA Tables 2.1, 6.5B, 6.5C, 6.5D, 6.9B, 6.9C, and 6.9D, summing "Compensation of employees" and "'Proprietors' income with inventory valuation and capital consumption adjustments," then dividing these values by total hours from full-time equivalent workers. According to the BEA total full-time equivalent workers are computed by dividing total labor hours by average hours for full-time workers only: $\left(\sum_i \ell_{it}\right)/\left(\frac{1}{\#\text{full-time}} \sum_i \ell_{it} \mathbf{1}\{i \text{ is full-time}\}\right)$.

by 89.1%. Note that after 1980 the rate of relative price decline appears to increase (the black line). This is partly due to the fact that declines to $MRTS_s$ have flattened since 1980. Meanwhile, $MRTS_g$ appears to have fallen at a constant rate since 1948.

This decomposition demonstrates that time can replace less market inputs than it could in the past, though this trend is more pronounced for physical, manufactured wares than services. The fact that this trend is more pronounced for goods than services may be attributable to the fact that many services are natural time/labor substitutes (eating out, hiring a maid or an accountant, day-time childcare, etc.). Thus, since $MRTS_s$ fell by only 49.2% since 1948, we conclude that the quality the consumer experiences from using services has improved at a slower rate than the quality experienced from using physical goods: while offering higher quality services than in the past, gains to experienced quality from hiring a maid or accountant today are less than gains to experienced quality from using a computer, watching television, or cooking with the latest kitchen appliances.

### 3.2.3. Income and Substitution Effects

Let us now briefly examine model-implied income and substitution effects in the context of classic consumer theory. For such an analysis we require the dual problem (EMP) associated with the utility-maximization problem (UMP) described above. As Blundell and Macurdy (1999) point out, the objective function for the EMP in models featuring at least one off-market time-utilization decision and elastic labor is just the left-hand side of (3.3) — the Beckerian budget constraint. Further, the marginal rates of substitution for the model's control variables along with the budget constraint contain all necessary information to relate off-market time-utilization decisions to market consumption. In our Beckerian framework with two market commodities, the control variables are $q_{igt}$, $q_{ist}$, $n_{igt}$, and $n_{ist}$, while $k_{i,t+1}$ is a dynamic choice variable. Denoting the right hand side of (3.3) by $y_{it}$, it is clear that this object is endogenous, a fact which makes estimating substitution and income elasticities difficult but does not preclude us from discussing their theoretical implications.

Let superscript $m$ index the Marshallian demand functions derived by solving the UMP. Let superscript $h$ index the Hicksian demand functions derived by solving the EMP, where total expenditure is equal to the variable $y_{it}$.[15] Marshallian demands are functions of consumption prices, wages, and full income, which is the value of income if all time were devoted to labor. We write the Marshallian demands as $q^m_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it})$. With elastic time use, just as in a standard $c/\ell$ model, the Hicksian is a function both of market prices and wages, $q^h_{ijt}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})$, since the opportunity cost of engaging in home production activities is $w_{it}$. Thus, $w_{it}$ is both the wage and a price.

In classic consumer theory it is assumed that the vector of prices faced by the consumer when making purchasing decisions is of the same cardinality as the vector of those decisions. Off-market time utilization is effectively a purchase decision: the consumer gives up a share of his possible income he could have earned working in exchange for more time. In the standard

[15] Assume, just for this exercise, the consumer engages in constant savings each period, so that $y_{it}$ (maximum income net of savings) is taken as given. We can think of this as looking at the second stage only of some two-stage budgeting decision problem, as described in Deaton and Muellbauer (1980).

$c/\ell$ model, there are thus two prices — one for each of the two purchasing decisions, $c$ and $\ell$. But with multiple off-market time-use decisions each weighted in the budget constraint by the same price, the cardinality of the price vector is less than the cardinality of the quantity vector, which includes time. Indeed, in Beckerian models the price vector is constrained to be one plus the size of the vector of market purchase prices, while the number of time-utilization decisions may grow as much as the modeler sees fit. Thus, Beckerian models do not conform to a fundamental assumption underlying classic consumer theory: if the left-hand side of the budget constraint contains $\mathcal{M}$ decisions then the corresponding vector of prices also has dimension $\mathcal{M}$.[16] In our case the consumer faces four effective purchase decisions — $q_{igt}$, $q_{ist}$, $n_{igt}$, and $n_{ist}$ — but only three prices — $P_{gt}$, $P_{st}$, and $w_{it}$.

Let $e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})$ be the expenditure function associated with the consumer's EMP. Since the prices of $n_{igt}$ and $n_{ist}$ are constrained to be identical, the model's version of Shepherd's Lemma is slightly different than the standard version.

**Lemma 1.** *Shepherd's Lemma for off-market time use and wages is*

$$n^h_{igt} + n^h_{ist} = \frac{\partial e_{it}}{\partial w_{it}}$$

All proofs are presented in Appendix C.1. Lemma 1 follows directly from the fact that $n_{igt}$ and $n_{ist}$ always have the same price but are separate decisions which will differ from each other due strictly to the structures of $f_g(\cdot,\cdot)$, $f_s(\cdot,\cdot)$, and $u(\cdot,\cdot)$.

The classical Shepherd's Lemma breaks here simply because the price set is smaller than the choice set. This has implications for the terms of the cross-price responsiveness of market commodities $q_{ijt}$ to wages $w_{it}$. Lemma 2 characterizes the Slutsky equation describing the responsiveness of market consumption to wage variation.

**Lemma 2.** *The Slutsky equations describing the responsiveness of demand $q_{ijt}$ to wages $w_{it}$ are*

$$\frac{\partial q^m_{ijt}}{\partial w_{it}} = \frac{\partial q^h_{ijt}}{\partial w_{it}} - \frac{\partial q^m_{ijt}}{\partial y_{it}}(n_{igt} + n_{ist}), \qquad \forall j \in \{g, s\}$$

This expression simply encodes cross-price responsiveness, where the price is the opportunity cost of off-market time utilization which is just $w_{it}$.

Notice that if $q_{ijt}$ is observed in the data to increase as wages rise, then Lemma 2 says that the substitution effect, not the income effect, must be dominating. Note, though, that Lemma 2 does not say anything about whether the classic $c/\ell$ income or substitution effect dominates. This is because demand may be linked in heterogeneous ways to the separate off-market time-use decisions.[17]

It is clear that $q_{igt}$ and $q_{ist}$ may respond to wage variation in different ways even if all of $f_j(\cdot,\cdot)$ and $u(\cdot,\cdot)$ are homothetic. This is because the effect of $w_{it}$ on $q_{ijt}$ is both an income effect and a substitution effect. In a model with inelastic labor, rising wages can only cause budget

[16]See, for example, Chapter 2.D of Mas-Colell, Winston, and Green (1995).

[17]For example, if $q_{ijt}$ and $n_{ijt}$ are strong complements and $c_{igt}$ and $c_{ist}$ are strong substitutes, then possibly $\frac{\partial q^m_{ijt}}{\partial w_{it}} < 0$. In such a case $q_{ijt}$ would appear in the data to be an inferior good if labor income rises. We demonstrate this for an explicit parameterization in Section 3.3.1.

shares to vary through the pure-income-effect channel. In the next section, amongst the many model features we explore, we show that under the Beckerian framework demand can vary in wages even if $u(\cdot,\cdot)$ composed with $f_j(\cdot,\cdot)$ yields a homothetic preference structure.

## 3.3. Comparative Statics for Household Decisions

To illustrate the important theoretical implications of the model, we engage in several comparative statics using fairly conventional parameterizations for home production and utility. The aim is to show that when consumers face multiple market-purchase decisions each complementary to a separate off-market time-use decision, even a homothetic preference structure can generate non-linear Engel curves for market purchases. Thus, in this section we focus only on household decisions in a static environment with no savings mechanism.[18] The household receives income only from supplying labor, $w\ell$. Assume there is no variation in home-production productivities so that $z_j = 1$ for all $j$.

For these exercises only, consider Cobb-Douglas home production functions, with process-specific output elasticities $\omega_j$. These functions are $f_j(q_j, n_j) = q_j^{\omega_j} n_j^{1-\omega_j}$. Consider constant elasticity of substitution (CES) utility over final consumption: $u(c_g, c_s) = \left(c_g^\rho + c_s^\rho\right)^{\frac{1}{\rho}}$. When $\rho \in (0,1)$, $c_g$ and $c_s$ are gross substitutes, and when $\rho < 0$ they are gross complements. The composite utility function $u\big(f_g(q_g, n_g), f_s(q_s, n_s)\big)$ is homothetic in both market quantities and time, but since $w$ is both income and the price of off-market time, expenditure shares will be affected by its variation.

Under this home-production parameterization, the infra-marginal rate of substitution between market purchases and time for activity $j$ is

$$\frac{n_j \omega_j}{q_j(1-\omega_j)} = \frac{P_j}{w} \tag{3.7}$$

We can use (3.7) to replace instances of $n_j$ from the marginal rate of substitution for market goods and services and instances of $q_j$ from the marginal rate of substitution between the two choices for off-market time use to derive expressions for relative market consumption and relative off-market time use as functions of prices and wages:

$$\left(\frac{q_g}{q_s}\right) = \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{3.8}$$

$$\left(\frac{n_g}{n_s}\right) = \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{3.9}$$

These represent ratios of Marshallian demands, though we ignore the $m$ superscript for notational simplicity.[19]

[18] We drop subscripts $i$ and $t$ in this section only.

[19] Since Marshallian demand functions are homogeneous of degree zero in prices and total income, their ratios are also homogeneous degree zero in prices, so that relative demand is aggregate-inflation neutral.

### 3.3.1. Income and Substitution Effects from Wage Variation

Relative market purchases will vary in wages as long as $\rho \neq 0$ and $\omega_s \neq \omega_g$, that is when utility (not home production) is *not* Cobb-Douglas and the home production processes associated with the consumption of goods and services have different time intensities. Whether the ratio of goods to services consumption rises or falls in $w$ will depend on whether the outputs of home production are complements or substitutes and whether services or goods are more time intensive. The same goes for relative time use. Labor supply responsiveness to wage variation will depend on the elasticity of substitution, $\frac{1}{1-\rho}$, for the home production outputs.

For this section only, assume $P_g$ and $P_s$ are fixed, and consider the responsiveness of consumer choices to wages. Propositions 1 through 3 characterize the responsiveness of Marshallian labor supply and demands for market goods and off-market time to wage variation.[20]

**Proposition 1.** *In a two-good, static economy with CES utility and Cobb-Douglas home production, the intensive margin of labor varies in wages as follows:*

*i. If the outputs of home production are substitutes so that $\rho \in (0,1)$, $\ell$ is increasing in $w$ and the classic $c/\ell$ substitution effect dominates.*

*ii. If the outputs of home production are complements so that $\rho < 0$, $\ell$ is decreasing in $w$ and the classic $c/\ell$ income effect dominates.*

**Proposition 2.** *Relative market purchases and off-market time use vary in wages as follows:*

*i. If $\rho \in (0,1)$ then market purchases and time use for the more time-intensive task fall relative to the less time-intensive task as $w$ rises.*

*ii. If $\rho < 0$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as $w$ rises.*

**Proposition 3.** *Marshallian demands for off-market time respond to wage increases as follows:*

*i. If $\rho \in (0,1)$ then time devoted to the more time-intensive task is inferior.*

*ii. If $\rho < 0$ then time devoted to the less time-intensive task is inferior.*

We now illustrate how the classic $c/\ell$ substitution and income effects are related to the inferiority of certain off-market time-use decisions under our parameterization. Consider the case where $\rho \in (0,1)$, so $c_g$ and $c_s$ are gross substitutes. Suppose that goods are more time intensive, so that $\omega_s > \omega_g$. By Proposition 1, as $w \uparrow$, $\ell \uparrow$, which implies total off-market time $\overline{n} - \ell$ falls. Thus, as $w \uparrow$, employing Propositions 1 and 3, note that since the total change in off-market time is $\mathrm{d}(\overline{n} - \ell) = \mathrm{d}n_g + \mathrm{d}n_s$ then it must be that $\mathrm{d}n_s < |\mathrm{d}n_g|$ since $\mathrm{d}n_s > 0$ and $\mathrm{d}n_g < 0$. It follows that the substitution effect driving $n_g$ down must be dominating the income effect driving $n_s$ up. This explains why when $\rho \in (0,1)$ the classic $c/\ell$ substitution effect dominates. When $\rho < 0$ the logic is the same, though the signs of total changes are the opposite: $|\mathrm{d}n_s| < \mathrm{d}n_g$ which implies that the income effect associated with increasing $n_g$ dominates the substitution effect associated with decreasing $n_s$.

[20] All proofs, again, are in Appendix C.1.

**Proposition 4.** *Marshallian demands for market purchases respond to wage increases as follows:*

*i. If $\rho \in (0,1)$ then the market purchase associated with the less time-intensive process is normal, but the market purchase associated with the more time-intensive process may, but need not, be inferior for certain prices and parameter combinations.*

*ii. If $\rho < 0$ then all market purchases are normal.*

Here, we discuss how relative consumption $q_g/q_s$ is impacted by classic $c/\ell$ substitution and income effects. Differences in time-use complementarities across the different home production processes play an important role. Indeed, if $\omega_s = \omega_g$ then clearly relative consumption is independent of wages, which can be seen by inspecting (3.9). Consider the case when goods are more time intensive, so that $\omega_s > \omega_g$. In this parameterization Proposition 4 states that Marshallian demand for services is always normal, though for goods it may be inferior if $\rho \in (0,1)$. The fact that the Beckerian model, even under a parameterization using conventional functional forms, can yield inferior market commodities has been little explored.[21] In our version, inferiority is more likely as $\rho \to 1$: that is, if the outputs of home production are strongly substitutable. When $\rho \in (0,1)$ time-spent using goods (more time-intensive) is always inferior by Proposition 3. When $q_g$ is also inferior, home production complementarities induce negative Hicksian substitution effects on $q_g$ as $w$ rises. These substitution effects, driven by the gross substitutability between $c_g$ and $c_s$, dominate the positive impact of rising income, and so $q_g$ manifests as an inferior good, while services consumption and time use increase.

When $\rho < 0$ the outputs of home production are gross complements, which eliminates the possibility that either market purchase may manifest as inferior. This is because, even though $n_s$ falls as $w$ rises, $c_g$ and $c_s$ co-move together. Thus, to make up for declining services time, consumers still increase services consumption, perhaps by purchasing more-valuable, higher-quality services: think about substituting bus travel for faster air travel, for example. $q_g$ and $q_s$ both rise due to wage increases, though $q_g$ rises faster as Proposition 2 states. While in the case of $\rho \in (0,1)$, the classic $c/\ell$ substitution effect may make both $q_g$ and $n_g$ manifest as inferior, when $\rho < 0$ the classic $c/\ell$ income effect works as would be expected: total income rises, total off-market time rises, and total consumption rise, with all components of the consumption vector increasing.

### 3.3.2. Income and Substitution Effects from Relative Price Variation

Generally, variation in the relative market price of goods to services $P_g/P_s$ can induce both classic income and substitution effects. In this model it can also induce classic $c/\ell$ income and substitution effects: relative price variation affects relative market consumption which in turn affects relative off-market time utilization via home production complementarities, which in turn affects the intensive margin of labor. The following propositions will outline these mechanics.

[21] Benhabib, Rogerson, and Wright (1991) discuss conditions under which leisure time is inferior, and Hymer and Resnick (1969) show that under certain conditions the activities themselves can be inferior, but to our knowledge nobody has used Beckerian models to address the possible inferiority of measured market purchases. This result thus has broad implications for more general models that contain even more varieties of products.

Assume $w/P_s$ is fixed, so that $w$ and $P_s$ grow at the same rate. For illustration consider the responsiveness of consumer choices to decreases in the relative price $P_g/P_s$ in the context of the two-good, static economy with CES utility and Cobb-Douglas home production.

**Proposition 5.** *Relative market purchases and off-market time use vary in the relative price of market purchases as follows:*

*i. If $\rho \in (0,1)$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as the more time-intensive task becomes cheaper.*

*ii. If $\rho < 0$ then market purchases for the more time-intensive task rise relative to the less time-intensive task, but time use for the more time-intensive task relative to the less time-intensive task falls as the more time-intensive task becomes cheaper.*

**Proposition 6.** *Marshallian demands for market purchases vary in relative prices as follows:*

*i. If $\rho \in (0,1)$, consumption of the less time-intensive market purchase falls while consumption of the more time-intensive purchase rises as the more time-intensive task becomes cheaper.*

*ii. If $\rho < 0$, consumption of both market purchases rises as the more time-intensive task becomes cheaper.*

**Proposition 7.** *Marshallian demands for off-market time vary in the relative price of market purchases as follows:*

*i. If $\rho \in (0,1)$, off-market time use for the less time-intensive task falls and time use for the more time-intensive task rises as the more time-intensive task becomes cheaper.*

*ii. If $\rho < 0$, off-market time use for the less time-intensive task rises and time use for the more time-intensive task falls as the more time-intensive task becomes cheaper.*

**Proposition 8.** *Marshallian labor supply varies in the relative price of market purchases as follows:*

*i. If $\rho \in (0,1)$, $\ell$ falls as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic $c/\ell$ income effect which dominates.*

*ii. If $\rho < 0$, $\ell$ rises as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic $c/\ell$ substitution effect which dominates.*

Unlike with straight wage variation, when relative prices change, relative market consumption and off-market time use can move in opposite directions if the outputs of home production are gross complements, $\rho < 0$. As more time-intensive tasks become cheaper, consumers may buy relatively more market inputs associated with those tasks but spend relatively more time engaged with less time-intensive activities. Indeed, if the outputs of home production are gross complements, then a decline in the relative price of the more time-intensive market purchase will lead to increased consumption across the board, though consumption of the more time-intensive commodity, whose price is falling, increases faster. This induces a substitution effect, where off-market time flows away from the more time-intensive task to the less time-intensive one as a result of gross complementarities in the preference structure. Further, the consumer's desire for more of each market commodity induces what manifests as a classic $c/\ell$ substitution effect as $\ell$ rises.

When $c_g$ and $c_s$ are gross substitutes so $\rho \in (0, 1)$, both off-market time and market consumption covary in the same manner. If more time-intensive market purchases become cheaper, consumers substitute both expenditure and time toward such purchases and away from less time-intensive, but more expensive, commodities. $\ell$ also declines which is actually a result of the Hicksian substitution effect brought on by a declining relative price: as the consumption basket has become more affordable, the consumer now need not work as much as before. He is thus better off buying more of the time-intensive commodity and spending more time using that commodity.

The mechanics outlined here thus show that linkages between off-market time utilization, labor supply, and market demand decisions can lead to rather complex co-movements of observables even under a fairly standard preference structure.

## 3.4. Empirical Regularities

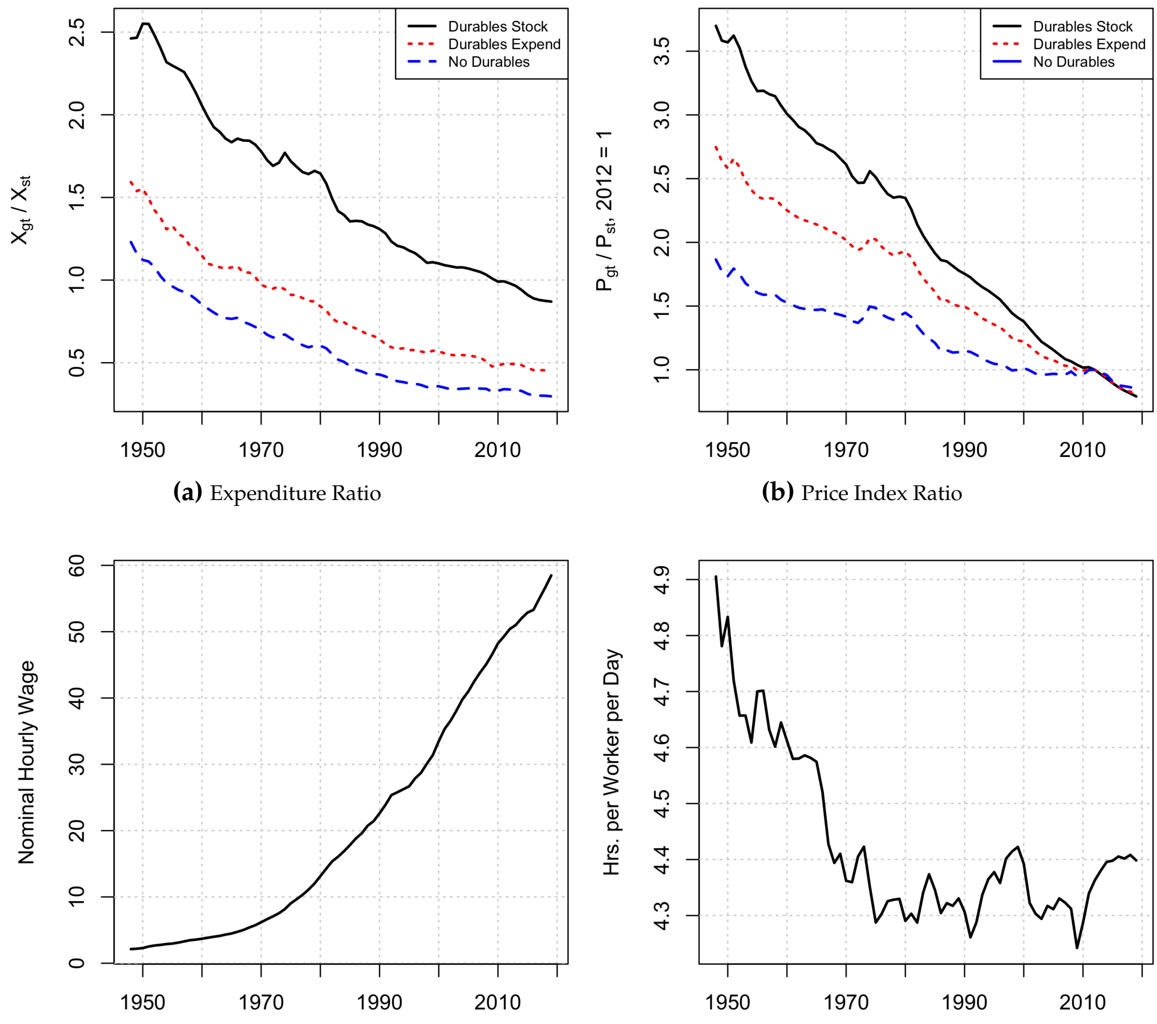
In this section we discuss several trends in both long-run, aggregate U.S. consumption expenditure and labor-hours data and dis-aggregated spending and time use data.[22] For details on how we build out our household-level and sectoral data series, see Online Technical Appendix A.

### 3.4.1. Aggregate U.S. Expenditure Data

Our quantitative exercises operate on several well-established long-run trends in U.S. economic activity from 1948-2019: the decline in the aggregate nominal consumption value of goods to services $X_{gt}/X_{st}$, the decline in aggregate relative goods to services prices $P_{gt}/P_{st}$, and the rise in labor income per hour, $w_t$. Both the signs and magnitudes of changes to spending, quantity, and price indices depend on the degree to which we account for the presence of consumer durables in the various goods series.

Figure 3.2 presents the aggregate data series of interest. Several facts stand out. First, spending and price ratios decline together, while wages rise. Second, focussing on just (a), (b), and (c) it is obvious that if hourly wage gains are highly correlated with income gains overall, including gains from capital income, then models with inelastic labor and non-homothetic preferences will match the expenditure series. In such models the rise in the services share of spending will be explained by pure income effects that are highly correlated with wage gains. Third, in simple $c/\ell$ models with elastic labor, it appears the classic $c/\ell$ income effect would dominate

[22] We draw aggregate non-durable and services data from the National Income and Product Accounts (NIPA). Consumer durables service flows and firms' capital utilization by sector are taken from the Bureau of Economic Analysis' (BEA) Fixed Asset Tables. Aggregate capital and labor income data, as well as sector-specific aggregate labor hours are taken from NIPA. All aggregate data are at annual frequencies from 1948-2019. Micro expenditure and time-use data are drawn from the Bureau of Labor Statistics' (BLS) Consumer Expenditure Survey (CEX) from 1984-2018 and American Time Use Survey (ATUS) from 2003-2019. Household-level wage data are from the annual March release of the Current Population Survey (CPS). At the time this paper was written a preliminary update to the CEX for 2019 had been released, but we found undocumented changes to the dataset with respect to how certain expenditures were classified. We await a reply to our correspondence with the BLS before incorporating the 2019 data into our analysis.




**(a)** Expenditure Ratio

**(b)** Price Index Ratio

**(c)** Nominal Wages

**(d)** Labor Hours

**Figure 3.2.:** Here we present the evolution of several long-run aggregate data series. The ratio of the aggregate nominal value of final goods to services consumption is in (a), the relative chain-weighted price of goods to services where 2012 = 1 is (b), average nominal labor-income per hour worked, including proprietors' incomes with inventory and capital consumption adjustments, is in (c), and total hours worked per day for each effective full-time worker is in (d). In (a) and (b) we show three data series each constructed to include different measures of consumer durables. The "Durables Stock" plots (solid black line) include the value of the entire stock of existing consumer durables in the goods series. The "Durables Expend" plots (dotted red line) include only new investment in durables. The "No Durables" plots (dashed blue line) only include non-durables in the goods series. All series are annual, 1948-2019.

the classic $c/\ell$ substitution effect at least over the period 1948-1970, as can be seen in panel (d). After 1970, average hours per worker per day are relatively flat, so that $c/\ell$ income effects may not be as strong as they were during mid-century. In our structural estimations we will attempt to assess the degree to which these various theoretical mechanisms have contributed to structural change.

### 3.4.2. Time-use and Expenditure in Micro Data

In this section, we examine cross-sectional variation in market demand and time use amongst different consumers at different income levels. We match the CEX summary cross-tabs by income quintile to the ATUS, which includes the annual March CPS wage data. Then, to construct separate spending series for goods and services, we roughly match CEX spending and ATUS activities to the detailed expenditure categorizations in NIPA Table 2.3.5 — spending by "major type of product."[23]

The ATUS dataset provides a potential way to distinguish between activities associated with using goods versus services for certain, but not all, tasks. As an example, the dataset contains both a variable that presents the time an individual respondent spent "Interior cleaning" and a separate variable that presents the time that same individual respondent spent "Using interior cleaning services." Many, but not all, tasks within the survey are classified in this manner. As an example of a task that cannot be assigned to either a goods or services task, the survey does not distinguish between traveling by car in one's own personal vehicle versus traveling by plane, bus, train, or rental car. The former would require the consumer to purchase gasoline which is classified as a good in the NIPA data, while the latter activities would be classified as services consumption. Given this particular inconsistency, in conjunction with the rather short length of the ATUS time series (2003-2019), we highlight how relative time use has evolved under our particular classification rubric but only use these series to construct prior estimates for hyper-parameters that we can then feed to our structural estimation routine which operates only on consumption data. Later, as a robustness check, we will compare the out-of-sample fit of our model's predictions for off-market time use against the ATUS data.

Figure 3.3 presents a breakdown by income quintile of observed time use and spending behavior from the micro data. Lower income consumers spend relatively more time using services than goods, tend to work less, and spend a larger fraction of their disposable income on goods compared to higher income consumers. While all workers spend, on average, more time using services than goods, it appears that the patterns of time use have not changed much since the ATUS was started in 2003. From Figure 3.3d, however, it is clear that relative goods to services expenditure has declined in a consistent manner for all consumers in all income quintiles. To compare the breakdown by income quintile with aggregates, use the dotted red line in Figure 3.2a as a reference point since new durables expenditure is included in the CEX measures but not the exact value of durable assets owned.

Finally, for a preliminary assessment of the relationship in data between labor hours and relative prices, we regressed labor hours $\ell_{it}$ on the hourly wage $w_{it}$ and the relative price $P_{gt}/P_{st}$, where $i$ indexes all respondents in the ATUS dataset, not just income quintiles. A one unit rise in the relative price $P_{gt}/P_{st}$ corresponds to a fall in labor hours per day of approximately $-0.396$ (10% significance level), while increases in wages correspond to an increase in hours per day of approximately 0.016 (1% significance level). Such a regression is obviously not causal, so we will not discuss the results here further. The main takeaway is that work time may be correlated with market prices in ways that suggest home production complementarities are at play. We

[23] For the CEX classification, we apply the same classification rubric as in Boppart (2014). The ATUS time-use classification details are in Online Technical Appendix A.2.


**Figure 3.3.:** In panel (a) we present the ratio of off-market goods to services time utilization from ATUS by income quintile, where personal-care time, including time spent sleeping, is categorized as goods utilization. In panel (b) we present the same ratio, except we exclude all activities in the personal care-time category except those associated with using shampoos, soaps, and personal hygiene products. Panel (c) shows total hours worked per day by income quintile from ATUS. Panel (d) features the ratio of goods to services expenditure from CEX. ATUS runs from 2003-2019 while the CEX runs from 1984-2018. The legend denoting which color and line type scheme correspond to which income quintile is included in the Expenditure Ratio plot.

will empirically demonstrate that our model can rationalize such relationships.

## 3.5. Quantitative Model and Estimation

We estimate the model separately with both aggregate and micro data. To account for general equilibrium effects we include the goods and services firms' marginal products of labor in the system of estimating equations. When estimating the dis-aggregated model with micro data we use synthetic expenditure and wage series indexed by income quintile. Our estimation procedures take a Bayesian approach, as we target the posterior distribution of structural parameters conditional upon observed data. Estimating this distribution requires computing an intractable integral, which we accomplish using Hamiltonian Monte Carlo (HMC) integration techniques.[24]

### 3.5.1. Structural Estimating Equations

For our quantitative exercises, we consider a more flexible CES parameterization for home production: $f_j(q_{ijt}, n_{ijt}) = \left(\omega_j q_{ijt}^{\nu_j} + (1-\omega_j) n_{ijt}^{\nu_j}\right)^{\frac{1}{\nu_j}}$. Preferences over final consumption are assumed to take the same CES form as in Section 3.3.[25] Composing home production functions featuring Hicks-neutral home productivities with preferences for final consumption, we get the nested CES structure:

$$u\big(c_{igt}(q_{igt}, n_{igt}), c_{ist}(q_{ist}, n_{ist})\big) = \left( \sum_{j\in\{g,s\}} z_{ijt}^{\rho} \left(\omega_j q_{ijt}^{\nu_j} + (1-\omega_j) n_{ijt}^{\nu_j}\right)^{\frac{\rho}{\nu_j}} \right)^{\frac{1}{\rho}} \tag{3.10}$$

After solving (3.1) using this preference structure, we can equate the marginal rate of substitution for market inputs $q_{igt}$ and $q_{ist}$ with the ratio of their prices:

$$\left(\frac{c_{igt}}{c_{ist}}\right)^{\rho-1} \frac{z_{igt}\omega_g q_{igt}^{\nu_g-1}}{z_{ist}\omega_s q_{ist}^{\nu_s-1}} \left(\omega_g q_{igt}^{\nu_g} + (1-\omega_g) n_{igt}^{\nu_g}\right)^{\frac{1-\nu_g}{\nu_g}} \left(\omega_s q_{ist}^{\nu_s} + (1-\omega_s) n_{ist}^{\nu_s}\right)^{\frac{\nu_s-1}{\nu_s}} = \frac{P_{gt}}{P_{st}} \tag{3.11}$$

The marginal rate of substitution for off-market time-utilization decisions equates to unity:

$$\left(\frac{c_{igt}}{c_{ist}}\right)^{\rho-1} \frac{z_{igt}(1-\omega_g) n_{igt}^{\nu_g-1}}{z_{ist}(1-\omega_s) n_{ist}^{\nu_s-1}} \left(\omega_g q_{igt}^{\nu_g} + (1-\omega_g) n_{igt}^{\nu_g}\right)^{\frac{1-\nu_g}{\nu_g}} \left(\omega_s q_{ist}^{\nu_s} + (1-\omega_s) n_{ist}^{\nu_s}\right)^{\frac{\nu_s-1}{\nu_s}} = 1 \tag{3.12}$$

There are four possible versions of the Euler equation describing consumption dynamics, each of which must simultaneously hold in equilibrium:[26]

$$\begin{aligned} &\left(\sum_{j\in\{g,s\}} c_{ijt}^{\rho}\right)^{\frac{1-\rho}{\rho}} c_{ikt}^{\rho-1} \left(\omega_k q_{ikt}^{\nu_k} + (1-\omega_k) n_{ikt}^{\nu_k}\right)^{\frac{1-\nu_k}{\nu_k}} z_{ikt}\omega_k q_{ikt}^{\nu_k-1} \\ &\quad = \beta\, \mathbb{E}_t \Bigg\{ R_{t+1} \left(\sum_{j\in\{g,s\}} c_{im,t+1}^{\rho}\right)^{\frac{1-\rho}{\rho}} c_{im,t+1}^{\rho-1} \\ &\quad \times \left(\omega_m q_{im,t+1}^{\nu_m} + (1-\omega_m) n_{im,t+1}^{\nu_m}\right)^{\frac{1-\nu_m}{\nu_m}} z_{im,t+1}\omega_m q_{im,t+1}^{\nu_m-1} \Bigg\}, \qquad \forall k, m \in \{g,s\} \end{aligned} \tag{3.13}$$

[24]We describe HMC integration techniques in Online Technical Appendix B.1. For detailed explanations of HMC techniques see Neal (2011), Betancourt and Stein (2011), Gelman et al. (2013b), and Gelman et al. (2013a).

[25]We choose to estimate the model using this parameterization so that elasticities of substitution between $q_{ijt}$ and $n_{ijt}$ are allowed to vary across processes. While this parameterization is indeed more flexible, it still yields a homothetic composite utility function.

[26]Due to stochastic singularity, which will be discussed in the forthcoming pages, each separate Euler equation describes the same equilibrium outcomes.

Using the infra-marginal rate of substitution between off-market time and market inputs for process $j$, we can write $n_{ijt}$ as an implicit function of $q_{ijt}$:

$$n_{ijt}(q_{ijt}) = q_{ijt}\left[\frac{w_{it}\omega_j}{P_{jt}(1-\omega_j)}\right]^{\frac{1}{\nu_j-1}}, \quad \forall j \in \{g, s\} \tag{3.14}$$

Note that we possess time-use data only for the period 2003-2019. Lacking time-use data for earlier years, we can use (3.14) to substitute out instances of $n_{ijt}$ in (3.11) and/or (3.13), allowing us to estimate the full model for the post-war years using only aggregate consumption data from 1948-2019 and CEX data from 1984-2018.

To recover the household's structural parameters we will focus on estimating (3.11). Relative home productivities ($z_{igt}/z_{ist}$) comprise the sole stochastic component of (3.11), while the Euler equations in (3.13) depend on productivity *levels*, not just relative productivities. Ultimately, we want to build the likelihood function around fluctuations in structural productivities without introducing additional model or measurement errors. Such a choice, however, comes with tradeoffs, namely that, depending on the home-productivity normalization we choose, (3.11) and (3.13) constitute a stochastically singular system. This is because, up to normalization, knowing the relative productivities means that we can back out a time series of productivity levels from one of the Euler equations so that it identically holds. Given the stochastic singularity, we thus choose to estimate the model using (3.11) while treating relative home productivities as the residual.

Issues pertaining to stochastic singularity arise in other applications involving estimation of dynamic stochastic general equilibrium (DSGE) models. The canonical example of a stochastically singular system is the stochastic growth model where output, consumption, and investment are all co-integrated and driven by a single, structural shock — TFP. To overcome the problem of having more endogenous variables than shocks, Komunjer and Ng (2011) point out that in practice it is common to either add measurement errors to the system or drop equations containing certain endogenous variables. The decision to exclude observables and equations in stochastically singular systems is usually motivated by the need to reduce computational complexity, rather than economic considerations (Qu 2018). This is because for most DSGE models that deal with aggregates, the data series are easily obtained. This is not the case, however, in our application where a long, high-quality off-market time-use data series is required: the ATUS data only extends back to 2003, and as we document in Section 3.4.2, there are issues with the way the ATUS survey may align with NIPA-categorized consumption activities. Further, the first-order conditions that describe households' decisions are many and each are highly non-linear, especially the Euler equations.

Meanwhile, it is common in most growth accounting exercises featuring multiple sectors to construct stylized stochastic processes which can be easily de-trended and which preserve balanced growth over aggregate consumption.[27] Such assumptions are restrictive for our exercise, since we seek to allow for the sectors to grow at different rates which may vary over time. While we are not contending that the data-generating process necessarily departs from a

[27]See, for example, the structural change analyses of Caselli and Coleman (2001), Kongsamut, Rebelo, and Xie (2001), Ngai and Pissarides (2007), and Boppart (2014). To ensure balanced growth, Kehoe, Ruhl, and Steinberg (2018) vary the subsistence parameters of a Stone-Geary utility function over time.

commodity-aggregate balanced growth path, our approach allows for the possibility that sectors do not grow at rates that are proportionally constant over time. Our quantitative approach thus most closely resembles that of Herrendorf, Rogerson, and Valentinyi (2013), who estimate preference parameters in a structural change model by focussing on household intratemporal equilibrium conditions. Rather than specifying a growth process for each $z_{ijt}$ and solving for a de-trended steady state, then linearizing the model around that steady state in order to arrive at estimating equations, we instead estimate (3.11) directly using the observed time series of relative expenditure data, accounting for price endogeneity using firms' marginal product conditions. Our estimates are consistent under the assumption that households are price-takers and the data was generated by our model. The Euler equations and household labor-supply functions are assumed to hold (because of stochastic singularity and the assumption that our model is the DGP), given some unknown and unspecified stochastic process for the levels of $z_{ijt}$. By estimating the model using only (3.11), we need only engage in weak assumptions as to how the relative productivities $z_{igt}/z_{ist}$ evolve over time.

While direct estimation of DSGE models is particularly attractive for those engaged in out-of-sample forecasting, moment-based calibration techniques (which usually require parameterizing the exogenous stochastic process and solving for a steady state) remain the gold standard for models whose primary purpose is an assessment of theory. Our methodological approach demonstrates that likelihood-based estimation techniques, where parameters are estimated by targeting an *entire data series* rather than moment-based summary statistics, can also be used to assess theory. Specifically, we can exploit the fact that our estimating system is stochastically singular to assess model performance. That is, we estimate the model's parameters using a subset of data associated with the general equilibrium variables (expenditure, prices, wages, and capital/labor ratios by sector). We then test model performance by using the model's equilibrium conditions and estimated parameters to simulate data series that were not targeted in the estimation, specifically $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$.[28] We can then use standard statistical methods to test the hypotheses that the model-simulated data series are equal to actual data, which we do in Sections 3.5.4.1 and 3.5.5.1.

Now, to arrive at an estimating equation for the household's parameters, we substitute out time use from (3.11). After collecting like terms we get an implicit expression, featuring only relative market quantities, that is consistent with the model's equilibrium:

$$\begin{aligned}\left(\frac{z_{igt}}{z_{ist}}\right)^{\rho}\left(\frac{q_{igt}}{q_{ist}}\right)^{\rho-1}\left(\frac{\omega_g}{\omega_s}\right)\left(\omega_g+(1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)^{\frac{\rho-\nu_g}{\nu_g}}& \\ \times\left(\omega_s+(1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)^{\frac{\nu_s-\rho}{\nu_s}}&=\frac{P_{gt}}{P_{st}}\end{aligned} \tag{3.15}$$

We can multiply both sides of (3.15) by $(P_{gt}/P_{st})^{\rho-1}$, so that the model can be estimated directly using relative expenditure data $x_{igt}/x_{ist}$. Then, to isolate $z_{igt}/z_{ist}$, so that relative productivities may be treated as a structural residual, we exponentiate both sides by $1/\rho$, take logs of both

[28]Hats are used to denote simulated data.

sides, and rearrange to get the expression:

$$
\begin{aligned}
&\frac{1-\rho}{\rho}\ln\left(\frac{x_{igt}}{x_{ist}}\right)-\frac{1}{\rho}\ln\left(\frac{\omega_g}{\omega_s}\right)+\ln\left(\frac{P_{gt}}{P_{st}}\right)\\
&\quad-\frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g+(1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)\\
&\quad+\frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s+(1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)=\ln\left(\frac{z_{igt}}{z_{ist}}\right)
\end{aligned}
\tag{3.16}
$$

Assume the log-ratio $\xi^1_{it}=\ln z_{igt}-\ln z_{ist}$ is first-difference stationary, and let $\Delta$ be the one-period, backwards first-difference operator. Define the residual term $\epsilon^1_{it}=\Delta\xi^1_{it}=\xi^1_{it}-\xi^1_{i,t-1}$, which is assumed mean zero. Taking first-differences of (3.16), we arrive at an estimating equation for household consumption decisions consistent with equilibrium utility maximization:

$$
\begin{aligned}
&\frac{1-\rho}{\rho}\Delta\ln\left(\frac{x_{igt}}{x_{ist}}\right)+\Delta\ln\left(\frac{P_{gt}}{P_{st}}\right)-\frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g+(1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)\\
&\quad+\frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g+(1-\omega_g)\left[\frac{w_{i,t-1}\omega_g}{P_{g,t-1}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)\\
&\quad+\frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s+(1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)\\
&\quad-\frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s+(1-\omega_s)\left[\frac{w_{i,t-1}\omega_s}{P_{s,t-1}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)=\epsilon^1_{it}
\end{aligned}
\tag{3.17}
$$

Goods and services producing firms have Cobb-Douglas technologies and face the same input prices $w_t$ and $r_t$. Equilibrium sectoral capital and labor inputs must satisfy:[29]

$$
\begin{aligned}
A_{jt}(1-\alpha_j)\left(\frac{K_{jt}}{L_{jt}}\right)^{\alpha_j}&=\frac{w_t}{\overline{P}_{jt}},\qquad\forall j\in\{g,s\}\\
A_{jt}\alpha_j\left(\frac{K_{jt}}{L_{jt}}\right)^{\alpha_j-1}&=\frac{r_t}{\overline{P}_{jt}},\qquad\forall j\in\{g,s\}
\end{aligned}
\tag{3.18}
$$

If $A_{jt}$ is the only residual term to the econometrician in the above two equations, then these equations, again, constitute a stochastically singular system. Absent introducing model or measurement error, we ignore one of the equations and estimate $\alpha_j$ using the other. We choose to focus on the marginal product of labor (MPL) conditions because we seek to provide a general equilibrium link for wages, as well as market prices.

Take logs of (3.18) and define the log-TFP term $\xi^2_{jt}=\ln A_{jt}$, which we assume is first-difference stationary. The residual term $\epsilon^2_{jt}=\Delta\xi^2_{jt}=\xi^2_{jt}-\xi^2_{j,t-1}$ is assumed mean zero. After taking first-differences, isolating $\epsilon^2_{jt}$, and rearranging, we get the estimating equations on firms'

[29] $\overline{P}_{jt}$ is used since prices here account only for new flows *produced* by the firms.

equilibrium conditions:

$$\Delta \ln\left(\frac{w_t}{\overline{P}_{jt}}\right) - \alpha_j \Delta \ln\left(\frac{K_{jt}}{L_{jt}}\right) = \epsilon^2_{jt}, \qquad \forall j \in \{g, s\} \tag{3.19}$$

Under our assumption that there exists a unit-mass of households in the economy, each household is a price-taker, so households do not consider how their decisions impact market prices, including wages. This assumption thus allows for replacement of agent-level variables with average aggregates. When only aggregates are considered, replace $\epsilon^1_{it}$ with $\epsilon^1_t$ and $w_{it}$ with $w_t$, labor income per hour worked.

**Table 3.1.:** Prior Distributional Assumptions

| *Firm Parameters* | *Household Parameters* |
|---|---|
| $\alpha_j \sim \text{Beta}(10, \underline{\alpha}_j)$ | $-\rho \sim \text{Lognormal}\big(-\frac{1}{2}, 1\big)$<br>$\omega_j \sim \text{Beta}(1, \underline{\omega}_j)$<br>$-\nu_j \sim \text{Lognormal}(\underline{\nu}_j, 1)$ |
| *Likelihood Variance/Covariance* | |
| $\text{chol}(\boldsymbol{\Sigma}) = \text{diag}(\boldsymbol{\chi}) \cdot \Xi$, with $\chi_k \sim \text{Cauchy}_{(0,\infty)}(0, 2)$, and $\Xi \sim \text{LKJ}(2)^a$ | |

[a] See Lewandowski, Kurowicka, and Joe (2009) for detailed derivation and discussion regarding the properties of the LKJ distribution.

### 3.5.2. Prior Distributional Assumptions

Let $\boldsymbol{\epsilon}_{it} = [\epsilon^1_{it}, \epsilon^2_{gt}, \epsilon^2_{st}]^\top$. We assume

$$\boldsymbol{\epsilon}_{it} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \qquad \forall i$$

with $\mathbb{E}[\epsilon^1_{it}\epsilon^1_{i',t}] = 0$ for $i \neq i'$. If households are indexed by wealth or income, this amounts to saying that households with access to different resources face idiosyncratic shocks to home production, though the variance of such shocks is constrained to be the same across households.

Table 3.1 defines the prior distributions imposed on the model parameters targeted in our estimation routine. We define $\underline{\omega}_j$, $\underline{\nu}_j$, and $\underline{\alpha}_j$ in the next section. $\nu_j$ are assumed negative because our prior estimates suggest that consumption and time-use are complementary inputs to the production processes $f_j(\cdot, \cdot)$. We assume $\rho < 0$, so that the outputs of home production are gross complements. This allows for situations where experiences from using goods and services are uniquely linked, like web browsing (internet telecommunication services) on a computer (durable good), for example. Further, $\rho < 0$ implies $c/\ell$ income effects dominate and thus $\ell_t$ falls as $w_t$ rises. This is consistent with what we observe over time in aggregate data as can be seen in Figures 3.2c and 3.2d. For easy computation of the variance/covariance matrix we impose priors on a Cholesky factorization of $\boldsymbol{\Sigma}$ using half-Cauchy and LKJ distributions

(Lewandowski, Kurowicka, and Joe 2009).

### 3.5.3. Retrieving Hyper-parameter Estimates from Data

We want to allow our limited series of time-use data to inform our prior distributional assumptions for the home production parameters $\omega_j$ and $\nu_j$. To do so, we run the following regressions of (3.14) using OLS on a panel of CEX and ATUS data (excluding personal care time)[30] with NIPA prices from 2003-2018.[31] We assume independent residuals, $\bar{\epsilon}_{ijt}$, over time and across households, indexed by income quintile $i \in \{1, 2, 3, 4, 5\}$, for each $j \in \{g, s\}$:

$$\ln P_{jt} n_{ijt} - \ln x_{ijt} = \frac{1}{\nu_j - 1} \ln \left( \frac{\omega_j}{1 - \omega_j} \right) + \frac{1}{\nu_j - 1} \ln \left( \frac{w_{it}}{P_{jt}} \right) + \bar{\epsilon}_{ijt} \tag{3.20}$$

These regressions generate prior estimates for the CES substitution elasticities that suggest time and market inputs are gross complements in both home production processes: $\widehat{\nu}_g = -2.187$ and $\widehat{\nu}_s = -1.535$. Preliminary estimates of the weights associated with market inputs in the two processes are rather small: $\widehat{\omega}_g = 0.016$ and $\widehat{\omega}_s = 0.002$.[32] We want $\widehat{\nu}_j$ to correspond to the mean of the log-normal distribution for $-\nu_j$, giving location hyper-parameter estimates of $\underline{\widehat{\nu}}_g = 0.283$ and $\underline{\widehat{\nu}}_s = -0.072$. Centering the Beta distributions for $\omega_j$ around these prior estimates, we get hyper-parameter estimates of $\underline{\widehat{\omega}}_g = 62.331$ and $\underline{\widehat{\omega}}_s = 437.566$.

To get prior estimates of $\underline{\alpha}_j$, we run OLS on (3.19) taking $\Delta \ln \left( \frac{w_t}{P_{jt}} \right)$ as the dependent variable. These regressions give estimates of $\widehat{\alpha}_g = 0.185$ and $\widehat{\alpha}_s = 0.089$. Again, placing the mean of the Beta distributions around these values, the shape hyper-parameters are $\underline{\widehat{\alpha}}_g = 44.028$ and $\underline{\widehat{\alpha}}_s = 102.180$.

### 3.5.4. Model Estimates with Aggregate Data

The HMC integration procedure yields estimates of the posterior distribution of model parameters given aggregate data, which we present in Table C.3.[33] We are surprised to find that services are slightly more time-intensive which can be seen by noting $\omega_g > \omega_s$ on average. Time is substantially more complementary with market consumption for services than goods, which can be seen by noting that $\nu_s << \nu_g$. Further, estimates of $\sigma_{1g}$ and $\sigma_{1s}$ suggest that we cannot reject the hypothesis that $\text{Cov}(\epsilon_t^1, \epsilon_{jt}^2) = 0, \forall j \in \{g, s\}$. While the orthogonality condition may hold, information is lost when only estimating the household condition in (3.17), which our robustness checks will reveal. Specifically, elasticity estimates are significantly affected by not accounting for price endogeneity, which can be seen by comparing the estimates for $\rho$, $\nu_g$, and

[30] Generally, this involves simply ignoring any category pertaining to "sleep" (see Online Technical Appendix A.2).

[31] We truncate at 2018 due to the truncated-nature of the CEX data.

[32] Including personal care time in $n_{igt}$ (e.g., sleeping on a bed — a good) only slightly changes the prior estimates: $\widehat{\nu}_g = -2.145$, $\widehat{\nu}_s = -1.535$, $\widehat{\omega}_g = 0.001$, and $\widehat{\omega}_s = 0.002$.

[33] Our estimates here account for the presence of consumer durable service flows. Online Technical Appendix B.2 features parameter estimates for data that includes only new durables expenditure and no service flows. Online Technical Appendix B.1, meanwhile, contains estimates for a regression that only uses the household MRS condition (not firm MPL conditions) to form the likelihood but still accounts for durable service flows. We will show that models without durable service flows in the data and which fail to account for general equilibrium effects by using firm's MPL conditions are poor specifications in that they fail to predict the evolution of the non-targeted aggregate labor series.

**Table 3.2.:** HMC Posterior Distribution, 1948-2019, Agg. Data Incl. Durable Svc. Flows

| | *Simultaneous Equations* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -24.1461 | 13.8701 | -59.1073 | -29.0357 | -20.7522 | -15.1617 | -8.8000 |
| $\nu_g$ | -1.0555 | 0.4939 | -2.0878 | -1.2212 | -0.9680 | -0.7718 | -0.4898 |
| $\nu_s$ | -3.0160 | 2.8209 | -10.1323 | -3.6312 | -2.2623 | -1.4174 | -0.5578 |
| $\omega_g$ | 0.0166 | 0.0156 | 0.0006 | 0.0054 | 0.0122 | 0.0229 | 0.0575 |
| $\omega_s$ | 0.0025 | 0.0024 | 0.0001 | 0.0008 | 0.0018 | 0.0035 | 0.0087 |
| $\alpha_g$ | 0.1264 | 0.0310 | 0.0706 | 0.1048 | 0.1250 | 0.1466 | 0.1914 |
| $\alpha_s$ | 0.0946 | 0.0215 | 0.0549 | 0.0795 | 0.0938 | 0.1089 | 0.1390 |
| $\sigma_1^2$ | 0.0002 | 0.0000 | 0.0001 | 0.0002 | 0.0002 | 0.0002 | 0.0003 |
| $\sigma_g^2$ | 0.0010 | 0.0002 | 0.0007 | 0.0009 | 0.0010 | 0.0011 | 0.0014 |
| $\sigma_s^2$ | 0.0003 | 0.0000 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0004 |
| $\sigma_{1g}$ | 0.0000 | 0.0001 | -0.0002 | -0.0000 | 0.0000 | 0.0001 | 0.0003 |
| $\sigma_{1s}$ | 0.0000 | 0.0001 | -0.0001 | 0.0000 | 0.0000 | 0.0001 | 0.0002 |
| $\sigma_{gs}$ | 0.0003 | 0.0001 | 0.0002 | 0.0003 | 0.0003 | 0.0004 | 0.0005 |
| $V(\mathcal{P})^a$ | 663.5566 | 2.6702 | 657.4420 | 661.9701 | 663.8869 | 665.4968 | 667.7878 |

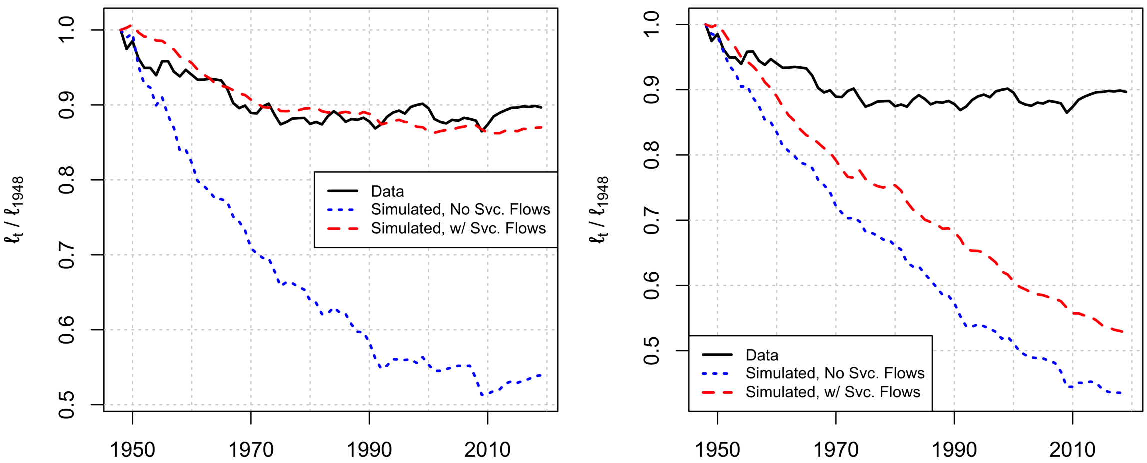
[a] $V(\mathcal{P})$ is the log posterior density of parameters, $\mathcal{P}$.

$\nu_s$ in Table C.3 with those featured in Online Technical Appendix B.2, where MPL conditions are excluded from the likelihood.

#### 3.5.4.1. Robustness Checks with Aggregate Data

In calibration exercises common practice is to simulate the model to match specific moments in the data, then check those simulations against data moments that were not used in the initial fitting procedure. We can perform a similar exercise in our likelihood-based approach by comparing posterior simulations against data series associated with the equilibrium conditions that were left out of the estimation procedure due to stochastic singularities. Specifically, we assess how our aggregate estimation performs with respect to predicting labor hours and off-market time use.

**Model-implied Aggregate Labor Hours:** Since the classic $c/\ell$ income effect appears in aggregate data to dominate for labor hours decisions, we want to ensure that our model estimates are consistent with this fact. Note that the household's Marshallian labor supply can be written as a function of prices, wages, maximum possible income net of savings which we




**(a)** Simultaneous Equations

**(b)** Household MRS Only

**Figure 3.4.:** Here, we compare the simulated fit of normalized $\ell_t$ relative to data when the parameters are estimated under a likelihood function formed around the MRS condition featuring relative expenditure. In panel (a) we show the fit of the simultaneous equations model, and panel (b) features the fit of the partial-equilibrium model, where only the household's MRS condition is used for estimation. In each figure the dashed red line corresponds to the mean of the posterior distribution of the simulated data that accounts for durables service flows while the dotted blue line is the same simulation performed against naive data containing only new durables expenditures. Each simulation uses the estimated model parameters for their respective regressions.

denote by $y_t$,[34] and relative home productivities: $\ell_t(P_{gt}, P_{st}, w_t, y_t, z_{gt}/z_{st})$.[35]

To test our model's performance with respect to predicting how household labor supply evolves over time, we simulate $\widehat{\ell_t}$ using estimated productivities and parameters. Figure 3.4 compares trend decline in the posterior mean of simulated $\widehat{\ell_t}$ against data from 1948-2019, normalized so that both series are unity in 1948. The simultaneous equations model with data accounting for durables service flows is a better fit for the non-targeted labor hours data. Table 3.3 provides $t$-tests of the hypotheses that the period-$t$ means of the posterior distribution of $\widehat{\ell_t}$ are the same as data — $H_0 : \widehat{\ell_t} - \ell_t = 0$. We cannot reject the null hypotheses for any of our models. Still, we prefer the full model accounting for general equilibrium price endogeneity and durable service flows because it yields a time series of posterior-mean predicted labor supply that appears to visually best match the data (the red dashed line in Figure 3.4a).

**Model-implied Relative Off-market Time Use, 2003-2019:** We use the weighted average of our limited 2003-2019 off-market time use data series to perform a similar model fit assessment as above. Given posterior parameter and productivity estimates, we simulate predicted $\widehat{n_{gt}/n_{st}}$

[34]We compute $y_t$ by taking the line "Personal saving as a percentage of disposable personal income" from NIPA Table 2.1, multiplying this value by the sum of capital and labor income and subtracting this savings level from our measure of capital income, which is the sum of lines titled "Rental income of persons with capital consumption adjustment" and "Personal income receipts on assets," also from NIPA Table 2.1. We add this value to wages multiplied by $\overline{n}$, which we take to be equal to 24-9, accounting for 9 hours of personal care time in one day. We then divide this value by total effective full time workers to arrive at a per-worker value for $y_t$.

[35]For derivation of the rather cumbersome parameterized version of this expression see Online Technical Appendix B.5.

**Table 3.3.:** Robustness Checks with Aggregate Data — Fit of Non-targeted Data Series

| | *Data Includes Durable Service Flows* | | | |
|---|---|---|---|---|
| | *Simultaneous Equations* | | *Household MRS Only* [a] | |
| Variable | S.E.[b] | Avg. Test Statistic | S.E.[b] | Avg. Test Statistic |
| $\ell_t$ | 0.325 | 0.004 | 0.326 | $-4.24 \times 10^{-4}$ |
| $n_{gt}/n_{st}$ | 11.521 | $1.586 \times 10^{-5}$ | 69.902 | $6.33 \times 10^{-5}$ |

| | *Data Does Not Include Durable Service Flows* [c] | | | |
|---|---|---|---|---|
| | *Simultaneous Equations* | | *Household MRS Only* | |
| Variable | S.E.[b] | Avg. Test Statistic | S.E.[b] | Avg. Test Statistic |
| $\ell_t$ | 0.249 | -0.006 | 0.297 | -0.003 |
| $n_{gt}/n_{st}$ | 15.733 | $5.610 \times 10^{-4}$ | 94,109.48 | $1.335 \times 10^{-4}$ |

Let $M$ denote the total number of atomic draws (epochs) from the posterior HMC sampler, and let $m$ index these draws. Suppose $\upsilon_t$ is the targeted data series and $\widehat{\upsilon}_t^m$ is a single epoch of the posterior sampler. For each $t$ we compute the distribution of $\widetilde{\upsilon}_t^m = (\widehat{\upsilon}_t^m - \upsilon_t)/SE(\widehat{\upsilon}_t^m)/\sqrt{M}$, where $\widetilde{\upsilon}_t^m$ represents the weighted error of the simulated draw relative to the actual data. We test the hypothesis $H_0 : \overline{\widetilde{\upsilon}_t^m} = 0$, i.e. the mean of $\widetilde{\upsilon}_t^m$ is zero. Since a separate hypothesis test is conducted for each period $t$, here we average over the test statistics associated with those hypothesis tests over time. This statistic technically follows a Student's-$t$ distribution with $M - 1$ degrees of freedom, where $M = 8000$, so the standard normal distribution is a sufficient benchmark for comparison.

[a] See Online Technical Appendix B.2 for estimates from this model.

[b] This is the average standard error over time: $\frac{1}{T} \sum_t SE(\widehat{\upsilon}_t^m)$.

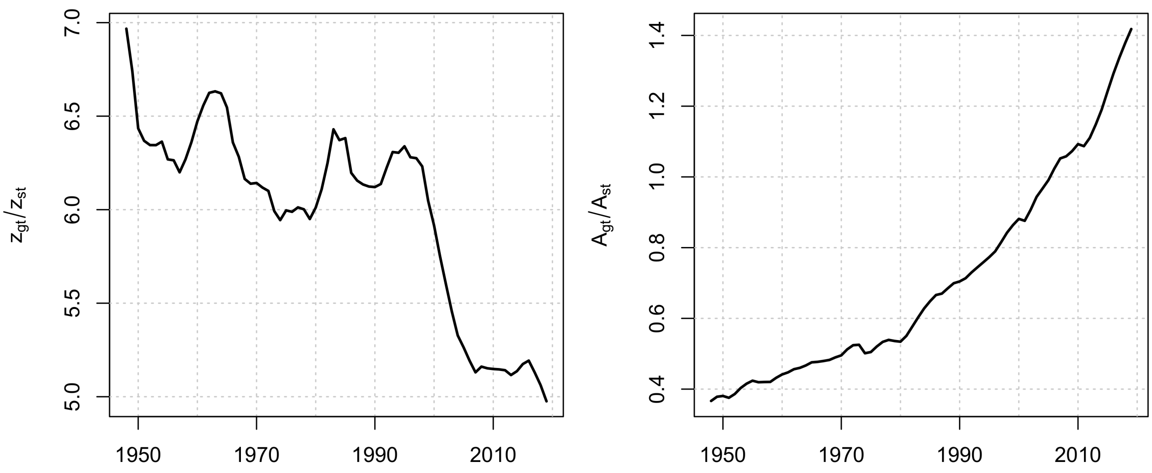
[c] See Online Technical Appendix B.3 for estimates from these models.

from the MRS condition for off-market time use in (3.12). We then use $t$-tests to compare the posterior distributions against data each period from 2003-2019. The results of these tests are presented in Table 3.3, alongside the results of the same tests performed for $\widehat{\ell}_t$. As with labor supply, we cannot reject the hypotheses that the posterior means of simulated relative off-market time-use are equal to data.

#### 3.5.4.2. Estimated Home and Sectoral Technological Change

Figure 3.5 compares the evolution of estimated in-home relative productivities versus sectoral relative productivities. In both panels we show the time series of estimated posterior means. Focussing first on Figure 3.5a we observe that the relative returns to in-home activities from using goods versus services have fallen since the late 1990s, despite the fact that goods are more efficiently produced over this period, as can be seen by noting the perpetual increase in $A_{gt}/A_{st}$. Thus, value-added to the consumer for services has improved relative to goods despite services becoming relatively less efficient to produce. Indeed, since 1997 we observe a 20.7% increase in the in-home efficiency of services relative to goods versus a 74% decline in the estimated efficiency of producing services relative to goods over this same period.

The analysis here is thus part of an emerging literature in economics that considers changes to the in-home efficiency of using certain types of products.[36] Our point has broad implications, which our model can help clarify: the often unmeasured returns to consumption can affect the ways individuals choose how to spend their time.




**(a)** Relative Home Productivities

**(b)** Relative Sectoral Productivities

**Figure 3.5.:** In this figure we present the posterior means of estimated relative in-home productivities (a) and relative sectoral productivities (b).

Theory suggests that as $P_{gt}/P_{st}$ declines, the sectoral TFP ratio $A_{gt}/A_{st}$ should rise (Ngai and Pissarides 2007; Herrendorf, Rogerson, and Valentinyi 2014). As Ngai and Pissarides (2007) show, rising sectoral TFP ratios can cause a decline in prices even if preferences are homothetic. Indeed, when sectoral production functions are Cobb-Douglas with $\alpha_j = \alpha$, $P_{gt}/P_{st} = (A_{gt}/A_{st})^{-1}$ (Herrendorf, Rogerson, and Valentinyi 2014). Note that the posterior distributions of $\alpha_g$ and $\alpha_s$ contain substantial overlap, suggesting we may not be able to reject the hypothesis that $\alpha_g = \alpha_s$. We estimate Pearson's correlation coefficient between observed relative prices and the posterior mean of estimated relative TFP, finding a correlation of $-0.936$, suggesting that sectoral capital-intensities are very similar. Thus, rather than differential sectoral capital-deepening, as proposed by Acemoglu and Guerrieri (2008) as a cause of structural change, our findings suggest that differential sectoral-productivity growth is the primary cause of changing relative prices and thus the rising services share. This result thus conforms with those in Ngai and Pissarides (2007) and Kehoe, Ruhl, and Steinberg (2018).

[36]See for example Boerma and Karabarbounis (2019), Fenton and Koenig (2020), and Kopytov, Roussanov, and Taschereau-Dumouchel (2020). As Fenton and Koenig (2020) point out, the quality-adjusted price of a television has fallen by over 1000% since the 1950s. Kopytov, Roussanov, and Taschereau-Dumouchel (2020) highlight this result along with the proliferation of new content streaming and production services like Netflix and Amazon Prime Video to argue that both the choice set of off-market activities and the ability of consumers to enjoy those choices has expanded dramatically. The increase in entertainment content, counted as services, along with consumers' ability to enjoy it, is one factor that Kopytov, Roussanov, and Taschereau-Dumouchel (2020) attribute to declining labor market participation and labor hours.

#### 3.5.4.3. Decomposing Contributions to Falling $MRS_t$

Recall that $MRS_t$ can be decomposed in two ways: 1) using the separate $MRTS_j$ conditions as was done in Section (3.2.2); 2) using the marginal rate of substitution for $c_{gt}$ and $c_{st}$ along with the exogenous productivities and endogenous marginal product ratios, as described in Section 3.2.1. The latter requires a parameterized model since the quantities of experiences/activities, $c_{jt}$, are not quantified in data. Here, we perform such a decomposition using the posterior distribution of parameter estimates from the simultaneous equations model with data accounting for durable services flows.

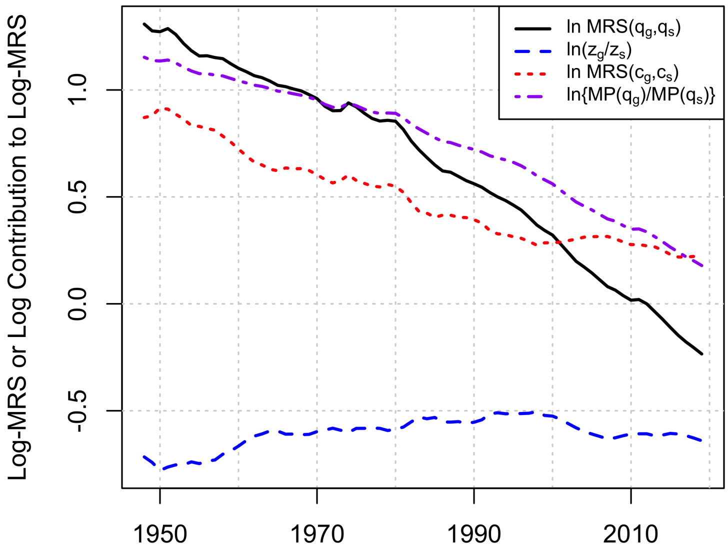



**Figure 3.6.:** We decompose the contributions to declines in the aggregate $MRS_t$ as outlined in Section 3.2.1. The black line is logged data, the dashed blue line is the log ratio of exogenous relative productivities, the dotted red line is the log $MRS_t$ of consumption experiences $c_{gt}$ and $c_{st}$, and the dashed purple line is the log ratio of marginal products. The black line is the sum of all other lines.

Figure 3.6 presents the decomposition. Exogenous changes to in-home productivities have made no contribution to the decline in the marginal rate of substitution (and thus relative prices) of $q_{gt}$ to $q_{st}$ since 1948. Because $z_{gt}/z_{st}$ is a residual object of our model, this implies that endogenous model mechanisms, namely the relative value of in-home experiences using goods versus services and the ratio of marginal products, are the primary drivers of declining $MRS_t(q_{gt}, q_{st})$. The relative value of using goods versus services (dotted red line) has increased: more services-related experiences are required to replace the value of a single goods experience in 2019 than were required in 1948. Meanwhile, in-home capital deepening from consumer durables has driven the decline in the ratio of marginal products (dashed purple line). Both have more-or-less equally contributed to declining $MRS_t(q_{gt}, q_{st})$, demonstrating the important role that technical improvements to durable goods, like computers, televisions, and smart phones, have had on changing consumers' consumption experiences.

**Table 3.4.:** HMC Posterior Distribution, 1984-2019, CEX Micro Data

| | *Simultaneous Equations* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -46.8980 | 24.6497 | -110.0319 | -55.9127 | -41.1374 | -30.7656 | -18.4952 |
| $\nu_g$ | -3.6960 | 3.6784 | -13.3039 | -4.5877 | -2.7209 | -1.5394 | -0.3646 |
| $\nu_s$ | -2.0353 | 1.7001 | -5.7722 | -2.3366 | -1.7300 | -1.2784 | -0.3176 |
| $\omega_g$ | 0.0169 | 0.0163 | 0.0004 | 0.0051 | 0.0120 | 0.0237 | 0.0600 |
| $\omega_s$ | 0.0025 | 0.0024 | 0.0001 | 0.0008 | 0.0018 | 0.0035 | 0.0088 |
| $\alpha_g$ | 0.1170 | 0.0213 | 0.0774 | 0.1023 | 0.1164 | 0.1312 | 0.1602 |
| $\alpha_s$ | 0.0612 | 0.0123 | 0.0383 | 0.0526 | 0.0608 | 0.0693 | 0.0865 |
| $\sigma_1^2$ | 0.0028 | 0.0003 | 0.0022 | 0.0026 | 0.0028 | 0.0030 | 0.0034 |
| $\sigma_g^2$ | 0.0009 | 0.0001 | 0.0007 | 0.0008 | 0.0009 | 0.0010 | 0.0011 |
| $\sigma_s^2$ | 0.0002 | 0.0000 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| $\sigma_{1g}$ | -0.0003 | 0.0002 | -0.0006 | -0.0004 | -0.0003 | -0.0002 | 0.0002 |
| $\sigma_{1s}$ | -0.0001 | 0.0001 | -0.0002 | -0.0001 | -0.0001 | -0.0001 | 0.0000 |
| $\sigma_{gs}$ | 0.0003 | 0.0000 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0004 |
| $V(\mathcal{P})$ | 1517.0680 | 2.7075 | 1510.8319 | 1515.4784 | 1517.4199 | 1519.0006 | 1521.3504 |

### 3.5.5. Model Estimates with Micro Data

In this section we estimate the household-only model using CEX spending ratios for the income quintiles as described in Section 3.4.2. Households are heterogeneous in wages $w_{it}$ and their relative productivities $z_{igt}/z_{ist}$. The first-differenced model error $\epsilon_{it}^1$ is heterogeneous across households, though the variance of the error distribution is the same across all households.

Table 3.4 presents the posterior distribution estimates for the micro-data simultaneous equations model.[37] Our estimates with micro data predict stronger complementarities between consumption and off-market time use for goods compared to our estimates with aggregate data. This could be a by-product of the fact that we do not have micro data on durable service flows, so that the prices and spending series with which we are estimating the model do not correspond exactly to those presented in the aggregate analysis. Indeed, when comparing Table 3.4 to parameter estimates from the aggregate model without durables service flows featured in Online Technical Appendix B.3, $\nu_g > \nu_s$ as here. For goods, complementarities between time and market commodities are stronger when durables are not accounted for, as suggested by evidence from both our macro and micro estimates. This suggests that including durable service flows is important to accurately estimate structural elasticities, lending validity to our estimates from aggregate data, given the behavioral assumptions underlying the model.[38]

[37] Online Technical Appendix B.4 contains estimates of the micro-data model where the likelihood is formed only around the household-MRS condition.

[38] Recall, CEX micro data do not account for durable service flows.

**Table 3.5.:** Robustness Checks with Micro Data — Fit of Non-targeted Data Series

| | *Aggregate Parameters* [a] | | | | *Micro Parameters* [b] | | | |
|---|---|---|---|---|---|---|---|---|
| | $\ell_{it}$ | | $n_{igt}/n_{ist}$ | | $\ell_{it}$ | | $n_{igt}/n_{ist}$ | |
| Quintile | S.E. | Test Statistic [c] | S.E. | Test Statistic [c] | S.E. | Test Statistic [c] | S.E. | Test Statistic [c] |
| $1^{st}$ | 0.388 | 0.006 | 1.796 | -0.011 | 0.361 | 0.012 | 1.424 | -0.015 |
| $2^{nd}$ | 0.354 | 0.005 | 1.274 | -0.027 | 0.355 | 0.010 | 1.041 | -0.015 |
| $3^{rd}$ | 0.318 | 0.005 | 0.893 | -0.044 | 0.330 | 0.007 | 0.742 | -0.054 |
| $4^{th}$ | 0.283 | 0.004 | 0.681 | -0.062 | 0.288 | 0.006 | 0.576 | -0.075 |
| $5^{th}$ | 0.227 | 0.003 | 0.469 | -0.087 | 0.206 | 0.004 | 0.409 | -0.101 |

This table compares posterior simulated $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$ against data, where $\widehat{\ell}_{it}$ is simulated from 1984-2019 and $\widehat{n_{igt}/n_{ist}}$ is simulated from 2003-2019 to match-up against CPS and ATUS data respectively.

[a] Parameters are taken from the simultaneous equations model estimated with aggregate data, excluding durables service flows. These estimates are featured in Online Technical Appendix B.3. Relative productivities are re-computed to fit CEX spending ratios, $x_{igt}/x_{ist}$, for all income quintiles.
[b] Parameters and relative productivities are taken from the simultaneous equations model estimated with micro data.
[c] These are the same posterior mean, summary test statistics as in Table 3.3.

### 3.5.5.1. Robustness Checks with Micro Data

We seek two assessments: the time-use fits by income quintile of the simultaneous equations model with parameters estimated from aggregate data and those same heterogeneous time-use fits with parameters estimated from the simultaneous equations model on micro data. Neither $\ell_{it}$ nor $n_{igt}/n_{ist}$ are targeted in the likelihood functions. If predicted labor hours and/or off-market time-use by income quintile are closer to observed data under the aggregate parameterization, this helps support our preference for the aggregate fit when moving on to counterfactuals.

Table 3.5 shows estimated $t$-statistics and standard errors for posterior means of $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$ against data. We find that there is little difference between the fit of the two models when comparing to micro data. The aggregate-estimated parameters applied to micro data provide a good fit for both $\ell_{it}$ and $n_{igt}/n_{ist}$, as does the micro-estimated model.[39]

### 3.5.5.2. Welfare Implications and Inequality

How have consumer experiences from engaging in consumption activities changed since 1984, and what are the implications of such changes for inequality? In this section we use the parameterized model, estimated with micro data, to understand how the in-home value of consumption/leisure activities has improved for consumers across the wage distribution. It

[39]Note that in simulating $\ell_{it}$ we require an estimate of maximum possible income net of savings $y_{it}$ per household, which is a function of capital income net of savings $\widetilde{y}_{it}$. To understand how sensitive our estimates of $\ell_{it}$ are to the level of $y_{it}$ we simulated $\ell_{it}$ for all income quintiles both with $\widetilde{y}_t$ per-capita computed from macro data and $\widetilde{y}_t = 0$. The results do not change, as $\ell_{it}$ appears not to be very sensitive to variation in $\widetilde{y}_{it}$.

is here that we thus attempt to understand the "most interesting and elusive dimension of inequality" (Díaz-Jiménez, Glover, and Ríos-Rull 2011).

Let $\psi_{ijt}$, $j \in \{g, s\}$ be the value (price) of in-home activities, $c_{ijt}$. Following directly from Becker (1965), this value captures both "direct costs" (market prices and wages) and "indirect costs" (activity-specific marginal products of inputs and time):[40]

$$\begin{aligned}\psi_{ijt}&(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist}) \\ &= P_{jt}\left(\frac{q_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})}{c_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})}\right) + w_{it}\left(\frac{n_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})}{c_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})}\right)\end{aligned} \tag{3.21}$$

The total value of final consumption is just $\sum_j \psi_{ijt} c_{ijt}$. Given wage and in-home productivity heterogeneity, $\psi_{ijt}$ is a *household-specific* price which reflects clearing of the internal household marketplace from home production to ultimate consumption. In this context, after market purchases have been made, each household itself encompasses a unique, closed marketplace with unique (different) internal market clearing conditions. In this sense, activities produced by household $i$ have fundamentally different valuations than those produced and consumed by household $i' \neq i$.[41]

Since each of $c_{ijt}$ and thus $\psi_{ijt}$ is a function of the productivity level $z_{ijt}$, and we only estimate relative productivities $z_{igt}/z_{ist}$, we must make some assumptions regarding how the levels grow over time. Since we primarily care about the activity-aggregated value of all in-home activities, $\sum_j \psi_{ijt} c_{ijt}$ and given that $z$'s enter linearly into $c$'s, it does not matter which $z_{ijt}$ we choose to normalize, as long as we assume that each of $z_{igt}$ and $z_{ist}$ grow at the same rate. In turn, for cross-sectional comparisons if the pair $(z_{igt}, z_{ist})$ grows at the same rate as $(z_{i',gt}, z_{i',st})$, then growth rates also will not matter when comparing welfare outcomes across the income distribution. We thus consider two situations pertaining to in-home productivity growth: 1) all $z$'s, regardless of their associated product type or household, grow at the same rate; 2) there is heterogeneity across households with respect to the growth rate of the pair $(z_{igt}, z_{ist})$.[42] For (1) growth does not matter. For (2) we allow the levels of each $z_{ijt}$ to grow at the same rate as real wages for households in income quintile $i$. From CPS data, we compute the following average annual real hourly wage growth rates by income quintile (1984-2018):

i. First quintile, 1.37%.

ii. Second quintile, 1.06%.

iii. Third quintile, 0.96%.

iv. Fourth quintile, 1.05%.

v. Fifth quintile, 1.27%.

[40] $q_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})$, $n_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})$, and $c_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it}, z_{igt}, z_{ist})$ represent the Marshallian demand functions for market quantities, time, and activities respectively.

[41] Note that each household uses the same production function $f_j(\cdot, \cdot)$ but faces different efficiencies $z_{ijt}$ and time-use prices $w_{it}$.

[42] A possible third situation, which we do not consider, is that there is simultaneous heterogeneity both across households and across product types with respect to in-home productivity growth. Lacking any reasonably justifiable baseline as to why this would be true, we choose not to speculatively simulate such a growth scenario.

Given between-household heterogeneity both with respect to in-home activities and prices, we seek to understand both how welfare has changed for households at each income quintile over time and how the cross-sectional welfare distribution in terms of in-home purchasing power parity (PPP) has evolved over time. Further, since $\psi_{ijt}$ and $c_{ijt}$ can each vary separately in market prices, wages, and in-home productivities, we will decompose $\psi_{ijt}$ over time for each income quintile $i$ to understand if (and if so, how) the separate contributions of changes to the in-home value of goods versus services activities and the relative contributions of direct and indirect costs have all affected welfare.

**Growth in Welfare Within Income Quintile (1984-2018):** Here, we compare how the non-normalized total value of final consumption, $\sum_j \psi_{ijt}c_{ijt}$, has evolved over time for consumers across the income distribution. To compute both $\psi_{ijt}$ and $c_{ijt}$, we use observed data series from 1984-2018 for $q_{ijt}$, $P_{jt}$, and $w_{it}$, while simulating the posterior atomic draws of $n^m_{ijt}$ and $c^m_{ijt}$ using estimated micro parameters and in-home productivities.[43] This exercise thus amounts to examining how much better off agents in income quintile $i$ are today relative to agents who belonged to the same income quintile in the past.

**Table 3.6.:** Pct. Change in the Value of In-home Activities, 1984-2018 [a]

| *Income (Wage) Quintiles* | | | | |
|---|---|---|---|---|
| $1^{st}$ | $2^{nd}$ | $3^{rd}$ | $4^{th}$ | $5^{th}$ |
| 160.981 | 208.270 | 213.520 | 218.319 | 286.950 |

[a] These values are: $100 \times (\sum_j \psi_{ij,2018}c_{ij,2018} / \sum_j \psi_{ij,1984}c_{ij,1984} - 1)$ in real \$2012.

Table 3.6 presents the overall growth rates by income quintile for the real value of in-home activities from 1984-2018.[44] We observe that growth in the value of in-home activities increases by income quintile. While the bottom 20% experienced a 161% improvement to their welfare over this period, the top 20% experienced over 1.75 times that amount of welfare growth. Thus, our model generates predictions for the fanning-out of the right tail of the welfare distribution that is consistent with income-based measures of the evolution of inequality, such as those discussed by Piketty, Saez, and Zucman (2018). However, our estimated magnitude of welfare growth, for all consumers, is considerably greater than those same income-based measures. For example, Piketty, Saez, and Zucman (2018) estimate that income has stagnated for the bottom 50% of U.S. households, while the top 10% have seen a 40% increase in pre-tax income. Income-based estimates thus may understate the degree to which livelihoods have improved for all consumers, especially those at the bottom of the income distribution.

**Changes to the Cross-sectional Welfare Distribution (1984-2018):** One difficulty presented by the Beckerian model is that the price of in-home consumption is household specific. Thus, the value of a particular activity level, $\underline{c} = c_{ijt}$, to household $i$ is not the same to household $i' \neq i$,

[43] In this context $m$ indexes the HMC sample epochs.

[44] Note that, for this exercise, since $\sum_j \psi_{ijt}c_{ijt}$ is not normalized and $z_{ijt}$ enters linearly into $c_{ijt}$, it does not matter which growth scheme we use to compare the value of in-home activities in 2018 to those in 1984.

**Table 3.7.:** In-home Laspeyres Price Index Ratios Across Income Quintiles

*No Productivity Growth Heterogeneity*

| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
|---|---|---|---|---|---|---|
| 1984 | 17.034 | 8.455 | 2.371 | 7.024 | 3.531 | 1.965 |
| 1990 | 16.984 | 9.084 | 2.406 | 6.987 | 3.751 | 1.858 |
| 2000 | 18.860 | 9.367 | 2.532 | 7.360 | 3.666 | 2.002 |
| 2010 | 21.609 | 10.429 | 2.819 | 7.585 | 3.673 | 2.056 |
| 2018 | 21.528 | 10.208 | 2.789 | 7.647 | 3.635 | 2.097 |

*Productivity Growth Equivalent to Wage Growth by Quintile*

| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
|---|---|---|---|---|---|---|
| 1984 | 17.034 | 8.455 | 2.371 | 7.024 | 3.531 | 1.965 |
| 1990 | 17.086 | 8.975 | 2.375 | 7.121 | 3.754 | 1.892 |
| 2000 | 19.165 | 9.070 | 2.446 | 7.741 | 3.674 | 2.101 |
| 2010 | 22.180 | 9.896 | 2.665 | 8.235 | 3.687 | 2.224 |
| 2018 | 22.275 | 9.532 | 2.592 | 8.514 | 3.652 | 2.324 |

even holding the activity level $\underline{c}$ fixed. This is because, in general, $\psi_{ijt} \neq \psi_{i',jt}$, since $\psi$'s depend on both relative in-home productivities and wages which are heterogeneous. Thus, to engage in cross-sectional comparisons, we must compute an index that accounts for this in-home purchasing power disparity, just as one might do when comparing consumption baskets of like goods under different pricing regimes between different countries.

To accommodate the normalization problem, we first construct aggregate Laspeyres price indices, $P^L_{it}$, for each series $\{\sum_j \psi_{ijt}c_{ijt}\}_t$, separately for each household $i$, letting the base year be 1984 for all households. For all $i \neq 3$, we then compute the normalized index $\widetilde{P}^L_{it} = P^L_{it}/P^L_{3t}$. This forces the median-income consumer's in-home price index to unity every period. We can then understand how the cross-sectional welfare distribution, in terms of the value of in-home activities, has evolved over time by comparing the values of $\left\{\{\widetilde{P}^L_{it}\}_t\right\}_i$, $\forall i \neq 3$.

Table 3.7 presents the ratio of in-home PPP-normalized Laspeyres indices between different quintiles under our two different growth schemes (no heterogeneity/growth heterogeneity). Meanwhile, Table 3.8 presents the percent-increase in the given quintile ratios relative to 1984. Three things stand out: 1) those already ahead continue to get further ahead; 2) those at the bottom of the distribution continue to fall further behind; 3) little is changed with regards to relative welfare of those in the middle of the distribution. The first observation can be seen by noting increases in the $5^{th}/1^{st}$ and $5^{th}/2^{nd}$ ratios over time, with a more subtle increase in the $4^{th}/1^{st}$ ratio over time. The second point is apparent when examining how the $2^{nd}/1^{st}$ ratio has grown over time. Meanwhile, the third point (relative stability between those closest to the middle of the distribution) can be seen by noting that the $4^{th}/2^{nd}$ ratio is relatively constant compared to others. In fact the $4^{th}/2^{nd}$ ratio appears to have even declined over the last decade.

**Table 3.8.:** In-home Laspeyres Price Index Ratios — % Change Since 1984 [a]

| | | | *No Productivity Growth Heterogeneity* | | | |
|---|---|---|---|---|---|---|
| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
| 1990 | -0.295 | 7.446 | 1.497 | -0.525 | 6.239 | -5.419 |
| 2000 | 10.720 | 10.792 | 6.795 | 4.781 | 3.819 | 1.903 |
| 2010 | 26.859 | 23.349 | 18.899 | 7.994 | 4.036 | 4.668 |
| 2018 | 26.384 | 20.741 | 17.661 | 8.874 | 2.937 | 6.746 |
| | | | *Productivity Growth Equivalent to Wage Growth by Quintile* | | | |
| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
| 1990 | 0.307 | 6.154 | 0.192 | 1.379 | 6.329 | -3.691 |
| 2000 | 12.510 | 7.276 | 3.172 | 10.214 | 4.054 | 6.946 |
| 2010 | 30.210 | 17.050 | 12.414 | 17.241 | 4.419 | 13.214 |
| 2018 | 30.766 | 12.742 | 9.340 | 21.222 | 3.432 | 18.283 |

[a] The units of the data are % increase in the Laspeyres ratio $\widetilde{P}^L_{it}/\widetilde{P}^L_{i',t}$ since 1984. For example, for the year 2000 the presented value for the $5^{th}/1^{st}$ ratio is $100 \times \left[(\widetilde{P}^L_{5,2000}/\widetilde{P}^L_{1,2000})/(\widetilde{P}^L_{5,1984}/\widetilde{P}^L_{1,1984}) - 1\right]$. In the top half of the table, this implies a 10.72% increase in inequality, while in the bottom half of the table it implies a 12.51% increase in inequality.

These ratios can be compared to the measured wealth gap. The ratio $\widetilde{P}^L_{it}/\widetilde{P}^L_{i',t}$, $i \neq i'$ and $i, i' \neq 3$, describes the rate of substitution in period $t$ between activities performed in household $i$ versus those in $i' \neq i$. For example, in 1984, the $5^{th}/1^{st}$ ratio is 17.034 with no growth heterogeneity, so that the value of consumption activities of a single type $i = 5$ household is equivalent to the value of all activities engaged-in by $> 17$ type $i = 1$ households. The in-home values of consumption/leisure activities of different households in the fourth and fifth quintiles are $> 7$ to $> 21$ times those in the first quintile and $> 3$ to $> 10$ times those in the second quintile. But this measure provides a relatively tame portrait of the welfare distribution when compared to wealth-based measures of inequality. Compare our measure here to the examples in Saez and Zucman (2016) who show that the top 10%, in terms of average wealth, have a net worth of $> 30$ times that of the average household in the bottom 90%. While not a direct comparison, qualitatively speaking our measure suggests the welfare distribution is less disperse than a distribution derived from wealth-based measures.

**Table 3.9.:** Share of Direct Cost of Market Inputs Contribution to $\psi_{ijt}$

| | *Goods Quantities, by Income Quintile* [a] | | | | |
|---|---|---|---|---|---|
| Year | $1^{st}$ | $2^{nd}$ | $3^{rd}$ | $4^{th}$ | $5^{th}$ |
| 1984 | 0.023 | 0.012 | 0.008 | 0.006 | 0.004 |
| 1990 | 0.017 | 0.011 | 0.008 | 0.006 | 0.003 |
| 2000 | 0.015 | 0.009 | 0.006 | 0.005 | 0.003 |
| 2010 | 0.012 | 0.007 | 0.005 | 0.004 | 0.002 |
| 2018 | 0.011 | 0.006 | 0.004 | 0.003 | 0.002 |

| | *Services Quantities, by Income Quintile* [a] | | | | |
|---|---|---|---|---|---|
| Year | $1^{st}$ | $2^{nd}$ | $3^{rd}$ | $4^{th}$ | $5^{th}$ |
| 1984 | 0.004 | 0.002 | 0.002 | 0.001 | 0.001 |
| 1990 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 |
| 2000 | 0.004 | 0.002 | 0.002 | 0.001 | 0.001 |
| 2010 | 0.003 | 0.002 | 0.001 | 0.001 | 0.001 |
| 2018 | 0.004 | 0.002 | 0.001 | 0.001 | 0.001 |

[a] These values are $P_{jt}(q_{ijt}/c_{ijt})/\psi_{ijt}$. Note that the corresponding share of indirect costs associated with time use is $1 - P_{jt}(q_{ijt}/c_{ijt})/\psi_{ijt}$.

**Decomposing Contributions of Changes to the Cross-sectional Distribution by Market Input, $j \in \{g, s\}$:** To what degree do goods activities relative to service activities contribute to our valuations of in-home production? Further, how do such contributions vary both over time and across the income distribution? What are the separate roles of time versus market inputs? To answer these questions, we provide several decompositions of (3.21) by quintile.

Specifically, we compute the contribution of changes to market quantity prices and the amount of market inputs per unit of in-home output to the evolution of the price, $\psi_{ijt}$. These estimates are presented in Table 3.9. Notice that the contribution of services input quantities to prices is flat over time for all income quintiles, while the in-home goods-activity price becomes increasingly associated with indirect costs pertaining to time use, especially for those at the lower end of the income distribution. As those who are relatively poorer have thus seen incomes increase (wages increase), the share of overall in-home value associated with the direct cost of buying market items has fallen. For those at the top, however, there is little change in the contribution of direct costs associated with using market goods versus indirect costs from allocating time toward their utilization.

## 3.6. Counterfactual Simulations

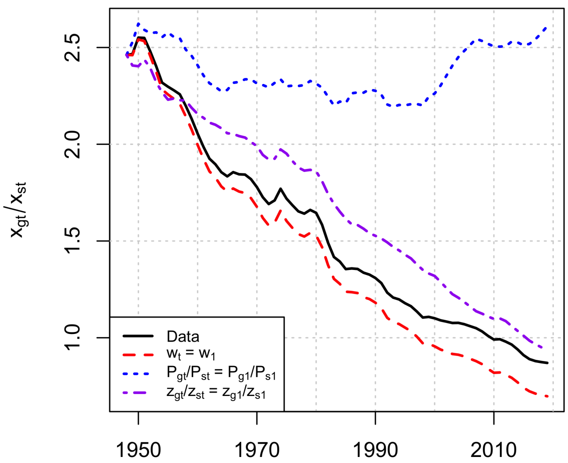
### 3.6.1. Structural Change with Aggregate Data

One goal of this paper is to better understand how wage growth and relative price changes affect aggregate demand allocations and labor hours when accounting for home production complementarities with off-market time. In this section we consider three counterfactual simulations. First, we fix aggregate wages at their 1948 levels $w_t = w_{1948}$, while allowing non-inflationary relative prices $P_{gt}/P_{st}$ to evolve according to data. Second, we allow $w_t$ to evolve as observed but fix relative prices $P_{gt}/P_{st} = P_{g,1948}/P_{s,1948}$. In each of these scenarios we simulate the counterfactual series of $x_{gt}/x_{st}$ and $\ell_t$ using the implied relative productivities as estimated from the model residuals. Third and finally, we assume relative home productivities remain fixed so that $z_{gt}/z_{st} = z_{g,1948}/z_{s,1948}$. These counterfactual exercises are partial equilibrium in nature and designed to isolate the effects of wage growth, relative price variation, and relative in-home productivity variation on aggregate demand.[45] The goal of these exercises is to succinctly quantify which channel has exhibited the strongest influence on structural change.

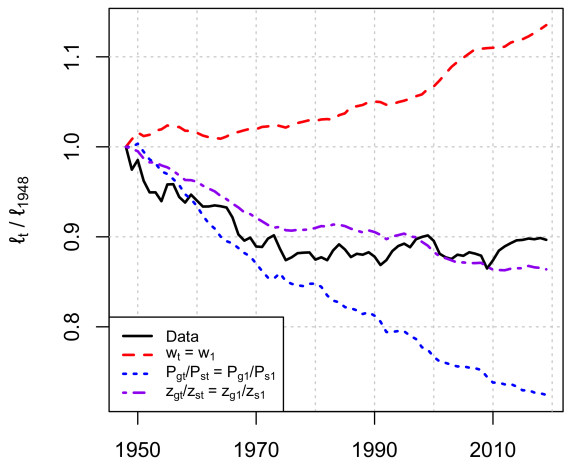
Figure 3.7 presents the posterior means of our counterfactual simulations for relative expenditure and the trend in labor hours per effective full-time worker. Data are featured using black lines. The main takeaway is that relative price effects dominate the effects of wage growth in determining the long-run decline in $x_{gt}/x_{st}$. This can be seen by noting that in the case where wages are allowed to grow but relative prices are held fixed at their 1948 level, counterfactual relative spending (dashed blue line) barely declines at all. The counterfactual results regarding labor hours are somewhat mixed, as relative price declines and wage growth seem to have been competing with each other in the context of our model.

For detailed intuition, consider first the fixed-wage scenario corresponding to the dashed red lines. When looking at the red lines we are observing how relative price and home-productivity variation have affected long-run outcomes, and theoretical intuition corresponds to that discussed in Section 3.3.2. From panel (b) it is clear that classic $c/\ell$ income effects have helped drive down labor hours since counterfactual $\ell_t$ rises instead of falls when wages do not grow. When $w_t$ is fixed, the rise in $\ell_t$ is driven by falling $P_{gt}/P_{st}$, and we are thus observing the classic $c/\ell$ substitution effect in relative prices. Clearly, the classic $c/\ell$ income effect has dominated the classic $c/\ell$ substitution effect since the data actually fall, so wage

[45] This exercise is identical to the counterfactual exercise in Herrendorf, Rogerson, and Valentinyi (2013), though we arrive at an opposing conclusion with respect to the cause of a rising services expenditure share.




**(a)** Relative Expenditure




**(b)** Labor Hours Per Worker

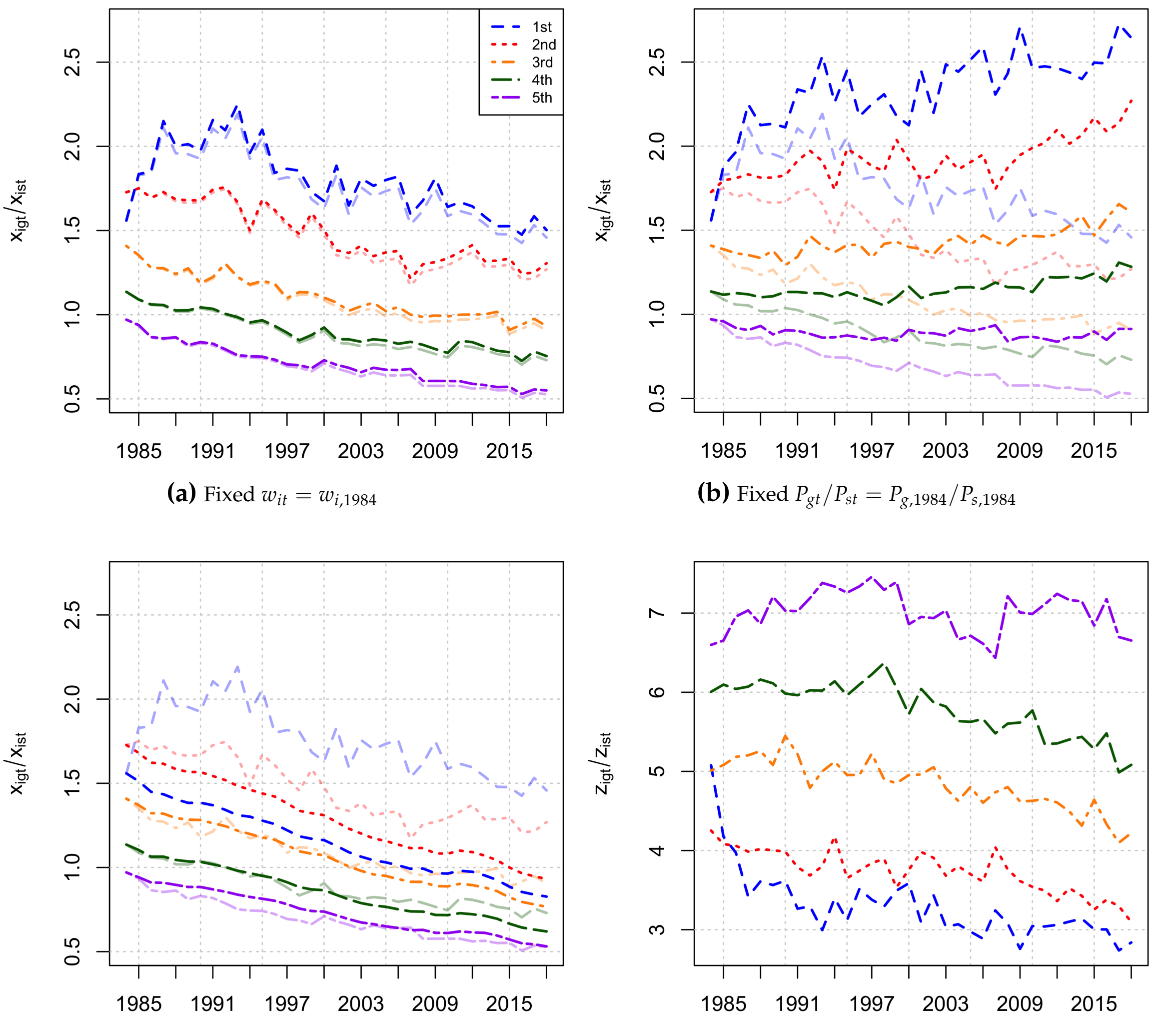
**Figure 3.7.:** Here we present data series for aggregate $x_{gt}/x_{st}$ and $\ell_t$ against the posterior means of our counterfactual simulations. The difference between dashed blue and black lines corresponds to the substitution effect from relative price variation. The difference between the dashed red and black lines corresponds to the simultaneous income and substitution effects from wage variation. The difference between the dashed purple and black lines represents the effect of evolving home productivities.

variation has stronger influence over labor hours than relative price variation. Nonetheless, the partial correlation between $P_{gt}/P_{st}$ and $\ell_t$ appears to be non-zero. By our parameter estimates $\nu_g > \nu_s$ and $\omega_g > \omega_s$ suggesting services are both more time-intensive and exhibit stronger complementarities with off-market time. Thus, as $P_{gt}/P_{st}$ falls the market commodity associated with the more time-intensive task is becoming relatively more expensive.

Now suppose $w_t$ evolves as observed and relative prices remain fixed at their 1948 level. Note that the classic $c/\ell$ income effect is dominating, sending $\ell_t$ down. Time devoted toward services consumption rises, but $x_{gt}/x_{st}$ changes very little, despite the fact that off-market time is being re-allocated to different tasks. The rise in $w_t$ induces increased consumption, but strong gross complementarities between $c_{gt}$ and $c_{st}$ appear to dominate differences between the underlying home production processes. This is because when relative prices are constant, re-allocations are driven by differences in the strength of off-market time complementarities between the different processes. If gross complementarities are stronger, the time allocation will respond more to wage variation than the expenditure allocation, which is what we observe here.

Note that fixing relative productivities has little affect on outcomes. Relative expenditure would have continued to decline, while labor hours would have still exhibited their partial, sideways J-shaped pattern. As an example, consider the increase in television quality captured by $z_{gt}$ alongside the availability of new streaming entertainment content captured by $z_{st}$. Strong gross complementarities suggest that consumers prefer that quality improvements to both televisions and content roughly keep pace with each other. That is, demand for new higher-quality services depends on the availability of new higher-quality goods.

These results suggest that neither pure income effects nor changes to relative in-home productivities are responsible for the rising services share of consumption expenditure. What we observe instead is that if wages had remained fixed at their 1948 level, the services share would have risen faster than the data, in contrast with other arguments in the literature. Declining relative prices are the primary cause of the rising services expenditure share.




**(a)** Fixed $w_{it} = w_{i,1984}$

**(b)** Fixed $P_{gt}/P_{st} = P_{g,1984}/P_{s,1984}$

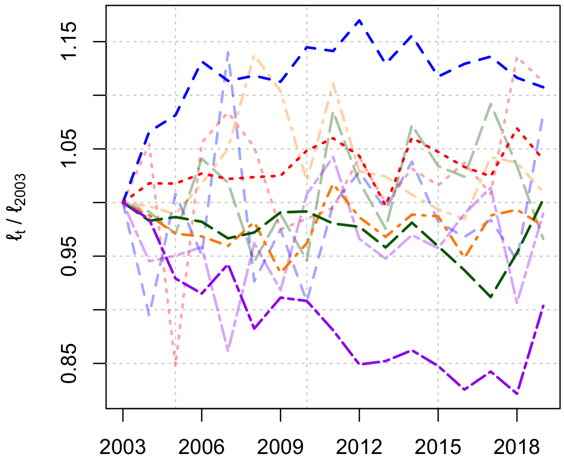
**(c)** Fixed $z_{igt}/z_{ist} = z_{ig,1984}/z_{is,1984}$

**(d)** $z_{igt}/z_{ist}$

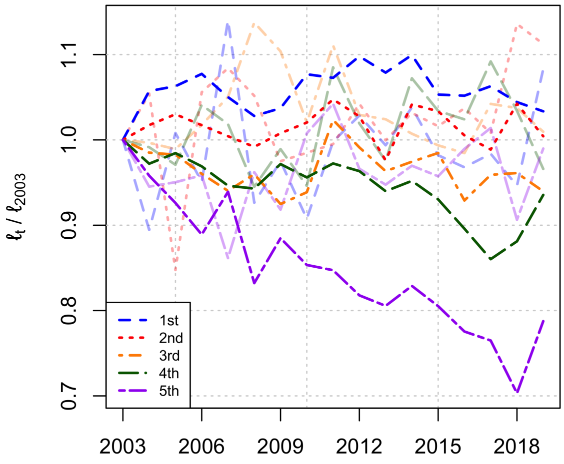
**Figure 3.8.:** In panels (a) through (c) we show the posterior means of counterfactual spending series in bold against their data counterparts in the same color and line-type scheme though faded in the background.

### 3.6.2. Structural Change with Micro Data

We draw the same conclusions regarding the causes of structural change when using the parameter and productivity estimates from micro data. Figure 3.8 shows the counterfactual series in bold against the corresponding data series in faded contrast, along with heterogeneous posterior means of $z_{igt}/z_{ist}$. Notice there is little difference between $x_{igt}/x_{ist}$ for all $i$ when holding wages fixed in panel (a). Relative price variation, shown in panel (b), again appears to have contributed to the rise in services share more than wage variation. In panel (c) we present the effect of holding $z_{igt}/z_{ist}$ fixed, while the raw relative home productivities for heterogeneous agents are presented in panel (d). Notice from panel (d) that $z_{igt}/z_{ist}$ are fairly flat, and so it is not surprising that holding $z_{igt}/z_{ist}$ fixed at its estimated 1984 level has little impact on the trends of the spending series.




**(a)** Fixed $w_{it} = w_{i,1984}$




**(b)** Fixed $P_{gt}/P_{st} = P_{g,1984}/P_{s,1984}$

**Figure 3.9.:** Here, we present counterfactual variation in $\ell_{it}$ for the fixed wage and fixed relative-price cases. The posterior means of simulated counterfactual $\ell_{it}$ are normalized so that $\ell_{i,1984} = 1$.

Perhaps more interesting is the responsiveness to wage and relative price variation of the intensive margin of labor across the income distribution. Figure 3.9 shows that high-income households are more sensitive to wage and relative price variation. In panel (a) we observe the effect of wage variation on $\ell_{it}$, and in panel (b) we observe the effect of relative price variation on $\ell_{it}$. The counterfactuals suggest that the classic $c/\ell$ substitution effect dominates for high-income workers but the classic $c/\ell$ income effect dominates for low-income workers. This is because in panel (a) we observe low-income workers working more hours had their wages stayed at 1984 levels, while high-income workers would have worked less hours, suggesting that wage growth places upward pressure on high-income workers' labor time.

The effect of relative prices on $\ell_{it}$ has similar variability across the income distribution. Recall that relative price variation is also associated with classic $c/\ell$ income and substitution effects due to differentials in the time-use intensities of off-market activities. In panel (b) had the goods-to-services price ratio remained fixed at its 1984 level, high-income workers would have worked less, as consumption would not have been substituted away from services to goods. Since their income is rising the classic $c/\ell$ income effect dominates here as the substitution effect from relative price variation is turned off. We see little change in $\ell_{it}$ from data for the first quintile, suggesting that lower income consumers' labor supply is less sensitive to relative price variation.

### 3.6.3. Welfare Inequality Under Different Rates of Price, Wage, and Productivity Growth

Section 3.5.5.2 shows that dispersion with respect to the cross-sectional distribution of the value of in-home activities is fanning out over time. To understand why, we perform three different counterfactual exercises, each of the same flavor as those in the preceding sections: 1) suppose all agents experience wage growth rates consistent with that of the median wage earner (average annual growth of 0.96%); 2) suppose relative prices remain fixed at their 1984 value, though price levels are allowed to inflate at the aggregate rate; 3) suppose all consumers face the same relative in-home productivity series as the median income consumer. For each counterfactual we simulate the percent change to the PPP-normalized (to the in-home activity basket of the median income consumer) mean-posterior Laspeyresian ratios as previously presented in Table 3.8.

**Table 3.10.:** In-home Laspeyres Price Index Ratios — Counterfactual % Change Since 1984 [a]

| *Constant Wage Growth* — 0.96% *Annually* | | | | | | |
|---|---|---|---|---|---|---|
| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
| 1990 | -32.778 | -9.994 | -5.269 | -29.149 | -5.004 | -25.549 |
| 2000 | -27.481 | -9.415 | -5.086 | -23.631 | -4.574 | -20.025 |
| 2010 | -34.484 | -12.982 | -4.033 | -31.714 | -9.331 | -24.874 |
| 2018 | -33.165 | -17.193 | -9.583 | -26.221 | -8.465 | -19.546 |
| *Constant Relative Prices* — $(P_{gt}/P_{st} = P_{g,1984}/P_{s,1984}, \forall t)$ | | | | | | |
| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
| 1990 | -33.373 | -10.482 | -5.555 | -29.591 | -5.242 | -25.862 |
| 2000 | -29.562 | -11.055 | -6.017 | -25.152 | -5.391 | -21.022 |
| 2010 | -37.987 | -15.908 | -5.731 | -34.321 | -10.837 | -26.667 |
| 2018 | -38.253 | -21.271 | -11.969 | -30.176 | -10.705 | -22.118 |
| *Constant Relative In-home Productivities Series* — $\{z_{igt}/z_{ist}\}_t \equiv \{z_{3,gt}/z_{3,st}\}_t$ | | | | | | |
| Year | $5^{th}/1^{st}$ | $5^{th}/2^{nd}$ | $5^{th}/4^{th}$ | $4^{th}/1^{st}$ | $4^{th}/2^{nd}$ | $2^{nd}/1^{st}$ |
| 1990 | 2.240 | 1.583 | 0.730 | 1.497 | 0.846 | 0.644 |
| 2000 | 6.457 | 4.514 | 2.054 | 4.299 | 2.402 | 1.839 |
| 2010 | 11.343 | 7.843 | 3.520 | 7.512 | 4.152 | 3.191 |
| 2018 | 16.014 | 10.961 | 4.859 | 10.557 | 5.775 | 4.457 |

[a] The units of the data are % increase in the counterfactually-simulated Laspeyres-index ratio $\widetilde{P}^L_{it}/\widetilde{P}^L_{i',t}$ since 1984. Note that we must re-simulate policies for $q$'s, $n$'s, $c$'s, and $\psi$'s under each separate counterfactual scenario.

Counterfactual percent-changes are presented in Table 3.10. Note that when wage growth is constant (top third of the table), we are observing the simultaneous impact of relative price evolution and heterogeneous relative in-home productivity evolution on the welfare distribution. If all consumers were to have experienced the same rate of wage growth as the median income consumer, differences in the standard of living would have fallen substantially from 1984-2018. This is primarily due to the fact that consumers in the fourth and fifth quintiles would have experienced slower rates of wage growth than they experienced in reality. The second third of Table 3.10 demonstrates the effect that fixing relative prices at their 1984 level would have had after allowing wages and in-home productivities to rise at their heterogeneous rates. Note that the compression of the welfare distribution is similar to that observed when wage growth rates are constant across society. Finally, the bottom third of the table shows that allowing for differences in in-home relative productivities actually dampens dispersion in the welfare distribution.

Differential wage growth rates, especially at the top of the income distribution, and the decline in the relative price of goods to services appear to have contributed fairly equally to rising welfare dispersion. Meanwhile, differential rates of relative in-home technical change appear to have had a dampening effect on increases to inequality. These results should provoke future research that continues to examine the role that differences in home production efficiencies play in determining welfare inequality.

## 3.7. Conclusion

We have shown that accounting for differential time-use complementarities in the consumption decision process can impact economic inference. This is especially true when considering which mechanisms are most responsible for the structural evolution of the U.S. economy from one previously dominated by the consumption of manufactured goods to today's service economy. The results presented here call into question the notion that rising incomes are responsible for changing tastes. Rather, the increase in the services share of expenditure appears to be a consequence of efficiency gains in goods production that have driven down relative prices. Further, welfare differences between different income quintiles may not be as stark as income- or wealth-based measures would suggest.

While our results here utilize the limited time-use data that is available, this paper should encourage the stewards of data collection to continue measuring the time-utilization decisions of consumers. A longer horizon of time-use data that easily matches to consumption activities could be used in the future to help validate our results. Indeed, we are encumbered by the relative shortness of the ATUS data series which limits analysis to the period since 2003, missing much of the major structural transformation that took place throughout the 70s, 80s, and 90s.

The relationship between time-use and consumption lends itself to exploring many questions at the frontier of our field. Some software services companies like Google and Facebook offer base-level products for free but their revenues, via advertisements, depend on consumers choosing to spend time and engage with their software. Similarly, the COVID-19 pandemic has ushered in major changes with respect to the way we communicate with each other —

an activity that is now directly associated with the utilization of a particular market service. What value do these specific services provide to the household beyond what is measurable from input and output data? To explore such a question requires models with rich, off-market time-utilization structures, since the time-utilization component is such an important part of the consumption activities associated with these products. We thus hope that our work encourages future exploration of these interesting questions and future utilization of the classic, but durable, Beckerian model of home production.

# A. Appendix: The Distribution of Bank Regulatory Capital

## A.1. Mathematical Appendix

**Derivation of the Default Probability:**
The default threshold is defined as:

$$\text{Prob}((1+r^f)e + (\tilde{r} - r^f)l + (r^f - r^d)d - c \leq \underline{r}^e)$$

where the left side of the inequality is updated net worth. It is equal to:

$$= \text{Prob}(\tilde{r}l \leq \underline{r}^e e - (1+r^f)e - (r^f - r^d)d + r^f l + c$$

$$= \text{Prob}\left(\tilde{r} \leq \frac{\underline{r}^e e - (1+r^f)e - (r^f - r^d)d + r^f l + c}{l}\right)$$

$$= G\left(\frac{\underline{r}^e e - (1+r^f)e - (r^f - r^d)d + r^f l + c}{l}\right)$$

Multiplying the inside fraction by $a/a$, defining $\psi = e/a$, $\phi = l/a$, $\bar{c} = c/a$, and recognizing that $a = e + d$, so that $(1-\psi) = d/a$, we get:

$$= G\left(\frac{\underline{r}^e \psi - (1+r^f)\psi - (r^f - r^d)(1-\psi) + r^f \phi + \bar{c}}{\phi}\right)$$

Rearranging and combining like terms yields:

$$G\left(r^f - \frac{(r^f - r^d) + (1 + r^d - \underline{r}^e)\psi - \bar{c}}{\phi}\right)$$

## A.2. Data Appendix

Call reports are filed quarterly. Balance sheet ("stock variables") items are reported as the level at the end of the quarter. Income statement ("flow variables") items are reported as the accumulated total for the year. To construct proper flow values for each year, the Q1 values are maintained. For Q2-Q4 of each year, the first difference for each income statement item is calculated and is used as the reported value for the quarter. The data was inspected for potential within year seasonality. No apparent patterns were observed.

For each quarter, bank's belonging to the same holding company are combined into a single observation.

### Interest Rates

Income here represents total asset (non-operational) income. For calculating interest rates, income $I$ is measured as:

$$I = A - B + C + D \tag{A.1}$$

where:

$$\begin{aligned} A &= \text{Interest Income} \\ B &= \text{Loan Loss Provisions} \\ C &= \text{Securities Gains/Losses} \\ D &= \text{Other Comprehensive Income} \end{aligned}$$

Total asset-generated income is given by the equation:

$$I = \tilde{r}l + r_f b \tag{A.2}$$

With portfolio weight $\psi$ and total assets $A$, this equation can be written as a risk-adjusted return:

$$I/(\psi A) = \tilde{r} + r_f(1-\psi)/\psi \tag{A.3}$$

Assuming the risky asset return can be represented as $\tilde{r} = \bar{r} + \epsilon$ where $\epsilon$ is drawn i.i.d. from $\mathcal{N}(0, \sigma^2)$, then if an individual bank is indexed by $i$, at each time period, the average risky return, the risk-free return, and the volatility of risky returns can be estimated via the cross-sectional regression:

$$I_i/(\psi_i A_i) = \bar{r} + r_f(1-\psi_i)/\psi_i + \epsilon_i \tag{A.4}$$

# B. Appendix: The Incidence of Capital Regulation

## B.1. Mathematical Appendix

### Decomposition surplus definitions

#### Direct effect

From the direct effect, total lending and deposit taking are:

$$L^{DE}(\gamma, \bar{\gamma}) = \int l(n, z, r(\gamma), \bar{\gamma}) \mathrm{d}\mu_\gamma(n, z)$$

$$D^{DE}(\gamma, \bar{\gamma}) = \int d(n, z, r(\gamma), \bar{\gamma}) \mathrm{d}\mu_\gamma(n, z)$$

with corresponding interest rates:

$$r_l^{DE}(\gamma, \bar{\gamma}) = r_l(L^{DE}(\gamma, \bar{\gamma})) \quad r_d^{DE}(\gamma, \bar{\gamma}) = r_d(D^{DE}(\gamma, \bar{\gamma}))$$

and the surplus of borrowers and savers is:

$$S_l^{DE}(\gamma, \bar{\gamma}) = \int_0^{L^{DE}(\gamma, \bar{\gamma})} (r_l(L) - r_l^{DE}(\gamma, \bar{\gamma})) \mathrm{d}L$$

$$S_d^{DE}(\gamma, \bar{\gamma}) = \int_0^{D^{DE}(\gamma, \bar{\gamma})} (r_d^{DE}(\gamma, \bar{\gamma}) - r_l(D)) \mathrm{d}D$$

To calculate banker surplus, define the lending supply and deposit demand functions:

$$\bar{L}(r_l, \gamma, \bar{\gamma}) = \int l(n, z, [r_l, r_f, r_d^{DE}(\gamma, \bar{\gamma})], \bar{\gamma}) \mathrm{d}\mu_\gamma(n, z)$$

$$\bar{D}(r_d, \gamma, \bar{\gamma}) = \int d(n, z, [r_l^{DE}(\gamma, \bar{\gamma}), r_f, r_d], \bar{\gamma}) \mathrm{d}\mu_\gamma(n, z)$$

where the inverses are defined as $\bar{r}_l(L, \gamma, \bar{\gamma}) = \bar{L}^{-1}(r_l, \gamma, \bar{\gamma})$ and $\bar{r}_d(D, \gamma, \bar{\gamma}) = \bar{D}^{-1}(r_d, \gamma, \bar{\gamma})$. The bankers' surplus in the lending and deposit markets are then given by:

$$S_{bl}^{DE}(\gamma, \bar{\gamma}) = \int_0^{L^{DE}(\gamma, \bar{\gamma})} (r_l^{DE}(\gamma, \bar{\gamma}) - \bar{r}_l(L, \gamma, \bar{\gamma})) \mathrm{d}L$$

$$S_{bd}^{DE}(\gamma, \bar{\gamma}) = \int_0^{D^{DE}(\gamma, \bar{\gamma})} (\bar{r}_d(D, \gamma, \bar{\gamma}) - r_d^{DE}(\gamma, \bar{\gamma})) \mathrm{d}D$$

### Short-run equilibrium effect

Using the industry structure $\mu_\gamma$, and the short-run equilibrium with policy $\bar{\gamma}$, total lending and deposit taking are:

$$L^S(\gamma,\bar{\gamma}) = \int l(n,z,r^S(\gamma,\bar{\gamma}),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z)$$

$$D^S(\gamma,\bar{\gamma}) = \int d(n,z,r^S(\gamma,\bar{\gamma}),\bar{\gamma})\mathrm{d}\mu_\gamma(n,z)$$

with corresponding interest rates:

$$r_l^S(\gamma,\bar{\gamma}) = r_l(L^S(\gamma,\bar{\gamma})) \quad r_d^S(\gamma,\bar{\gamma}) = r_d(D^S(\gamma,\bar{\gamma}))$$

and the surplus of borrowers and savers is:

$$S_l^S(\gamma,\bar{\gamma}) = \int_0^{L^S(\gamma,\bar{\gamma})} (r_l(L) - r_l^S(\gamma,\bar{\gamma}))\mathrm{d}L$$

$$S_d^S(\gamma,\bar{\gamma}) = \int_0^{D^S(\gamma,\bar{\gamma})} (r_d^S(\gamma,\bar{\gamma}) - r_l(D))\mathrm{d}D$$

To calculate banker surplus, define the lending supply and deposit demand functions:

$$\bar{L}(r_l,\gamma,\bar{\gamma}) = \int l^S(n,z,\left[r_l,r_f,r_d^S(\gamma,\bar{\gamma})\right],\bar{\gamma})\mathrm{d}\mu_\gamma(n,z)$$

$$\bar{D}(r_d,\gamma,\bar{\gamma}) = \int d^S(n,z,\left[r_l^S(\gamma,\bar{\gamma}),r_f,r_d\right],\bar{\gamma})\mathrm{d}\mu_\gamma(n,z)$$

where the inverses are defined as $\bar{r}_l(L,\gamma,\bar{\gamma}) = \bar{L}^{-1}(r_l,\gamma,\bar{\gamma})$ and $\bar{r}_d(D,\gamma,\bar{\gamma}) = \bar{D}^{-1}(r_d,\gamma,\bar{\gamma})$. The bankers' surplus in the lending and deposit markets are then given by:

$$S_{bl}^S(\gamma,\bar{\gamma}) = \int_0^{L^S(\gamma,\bar{\gamma})} (r_l^S(\gamma,\bar{\gamma}) - \bar{r}_l(L,\gamma,\bar{\gamma}))\mathrm{d}L$$

$$S_{bd}^S(\gamma,\bar{\gamma}) = \int_0^{D^S(\gamma,\bar{\gamma})} (\bar{r}_d(D,\gamma,\bar{\gamma}) - r_d^S(\gamma,\bar{\gamma}))\mathrm{d}D$$

## Equilibrium Solution Algorithm

### Long-run stationary equilibrium

Using value function iteration, I first solve for the discrete approximation to the value function $V(n,z,\{r_l,r_f.r_d\},\gamma)$ with associated policy functions. The continuous versions are then approximated using a linear spline. From here, with the value and policy functions, solving for equilibrium involves iterating over values of $r_d$. Given a value for $r_d$, the corresponding equilibrium $r_l$ is determined via the entry condition:

$$\sum_z V(\bar{n},z,\{r_l,r_f,r_d\},\gamma) - c^e = 0$$

which is solved using a univariate root solving algorithm. Total lending $L$ is determined via the equation $L = r_l^{-1}(r_l)$.

For the remainder, policy functions of the form $y(n, z, \{r_l, r_f, r_d\}, \gamma)$ are written $y(n, z)$ taking the previously determined rates as given. Using the policy functions, I can then construct the numerical transitions $q(n', z'|n, z)$ as:

$$q(n', z'|n, z) = \Gamma(z'|z)(1 - x(n, z)) \left[ \sum_{\epsilon} \mathbb{I}_{n'=\bar{n}(n,z,\epsilon)} g(\epsilon) \right]$$

and $\bar{n}$ is rounded up to the next highest gridpoint. Note here the distinction between the numerical discrete case, and the theoretical continuous case. Here, I have an equality indicator, where in the continuous case I had an inequality indicator. The difference is due to the measure zero probability of a particular $n'$ in the continuous case. In the numerical problem, the equality version is equivalent to first calculating the cummulative values (i.e. the direct numerical version of the theory), and then marginalizing the distribution, or differencing the distribution across the discrete gridpoints.

Solving for the stationary distribution $\mu$ and entry margin $M$ then requires constructing the appropriate linear mapping. Consider an object $Y$ as an $N \times Z$ matrix representing the discretized version of a function $Y(n, z)$ of net worth and productivity. Define $Y_v$ as the vectorized version of $X$ such that:

$$Y_v = \left[ Y(n_1, z_1), Y(n_2, z_1), \cdots, Y(n_N, z_1), Y(n_1, z_2), \cdots, Y(n_N, z_Z) \right]'$$

meaning that $Y_v$ is an $N \times Z$ element column vector containing the elements of $Y$. I then define the following objects:

$$L_v = \left[ (1 - x(n_1, z_1)) l(n_1, z_1), \cdots, (1 - x(n_N, z_Z)) l(n_N, z_Z) \right]'$$

$$D_v = \left[ (1 - x(n_1, z_1)) d(n_1, z_1), \cdots, (1 - x(n_N, z_Z)) d(n_N, z_Z) \right]'$$

$$\mu_v = \left[ \mu(n_1, z_1), \cdots, \mu(n_N, z_Z) \right]'$$

$$\lambda_v = \left[ 0, \cdots, 0, 1(n_i = \bar{n}, z_1), \cdots, 1(n_i = \bar{n}, z_Z), 0, \cdots, 0 \right]$$

$$Q = \begin{bmatrix} q(n_1, z_1|n_1, z_1) & \cdots & q(n_1, z_1|n_N, z_Z) \\ \vdots & \ddots & \vdots \\ q(n_N, z_Z|n_1, z_1) & \cdots & q(n_N, z_Z|n_N, z_Z) \end{bmatrix}$$

Not that $L_v$, $D_v$, $\mu_v$, and $\lambda_v$ are all $N \times Z$ element column vectors, where $Q$ is a $(N \times Z) \times (N \times Z)$ matrix. Given interest rates, the stationary distribution $\mu$ and entry margin $M$ represent a solution to the following two equations:

$$L = L_v' \mu_v$$

$$\mu_v = Q\mu_v + M\lambda_v$$

If $\tilde{Q} = Q - \mathbb{I}$ where $\mathbb{I}$ is an $(N \times Z) \times (N \times Z)$ identity matrix, $\mu_v$ and $M$ are:

$$\begin{bmatrix} \mu_v \\ M \end{bmatrix} = \begin{bmatrix} L_v' & 0 \\ \tilde{Q} & \lambda_v \end{bmatrix}^{-1} \begin{bmatrix} L \\ 0_v \end{bmatrix}$$

where $0_v$ is an $N \times Z$ column vector of zeros. Using $\mu_v$ from this solution implies total deposits of:

$$D = D_v' \mu_v$$

and a deposit rate equal to $r_d(D)$. Solving for equilibrium then requires finding the fixed point of this algorithm for $r_d$, which is done using a root solver over the entire operation.

### Short-run equilibrium

For short-run equilibrium, I use the solution to the Bellman iteration $V(n, z, \{r_l, r_f, r_d\}, \gamma)$ and corresponding policy functions. Given a fixed distribution $\bar{\mu}(n, z)$, the short-run equilibrium is simply the solution to the following fixed point mapping:

$$\begin{bmatrix} r_l \\ r_d \end{bmatrix} = \begin{bmatrix} r_l\left( \int (1 - x(n, z, \{r_l, r_f, r_d)\gamma\}))l(n, z, \{r_l, r_f, r_d\}, \gamma)) \mathrm{d}\bar{\mu}(n, z) \right) \\ r_d\left( \int (1 - x(n, z, \{r_l, r_f, r_d)\gamma\}))d(n, z, \{r_l, r_f, r_d\}, \gamma)) \mathrm{d}\bar{\mu}(n, z) \right) \end{bmatrix}$$

which is solved via two dimensional fixed point solver. In the counterfactual exercises, the fixed distribution $\bar{\mu}$ will be the distribution in the benchmark model.

## B.2. Data Appendix

| Time Period | Number of Banks | ln(Assets) | Concentration |
|---|---|---|---|
| 1993-2019 | 7549 | 11.87 | 45.64% |
| 1993-1995 | 10338 | 11.20 | 26.25% |
| 1996-1998 | 9024 | 11.36 | 33.41% |
| 1999-2001 | 8300 | 11.52 | 40.47% |
| 2002-2004 | 7902 | 11.73 | 44.31% |
| 2005-2007 | 7574 | 11.90 | 49.29% |
| 2008-2010 | 7182 | 12.07 | 54.15% |
| 2011-2013 | 6580 | 12.20 | 55.47% |
| 2014-2016 | 5876 | 12.34 | 54.65% |
| 2017-2019 | 5163 | 12.51 | 52.75% |

**Table B.1.:** Bank Summary Statistics: Annual Averages

# C. Appendix: Home Production with Time to Consume

## C.1. Mathematical Appendix

**Lemma 1.** *Shepherd's Lemma for off-market time use and wages is*

$$n^h_{igt} + n^h_{ist} = \frac{\partial e_{it}}{\partial w_{it}}$$

*Proof.* Note that

$$e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it}) = P_{gt}q^h_{igt} + P_{st}q^h_{ist} + w_{it}n^h_{igt} + w_{it}n^h_{ist} \tag{C.1}$$

where $n^h_{ijt}$ are the Hicksian demands for off-market time use in process $j$. Differentiating (C.1) in $w_{it}$ we get

$$\frac{\partial e_{it}}{\partial w_{it}} = P_{gt}\frac{\partial q^h_{igt}}{\partial w_{it}} + P_{st}\frac{\partial q^h_{ist}}{\partial w_{it}} + w_{it}\frac{\partial n^h_{igt}}{\partial w_{it}} w_{it}\frac{\partial n^h_{ist}}{\partial w_{it}} + n^h_{igt} + n^h_{ist} \tag{C.2}$$

Letting $\lambda_{it}$ be the multiplier on the budget constraint, we replace prices with the first-order conditions from the UMP, where we set $w_{it} = \frac{\partial u}{\partial n^h_{ijt}}\frac{1}{\lambda_{it}}$, next to its corresponding Hicksian partial derivative:

$$\frac{\partial e_{it}}{\partial w_{it}} = \frac{\partial u}{\partial q^h_{igt}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial q^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{ist}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{ist}}{\partial w_{it}} + n^h_{igt} + n^h_{ist} \tag{C.3}$$

By the fact that $u(q^h_{igt}, q^h_{ist}, n^h_{igt}, n^h_{ist}) = \overline{u}_{it}$ holds for all prices including wages, and given the Hicksian demand functions minimize the Lagrangian for the EMP:

$$\frac{\partial u}{\partial q^h_{igt}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial q^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{ist}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{ist}}{\partial w_{it}} = 0 \tag{C.4}$$

□

**Lemma 2.** *The Slutsky equations describing the responsiveness of demand $q_{ijt}$ to wages $w_{it}$ are*

$$\frac{\partial q^m_{ijt}}{\partial w_{it}} = \frac{\partial q^h_{ijt}}{\partial w_{it}} - \frac{\partial q^m_{ijt}}{\partial y_{it}}(n_{igt} + n_{ist}), \qquad \forall j \in \{g, s\}$$

*Proof.* The proof is the standard one, where the version of Shepherd's Lemma used is that of Lemma 1. Note that

$$q^m_{ijt}\big(P_{gt}, P_{st}, w_{it}, e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})\big) = q^h_{ijt}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it}) \tag{C.5}$$

and totally differentiate it to get

$$\frac{\partial q^m}{\partial w_{it}} + \frac{\partial q^m}{\partial y_{it}}\frac{\partial e_{it}}{\partial w_{it}} \tag{C.6}$$

Use Lemma 1 to replace $\frac{\partial e_{it}}{\partial w_{it}}$ then rearrange to get the result. □

**Lemma 3.** *In a two-good, static economy with CES utility and Cobb-Douglas home production, suppose $\omega_s > \omega_g$. Then*

$$\phi^n = \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} > \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} = \phi^q$$

*Proof.* Start with $1 > \omega_s > \omega_g > 0$.

$$\Rightarrow \quad \frac{\omega_g}{\omega_s} < \frac{1-\omega_g}{1-\omega_s} \tag{C.7}$$

Since $1-\rho > 0$ for all $\rho < 1$:

$$\Rightarrow \quad \left(\frac{\omega_g}{1-\omega_g}\right)^{1-\rho-\rho\omega_g+\rho\omega_g} < \left(\frac{\omega_s}{1-\omega_s}\right)^{1-\rho-\rho\omega_s+\rho\omega_s} \tag{C.8}$$

$$\Leftrightarrow \quad \left(\frac{\omega_g}{1-\omega_g}\right)^{1-\rho\omega_g}\left(\frac{\omega_g}{1-\omega_g}\right)^{(\omega_g-1)\rho} < \left(\frac{\omega_s}{1-\omega_s}\right)^{1-\rho\omega_s}\left(\frac{\omega_s}{1-\omega_s}\right)^{(\omega_s-1)\rho} \tag{C.9}$$

$$\begin{aligned}\Leftrightarrow \quad & \left(\frac{\omega_g}{1-\omega_g}\right)\left(\frac{\omega_g}{1-\omega_g}\right)^{-\rho\omega_g}\left(\frac{\omega_g}{1-\omega_g}\right)^{(\omega_g-1)\rho} \\ & < \left(\frac{\omega_s}{1-\omega_s}\right)\left(\frac{\omega_s}{1-\omega_s}\right)^{-\rho\omega_s}\left(\frac{\omega_s}{1-\omega_s}\right)^{(\omega_s-1)\rho}\end{aligned} \tag{C.10}$$

$$\Leftrightarrow \quad \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right] < \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right] \tag{C.11}$$

Since $\frac{1}{\rho-1} < 0$ for all $\rho < 1$:

$$\left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} > \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} \tag{C.12}$$

Thus, $\phi^n > \phi^q$. □

**Lemma 4.** *In a two-good, static economy with CES utility and Cobb-Douglas home production, suppose $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Define the function*

$$\Upsilon^n(P_g, P_s, w) = \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}}$$

*i. If $\rho \in (0,1)$ then $\Upsilon^n$ is decreasing in $w$.*

*ii. If $\rho < 0$ then $\Upsilon^n$ is increasing in $w$.*

*Proof.* Note that

$$\frac{\partial \Upsilon^n}{\partial w} = \Upsilon^n(P_g, P_s, w)\left(\frac{\rho(\omega_s-\omega_g)}{\rho-1}\right)\left(\frac{1}{w}\right) \tag{C.13}$$

Clearly, $\Upsilon^n > 0$ and $w > 0$ always, so the sign of $\frac{\partial \Upsilon^n}{\partial w}$ hinges on the term $\frac{\rho(\omega_s-\omega_g)}{\rho-1}$.

i. If $\rho \in (0,1)$, $\rho - 1 < 0$ and $\rho(\omega_s - \omega_g) > 0$, so $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 0$.

ii. If $\rho < 0$, $\rho - 1 < 0$ and $\rho(\omega_s - \omega_g) < 0$, so $\frac{\rho(\omega_s-\omega_g)}{\rho-1} > 0$.

□

**Lemma 5.** *In a two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian off-market time-utilization functions are*

$$\begin{aligned} n_s(P_g, P_s, w) &= \overline{n}\left[\Upsilon^n(P_g, P_s, w)\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right) + \frac{\omega_s}{1-\omega_s} + 1\right]^{-1} \\ n_g(P_g, P_s, w) &= \Upsilon^n(P_g, P_s, w) n_s(P_g, P_s, w) \end{aligned}$$

*Proof.* Define the function

$$\Upsilon^q(P_g, P_s, w) = \phi^q P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{C.14}$$

$$\Rightarrow \quad \Upsilon^q(P_g, P_s, w) = \frac{\phi^q}{\phi^n}\Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{P_g}\right) \tag{C.15}$$

Dropping dependencies on prices and wages, we have the following implicit functions using (4), (5), and (6) from the main text:

$$q_j(n_j) = \left(\frac{\omega_j}{1-\omega_j}\right)\left(\frac{w}{P_j}\right)n_j \tag{C.16}$$

$$q_g(q_s) = \Upsilon^q q_s \tag{C.17}$$

$$n_g(n_s) = \Upsilon^n n_s \tag{C.18}$$

Starting with the derivation of $n_s$, note that $q_g(q_s(n_s)) = \Upsilon^q\left(\frac{\omega_s}{1-\omega_s}\right)\left(\frac{w}{P_s}\right)n_s$. Using the Beckerian budget constraint, $P_g q_q + P_s q_s + w(n_g + n_s) = \overline{n}$, we can substitute out $q_g$, $q_s$, and $n_g$ using the implicit functions derived here, divide by $w$, substitute $\Upsilon^q$ for the expression in (C.15), and then rearrange to get the expression for $n_s(P_g, P_s, w)$. The expression for $n_g(P_g, P_s, w)$ follows directly from the relative off-market time use expression in (6). $\square$

**Corollary 1.** *In the two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian demand functions for market services and goods are*

$$q_s(P_g, P_s, w) = w\overline{n}\left[P_s\frac{\phi^q}{\phi^n}\Upsilon^n(P_g, P_s, w)\left(1 + \frac{1-\omega_s}{\omega_s}\right) + P_s + P_s\left(\frac{1-\omega_s}{\omega_s}\right)\right]^{-1}$$
$$q_g(P_g, P_s, w) = \frac{\phi^q}{\phi^n}\Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{P_g}\right)q_s(P_g, P_s, w)$$

*Proof.* Given the first-order conditions from the utility-maximization problem and using the definitions of $\Upsilon^n$ and $\Upsilon^q$ from Lemmas 4 and 5, we can write

$$q_g(q_s) = \Upsilon^q(P_g, P_s, w)q_s \tag{C.19}$$
$$n_s(q_s) = \left(\frac{P_s}{w}\right)\left(\frac{1-\omega_s}{\omega_s}\right)q_s \tag{C.20}$$
$$n_g\big(n_s(q_s)\big) = \Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{w}\right)\left(\frac{1-\omega_s}{\omega_s}\right)q_s \tag{C.21}$$

Then we can plug the above objects into the Beckerian budget constraint to get:

$$P_g\Upsilon^q q_s + P_s q_s + P_s\Upsilon^n\left(\frac{1-\omega_s}{\omega_s}\right)q_s + P_s\left(\frac{1-\omega_s}{\omega_s}\right)q_s = w\overline{n} \tag{C.22}$$

Isolate $q_s$ to get the result.

For $q_g$ just plug in $q_s(P_g, P_s, w)$ to (C.19), substitute (C.15) for $\Upsilon^q$, and the result is attained. $\square$

**Lemma 6.** *In a two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian labor supply function is*

$$\ell(P_g, P_s, w) = \overline{n} - \Upsilon^n(P_g, P_s, w)n_s(P_g, P_s, w) - n_s(P_g, P_s, w)$$

*Proof.* From Lemmas 4 and 5 note that

$$n_g(P_g, P_s, w) = \Upsilon^n(P_g, P_s, w)n_s(P_g, P_s, w) \tag{C.23}$$

Using the time use constraint:

$$\ell(P_g, P_s, w) = \overline{n} - n_g(P_g, P_s, w) - n_s(P_g, P_s, w) \tag{C.24}$$
$$\Rightarrow \quad \ell(P_g, P_s, w) = \overline{n} - \Upsilon^n(P_g, P_s, w)n_s(P_g, P_s, w) - n_s(P_g, P_s, w) \tag{C.25}$$

$\square$

**Proposition 1.** *Fix prices $P_g$ and $P_s$. In a two-good, static economy with CES utility and Cobb-Douglas home production, the intensive margin of labor varies in wages as follows:*

*i. If the outputs of home production are substitutes so that $\rho \in (0,1)$, $\ell$ is increasing in $w$ and the classic $c/\ell$ substitution effect dominates.*

*ii. If the outputs of home production are complements so that $\rho < 0$, $\ell$ is decreasing in $w$ and the classic $c/\ell$ income effect dominates.*

*Proof.* Assume $\omega_s > \omega_g$, so that home production using goods is more time-intensive. For $\omega_s < \omega_g$ just exchange indices. Let $w' > w > 0$. We will prove each case separately. For this proof we will ignore the dependency of $\Upsilon^n$ on prices $P_g$ and $P_s$ to reduce notational clutter.

i. Suppose $\rho \in (0,1)$. Start with the fact that $\Upsilon^n(w') < \Upsilon^n(w)$ as shown in Lemma 4. Denote the constants $a = 1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}$ and $b = 1 + \frac{\omega_s}{1-\omega_s}$. Since, by Lemma 3, $\phi^n > \phi^q$ then $(b-a) > 0$ which implies

$$\Upsilon^n(w')(b-a) < \Upsilon^n(w)(b-a) \tag{C.26}$$

$$\Leftrightarrow \quad \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w')(b-a) + b < \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w)(b-a) + b \tag{C.27}$$

$$\begin{aligned} \Leftrightarrow \quad & \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w')b + \Upsilon^n(w)a + b \\ & < \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w)b + \Upsilon^n(w')a + b \end{aligned} \tag{C.28}$$

$$\Leftrightarrow \quad \big(\Upsilon^n(w)a + b\big)\big(\Upsilon^n(w') + 1\big) < \big(\Upsilon^n(w')a + b\big)\big(\Upsilon^n(w) + 1\big) \tag{C.29}$$

$$\Leftrightarrow \quad \overline{n}\left(\frac{\Upsilon^n(w') + 1}{\Upsilon^n(w')a + b}\right) < \left(\frac{\Upsilon^n(w) + 1}{\Upsilon^n(w)a + b}\right)\overline{n} \tag{C.30}$$

Since $n_s(w) = \frac{\overline{n}}{\Upsilon^n(w)a+b}$ by Lemma 5

$$\Upsilon^n(w')n_s(w') + n_s(w') < \Upsilon^n(w)n_s(w) + n_s(w) \tag{C.31}$$

$$\Leftrightarrow \quad \overline{n} - \Upsilon^n(w')n_s(w') - n_s(w') > \overline{n} - \Upsilon^n(w)n_s(w) - n_s(w) \tag{C.32}$$

Since $\ell(w) = \overline{n} - \Upsilon^n(w)n_s(w) - n_s(w)$ by Lemma 6, then $\ell(w') > \ell(w)$.

ii. Suppose $\rho < 0$. Note that $\Upsilon^n(w') > \Upsilon^n(w)$ when $\rho < 0$ by Lemma 4, but $\phi^n > \phi^q$ by Lemma 3, so just flip the inequalities from case (i).

□

**Proposition 2.** *Relative market purchases and off-market time use vary in wages as follows:*

*i. If $\rho \in (0,1)$ then market purchases and time use for the more time-intensive task fall relative to the less time-intensive task as $w$ rises.*

*ii. If $\rho < 0$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as $w$ rises.*

*Proof.* Assume throughout the proof that $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Note that:

$$\frac{\partial(q_g/q_s)}{\partial w} = \Upsilon^q \frac{\rho(\omega_s - \omega_g)}{\rho - 1}\left(\frac{1}{w}\right) \tag{C.33}$$

$$\frac{\partial(n_g/n_s)}{\partial w} = \Upsilon^n \frac{\rho(\omega_s - \omega_g)}{\rho - 1}\left(\frac{1}{w}\right) \tag{C.34}$$

where $\Upsilon^q, \Upsilon^n > 0$ always.

i. Suppose $\rho \in (0,1)$. Clearly $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 0$ so relative consumption and relative time use both fall.

ii. Suppose $\rho < 0$. Then $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1} > 0$ so relative consumption and relative time use both rise.

Since $g$ is assumed more time-intensive, this completes the proof. □

**Corollary 2.** *If the more time-intensive market commodity is more expensive than the less time-intensive commodity, and the relative price of the two exceeds the ratio $\frac{\phi^q}{\phi^n}$ then relative time use changes faster than relative consumption in response to wage increases.*

*Proof.* Assume throughout the proof that $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Since $g$ is more time intensive, $P_g > P_s$. Further, we are given:

$$\frac{P_g}{P_s} > \frac{\phi^q}{\phi^n} \tag{C.35}$$

By inspecting Proposition 2, it is clear that $\frac{n_g}{n_s}$ will change more than $\frac{q_g}{q_s}$ if and only if $\Upsilon^q < \Upsilon^n$. Starting with (C.35):

$$\Leftrightarrow \quad \phi^q P_g^{-1} < \phi^n P_s^{-1} \tag{C.36}$$

$$\Leftrightarrow \quad \phi^q P_g^{\frac{1-\rho+\rho\omega_g-\rho\omega_g}{\rho-1}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} < \phi^n P_s^{\frac{1-\rho+\rho\omega_s-\rho\omega_s}{\rho-1}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{C.37}$$

$$\Leftrightarrow \quad \phi^q P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} < \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{C.38}$$

$$\Leftrightarrow \quad \Upsilon^q < \Upsilon^n \tag{C.39}$$

□

**Proposition 3.** *Marshallian demands for off-market time respond to wage increases as follows:*

*i. If $\rho \in (0,1)$ then time devoted to the more time-intensive task is inferior.*

*ii. If $\rho < 0$ then time devoted to the less time-intensive task is inferior.*

*Proof.* Inferiority amounts to showing that Marshallian demand for time in category $j$ is decreasing in $w\overline{n}$, which is total income. Since $\overline{n}$ is fixed we can simply show that demand for time is decreasing in $w$. Dropping dependencies on $P_g$ and $P_s$ for notational convenience, from Lemma 5, note that

$$\frac{\partial n_s}{\partial w} = -\frac{\left[n_s(w)\right]^2}{\overline{n}}\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right)\frac{\partial \Upsilon^n}{\partial w} \tag{C.40}$$

Suppose $\omega_s > \omega_g$ so process $g$ is more time intensive.

i. Suppose $\rho \in (0,1)$. Then $\frac{\partial \Upsilon^n}{\partial w} < 0$ by Lemma 4, and $\frac{\partial n_s}{\partial w} > 0$. By Proposition 1 since $\ell$ is increasing in $w$, $\overline{n} - \ell$ is decreasing in $w$, which implies that $n_g$ is decreasing in $w$ since $n_s$ is increasing. Thus, $n_g$ is inferior.

ii. Suppose $\rho < 0$. Then $\frac{\partial \Upsilon^n}{\partial w} > 0$ by Lemma 4, and $\frac{\partial n_s}{\partial w} < 0$. By Proposition 1 since $\ell$ is decreasing in $w$, $\overline{n} - \ell$ is increasing in $w$, but $n_s$ falls as $w$ rises, so $n_g$ must be increasing in $w$.

□

**Proposition 4.** *Marshallian demands for market purchases respond to wage increases as follows:*

*i. If $\rho \in (0,1)$ then the market purchase associated with the less time-intensive process is normal, but the market purchase associated with the more time-intensive process may, but need not, be inferior for certain prices and parameter combinations.*

*ii. If $\rho < 0$ then all market purchases are normal.*

*Proof.* Differentiating the Marshallian demand for services in $w$ from Corollary 1, we get:

$$\frac{\partial q_s}{\partial w} = \left(\frac{q_s}{w}\right)\left[1 - \left(\frac{\rho(\omega_s - \omega_g)}{\rho - 1}\right) \underbrace{\frac{P_s \frac{\phi^q}{\phi^n}\Upsilon^n + P_s \frac{\phi^q(1-\omega_g)}{\phi^n \omega_g}\Upsilon^n}{P_s \frac{\phi^q}{\phi^n}\Upsilon^n + P_s \frac{\phi^q(1-\omega_g)}{\phi^n \omega_g}\Upsilon^n + P_s + \frac{1-\omega_s}{\omega_s}P_s}}_{\kappa}\right] \tag{C.41}$$

First, suppose that $\omega_s > \omega_g$, so that goods are more time intensive. Note that $0 < \kappa < 1$, clearly. When $\rho \in (0,1)$, $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 0$, so the second term in (C.41) is positive. Thus $\frac{\partial q_s}{\partial w} > 0$, always.

Now, suppose without loss of generality, $\omega_s < \omega_g$, so that now services are more time intensive. We need only show that there exists one combination of parameters and prices, such that $\frac{\partial q_s}{\partial w} < 0$, so that the market purchase associated with the more time intensive task is inferior. Consider $P_g = P_s = w = \overline{n} = 1$, $\omega_s = 0.2$, $\omega_g = 0.75$, and $\rho = 0.9$. In this case $\frac{\partial q_s}{\partial w} \approx -0.102 < 0$. Thus, market purchases associated with the more time-intensive task are inferior. To show they need not be inferior, consider the same parameterization, except now let $\rho = 0.2$. Then $\frac{\partial q_s}{\partial w} \approx 0.094 > 0$.

ii. Again, first suppose that $\omega_s > \omega_g$, so that goods are more time intensive. Since $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} > 0$ and $0 < \kappa < 1$ always, it is sufficient to show that $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 1$ always. Note that since $0 < \omega_s - \omega_g < 1$ then $|\rho(\omega_s - \omega_g)| < |\rho - 1|$ which implies $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 1$.

Now suppose services are more time intensive so that $\omega_s < \omega_g$. Then $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 0$ and the result is proven.

□

**Lemma 7.** *Since $\Upsilon^n(P_g, P_s, w)$ is homogeneous of degree 0, we can equivalently write the function with two relative-price arguments: $\widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right) = \Upsilon^n(P_g, P_s, w)$. Assume $\frac{w}{P_s}$ is fixed, so that $w$ and $P_s$ have the same rate of inflation. $\widehat{\Upsilon}^n$ varies in $\frac{P_g}{P_s}$ as follows:*

i. *If* $\rho \in (0,1)$, $\widehat{\Upsilon}^n$ *is decreasing in* $\frac{P_g}{P_s}$.

ii. *If* $\rho < 0$, $\widehat{\Upsilon}^n$ *is increasing in* $\frac{P_g}{P_s}$.

*Proof.* First, note that

$$\Upsilon^n(P_g, P_s, w) = \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} = \phi^n \left(\frac{P_g}{P_s}\right)^{\frac{\rho\omega_g}{\rho-1}} \left(\frac{w}{P_s}\right)^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} = \widehat{\Upsilon}^n(P_g/P_s, w/P_s) \quad \text{(C.42)}$$

Differentiating $\widehat{\Upsilon}^n$ in $P_g/P_s$:

$$\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} = \widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho-1}\right)\left(\frac{P_s}{P_g}\right) \quad \text{(C.43)}$$

which is clearly $< 0$ if $\rho \in (0,1)$ and $> 0$ if $\rho < 0$. ◻

**Proposition 5.** *Relative market purchases and off-market time use vary in the relative price of market purchases as follows:*

i. *If* $\rho \in (0,1)$ *then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as the more time-intensive task becomes cheaper.*

ii. *If* $\rho < 0$ *then market purchases for the more time-intensive task rise relative to the less time-intensive task, but time use for the more time-intensive task relative to the less time-intensive task falls as the more time-intensive task becomes cheaper.*

*Proof.* By Lemma 7, $\Upsilon^n(P_g, P_s, w)$ is homogeneous of degree 0. By the fact that $\Upsilon^q = \frac{\phi^q}{\phi^n}\Upsilon^n\left(\frac{P_s}{P_g}\right)$ it is also homogeneous of degree 0. Therefore, we can rewrite the relative demand and time use functions as follows:

$$\left(\frac{q_g}{q_s}\right) = \frac{\phi^q}{\phi^n}\widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right)\left(\frac{P_s}{P_g}\right) \quad \text{(C.44)}$$

$$\left(\frac{n_g}{n_s}\right) = \widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right) \quad \text{(C.45)}$$

Suppose without loss of generality $\omega_s > \omega_g$, so $g$ is more time intensive. If this were not the case, just exchange indices. By Lemma 7, as $P_g/P_s$ falls $n_g/n_s$ rises when $\rho \in (0,1)$ and falls when $\rho < 0$. Relative quantities vary in $P_g/P_s$ as follows:

$$\frac{\partial(q_g/q_s)}{\partial(P_g/P_s)} = \frac{\phi^q}{\phi^n}\left(\frac{P_s}{P_g}\right)\left[\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} - \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right)\right] \quad \text{(C.46)}$$

As long as the second term is $< 0$ then $q_g/q_s$ will rise as $P_g/P_s$ falls. When $\rho \in (0,1)$ this is clearly true since $\widehat{\Upsilon}^n(P_s/P_g) > 0$. Note that

$$\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} - \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right) = \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right)\left[\frac{\rho\omega_g}{\rho-1} - 1\right] \quad \text{(C.47)}$$

which is $< 0$ as long as

$$\frac{\rho\omega_g}{\rho - 1} < 1 \tag{C.48}$$

$$\Leftrightarrow \quad \frac{\rho\omega_g}{1-\rho} > -1 \tag{C.49}$$

$$\Leftrightarrow \quad \rho\omega_g > \rho - 1 \tag{C.50}$$

$$\Leftrightarrow \quad \omega_g < \frac{\rho - 1}{\rho} \tag{C.51}$$

Since $\frac{\rho-1}{\rho} > 1$ this is always true. Thus $q_g/q_s$ is declining in $P_g/P_s$ always. □

**Proposition 6.** *Marshallian demands for market purchases vary in relative prices as follows:*

*i. If $\rho \in (0, 1)$, consumption of the less time-intensive market purchase falls while consumption of the more time-intensive purchase rises as the more time-intensive task becomes cheaper.*

*ii. If $\rho < 0$, consumption of both market purchases rises as the more time-intensive task becomes cheaper.*

*Proof.* Suppose without loss of generality $\omega_s > \omega_g$, so $g$ is more time intensive. If this were not the case, just exchange all indices. By the fact that they are classic Marshallian demand functions, $q_s(P_g, P_s, w)$ and $q_g(P_s, P_g, w)$ are homogeneous of degree 0. We can thus write $\widehat{q}_j(P_g/P_s, w/P_s) = q_j(P_g, P_s, w)$ for all $j \in \{g, s\}$. Referring back to Corollary 1 and Lemma 7, note that:

$$\frac{\partial \widehat{q}_s}{\partial (P_g/P_s)} = -(\widehat{q}_s)^2 \left(\frac{P_s}{\overline{n}w}\right)\left(\frac{\phi^q}{\phi^n}\right)\left(\frac{1}{\omega_s}\right)\frac{\partial \widehat{\Upsilon}^n}{\partial (P_g/P_s)} \tag{C.52}$$

Since $\frac{\partial \widehat{\Upsilon}^n}{\partial (P_g/P_s)} < 0$ when $\rho \in (0, 1)$, $q_s$ falls as $P_g/P_s$ falls, and the classic substitution effect appears to dominate for services consumption. This is only true when final goods are gross substitutes. When final goods are gross complements, the classic income effect dominates and $\widehat{q}_s$ rises as $P_g/P_s$ falls, again by Lemma 7.

For $q_g$:

$$\begin{aligned}
\frac{\partial \widehat{q}_g}{\partial (P_g/P_s)} &= \left(\frac{\phi^q}{\phi^n}\right)\left[-\left(\frac{P_s}{P_g}\right)^2 \widehat{\Upsilon}^n \widehat{q}_s + \left(\frac{P_s}{P_g}\right)\frac{\partial \widehat{\Upsilon}^n}{\partial (P_g/P_s)}\widehat{q}_s + \left(\frac{P_s}{P_g}\right)\widehat{\Upsilon}^n \frac{\partial \widehat{q}_s}{\partial (P_g/P_s)}\right] \\
&= \left(\frac{\phi^q}{\phi^n}\right)\left(\frac{P_s}{P_g}\right)\left[\widehat{\Upsilon}^n \widehat{q}_s \left(\frac{P_s}{P_g}\right)\left(\frac{\rho\omega_g}{\rho - 1} - 1\right) + \widehat{\Upsilon}^n \frac{\partial \widehat{q}_s}{\partial (P_g/P_s)}\right] \\
&= \left(\frac{\phi^q}{\phi^n}\right)\left(\frac{P_s}{P_g}\right)^2 \widehat{\Upsilon}^n \widehat{q}_s \left[\left(\frac{\rho\omega_g}{\rho - 1} - 1\right) - \underbrace{\frac{\widehat{\Upsilon}^n \frac{\phi^q}{\phi^n \omega_s}}{\widehat{\Upsilon}^n \frac{\phi^q}{\phi^n \omega_s} + \frac{1}{\omega_s}}}_{\kappa}\left(\frac{\rho\omega_g}{\rho - 1}\right)\right]
\end{aligned} \tag{C.53}$$

where the sign hinges on the term.

$$\frac{\rho\omega_g}{\rho - 1}(1 - \kappa) - 1 \tag{C.54}$$

$0 < \kappa < 1$ always. Note that

$$\frac{\rho\omega_g}{\rho - 1}(1-\kappa) < 1 \tag{C.55}$$

$$\Leftrightarrow \quad \frac{\rho\omega_g}{\rho - 1}(1-\kappa) > -1 \tag{C.56}$$

$$\Leftrightarrow \quad \rho\omega_g(1-\kappa) > \rho - 1 \tag{C.57}$$

When $\rho \in (0,1)$ the left side is positive and the right side is negative so the inequality holds. When $\rho < 0$, note that $|\rho\omega_g(1-\kappa)| < |\rho - 1|$, and the inequality holds. □

**Proposition 7.** *Marshallian demands for off-market time vary in the relative price of market purchases as follows:*

*i. If $\rho \in (0,1)$, off-market time use for the less time-intensive task falls and time use for the more time-intensive task rises as the more time-intensive task becomes cheaper.*

*ii. If $\rho < 0$, off-market time use for the less time-intensive task rises and time use for the more time-intensive task falls as the more time-intensive task becomes cheaper.*

*Proof.* Suppose without loss of generality $\omega_s > \omega_g$, so $g$ is more time intensive. If this were not the case, just exchange all indices. By Lemma 5 prices only enter $n_s(P_g, P_s, w)$ via $\Upsilon^n(P_g, P_s, w)$. Since $\Upsilon^n$ is homogeneous of degree 0 by Lemma 7, then $n_s$ is also homogeneous of degree 0. Thus we can write $n_s(P_g, P_s, w) = \widehat{n}_s(P_g/P_s, w/P_s)$. Differentiating in $P_g/P_s$:

$$\frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} = -\frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} \tag{C.58}$$

Note that the sign of (C.58) hinges solely on the sign of $\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}$. By Lemma 7, it thus follows that the level of $n_s$ falls as $P_g/P_s$ falls when $\rho \in (0,1)$ and it rises when $\rho < 0$.

For $n_g$ note that

$$\frac{\partial \widehat{n}_g}{\partial(P_g/P_s)} = \frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s + \widehat{\Upsilon^n}\frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} \tag{C.59}$$

$$= \frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s\left[1 - \widehat{\Upsilon^n}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\left(\widehat{\Upsilon^n}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right) + \frac{\omega_s}{1-\omega_s} + 1\right)^{-1}\right] \tag{C.60}$$

Note that the second term in (C.60) is $> 0$ always. Thus, the sign of $\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}$ also governs how $\widehat{n}_g$ varies in $P_g/P_s$. When $P_g/P_s$ falls and $\rho \in (0,1)$, $\widehat{n}_g$ rises. When $P_g/P_s$ falls and $\rho < 0$, $\widehat{n}_g$ falls. □

**Proposition 8.** *Marshallian labor supply varies in the relative price of market purchases as follows:*

*i. If $\rho \in (0,1)$, $\ell$ falls as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic $c/\ell$ income effect which dominates.*

*ii. If $\rho < 0$, $\ell$ rises as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic $c/\ell$ substitution effect which dominates.*

*Proof.* Suppose without loss of generality $\omega_s > \omega_g$, so $g$ is more time intensive. If this were not the case, just exchange all indices. Note that the Marshallian labor supply function described in Lemma 6 is homogeneous of degree 0 and can be written:

$$\ell(P_g/P_s, w/P_s) = \overline{n} - \widehat{\Upsilon}^n(P_g/P_s, w/P_s)\widehat{n}_s(P_g/P_s, w/P_s) - \widehat{n}_s(P_g/P_s, w/P_s) \tag{C.61}$$

$$\Rightarrow \quad \frac{\partial \widehat{\ell}}{\partial(P_g/P_s)} = -\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s - \widehat{\Upsilon}^n \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} - \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} \tag{C.62}$$

$$= -\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s - \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)}(\widehat{\Upsilon}^n + 1) \tag{C.63}$$

$$= -\widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s + \frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}(\widehat{\Upsilon}^n + 1) \tag{C.64}$$

$$\begin{aligned} &= -\widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s \\ &\qquad + \frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)(\widehat{\Upsilon}^n + 1) \\ &= \widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s \underbrace{\left(\frac{\widehat{\Upsilon}^n\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right) + 1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}}{\widehat{\Upsilon}^n\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right) + 1 + \frac{\omega_s}{1-\omega_s}} - 1\right)}_{\kappa} \end{aligned} \tag{C.65}$$

Note that $\kappa < 0$ always since $\phi^n > \phi^q$ by Lemma 3. Thus the sign of the derivative is governed by $\frac{\rho\omega_g}{\rho-1}$ which is $< 0$ when $\rho \in (0, 1)$ and $> 0$ when $\rho < 0$. It thus follows that as $P_g/P_s$ falls, $\ell$ falls when $\rho \in (0, 1)$ and $\ell$ rises when $\rho < 0$. □

## C.2. Data Appendix

### Constructing Expenditure, Price, and Quantity Series

We will illustrate how to unwind chain-weighted price and quantity indices with an example showing how to combine durables stock data with new consumption expenditure data. Our consumption expenditure data series are taken from the Bureau of Economic Analysis' (BEA) National Income and Product Account (NIPA) tables. Specifically, we take the non-durable goods and services nominal expenditure series from NIPA Table 1.1.5. To construct real data series, we download the chain-type price indices from NIPA Table 1.1.4. and chain type quantity indices from NIPA Table 1.1.3. for non-durable goods and services. To account for the fact that consumers enjoy service flows from durable expenditure over more than one period we turn to BEA Fixed Asset Table 1.1. which gives the current dollar value of the nominal capital stock, including consumer durables. BEA Fixed Asset Table 1.2. provides a corresponding quantity index. From each of these, we use only the "Consumer durable goods" series.

Now in possession of data series for nominal expenditure of non-durable goods and services, the nominal value of the stock of consumer durables and corresponding price and quantity indices where available, we can construct our aggregate "goods" and "services" consumption series in real chained 2012 dollars. Note that construction of the real services consumption series requires no additional steps beyond a standard deflationary procedure dividing the nominal services expenditure series from Table 1.1.5. by the chain type price index from Table 1.1.4. The units of this series should be read as "the real value of services consumption expenditure in 2012 chained dollars."[1] Since "goods" consumption is the sum of non-durable consumption and the consumption of service flows from the net stock of durable assets, we follow the procedure outlined in Online Appendix C of Herrendorf, Rogerson, and Valentinyi (2013) and discussed in Whelan (2002) to construct a measure of real goods consumption in units of 2012 chained dollars. Unfortunately we cannot simply sum expenditure of non-durables and durables and divide this number by the sum of 2012 chain-weighted real consumption since chain-weighted series are generally not additive (Whelan 2000; Whelan 2002). Instead we require an aggregate "goods" price index that accounts for changing relative prices of non-durables and the price associated with the stock of all durables. Note that the price index the BEA uses to construct the quantity index associated with the stock of consumer durables from BEA Fixed Asset Table 1.2. is the same 2012 chained dollar index for durables *expenditure* presented in NIPA Table 1.1.4.[2] Using this durables expenditure price index, we can construct a total "goods" quantity index that accounts for both aggregate non-durable consumption and service flows derived from the entire stock of consumer durables.

Consider only aggregates, so that all variables are presented in capital letters. Let $\widetilde{Q}_{gt}$ be a chain-weighted "goods" quantity index. Let the subscript $nd$ denote non-durables and $d$ durables. Let $X_{jt}$ denote current dollar expenditure, $P_{jt}$ be a chain-weighted price such that $P_{j,2012} = 100$, and $Q_{jt}$ be the real value of consumption in 2012 chained dollars for all $j \in \{nd, d\}$. Note that $Q_{jt} = \frac{X_{jt}}{P_{jt}}$. Set the 2012 base year aggregate quantity index, $\widetilde{Q}_{g,2012} = 1$. We compute

$$\widetilde{Q}_{gt} = \widetilde{Q}_{g,t-1} \sqrt{\frac{(\sum_j P_{j,t-1} Q_{jt})(\sum_j X_{jt})}{(\sum_j X_{j,t-1})(\sum_j P_{jt} Q_{j,t-1})}} \quad \forall t > 2012 \quad \text{where} \quad j \in \{nd, d\} \tag{C.66}$$

$$\widetilde{Q}_{gt} = \widetilde{Q}_{g,t+1} \Big/ \sqrt{\frac{(\sum_j P_{jt} Q_{j,t+1})(\sum_j X_{j,t+1})}{(\sum_j X_{jt})(\sum_j P_{j,t+1} Q_{jt})}} \quad \forall t < 2012 \quad \text{where} \quad j \in \{nd, d\} \tag{C.67}$$

Aggregate goods consumption in chained 2012 dollars is then

$$Q_{gt} = \widetilde{Q}_{gt} \sum_{j \in \{nd,d\}} X_{j,2012} \tag{C.68}$$

Finally, the aggregate chain-weighted goods price index with $P_{g,2012} = 1$ is just

$$P_{gt} = \frac{\sum_{j \in \{nd,d\}} X_{jt}}{Q_{gt}} \tag{C.69}$$

In addition to a composite goods price index, we also need an aggregate consumption price index that accounts for relative changes in the value of both services and goods over

[1]See Whelan (2000) and Whelan (2002) for further discussion of chain-weighted units.

[2]The BEA only presents current dollar value $X_{dt}$ (nominal values are $X$'s) and 2012 chain-weighted quantity indices $\widetilde{Q}_{dt}$ (quantity index) for durable stocks in the fixed asset tables, not prices. Nonetheless, it can be confirmed that the price index associated with the stock of durables is the same as that associated with the flow of durables expenditure by performing the following procedure. First, compute the 2012 chain-weighted real value of durables $Q_{dt} = \frac{\widetilde{Q}_{dt} X_{d,2012}}{100}$, since $\widetilde{Q}_{d,2012} = 100$. Then compute $P_{dt} = \frac{X_{dt}}{Q_{dt}}$ and compare this series to the price index for durables expenditure in NIPA Table 1.1.4. They are the same.

time in order to properly place aggregate wages in the same 2012 chain-weighted units as consumption prices. To construct this series, repeat the above procedure except this time sum over $j \in \{s, nd, d\}$ to get $\widetilde{Q}_t$, an aggregate consumption quantity index in 2012 chain-weighted units. From there, an aggregate consumption price index in 2012 chain-weighted units can be easily derived by first computing real 2012 chain-weighted aggregate consumption $Q_t = \widetilde{Q}_t \sum_{j \in \{s,nd,d\}} X_{j,2012}$, then using that value to get an aggregate price index $P_t = \frac{\sum_{j \in \{s,nd,d\}} X_{jt}}{Q_t}$.

## Time-use Task Classifications

The ATUS classifies consumer time use into hundreds of different activities. Here, we list activities by their activity number that we assume are complementary either with using goods or services. We classify activities as follows, where the six-digit numbers listed in parentheses after the top-level activity category correspond to the sub-categories included in our classification. First, we consider the classification of activities when personal care time is included as part of time spent using goods. Second, we consider the same classification rubric except we exclude personal care time. The services activity classification is invariant to the inclusion of personal care activities.

**Services:** Education (060101 — 069999); Professional, Personal Care, and Household Services (080101 — 099999); Government Services & Civic Obligations (100101 — 109999); Waiting Associated w/ Eating & Drinking (110281 — 110289); Religious & Spiritual Activities (140101 — 149999); Volunteer Activities (150101 — 159989); Telephone Calls (160101 — 169989); Travel Related to Personal Care, Household Activities, and Helping Others (180101 — 180499); All Other Non-work Travel (180601 — 189999).

**Goods, incl. All Personal Care Time:** Personal Care Activities Including Sleep (010101 — 019999); Household Activities (020101 — 020999); Caring for & Helping Both Household and Non-household Members (030101 — 049999); Consumer Purchases (070101 — 079999); Eating & Drinking ( 110101 — 110199, 119999); Socializing, Relaxing, Leisure, Sports, Exercise, & Recreation (120101 — 139999).

**Goods, excl. Personal Care Time:** For this classification, we include only personal care activities pertaining to Grooming (010201, 010299). All other classifications from household activities through recreation are included.

## Aligning NIPA Production Sectors with Expenditure

We must classify the sectors from the capital-distribution breakdowns in BEA Fixed Asset Tables 3.1 and 3.2 and the labor-distribution breakdowns from NIPA Tables 6.5B, 6.5C, 6.5D, 6.9B, 6.9C, and 6.9D to align with the product-type expenditure classifications in NIPA Table 2.4.5. We admittedly take some liberties, making rather broad assumptions: namely, we assume that all production is final (there are no intermediaries), and that wholesale and retail trade sectors primarily deal in goods. The classification rubrics are as follows.

### Labor Categorization

**Services (Series B):** Transportation; Communication; Electric, gas, and sanitary services; Finance, insurance, and real estate; Services.

**Services (Series C):** Transportation; Communications; Electric, gas, and sanitary services; Finance, insurance, and real estate; Services.

**Services (Series D):** Utilities; Transportation and warehousing; Information; Finance and insurance, real estate, rental, and leasing; Professional and business services; Educational services, health care and social assistance; Arts, entertainment, recreation, accommodation, and food services; Other services, except government.

**Goods (Series B and C):** Farms; Agricultural services, forestry, and fishing; Mining; Construction; Durable goods; Nondurable goods; Wholesale trade; Retail trade.

**Goods (Series D):** Farms; Forestry, fishing, and related activities; Mining; Construction; Durable goods; Nondurable goods; Wholesale trade; Retail trade.

### Capital Categorization

**Services:** Utilities; Air transportation; Railroad transportation; Water transportation; Truck transportation; Transit and ground passenger transportation; Pipeline transportation; Other transportation and support activities; Warehousing and storage; Publishing industries (includes software); Motion picture and sound recording industries; Broadcasting and telecommunications; Information and data processing services; Federal Reserve banks; Credit intermediation and related activities; Securities, commodity contracts, and investments; Insurance carriers and related activities; Funds, trusts, and other financial vehicles; Real estate; Rental and leasing services and lessors of intangible assets; Legal services; Computer systems design and related services; Miscellaneous professional, scientific, and technical services; Management of companies and enterprises; Administrative and support services; Waste management and remediation services; Educational services; Ambulatory health care services; Hospitals; Nursing and residential care facilities; Social assistance; Performing arts, spectator sports, museums, and related activities; Amusements, gambling, and recreation industries; Accommodation; Food services and drinking places; Other services, except government.

**Goods:** Farms; Forestry, fishing, and related activities; Oil and gas extraction; Mining, except oil and gas; Support activities for mining; Construction; Wood products; Nonmetallic mineral products; Primary metals; Fabricated metal products; Machinery; Computer and electronic products; Electrical equipment, appliances, and components; Motor vehicles, bodies and trailers, and parts; Other transportation equipment; Furniture and related products; Miscellaneous manufacturing; Food and beverage and tobacco products; Textile mills and textile product mills; Apparel and leather and allied products; Paper products; Printing and related support activities; Petroleum and coal products; Chemical products; Plastics and rubber products; Wholesale trade; Retail trade.

## C.3. Estimation Appendix

### Description of Hamiltonian Monte Carlo

For a thorough and readable, detailed treatment of Hamiltonian Monte Carlo (HMC) sampling, we recommend reading Neal (2011). For an equally thorough, more technical treatment, read Betancourt and Stein (2011). For a brief overview of HMC see Gelman et al. (2013b). Here, we provide the "Reader's Digest" overview of the sampler's properties and benefits since there are very few examples of this particular estimation technique being used in the econometrics literature.[3] This appendix is not intended to be a full treatment, only an outline.

Recall, in traditional Bayesian MCMC posterior sampling (i.e. Gibbs sampling or Metropolis-Hastings sampling), the state of the sampler at iteration $m$ is the $m^{th}$ draw of the parameter set $\mathcal{P}^m$ from the posterior distribution with density $\pi(\mathcal{P} \mid \mathcal{D})$, where $\mathcal{D}$ are data. Thus a traditional sampler is a Markov Chain operating in a discrete, countable sampling space, with discrete dynamics. For example, the $m^{th}$ draw of a Metropolis-Hastings sampler depends on the $m-1$ draw, and the econometrician knows this distribution has converged when autocorrelation between draws has been sufficiently minimized, depending on the nature of the estimation problem. HMC extends this concept to a sample space with continuous dynamics.[4] In Hamiltonian dynamics commonly employed in physical mechanics, the state of the system depends on both the forward "momentum" of the system $\mathcal{Q}$ and the "position" of the system $\mathcal{P}$ (itself, the parameter vector), each of the same dimension. The Hamiltonian equation associated with this system is the sum of "potential" $V(\cdot)$ and "kinetic" $K(\cdot)$ energy:

$$H(\mathcal{Q}, \mathcal{P}) = V(\mathcal{P}) + K(\mathcal{Q}) \tag{C.70}$$

The partial derivatives of this equation will determine how the parameter space $\mathcal{P}$ evolves as the sampler proceeds, as well as the rate of this evolution $\mathcal{Q}$.

Since the "dynamics" of our particular task involve traversing a parameter space over which some posterior density function $\pi(\mathcal{P} \mid \mathcal{D})$ is defined, the potential energy function for HMC is just the negative log of the right hand side of $\pi(\mathcal{P} \mid \mathcal{D}) = \pi(\mathcal{D} \mid \mathcal{P})\pi(\mathcal{P})$:

$$V(\mathcal{P}) = -\ln\Big[\pi(\mathcal{D} \mid \mathcal{P})\pi(\mathcal{P})\Big] \tag{C.71}$$

Note that we can sample from this using HMC only if $V(\mathcal{P})$ is continuously differentiable over the entire parameter space since HMC operates on Hamilton's equations which require computation of the gradient vectors for both $V(\cdot)$ and $K(\cdot)$. Let # denote the cardinality of a countable set. In practice, the kinetic energy function is defined conditional on $\mathcal{P}$ and taken to

[3]The only published example in the economics literature we were able to find is Burda (2015). Martin Burda also has a recent working paper providing an application of HMC to a discrete choice model (Burda and Daviet 2018).

[4]Recall, "dynamics" in this context refer to the dynamics of the parameter sampler, *not* actual time, as in the modeled structural dynamics of the underlying agent's decision process. One dynamic operates over the sampler dimension $m$ while the other over the data dimension $t$. This can be confusing when estimating models wrought from a dynamic decision process or which have dynamic time series components.

be a quadratic of the form:[5]

$$K(\mathcal{Q} \mid \mathcal{P}) = \frac{1}{2} \sum_{k=1}^{\#\mathcal{Q}} \sum_{r=1}^{\#\mathcal{Q}} \mathcal{Q}_k \mathcal{Q}_r \Lambda_{k,r}(\mathcal{P}) - \frac{1}{2} \ln \Big[ \det\big(\boldsymbol{\Lambda}(\mathcal{P})\big) \Big] \tag{C.72}$$

where $\boldsymbol{\Lambda}(\mathcal{P})$ is what is called the mass matrix and may be constant. It could also be restricted to the identity matrix or simply be diagonal. At most it is a dense symmetric positive definite matrix that represents the variance/covariance of an underlying conditional Gaussian distribution function for the momentum vector:

$$\mathcal{Q} \mid \mathcal{P} \sim \mathcal{N}\big(\mathbf{0}, \boldsymbol{\Lambda}(\mathcal{P})\big) \tag{C.73}$$

A dense $\boldsymbol{\Lambda}(\mathcal{P})$ can help account for high local non-linearities in $V(\mathcal{P})$, though can be difficult to compute (Betancourt and Stein 2011). In our estimation, we allow $\boldsymbol{\Lambda}(\mathcal{P})$ to be diagonal and tune $\boldsymbol{\Lambda}(\mathcal{P})$ during a warm-up period using Stan's HMC implementation.[6]

Having defined the objects on which we operate, we can summarize the algorithm described in detail in Neal (2011). Here is where the kinetic energy component of the Hamiltonian helps greatly speed up convergence and reduce autocorrelation in our sampling routine. After a sufficient warm-up period,[7] a sampling step proceeds as follows:

i. Given the position from the previous iteration $\mathcal{P}$, draw a new $\mathcal{Q}$ from (C.73). In a sense (C.73) is thus the HMC analog of what is commonly called a "proposal distribution" for an MCMC Metropolis-Hastings algorithm.

ii. Given $(\mathcal{Q}, \mathcal{P})$, iterate on Hamilton's equations using the leapfrog method for $L$ steps to get a pair $(\mathcal{Q}', \mathcal{P}')$.[8] This is your proposed new state.

iii. Similar to the Metropolis-Hastings algorithm, draw a uniform random deviate $u \sim \mathcal{U}[0,1]$ and accept $(\mathcal{Q}', \mathcal{P}')$ as the next state if

$$u < \exp\big\{ - H(\mathcal{Q}', \mathcal{P}') + H(\mathcal{Q}, \mathcal{P}) \big\} \tag{C.74}$$

HMC is most useful when sampling from $\pi\big(\mathcal{P} \mid \mathcal{D}\big)$ is computationally burdensome and may require an incredibly long chain of discrete draws in order to achieve convergence. This happens in cases where posterior draws are likely to be highly autocorrelated in a traditional MCMC, especially when using Metropolis-Hastings algorithms where high autocorrelation may also require low acceptance probabilities, thus further giving reason for a long sampling chain.

[5]This is a more flexible form of the kinetic energy function than that in Neal (2011). See Betancourt and Stein (2011) for more details.

[6]Stan is free software for high-performance HMC which utilizes automatic differentiation, LAPACK, and BLAS libraries for computational efficiency and which can be implemented and executed in a number of top-end scientific computing programs like R, Python, Matlab, or executed simply from the shell. See http://mc-stan.org. For examples using the Stan language to execute HMC models, see Gelman et al. (2013a).

[7]This is akin to the "burn-in" period for an MCMC operating under discrete dynamics.

[8]See Neal (2011) for a detailed description of the iterative "leapfrog" method and how it operates on Hamilton's equations.

## Aggregate Estimates of MRS-only Model with Durable Service Flows

**Table C.1.:** HMC Posterior Distribution, 1948-2019, Agg. Data Incl. Durable Svc. Flows

| | *Household MRS Only* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -1.4546 | 1.5051 | -5.3864 | -1.7257 | -0.9946 | -0.6117 | -0.2909 |
| $\nu_g$ | -2.2046 | 2.8338 | -9.5480 | -2.5939 | -1.3639 | -0.7251 | -0.2125 |
| $\nu_s$ | -1.5060 | 1.9690 | -6.4886 | -1.7818 | -0.9139 | -0.4754 | -0.1346 |
| $\omega_g$ | 0.0157 | 0.0157 | 0.0004 | 0.0046 | 0.0108 | 0.0216 | 0.0578 |
| $\omega_s$ | 0.0023 | 0.0023 | 0.0001 | 0.0007 | 0.0016 | 0.0032 | 0.0083 |
| $\sigma_1^2$ | 0.1156 | 0.0196 | 0.0838 | 0.1016 | 0.1135 | 0.1270 | 0.1599 |
| $V(\mathcal{P})^a$ | 26.7661 | 1.7839 | 22.4548 | 25.8498 | 27.0997 | 28.0706 | 29.2112 |

In this appendix we present the estimation results from the single-equation model, excluding equations describing the marginal products of labor, while forming the likelihood strictly around the household MRS condition. The aggregate data we use include the value of durable service flows. Let $\sigma_1^2$ be the first term on the diagonal of $\mathbf{\Sigma}$, the variance/covariance matrix of the multiple-equation model. The likelihood of this model is formed around $\epsilon_t^1 \sim \mathcal{N}(0, \sigma_1^2)$, with an inverse gamma prior on the variance term: $1/\sigma_1^2 \sim \text{Gamma}(2, 4)$.[9] All other prior distributions are as presented in Table 1 of the main text.

Note that, compared to the full, simultaneous equations model with parameter estimates featured in Table 2 of the main text, the production-specific elasticities of substitution ($\nu_j$) are of different relative magnitudes when compared across processes. When price endogeneity is accounted for by including firm MPL conditions in the likelihood function, services are substantially more complementary with off-market time than goods. However, in the household MRS-only model presented here, the opposite is true. Thus, failing to account for price endogeneity may lead to biased estimates of the relative time complementarities associated with different types of market purchases. We also observe substantial possible upward bias of estimates of $\rho$ when comparing the two tables.

## Aggregate Estimates Without Durable Service Flows

In this appendix, we present estimates from both a simultaneous equations model and a household-MRS-only model where data account for new durable expeditures but not the value of service flows from previously-purchased durable goods. Estimates of $\rho$ in these models do not appear affected by leaving out durable service flows, yet estimates of production elasticities

[9] When using gamma distributions we use the shape/rate parameterization:

$$\text{Gamma}(a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$$

**Table C.2.:** HMC Posterior Distribution, 1948-2019, No Durables Service Flows

| | *Simultaneous Equations* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -24.2078 | 13.5695 | -59.8951 | -29.0989 | -20.9408 | -15.1883 | -8.7876 |
| $\nu_g$ | -2.8653 | 3.0155 | -10.3487 | -3.2837 | -1.9921 | -1.3376 | -0.6749 |
| $\nu_s$ | -1.5318 | 1.9357 | -6.5066 | -1.7688 | -0.9502 | -0.5311 | -0.1611 |
| $\omega_g$ | 0.0171 | 0.0160 | 0.0006 | 0.0053 | 0.0124 | 0.0239 | 0.0611 |
| $\omega_s$ | 0.0023 | 0.0023 | 0.0001 | 0.0006 | 0.0016 | 0.0031 | 0.0084 |
| $\alpha_g$ | 0.1371 | 0.0314 | 0.0802 | 0.1151 | 0.1356 | 0.1573 | 0.2020 |
| $\alpha_s$ | 0.0924 | 0.0206 | 0.0542 | 0.0780 | 0.0917 | 0.1061 | 0.1353 |
| $\sigma_1^2$ | 0.0005 | 0.0001 | 0.0004 | 0.0005 | 0.0005 | 0.0006 | 0.0008 |
| $\sigma_g^2$ | 0.0010 | 0.0002 | 0.0007 | 0.0009 | 0.0010 | 0.0011 | 0.0014 |
| $\sigma_s^2$ | 0.0003 | 0.0000 | 0.0002 | 0.0002 | 0.0003 | 0.0003 | 0.0004 |
| $\sigma_{1g}$ | -0.0001 | 0.0002 | -0.0004 | -0.0002 | -0.0002 | -0.0001 | 0.0002 |
| $\sigma_{1s}$ | 0.0000 | 0.0001 | -0.0001 | -0.0000 | 0.0000 | 0.0001 | 0.0002 |
| $\sigma_{gs}$ | 0.0003 | 0.0001 | 0.0002 | 0.0003 | 0.0003 | 0.0004 | 0.0005 |
| $V(\mathcal{P})$ | 632.0018 | 2.6416 | 625.9992 | 630.4777 | 632.3368 | 633.9314 | 636.2049 |
| | *Household MRS Only* | | | | | | |
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -1.7101 | 1.6493 | -6.0459 | -2.0374 | -1.2158 | -0.7688 | -0.3760 |
| $\nu_g$ | -2.2436 | 2.7713 | -9.4585 | -2.6835 | -1.3855 | -0.7268 | -0.2108 |
| $\nu_s$ | -1.5064 | 1.9700 | -6.7073 | -1.7841 | -0.8946 | -0.4539 | -0.1302 |
| $\omega_g$ | 0.0159 | 0.0158 | 0.0003 | 0.0046 | 0.0111 | 0.0222 | 0.0586 |
| $\omega_s$ | 0.0023 | 0.0023 | 0.0001 | 0.0007 | 0.0016 | 0.0031 | 0.0085 |
| $\sigma_1^2$ | 0.1162 | 0.0199 | 0.0838 | 0.1019 | 0.1142 | 0.1281 | 0.1612 |
| $V(\mathcal{P})$ | 26.3337 | 1.8165 | 21.8926 | 25.3933 | 26.6738 | 27.6677 | 28.8357 |

$\nu_j$ do. We contend that including durable service flows is theoretically most consistent with the behavioral realities underscoring the micro-founded optimization problem of households. Our estimates suggest that failing to account for how households use service flows may significantly affect structural parameter estimates.

### Micro Estimates of MRS-only Model

As with aggregate data, failing to account for general equilibrium effects appears to lead to (possibly) upwardly biased estimates of the gross complementarity coefficient, $\rho$. This can be seen by comparing the results presented here with the simultaneous equations model in Table 4 of the main text.

**Table C.3.:** HMC Posterior Distribution, 1948-2019, Agg. Data Incl. Durable Svc. Flows

| | *Households' MRS Only* | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | S.D. | 2.5% | 25% | 50% | 75% | 97.5% |
| $\rho$ | -6.3622 | 4.2810 | -17.5516 | -7.6882 | -5.1773 | -3.6557 | -2.0940 |
| $\nu_g$ | -2.1912 | 2.7484 | -9.0035 | -2.6016 | -1.3432 | -0.7337 | -0.2450 |
| $\nu_s$ | -1.6118 | 2.0024 | -6.6608 | -1.8956 | -1.0387 | -0.5738 | -0.1718 |
| $\omega_g$ | 0.0157 | 0.0155 | 0.0004 | 0.0047 | 0.0110 | 0.0218 | 0.0574 |
| $\omega_s$ | 0.0023 | 0.0022 | 0.0001 | 0.0007 | 0.0016 | 0.0031 | 0.0082 |
| $\sigma_1^2$ | 0.0512 | 0.0056 | 0.0413 | 0.0473 | 0.0508 | 0.0547 | 0.0635 |
| $V(\mathcal{P})$ | 151.6116 | 1.8617 | 146.9853 | 150.6559 | 151.9514 | 152.9522 | 154.1710 |

## Derivation of Marshallian Demand and Labor Supply Functions

We derive the parameterized expression for the household labor supply function here. Let $\widetilde{y}_{it}$ be capital income net of depreciation and re-investment: $\widetilde{y}_{it} = R_t k_{it} - k_{i,t+1}$. Note that since $y_{it} = w_{it}\overline{n} + R_t k_{it} - k_{i,t+1}$ then $y_{it} = w_{it}\overline{n} + \widetilde{y}_{it}$. Start with (15) in the main text and re-arrange to get:

$$\frac{q_{igt}}{q_{ist}} = \left(\frac{z_{igt}}{z_{ist}}\right)^{\frac{\rho}{1-\rho}} \times \underbrace{\left(\frac{P_{st}\omega_g}{P_{gt}\omega_s}\right)^{\frac{1}{1-\rho}} \left(\omega_g + (1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)^{\frac{\nu_g-\rho}{\nu_g(\rho-1)}} \left(\omega_s + (1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)^{\frac{\rho-\nu_s}{\nu_s(\rho-1)}}}_{\Pi^q(P_{gt},P_{st},w_{it})} \tag{C.75}$$

We can then invert (14) in the main text and substitute the $q_{ijt}$ out of (12), and get $n_{igt}$ as an implicit function of $n_{ist}$:

$$\frac{n_{igt}}{n_{ist}} = \left(\frac{z_{igt}}{z_{ist}}\right)^{\frac{\rho}{1-\rho}} \times \underbrace{\left(\frac{1-\omega_g}{1-\omega_s}\right)^{\frac{1}{1-\rho}} \left(\omega_g\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{1-\nu_g}} + 1 - \omega_g\right)^{\frac{\nu_g-\rho}{\nu_g(\rho-1)}} \left(\omega_s\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{1-\nu_s}} + 1 - \omega_s\right)^{\frac{\rho-\nu_s}{\nu_s(\rho-1)}}}_{\Pi^n(P_{gt},P_{st},w_{it})} \tag{C.76}$$

Using the infra-marginal condition in (14), along with (C.75) and (C.76), we can use the budget constraint to get the Marshallian demand for off-market services time, $n_{ist}$:

$$
\begin{aligned}
n_{ist} &= (w_{it}\overline{n} + \widetilde{y}_{it}) \\
&\times \left( P_{gt} \left( \frac{z_{igt}}{z_{ist}} \right)^{\frac{\rho}{1-\rho}} \Pi_t^q(P_{gt}, P_{st}, w_{it}) \left[ \frac{w_{it}\omega_s}{P_{st}(1-\omega_s)} \right]^{\frac{1}{1-\nu_s}} \right. \\
&\qquad \left. + P_{st} \left[ \frac{w_{it}\omega_s}{P_{st}(1-\omega_s)} \right]^{\frac{1}{1-\nu_s}} + w_{it} + w_{it} \left( \frac{z_{igt}}{z_{ist}} \right)^{\frac{\rho}{1-\rho}} \Pi_t^n(P_{gt}, P_{st}, w_{it}) \right)^{-1}
\end{aligned}
\tag{C.77}
$$

Note that $\ell_{it} = \overline{n} - n_{ist} - n_{igt}$ where $n_{igt} = (z_{igt}/z_{ist})^{\frac{\rho}{1-\rho}} \Pi^n(P_{gt}, P_{st}, w_{it}) n_{ist}(P_{gt}, P_{st}, w_{it}, \widetilde{y}_{it}, z_{igt}/z_{ist})$. Model-implied labor supply can thus be simulated using posterior parameter estimates and data for prices, wages, and capital income net of investment only. Finally, now that we have the expressions for off-market time use, we can simply use the infra-marginal condition in (14) to get the Marshallian demand functions for quantities.

# Bibliography


Acemoglu, Daron and Veronica Guerrieri (2008). "Capital Deepening and Nonbalanced Economic Growth". *Journal of Political Economy* 116.3 (cit. on pp. 50, 55, 78).

Aggarwal, Raj and Kevin T Jacques (2001). "The impact of FDICIA and prompt corrective action on bank capital and risk: Estimates using a simultaneous equations model". *Journal of Banking & Finance* 25.6, pp. 1139–1160 (cit. on p. 4).

Aguiar, Mark and Erik Hurst (2007). "Life-Cycle Prices and Production". *The American Economic Review* 97.5 (cit. on p. 51).

Aguiar, Mark et al. (2021). "Leisure Luxuries and the Labor Supply of Young Men". *Journal of Political Economy* 129.2, pp. 337–382 (cit. on p. 52).

Ahmad, Rubi, Mohamed Ariff, and Michael J Skully (2008). "The determinants of bank capital ratios in a developing economy". *Asia-Pacific financial markets* 15.3-4, pp. 255–272 (cit. on p. 3).

Aiyar, Shekhar et al. (2014). "The international transmission of bank capital requirements: Evidence from the UK". *Journal of Financial Economics* 113.3, pp. 368–382 (cit. on p. 4).

Akhigbe, Aigbe, Jeff Madura, and Marek Marciniak (2012). "Bank capital and exposure to the financial crisis". *Journal of Economics and Business* 64.5, pp. 377–392 (cit. on p. 4).

Akinci, Ozge and Jane Olmstead-Rumsey (2018). "How effective are macroprudential policies? An empirical investigation". *Journal of Financial Intermediation* 33, pp. 33–57 (cit. on p. 5).

Altman, Edward I and Gabriele Sabato (2005). "Effects of the new Basel capital accord on bank capital requirements for SMEs". *Journal of financial services research* 28.1, pp. 15–42 (cit. on p. 4).

Altunbas, Yener, Mahir Binici, and Leonardo Gambacorta (2018). "Macroprudential policy and bank risk". *Journal of International Money and Finance* 81, pp. 203–220 (cit. on p. 4).

Altunbas, Yener et al. (2007). "Examining the relationships between capital, risk and efficiency in European banking". *European financial management* 13.1, pp. 49–70 (cit. on p. 3).

Anginer, Deniz, Asli Demirgüç-Kunt, and Davide S Mare (2018). "Bank capital, institutional environment and systemic stability". *Journal of Financial Stability* 37, pp. 97–106 (cit. on p. 5).

Athanasoglou, Panayiotis P (2011). "Bank capital and risk in the South Eastern European region". *Available at SSRN 1774585* (cit. on p. 3).

Autor, David and David Dorn (2013). "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market". *The American Economic Review* 103.5, pp. 1553–1597 (cit. on p. 50).

Autor, David et al. (2020). "The Fall of the Labor Share and the Rise of Superstar Firms". *Quarterly Journal of Economics* 135.2, pp. 645–709 (cit. on p. 50).

Baker, Malcolm and Jeffrey Wurgler (2015). "Do strict capital requirements raise the cost of capital? Bank regulation, capital structure, and the low-risk anomaly". *American Economic Review* 105.5, pp. 315–20 (cit. on p. 4).

Barnett, William (1977). "Pollak and Wachter on the Household Production Function Approach". *Journal of Political Economy* 85.5 (cit. on p. 55).

Barro, Robert (1984). *Macroeconomics*. Cambridge, MA: MIT Press (cit. on p. 52).

Barth, James R et al. (2013). "Do bank regulation, supervision and monitoring enhance or impede bank efficiency?" *Journal of Banking & Finance* 37.8, pp. 2879–2892 (cit. on p. 4).

Becker, Gary (1965). "A Theory of the Allocation of Time". *The Economic Journal* 75.299 (cit. on pp. 48, 51, 55, 56, 82).

Begenau, Juliane (2020). "Capital requirements, risk choice, and liquidity provision in a business-cycle model". *Journal of Financial Economics* 136.2, pp. 355–378 (cit. on p. 25).

Begenau, Juliane and Tim Landvoigt (2018). "Financial regulation in a quantitative model of the modern banking system". *Available at SSRN 2748206* (cit. on p. 25).

Behn, Markus, Rainer Haselmann, and Paul Wachtel (2016). "Procyclical capital regulation and lending". *The Journal of Finance* 71.2, pp. 919–956 (cit. on p. 4).

Behr, Patrick, Reinhard H Schmidt, and Ru Xie (2010). "Market structure, capital regulation and bank risk taking". *Journal of Financial Services Research* 37.2, pp. 131–158 (cit. on p. 4).

Benhabib, Jess, Richard Rogerson, and Randall Wright (1991). "Homework in Macroeconomics: Household Production and Aggregate Fluctuations". *Journal of Political Economy* 99.6 (cit. on pp. 51, 63).

Berger, Allen N et al. (2008). "How do large banking organizations manage their capital ratios?" *Journal of Financial Services Research* 34.2-3, pp. 123–149 (cit. on p. 3).

Bernanke, Ben (1985). "Adjustment Costs, Durables, and Aggregate Consumption". *Journal of Monetary Economics* 15, pp. 41–68 (cit. on p. 51).

Berrospide, Jose M and Rochelle M Edge (2010). "The effects of bank capital on lending: What do we know, and what does it mean?" (Cit. on p. 5).

Betancourt, Michael and Leo Stein (2011). "The Geometry of Hamiltonian Monte Carlo". *arXiv:1112.4118* (cit. on pp. 69, 115, 116).

Blundell, Richard and Thomas Macurdy (1999). "Labor Supply: A Review of Alternative Approaches". *Handbook of Labor Economics*. Ed. by O. Ashenfelter and D. Card. Vol. 3. Elsevier Science. Chap. 27 (cit. on p. 59).

Boar, Codruta et al. (2017). "What are the effects of macroprudential policies on macroeconomic performance?" *BIS Quarterly Review September* (cit. on p. 5).

Boerma, Job and Loukas Karabarbounis (2019). "Inferring Inequality with Home Production". *Working Paper* (cit. on pp. 51, 52, 78).

Boppart, Timo (2014). "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences". *Econometrica* 82.6 (cit. on pp. 50, 53–55, 67, 70).

Boppart, Timo and Per Krusell (2020). "Labor Supply in the Past, Present, and Future: A Balanced-Growth Perspective". *Journal of Political Economy* 128.1, pp. 118–157 (cit. on pp. 51, 52).

Boppart, Timo and Rachel Ngai (2021). "Rising inequality and trends in leisure". *Journal of Economic Growth* Accepted (cit. on p. 52).

Boskin, Michael et al. (1998). "Consumer Prices, the Consumer Price Index, and the Cost of Living". *Journal of Economic Perspectives* 12.1, pp. 3–26 (cit. on p. 57).

Bridgman, Benjamin, George Duernecker, and Berthold Herrendorf (2018). "Structural transformation, marketization, and household production around the world". *Journal of Development Economics* 133, pp. 102–126 (cit. on p. 51).

Bruno, Valentina, Ilhyock Shim, and Hyun Song Shin (2017). "Comparative assessment of macroprudential policies". *Journal of Financial Stability* 28, pp. 183–202 (cit. on p. 5).

Buera, Francisco and Joseph Kaboski (2009). "Can Traditional Theories of Structural Change Fit the Data?" *Journal of the European Economic Association* 7.2-3, pp. 469–477 (cit. on p. 50).

– (2012). "The Rise of the Service Economy". *The American Economic Review* 102.6 (cit. on p. 50).

Burda, Martin (2015). "Constrained Hamiltonian Monte Carlo in BEKK GARCH with Targeting". *Journal of Time Series Econometrics* 7.1 (cit. on p. 115).

Burda, Martin and Remi Daviet (2018). "Hamiltonian Sequential Monte Carlo with Application to Consumer Choice Behavior". *University of Toronto, Department of Economics Working Paper #618* (cit. on p. 115).

Camara, Boubacar, Laetitia Lepetit, and Amine Tarazi (2013). "Ex ante capital position, changes in the different components of regulatory capital and bank risk". *Applied Economics* 45.34, pp. 4831–4856 (cit. on p. 4).

Carlson, Mark, Hui Shan, and Missaka Warusawitharana (2013). "Capital ratios and bank lending: A matched bank approach". *Journal of Financial Intermediation* 22.4, pp. 663–687 (cit. on p. 4).

Caselli, Francesco and Wilbur Coleman (2001). "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation". *Journal of Political Economy* 109.3 (cit. on pp. 50, 70).

Cerutti, Eugenio, Stijn Claessens, and Luc Laeven (2017). "The use and effectiveness of macroprudential policies: New evidence". *Journal of Financial Stability* 28, pp. 203–224 (cit. on p. 5).

Comin, Diego, Danial Lashkari, and Martí Mestieri (2015). "Structural Change with Long-run Income and Price Effects". *NBER Working Paper No. 21595* (cit. on p. 50).

Cooley, Thomas F. and Vincenzo Quadrini (2001). "Financial markets and firm dynamics". *American Economic Review* 91.5, pp. 1286–1310 (cit. on pp. 25–27, 39).

Corbae, Dean and Pablo D'Erasmo (2019). *Capital requirements in a quantitative model of banking industry dynamics*. Tech. rep. National Bureau of Economic Research (cit. on p. 26).

Corbae, Dean, Pablo D'erasmo, et al. (2013). "A quantitative model of banking industry dynamics". *Federal Reserve Bank of Philadelphia Working Paper* 14 (cit. on pp. 26, 38).

Das, Abhiman and Saibal Ghosh (2006). "Financial deregulation and efficiency: An empirical analysis of Indian banks during the post reform period". *Review of Financial Economics* 15.3, pp. 193–221 (cit. on p. 4).

Davydiuk, Tetiana (2017). "Dynamic bank capital requirements". *Available at SSRN 3110800* (cit. on p. 25).

De Jonghe, Olivier and Özde Öztekin (2015). "Bank capital management: International evidence". *Journal of Financial Intermediation* 24.2, pp. 154–177 (cit. on p. 3).

De Nicolo Gianni, Andrea Gamba and Marcella Lucchetta (2014). "Microprudential regulation in a dynamic model of banking". *The Review of Financial Studies* 27.7, pp. 2097–2138 (cit. on pp. 25, 38, 39).

DeAngelo, Harry and Rene M Stulz. (2015). “Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks.” *Journal of Financial Economics* 116.2, pp. 219–236 (cit. on p. 29).

Deaton, Angus and John Muellbauer (1980). *Economics and Consumer Behavior*. Cambridge University Press: New York (cit. on p. 59).

Delis, Manthos D, Kien C Tran, and Efthymios G Tsionas (2012). “Quantifying and explaining parameter heterogeneity in the capital regulation-bank risk nexus”. *Journal of Financial Stability* 8.2, pp. 57–68 (cit. on p. 4).

Demirguc-Kunt, Asli, Enrica Detragiache, and Ouarda Merrouche (2013). “Bank capital: Lessons from the financial crisis”. *Journal of Money, Credit and Banking* 45.6, pp. 1147–1164 (cit. on p. 4).

Diamond, Douglas W and Philip H Dybvig (1983). “Bank runs, deposit insurance, and liquidity”. *Journal of political economy* 91.3, pp. 401–419 (cit. on p. 22).

Dias, Roshanthi (2021). “Capital regulation and bank risk-taking–new global evidence”. *Accounting & Finance* 61.1, pp. 847–884 (cit. on p. 4).

Díaz-Jiménez, Javier, Andy Glover, and José-Victor Ríos-Rull (2011). “Facts on the distributions of earnings, income, and wealth in the United States: 2007 update”. *Federal Reserve Bank of Minneapolis Quarterly Review* 34.1, pp. 2–31 (cit. on pp. 52, 82).

Dietrich, J Kimball and Christopher James (1983). “Regulation and the determination of bank capital changes: A note”. *The Journal of Finance* 38.5, pp. 1651–1658 (cit. on p. 3).

Distinguin, Isabelle, Caroline Roulet, and Amine Tarazi (2013). “Bank regulatory capital and liquidity: Evidence from US and European publicly traded banks”. *Journal of Banking & Finance* 37.9, pp. 3295–3317 (cit. on p. 4).

Duca, John V (2016). “How capital regulation and other factors drive the role of shadow banking in funding short-term business credit”. *Journal of Banking & Finance* 69, S10–S24 (cit. on p. 5).

Ericson, Richard and Ariel Pakes (1995). “Markov-perfect industry dynamics: A framework for empirical work”. *The Review of economic studies* 62.1, pp. 53–82 (cit. on p. 26).

Fang, Lei, Anne Hannusch, and Pedro Silos (2020). “Bundling Time and Goods: Implications for Hours Dispersion”. *Working Paper* (cit. on p. 51).

Fenton, George and Felix Koenig (2020). *Labor Supply and Innovation in Entertainment: Evidence from the US TV Rollout*. Tech. rep. Working Paper (cit. on pp. 52, 78).

Flannery, Mark J and Kasturi P Rangan (2006). “Partial adjustment toward target capital structures”. *Journal of financial economics* 79.3, pp. 469–506 (cit. on p. 3).

– (2008). “What caused the bank capital build-up of the 1990s?” *Review of Finance* 12.2, pp. 391–429 (cit. on p. 3).

Foellmi, Reto and Josef Zweimuller (2008). “Structural change, Engel’s consumption cycles and Kaldor’s facts of economic growth”. *Journal of Monetary Economics* 55 (cit. on p. 53).

Fonseca, Ana Rosa and Francisco González (2010). “How bank capital buffers vary across countries: The influence of cost of deposits, market power and bank regulation”. *Journal of banking & finance* 34.4, pp. 892–902 (cit. on p. 3).

Francis, William, Matthew Osborne, et al. (2009). “Bank regulation, capital and credit supply: measuring the impact of prudential standards”. *Occasional paper* 36 (cit. on p. 4).

Frazis, Harley and Jay Stewart (2011). “How does household production affect measured income inequality?” *Journal of Population Economics* 24, pp. 3–22 (cit. on p. 52).
Freixas, Xavier and Jean-Charles Rochet (2008). *Microeconomics of banking*. MIT press (cit. on p. 28).
Furlong, Frederick T (1992). “Capital regulation and bank lending”. *Economic Review-Federal Reserve Bank of San Francisco* 3, p. 23 (cit. on p. 4).
Gambacorta, Leonardo (2011). “Do bank capital and liquidity affect real economic activity in the long run? A VECM analysis for the US”. *Economic notes* 40.3, pp. 75–91 (cit. on p. 5).
Gambacorta, Leonardo and Paolo Emilio Mistrulli (2004). “Does bank capital affect lending behavior?” *Journal of Financial intermediation* 13.4, pp. 436–457 (cit. on p. 5).
Gambacorta, Leonardo and Hyun Song Shin (2018). “Why bank capital matters for monetary policy”. *Journal of Financial Intermediation* 35, pp. 17–29 (cit. on p. 5).
Gauthier, Céline, Alfred Lehar, and Moez Souissi (2010). *Macroprudential regulation and systemic capital requirements*. Tech. rep. Bank of Canada Working Paper (cit. on p. 5).
Geary, R.C. (1950). “A Note on “A Constant-Utility Index of the Cost of Living””. *The Review of Economic Studies* 18.1 (cit. on p. 53).
Gelman, Andrew et al. (2013a). “Computation in R and Stan”. *Bayesian Data Analysis*. Third. Chapman & Hall. Chap. Appendix C (cit. on pp. 69, 116).
– (2013b). “Computationally efficient Markov chain simulation”. *Bayesian Data Analysis*. Third. Chapman & Hall. Chap. 12 (cit. on pp. 69, 115).
Godlewski, Christophe J (2005). “Bank capital and credit risk taking in emerging market economies”. *Journal of banking Regulation* 6.2, pp. 128–145 (cit. on p. 3).
Goel, Tirupam (2016). “Banking industry dynamics and size-dependent capital regulation” (cit. on p. 26).
Gomme, Paul, Finn Kydland, and Peter Rupert (2001). “Home Production Meets Time to Build”. *Journal of Political Economy* 109.5 (cit. on p. 51).
Goodhart, Charles, Boris Hofmann, and Miguel Segoviano (2004). “Bank regulation and macroeconomic fluctuations”. *Oxford review of economic Policy* 20.4, pp. 591–615 (cit. on p. 5).
Goolsbee, Austan and Peter Klenow (2006). “Valuing consumer products by the time spent using them: An application to the Internet”. *American Economic Review* 96.2, pp. 108–113 (cit. on p. 51).
Gordon, Robert (2016). *The Rise and Fall of American Growth*. Princeton University Press (cit. on pp. 49, 52).
Gottschalk, Peter and Susan Mayer (2002). “Changes in home production and trends in economic inequality.” *The new economics of rising inequality*, pp. 265–284 (cit. on p. 52).
Graham, John and Carole Green (1984). “Estimating the Parameters of a Household Production Function with Joint Products”. *The Review of Economics and Statistics* 66.2 (cit. on p. 51).
Greenwood, Jeremy and Zvi Hercowitz (1991). “The Allocation of Capital and Time over the Business Cycle”. *Journal of Political Economy* 99.6, pp. 1188–1214 (cit. on p. 51).
Greenwood, Jeremy, Richard Rogerson, and Randall Wright (1995). “Household Production in Real Business Cycle Theory”. *Frontiers of Business Cycle Research*. Ed. by Thomas Cooley. Princeton: Princeton University Press, pp. 157–174 (cit. on p. 51).

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu (2005). “Engines of Liberation”. *Review of Economic Studies* 72, pp. 109–133 (cit. on p. 51).

Gronau, Reuben (1977). “Leisure, Home Production, and Work – the Theory of the Allocation of Time Revisited”. *Journal of Political Economy* 85.6 (cit. on p. 51).

– (1986). “Home production — a survey.” *Handbook of Labor Economics*. Ed. by Orley Ashenfelter and Richard Layard. North Holland, Amsterdam., pp. 273–304 (cit. on p. 52).

Gropp, Reint and Florian Heider (2010). “The determinants of bank capital structure”. *Review of finance* 14.4, pp. 587–622 (cit. on p. 3).

Hancock, Diana, Andrew J Laing, and James A Wilcox (1995). “Bank capital shocks: dynamic effects on securities, loans, and capital”. *Journal of Banking & Finance* 19.3-4, pp. 661–677 (cit. on p. 4).

Heid, Frank, Daniel Porath, and Stéphanie Stolz (2003). *Does capital regulation matter for bank behavior? Evidence for German savings banks*. Tech. rep. Kiel Working Paper (cit. on p. 3).

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi (2015). “Sectoral Technology and Structural Transformation”. *American Economic Journal: Macroeconomics* 7.4, pp. 104–133 (cit. on p. 50).

Herrendorf, Berthold, Richard Rogerson, and Ákos Valentinyi (2013). “Two Perspectives on Preferences and Structural Transformation”. *American Economic Review* 103.7 (cit. on pp. 50, 53, 54, 71, 87, 112).

– (2014). “Growth and structural transformation”. *Handbook of economic growth*. Vol. 2. Elsevier, pp. 855–941 (cit. on pp. 55, 78).

– (2018). *Structural change in investment and consumption: A unified approach*. Tech. rep. National Bureau of Economic Research (cit. on p. 55).

Hoggarth, Glenn, Lavan Mahadeva, and Jeremy HBC Martin (2010). “Understanding international bank capital flows during the recent financial crisis”. *Bank of England Financial Stability Paper* 8 (cit. on p. 5).

Hopenhayn, Hugo and Richard Rogerson (1993). “Job turnover and policy evaluation: A general equilibrium analysis”. *Journal of political Economy* 101.5, pp. 915–938 (cit. on p. 26).

Hopenhayn, Hugo A (1992). “Entry, exit, and firm dynamics in long run equilibrium”. *Econometrica: Journal of the Econometric Society*, pp. 1127–1150 (cit. on pp. 24, 26, 30).

Horváth, Roman, Jakub Seidler, and Laurent Weill (2014). “Bank capital and liquidity creation: Granger-causality evidence”. *Journal of Financial Services Research* 45.3, pp. 341–361 (cit. on p. 4).

Hovakimian, Armen and Edward J Kane (2000). “Effectiveness of capital regulation at US commercial banks, 1985 to 1994”. *the Journal of Finance* 55.1, pp. 451–468 (cit. on p. 4).

Hymer, Stephen and Stephen Resnick (1969). “A Model of an Agrarian Economy with Nonagricultural Activities”. *The American Economic Review* 59.4, pp. 493–506 (cit. on p. 63).

Ingram, Beth, Narayana Kocherlakota, and N.E. Savin (1997). “Using theory for measurement: An analysis of the cyclical behavior of home production”. *Journal of Monetary Economics* 40, pp. 435–456 (cit. on p. 51).

Irani, Rustom M et al. (2021). “The rise of shadow banking: Evidence from capital regulation”. *The Review of Financial Studies* 34.5, pp. 2181–2235 (cit. on p. 5).

Jacques, Kevin and Peter Nigro (1997). “Risk-based capital, portfolio risk, and bank capital: A simultaneous equations approach”. *Journal of Economics and business* 49.6, pp. 533–547 (cit. on p. 3).

Jenkins, Stephen and Nigel O’Leary (1996). “Household income plus household production: the distribution of extended income in the U.K.” *Review of Income and Wealth* 42.4, pp. 401–419 (cit. on p. 52).

Jiménez, Gabriel et al. (2017). “Macroprudential policy, countercyclical bank capital buffers, and credit supply: Evidence from the Spanish dynamic provisioning experiments”. *Journal of Political Economy* 125.6, pp. 2126–2177 (cit. on p. 5).

Jokipii, Terhi and Alistair Milne (2011). “Bank capital buffer and risk adjustment decisions”. *Journal of Financial Stability* 7.3, pp. 165–178 (cit. on p. 3).

Jones, Charles (2016). “The Facts of Economic Growth”. *Handbook of Macroeconomics*. Ed. by John Taylor and Harald Uhlig. Vol. 2. Amsterdam: North-Holland (cit. on p. 52).

Jordà, Òscar et al. (2021). “Bank capital redux: solvency, liquidity, and crisis”. *The Review of Economic Studies* 88.1, pp. 260–286 (cit. on p. 5).

Jovanovic, Boyan (1982). “Selection and the Evolution of Industry”. *Econometrica: Journal of the Econometric Society*, pp. 649–670 (cit. on p. 26).

Kashyap, Anil and Jeremy Stein (2000). “What do a million observations say about the transmission of monetary policy?” *American Economic Review* 90.3, pp. 407–428 (cit. on p. 5).

Kehoe, Timothy, Kim Ruhl, and Joseph Steinberg (2018). “Global Imbalances and Structural Change in the United States”. *Journal of Political Economy* 126.1, pp. 761–796 (cit. on pp. 50, 54, 70, 78).

Kim, Dohan and Wook Sohn (2017). “The effect of bank capital on lending: Does liquidity matter?” *Journal of Banking & Finance* 77, pp. 95–107 (cit. on p. 4).

Kim, Teakdong, Bonwoo Koo, and Minsoo Park (2013). “Role of financial regulation and innovation in the financial crisis”. *Journal of Financial stability* 9.4, pp. 662–672 (cit. on p. 5).

King, Robert, Charles Plosser, and Sergio Rebelo (1988). “Production, Growth and Business Cycles I. The Basic Neoclassical Model”. *Journal of Monetary Economics* 21, pp. 195–232 (cit. on p. 51).

Kishan, Ruby P and Timothy P Opiela (2000). “Bank size, bank capital, and the bank lending channel”. *Journal of Money, credit and banking*, pp. 121–141 (cit. on p. 4).

– (2006). “Bank capital and loan asymmetry in the transmission of monetary policy”. *Journal of Banking & Finance* 30.1, pp. 259–285 (cit. on p. 4).

Kleff, Volker and Martin Weber (2008). “How do banks determine capital? Evidence from Germany”. *German Economic Review* 9.3, pp. 354–372 (cit. on p. 3).

Komunjer, Ivana and Serena Ng (2011). “Dynamic Identification of Dynamic Stochastic General Equilibrium Models”. *Econometrica* 79.6, pp. 1995–2032 (cit. on p. 70).

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie (2001). “Beyond Balanced Growth”. *Review of Economic Studies* 68, pp. 869–882 (cit. on pp. 50, 53, 70).

Kopytov, Alexandr, Nikolai Roussanov, and Mathieu Taschereau-Dumouchel (2020). “Cheap Thrills: the Price of Leisure and the Global Decline in Work Hours”. *Working Paper* (cit. on pp. 51, 52, 78).

Košak, Marko et al. (2015). “Quality of bank capital and bank lending behavior during the global financial crisis”. *International review of financial analysis* 37, pp. 168–183 (cit. on p. 4).
Kwan, Simon and Robert A Eisenbeis (1997). “Bank risk, capitalization, and operating efficiency”. *Journal of financial services research* 12.2, pp. 117–131 (cit. on p. 3).
Laeven, Luc and Ross Levine (2009). “Bank governance, regulation and risk taking”. *Journal of financial economics* 93.2, pp. 259–275 (cit. on p. 4).
Lee, Chien-Chiang and Meng-Fen Hsieh (2013). “The impact of bank capital on profitability and risk in Asian banking”. *Journal of international money and finance* 32, pp. 251–281 (cit. on p. 4).
Lepetit, Laetitia, Nadia Saghi-Zedek, and Amine Tarazi (2015). “Excess control rights, bank capital structure adjustments, and lending”. *Journal of Financial Economics* 115.3, pp. 574–591 (cit. on p. 3).
Lewandowski, Daniel, Dorota Kurowicka, and Harry Joe (2009). “Generating random correlation matrices based on vines and extended onion method”. *Journal of Multivariate Analysis* 100.9 (cit. on pp. 73, 74).
Lindquist, Kjersti-Gro (2004). “Banks’ buffer capital: how important is risk”. *Journal of International Money and Finance* 23.3, pp. 493–513 (cit. on p. 3).
Locay, Luis (1990). “Economic development and the division of production between households and markets”. *Journal of Political Economy* 98.5, pp. 965–982 (cit. on p. 51).
Lucas Jr., Robert (1988). “On the Mechanics of Economic Development”. *Journal of Monetary Economics* 22, pp. 3–42 (cit. on p. 51).
Mankart, Jochen, Alexander Michaelides, and Spyros Pagratis (2020). “Bank capital buffers in a dynamic model”. *Financial Management* 49.2, pp. 473–502 (cit. on p. 25).
Mankiw, Gregory (2010). *Macroeconomics*. 7th ed. New York: Worth (cit. on p. 52).
Marcus, Alan J (1983). “The bank capital decision: a time series—cross section analysis”. *The Journal of Finance* 38.4, pp. 1217–1232 (cit. on p. 3).
Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green, et al. (1995). *Microeconomic theory*. Vol. 1. Oxford university press New York (cit. on p. 33).
Mas-Colell, Andreu, Michael Winston, and Jerry Green (1995). *Microeconomic Theory*. New York: Oxford University Press (cit. on p. 60).
Matsuyama, Kiminori (2009). “Structural Change in an Interdependent World: A Global View of Manufacturing Decline”. *Journal of the European Economic Association* 7.2-3, pp. 478–486 (cit. on pp. 50, 53).
McGrattan, Ellen, Richard Rogerson, and Randall Wright (1993). “Household Production and Taxation in the Stochastic Growth Model”. *Federal Reserve Bank of Minneapolis: Staff Report #166* (cit. on p. 51).
Meeks, Roland (2017). “Capital regulation and the macroeconomy: Empirical evidence and macroprudential policy”. *European Economic Review* 95, pp. 125–141 (cit. on p. 5).
Muellbauer, John (1975). “Aggregation, Income Distribution, and Consumer Demand”. *Review of Economic Studies* 62.4, pp. 526–542 (cit. on p. 53).
– (1976). “Community Preferences and the Representative Consumer”. *Econometrica* 44.5, pp. 979–999 (cit. on p. 53).

Neal, Radford (2011). "MCMC using Hamiltonian dynamics". *Handbook of Markov Chain Monte Carlo*. Ed. by Steve Brooks et al. Chapman & Hall. Chap. 5 (cit. on pp. 69, 115, 116).

Ngai, Rachel and Christopher Pissarides (2007). "Structural Change in a Multisector Model of Growth". *American Economic Review* 97.1 (cit. on pp. 50, 54, 55, 70, 78).

– (2008). "Trends in hours and economic growth". *Review of Economic Dynamics* 11, pp. 239–256 (cit. on pp. 50–52).

Nguyen, Thien Tung (2015). "Bank capital requirements: A quantitative analysis". *Charles A. Dice Center Working Paper* 2015-14 (cit. on p. 25).

Nordhaus, William (1998). "Quality change in price indexes". *Journal of Economic Perspectives* 12.1, pp. 59–68 (cit. on p. 57).

Noss, Joseph and Priscilla Toffano (2016). "Estimating the impact of changes in aggregate bank capital requirements on lending and growth during an upswing". *Journal of Banking & Finance* 62, pp. 15–27 (cit. on p. 5).

Peek, Joe and Eric Rosengren (1995). "Bank regulation and the credit crunch". *Journal of Banking & Finance* 19.3-4, pp. 679–692 (cit. on p. 4).

Perli, Roberto and Plutarchos Sakellaris (1998). "Human capital formation and business cycle persistence". *Journal of Monetary Economics* 42, pp. 67–92 (cit. on p. 51).

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman (2018). "Distributional National Accounts: Methods and Estimates for the United States". *The Quarterly Journal of Economics* 133.2, pp. 553–609 (cit. on pp. 52, 83).

Pollak, Robert and Michael Wachter (1975). "The Relevance of the Household Production Function and Its Implications for the Allocation of Time". *Journal of Political Economy* 83.2 (cit. on p. 55).

Porzio, Tommaso, Federico Rossi, and Gabriella Santangelo (2020). "The Human Side or Structural Transformation". *Warwick Economics Research Papers, No. 1297* (cit. on pp. 50, 51).

Qu, Zhongjun (2018). "A Composite Likelihood Framework for Analyzing Singular DSGE Models". *The Review of Economics and Statistics* 100.5, pp. 916–932 (cit. on p. 70).

Ramey, Valerie (2009). "Time Spent in Home Production in the Twentieth-Century United States: New Estimates from Old Data". *The Journal of Economic History* 69.1, pp. 1–47 (cit. on p. 51).

Ramey, Valerie and Neville Francis (2009). "A Century of Work and Leisure". *American Economic Journal: Macroeconomics* 1.2, pp. 189–224 (cit. on pp. 51, 52).

Rime, Bertrand (2001). "Capital requirements and bank behaviour: Empirical evidence for Switzerland". *Journal of banking & finance* 25.4, pp. 789–805 (cit. on p. 3).

Ríos-Rull, José-Víctor (1993). "Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach". *The American Economic Review* 83.4, pp. 893–907 (cit. on p. 51).

Rıos-Rull, José-Vıctor, Tamon Takamura, and Yaz Terajima (2020). "Banking Dynamics, Market Discipline and Capital Regulations" (cit. on pp. 25–27).

Rupert, Peter, Richard Rogerson, and Randall Wright (1995). "Estimating Substitution Elasticities in Household Production Models". *Economic Theory* 6.1 (cit. on p. 51).

Saez, Emmanuel and Gabriel Zucman (2016). “Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data”. *The Quarterly Journal of Economics* 131.2, pp. 519–578 (cit. on p. 86).

Santos, Joao AC and Andrew Winton (2019). “Bank capital, borrower power, and loan rates”. *The Review of Financial Studies* 32.11, pp. 4501–4541 (cit. on p. 4).

Schwert, Michael (2018). “Bank capital and lending relationships”. *The Journal of Finance* 73.2, pp. 787–830 (cit. on p. 4).

Sealey Jr, Calvin W. and James T Lindley (1977). “Inputs, outputs, and a theory of production and cost at depository financial institutions.” *Journal of Finance* 32.4, pp. 1251–1266 (cit. on p. 28).

Shim, Jeungbo (2013). “Bank capital buffer and portfolio risk: The influence of business cycle and revenue diversification”. *Journal of banking & finance* 37.3, pp. 761–772 (cit. on p. 3).

Shrieves, Ronald E and Drew Dahl (1992). “The relationship between risk and capital in commercial banks”. *Journal of banking & finance* 16.2, pp. 439–457 (cit. on p. 3).

Stone, Richard (1954). “Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand”. *The Economic Journal* 64.255 (cit. on p. 53).

Teixeira, João CA et al. (2014). “Banks’ capital, regulation and the financial crisis”. *The North American Journal of Economics and Finance* 28, pp. 33–58 (cit. on p. 3).

Utrero-González, Natalia (2007). “Banking regulation, institutional framework and capital structure: International evidence from industry data”. *The Quarterly Review of Economics and Finance* 47.4, pp. 481–506 (cit. on p. 5).

Uy, Timothy, Kei-Mu Yi, and Jing Zhang (2013). “Structural change in an open economy”. *Journal of Monetary Economics* 60, pp. 667–682 (cit. on p. 50).

Van denHeuvel, Skander et al. (2016). “The welfare effects of bank liquidity and capital requirements”. *Unpublished manuscript* (cit. on p. 25).

Van denHeuvel, Skander J (2008). “The welfare cost of bank capital requirements”. *Journal of Monetary Economics* 55.2, pp. 298–320 (cit. on p. 25).

Van Roy, Patrick (2005). “The impact of the 1988 Basel Accord on banks’ capital ratios and credit risk-taking: an international study”. *EFMA 2004 Basel Meetings* (cit. on p. 3).

Wall, Larry D and David R Peterson (1995). “Bank holding company capital targets in the early 1990s: The regulators versus the markets”. *Journal of Banking & Finance* 19.3-4, pp. 563–574 (cit. on p. 3).

Whelan, Karl (2000). “A Guide to the Use of Chain Aggregated NIPA Data”. *Federal Reserve Board Working Paper* (cit. on p. 112).

– (2002). “A Guide to U.S. Chain Aggregated NIPA Data”. *Review of Income and Wealth* 48.2, pp. 217–233 (cit. on p. 112).

Zhu, Haibin (2007). “Capital regulation and banks’ financial decisions” (cit. on pp. 25–27, 39).